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In this paper, a procedure is proposed which analyzes non-geolocation attribute information and yields a posterior possibility distribution of target correlations. In turn, this result may be utilized to compute the overall posterior probability distribution of correlations. The key factor in this procedure is a theorem which shows that a uniformly most accurate confidence set exists which is determined by a single possibility distribution that is feasible to compute.

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APPLICATIONS OF POSSIBILITY THEORY TO OCEAN SURVEILLANCE CORRELATION (U)

(THIS PAPER IS UNCLASSIFIED)

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ABSTRACT

The multiple target ocean surveillance contact correlation problem can be decoupled roughly into two parts. The first, extensively treated in the literature, involves geolocation information only, and is normally analyzed through use of a bank of Kalman filters. The second is concerned with non-geolocation attribute information. Typically, the latter includes all data obtained through linguistic, visual, or discrete valued numerical sources.

In this paper, a procedure is proposed which analyzes non-geolocation attribute information and yiels a posterior possibility distribution of target correlations. In turn, this result may be utilized to compute the overall posterior probability distribution of correlations. The key factor in this procedure is a theorem which shows that a uniformly most accurate confidence set exists which is determined by a single possibility distribution that is feasible to compute.

INTRODUCTION

The theme of the 48th MORS, "Military Operations Research Techniques for the 80's", is being addressed in many different ways throughout the symposium. Most of the approaches are either deterministic or probabilistic in nature, reflecting the trend in current analysis, yet there are many problems, military and non-military, that can not be easily formulated in probabilistic (nor deterministic) terms. For example, when information is gleaned from strictly human operator sources such as through visual sitings or judgments based on experience, linguistic discriptions may be the prime data base. Such information, although often ambiguous or vague, certainly may prove useful - often in conjuntion with "harder" statistical information.

In the 39th MORS, Dr. J.T. Dockery most aptly showed how fuzzy set techniques could be used in military problems (1). Among other papers demonstrating the feasibility of fuzzy set theory and techniques in a military context, one may include the basic work of Watson et al. (2) and this author ((3), Examples 2 and 4). This paper, too, will deal with one aspect of a class of military problems - the multiple target, multiple sensor contact correlation problems - through the use of fuzzy set theory.

The theory of fuzzy sets is relatively new, being formulated fully for the first time by Zadeh in 1965 (4). (For earlier attempts, see for example, the interesting work (1937) of Shirai (5) and (independently) Black (6); Sheppard's psychological quantifications (7); Watanabe's contributions (aimed toward quantum mechanics)(8); and Klaua's work based on many-valued set theory (9).) At present, much work has been done justifying rigorously the important role that fuzzy set theory plays with respect to probability theory((10), (11)). (Future work will address the fuzzy set approach to data sampling and the Laws of Large Numbers and Central Limit Theorems) Essentially, it can be shown-emphasizing the connection of fuzzy set modeling to vagueness of information- that any fuzzy set can be identified in a natural way with an equivalence class of random sets (in general, many, unless the fuzzy set is an ordinary set) and that fuzzy and random set operations correspond. Nevertheless, many of the difficulties inherent in the probabilistic approach can be avoided by use of the well-developed calculus of fuzzy set operations and relations. Specifically, determination of joint distributions and integration of functions, a necessary factor in probabilistic modeling, is replaced by much simpler operations. (See, for example, Dubois and Prade's comprehensive text (12), illustrating these procedures.)

On the other hand, it is not the intent of the above exposition to claim that fuzzy set techniques should replace probabilistic ones. Rather, it is that the two approaches to modeling and manipulating uncertainties may be used compatibly (see (13)): when numerical descriptions are available with well defined probability distributions present, use a probabilistic approach; when linguistic descriptions or other vague information is present, use a fuzzy set - or, to establish an analagous terminology - possibilistic approach.

Although an elementary treatment of possibility theory and techniques will not be given here (see, for example, (12) or (14)), one basic property contrasting with probability theory must be presented. In probability theory, the well-known concept of a probability distribution plays a key role. In possibility theory, the

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possibility distribution plays a key role. In the latter, equivalently described by a fuzzy set membership function generalizing the concept of the membership or characteristic function of an ordinary set (by allowing values that may often be neither zero nor one, but intermediate), the sum (or integral, if appropriate) of the possible values need not add up to one - as opposed to the sum of values described by a probability distribution. This concept will be clarified further through the procedure developed later in the paper.

Consider now the general contact correlation problem. This military problem has had a long history, beginning with the early formulations of Sittler (1964)(15) and Wax (1955)(16) down through the present. In 1979, well over 300 papers were gathered at the Naval Research Laboratory as part of the Naval Ocean-Surveillance Correlation Handbook Project. (See (17) for a compendium, overview, and analysis of many of these papers in the correlation field. See also (18) and (19) for further analysis of the general problem.) Basically, the problem can be stated as a data partitioning one, where that partitioning is sought where each component represents sensor or human source-gathered information (over some sampling time period) pertaining to the same target or object. The information may be roughly divided into three classes: 1, geolocation sensor-obtained, such as bearing and range measurements; 2, false alarm, an encompassing term used to denote data arising from sources which are either of no interest-icebergs, neutral ships - or false signals due to reflection and scattering, for example; and 3, attribute information. Models which address the first two types of information dominate the correlation field. (Again, see (17) for further details.) However, the work of Reid (20) and Bowman (21), among others, in attempting to incorporate the third category of data should be cited. Nevertheless, to the author's knowledge, no approach to this problem through possibility theory until now, has been undertaken. (Figure 1 illustrates a typical correlation situation.)

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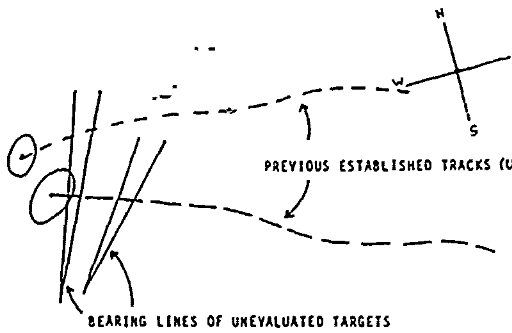

SOME INFORMATION AVAILABLE DESCRIBING EVALUATED AND UNEVALUATED TARGETS INCLUDES CLASSIFICATION, BEARINGS, FREQUENCY INFORMATION AND LITERAL COMMENTS SUCH AS SEA STATE LEVELS, MERCHANT SHIPPING DENSITY, ETC.

(U)Typical Correlation Problem
Figure 1

What is meant by attributes in the context of the contact correlation problem? An example should suffice: Target A may have associated with it maneuvering characteristics, signal frequency information, and a visual siting indicating an irregular design. Target B may also have similar maneuvering characteristics, signal pattern, and a tentative classification. In addition, both targets have been related by intelligence information, although certain discrepancies appear. Finally, Target A has been determined to have on-board three radar sensors, while Target B is known to have "several sensor systems operating, and appears to be heading to port". Should Targets A and B be correlated, tentatively correlated until statistical data (geolocational) is available, or should the two be ruled out for possible correlation? Another example could be generated, where a mix of statistical and verbal descriptions are present for both targets in question. In any case, certainly, wherever possible, the experience of human decision makers should also be taken into account - as well as available automatic correlation algorithms - in order to utilize fully the information. (See figure 2.)

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ATTRIBUTE OF TARGET A	TYPICAL VALUE, LITERAL OR NUMERICAL θ	TYPICAL CONFIDENCE α
BEARING LINE	1158°	±40°, for 95%
CLASSIFICATION	PROBABLY TYPE 4, BUT COULD BE TYPE 3, OR, LESS LIKELY, TYPE 5	MEDIUM
RANGE LIMITATION	11000 MILES	95% LEVEL
FUNDAMENTAL STRENGTH	24198 Hz	±10Hz for 90%
SIGNAL STRENGTH	MEDIUM	HIGH
SIGNAL STABILITY	UNSTABLE-WAVY	HIGH
OBSERVED MANEUVERING	SOME	MEDIUM
OBSERVED HARMONICS	2117,4234	LOW

(U)Example of Attributes
Figure 2

Following simplifying assumptions concerning the statistical dependencies of the relevant variables and the accuracy of the overall modeling (which may change drastically over a sufficient period of time), the following theorem concisely shows the role that attribute probabilities play:

<u>Theorem</u> Structural decomposition of correlation problem. Let $Q=(Q_+,Q_f)$ denote any partitioning of data $Z=(Z_g,Z_a,Z_f)$ up to time t_j, where the subscript f indicates false alarm, g indicates geolocational, a indicates attributes, and + denotes the collection of all components of Q which correspond to targets of interest. Then assuming statistical dependencies only occurring in the conditional forms remaining,

$$p(Q|Z) = p(Z_g|Z_a,Q)\cdot p(Z_f|Q_f)\cdot p(Q|Z_a)/p(Z_g,Z_f|Z_a). \qquad (1)$$

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(The proof of the theorem follows directly from the properties of conditional probabilities.) ∎

See (17) and (18) for background and related results. Note that the theorem states that the posterior probability distribution of the data partitionings depends directly on the posterior distribution of the data partitionings given the attributes. The first factor in eq. (1) corresponds to the Kalman filter innovations (under the standard assumptions of Gauss-Markov linear target state relations and measurements), while the second factor may be obtained in several different ways, according to the model assumed for the dispersion and occurence of false alarms. The divisor is a function of Z only and not of Q. Hence, this term plays no role when optimum Q is sought, i.e. that value of Q - subject to feasible search constraints - which maximizes the posterior probability $p(Q|Z)$. Finally, it should be noted that the above theorem is concerned solely with probabilities, not possibilities. However, the remainder of this paper will be devoted to showing how possibilities (and fuzzy set theory) can be used to obtain the desired probabilities.

The main goal of this paper can now be stated: To show that there exists a feasible and mathematically justifiable procedure for obtaining the posterior possibility distribution of Q given Z_a, and in turn using this expression to generate a naturally corresponding evaluation for $p(Q|Z_a)$, which from eq. (1) may be used in determining the overall value of $p(Q|Z)$.

BASIC MODEL

The procedure for obtaining the posterior possibility distribution of data partitionings given the relevant attribute information, can be conveniently divided into eight steps.

I. A taxonomy of attribute relations is developed using heuristic or statistical procedures. (See for example, the approach of Novakowska (22). The goal here is the definition of a set of relatively independent critical attributes for the correlation problem at hand. For example, this list could include A_1= degree of observed maneuvering, A_2= signal frequency characteristics (this could be further subdived), A_3= bearing information, A_4= classification (although this attribute may depend to a large degree on more primary ones), A_5= number of sensor systems on-board, A_6= target identification number, A_7= visual characteristics (again, subdivision into more specific categories is more meaningful). Call the final set of primary attributes $\{A_1, A_2, \ldots, A_M\}$.

II. The domain of possible values or confusables that each attribute can assume is determined. In order to accomplish this and related tasks, a panel of experts must be available for querying. The objective in this phase is the modeling of the posterior attribute possibility distributions given typically by

$$\phi_{(A_j|Z_j)} : D_j \rightarrow [0,1] \quad , \qquad (2)$$

$j = 1,\ldots,M$, where each A_j is an attribute, $Z_j \in D_j$ is arbitrary representing any potential observed (data) value of A_j , D_j being the domain or set of all possible values of A_j, usually measured in some convenient dimensional units. (See figure 3.)

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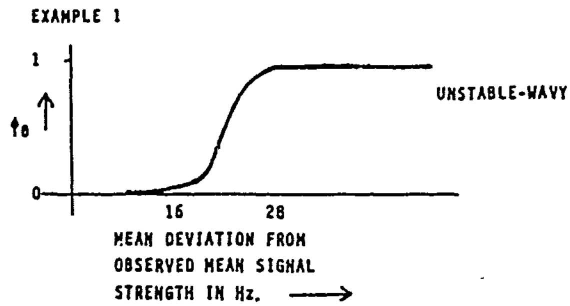

(U)Example 1 of a Posterior Possibility Distribution for an Attribute
Figure 3

III. Consider now the set of experts $y_1, y_2, \ldots$ Each expert y_k is asked the following question:

"What is the possibility that given you have observed data Z_j as a value for attribute A_j, that, say, V_j is the actual (or equivalently, a confusable) value for attribute A_j?"

Each value V_j is then moved around freely in domain D_j, and in turn, the Z_j 's are moved about in D_j, for each j, j=1,2,..,M. As a check, when $V_j = Z_j$, the response possibility should be unity or nearly so. Thus, symbolically $\phi_{(A_j|Z_j,y_k)}(V_j)$ represents the possibility that V_j has attribute A_j, given Z_j is observed by expert y_k. Bayes' theorem (see phase VI) in a fuzzy context could have been used here, but would entail prior distributional assumptions as well as additional calculations. Instead, the posterior possibility distributions are obtained directly. (See also figure 4.)

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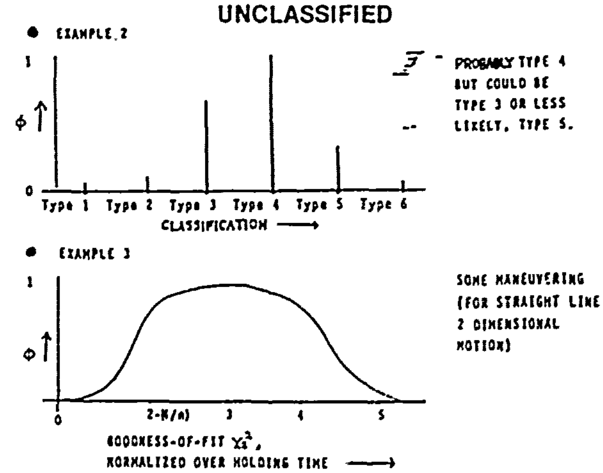

(U)Examples 2 and 3 of Posterior Posibility Distributions for Attributes
Figure 4

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IV. A simple calculation shows that if s_j is the number of elements in domain D_j, then $(s_j)^2$ calculations are required in step III for each corresponding posterior possibility distribution. One way of avoiding all of these calculations is to use - where appropriate, of course, such as for attributes A_1 and A_3 in the example in step I - a translation parameter family of functions. Thus, some of the possibility distributions obtained in III may be put in the form

$$\phi_{(A_j \mid z_j, y_k)}(v_j) = \phi_{(A_j \mid y_k)}(v_j - z_j). \quad (3)$$

V. The next step in the procedure is the determination of the set of relevant rules used in the decision procedure by the experts when correlation is to be carried out based on the available information. More specifically, the rules considered here are of the form

"If attributes A_{j_1} and A_{j_2} and.. or A_{i_1} or A_{i_2} or.. and ... or ... etc. are present at actual values z_{j_1}, $z_{j_2}, \ldots, z_{i_1}, z_{i_2}, \ldots$, respectively, then correlations or equivalently data partitionings $Q_1, Q_2, \ldots$ are possible with Q_1 being most likely, Q_2 less likely, etc."

In terms of attributes or fuzzy sets the above expression may be stated as

$$E_i = ((\underset{k \in R_i''}{\&}(\underset{j \in R_{ik}'}{\mathrm{or}}(A_j))) \Longrightarrow B_i), \quad (4)$$

where the arrow indicates fuzzy set implication. Reducing the implication to more primative fuzzy set operations and employing fuzzy set membership function notation, one obtains

$$\phi_{E_i}(\bar{V}, Q_+) = \psi_{\mathrm{or}}(1 - \underset{k \in R_i}{\psi_\&}(\underset{j \in R_{ik}}{\psi_{\mathrm{or}}}(\phi_{A_j}(v_j))), \phi_{B_i}(Q_+)) \quad (5)$$

where $\bar{V}$ is a large vector consisting of all the V_j's and B_i is some attribute delineating the possibility distribution of the correlations.

Many of these implication (or modus ponens) rules, before being put into the forms given in eq.(4) or above, are typically obtained from the panel of experts in an observed data-decision operational framework, as the following examples show:

1. If targets A and B are such that their (observed) signal characteristics match reasonably well (this can be made more specific), then they probably correlate.
2. If targets A and B are such that when updated, their regions of uncertainty reasonably overlap (again, this can be more specifically quantified), then they are candidates for correlating.
3. If targets A and B match on certain characteristics but not on others, then correlating may or may not occur. (In practise, this rule would be replaced by a number of rules containing various combinations of matching attributes and various conclusions as to the possible correlation levels.)
4. If targets A and B are such that their positions match up to some gating level C and their visual forms appear to match up to some gating level D (as for example by comparing their lengths, shapes, markings, etc.), then they most likely correlate.

Other rules may take into account geographical barriers or physical constraints.

Care must be taken here in the modeling of each implication (or modus ponens) rule E_i that the implication is based on an ideal situation, i.e., no error is assumed for the observation of the attribute values; errors are accounted for separately (and then combined optimally with the rules in step VIII). Although the model can handle match-no match situations in a non-trivial manner, analagous to the testing of hypotheses in a probabilistic-statistical situation, more flexibility is achieved when the match-no match situations are replaced by fuzzy gates (analagous to the introduction of randomness for the parameters involved in the testing of hypotheses analogy).

A brief comment on the operators $\psi_\&$ and ψ_{or} is appropriate here. These fuzzy set operators represent 'and' and 'or', respectively. (Incidently, 'not' is represented by the operation 1-(·), throughout, but will not be explicitly used.) In the initial period of fuzzy set theory (circa 1965-1975), these operations were usually interpreted as min and max, respectively. However, more recently, it has been shown from both an empirical viewpoint and theoretical considerations that these operators should have a more flexible interpretation. One class of such interpretations (theoretically justified) consists of the triangular or t-norms and conorms. Thus the functions prod and probsum as well as many others may well be used in evaluating these operations. (See Zimmermann (23) for empirical studies and Klement (24) and Goodman (11) for theoretical work in this area.) Throughout this paper we do not specifically evaluate the 'and' and 'or' operations. Work is currently being carried out to determine which evaluations are most appropriate for the ocean surveillance correlation problem and a future paper by the author will discuss this. (See RESEARCH ISSUES.)

Analagous to the situation in step III, each rule is actually obtained from the panel of experts and thus may vary somewhat from individual to individual. Thus, initially, equation (5) should be modified to reflect y_k as is the case in equation (3).

VI. In steps III and V, the possibility distributions for the attributes and the rules depend on each expert's interpretation. A procedure is required which will average out these variations, yet retain all of the information. Two options are available. The first employs a probabilistic approach. Each expert's response to a particular attribute or rule is weighted and the corresponding possibility distributions are summed pointwise. The convergence of this expression to the 'true' value can be justified by appealing to the Law of Large Numbers. (See Goodman (25), Theorem 2.2) for a more thorough discussion and results.) The second option in combining the possibility distributions is to use a fuzzy set-multiple valued truth theory approach. In this approach, each response by an expert is considered a conditional one and corresponds to a conditional fuzzy set (hence, the notation in eq. (3), for example) relative to the interpretation of the operators $\psi_\&$ and ψ_{or}. By developing a general theory for such conditional fuzzy sets, and, in turn, deriving an extended fuzzy set Bayes' theorem, a fuzzy set form of the Law of Large Numbers and Central Limit Theorem can be otained. (Again, see (25), Theorems 2.1 and 2.2) as

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well as (11), section 3). In particular, non-trivial results may be derived for the interpretation $\psi_{\&}$, ψ_{or}) = (prod,probsum), but ironically, not for (min,max)!

As a consequence of the above discussion, from now on, the individual expert variation response will be omitted.

VII. The heart of the procedure utilizing attribute information is based on a deductive logic theorem first exhibited in a more narrow form in (3), and then extended to a very general setting in (25), Theorem 2.3 .

<u>Theorem</u> Uniformly most accurate confidence sets.

Let $C = \{C_j \mid j=1,\ldots,m\}$ be any collection of fuzzy subsets of some fixed base space X. Thus, $\phi_{C_i} : X \to [0,1]$, $j=1,2,\ldots,m$, are the corresponding possibility distribution functions. Let L_m = the set of all 1 by m row vectors $a = (a_1,\ldots,a_m)$, where each a_i is such that $0 \le a_i \le 1$. Let g be any function where $g: L_m \to [0,1]$ is such that g is non-decreasing, i.e., if $a' = (a'_1,\ldots,a'_m) \le a'' = (a''_1,\ldots,a''_m)$ that is, $a'_i \le a''_i$, for all i), then $g(a') \le g(a'')$

Define the fuzzy subset of X, G(C,g) by the possibility distribution function $\phi_{G(C,g)} \to [0,1]$, where for any $W \in X$,

$$\phi_{G(C,g)}(W) = g((\phi_{C_1}(W),\ldots,\phi_{C_m}(W))). \tag{6}$$

Next, for any $a \in L_m$, define the ordinary subset of X

$$H(C,a) = \{W \mid \phi_{C_j}(W) \ge a_j,\ W \in X,\ \text{for all } j\}. \tag{7}$$

and for any fuzzy subset A of X and real number u, $0 \le u \le 1$, define the ordinary subset of X

$$K(A,u) = \{W \mid \phi_A(W) \ge u,\ W \in X\} . \tag{8}$$

Then:

(i) $H(C,a) \subseteq K(G(C,g),g(a))$, for all C,a as above.

(ii) If A is any fuzzy subset of X such that $H(C,a) \subseteq K(A,g(a))$, for all a as above, then necessarily $K(G(C,g),g(a)) \subseteq K(A,g(a))$, for all C,a as before.

(For a proof, see (25).) ∎

Thus, K(G(C,g),g(a)) can be considered to be the '<u>tightest</u>' (ordinary) g(a)-<u>level confidence subset</u> of X containing <u>hypotheses</u> set H(C,a), simultaneously for all $a \in L_m$. Examples of function g in the theorem include: any weighted average, prod, min, max, and many other functions, including all t-norms. Indeed, it can be shown that if g is chosen to be the t-norm used for the interpretation of the 'and' operator $\psi_{\&}$, then the confidence set K(G(C,g),g(a)) also represents the fuzzy set interpretation of the <u>entire</u> hypotheses set H(C,a). On the other hand, use of the weighted average leads to more stable values for the confidence g(a) than, for example, use of the t-norm min (which can be lowered drastically by merely one small confidence value).

<u>Corollary</u>

With the same assumptions in the above theorem, all results carry over with the obvious modifications when G(C,g) is replaced by any proj(G(C,g)), that is any (fuzzy set) projection into any subspace of X.

(Proof: Follows easily from the theorem, again, see (25)) ∎

VIII. The theorem in step VII is applied to the situation at hand: Specific rules are selected as appropriate and attribute data is observed. Thus

(i) Z_j is observed with some confidence α_j , where any confusable attribute value V_j for A_j (in domain D_j) satisfies the confidence relation

$$\phi_{(A_j|Z_j)}(V_j) \ge \alpha_j \quad , j=1,\ldots,M. \tag{9}$$

using the modeling from step III.

(ii) Rule E_i is selected, for say, $i=1,\ldots,N$. Each such rule is assigned some confidence level so that the arguments $\hat{V}$ and Q_+ jointly satisfy the relation

$$\phi_{E_i}(\hat{V},Q_+) \ge \beta_i \quad , i=1,\ldots,N. \tag{10}$$

(iii) If any rules or attribute data is initially described in terms of <u>random</u> confidence sets - which are usually in a conditional parameter form-a procedure exists for first replacing these confidence sets by those not in conditional form and then converting to possibilistic forms. (See (13), Appendix A and eq.(2.1))

(iv) In the theorem, replace each C_j by an $(A_j|Z_j)$ or E_i, whichever are appropriate, and similarly replace each a_j by either α_j or β_i .

(v) Choose for g some convenient function such as an averaging one, min, or prod. The appropriate choice for g can be determined, for example, by the choice of t-norm for the 'and' operation. Although weighted averaging does <u>not</u> yield a t-norm, it is also a natural way to obtain a <u>single</u> figure-of-merit confidence level from the given ones (i.e., the α_j's and β_i's), from a statistical view point. (See the remarks following the theorem in step VII.)

(vi) Use the corollary following the theorem in step VII, where the projection operation is into the space $\mathcal{Q}$ of all possible correlations (or data partitionings) Q_+ , eliminating finally the 'nuisance' vector domain correspond-to all possible values for the $\hat{V}$.

Specifically, substeps (i)-(v) above lead to the fuzzy set G(C,g) which is described by possibility distribution $\phi_{G(C,g)}$ described in eq. (6), where argument $W=(\hat{V},Q_+)$. Then $proj_{\mathcal{Q}}(G(C,g))$ here is determined by the relation

$$\phi_{proj_{\mathcal{Q}}(G(C,g))}(Q_+) = \underset{V \in \mathcal{V}}{\phi_{or}} (\phi_{G(C,g)}(\hat{V},Q_+)). \tag{11}$$

where $\mathcal{V}$ is the set of all possible V's, i.e., $\mathcal{V} = \overset{M}{\underset{j=1}{\times}} (D_j)$, and ϕ_{or} is some appropriately chosen t-conorm for the fuzzy set system. (See the comments near the end of step V.)

The final result of the above eight steps is (via the corollary of step VII) that $K(proj_{\mathcal{Q}}(G(C,g)),g(a))$ is the tightest or uniformly most accurate g(a)-level confidence subset of $\mathcal{Q}$ containing the hypotheses set projection into $\mathcal{Q}$, $proj_{\mathcal{Q}}(H(C,a))$. Thus, in an optimal

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mathematical way, this confidence set describes the possible values for Q_+, given all of the relevant attribute information and rules. (The following example, gives a simple geometric illustration of the above.)

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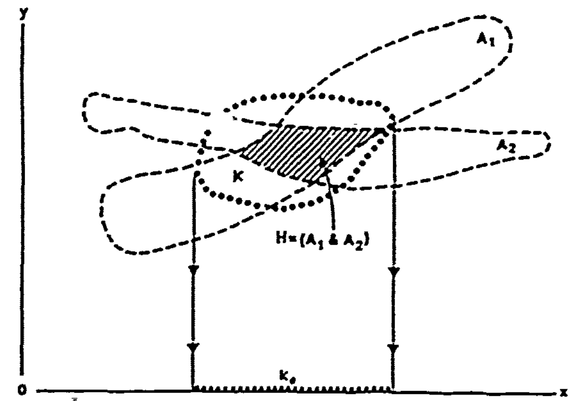

Conclusion K, with boundary ⁙, contains as a proper subset, premise (A_1 & A_2). Indicated by ////. Projection K_o = $PROJ_1$ (K) is indicated by ......

(U)Geometric Interpretation of Conjunctive Premises
Figure 5

Clearly, this procedure can be expanded directly to include all information concerning correlation; in particular, geolocational information can be easily integrated into the scheme. However, as stated at the outset of this paper, since geolocational and false alarm information has been treated historically from a different viewpoint, namely, a probabilistic approach, which also has many justifications for use, an all-encompassing fuzzy set approach to the contact correlation will not be persued here. Future work, however, may lead at least partially in this direction.

DETERMINATION OF POSTERIOR POSSIBILITY AND PROBABILITY DISTRIBUTIONS

With the determination of the optimal confidence set for Q_+, given all attribute data and rules, the posterior possibility distribution function for Q_+ can then be determined. Clearly, by inspection of the form of the optimal confidence set, it follows that the desired distribution function is given by the truncated form

$$\phi_p(Q_+) = \begin{cases} \phi_{proj_2(G(C,g))}(Q_+), & \text{if } Q_+ \in K_o \\ 0 & , \text{if } Q_+ \notin K_o \,, \end{cases} \quad (12)$$

where $K_o \equiv K(proj_2(G(C,g)),g(a))$, noting that

$$Q_+ \in K_o \text{ iff } \phi_{proj_2(G(C,g))}(Q_+) \geq g(a).$$

Determination of the posterior probability distribution function corresponding to ϕ_p can be accomplished in two different ways.

The first procedure is an immediate consequence of eq. (12): Either identify directly or renormalize the possibility distribution ϕ_p.

The second procedure requires additional computations, but is based on more rigorous grounds.

Recall that $\mathfrak{Q}$ is the class of all possible correlations Q_+. Let $\mathcal{F}(\mathfrak{Q})$ denote the class of all fuzzy subsets A of $\mathfrak{Q}$ with corresponding possibility distribution function $\phi_A : \mathfrak{Q} \to [0,1]$. Note that $A_o \equiv proj_2(G(C,g)) \in \mathcal{F}(\mathfrak{Q})$. Let $\mathcal{R}(\mathfrak{Q})$ denote the class of all random subsets of $\mathfrak{Q}$. (See (13) for background.) Then it is known ((10),(11)) that a number of such mappings S exist, called choice functions, such that

$$S: \ \mathcal{F}(\mathfrak{Q}) \to \mathcal{R}(\mathfrak{Q}) \text{ (onto)} . \quad (13)$$

where, for any $A \in \mathcal{F}(\mathfrak{Q})$, $S(A) \in \mathcal{R}(\mathfrak{Q})$ is such that, for all $Q_+ \in \mathfrak{Q}$,

$$p(Q_+ \in S(a)) = \phi_A(Q_+), \quad (14)$$

and such that certain homomorphic fuzzy set and random set operator relations hold for S and a related family of mappings. (See (10) and (11).) Each S generates the equivalence class of random sets mentioned briefly in the Introduction.

Two of the most important choice functions relating fuzzy set theory and probability theory as outlined above are:

(i) $S = S_U$, where U is a random variable distributed uniformly over [0,1], and for any $A \in \mathcal{F}(\mathfrak{Q})$,

$$S_U(A) = \phi_A^{-1}([U,1]) = \{Q_+ \mid \phi_A(Q_+) \geq U\} \quad (15)$$

and

(ii) $S = T$, where, for any $A \in \mathcal{F}(\mathfrak{Q})$, T(A) is determined by considering the corresponding (ordinary random) membership function $\phi_{T(A)}$, where each of the latter's one dimensional zero-one marginal random variables $\phi_{T(A)}(Q_+)$, $Q_+ \in \mathfrak{Q}$, are all mutually statistically independent with

$$\begin{cases} p(\phi_{T(A)}(Q_+) = 1) = p(Q_+ \in T(A)) = \phi_A(Q_+) \\ p(\phi_{T(A)}(Q_+) = 0) = p(Q_+ \notin T(A)) = 1-\phi_A(Q_+) \,. \end{cases} \quad (16)$$

Suppose now that, without loss of generality,

$K_o = \{Q_+^{(1)},\ldots,Q_+^{(m)}\}$, where

$$0<g(a)\leq\phi_{A_o}(Q_+^{(1)})<\phi_{A_o}(Q_+^{(2)})<\ldots<\phi_{A_o}(Q_+^{(m)})\leq 1 \,. \quad (17)$$

(The following results can be modified if equality occurs in places in eq. (17).) If S is any choice function, there is a naturally corresponding random variable $\mathcal{U}(S(A_o))$ over $\mathfrak{Q}$, representing the maximal possible element of random set $S(A_o)$ with respect to K_o given by

$\mathcal{U}(S(A_o)) \to$ that unique value of Q_+, i.e., some $Q_+^{(j)}$, for $1\leq j\leq m$ such that $\max\limits_{(Q_+ \in (S(A_o)\cap K_o))}(\phi_{A_o}(Q_+))$,

occurs for $Q_+^{(j)}$, provided that $S(A_o) \cap K_o \neq \emptyset$. (18)

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and

$$\mathcal{U}(S(A_o)) = \emptyset \text{ ; when } S(A_o) \cap K_o = \emptyset \quad . \quad (19)$$

It follows that the probability function for $\mathcal{U}(S(A_o))$ is given by, for any j, $1 \leq j \leq m$,

$$p(\mathcal{U}(S(A_o)) = Q_+^{(j)}) = \sum_{\alpha \in \mathcal{G}} p(S(A_o) = \alpha) \ , \quad (20)$$

$$\mathcal{G} = \{\alpha \mid \alpha \subseteq K_o \text{ and } \mathcal{U}(\alpha) = Q_+^{(j)}\} \ .$$

Thus, the following identification may be made:

$$p(Q_+ \mid Z_a) = p(\mathcal{U}(S(A_o)) = Q_+) \quad . \quad (21)$$

In particular, specialization of the above results to cases (i) and (ii) for choice function S yields:

(iii) For $S = S_U$,

$$p(\mathcal{U}(S(A_o)) = Q_+^{(m)}) = \phi_{A_o}(Q_+^{(m)}) \ , \quad (22)$$

$$p(\mathcal{U}(S(A_o)) = Q_+^{(j)}) = 0 \ , \ 1 \leq j \leq m-1 \ , \quad (23)$$

$$p(\mathcal{U}(S(A_o)) = \emptyset) = 1-\phi_{A_o}(Q_+^{(m)}) \ . \quad (24)$$

(iv) For $S = T$,

$$p(\mathcal{U}(S(A_o)) = Q_+^{(j)}) = \phi_{A_o}(Q_+^{(j)}) \cdot \prod_{i=j+1}^{m} (1-\phi_{A_o}(Q_+^{(i)})) \ , \quad (25)$$

for $0 \leq j \leq m$, where if $j = m$, the product term in eq.(25) is defined to be unity, and if $j = 0$, $Q_+^{(0)}$ is defined to be equal to the null set $\emptyset$.

Clearly, use of T leads to a more tractable result than the one point mass result due to S_U. However, for either choice function, the resulting evaluation for maximal $p(Q_+ \mid Z_a)$ coincide, the desired value occurring at $Q_+ = Q_+^{(m)}$, the most possible value of Q_+ .

Finally, tabulate the possibility distribution $\phi_P(Q_-)$ versus Q_+ (by first tabulating $\phi_A(Q_+)$ versus Q_+) for all feasible $Q_+ \in \mathcal{Q}$, and substitute these values into eqs.(22)-(25), and using eq.(21), finally into eq.(1).

RESEARCH ISSUES

Throughout this paper, it has been emphasized that for any reasonable determination of operator pair $(\phi_\&, \phi_{or})$, the entire procedure remains valid, conditional upon the interpretation of these operators, such as (min,max) or (prod,probsum). Yet in order to implement the scheme, a specific evaluation is obviously needed for the 'and' and 'or' operators.

It has been shown in (11), and mentioned briefly in step 7 of the basic modeling, that a reasonable family of fuzzy set operators (justified by, e.g., relations with multiple-valued logic and set theory-see (24)) to consider is that of pairs of t-norms and t-conorms (usually restricted to have DeMorgan's property- see (11)). (See (12) and (24) for background and further properties.) Furthermore, it has been shown in (11) that a particular subfamily of such operators yields especially close (homomorphic) relations between all fuzzy set systems determined by these operations and corresponding random set systems. This subfamily of pairs of operations (for 'and' and 'or') consists essentially of (min, max).(prod, probsum), and all countably infinite or finite convex weighted sums of the above operations restricted to disjoint regions (though somewhat restricted in form), which include the first two pairs as special cases. Currently, work is going on in determining empirically what values the above-mentioned weights should be assigned. (This is somewhat analagous to the work of Zimmermann (23), who used a different family of operators and empirically estimated certain adjustable parameters in the family.)

Alternatively, another family of operators, possessing some (but not all) of the desirable properties of the first-mentioned family, has been shown to yield very desirable fuzzy set analogues of the Laws of Large Numbers and Central Limit Theorem. (This family is denoted as the Archimedian Frankian family and is discussed in (11).) On the other hand, (min, max) does not yield desirable asymptotic system properties (although (prod, probsum) does, as an internal member of the Archimedian Frankian family), nor apparently do any of the weighted sums previously referred to. A paper on this topic will be forthcoming.

Trade-offs need be established between utilities of choice of the various plausible operator pairs for $(\phi_\&, \phi_{or})$, before determining a final candidate pair.

SUMMARY

An approach to the utilization of attribute information for the contact correlation problem has been outlined in this paper. The novelty of the technique lies in the use of possibility theory in the modeling.

The required steps in developing this technique were:

(I) Establishment of a set of relatively primative attributes.

(II) Determination of attribute domains

(III) Querying of a panel of experts to establish posterior possibility distributions directly, in place of a Bayesian approach, for the attribute values.

(IV) Reducing the calculation load by use of certain analytic models such as those of the translation type.

(V) Determination of modus ponens rules which delineate the possible correlations, by again questioning the available experienced personnel.

(VI) Smoothing out of the variability in the models obtained in steps (III) and (V) due to individual responses. Either probabilistic or analagous fuzzy set asymptotic results are deployed.

(VII) Demonstration of a general theorem which shows that given a set of hypotheses H formed by the conjunction of individual confidence sets (described by possibility, or, in effect, probability distributions), a uniformly most accurate confidence set K exists described by a single possibility distribution, which contains H. The latter possibility distribution was shown to be constructed by a simple application of any one of a large class of (non-decreasing) functions which also determine the confidence level for K. Modification of this theorem for projection operations was displayed in the form of a corollary.

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(VIII) Application of the corollary given in step (VII) to the attribute problem, assumed to be in the form of a conjunction of attribute data and available modus ponens rules (modeled in step (V)).

(IX) Derivation of posterior possibility distribution for the correlations given the attribute information. In turn, the posterior probability distribution function for the correlations was also obtained by two different approaches, the first being relatively simple, the second, requiring more computations, but derivable from a sounder basis (using relations between fuzzy and random sets).

(X) Substitution of the results from step (IX) into the basic factor model for the overall posterior probability distribution of the correlations.

Implementation of the technique demands a specific choice for the 'and' and 'or' fuzzy set operations used throughout. Some discussion was presented concerning this problem.

The same procedure developed in this paper could also be used in a wide variety of problems involving the estimation of an unknown parameter when some of the available information is given in linguistic form. (See some earlier related work in (3), especially section 3.)

Future work will be greatly concerned with real-world implementations and modifications of the technique presented here. The reported successful implementation of a number of fuzzy set-logical approaches to non-military problem areas such as in medical diagnosis (26), library search systems (27), and fault analysis (28), may well serve as an impetus for the treatment of military problems by such techniques, as presented in this paper.

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