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"Parallel and Sequential Iterative Methods for Linear and Nonlinear Systems." Much of the work on this topic concentrated on the convergence and rate of convergence of parallel asynchronized methods for solving linear systems arising, on the one hand, from the numerical solution to partial differential equations and, on the other hand, from least squares solution to rectangular systems which arise in application such as image reconstruction from incomplete tomographical data. The mathematics behind the analysis of these two applications of the asynchronized parallel methods is quite different. Recently they have been able to extend their convergence results to asynchronized methods for solving nonlinear systems. One application now consists of tomographic reconstruction from incomplete data where the image is constrained to lie in a bounded convex set such as an n dimensional box.

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Convergence and Performance of Synchronous and Asynchronous Parallel and Conventional Iterative Methods

FINAL TECHNICAL REPORT

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1 RESEARCH PROGRESS AND ACTIVITIES REPORT

During the last eighteen months of the grant our work concentrated on two main research topics:

(i) **"Parallel and Sequential Iterative Methods for Linear and Non-linear Systems"**. Much of the work on this topic concentrated on the convergence and rate of convergence of parallel asynchronized methods for solving linear systems arising, on the one hand, from the numerical solution to partial differential equations and, on the other hand, from least squares solution to rectangular systems which arise in application such as image reconstruction from incomplete tomographical data. The mathematics behind the analysis of these two applications of the asynchronized parallel methods is quite different.

Recently we have been able to extend our convergence results to asynchronized methods for solving nonlinear systems. One application now consists of tomographic reconstruction from incomplete data where the image is constrained to lie in a bounded convex set such as an n dimensional box.

(ii) **"Reachability Problems for Dynamical Systems"**. Here we concentrated on developing numerical methods to test whether the trajectory of a linear differential system emanating from a given initial state becomes from some point onwards nonnegative. We particularly characterized when such intial states are *symbioisis points*, meaning that from some point onwards all populations become **nondecreasing**.

As a by-product of the work on these topics we also had to solve various theoretical problems which can be described under the heading:

(iii) "Problems in Nonnegative Matrices and their Applications".

We shall describe the main results which were obtained on these topics in the next section of this report. We strongly believe that an examination of the results which were achieved over the life of the grant shows that many of the goals which were suggested in the intial 1987 proposal and in the annual

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and progress reports which have been submitted since, have been realized. Quite a few of the questions that have been raised have been answered, but not always with the solution that was conjectured.

Since the beginning of the work on the proposal, 15 papers which summarize our results on the above three topics have been submitted for publication. Their titles are as follows:

On parallel and sequential iterative methods

1) (with P. J. Kavanagh) "Consistency and convergence of the parallel multisplitting method for singular M-matrices," SIAM J. Matrix Analysis Appl., 10(1989), pp. 210-218.

2) (with L. Elsner and I. Koltracht) "On the convergence of asynchronous paracontractions with application to tomographic reconstruction from incomplete data," Lin. Alg. Appl., 130(1990), pp.65–82.

3) (with A. Hadjidimos) "Convergence domains of the SSOR method for generalized consistently ordered matrices", J. Comp. Appl. Math., 33(1990), pp. 35–52.

4) (with M. Hanke) "Preconditioning and splittings for rectangular systems", Numer. Math., 57(1990), pp.85–95.

5) (with E. Elsner and B. Vemmer) "The effect of the number of processors on the convergence of the parallel block Jacobi method", Lin. Alg. Appl., 154–156(1991), pp.311–330.

6) (with M. Hanke and W. Niethammer) "On the SOR method for symmetric positive definite systems", Lin. Alg. Appl., 154–156(1991), pp.457–472.

7) (with L. Elsner) "Monotonic sequences and rates of convergence of asynchronized iterative methods", submitted to Numer. Math.

8) (with L. Elsner and I. Koltracht) "Convergence of sequential and asynchronous nonlinear paracontractions", submitted to Numer. Math.

On the reachability problem

9) (with R. J. Stern) " Discrete approximations to reachability cones of linear differential equations," Lin. Alg. Appl., 120(1989), pp. 65–79.

10) (with R. J. Stern and M. Tsatsomeros) "The reachability cones of essentially nonnegative matrices", Lin. Multilin. Alg., 28(1991), pp.213–224.

11) (with M. Tsatsomeros) "Symbiosis points for linear differntial systems", Lin. Multilin. Alg., 30(1991), pp.49–59.

On nonnegative matrices and applications

12) (with R. E. Hartwig and N. J. Rose) " An algebraic-analytic approach to nonnegative basis, " Lin. Alg. Appl., 133(1990), pp.77–88.

13) (with I. Koltracht) "On the inverse M–matrix problem for real symmetric positive definite Toeplitz matrices", SIAM J. Matrix Anal. Appl., 12(1991), pp.310–320.

14) (with H. J. Werner) "Nonnegative group inverses", Lin. Alg. Appl., 151(1991), pp.85–96.

15) (with H. Schneider) "Principal components of minus M–matrices", submitted to Lin. Multilin. Alg.

During the first half of the period in which this grant has been in effect we have also co–authored a book in connection with the second research topic listed above. The title of the book is " Nonnegative Matrices in Dynamic Systems". Its other authors are A. Berman and R. J. Stern and it was published in the Series in Pure and Applied Mathematics, Wiley Interscience, New York, 1989.

Since the start of the grant we have presented results from our work and acknowledged the support of the Air Force in conferences and colloquia as follow: "Special Session on Modern Trends in Matrix Theory and its Applications", AMS Annual Meeting, January, 1988; "Oberwolfach Meeting on Numerical Algebra and Parallel Computations", Oberwolfach, February 1988; "Workshop on Iterative Solutions to Singular Systems", Univ. of Karlsruhe, Karlsruhe, West Germany, March 1988; "SIAM 3rd Conference on Applied Linear Algebra", Madison, May 1988; "International Symposium on Computational Applied Math.", Leuven, Belgium, July 1988; "NATO Advanced Study Institute on Numerical Linear Algebra, Digital Signal Processing and Parallel Algorithms", Leuven, Belgium, August 1988; "Conference on Iterative Methods for Large Linear Systems (dedicated to David

M. Young)", University of Texas, Austin, October 1988; "A Conference on Approximation Theory and Numerical Linear Algebra (dedicated to R. S. Varga)", Kent State University, Kent, Ohio, March 1989; Northern Illinois University Conference on "Linear Algebra, Numerical Linear Algebra and Applications", DeKalb, Illinois, April 1989; "The Householder Symposium XI on Numerical Linear Algebra", Tylösand, Sweden, June 1990; Meeting on "Numerical Linear Algebra", Oberwolfach, Germany, April 1991; "Haifa VII Matrix Theory Conference", Haifa, Israel, June 1991.

The referencing within the report is as follows. Papers cited, but not co–authored by the P.I., are referenced by numbers in the text and the key is given after section 3. References to papers co–authored by the P.I. are numbered by [Nxx], where xx refers to the paper number in the P.I.'s vitae which is attached at the end of this report.

2 ASYNCHRONOUS ALGORITHMS FOR LARGE LINEAR SYSTEMS

In this section we describe our research concerning the convergence and the acceleration of convergence of a certain model of a *parallel chaotic* (also known as *asynchronized*) iteration scheme. Some of the problems we encountered come from the fact that we tried to apply the same asynchronized model to linear systems whose coefficient matrices arises in different applications and also to nonlinear systems. This means that for each application we had to find the inherent mathematical properties which make the convergence of the algorithm and its acceleration possible. Each type of system, in turn, gives rise to different problems in the actual implementation of the algorithm.

The chaotic iteration method which we have in mind has the following form: We are given

i) m linear or nonlinear operators $B_1, \ldots, B_m$.

ii) A *computation cycle*, namely, a fixed time period $T > 0$, and a *regulated sequence integers on* m, that is a sequence of integers $\{i_j\}_{j=1}^{\infty}$ with $1 \leq i_j \leq m$ and such that

$$\{1, 2, \ldots, m\} \subseteq \{i_j, \ldots, i_{j+T-1}\}, \ \forall j \geq 1. \tag{2.1}$$

iii) m nonnegative diagonal matrices E_l, $l = 1, \ldots, m$, whose sum is the identity matrix. (They are sometimes called *weighting* or *masking* matrices.)

iv) A parallel machine with k processors and a host node.

We perform the iteration:

$$x^{(j+r_j)} = (I - E_{i_j}) x^{(j+r_j-1)} + E_{i_j} B_{i_j} x^{(j)}, \ j = 1, 2, \ldots. \tag{2.2}$$

Our model works as follows: At time j a processor, call it for now the **subject** processor, which has just completed a previous task is assigned the task specified by i_j, namely, by the operators E_{i_j} and B_{i_j}. This means that it begins to calculate the vector $u = E_{i_j} B_{i_j} x^{(j)}$. Note that only the entries of u corresponding to the nonzero diagonal entries in E_{i_j} need be computed. The number $r_j - 1$, $r_j \leq T$, then represents the number of

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similar tasks completed by other processors before the subject processor completes its present computation. When this computation is done, the sum $x^{(j+r_j)} = u + (I - E_{i_j})x^{(j+r_j-1)}$ is formed by the host processor and the subject processor is then assigned task i_{j+r_j}. The existence of a computation cycle $T > 0$ as given by (2.1) means that there exists a time period T such that in every T successive iteration, the global approximation to the solution is corrected at least once by each of the operators $B_1, \ldots, B_m$.

We have proved the convergence of (2.2) when the chaotic process is used to solve iteratively two quite different types of linear systems

$$Ax = b. \tag{2.3}$$

The first type usually arises from a finite differences approximation to second order partial differential equations subject to boundary value conditions. There A is frequently a *monotone* matrix, meaning it is nonsingular with $A^{-1} \geq 0$. The operators B_l, $l = 1, \ldots, m$, are then iteration matrices induced by m *weak regular splittings* of the matrix A, that is, by m splittings

$$A = M_l - N_l, \tag{2.4}$$

satisfying M_l is invertible with

$$M^{-1} \geq 0 \quad \text{and} \quad B_l = M_l^{-1}N_l \geq 0. \tag{2.5}$$

The second type of linear systems to which we have applied (2.2) are rectangular systems (2.3) which arise in image reconstruction from incomplete data as, for example, in well–to–well tomography used in geophysics. Previously the *cyclic Algebraic Reconstruction Technique* (ART), which itself is a generalization of the Kaczmarz projection method, has been applied to find the least squares solution of minimal norm to such systems. The cyclic ART method, which is also closely related to the *successive overrelaxation* (SOR) method (see Koltracht and Lancaster [1] and Hanke and Niethammer [2]), is a sequential method where we apply in a cyclic order the m operators B_l, each of which corresponds to an orthogonal projection onto a subspace spanned by a row or a group of rows of the coefficient matrix A with each row of the A appearing in at least one of the groups (thus overlapping is allowed). The proof that the cyclic ART–SOR can be parallelized according to (2.2) is more intricate than in the case of chaotic iteration for solving monotone system. It requires certain norm considerations and restrictions on the weighting diagonal matrices which are not necessary in the case of

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chaotic iterations for monotone systems.

For both types of linear systems mentioned above we have found various proofs for the convergence of (2.2) all of which involve the embedding of the process as a sequential iteration process which takes place in higher dimensional space. In one of the types of embedding, we iterate sequentially in the kn-dimensional space (recall k is the number of processors and n is the dimension of the iterates in (2.2)) and produce a sequence of iteration vectors $z^{(j)}, j = 1, 2, \ldots$. The idea now is to prove that the k subvectors of $z^{(j)} \in R^{nk}$ each has a limit point, as $j \to \infty$, equal to the solution of (2.3). Although we have not used this name **formally** in any of our reports, we like to refer **informally** to this approach to the proof as the "**logbook**" approach. This is because for any processor, say the ν-th, we think of the sequence of n-vectors which we can form from the ν-th subvectors of the $z^{(j)}$ as keeping a "logbook" of the local approximations in the ν-th processor at each time step of the global iteration. Thus much of the time only the index of the iteration in the processor is advanced, but the actual value of the local approximation is unchanged. It only changes when the ν-th processor updates to and downdates from the host processor.

The second type of embedding we have used to prove the convergence of (2.2) is by blowing up the process in the n-dimensional space to a sequential process in the nT-dimensional space, where T is the computation cycle. This is achieved by looking at the iteration

$$y^{(j+r_j)} = C_{j+r_j} y^{(j+r_j-1)} + \tilde{b},$$

where

$$C_{j+r_j} = \begin{pmatrix} (I - E_{i_j}) & 0 & \ldots & 0 & E_{i_j} B_{i_j} & 0 & \ldots & 0 \\ I & & 0 & & 0 & & & \\ 0 & & I & & & & & \\ & & & & \ddots & 0 & & \\ 0 & & \ldots & \ldots & 0 & I & & \\ & & & & & & \ddots & \\ & & & & & & & \ddots \\ 0 & & \ldots & & \ldots & 0 & & I & 0 \end{pmatrix} \quad \overbrace{}^{r_j \text{ blocks}} \tag{2.6}$$

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and where

$$y^{(s)} = \begin{pmatrix} x^{(s)} \\ x^{(s-1)} \\ \vdots \\ x^{(s-T+1)} \end{pmatrix} \quad \text{and} \quad \bar{b} = \begin{pmatrix} b \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

Above, under the appropriate assumptions on A and the B_l's, a contraction occurs at least every time a succession of $2T$ iterations has been applied. In otherwords, in an appropriately induced norm on R^{nT}, $\|C_{k+2T-1} \cdots C_k\| < 1, \forall k \geq 1$.

One of the goals of our research was to obtain a better understanding of the meaning and the interpretation of the two embeddings. First, we can view r_j as the time elapsed (= time-lag) between two updates of the global approximation by the operator i_j. Thus if r_j is large, then the subject processor which received the global approximation $x^{(j)}$ is, by the time it has finished computing u, using a correction based on a relatively old approximation to update the current global approximation in the host node. Using special nested subcones of monotonic vectors in R_+^{nT}, the cone of nonnegative vectors in the R^{nT}-dimensional space, we were able to prove in [N61] the following result on the rate of converegence of the model given in (2.2).

Theorem 1 *Let $\{s_j\}_{j=1}^{\infty}$ and $\{r_j\}_{j=1}^{\infty}$ be two sequences of time-laggs such that*

$$1 \leq s_j \leq r_j \leq T, \quad j = 1, 2, \ldots . \tag{2.7}$$

If

$$s_{j+1} \leq s_j + 1, \quad j = 1, 2, \ldots, \tag{2.8}$$

then

$$\sup_{v \in R^{nT}} \limsup_{j \to \infty} \|\tilde{C}_j \ldots \tilde{C}_1 y - \xi\|^{1/j} \leq \sup_{v \in R^{nT}} \limsup_{j \to \infty} \|C_j \ldots C_1 y - \xi\|^{1/j}, \tag{2.9}$$

where

$$
\tilde{C}_{j+s_j} = \begin{pmatrix}
(I - E_{i_j}) & 0 & \ldots & 0 & E_{i_j} B_{i_j} & 0 & \ldots & 0 \\
I & 0 & & & 0 & & & \\
0 & I & & & & & & \\
& & & \ddots & 0 & & & \\
0 & & \ldots & \ldots & 0 & I & & \\
& & & & & & \ddots & \\
& & & & & & & \ddots \\
0 & & \ldots & & \ldots & 0 & I & 0
\end{pmatrix}
$$

with an overbrace labeled "s_j blocks"

and where $\xi = (\bar{x}^T, \ldots, \bar{x}^T)^T \in R^{nT}$ with $\bar{x}$ being the solution to (2.3).

Let us refer to the iterative processes induced by the sequences of time–laggs $\{s_j\}_{j=1}^{\infty}$ and $\{r_j\}_{j=1}^{\infty}$ as the **more frequently updtaing process** and the **more infrequently updating process,** respectively. What the above result says is this: When $s_{j_0+1} > s_{j_0} + 1$ for some $j_0 \geq 1$, then the more frequently updating process uses an older approximation to compute the j_0–th iteration than the approximation it has used in computing the immediately prcceeding iteration. Therefore condition (2.8) means that when the more frequently updating process never "suddenly" uses an older approximation in computing some iterate than the approximation it has used in computing the previous iterate, then the rate of convergence of the more frequently updating iteration is more favorable than the rate of convergence of the more infrequently updating. We have shown by means of examples that if condition (2.7) holds, but condition (2.8) does not, then the result of the theorem is not true.

We have carried out many numerical experiments in connection with Theorem 1 and several such are given in [N57]. There we considered the special case when:

$$
s_j = k = const. \quad and \quad r_j = k' = const., \quad \forall j \geq 1
$$

and with

$$
k \leq k'.
$$

Such a delay will occur when the work among the processors is equally distributed in which case the constant delay is just the number of processors minus 1. In the following table, for a typical 80 × 80 diagonally dominant matrix, we shcw how the increasing the number of processors (viz. increasing the delay) effects the number of iteration which are necessary to reach a given accuracy of 10^{-6}. In the table k is the number of processors used by the machine and $J = J(k)$ is the number of iterations required to achieve a prescribed accuracy to the solution. The last column is the ratio of the number of iterations to the number of processors, indicating, roughly, the number of iterations which each of the processors would have to execute in parallel if communication overheads are reasonable:

k	J	J/k	k	J	J/k
1	101	101.0	11	291	26.5
2	134	67.0	12	324	27.0
3	162	54.0	13	352	27.1
4	194	48.5	14	384	27.4
5	204	40.8	15	394	26.3
6	212	35.3	16	402	25.1
7	244	34.9	17	434	25.5
8	261	32.6	18	454	25.3
9	282	31.3	19	474	24.9
10	291	29.1	20	484	24.2

To give a better illustration of this table of iterates let us graph k versus

J/k:
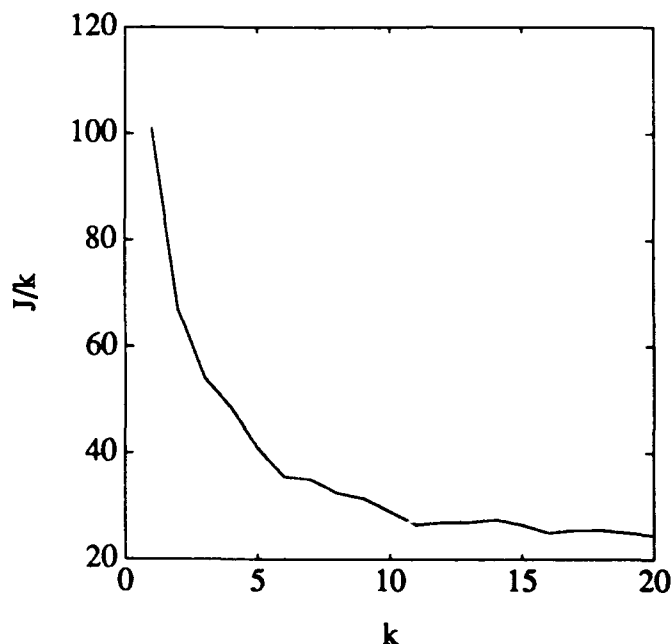
The graph clearly points to a conjecture that we have that as the number of processors increase the speed-up tends to a linear constant and no gain is achieved by increasing heavily the number of processors.

For rectangular systems of equations we have considered chaotic iterations for computing least squares solutions. As mentioned erlier, the mathematics that is needed to demonstarte that such iterations converge is quite different than for monotone systems. We attach to this report a recent paper in which we show that chaotic methods of the form (2.2) can be applied also to nonlinear systems of equations. This allows us also to consider applications to finding linear least squares solutions lying in some closed convex set which represents a nonlinear constraint on the solution. Such an application arises in computed tomography from incomplete data.

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3 The Determination of Nonnegative Solutions to Linear Systems of Differential Equations by Finite Differences Methods

Consider the system of linear differential equations

$$\dot{x} = Ax, \tag{3.1}$$

where $A = (a_{ij})$ is an $n \times n$ real matrix. In many engineering, biological, and other applications the vector $x(t)$ represents the state of a system at time t and its components frequently represent the sizes of *populations* or *species* at time t. In some applications (see Luenberger [1979]) the matrix A is essentially nonnegative, that is, $a_{ij} \geq 0$, $i \neq j$. Such a constraint ensures that trajectories which emanate from nonnegative initial states remain nonnegative. Moreover interest centers on trajectories which eventually become and remain nonnegative or trajectories whose velocities (derivatives) become and remain nonnegative. **If the latter condition holds, then, in time, the system reaches a state from which every species will not decrease in size thereafter.** We call initial points whose trajectories and their velocities eventually become and remain nonnegative it **symbiosis points.**

From now on we shall suppose that A is essentially nonnegative. Denote by $X_A(R_+^n)$ the set of all points in R^n such that the trajectories emanating from these points become and, due to the essential nonnegativity of A, remain nonnegative. In [N33] we showed that $X_A(R_+^n)$ is a convex cone which, however, need *not* be closed or pointed. In a sequence of papers [N31], [N33], and [N37] we gave formulas for the closure of $X_A(R_+^n)$ under various further assumptions on A such as diagonalizability, real spectrum, etc. These formulas were very difficult to apply for two reasons: (i) they were too complicated as they involved the intersections of the eigenspaces of A with various projections of the nonnegative orthant, and (ii) as only the closure of the reachability cone was determined, there were further complications in applying the formulas to determine whether a given boundary point of $X_A(R_+^n)$ is also a reachability point.

Because of the difficulties we described above we thought of the possibility of applying finite differences methods in order to determine whether a given point $x_0 \in R^n$ is a reachability point. In the simplest finite differences

schemes, the so called *Cauchy-Euler method*, we approximate the solution at times $k = 1, 2, \ldots$ from the quotients

$$\frac{\hat{x}_k - \hat{x}_{k-1}}{h} = A\hat{x}_{k-1}, \qquad (3.2)$$

where h is the **time–step** used by the method. After some rearrangement we obtain from (3.2) the *discrete trajectory of points* emanating from $\hat{x}_0 = x_0$ given by

$$\hat{x}_k = (I + hA)^k \hat{x}_0, \quad k = 1, 2, \ldots \qquad (3.3)$$

We see that discretization schemes for systems of ordinary differential equations thus resemble error analysis for iterative solutions to linear systems of equations in the sense that both procedures involve powering–up matrices. It is therefore of no surprise that underlying both are basic features and problems of the power method for determining eigenvalues and invariant subspaces.

Observe that if the time step h is small enough to make the matrix $I + hA$ nonnegative, then if $\hat{x}_0 \in R^n$ is a point for which there exists an an exponent k_0 such that $\hat{x}_{k_0}$ is nonnegative, then all subsequent points in the trajectory emanating from $\hat{x}_0$ will remain nonnegative. One result that we completed proving during the course of this grant represents a considerable improvement over results which we obtained previously in [N48]. It is the following:

Theorem 2 ([N54]) *Let A be an $n \times n$ essentially nonnegative matrix and consider the linear differential system* (3.1). *Let*

$$h(A) := \sup\{h > 0 \mid I + hA \geq 0\}. \qquad (3.4)$$

Then $x_0 \in X_A(R_+^n)$ if and only if for any $0 < h < h(A)$ there exists an index k_0 such that the discrete trajectory of points (3.3) *generated from $\hat{x}_0 = x_0$ satisfies that*

$$\hat{x}_k \geq 0, \quad \forall k \geq k_0.$$

Notice that $h(A)$ can be very large, it is $+\infty$ if A is nonnegative, but in any case it depends only in the size of the diagonal entries of A and is not "infinitesimal". What the result means is this: **"regardless of the extent to which the continuous and discrete trajectories diverge from each other, one becomes nonnegative if and only if the other one**

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does, provided only that the time–step h **satisfies** $0 < h < h(A)^"$.
This is a qualitative as well as a numerical statement about the behavior of
the solutions to systems of ordinary differential equations whose coefficient
matrix is essentially nonnegative. The proof of the above theorem is quite
involved and is the sub... t of the manuscript "Reachability cones of essen-
tially nonnegative matrices" which has just been accepted for publication in
the journal of Linear and Multilinear Algebra. We mention that quite an
important tool which was used in the proof of the theorem is taken from
an earlier paper [N52] in which we considered an analytic approach to the
question of existence of a nonnegative basis for the eigenspace of a nonneg-
ative matrix corresponding to its Perron root.

The characterization of symbiosis points is done in [N59]. Decompose a
point $v \in X_A(R_+^n)$ into

$$v = v_+ - v_-,$$

where v_+ is the join of all eigenspaces of A corresponding to eigenvalues
with a nonnegative real part and v_- is in the join of all eigenspaces of A
corresponding to eigenvalues with a negative real part. Thus v_- is in the
stability part of the space since $\lim_{t \to \infty} e^{tA} v_- = 0$. For v we define the
invariant set of components of v as the set

$$\mathcal{I}(v) = \{1 \leq i \leq n \; : \; (e^{tA})_i = (v_+)_i, \; \forall t \geq 0 \}. \tag{3.5}$$

Thus $\mathcal{I}(v)$ consists of the indices of the components of the vector v_+ which
remain invariant throughout the entire trajectory emanating from v_+. We
prove the following characterization:

Theorem 3 ([N59]) *Let* A *be an* $n \times n$ *essentially nonnegative matrix. Then
a vector* $v \in X_A(R_+^n)$ *is a symbiosis point for* (3.1) *if and only if there exists
a sufficiently large time* t_0 *such that*

$$j \in \mathcal{I}(v) \implies 0 \leq (e^{tA} v_-)_j \downarrow 0, \quad \forall t \geq t_0, \tag{3.6}$$

where $\mathcal{I}(v)$ *is given in* (3.5). *Furthermore, if* $v \in X_A(R_+^n)$ *is a symbiosis
point, then for any* $j \in \mathcal{I}(v)$, $(v_+)_j = 0$ *if and only if* $(e^{tA} v)_j = 0$ *for all*
$t \geq 0$.

We have two comments. First, in the spirit of Theorem 2 we can also
characterize a symbiosis point $v \in X_A(R_+^n)$ in terms of the nondecreasing-
ness of finite differences sequences generated from v (similar to the way

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in which the sequence in (3.3) is generated from x_0) where the step size h satisfies (3.4). Second in the case when A is **weakly stable**, meaning its eigenvalue with the largest real part is the origin, then symbiosis points admit a matrix–combinatorial structure in the sense that it is possible to determine apriori which indices $1 \leq i \leq n$ lie in $\mathcal{I}(v)$ from a certain block directed graph of the matrix A. Both of these issues are addressed in [N59].

References

[1] L. Elsner, I. Koltracht, and P. Lancaster. Convergence properties of the ART and SOR algorithms. 1989, to appear.

[2] M. Hanke and W. Niethammer. On the acceleration of Kaczmarz's method for inconsistent linear systems. 1989. To appear.

Conference lectures:

"SIAM National Conferences on Applied Linear Algebra", Raleigh, North Carolina, 1983 and 1985; "Dundee Conference on Numerical Analysis"; "Auburn Matrix Theory Meeting"; "Oberwolfach Meeting on Matrix Theory & Numer. Algebra", Oberwolfach, February 1983; "The Gatlinburg IX Conference on Numerical Linear Algebra", University Waterloo, Canada, August 1984; The AMS-SIM-SIAM Joint Summer Research Conference on the "Role of Linear Algebra in Systems Theory", Bowdin College, Maine, July 1984; "International Conference on Linear Algebra and its Applications", Coimbra, Portugal, October 1984; "MTNS '85" (Mathematical Theory of Networks and Systems) Stockholm, Sweden, June 1985; "Haifa Third Matrix Theory Conference", Haifa, Israel, January 1987; "The Gatlinburg X Conference on Numerical Linear Algebra", Fairfield, Tennessee, October 1987; "The Valencia International Conference on Linear Algebra and its Applications", Valencia, Spain, September 1987; "Special Session on Modern Trends in Matrix Theory and its Applications", AMS Annual Meeting, January, 1988; "Oberwolfach Meeting on Numerical Algebra and Parallel Computations", Oberwolfach, February 1988; "Workshop on Iterative Solutions to Singular Systems", Univ. of Karlsruhe, Karlsruhe, West Germany, March 1988; "SIAM 3rd Conference on Applied Linear Algebra", Madison, May 1988; "International Symposium on Computational Applied Math.", Leuven, Belgium, July 1988; "NATO Advanced Study Institute on Numerical Linear Algebra, Digital Signal Processing and Parallel Algorithms", Leuven, Belgium, August 1988; "Conference on Iterative Methods for Large Linear Systems (dedicated to David M. Young)", University of Texas, Austin, October 1988; "A Conference on Approximation Theory and Numerical Linear Algebra (dedicated to R. S. Varga)", Kent State University, Kent, Ohio, March 1989; Northern Illinois University Conference on "Linear Algebra, Numerical Linear Algebra and Applications", DeKalb, Illinois, April 1989; "The Householder Symposium XI on Numerical Linear Algebra", Tylösand, Sweden, June 1990; Meeting on "Numerical Linear Algebra", Oberwolfach, Germany, April 1991; "Haifa Seventh Matrix Theory Conference", Haifa, Israel, June 1991.

Hall Tutor: Wortley Hall, The Univ. of Nottingham, Nottingham, England: Oct. 1976 - August 1980. (Hall Tutor is an undertaking of some social responsibility for a group of students living on campus).

Organizer of:

"The University of South Carolina Mini-Conference on Linear Algebra &

Matrix Theory", February, 1981.

"The Valencia International Conference on Linear Algebra and its Applications", Valencia, Spain, September, 1987, (member of Scientific Organizing Committee).

Teaching of Undergraduate and Graduate Courses:

Taught most of the undergraduate curriculum and many courses in the graduate curriculum in both pure and applied mathematics. Courses which have been taught most frequently have been in Numerical Analysis, Partial Differential Equations, The Theory of Nonnegative Matrices and their Applications, Linear Programming, and Methods of Applied Mathematics.

Ph.D. Student: (1) Dr. Valerie Miller. Her thesis title was: "Successive Overrelaxation Methods for Solving Large Scale Rank Deficient Least Square Problems.", (1985). (2) Dr. Michael J. Tsatsomeros. His thesis title was: "Reachability of Nonnegative and Symbiotic States for Linear Differential Systems".

Computing Skills:

FORTRAN, BASIC, APL, UNIX, VMS, CMS, MATLAB.

Membership of Professional Societies:

Society for Industrial and Applied Mathematics

The American Mathematical Society

The Institute of Mathematics and its Applications (Associate Fellow)

Awards:

1981/82 National Science Foundation Grant for research on "Special Matrices", (co-investigator).

1983/84 National Science Foundation. "Mathematical Sciences Research Equipment" Grant. Co-proposer of "Numerical Linear Algebra" project in the proposal.

1984/86 National Science Foundation "Functions and Applications and Nonnegative and Cone Preserving Maps."

1986 NSF "Mathematical Research Equipment Grant", (P.I.).

7/1987–6/1988 NSF Research Grant "Computational and Applied Processes in Linear Algebra."

Publications:

a) Published:

1) "Subproper splitting for rectangular matrices," Lin. Alg. Appl. 14 (1976), pp.41-51.

2) (with A. Berman) "Proper splittings of rectangular matrices," SIAM J. Appl. Math. 31(1976), pp.307-312.

3) (with A. Berman) "Consistency and splittings," SIAM J. Numer. Anal. 13(1976), pp.877-888.

4) (with S. P. Gudder) " Splittings and iterative methods for approximate solutions to singular operator equations in Hilbert spaces," J. Math. Anal. Appl. 62(1978), pp.272-297.

5) "Some applications of partial orderings to iterative methods for rectangular linear systems," Lin. Alg. Appl. 19(1978), pp .95-116.

6) " 3-part splittings for singular and rectangular linear systems," J. Math. Anal. Appl. 64(1978), pp.297-318.

7) (with R. J. Plemmons) " Convergent nonnegative matrices and iterative methods for consistent linear systems," Numer. Math. 31(1978), pp.265-279.

8) (with R. J. Plemmons) "Generalized inverse-positivity and splittings of M-matrices," Lin. Alg. Appl. 23(1979), pp.21-36.

9) "Weak stability for matrices," Lin. Multilin. Alg. 7(1979), pp.257-263.

10) "A note on generalizations of strict diagonal dominance for real matrices," Lin. Alg. Appl. 26(1979), pp.3-14.

11) (with D. W. Masser) "On the square roots of quasiaccretive matrices," Lin. Alg. Appl. 28(1979), pp.135-140.

12) (with C. R. Johnson) "Square roots with positive definite hermitian part," Lin. Multilin. Alg. 8(1980), pp.353-355.

13) (with M. Lewin) "The inverse M-matrix problem for (0,1)-matrices," Lin. Alg. Appl. 30(1980), pp.41-50.

14) (with R. J. Plemmons) "M-matrix characterizations II: General M-matrices," Lin. Multilin. Alg. 9(1980), pp.211-225.

15) (with R. S. Varga) "On the sharpness of some upper bounds for the spectral radii of S.O.R. iteration matrices," Numer. Math. 35(1980), pp.69-79.

16) "A combined direct-iterative approach for solving large scale singular and rectangular consistent systems of linear equations," Lin. Alg. Appl. 34(1980), pp.85-101. (Also appeared in: Large Scale Matrix Problems, A. Bjorck, R. J. Plemmons and H. Schneider, editors, Elsevier N. Holland, N.Y. 1980.)

17) (with J. de Pillis) "A noncommutative spectral theorem for operator entried companion matrices," Lin. Multilin. Alg. 10(1981), pp.45-51.

18) "On the Schur complement and the LU-factorization of a matrix," Lin. Multilin. Alg. 9(1981), pp.241-254.

19) (with J. de Pillis) "Iterative methods with k-part splittings," IMA J. Numer. Anal. 1(1981), pp.65-79.

20) "The Kahan S.O.R. convergence bound for nonsingular and irreducible M-matrices," Lin. Alg. Appl. 39(1981), pp.205-222.

21) (with L. Elsner and C. R. Johnson) "The effect of the perturbation of a nonnegative matrix on its Perron eigenvector," Czech. Math. J. 32(1982), pp.99-109.

22) (with J. Buoni and R. S. Varga) "Theorems of Stein-Rosenberg type III: The singular case," Lin. Alg. Appl. 42(1982), pp.183-198.

23) (with G. Poole and H. Werner) "More on the generalizations of matrix monotonicity," Lin. Alg. Appl. 48(1982), pp.413-435.

24) (with R. E. Funderlic and R. J. Plemmons) "LU decompositions of generalized diagonally dominant matrices," Numer. Math. 40(1982), pp.57-69.

25) (with M. Fiedler and T. L. Markham) "Classes of products of M-matrices and inverse M-matrices," Lin. Alg. Appl., 52/53 (1983), pp.265-287.

26) "On bounds for the convergence of the SSOR method for H-matrices," Lin. Multilin. Alg., 15(1984), pp. 13-21.

27) (with S. R. Mohan and K. G. Ramamurthy) "Nonnegativity of principal minors of generalized inverses of M-matrices," Lin. Alg. Appl., 58 (1984), pp.247-259.

28) (with E. Deutsch) "Derivatives of the Perron root at an essentially nonnegative matrix and the group inverse of an M-matrix," J. Math. Anal. Appl., 102(1984), pp.1-29.

29) (with R. J. Plemmons) "Backward error analysis for linear systems associated with inverses of H-matrices," BIT, 24(1984), pp.102-112.

30) (with J. de Pillis) " The effect of the perturbation of hermitian matrices on their eigenvectors," SIAM J. Alg. Discrete Methods, 6(1985), pp.201-209.

31) (with R. Stern) "Boundary results for positively invariant cones and their reachability cones," Lin. Multilin. Alg. 17(1985), pp.143-154.

32) (with T. L. Markham and R. J. Plemmons) "Convergence of a direct-iterative method for large-scale least squares problems." Lin. Alg. Appl., 69(1985), pp.155-167.

33) (with Ronald J. Stern) "Cone reachability for linear differential systems," Applicable Analysis, 20(1985), pp.57-71.

34) (with M. Fiedler, C. R. Johnson, and T. L. Markham) "A trace inequality for M-matrices and symmetrizability of a real matrix by a positive diagonal matrix," Lin. Alg. Appl., 71(1985), pp.81-94.

35) (with E. Deutsch) "On the first and second derivatives of the Perron vector," Lin. Alg. Appl., 71(1985), pp.57-76.

36) (with V. Miller) "A note on comparison theorems for nonnegative iteration matrices," Numer. Math., 47(1985), pp.427-434.

37) (with A. Berman and R. J. Stern) "Cone reachability for nondiagonable linear differential systems," Lin. Alg. Appl., 81(1986), pp.263-280.

38) "Neighborhoods of dominant convergence for the SSOR method," SIAM J. Alg. Discrete Methods, 7(1986), pp.551-559.

39) (with V. Miller) "Successive overrelaxation methods for solving the rank deficient linear least squares problem," Lin. Alg. Appl. 88/89(1987), pp.533-557.

40) (with R. J. Plemmons) "Convergence of parallel multisplitting iterative methods for M-matrices," Lin. Alg. Appl. 88/89(1987), pp.533-557.

41) (with E. Deutsch) "On the derivative of the Perron vector whose infinity norm is fixed," Lin. Mutltilin. Alg., 21(1987), pp.75-85.

42) (with S. Nelson) "Generalizations of the projection method with applications to SOR theory for hermitian positive semidefinite systems," Numer. Math., 51(1987), pp.123-141.

43) (with L. Elsner and R. Bru) "Models of parallel chaotic iteration methods," Lin. Alg. Appl., 102(1988), pp.175-192.

44) (with R. Bru) "Nonnegative Jordan Basis," Lin. Multilin. Alg., 23(1988), pp.95-109.

45) (with A. Hadjidimos) "A note on the SSOR convergence domain due to Neumaier and Varga," Lin. Alg. Appl., 107(1988), pp.207-217.

46) (with G. Avdelas, J. de Pillis, and A. Hadjidimos) "A guide to the acceleration of iterative methods whose iteration matrix is nonnegative and convergent," SIAM J. Matrix Analysis Appl., 9(1988), pp.329-342.

47) (with P. J. Kavanagh) "Consistency and convergence of the parallel multisplitting method for singular M-matrices," SIAM J. Matrix Analysis Appl., 10(1989), pp. 210-218.

48) (with R. J. Stern) " Discrete approximations to reachability cones of linear differential equations," Lin. Alg. Appl., 120(1989), pp. 65-79.

49) (with A. Hadjidimos) "Precise domains of convergence for the block SSOR method associated with p-cyclic matrices," BIT, 29(1989), pp. 311–320.

50) (with M. Hanke) "Preconditioning and splittings for rectangular systems", Numer. Math., 57(1990), pp.85–95.

51) (with L. Elsner and I. Koltracht) "On the convergence of asynchronous paracontractions with application to tomographic reconstruction from incomplete data," Lin. Alg. Appl., 130(1990), pp.65–82.

52) (with R. E. Hartwig and N. J. Rose) " An algebraic-analytic approach to nonnegative basis, " Lin. Alg. Appl., 133(1990), pp.77–88.

53) (with A. Hadjidimos) "Convergence domains of the SSOR method for generalized consistently ordered matrices", J. Comp. Appl. Math., 33(1990), pp. 35–52.

54) (with R. J. Stern and M. Tsatsomeros) "The reachability cones of essentially nonnegative matrices", Lin. Multilin. Alg., 28(1991), pp.213–224.

55) (with I. Koltracht) "On the inverse M–matrix problem for real symmetric positive definite Toeplitz matrices", SIAM J. Matrix Anal. Appl., 12(1991), pp.310–320.

56) (with H. J. Werner) "Nonnegative group inverses", Lin. Alg. Appl., 151(1991), pp.85–96.

57) (with E. Elsner and B. Vemmer) "The effect of the number of processors on the convergence of the parallel block Jacobi method", Lin. Alg. Appl., Lin. Alg. Appl., 154–156(1991), pp.311–330.

58) (with M. Hanke and W. Niethammer) "On the SOR method for symmetric positive definite systems", Lin. Alg. Appl., 154–156(1991), 457–472.

59) (with M. Tsatsomeros) "Symbiosis points for linear differntial systems", Lin. Multilin. Alg., 30(1991), pp49–59.

b) **Accepted for publication**:

60) (with H. Schneider) "Principal components of minus M–matrices", submitted to Lin. Multilin. Alg.

c) **Submitted for publication**:

61) (with L. Elsner) "Monotonic sequences and rates of convergence of asynchronized iterative methods", submitted to Numer. Math.

62) (with L. Elsner and I. Koltracht) "Convergence of sequential and asynchronous nonlinear paracontractions", submitted to Numer. Math.

63) (with L. Elsner, I. Koltracht, and D. Xiao) "On computations of the Perron root", submitted to SIAM J. Matrix Anal. Appl.

Books:

Additional Material

1) (with A. Berman) "Monotone submatrices," a problem in SIAM Rev., 18(1976), p.490.

2) (with M. Newborn and A. Ziv) "An analysis of the Alpha-Beta algorithm for trees of depth two," Technical report No. 58, Computer Science Department, Technion, Haifa, Israel, 1976.

3) "The University of South Carolina mini-conference on linear algebra and matrix theory," Letters in Linear Algebra, Lin. Alg. Appl., 43(1982), pp.273-284.