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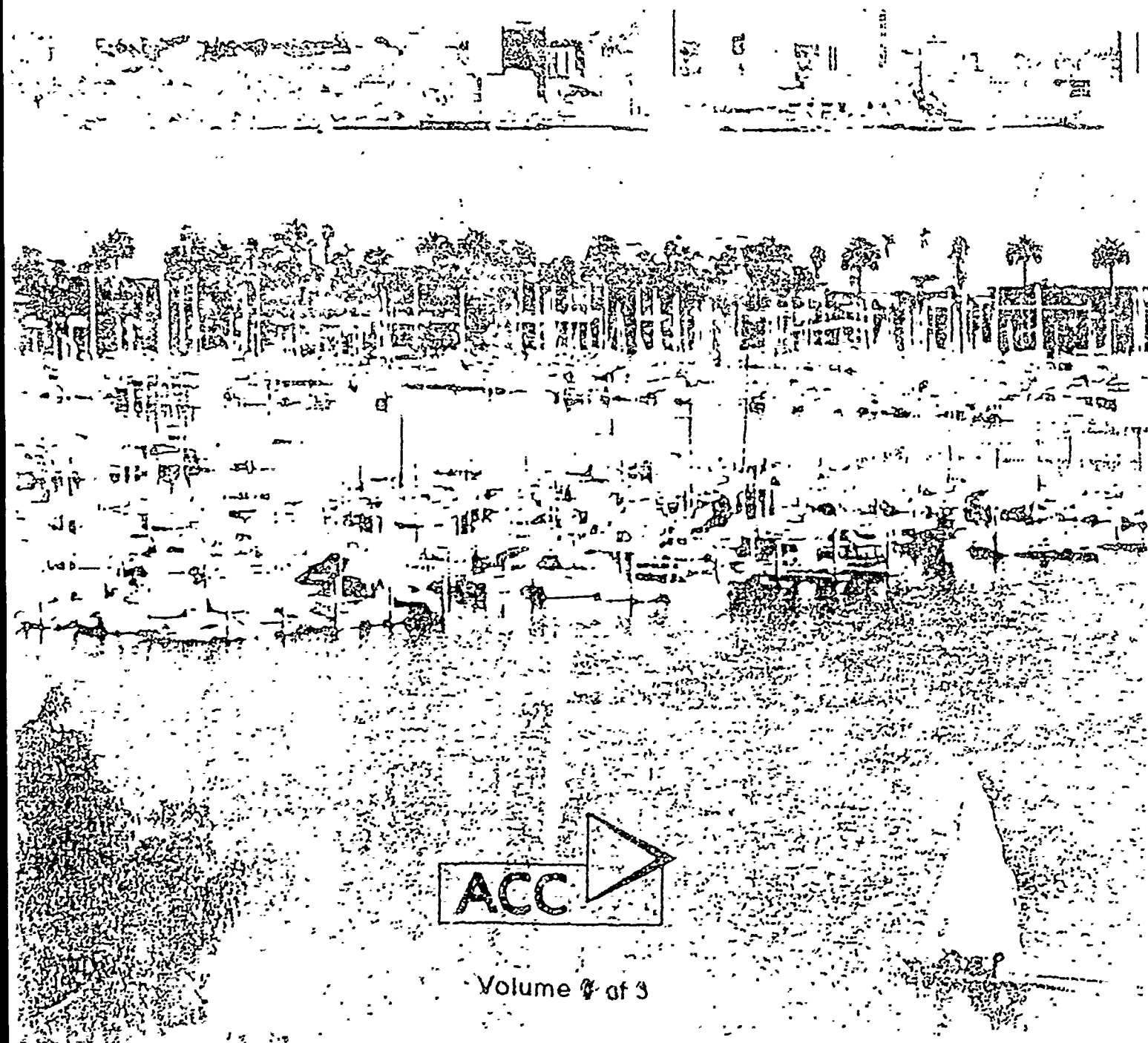
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AN APPROACH TO THE TARGET DATA ASSOCIATION PROBLEM USING
SUBJECTIVE AND STATISTICAL INFORMATION

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Abstract

A general approach is presented to the data association problem in tracking which incorporates both linguistic-based and statistical information, extending classical results. The technique continues the previous effort of the PACT (Possibilistic Approach to Correlation and Tracking) algorithm. In addition, asymptotic consistency results are obtained relative to increasing sample sizes and model refinements.

corresponding uniquely to attribute or fuzzy subset A of space X having domain $\text{dom}(A) = \text{support}(\phi_A) \subseteq X$; for various A and X . Such functions include ordinary probability functions when X is discrete, ordinary cdf's when $X \subseteq \mathbb{R}^n$, unimodal normable continuous or not (normable=having-unit value somewhere), etc. (See, e.g. [8] for various basic properties of fuzzy sets and possibility functions as well as many applications.)

The collection S of all strings produced by F consists of all well-defined combinations of operators from F evaluated at arbitrary possibility functions chosen from U and possibly further evaluated at functional arguments. A typical string is denoted $\text{comb}(\dots \psi_n \dots; \dots \psi_g \dots; \dots \psi_0 \dots; \dots \phi_A \dots)(\dots, x, \dots)$ (3)

The following principles involving linguistic/semantic evaluations are assumed:

Principle 1 - Parsing

All statements in natural language may be expressed in a formal language in terms of memberships or degrees of compatibility of elements w.r.t. attributes and the use of operators forming formal strings representing (possible different) not, &, or. Symbolically, a typical string here is $s = \text{comb}(\dots \text{not} \dots; \dots \& \dots; \dots \text{or} \dots; \dots (x \in A) \dots)$ (4)
 $= (\dots, x, \dots) \in (\text{comb}(\dots \text{not} \dots; \dots \& \dots; \dots \text{or} \dots)(\dots A \dots))$

Principle 2 - Abstraction

The truth or semantic content of any string (or proposition) as in eq.(4) is $\text{tr}(s)$, as in eq.(3). If the system is well-defined, then (by definition) $\text{tr}(s) = \text{comb}(\dots \psi_n \dots; \dots \psi_g \dots; \dots \psi_0 \dots)(\dots \phi_A(x), \dots)$ (5)

Other concepts associated with natural language may be described and evaluated, including: conditioning, quantification, interaction/independence as well as dependence, projections and marginal attributes; tense, mode, case and prepositional and predicative relations; and functional and relational transforms, including arithmetic operations and linguistic modifications. (See for example, [4] or [9].) Examples of the above will be given later in association with the PACT algorithm essentially as error distribution and inference rules.

One example of a hybrid system is simple probability logic where $F = (\text{neg}; \psi_g; \psi_0; U_{\text{cdf}})$, where $\text{neg}(x) = 1-x$; ψ_g is a fixed n (argument) copula, ψ_0 is the DeMorgan transform of ψ_g and thus a co-copula, and U_{cdf} is the collection of all cdf's over \mathbb{R} . Then by Sklar's Theorem [7] for any F_1, \dots, F_n $\psi_g(F_1, \dots, F_n)$ (in compositional form) is a simple string representing a legitimate cdf. By replacing F_j 's by 1, marginal cdf's or cdf's of number less

1. Introduction

The PACT algorithm has been documented in previous papers, [1],[2],[3]. It is an attempt to perform track-to-track or report-to-track data association or "correlation" based upon varying information sources. The procedure in an obvious way may be extended to problems of medical diagnosis, fault determination, and classification [1]. The entire technique is based upon the idea that attributes may be represented by fuzzy sets and that such objects represent a weakening of probabilistic descriptions and hence suitable for the interpretation of natural language information ([3]; [+ for other work]).

2. Hybrid Logical Systems

A hybrid logical system consists of the tuple $F = (O_{\text{not}}, O_{\&}, O_{\text{or}}; U)$ where O_{not} is a collection of negation operators $\psi_n: [0,1] \rightarrow [0,1]$ nonincreasing with $\psi_n(0) = 1, \psi_n(1) = 0$. Other properties that ψ_n can possess include involution and continuity [5]. $O_{\&}$ is a collection of conjunction or general cartesian product operators $\psi_g: [0,1]^2 \rightarrow [0,1]$ nondecreasing, continuous, bounded above by min, and such that for all u, v : $\psi_g(0, v) = \psi_g(u, 0) = 0$; $\psi_g(1, v) = v = \psi_g(v, 1)$, (1) as well as higher argument operators with similar properties. When ψ_g as a two place operator is symmetric and associative, then it is called a t-norm, and possesses many useful properties, including an unambiguous recursive extension to multiple arguments and canonical representations [5],[6]. When n -argument ψ_g is the cumulative distribution function (c.d.f.) of uniform $[0,1]$ random variables (r.v.) (not necessarily independent, nor ψ_g need be symmetric or associative), it is called a copula [6]. Copulas and t-norms may coincide or not. (See [7].) A similar situation holds dually for O_{or} , a collection of disjunction or general cartesian sum operators $\psi_0: [0,1]^m \rightarrow [0,1]$, for various m , nondecreasing, continuous, bounded below by max, and e.g., for $m=2$ all u, v : $\psi_0(1, v) = \psi_0(u, 1) = 1$; $\psi_0(0, v) = v = \psi_0(v, 0)$. (2) t-conorms and co-copulas are also defined analogously [5],[6]. Finally, U is a collection of fuzzy set membership or possibility functions $\phi_A: X \rightarrow [0,1]$.

than π can be formed. More generally, non-cdf string

$$\text{comb}(\text{neg}; \psi_g, \psi_0)(F_1, \dots, F_m)(x_1, \dots, x_m) \quad (6)$$

$$= \psi_g(F_1, \dots, F_m)(\text{comb}(c, X, t)((-\infty, x_1], \dots, (-\infty, x_m])),$$

which may be extended uniquely above the ray level to arbitrary Borel sets, but for the latter situation, F is no longer truth functional in general (a characteristic of jointness of events in probability- see, e.g. Rescher [10]).

More generally, a Lebesgue-Stieltjes logic may be developed, where cdf's are replaced by functions of bounded variation and probability measures by signed measures at the Borel set level [9]. Of course, in the probability logic example, the single ψ_g could be replaced by a collection of copulas and related functions (noting that in general copulas composed with multiple copulas are not copulas, although they do remain conjunction operators).

A single natural language concept may lead to a variety of possible semantic evaluations. One contributing factor to this ambiguity lies in the nonuniqueness of extending an ordinary function $f: X_1 \times X_2 \rightarrow Y$ to $\tilde{f}: P(X_1) \times P(X_2) \rightarrow P(Y)$, the lifting of f ,

where $F(X)$ denotes an appropriate class of attributes of X (or all attributes on X). Two candidates for \tilde{f} are, using Principles 1 and 2 and the unique representation of attributes through their membership functions for a simple logical system:

$$\phi_{\tilde{f}(A,B)}(y) = \text{tr}(y \circ f(A,B)) = \text{tr}(\exists(x,z) \in A \times B \& y = f(x,z)) \\ = \psi_0(\psi_g(\phi_A(x), \phi_B(z))) \quad (7) \\ (\text{all } (x,z) \in f^{-1}(y))$$

and

$$\phi_{\tilde{f}(A,B)}(y) = \text{tr}(f^{-1}(y) \circ A \times B) = \psi_g(\phi_A, \phi_B)(f^{-1}(y)), \quad (8)$$

where in the first case, $A \in F(X_1), B \in F(X_2)$, and in the second case, $A \in F(P(X_1)), B \in F(P(X_2))$; related to the measure theory concept of transformation of measure.

In any case, once the functional extension problem is determined, other ambiguities arise: Let \mathcal{BP} be the class of all multiple and single argument Boolean truth polynomials, i.e.,

$$\mathcal{BP} = \bigcup_{n=1}^{\infty} \{(0,1)^n\}. \quad (9)$$

Thus each function f in \mathcal{BP} may be extended to $\tilde{f}: F[0,1] \times \dots \times F[0,1] \rightarrow P[0,1]$. Call this class \mathcal{BP} . Now, $f = \text{comb}(\psi_n; \psi_g; \psi_0)$ restricted to $\{(0,1)^n\}$ for suitable

choices of n yields a classical logical connective, and in turn the extension \tilde{f} thus provides an alternative natural evaluation to eq.(5)

$$\text{tr}(s) = (\phi_{\tilde{f}}((A,CA) \dots)^{(0)}, \phi_{\tilde{f}}((A,CA) \dots)^{(1)}). \quad (10)$$

In general, distinct in form from that in eq.(5).

Another problem is the choice of the operators for F to model a given situation. Restricting these operators to be, e.g., t-norms or copulas may be confining, despite the appeal. This is closely related to Hilbert's 13th problem, and more generally, to the problem of representing multiple argument functions by superpositions of lower argument functions [11], when associative copulas or t-norms are employed, because of their canonical representations. On the positive side, it can be shown that Sprecher's refinement of Kolmogorov's original form for an arbitrary continuous function of multiple arguments may

be formally couched as the string for constants c_j , $\text{bndsum}_{j=1, \dots, n} (g(\psi_g(\psi_g^{(\lambda)}(\phi_{A_1}(x_1)), \dots, \psi_g^{(\lambda)}(\phi_{A_n}(x_n)), c_j)))$, where bndsum is ordinary sum bounded by n unity, (11) a t-conorm, g is monotone decreasing, and ψ_g is a strict (and hence Archimedean) t-norm having canonical representation

$$\psi_g(x_1, \dots, x_n) = h^{-1}(h(x_1) + \dots + h(x_n)), \quad (12)$$

where $h: [0,1] \rightarrow \mathbb{R}^+$ is continuous decreasing with $h(0) = +\infty$ and $h(1) = 0$. Also, the λ -intensification or modification of A through ψ_g is given by

$$\psi_g^{(\lambda)}(\phi_A(x)) = h^{-1}(\lambda h(\phi_A(x))), \quad (13)$$

noting that these intensifications obey the exponential law

$$(\psi_g^{(\lambda')})^{(\lambda'')} = (\psi_g^{(\lambda'')})^{(\lambda')} = \psi_g^{(\lambda' \lambda'')}, \quad (14)$$

for all nonnegative real λ , extending the integer-valued repetition of operator ψ_g . A reasonable substitution for the above form is the use of ordinary exponentiation or translation of arguments. (See [3])

3. Combination of Evidence

Let F be a given hybrid logical system to be used for the relevant modeling. Let $\theta \in X$ be an unknown parameter vector to be estimated or decided upon. Suppose that $\phi_{A_j}: X \times Y_j \rightarrow [0,1]$, $j=1, \dots, m$, are m possibility functions describing θ as well as introducing nuisance parameters $Z_j \in Y_j$, $j=1, \dots, m$.

Then, if for example a single m -copula ψ_g is chosen for combination, one has the joint non-interactive function

$$\phi_A(\theta, Z) = \psi_g(\phi_{A_j}(\theta_j, Z_j)), \quad (15) \\ (j=1, \dots, m)$$

$$\theta \in \mathcal{D}(\theta_1, \dots, \theta_m); Z \in \mathcal{D}(Z_1, \dots, Z_m). \quad (16)$$

This representation of the joint information and all nuisance values may be interpreted in terms of either ordinary operations upon nested level sets or equivalently upon cartesian products; in particular, on nested random sets. (Indeed, a strong isomorphic-like relation holds between many common set operations on nested (random) sets and corresponding possibility functions, including intersections, unions, set differences, projections, and functional transforms. See [6] and [12].) Thus, for all θ, Z ,

$$\phi_A(\theta, Z) = \text{Pr}(\{(\theta_1, Z_1) \in S_{U_1}(A_1) \& \dots \& (\theta_m, Z_m) \in S_{U_m}(A_m)\}) \quad (17)$$

where $S_{U_j}(A_j) = \phi_{A_j}^{-1}[U_j, 1]$ is the unique nested random

subset of $X \times Y_j$ which is one point coverage equivalent to ϕ_{A_j} , where U_1, \dots, U_m are, marginally, rv's distributed uniformly over $[0,1]$ with joint cdf ψ_g . (See [6], [7] for other properties and results.)

Next, consider the concept of conditional possibility functions: Let X and Y be given spaces with attribute $B \in F(X \times Y)$ dominated by attribute $C \in F(Y)$ i.e.,

$$\sup_{(x,y) \in B} \phi_B(x,y) \leq \phi_C(y); \text{ all } y \in Y. \quad (18)$$

Then it follows that there exists (uniquely, if, eg, ψ_g is a strict t-norm) conditional attributes $(B|_Y)$ satisfying (for given operator ψ_g) for all $x \in X, y \in Y$

$$\phi_B(x,y) = \psi_g(\phi_{(B|_Y)}(x), \phi_C(y)). \quad (19)$$

In particular, let the diagonal possibility descrip-

tion of θ be \dots , assuming here a simple system,

$$\phi_{d(A)}(\theta) = \text{tr}(\theta_1 \dots \theta_m = \epsilon \prod_{1 \leq j \leq m} \text{proj}_X(A_j))$$

$$= \phi_{\theta}(\phi_A(\theta, \dots, \theta; Z)) \quad (20)$$

(all Z)

Let $X = X$ and Y be the space of all possible descriptions (or "possible worlds") of θ . Hence, $d(A) \in Y$ and the universal inclusion description $\mathcal{C} \in F(Y)$ with $\phi_{\mathcal{C}}(d(A)) = \text{tr}(\exists \theta (\theta \in d(A))) = \phi_{\theta}(\phi_{d(A)}(\theta))$. (21)

Since the compound attribute representing θ and $d(A)$ has the same evaluation, it follows that

$$\phi_{d(A)}(\theta) = \phi_{\theta}(\phi_{\theta|d(A)}, \phi_{d(A)}) \quad (22)$$

determines the final combination of evidence function $\phi_{\theta|d(A)}$ as a function of $\theta \in X$.

As a check on this procedure reducing to the classical combination of statistical evidence, if $\phi_{\theta} = \text{prod}$, $\phi_{\text{or}} = \text{bndsum}$, Y_j is vacuous, and $X = R^n$ suitably discretized where each ϕ_{A_j} is approximated by

$$\phi_{A_j}(\theta_j) = \varphi(\theta_j - u_j, \lambda_j) \cdot \Delta(\theta_j) \quad (23)$$

φ being the standard probability density function (p.d.f.) for the gaussian distribution $N(\mu, I)$, then

$$\phi_{\theta|d(A)}(\theta) = \varphi(\theta - \bar{u}, \bar{\lambda}) \cdot \Delta(\theta) \quad (24)$$

$$\bar{u} = \frac{1}{m} \sum_{j=1}^m \lambda_j^{-1} u_j; \quad \bar{\lambda} = \left(\sum_{j=1}^m \lambda_j^{-1} \right)^{-1} \quad (25)$$

the same formally as the BLUE (best linear unbiased estimator) of μ , and by a fiducial argument, of θ .

4. Application to Data Association

Suppose now that attributes A_1, \dots, A_m describing nuisance parameters concerned with θ (but not θ directly) are available in the form of compound attributes represented by error functions and by inference rules. A_1 could represent geolocation, A_2 a sensor system state, A_3 another state, and for $m=4$, A_4 could represent visual descriptions, where θ represents the level of correlation between a pair of track histories, say i and j . Thus, Z is described separately by error functions $\phi_{P_k}(Z_k^{(i)} | Z_k^{(j)})$ for i, j and $k=1, \dots, m$, where $Z_k^{(i)}$ are observed or predicted outcomes of data associated with A_k - in fact, $Z_k^{(i)} \in \text{dom}(A_k)$ - and $Z_k^{(j)} \in \text{dom}(A_k)$ can be arbitrary. Denote

$$Z = (Z_1, \dots, Z_m); \quad Z_k = (Z_k^{(1)}, Z_k^{(j)}), \quad k=1, \dots, m \quad (26)$$

with similar notation for Z and Z_k .

In addition, Z and θ are described jointly (or perhaps, more accurately, conditionally) by r inference rules R_t , $t=1, \dots, r$, where

$$\phi_{R_t}(\theta | Z) = \text{tr}((Z \in G_t) \text{ iff } (\theta \in \text{Corr}_{Y_t}))$$

$$= \phi_{\theta}(\phi_{G_t}(Z), \phi_{\text{Corr}_{Y_t}}(\theta)), \quad (27)$$

where implication operator $\phi = \phi_{\rightarrow}$ or ϕ_{\leftrightarrow} , corresponding to "if-then" and "iff", respectively. For example, one can define for all $u, v \in [0, 1]$,

$$\phi_{\rightarrow}(u, v) = \phi_{\rightarrow}(u, v), \phi_{\rightarrow}(v, u); \quad \phi_{\leftrightarrow}(u, v) = \phi_{\rightarrow}(\phi_{\rightarrow}(u, v), v)$$

where antecedent

$$\phi_{G_t}(Z) = \text{tr}(\bigotimes_{(k \in J_t)} \delta(Z_k^{(1)} \& Z_k^{(j)} \text{ match to degree } \beta_{t,k}))$$

$$= \phi_{\&}(\phi_{F_t}(\phi_{M_k}(Z_k))) \quad (28)$$

($k \in J_t$) $\beta_{t,k}$

$J_t \subseteq \{1, \dots, m\}$ is index set of attributes R_t operates

on. M_k is a matching 'set' for attribute A_k so that $\phi_{M_k}(Z_k^{(i)})$ is the degree to which $Z_k^{(i)}$ and $Z_k^{(j)}$ are compatible, with intensification (or extensification) determined by ϕ_F for some $\beta_{t,k} \geq 0$, either

in the form as given in eq.(13) or (as previously mentioned) as exponential or translational argument form. Corr_{Y_t} similarly represents γ_t -degree correlation. A table can be established relating linguistic intensifications with numerical ones - $\beta_{t,k}, \gamma_t$. From now on, simplifying notation for all relevant error and matching functions and inference rules will be employed. The basic problem here is to apply the combination of evidence procedure to the data association problem for an appropriately chosen hybrid logical system. First note that:

- A. Attributes may be grouped roughly into two parts: (A_1, \dots, A_m) , statistical attributes ;
 (A_{m+1}, \dots, A_n) , subjective attributes.

For each statistical attribute A_k such as geolocation or sensor state parameter, P_k arises typically as the discretization of pdf f_k , say, obtained by various geometric and physical considerations. For each subjective attribute A_k such as identification or visual description of some type, P_k is obtained from a panel of experts and represents possibility values conditioned on potential observed data and in general will be approximately a symmetric binary function, so that a reasonable choice for M_k in any R_t is $M_k(Z_k) = P_k(Z_k^{(1)} | Z_k^{(j)})$.

B. On the other hand, for any statistical attribute A_k , in general M_k is not the same as P_k , since statistical hypotheses testing (involving P_k) presents a natural candidate for M_k : the significance level function for one-dimensional test statistic n_k , say, where $n_k = n_k(Z_k)$ for any outcome of rv Z_k , noting again that P_k is a discretization of f_k the pdf of $(Z_k^{(1)} | Z_k^{(j)})$, $j=1, \dots, m$. Thus, for the hypotheses

$H_0: i, j$ match vs. $H_1: i, j$ don't match, one obtains

$$M_k(Z_k) = \text{Pr}(n_k(Z_k) > n_k(Z_k) | H_0) = 1 - T_k(n_k(Z_k)), \quad (29)$$

where T_k is the cdf of $(n_k(Z_k) | H_0)$.

C. Inference rules are generally selected from a panel of experts and are thus uniquely determined by -once a choice of F is made- J_t and the tuple $(\beta_{t,k})_{k \in J_t}; \gamma_t$, although modifications to this

may occur if negative matching or negative correlation relations are introduced.

The following three theorems will be useful in developing principles for choosing appropriate classes of operators for F .

Theorem 1.

Let $\theta \in X$ with $\theta_1, \theta_2, \dots \in F(X)$, $\nu_{\&}$ a conjunction operator, and form for any possible value x of θ ,

$$\phi_{\&}(x) = \phi_{\&}(\bigcap_{k=1, 2, \dots} \beta_k(x) = \phi_{\&}(\phi_{\beta_1}(x), \phi_{\beta_2}(x), \dots)) \quad (30)$$

Then:

(i) If $\phi_{\&}$ is either an Archimedean t-norm or min

$$(x | x \in X \& \lim_{k \rightarrow \infty} \phi_{\&}(x) = 1) = \beta \quad (31)$$

then, unless $\phi_{\&} = \min$, $\phi_{\&}(x) = 0$, for all $x \in X$.

(ii) The α -level description of θ by level sets equivalent to B_1, B_2, \dots is

$$g(\alpha) = \bigcap_{1 \leq k \leq \infty} (\phi_{B_k}^{-1}[\alpha, 1]) \subseteq X; 0 \leq \alpha \leq 1. \quad (32)$$

Then

$$\phi_{B_k}^{-1}[\alpha, 1] \subseteq g(\alpha), \text{ all } 0 \leq \alpha \leq 1, \quad (33)$$

with proper subset relation holding in general, and with equality holding iff $\phi_{B_k} = \min$, i.e., the only case where no information is lost concerning θ is \min .

Theorem 2.

Let $C \subseteq P(R^n)$ with the p^{th} discretization $C_p \subseteq F(D_p)$ of C defined by letting D_p be the domain of C_p formed by making a p^{th} discretization of R^n with mesh fineness strictly decreasing as p grows, etc., and where

$$\phi_{C_p}(x) = \phi_C(x); \text{ all } x \in D_p. \quad (34)$$

Then,

(i) If ψ_0 is an Archimedean t-conorm,

$$\lim_{p \rightarrow \infty} \psi_0(\phi_{C_p}(x)) = 1. \quad (35)$$

(ii) If $\psi_0 = \max$,

$$\lim_{p \rightarrow \infty} \psi_0(\phi_{C_p}(x)) \leq 1, \quad (36)$$

with, in general, strict inequality holding.

Theorem 3. (See also [13].)

(i) Let $f_k: R^{m_k} \rightarrow R^+$, $k=1, \dots, a$, for a , fixed finite, be bounded pdf's.

(ii) Let $\psi_{k,p}$ be a conjunction. Define the p^{th} modified discretization $f_{k,p}$ of f_k as

$$f_{k,p}(x_k) \triangleq f_k(x_k) \cdot \Delta(x_k); x_k \in D_{k,p}, \quad (37)$$

where $D_{k,p}$ is the corresponding discrete domain and Δ is the mesh function so that

$$\sum_{x_k \in D_{k,p}} f_{k,p}(x_k) = 1; \quad (38)$$

and finally, let for any $x = (x_1, \dots, x_a) \in D(p) \triangleq D_{1,p} \times \dots \times D_{a,p}$,

$$f_{(p)}(x) \triangleq \psi_{k,p}(f_{1,p}(x_1), \dots, f_{a,p}(x_a)), \quad (39)$$

(iii) Suppose also that

$$h_a \triangleq (\partial^a \psi_{k,p}(u_1, \dots, u_a) / \partial u_1 \dots \partial u_a)_{u_1 = \dots = u_a = 0}$$

exists finitely.

(iv) Suppose that $\psi_{k,2}$ is a conjunction such that there is δ , $0 < \delta < 1$, such that for all $u \in [0, 1]$, $\partial \psi_{k,2}(u, v) / \partial v$ and $\partial^2 \psi_{k,2}(u, v) / \partial^2 v$ are bounded for all $v \in [0, \delta]$.

(v) $\phi_{\Gamma}: X \times R^{m_0} \rightarrow [0, 1]$ is a given continuous function where ΓX is the space that contains θ and where

$$m_0 \triangleq m_1 + \dots + m_a. \quad (41)$$

(vi) It is assumed that ϕ is an Archimedean t-conorm with canonical generator h : h is continuous with

$$\phi(u, v) = 1 - h^{-1}(\min(h(1-u) + h(1-v), h(0))), \quad (42)$$

for all $u, v \in [0, 1]$, and $h: [0, 1] \rightarrow R^+$ is nonincreasing with $h(1) = 0$. (See [5] for further properties.)

It is further assumed that there is a κ , $0 < \kappa < 1$, such that $dn(v)/dv$ and $d^2h(v)/d^2v$ are bounded functions of $v \in [\kappa, 1]$.

Then, for all $\theta \in X$,

$$\lim_{p \rightarrow \infty} \psi_0(\phi_{\Gamma}(\phi_{\Gamma}(\theta, x), f_{(p)}(x))) = 1 - h^{-1}(\min(\tau(\theta), h(0))), \quad (43)$$

where

$$\tau(\theta) \triangleq \int_{x \in R^{m_0}} (h(\phi_{\Gamma}(\theta, x)) \cdot \prod_{k=1}^a f_k(x_k) dx_k), \quad (44)$$

$$h(u) \triangleq -(dh(v)/dv)_{v=1} \cdot (\partial \psi_{k,2}(u, v) / \partial v)_{v=0}, \quad (45)$$

for all $u \in [0, 1]$.

Remark.

Frank's family of modular t-norms (see [5] or [6] for various properties) and t-conorms can be shown to satisfy the conditions of Theorem 3 for any $\psi_{k,1}$, $\psi_{k,2}$ and ϕ_0 chosen from the Archimedean part of the family.

Theorems 1, 2, 3 lead to the following:

Principle 3

If $\phi_{B_k}: X \rightarrow [0, 1]$, $k=1, 2, \dots$ is a sequence of descriptions of $\theta \in X$ with either high redundancy or much irregularity occurs in the sequence $\lim_{p \rightarrow \infty} \phi_{B_k}(\theta) = 1$ does not appear viable, for combining this sequence $\psi_{k,1} = \min$. In addition, \min allows for an uncountable number of descriptions of θ .

Principle 4

If ϕ_{B_k} as above are now relatively non-interactive/non-overlapping, then $\psi_{k,1}$ and $\psi_{k,2}$ as Archimedean t-norms and t-conorms, respectively, may be reasonable.

Principle 5

If C_p is the p^{th} discretization of C as in Theorem 2, then for the model, choose $\psi_{k,p} = \max$ so that $\lim_{p \rightarrow \infty} \psi_0(\phi_{C_p}(x))$ becomes non-trivial.

Thus, returning to the basic problem, Principle 4 implies that if A_1, \dots, A_m are all relatively non-overlapping, then the total error effect may be obtained by simply choosing an appropriate conjunction $\psi_{k,1}$ which may well be an Archimedean t-norm such as from Frank's family. This yields the overall statistical error effect

$$P(Z' | Z') \triangleq \psi_{k,1}(P_k(Z_k | Z_k^j)), \quad (46)$$

where

$$P_k(Z_k | Z_k^j) \triangleq (P_k(Z_k^{(1)} | Z_k^{(1)}), P_k(Z_k^{(j)} | Z_k^{(j)})) \quad (47)$$

for all $k, j=1, \dots, m$.

Similarly, for A_{m+1}, \dots, A_m , the overall subjective error effect is

$$P(Z'' | Z'') \triangleq \psi_{k,2}(P_k(Z_k | Z_k^j)), \quad (48)$$

with possibly the two conjunctions being chosen the same, and further, as an Archimedean t-norm.

On the other hand, since inference rules typically operate on combinations of common or similar attributes - by intensification modifications - Prin-

principle 3 may be invoked to yield the overall inference rule effect as

$$R(\theta|Z) = \min_{1 \leq t \leq r} (R_t(\theta|Z)) \quad (49)$$

Again using Principle 4, it may be appropriate to choose Archimedean t-norm ψ_{g3} to obtain the joint posterior function

$$\begin{aligned} \phi(\theta, Z|\tilde{Z}, R, P) &\stackrel{d}{=} \psi_{g3}(P(Z|\tilde{Z}), R(\theta|Z)) \\ &= \psi_{g3}(P(Z|\tilde{Z}'), \phi_T(\theta, Z)) \end{aligned} \quad (50)$$

where overall error effect is

$$P(Z|\tilde{Z}) \stackrel{d}{=} \psi_{g3}(P(Z|\tilde{Z}'), P(Z''|\tilde{Z}'')) \quad (51)$$

and, as in Theorem 3,

$$\phi_T(\theta, Z) \stackrel{d}{=} \psi_{g3}(P(Z''|\tilde{Z}''), R(\theta|Z)) \quad (52)$$

Note that a sufficiency condition holds here for

$$\phi(\theta|Z, \tilde{Z}, R, P) = \phi(\theta|Z, R) = R(\theta|Z) \quad (53)$$

as well as for

$$\phi(Z|\tilde{Z}, R, P) = \phi(Z|\tilde{Z}, P) = P(Z|\tilde{Z}), \quad (54)$$

which also when substituted into (50) yields a form which may be independently derived by use of conditioning of possibility functions.

Next, Principle 5 may be used to obtain the marginal posterior function for θ , before normalization, using the universal inclusion description:

$$\phi_{d(A)}(\theta) = \phi(\theta|\tilde{Z}, R, P) = \max_{\substack{\text{all } Z'' \\ \text{all } Z'}} (\psi_0(\phi(\theta, Z|\tilde{Z}, R, P))), \quad (55)$$

for some choice of Archimedean t-conorm ψ_0 , followed in turn by again invoking Principle 5,

$$\phi_{d(A)} = \sup_{0 \leq \theta \leq 1} (\phi(\theta|\tilde{Z}, R, P)) \quad (56)$$

which finally yields the normalized posterior $\phi(\theta|d(A))$ (dependent upon \tilde{Z}, R, P), by use of (22).

Note that if $P_k(Z_k|\tilde{Z}_k) = \delta_{Z_k, \tilde{Z}_k}$ (Kronecker delta), then the evaluation of all R_t involving that attribute A_k , in effect have $M_k(Z_k)$ replaced by $M_k(\tilde{Z}_k)$ in the antecedent and the term $P_k(Z_k|\tilde{Z}_k)$ is dropped from the calculations for the (normalized) posterior. In the above computations, for simplicity, the ψ_{g3} for $\ell=1, 2, 3$ could be set equal, if other conditions do not arise.

5. Asymptotic Behavior

Suppose that the combination of evidence problem relative to data association is carried out as outlined in the previous section, where P_k is the p^{th} discretization (modified) of pdf f_k , for $k=1, \dots, m'$, and all corresponding conditions for parts (i)-(iv) and (vi) for Theorem 3 hold ($a=2m'$). Also, note that part (v) of Theorem 3 is satisfied by use of eq.(52) and that any given inference rule R_t may use implication operator \rightarrow being either \rightarrow or \leftrightarrow .

(A) Under the above general conditions, for all $\theta \in [0, 1]$,

$$\lim_{p \rightarrow \infty} (\phi(\theta|\tilde{Z}, R, P)) = 1 - h^{-1}(\min(\tau(\theta), h(\theta))) \quad (57)$$

as in eq.(43), but changed to account for max, where

$$\tau(\theta) = h_{2m'} \cdot \max_{\text{all } Z''} (E_Y(k(\phi_T(\theta, (Y, Z''))))), \quad (58)$$

where Z in eq.(52) is replaced by (Y, Z'') , Y being a r.v. representing formally Z' and having pdf f' :

$$f'(Z'|\tilde{Z}') = \prod_{k=1}^{m'} (f_k(Z_k^{(i)}|\tilde{Z}_k^{(i)}) \cdot f_k(Z_k^{(j)}|\tilde{Z}_k^{(j)})), \quad (59)$$

(B) If also both statistical and possibilistic consistency holds, i.e.,

$$f_k(Z_k^{(\ell)}|\tilde{Z}_k^{(\ell)}) \rightarrow \delta(Z_k^{(\ell)} - \tilde{Z}_k^{(\ell)}) \text{ (dirac delta)} \quad (60)$$

for $k=1, \dots, m'$, and

$$P_k(Z_k^{(\ell)}|\tilde{Z}_k^{(\ell)}) \rightarrow \delta_{Z_k^{(\ell)}, \tilde{Z}_k^{(\ell)}} \text{ (Kronecker delta)} \quad (61)$$

for $k=m'+1, \dots, m$, and if k is such that k is non-decreasing with $k(0)=0$ (as is true for the members of Frank's family-see [9]), then

$$\tau(\theta) = h_{2m'} \cdot k(R(\theta|\tilde{Z})) \quad (62)$$

and hence

$$\lim_{p \rightarrow \infty} (\phi(\theta|\tilde{Z}, R, P)) \leq 1 - h^{-1}(h_{2m'} \cdot k(1)). \quad (63)$$

(C) Suppose the conditions in (A) and (B) obtain. Then

$$\begin{aligned} R(\theta|\tilde{Z}) &\leq \min_{1 \leq t \leq r} (\psi_0(1 - G_t(\tilde{Z}), \theta^{Y_t})) \\ &\leq \min_{1 \leq t \leq r} (\text{card}(J_t) - \sum_{k \in J_t} M_k(\tilde{Z}_k)^{\beta_t, k}) + \theta^{Y_t} \\ &\stackrel{d}{=} \psi_0 \end{aligned} \quad (64)$$

(D) Thus, if also for $k=1, \dots, m$,

$$1 \geq M_k(\tilde{Z}_k) \geq 1 - \omega; \quad 0 \leq \omega < 1 \text{ constant}, \quad (65)$$

then

$$\begin{aligned} R(\theta|\tilde{Z}) &\leq \min_{1 \leq t \leq r} (\text{card}(J_t)(1 - (1 - \omega)^{\beta_t}) + \theta^{Y_t}) \\ &\leq m \cdot (1 - (1 - \omega)^{\beta_0}) + \theta^{Y_0} \\ &\stackrel{d}{=} \epsilon_0 \end{aligned} \quad (66)$$

where

$$\beta_t \stackrel{d}{=} \max_{k \in J_t} (\beta_{t, k}); \quad \beta_0 \stackrel{d}{=} \max_{1 \leq t \leq r} (\beta_t); \quad \gamma_0 \stackrel{d}{=} \max_{1 \leq t \leq r} (\gamma_t) \quad (67)$$

Hence, for all $\theta \in [0, 1]$,

$$\lim_{p \rightarrow \infty} (\phi(\theta|\tilde{Z}, R, P)) \leq 1 - h^{-1}(h_{2m'} \cdot k(\epsilon_0)). \quad (68)$$

Thus, if also $M_k(\tilde{Z}_k) \rightarrow 1$ uniformly, for $k=1, \dots, m$, β_0 is bounded positively below, and γ_0 increases without bound, then it follows from (66) and (68) that for all $0 \leq \theta < 1$,

$$\lim_{p \rightarrow \infty} (\phi(\theta|\tilde{Z}, R, P)) = 1 - h^{-1}(0) = 0. \quad (69)$$

Finally, since in general $\phi_{d(A)} > 0$, (22)

implies that the final normalized posterior is

$$\lim_{p \rightarrow \infty} (\phi(\theta|d(A))) = \delta_{\theta, 1}; \text{ all } \theta \in [0, 1]. \quad (70)$$

This may be interpreted as describing the correlation condition "extremely likely correlated", since this can be considered, e.g., a limiting case of the function $\phi_c(\theta) = \theta^q$, for all θ , as $q \rightarrow \infty$, where all intensification is by exponentiation applied to a standard membership function-in this case, the 1-

density function over [0,1] representing the attribute "correlation level" with domain being probability of correlation or possibility of correlation.

(E) On the other hand, with the conditions in (A) and (B) holding, the lower bound for R is

$$R(\theta|\hat{Z}) \geq \min_{(1 \leq t \leq r)} (\psi_{\theta}(\psi_{\theta}(1-G_t(\hat{Z}), \theta^t), \psi_{\theta}(1-\theta^t, G_t(\hat{Z}))))$$

$$\geq \min_{(1 \leq t \leq r)} (1-G_t(\hat{Z})-\theta^t)$$

$$\geq \min_{(1 \leq t \leq r)} (1 - \min_{(k \in J_t)} (M_k(\hat{Z}_k)^{\beta_t, k}) - \theta^t)$$

$$\geq \rho_0 \quad (71)$$

(F) Thus, if also for $k=i, \dots, m$

$$\omega M_k(\hat{Z}_k) \geq 0; 0 \leq \omega \leq 1 \text{ constant}, \quad (72)$$

then

$$R(\theta|\hat{Z}) \geq \min_{(1 \leq t \leq r)} (1-\omega^{\beta_t} - \theta^t)$$

$$\geq 1-\omega^{\beta_0} - \theta^{\gamma_0}$$

$$\geq \alpha_0 \quad (73)$$

where

$$\beta_t \stackrel{d}{=} \min_{(k \in J_t)} (\beta_{t,k}); \beta_0 \stackrel{d}{=} \min_{(1 \leq t \leq r)} (\beta_t); \gamma_0 \stackrel{d}{=} \min_{(1 \leq t \leq r)} (\gamma_t) \quad (74)$$

Hence, for all $\theta \in [0,1]$,

$$\lim_{p \rightarrow \infty} (\phi(\theta|\hat{Z}, R, P)) \geq 1-h^{-1}(h_{2m} \cdot k(\alpha_0)) \quad (75)$$

Thus, if also $M_k(\hat{Z}_k) \rightarrow 0$ uniformly, for $k=1, \dots, m$, β_0 is bounded positively below, and γ_0 increases without bound, then it follows from (73) and (75) that for all $0 \leq \theta < 1$,

$$\lim_{p \rightarrow \infty} (\phi(\theta|\hat{Z}, R, P)) \geq 1-h^{-1}(h_{2m} \cdot k(1)) \quad (76)$$

But, eq.(63) shows the other direction, and hence

$$\lim_{p \rightarrow \infty} (\phi(\theta|\hat{Z}, R, P)) = 1-h^{-1}(h_{2m} \cdot k(1)) > 0 \quad (77)$$

In general (as is true, e.g., for Frank's family [7]), Also note that for $\theta=1$,

$$R(1|\hat{Z}) = \min_{\substack{(1 \leq t \leq r, \\ R_t \text{ uses } \leftrightarrow)}} (G_t(\hat{Z})) \leq \omega^{\beta_0} \quad (78)$$

where

$$\beta_0 \stackrel{d}{=} \min_{\substack{(k \in J_t, \\ 1 \leq t \leq r; R_t \text{ uses } \leftrightarrow)}} (\beta_{t,k}) \quad (79)$$

Thus, if there is an R_t using \leftrightarrow , it follows

that if β_0 is bounded positively from below, then the above conditions imply that

$$\lim_{p \rightarrow \infty} (\phi(1|\hat{Z}, R, P)) = 1-h^{-1}(0) = 0 \quad (80)$$

and hence from (22) it follows that for all $0 \in [0,1]$,

$$\lim_{p \rightarrow \infty} (\phi(\theta|d(A))) = 1 - \delta_{\theta,1} \quad (81)$$

which, analogous to the result in (D), indicates linguistically that the condition "not extremely correlated" holds, corresponding to the limiting case for possibility function $\phi_{\theta}(\theta) = 1-\theta^q$ as a function of θ , as $q \rightarrow \infty$.

In summary, the above results show that sufficiently good information and matching of data (as in (D)) leads to asymptotic consistent results, while relatively poor matching leads to the unde-

sirable results of (F).

6. Concluding Remarks

This paper has shown how a combination of evidence procedure may be established with a general application to the data association problem. Some asymptotic consistency properties of the procedure were established. Implementation of the procedure is necessarily deeply involved with the use of Kalman filters for updating geolocation state vectors, as well as other real-world procedures, including sensor system models and schemes for the clustering of ordered pairs based upon their level of correlation, often presented in the form of a "correlation matrix". The latter depends upon a single figure-of-merit representing the average correlation level (or probability) between any two track histories i, j . One suggestion for this has been to use, in effect, the two stage r.v. $(W|S_U(C))$, where $S_U(C)$ is the naturally corresponding random set (discussed before - see section 3) for output $\phi = \phi(\cdot|d(A))$, a function of $\theta \in [0,1]$, as obtained above, and where W in its conditional form is a r.v. uniformly distributed over the outcome $S_U(C)$ [14]. In particular, if ϕ_C is unimodal, then the measure of mean is

$$E(W|S_U(C)) = E(1/2 (\phi_C^{-1}(U) + \phi_C^{-1}(U)))$$

$$= 1/2 (\int_{x=0}^u x d\phi_C(x) + \int_{x=u}^1 x d(1-\phi_C(x)))$$

$$= u\phi_C(u) + (1/2) (\int_{x=u}^1 \phi_C(x) dx - \int_{x=0}^u \phi_C(x) dx - \phi_C(1)) \quad (82)$$

where u is the mode. An open question connected with this value is the determination of a related figure-of-merit which is invariant with respect to any particular random set, one point coverage equivalent to ϕ_C .

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