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A long unrecognized problem in probability and statistics has been the inability to treat inference statements – such as "if b then a" or "a given b" – so that logical combinations of them can be evaluated, compatible with conditional probability. Thus, in the situation where no conditioning occurs – or everything is conditioned on a common antecedent – statements such as "if b then a or if b then c" can be readily addressed with the typical evaluation: $p(a \vee c \mid b) = p_b(a \vee c) = p_b(a) + p_b(c) - p_b(a.c)$, etc., for any probability measure p over the space of events.

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21a. NAME OF RESPONSIBLE INDIVIDUAL	21b. TELEPHONE (include Area Code)	21c. OFFICE SYMBOL
I. R. Goodman	(619) 553-4014	Code 421

ASSIGNMENT OF SINGLE VALUES TO PROBABILITY INTERVALS, EVALUATION OF CONDITIONAL EVENTS, AND APPLICATIONS TO COMBINATION OF EVIDENCE

DR. I.R. GOODMAN

COMMAND & CONTROL DEPARTMENT,
CODE 421
NAVAL OCEAN SYSTEMS CENTER
SAN DIEGO, CALIFORNIA 92152-5000

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ASSIGNMENT OF SINGLE VALUES TO PROBABILITY INTERVALS, EVALUATION OF CONDITIONAL EVENTS AND APPLICATIONS TO COMBINATION OF EVIDENCE

ABSTRACT

A long unrecognized problem in probability and statistics has been the inability to treat inference statements - such as "if b then a" or "a given b" - so that logical combinations of them can be evaluated, compatible with conditional probability. Thus, in the situation where no conditioning occurs - or everything is conditioned on a common antecedent - statements such as "if b then a or if b then c" can be readily addressed with the typical evaluation: $p(a \vee c \mid b) = p_b(a \vee c) = p_b(a) + p_b(c) - p_b(a \cdot c)$, etc., for any probability measure p over the space of events. On the other hand, until recently, such simple appearing statements as Δ = "(if b then a) and (if d then not c)" could not be analyzed within the standard pervue of probability so that one could make sense of the evaluation $p(\Delta)$, compatible with conditional probability, i.e., p("if b then a") = $p(a|b)$ and p("if d then not c") = $p(c'|d)$. (This excludes material implication - and indeed, as Goodman & Nguyen have recently demonstrated [*Conditional Inference and Logic for Intelligent Systems: A Theory of Measure-Free Conditioning*, Chapter 1, North-Holland Press, to appear], no closed operator over a finite boolean algebra of events will also work.) This has lead to the development of a syntactic / algebraic approach to conditioning in probability - much as Boole originally envisioned with his "division" operator, but which was only partially developed by him (although later justified by Hailperin - *Boole's Logic and Probability*) and independently considered from time to time. Only Schay (1968) and, independently, Calabrese (1985), prior to the work here considered, have attempted to develop full-blown conditional event algebras, but their efforts are frought with empirical and ad hoc components.

In the establishment of such an algebra of conditional events (as in the above reference of Goodman & Nguyen), a program of four parts is required as follows (though not necessary in that order at all times): 1 What algebraic forms, if any, must conditional events take? (Answer: all principal ideal cosets generated from all principal ideal quotient boolean algebras of the original boolean algebra of unconditional events); 2 What functional forms must the conditonal event extensions of boolean (unconditional) operators take? (Answer: functional image extensions of all the unconditional pointwise operators of the original boolean algebra to the coset domains), 3 What properties do conditional events and their operators and relations possess? (Answer: Feasible calculus of extended boolean-like operators and partial order extending ordinary subet relations leading to: a bounded, distributive, idempotent, DeMorgan, involutive, pseudo-complemented Stone lattice which is also a semi-simple Chang algebra isomorphic to certain variations of Lukasiewicz three-valued logic; and which is a form of Koopman qualitative conditional probability structure; also, which has a full algebraic characterization, extending the Stone Representation Theorem to conditional form); 4 What numerical or semantic properties do these entities possess and what is the nature of assigning a single number - the conditional probability - to a coset of events, which under functional image extensions of probability becomes an interval of numbers in the unit interval? It is this last issue that has not yet been fully satisfactorally addressed.

Given that the assignment is simply $p((a|b)) = p(a|b)$ to the conditional event $(a|b)$, one in effect is attaching a single most representative number in some sense to the interval $\{p(x) : x \in (a|b)\} = \{p(x \cdot b') + p(a \cdot b) : x \text{ arb.} \in \text{boolean alg.}\}$ = closed interval $[p(a \cdot b), p(a \cdot b) + 1 - p(b)]$, provided p is non-atomic. With this evaluation, one can show a resulting conditional event probability logic which is sound and complete and monotonically preseving partial order of conditional events, etc. But all of this hinges upon the "natural" interpretation of $p((a|b)) = p(a|b)$. Some characterizations for this relation are presented, including a fixed point weighting representation, a modified Renyi-Aczel property, DeFinetti-Lindley uncertainty game approach, and others. But the basic question remains: Why should s/t be assigned to [s,1-t+s], or equivalently, s/(1-t+s) to [s,t], for all $0 \le s \le t \le 1$?