

AD-A241 005



2

DTIC  
ELECTE  
OCT 03 1991  
S D D



# A Recursive Algorithm for Computing Ray-Parameter Derivatives

by

Zhenyue Liu

This document has been approved  
for public release and sale; its  
distribution is unlimited.

Center for Wave Phenomena  
Colorado School of Mines  
Golden, Colorado 80401  
(303)273-3557

91 10 1 047

91-12066



# A Recursive Algorithm For Computing Ray-parameter Derivatives

*Zhenyue Liu*

## 1. INTRODUCTION

Under the assumption of high frequency, wave propagation can be described by the ray method. Many quantities in wave phenomena are characterized by some ray parameters and their derivatives. For example, the slope of arrival time-offset curve is characterized by the emergence angle of a ray; the curvature is characterized by derivative of that angle, and the amplitude of the signal is characterized by the derivative of the taking-off angle. Moreover, when we solve an equation containing ray parameter, we also require analytic expressions for the derivatives. Generally, it is difficult to obtain analytic expressions for these derivatives. We can use finite difference approximation for these derivatives. This type of approximation fails at caustics. And also the accuracy and stability of the calculation are influenced by the smoothness of interfaces and the spacing between adjacent rays.

In this paper, we give a recursive algorithm for ray-parameter derivatives, when the medium is made up of constant-velocity layers separated by arbitrary, smooth interfaces. The algorithm uses derivatives of interfaces, intersection points of rays with the interfaces, and angles of incidence and transmission, all of which are easily computed. A conclusion of these formulas is that no ray caustic exists for transmission or for the reflection from any one of the reflectors when the interfaces are flat.

## 2. RECURSIVE ALGORITHM

Suppose that an acoustic earth model is made up of constant-velocity layers separated by arbitrary, smooth interfaces. We are going to derive a recursive formula for computing ray-parameter derivatives when a ray propagates through the interfaces.

We consider two interfaces  $L_1$  and  $L_2$ , defined respectively by  $z = f_1(x)$  and  $z = f_2(x)$ . A ray from  $L_1$  arrives at  $L_2$ , with intersection points  $(x_1, z_1)$  and  $(x_2, z_2)$ . The incident angle from the vertical at  $L_1$  is  $\alpha_1$  and the outgoing angle is  $\alpha_2$ . The angle of the normal at  $L_1$  is  $\theta$ . When the ray arrives to  $L_1$ , there two possibilities:

reflection and transmission. Suppose that for this ray the initial angle from the vertical is  $\beta$  and the initial depth is  $z_0$ .

Starting from the tangent equation

$$-\frac{df_1}{dx} \Big|_{x=x_1} = \tan \theta,$$

the Snell' law

$$\frac{\sin(\alpha_1 - \theta)}{c_1} = \frac{\sin(\alpha_2 - \theta)}{c_2} \text{ for transmission,}$$

$$\alpha_2 - \theta = \theta - \alpha_1 \text{ for reflection,}$$

and the stright line equation

$$x_2 - x_1 = (z_2 - z_1) \tan \alpha_2,$$

and differentiating them with respect to  $\beta$ , we find a recursive algorithm as follows:

$$\frac{d\theta}{d\beta} = -\frac{d^2 f_1}{dx^2} \Big|_{x=x_1} \frac{dx_1}{d\beta} \cos^2 \theta. \quad (1)$$

$$\frac{d\alpha_2}{d\beta} = \frac{d\theta}{d\beta} + \frac{c_2 \cos(\alpha_1 - \theta)}{c_1 \cos(\alpha_2 - \theta)} \left( \frac{d\alpha_1}{d\beta} - \frac{d\theta}{d\beta} \right) \text{ for transmission,} \quad (2)$$

or

$$\frac{d\alpha_2}{d\beta} = 2 \frac{d\theta}{d\beta} - \frac{d\alpha_1}{d\beta} \text{ for reflection,} \quad (3)$$

$$(1 - \tan \alpha_2 \frac{df_2}{dx} \Big|_{x=x_2}) \frac{dx_2}{d\beta} = \frac{z_2 - z_1}{\cos^2 \alpha_2} \frac{d\alpha_2}{d\beta} + (1 - \tan \alpha_2 \frac{df_1}{dx} \Big|_{x=x_1}) \frac{dx_1}{d\beta}. \quad (4)$$

The initial values of the relevant parameters are

$$\frac{d\alpha_1}{d\beta} = 1,$$

$$\frac{dx_1}{d\beta} = \frac{z_1 - z_0}{\cos^2 \alpha_1} / (1 - \tan \alpha_1 \frac{df_1}{dx} \Big|_{x=x_1}).$$

### 3. APPLICATION

#### Geometry spreading factor $\partial\beta / \partial\xi$

From Docherty (1987), we require the calculation of  $\partial\beta / \partial\xi$  in order to obtain the amplitude of the signal. Here,

$$\frac{\partial\beta}{\partial\xi} = 1 / \frac{dx_2}{d\beta} \Big|_{x_2=\xi},$$

where  $\beta$  is the taking-off angle of the ray and  $\xi$  is the horizontal coordinate of the end of the ray. The factor  $dx_2/d\beta$  can be calculated by using equation (4).

### Curvature of arrival time-offset curve

Starting from the equation

$$\frac{\partial \tau}{\partial x} = \frac{\sin \alpha}{v},$$

where  $\alpha$  is the emergence angle of the ray and  $v$  is the velocity, it follows that

$$\frac{\partial^2 \tau}{\partial x^2} = \frac{\cos \alpha}{v} \frac{\partial \alpha}{\partial x}.$$

Since the upper surface is horizontal,  $\alpha = \alpha_2$  at  $z = 0$ . From equations (2) to (4), we can compute  $d\alpha_2/d\beta$  and  $dx_2/d\beta$  from which we find

$$\frac{\partial \alpha}{\partial x} = \frac{d\alpha_2}{d\beta} \Big|_{\alpha_2=\alpha} / \frac{dx_2}{d\beta} \Big|_{x_2=x},$$

### Caustic for flat interfaces

An interesting conclusion of equations (1) to (4) is that when interfaces are flat, no caustic exists. i.e.,  $\Delta\beta \neq 0$  implies  $\Delta x \neq 0$ . In fact, for flat interfaces,  $d\theta/d\beta = 0$  from equation (1). For the transmission, all coefficients are positive in equations (2) and (4), so  $dx_2/d\beta$  is positive. For the reflection,  $d\beta_2$  and  $z_2 - z_1$  change the sign at same time, so  $dx_2/d\beta$  still keeps the positive sign. This implies that rays from two different taking-off angles must not intersect each other for transmission or reflection from one reflector.

## 4. CONCLUSION

The recursive algorithm has higher accuracy and more stability than the finite difference approximation. Compared the ray tracing, only a little additional computation is required for the recursive algorithm. Furthermore, because of its analytic expression, the recursive algorithm provides a useful tool for the theoretical analysis.

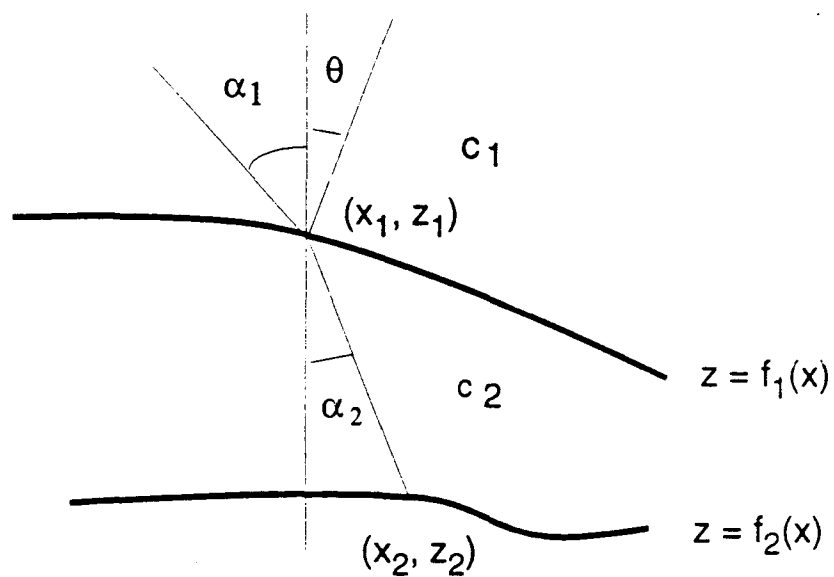
## ACKNOWLEDGMENTS

Financial support for this work was provided in part by the Office of Naval Research, Mathematics Division and by the Center for Wave Phenomena Consortium Project.

**REFERENCES**

Docherty, P. C., 1987, Two-and-one-half dimensional common shot modeling: Center for wave phenomena Research Report No. CWP-050.

## Transmission



## Reflection

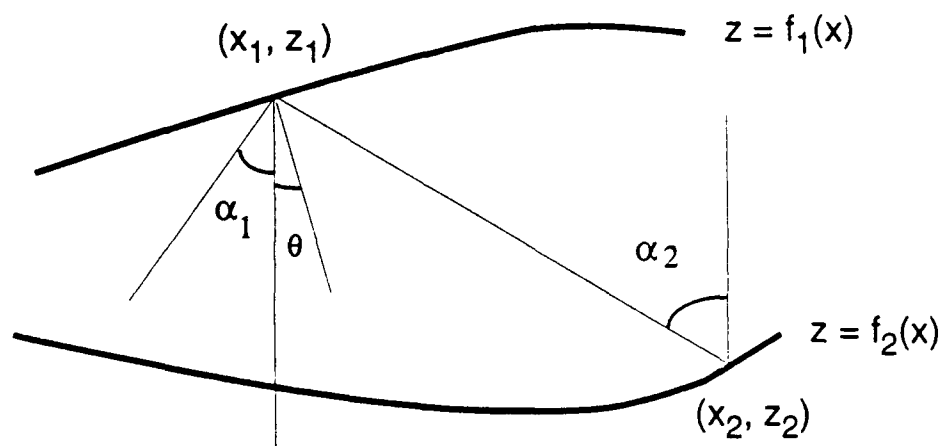


FIG. 1. Paths of arriving ray for transmission and reflection.

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION Unclassified			1b RESTRICTIVE MARKINGS None.			
2a SECURITY CLASSIFICATION AUTHORITY			3 DISTRIBUTION/AVAILABILITY OF REPORT			
2b DECLASSIFICATION/DOWNGRADING SCHEDULE						
4 PERFORMING ORGANIZATION REPORT NUMBER(S)			5 MONITORING ORGANIZATION REPORT NUMBER(S)			
6a NAME OF PERFORMING ORGANIZATION Center for Wave Phenomena Colorado School of Mines		6b OFFICE SYMBOL (If applicable)	7a NAME OF MONITORING ORGANIZATION Mathematial Sciences			
6c. ADDRESS (City, State, and ZIP Code) Golden, CO 80401			7b. ADDRESS (City, State, and ZIP Code) 800 N. Quincy Street Arlington, VA 22217-5000			
8a. NAME OF FUNDING/SPONSORING ORGANIZATION		8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N00014-91-J-1267			
8c. ADDRESS (City, State, and ZIP Code)			10. SOURCE OF FUNDING NUMBERS			
			PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
11 TITLE (Include Security Classification) A Recursive Algorithm for Computing Ray-Parameter Derivatives						
12. PERSONAL AUTHOR(S) Zhenyue Liu						
13a. TYPE OF REPORT Technical		13b. TIME COVERED FROM 10/17/90 TO 9/30/91		14. DATE OF REPORT (Year, Month, Day) 5/20/91	15. PAGE COUNT 5	
16. SUPPLEMENTARY NOTATION						
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) recursive algorithm, ray parameters			
FIELD	GROUP	SUB-GROUP				
19. ABSTRACT (Continue on reverse if necessary and identify by block number)  See Reverse.						
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION			
22a. NAME OF RESPONSIBLE INDIVIDUAL Norman Bleistein			22b. TELEPHONE (Include Area Code) (303) 273-3557		22c. OFFICE SYMBOL	

### Abstract

Under the assumption of high frequency, wave propagation can be described by the ray method. Many quantities in wave phenomena are characterized by some ray parameters and their derivatives. When we solve an equation containing ray parameter, we also require analytic expressions for the derivatives. We can use finite difference approximation for these derivatives. This type of approximation fails at caustics. Also, the accuracy and stability of the calculation are influenced by the smoothness of interfaces and the spacing between adjacent rays.

In this paper, a recursive algorithm for ray-parameter derivatives is given when the medium is made up of constant-velocity layers separated by arbitrary, smooth interfaces. The algorithm uses derivatives of interfaces, intersection points of rays with the interfaces, and angles of incidence and transmission, all of which are easily computed. A conclusion of these formulas is that no ray caustic exists for transmission or for the reflection from any one of the reflectors when the interfaces are flat.

Statement A per telecon Dr. Lau  
 ONR/Code 1111MA  
 Arlington, VA 22217-5000

NWW 10/2/91

Accession For	
NTIS CRA&I	J
DTIC TAB	
Unannounced	
Justification	
By	
Distribution/	
Availability	
Dist	Availability of Special
A-1	

