

①

Office of Naval Research

Contract N00014-90-J-1726

Technical Report 1

AD-A240 835



The Relationship Between Fractal Geometry and Fractography

by

J. J. Mecholsky and S. W. Freiman

Prepared for Publication

in

J. American Ceramic Society

Department of Materials Science and Engineering
University of Florida
Gainesville, FL 32611

91-11419



August 28, 1991

Reproduction in whole or in part is permitted for
any purpose of the United States Government

91 02 063

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED		1b RESTRICTIVE MARKINGS	
2a SECURITY CLASSIFICATION AUTHORITY		3 DISTRIBUTION/AVAILABILITY OF REPORT UNLIMITED	
2b DECLASSIFICATION/DOWNGRADING SCHEDULE			
4 PERFORMING ORGANIZATION REPORT NUMBER(S)		5 MONITORING ORGANIZATION REPORT NUMBER(S) N00014-90-J-1726	
6a NAME OF PERFORMING ORGANIZATION University of Florida	6b OFFICE SYMBOL (If applicable)	7a NAME OF MONITORING ORGANIZATION ONR / Dr. Robert Schwartz	
6c ADDRESS (City, State, and ZIP Code) 256A Rhines Hall Gainesville, FL 32611		7b ADDRESS (City, State, and ZIP Code) Code 373 Naval Weapons Center China Lake, CA 93555-6001	
8a NAME OF FUNDING/SPONSORING ORGANIZATION ONR	8b OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c ADDRESS (City, State, and ZIP Code) 800 North Quincy Street Arlington, VA 22217-5000		10. SOURCE OF FUNDING NUMBERS	
		PROGRAM ELEMENT NO	PROJECT NO
		TASK NO	WORK UNIT NO
11 TITLE (Include Security Classification) The Relationship Between Fractal Geometry and Fractography (U)			
12 PERSONAL AUTHOR(S) J. J. Mecholsky, Jr. and S. W. Freiman			
13a TYPE OF REPORT Technical	13b. TIME COVERED FROM TO	14 DATE OF REPORT (Year, Month, Day) 91-8-28	15 PAGE COUNT 10
16 SUPPLEMENTARY NOTATION			
17 COSATI CODES		18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)	
FIELD	GROUP	SUB-GROUP	
		This document has been approved for public release and sale; its distribution is unlimited.	
19 ABSTRACT (Continue on reverse if necessary and identify by block number) Fractal geometry has been used to describe irregular fracture surfaces in a quantitative way. The fractal dimensional increment has been related to the fracture toughness of the material through the elastic modulus and a characteristic structure parameter, a_0 . The study of fractography has shown the relationship between the flaw-to-mirror size ratio and the fracture toughness. An experimental observation has shown that the fracture toughness is related to the elastic modulus through another structure parameter, b_0 . Combining all of these relationships leads to the conclusion that the fractal dimensional increment, D^* , is directly related to the flaw-to-mirror size ratio. This note shows that experimental measurements of the fractal dimension and the flaw-to-mirror size ratio on glasses, a glass ceramic, polycrystalline ceramics and a single crystal all agree with the prediction. The implication of this finding is that there is a linear scaling law in operation at fracture between the energy of the crack initiation and branching and is reflected in the features on the fracture surface.			
20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS		21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED	
22a NAME OF RESPONSIBLE INDIVIDUAL		22b TELEPHONE (Include Area Code)	22c OFFICE SYMBOL

The Relationship Between Fractal Geometry and Fractography

J. J. Mecholsky, Jr.

Department of Materials Science & Engineering
University of Florida
Gainesville, FL 32611



and

S. W. Freiman

National Institute of Standards & Technology
Gaithersburg, MD 20899

ABSTRACT

A-1

Fractal geometry has been used to describe irregular fracture surfaces in a quantitative way. The fractal dimensional increment has been related to the fracture toughness of the material through the elastic modulus and a characteristic structure parameter, a_0 . The study of fractography has shown the relationship between the flaw-to-mirror size ratio and the fracture toughness. An experimental observation has shown that the fracture toughness is related to the elastic modulus through another structure parameter, b_0 . Combining all of these relationships leads to the conclusion that the fractal dimensional increment, D^* , is directly related to the flaw-to-mirror size ratio. This note shows that experimental measurements of the fractal dimension and the flaw-to-mirror size ratio on glasses, a glass ceramic, polycrystalline ceramics and a single crystal all agree with the prediction. The implication of this finding is that there is a linear scaling law in operation at fracture between the energy of crack initiation and branching and is reflected in the features on the fracture surface.

Fractal geometry is being used in many fields of materials science, physics, chemistry and engineering because it can be applied to describe, relatively easily, shapes and processes which are non-linear and seemingly complex^{1,2}. Self-similarity and scale invariance are characteristics of (scaling) fractals. Self-similarity means that multiple features on the surface appear the same. Scale invariance means that a feature at one level of magnification is related to another feature at another magnification through a scalar magnification constant. Fracture is one of the phenomena that has been modelled using fractal geometry³⁻⁸. Fractography has been shown to be quantitatively related to the stress at failure, the nature of the stress state, the amount

of residual stress and the size of the fracture initiating crack⁹ The purpose of this note is to demonstrate the relationship between fractography and fractal geometry.

Fractal objects are characterized by their fractal dimension, D , which is the dimension in which the proper measurement of a fractal object is made. For example, if we had a plane square, then the only dimension in which to make a meaningful measurement is 2, i.e., the area of the square. The length or volume of a square is meaningless. This same concept is generalized for fractals which are allowed to have non-integer dimensions. Thus, a plane square with "bumps" out of the plane would have dimension, $2.D^*$, where D^* is the fractional part of the fractal dimension and represents the amount of tortuosity out of the plane. An object with a fractal dimension of 2.1 ($D^* = 0.1$) would be relatively flat and an object with $D=2.9$ ($D^* = 0.9$) would almost be a volume filling object. If the same relatively flat "bumpy" plane was measured by a contour line, then the fractal dimension would be $D = 1.1$, but D^* would still be 0.1. Of course, this latter relationship only holds for self-similar fractals of which fracture surfaces appear to be a representation^{8,10}. The fractal dimension can be measured in a number of ways. One of these has been called the slit-island technique⁸ in which the length of part, or all, of the contour of an "island" that appears from polishing of an embedded fracture surface is measured. This measurement is made repeatedly using different starting measurement rulers. The highest point of the fracture surface is measured first and subsequently (lower) areas are measured. Generally, this is a random section on the fracture surface. Thus, an average representation of the roughest portion on the fracture surface is usually obtained.

It has been shown previously that fundamental relationships can be derived between the fractal dimensional increment, D^* , and the fracture toughness of a material in the form of the critical stress intensity factor, K_{IC} ^{8,10}:

$$K_{IC} = E a_0^{1/2} D^{*1/2} = Y(\theta) \sigma_f c^{1/2} \quad (1)$$

where E is the elastic (Young's) modulus, a_0 is a parameter having the units of length, $Y(\theta)$ is a geometric constant dependent on geometry of the crack and loading condition, σ_f is the applied stress at fracture and c is the crack size. The relationship between K_{IC} and D^* is based on experimental observations^{8,11,12} and the relationship between K_{IC} and c is based on fracture mechanics and experimental confirmation^{13,14}. The measurement of D^* is an average property of the entire fracture surface and a measure of its tortuosity. The flaw size is a measure of the critical flaw area locally

around the fracture origin. Figure 1 shows a schematic of a fracture surface depicting the origin and the surrounding tortuous topography.

There are regions surrounding the fracture initiating crack known as mirror, mist and hackle which are related to the stress at fracture¹⁵. The radial distances from the origin to the demarcations between the regions have become known collectively as "mirror" radii. The first region is generally a relatively smooth (mirror) region, the second region is a slightly stippled (mist) region and the third region is a very coarse (hackle) region. This last region ends with macroscopic crack branching. The distances r_1, r_2 and r_3 represent the boundaries at which mist, hackle and crack branching occur, respectively. These boundaries may or may not be symmetric about the fracture origin depending on the stress distribution, shape of the crack or elastic anisotropy (in the case of single crystals)¹⁶. It was first observed experimentally¹⁷ that the stress at failure, σ_f , is related to these features:

$$\sigma_f r_j^{1/2} = M_j \quad (2)$$

where r_j is either r_1, r_2 or r_3 corresponding to the "mirror" constant M_j . This relationship has been shown to hold for a wide variety of materials. Kirchner and Kirchner¹⁸ and Kirchner and Conway¹⁹ have shown that the above is a special case of a more general fracture mechanics approach. They assumed that the formation of mist, hackle and crack branching occur at a constant characteristic stress intensity factor. This approach makes it necessary to modify Equation (2) through a crack border correction factor, $Y(\theta)$, where θ is the angle from the surface to the interior, i.e. $\theta = 0$ to 90° , i.e.,

$$K_{Bj} = Y(\theta) \sigma_f r_j^{1/2} \quad (3)$$

where K_{Bj} is the crack-branching stress intensity factor. This correction accounts for the fact that the stress intensity is not constant along the crack front. Equation (2) is only valid for measurements made along the tensile surface for materials fractured in flexure, whereas, if we use Eq. (3), the measurement may be taken anywhere along the fracture mirror boundary. Notice that at the surface, $K_{Bj} = Y(0) M_j$. [$Y(0) = 1.24$ for a semi-circular crack in flexure or tension which is small relative to the thickness of the beam]. In addition to Eq. (3), K_{Bj} has been experimentally related to E ^{15,20}:

$$K_{Bj} = E b_0^{1/2} \quad (4)$$

where b_0 is a parameter with the units of length. Notice if we assume that the length parameters a_0 and b_0 are in some way related to one another, i.e., $a_0 \propto b_0$, then the combination of Eqs. (1), (3) and (4) results in the relation:

$$r_{j/c} \propto 1/D^* \quad (5)$$

If we compare $r_{j/c}$ to $1/D^*$ as in Table 1 for a variety of ceramic materials, we see that the proportionality constant between a_0 and b_0 appears to be 1 for the mirror-mist boundary.

There are several interesting implications of the relationship demonstrated in Eq.(5) and shown in Table 1. The fractal dimensional increment is obtained from the analysis of the tortuosity of the entire fracture surface whereas the $r_{j/c}$ ratio is obtained from a measure of the "in plane" distance from the initiation of fracture to the point of optically observed branching. At first, these quantities may seem unrelated. However, if we realize that the tortuosity that we observe optically merely means that the perturbations above a smooth plane are large enough to reflect light, then we can see a direct relation between the out-of-plane tortuosity and the "in-plane" distance at which this occurs.

In the past, there has been disagreement among investigators as to the form of the scaling law during fracture⁴⁻⁸. Investigators have the choice of scaling characteristic microstructural features⁴ which occur at different levels such as atom clusters, microcracks, pores, grains, etc. or of scaling energy^{5,6}, i.e., the energy of bond breaking may be some multiple of the energy to initiate crack propagation and this, in turn, is the same multiple of macroscopic branching. It has been debatable as to the form of scaling, i.e., power law⁴⁻⁷ or linear⁸. The agreement of experimental data with Eq. (5) (Table 1) implies that there is a scaling between the crack size and the point of branching and between the energy of crack initiation and of crack branching. Thus, analyses considering an energy scaling or a structural (geometric) scaling are equivalent. Moreover, the scaling is linear in energy or geometry. It appears significant that two parameters obtained by very different analyses on the fracture surface are similar in magnitude. More specifically, a_0 is obtained from a measurement of the tortuosity of the fracture surface, while b_0 relates the bonding strength (at a strain level approximately zero) to the energy required to branch. The schematic in Figure 1 shows the regions from which the fractal dimensional increment is obtained and that from which the toughness measurement is made from the fracture

initiating crack size or mirror regions. In general, the fractal dimensional increment data are obtained from a random section on the fracture surface and usually in a macroscopically tortuous region. The agreement between D^* and c/r_1 shown in Table 1 suggests that there is a relationship between the local region of fracture at the origin and the resulting topography all across the fracture surface. We can speculate that the absolute magnitudes of a_0 and b_0 are in some way related to the volume of material surrounding the crack tip in which the fracture process takes place, i.e., a process-zone size. Passoja²¹ has suggested that this scaling is a general phenomena that is observed in many materials and is related to the atomic structure.

Acknowledgements

The authors thank Drs. D. E. Passoja and T. J. Mackin for many useful discussions. One of the authors (JJM) thanks the Office of Naval Research for financial support.

References

1. B. B. Mandelbrot, "The Fractal Geometry of Nature," W. H. Freeman, New York (1982).
2. H. Takayasu, Fractals in the Physical Sciences, Manchester University Press, Manchester, UK (1990).
3. Disorder and Fracture, eds. J. C. Charmet, S. Roux and E. Guyon, Plenum Press, NY 1990.
4. A. R. Rosenfield, Fractal Mechanics, Scripta Metallurgica, V.21, pp.1359-1361, (1987).
5. K. Sieradzki, "The Fracture Strength of Solids Near The Percolation Threshold", J. Phys. C: Solid State Phys. 18 L855-L856 (1985).
6. J. C.M. Li, "A theoretical Limit of Fracture Toughness," Scripta Metallurgica, V.22, pp. 837-838(1988).

7. R. F. Cook, Effective-Medium Theory For Fracture of Fractal Porous Media, in Extended Abstracts Fractal Aspects of Materials:Disordered Systems eds. D. A. Weitz, L. M. Sander and B. B. Mandelbrot, Materials Research Society (1988).
8. J. J. Mecholsky, D. E. Passoja and K. S. Feinberg-Ringel, "Quantitative Analysis of Brittle Fracture Surfaces Using Fractal Geometry," J. Am. Ceram. Soc., 72 [1] (1989).
9. B. B. Mandelbrot, D. E. Passoja, and A. J. Paulley, "Fractal Character of Fracture Surface of Metals", Nature 308, [5961] 721-22 (1984).
10. S. W. Freiman, J. J. Mecholsky and P. F. Becher, "Fractography: A Quantitative Measure of the Fracture Process," Ceramic Transactions 17, Fractography of Glasses and Ceramics II, eds. V. D. Frechette and J. Varner, The American Ceramic Soc., (1991).
11. J. J. Mecholsky, T. J. Mackin and D. E. Passoja, "Self-similar Crack Propagation in Brittle Materials," Adv. in Ceramics, v22, Fractography of Glasses and Ceramics, The American Ceramic Society, Inc., pp.127-134 (1988).
12. J. J. Mecholsky, Jr. and T. J. Mackin, "Fractal Analysis of Fracture in Ocala Chert," J. Mater. Sci., 7, 1145 -47 (1988).
13. T. J. Mackin, D. E. Passoja and J. J. Mecholsky, Jr. "Fractal Analysis of Bond Breaking Processes in Brittle Materials," in Extended Abstracts - Fractal Aspects of Materials: Disordered Systems, eds. A. J. Hurd, D. A. Weitz and B. B. Mandelbrot, MRS (1987).
14. G. R. Irwin, "Relation of Crack-Toughness Measurements to Practical Applications," ASME Paper No. 62-MET-15, 1-13 (1962).
15. J. J. Mecholsky, Jr. and S. W. Freiman, "Determination of Fracture Mechanics Parameters Through Fractographic Analysis of Ceramics," pp. 136-150 in Fracture Mechanics Applied to Brittle Materials, ASTM STP

678, eds. S.W. Freiman, American Society for Testing and Materials, Philadelphia, PA (1980).

16. J. J. Mecholsky, Jr., "Quantitative Fractography: An Assessment," Cer. Trans. 17, Fractography of Glasses and Ceramics II, op. cit.
17. Y. L. Tsai and J. J. Mecholsky, Jr., Fractal Fracture in Single Crystal Silicon," J. Materials Res. 6 [6] 1248-1263 (1991).
18. E. B. Shand, "Breaking Stresses of Glass Determined from Dimensions of Fracture Mirrors", J. Am. Ceram. Soc., 42,[10]474-77(1959).
19. H. P. Kirchner and J. W. Kirchner, "Fracture Mechanics of Fracture Mirrors," J. Am. Cer. Soc. 62 [3-4] 198-202 (1979).
20. H. P. Kirchner and J. C. Conway, Jr., "Comparison of the Stress Intensity and Johnson-and-Holloway Criteria for Crack Branching in Rectangular Bars," J. Am. Cer. Soc. 70 [8] 565-9(1987).
21. D. E. Passoja, "Fundamental Relationships Between Energy and Geometry in Fracture," Adv. in Ceramics 22: Fractography of Glasses and Ceramics, The American Ceramic Soc. (1988).
22. J. J. Mecholsky, Jr. and J. Plaia, "Fractal Analysis of Glass Using Replication Techniques" to be published.
23. J. J. Mecholsky, S.W. Freiman and R. W. Rice, "Fracture Surface Analysis of Ceramics", J. Mater. Sci., 11, 1310-19 (1976).
24. T. J. Mackin, D. E. Passoja and J. J. Mecholsky, Jr., " Fractal Analysis of Bond Breaking Processes in Brittle Materials," in Extended Abstracts- Fractal Aspects of Materials: Disordered Systems, eds. A. J. Hurd, D. A. Weitz and B. B. Mandelbrot, Mat'ls Res. Soc., Pittsburgh, PA (1987).
25. P. L. Yoder, "The Effect of Grain Size on the Mechanical Properties of Zinc Sulfide" Master's Thesis, Penn State Univ., University Park, PA (1989).

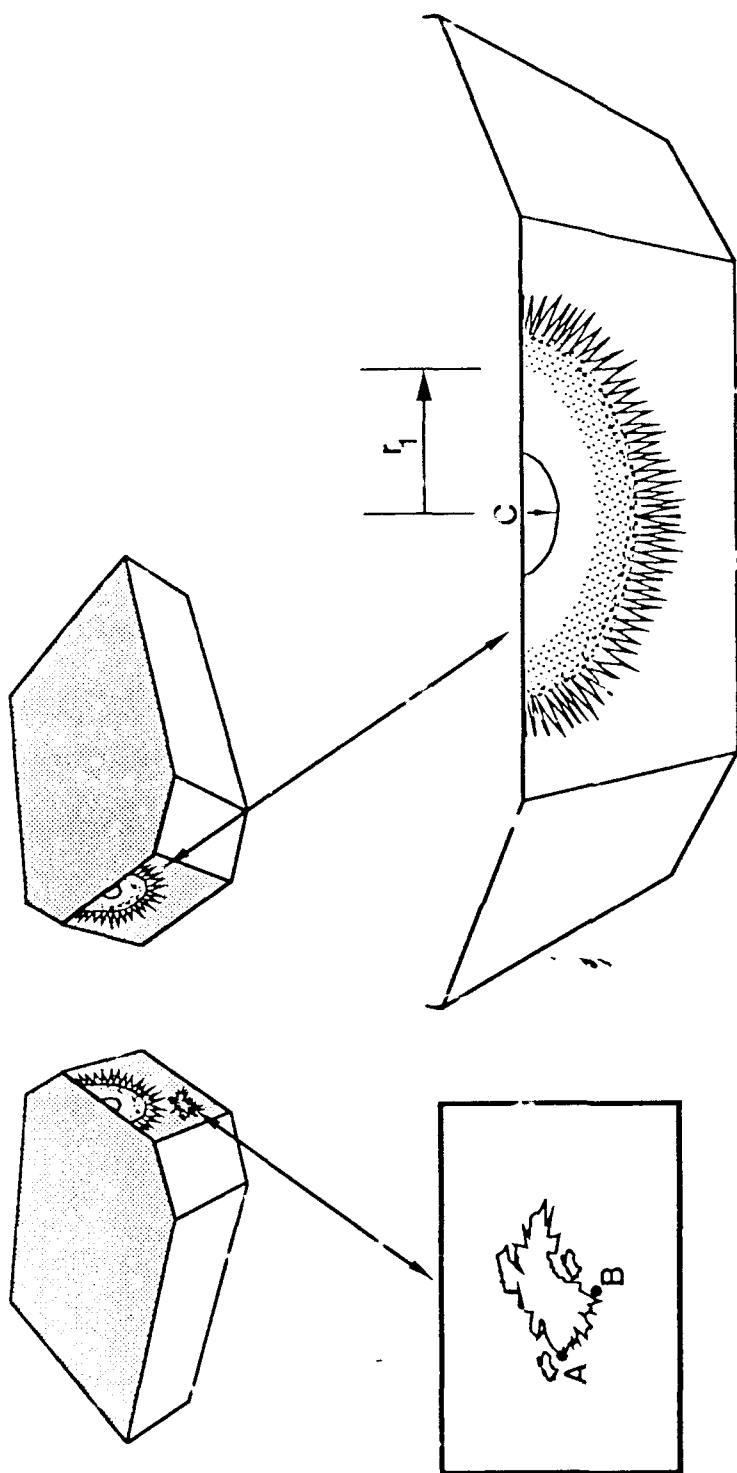
26. J. J. Mecholsky and J. J. Mecholsky, Jr., "Fractal Analysis of WC-Co Composites", to be published.
27. D. Han and J. J. Mecholsky, Jr., "Fracture Analysis of Cobalt-bonded Tungsten Carbide Composites" J. Mater. Sci. 25, 4949-56 (1990).

Table I - Relationship Between D^* and c/r_1 .

MATERIAL*	$1/D^*$	r_1/c
Borosilicate Glass [22]	11	10
Calcium Aluminosilicate Glass [22]	11	10
Pyroceram 9606 [23,24]	6	3
Alumina [23,24]	3	4
PZT [23,24]	5	7
ZnS [24,25]	6	6
ZnSe [23,24]	3	4
Si (100)(110) [17,22]	8	8
WC-Co [26, 27]	4-5	4-6

*Data from references shown in [].

Figure 1 - Schematic of a Fracture Surface Showing the Origin and "Slit Island" Region. The fracture origin is local to one point on the fracture surface. The contour on the fracture surface known as the slit island, in general, is taken from the more tortuous region of the fracture surface. The length, L , of the contour perimeter between A and B is measured using different scales, ϵ . The slope of $\log L$ versus $\log \epsilon$ is the fractal dimensional increment, D^* .



$$\frac{C}{r_i} = \text{Constant}$$

$$D^* = \frac{C}{r_i}$$

