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AUGUST 1, 1987 - MAY 31, 1991

INERTIAL MANIFOLDS FOR NAVIER-STOKES EQUATIONS AND RELATED DYNAMICAL SYSTEMS

Principal Investigators: Mitchell Luskin and George R. Sell

SUMMARY

The support from AFOSR/DARPA received over this four year period has enabled the research team at the University of Minnesota to make substantial strides in their efforts to better understand the long-time dynamics of the Navier-Stokes equations and related dynamical systems. The accomplishments on this project, which are described below, have had the effect of giving the University of Minnesota the reputation of being one of the leading research centers in the world in the area of the dynamics of the Navier-Stokes equations. Two major scientific breakthroughs were recently made in the study of the dynamics of the Navier-Stokes equations by scientists working on this project.

Research Highlights. (1) The first, which deals with the study of the dynamics of the Navier-Stokes equations in 3D, was made recently by George R. Sell in collaboration with Geneviève Raugel, (see Raugel and Sell (1989, 1990)). Specifically they have proved the existence of a global, regular attractor for the weak solutions of the Navier-Stokes equations on thin 3D domains. The existence of a global, regular attractor for a 3D problem, even on a thin domain, is a surprising result. This is especially interesting because, in addition, the dynamics on the attractor are far from trivial. This is the first time that anyone has been able to prove the existence of a global, regular attractor (with nontrivial dynamics) for any Navier-Stokes problem in 3D.

(2) The second major breakthrough was in the area of the 2D Navier-Stokes equations. It had been unknown whether the long-time dynamics of the 2D Navier-Stokes equation could be completely described by the dynamics of a finite system of ODEs. By using a very ingenious nonlinear change of variables, Dr. Minkyu Kwak, a recent Ph.D student of George Sell, solved this problem. His approach was to use the nonlinear change of variables to imbed the Navier-Stokes equations into a system of reaction diffusion equations. This imbedding, which is now being called the Kwak Transformation by some experts, is chosen so that it preserves all the dynamics of the Navier-Stokes equation. More importantly, the system of reaction diffusion equations has only algebraic nonlinearities (i.e., no derivative terms), and consequently, it has an inertial manifold. As a result, the dynamics of the associated inertial form then completely describes the long-time dynamics of the original 2D Navier-Stokes equations. The work of Kwak has already attracted wide spread interest. For example, his paper was recently presented at an International Conference on Turbulence Modeling at Arizona State University, and it was generally felt to be the most important new result presented at this meeting.

The one-year no-cost extension for this project was very valuable, because it enabled the researchers to begin a serious numerical investigation of the long-time dynamics of the Kolmogorov flow. The results of this investigation are now being recorded on a video, and a copy of this video will be sent to the AFOSR/DARPA Program Directors in the very near future.

ACCOMPLISHMENTS

The major scientific accomplishments on this project are the following:

- NEW DEVELOPMENTS IN NAVIER-STOKES DYNAMICS.
- GLOBAL REGULARITY FOR THE 3D NAVIER-STOKES EQUATIONS.
- INERTIAL FORMS FOR THE 2D NAVIER-STOKES EQUATIONS.
- NUMERICAL STUDY OF THE KOLMOGOROV FLOW.
- THEORY OF APPROXIMATION DYNAMICS
- PRINCIPLE OF SPATIAL AVERAGING AND INERTIAL MANIFOLDS.
- APPROXIMATION OF INERTIAL MANIFOLDS.
- EULER-GALERKIN METHOD.
- ELLIPTIC REGULARIZATION METHOD.

Global Regularity for the 3D Navier-Stokes Equations. One of the oldest outstanding problems in the theory of the Navier-Stokes equations (for three-dimensional flows) is that of the global regularity of the solutions of these equations. A related problem is the question of the existence of a global attractor for the solutions of these equations. These problems, which go back to the pioneering work of Leray in the 1930s, are closely connected to engineering and physical problems, such things as wind-shear in atmospheric flows. They are considered by many persons to be two of the really difficult unsolved problems in the area of fluid dynamics.

What was known on this problem (before this recent work) is that the solutions of the Navier-Stokes equations are regular for a finite time interval. What one would like to know is whether or not the solutions are regular for all time, i.e., one wants to understand the global (in time) regularity of solutions. Some results on global regularity had been known for small data (i.e., initial data and boundary data), but nothing was known about the large data problem.

The break through made by the scientists on this project (George R. Sell in collaboration with Geneviève Raugel) is to prove the global regularity of solutions with large data for the three dimensional Navier-Stokes equations on thin domains. (An example of such a thin domain is the region between two spheres of large radius. As a result this theory applies to the study of atmospheric flows.) These results are reported on in References numbered 10 and 12.

It is expected that future extensions of this work will have direct impact on the study

of wind-shear phenomena. This is important for the design and control of all types of aircraft, and for weather prediction.

Inertial Forms for the 2D Navier-Stokes Equations. The proof of the existence of finite dimensional inertial forms by Dr. Minkyu Kwak for the 2D Navier-Stokes equations was rather surprising. Most of the leading experts in the world in inertial manifold theory had tried to do this, but with no success. In a related paper, Kwak showed that another class of partial differential equations, a class which includes Burgers equation, has the finite dimensional inertial form property.

Kolmogorov Flow. George Sell, together with his postdoctoral fellow, Yin Yan, set out to compute the flow of 2D incompressible Navier-Stokes equations using a Cray-2. The computational result is animated on the Silicon Graphics IRIS (SGI). The RLE files for the animation are loaded to an Abekas and recorded to tape by Betacam. This computational work was done by using the facilities of the Army High Performance Computing Research Center at the University of Minnesota.

The theoretical study proves that there exists absorbing balls for the spatially discretized Navier-Stokes equations using finite difference schemes. The radii of this ball for the discretized Sobolev norms are bounded from above by a constant independent of the spatial mesh sizes, and it attracts all trajectories at the exponential rate. The existence of discrete global attractors is implied. Further study shows that the Hausdorff dimensions of these discrete global attractors are also bounded from above by a constant independent of the spatial mesh sizes, see References 19 and 20.

For accuracy, it is proved that for trajectories, with the finite difference discretization, the error for a semi-discrete solution can be reduced to one order of the spatial mesh size, with multiplication of a constant dependent of the time, plus a half order of the spatial mesh size, due to the strong non-conforming property of the finite difference scheme. The constant involved in the error estimate grows exponentially in time. This is expected in practice: even in the case that there is *no* computational error at all, generally a small perturbation of the initial condition can cause two trajectories to split apart at the exponential rate. This is particularly true for turbulent cases. The exponential growth in time involved in the error estimates is unavoidable. In practice, this kind of estimate is unapplicable. It is shown that in combining the dissipativity of semi-discrete and continuous solutions, *local* error estimates imply *global* error estimates. Instead of considering local objects (points, trajectories, etc.), one may consider global objects (sets, attractors, etc.). The theory proves that under the assumption of moderate smoothness of the solutions and dissipativity (existence of global attractors) for both continuous and semi-discrete systems, the *time-dependent* n -th order error estimates for trajectories imply the *time-free* n -th order error estimates to a tolerance for attractors. This gives a definitive answer for long-time flow simulation by supercomputers. Starting from an initial value, one computes the flow at fine meshes for a long period of time. The accumulation of the error makes

the distance between the discrete trajectory and the true trajectory bigger and bigger, possibly at the exponential rate. The computational result does not show the behavior of the original theoretical *trajectory* any more. But it still gives valuable information of the *system*. It eventually reflects the dynamical behavior *around the global attractor* of a dissipative system. If the initial data is close enough to the global attractor, one is assured that the simulation gives the information of the global attractor from the very beginning.

A special case of the Navier-Stokes flow, the so-called Kolmogorov flow, is simulated. The external force is the Kolmogorov force, where the first component is a sine function of the second spatial variable, and the second component is zero. The Kolmogorov flow is interesting for dynamical studies because it has an unstable stationary state. Two groups of data are chosen for the viscosity constant and the force frequency. The first group of parameters gives a bigger viscosity constant, lower frequency and milder force; the second group gives smaller viscosity, higher frequency and stronger force. The simulation starts from an initial state which is obtained by a small perturbation of the stationary solution along its unstable direction. The dissipativity and the time-free error estimates to a tolerance for the global attractors imply that the simulating flow is close to the global attractor. For the first group of parameters, the computation shows that the flow leaves the stationary solution quickly and enters a chaotic region. The solution is very oscillatory in the chaotic region. After a short while, the oscillatory components of the flow are damped out and it approaches another stationary solution and wanders around for quite a while. It finally leaves the stationary solution. The animations show an interesting evidence for the time-delay property studied in Pliss and Sell (1990) and Sell and Yan, (1991). For both groups of data, four animations of meshes 32×32 , 64×64 , 128×128 and 256×256 are synchronized to a big animation. As is heuristically predicted by the theory, for the two groups of parameters, the degree of freedom is bounded by 1.44×10^5 and 2.30×10^6 , which suggests the meshes are coarser than 380×380 and 1520×1520 for two cases. The computation shows that 128×128 and 256×256 already give fairly accurate results. The theory is demonstrated by the simulation that *beyond certain limits, finer meshes do not give more details of the turbulence*. It is worth pointing out that theoretical bounds for prediction are generally *not* optimal. Even optimal bounds are for the worst case. This is the reason for the fact that the flows can be simulated at meshes coarser than what theories predict.

The giant 512 million-word Cray-2 memory and its great speed of computation are big advantages for the large-scale flow simulation. Using the simple and efficient finite difference scheme for spatial discretization and explicit method for time discretization, the data files for the animations of vorticity, usually of hundreds of megabytes, are generated from Cray-2 at the speed of 70 – 100 Mflops. Real valued data files are converted into byte (gray scaled) raster files by calling the "fltras" subroutine on Cray-2 at a great speed. Raster files are passed to the local SGI workstations and are loaded to the SGIs memory or raw disks to display the animations on SGIs.

Using the Utah Raster Toolkit (URT), the animations, composited with scripts and comments, are loaded to the Abekas-A60 digital image storage device, and then recorded to the Betacam BVW-75 analog tape recorder. Static pictures of some frames of the animations are taken by using the Solitaire Image Recorder.

The study for the computation of Navier-Stokes is being carried out on the Connection Machine (CM 2). In addition to the finite difference discretization, spectral projection methods are particularly considered, for the convenience of the existing multi-dimensional FFT library routine in the Connection Machine Scientific Software Library.

Codes are massively parallelized and executed by using the newest CM slicewise mode software and by attaching to CM's 8K, 16K or even 32K processors. With FORTRAN-C interfacing, huge data files are output to the Data Vault parallelly with little cost. In addition to the SGIs, Abekas, Betacam and Solitaire, the Framebuffer of CM-2 will be used for more efficient graphics production.

As part of the joint project with Michael Jolly and George Sell, our main purpose is to apply the inertial manifold and approximate inertial manifold theory to solve the Navier-Stokes equations, mainly by using the Connection Machine, together with the Cray-2 and the Cray-XMP. Rather than solving a system of thousands or even millions of algebraic equations obtained from discretizing the continuous Navier-Stokes equations, one expects to solve much fewer equations to replicate the dynamics obtained by solving the Navier-Stokes equations by using classical methods.

For the traditional discretization schemes of partial differential equations, one replaces a partial differential equation by a system of ordinary differential equations. When the mesh size is small, this system of ordinary equations is very large. In the current work, systems of up to 200,000 ordinary equations are solved. Theoretically, solving the Navier-Stokes equations is equivalent to solving a system of infinitely (countably) ordinary differential equations. The inertial and approximate inertial manifold theory shows that this tedious work can be replaced by solving a relatively smaller system of ordinary differential equations. The approximate inertial manifold theory will be applied to solve the Navier-Stokes equations. This theory proves that despite the fact that it is still an open problem for the existence of the inertial manifolds for Navier-Stokes equations, one can construct approximate inertial manifolds to greatly reduce the amount of computation for the study of the dynamical behavior of the Navier-Stokes equations Sell (1988).

Approximation Dynamics. In References Number 11, 13, and 18, we present a new theory of approximation dynamics. In particular, we present a general theory of approximate inertial manifolds (AIM) for nonlinear dissipative dynamical systems on infinite dimensional Hilbert spaces. The goal of this theory is to prove the Basic Theorem of Approximation Dynamics, wherein we show that there is a fundamental connection between the order of the approximating manifold and the amount of long-time dynamical information which is preserved by the approximation. We also present a new general method

(the Gamma Method) for the construction of an AIM. We show that this construction applies to the Navier-Stokes equations on any bounded region in 2D (and on certain thin 3D regions) as well as to reaction diffusion equations in any space dimension. All these equations have good AIMs which preserve the essential dynamics of the global attractor.

Principle of Spatial Averaging. The Principle of Spatial Averaging is described in the paper by John Mallet-Paret and George R. Sell, *Inertial manifolds for reaction diffusion equations in higher space dimensions*, see Reference Number 1 below. This is an important contribution to the theory of inertial manifolds because it allows one to prove the existence of inertial manifolds in certain situations (such as space dimension 3) where the spectral gap condition is not satisfied. This paper contains some of the sharpest known results on the theory of inertial manifolds.

Approximation of Inertial Manifolds. The major thrust of the research efforts on the current DARPA project over the last two years has been in the area of the approximation of inertial manifolds. Seven of the thirteen research papers described below deal with various aspects of this topic. (See Numbers 6-9, 11, 13, and 18.) One of the major goals of our research is the attempt to discover new algorithms for the approximation of inertial manifolds. Three such algorithms, the EULER-GALERKIN METHOD, the METHOD OF ELLIPTIC REGULARIZATION, and the GAMMA METHOD are announced and analyzed in these papers. The Gamma Method is an important feature of a new theory, which we call *Approximation Dynamics* and which is described below.

RESEARCH PAPERS COMPLETED

1. John Mallet-Paret and George R. Sell, *Inertial manifolds for reaction diffusion equations in higher space dimensions*, IMA Preprint No. 331, June, 1987, Journal American Mathematical Society, vol 1, 1989, pp. 805-866.

Summary: In this paper we show that the scalar reaction diffusion equation

$$u_t = \nu \Delta u + f(x, u), \quad u \in R$$

with $x \in \Omega_n \subset R^n$ ($n=2,3$) and with Dirichlet, Neumann, or Periodic Boundary conditions, has an inertial manifold when (1) the equation is dissipative, and (2) f is of class C^3 and for $\Omega_3 = (0, 2\pi)^3$ or $\Omega_2 = (0, 2\pi/a_1) \times (0, 2\pi/a_2)$, where a_1 and a_2 are positive. The proof is based on an (abstract) Invariant Manifold Theorem for flows on a Hilbert space. It is significant that on Ω_3 the spectrum of the Laplacian Δ does not have arbitrary large gaps, as required in other theories of inertial manifolds. Our proof is based on a crucial property of the Schroedinger operator $\Delta + v(x)$, which is valid only in space dimension $n \leq 3$. This property says that $\Delta + v(x)$ can be well approximated by the constant coefficient problem $\Delta + \bar{v}$ over large segments of the Hilbert space $L^2(\Omega)$, where $\bar{v} = (\text{vol } \Omega)^{-1} \int_{\Omega} v dx$ is the average value of v . We call this property the Principle of Spatial Averaging. The proof

that the Schroedinger operator satisfies the Principle of Spatial Averaging on the regions Ω_2 and Ω_3 described above follows from a Gap Theorem for Finite Families of Quadratic Forms, which we present in an Appendix to this paper.

2. Kenneth R. Meyer and George R. Sell, *Melnikov transforms, Bernoulli bundles and almost periodic perturbations*, IMA Preprint 358, Transactions of the American Mathematical Society, vol 314, 1989, pp. 63-105.

Summary: In this paper we study nonlinear time-varying perturbations of an autonomous vector field in the plane R^2 . We assume that the unperturbed equation, i.e. the given vector field has a homoclinic orbit and we present a generalization of the Melnikov method which allows us to show that the perturbed equation has a transversal homoclinic trajectory. The key to our generalization is the concept of the Melnikov transform, which is a linear transformation on the space of perturbation functions. The appropriate dynamical setting for studying these perturbation is the concept of a skew product flow. The concept of transversality we require is best understood in this context. Under conditions whereby the perturbed equation admits a transversal homoclinic trajectory, we also study the dynamics of the perturbed vector field in the vicinity of this trajectory in the skew product flow. We show the dynamics near this trajectory can have the exotic behavior of the Bernoulli shift. The exact description of this dynamical phenomenon is in terms of a flow on a fiber bundle, which we call, the Bernoulli bundle. We allow all perturbations which are bounded and uniformly continuous in time. Thus our theory includes the classical periodic perturbations studied by Melnikov, quasi periodic and almost periodic perturbations, as well as toroidal perturbations which are close to quasi periodic perturbations.

3. Ciprian Foias, Basil Nicolaenko, George R. Sell, and Roger Témam, *Inertial manifolds for the Kuramoto-Sivashinsky equation and the lowest estimate for their dimensions*, J. Math. Pures Appl., vol 67, 1988, pp. 197-226.

Summary: In this paper we show that the Kuramoto-Sivashinsky (KS) equation in one space dimension has inertial manifolds for every value of the cell size L . One objective is to carefully estimate the lowest dimension of the inertial manifold $\mathfrak{M}$. We show that there is a constant C_1 , which does not depend on L , such that $\dim \mathfrak{M} \leq (1 + C_1 L^{7/2})$. This can be compared with known estimates of the Hausdorff dimension d_H of the global attractor A for the KS equation, which is given by $d_H \leq C_2 L^{3/2}$, for some constant C_2 .

4. Mario Taboada, *Finite dimensional asymptotic behavior for the Swift-Hohenberg model of convection*, Nonlinear Analysis, TMA.

Summary: We study the asymptotic behavior of the equation

$$u_t + u_{xxxx} + u_{xx} + \alpha u + uu_x = 0$$

for $x \in I = [-L/2, L/2]$ with boundary condition

$$u = u_x = 0 \quad \text{on } \partial I.$$

This equation was introduced by Swift and Hohenberg as a model for convection. We prove that all solutions enter and are confined within a fixed ball in $L^2(I)$ and, by applying methods developed by Foias, Sell and Témam, we show that the equation has an inertial manifold. This confirms the conjecture of Chaté and Manneville about the low-dimensional behavior of the system.

5. George R. Sell, *Hausdorff and Lyapunov dimensions for gradient systems*, IMA Preprint No. 399, Contemporary Math., vol 99, 1989, pp. 85-92.

Summary: We consider a generic class of gradient systems on a suitable bounded region $\Omega \subset R^m$, where $m = 1, 2, \dots$. Let d_H and d_L denote, respectively, the Hausdorff and Lyapunov dimensions of the global attractor of the gradient system. Our objective in this paper is to derive an asymptotic formula which implies that

$$\frac{d_L}{d_H} \rightarrow \left(\frac{2}{m} + 1 \right)^{m/2} \quad \text{as } d_H \rightarrow \infty.$$

6. Ciprian Foias, George R. Sell, and Edriss S. Titi, *Exponential tracking and approximation of inertial manifolds for dissipative nonlinear systems*, J. Dynamics and Differential Equations, vol 1, 1989, pp. 199-244.

Summary: In this paper we study the long time behavior of solutions for a class of nonlinear dissipative partial differential equations. By means of the Lyapunov-Perron method we show that these equations have an inertial manifold, provided that a certain gap condition in the spectrum of the linear part is satisfied. We verify that the constructed inertial manifold has the property of exponential tracking (i.e., stability with asymptotic phase, or asymptotic completeness), which makes it a faithful representative to the relevant dynamics of the equation. This theory of inertial manifolds allows us to introduce a modified Galerkin approximation for analyzing the original PDE. In an illustrative example (which we believe to be typical), we show that this modified Galerkin approximation yields a smaller error than the standard Galerkin approximation.

7. Ciprian Foias, Michael S. Jolly, I. G. Kevrekidis, George R. Sell, and Edriss S. Titi, *On the computation of inertial manifolds*, Physics Letters A, vol 131, 1988, pp. 433-436.

Summary: A modified Galerkin (the *Euler-Galerkin*) algorithm for the computation of inertial manifolds is described and applied to a reaction diffusion equation and the Kuramoto-Sivashinsky (KS) equation. In the context of the KS equation, a low-dimensional Euler-Galerkin approximation ($n = 3$) is distinctly superior to the traditional

Galerkin of the same dimension, and comparable to a traditional Galerkin of a much higher dimension ($n = 16$).

8. Mitchell Luskin and George R. Sell, *Approximation theories for inertial manifolds*, Proceedings of the Luminy Conference on Infinite Dimensional Dynamical Systems, Math. Modelling and Numerical Anal.

Summary: During the last few years it has been shown that some infinite dimensional nonlinear dissipative evolutionary equations have inertial manifolds. This discovery has profound significance in the study of the long-time behavior of the solutions of these equations for the following reasons:

- The inertial manifold $\mathfrak{M}$ is a positively invariant finite dimensional manifold in the ambient infinite dimensional phase space, and the given evolutionary equations reduces to a finite dimensional ordinary differential equation, an ODE, on $\mathfrak{M}$.
- Every attractor, including the global attractor, lies in $\mathfrak{M}$.
- Every solution of the nonlinear evolutionary equation is tracked at a exponential rate by a solution on $\mathfrak{M}$. This means that there is an $\eta > 0$ such that for every solution $u(t)$ of the original evolutionary system, there is a solution $v(t)$ on $\mathfrak{M}$ such that

$$(0) \quad \|u(t) - v(t)\| \leq K e^{-\eta t}, \quad t > 0,$$

where K depends on $u(0)$.

Because the existence of an inertial manifold implies that the dynamics of the original evolutionary equation is completely described by a finite dimensional ODE, with no error, this should lead to substantial improvements in the computational efficiency of numerical methods used to study the evolutionary equation. In order to realize this efficiency, it is important to find good algorithms for approximating the inertial manifolds. The main objective in this paper is to examine several approximation theories for inertial manifolds. Since every existence theory is a potential spawning ground for an approximation theory, we begin with a brief review of the three known classes of existence theories for inertial manifolds.

The first existence theory uses the Lyapunov-Perron method, which is based on the variation of constants formula. While the Lyapunov-Perron method is very useful for deriving properties of inertial manifolds (in addition to proving existence), it is not a very promising arena for finding a good approximation theory. The main fault of the Lyapunov-Perron method is that it uses backward integration of the evolutionary equation. Since the backward integration is in the “unstable” direction of the evolutionary equation, one will encounter a blow-up of the solutions, which in turn is an inherent source of computational inefficiency.

The second class of existence theories use the Hadamard method, or the graph transform method. The basic idea here is to start with some initial approximation to the inertial manifold. This initial approximation is an easily computed manifold of the correct dimension, call it $\mathfrak{M}_0$. One then lets the dynamics of the given evolutionary equation act on $\mathfrak{M}_0$, thereby obtaining a set $\mathfrak{M}_t$ at each time $t > 0$. One then proves, under suitable hypotheses of course, that each $\mathfrak{M}_t$ is representable as the graph of some function, that the limit $\lim_{t \rightarrow \infty} \mathfrak{M}_t = \mathfrak{M}$ exists, and that $\mathfrak{M}$ is the inertial manifold.

Approximation theories based on the Hadamard method will be better than theories based on the Lyapunov-Perron method because one is integrating forward in time, i.e., in the stable direction. Because of inequality (0) one expects that $\mathfrak{M}_\tau \approx \mathfrak{M}$, for an appropriate $\tau > 0$. Approximation theories based on the Hadamard method try to approximate $\mathfrak{M}_\tau$. Such approximations can be easily implemented when τ is small, or when the constant η in (0) is large. The Euler-Galerkin method, which is introduced in Foias, Sell and Titi (1988) and described in Section 3 below, is an illustration of a Hadamard-type approximation. If the convergence of $\mathfrak{M}_t$ to $\mathfrak{M}$ is slow, then the Hadamard-type approximation theories will require the time parameter τ to be large in order to get good approximations. We expect that in these situations, one will get better approximations by using the following alternative.

The third method for proving the existence of inertial manifolds is based on the method of elliptic regularization which Sacker (1964, 1965, 1969) used in the study of finite dimensional invariant manifolds. The extension of the Sacker method to infinite dimensional dynamical systems is presented in Fabes, Luskin and Sell (1988), and Luskin and Sell (1988).

9. Eugene Fabes, Mitchell Luskin and George R. Sell, *Construction of inertial manifolds by elliptic regularization*, IMA Preprint No. 459, J. Differential Equations, vol 89, 1991, pp. 355-387.

Summary: In many cases an inertial manifold $\mathfrak{M}$ for an infinite dimensional dissipative dynamical system can be represented as the graph of a smooth function Φ from a finite dimensional Hilbert space H^p to another Hilbert space H^q . The invariance property of $\mathfrak{M}$ means that Φ can be written as the solution of a first order partial differential equation

$$\nabla \Phi(p) G_1(p, \Phi(p)) + A\Phi(p) = G_2(p, \Phi(p))$$

over H^p , where G_1 and G_2 are nonlinear functions which depend on the original dynamical system and A is a suitably "stable" linear operator. In this paper we use a method introduced by Sacker (1965), for the study of finite dimensional dynamical systems, to find inertial manifolds in the infinite dimensional setting. This method involves replacing the first order equation for Φ by the regularized elliptic equation

$$-\varepsilon \Delta \Phi + \nabla \Phi(p) G_1(p, \Phi(p)) + A\Phi(p) = G_2(p, \Phi(p)),$$

with suitable boundary conditions. It is shown that if A satisfies a spectral gap condition, then the solutions Φ_ε of the elliptic equation converge as $\varepsilon \rightarrow 0^+$.

10. Geneviève Raugel and George R. Sell, *Équations de Navier-Stokes dans des domaines minces en dimension trois: régularité globale*, Comptes Rendu Paris, vol 309, 1989, pp. 299-303.

Summary: Dans cette note, nous présentons un résultat d'existence et de régularité globales de solutions des équations de Navier-Stokes avec conditions limites périodiques dans un domaine mince, en dimension trois. En outre nous comparons l'attracteur de ces équations avec l'attracteur global d'un système *réduit* d'équations de Navier-Stokes en dimension deux.

11. George R. Sell, *Approximation dynamics: Hyperbolic sets and inertial manifolds*, Minnesota Supercomputer Institute Preprint No. 89/39, March 1989.

Summary: There are three objectives in this paper. First we present a general theory of approximate inertial manifolds (AIM) for nonlinear dissipative dynamical systems on infinite dimensional Hilbert spaces. The goal of this theory is to prove the Basic Theorem of Approximation Dynamics, wherein we show that there is a fundamental connection between the order of the approximating manifold and the amount of long-time dynamical information which is preserved by the approximation. The second objective is to present a new general method for the construction of AIM. Thirdly we show that this construction applies to the Navier-Stokes equations on any bounded region in 2D (and on certain thin 3D regions) as well as to reaction diffusion equations in any space dimension. All these equations have good AIMs which preserve the essential dynamics of the global attractor.

12. Geneviève Raugel and George R. Sell, *Navier-Stokes equations on thin three dimensional domains: Global regularity of solutions I*, IMA Preprint No. 662, May 1990, submitted for publication.

Summary: We examine the Navier-Stokes equations (NS) on a thin 3D domain $\Omega_\varepsilon = Q_2 \times (0, \varepsilon)$, where Q_2 is a suitable bounded domain in R^2 and ε is a small, positive, real parameter. We consider these equations with various homogeneous boundary conditions, especially spatially periodic boundary conditions. We show that there are *large* sets $R(\varepsilon)$ in $H^1(\Omega_\varepsilon)$ and $\delta(\varepsilon)$, in $W^{1,\infty}((0, \infty), L^2(\Omega_\varepsilon))$ such that if $U_0 \in R(\varepsilon)$ and $F \in \delta(\varepsilon)$, then (NS) has a strong solution $U(t)$ that remains $H^1(\Omega_\varepsilon)$ for all $t \geq 0$ and in $H^2(\Omega_\varepsilon)$ for all $t > 0$. We show that the set of strong solutions of (NS) has a local attractor $\mathfrak{A}_\varepsilon$ in $H^1(\Omega_\varepsilon)$, which is compact in $H^2(\Omega_\varepsilon)$. This local attractor $\mathfrak{A}_\varepsilon$ is the global attractor for all the weak solutions (in the sense of Leray) of (NS). We also show that, under reasonable assumptions, $\mathfrak{A}_\varepsilon$ is upper semicontinuous at $\varepsilon = 0$.

13. Victor A. Pliss and George R. Sell, *Perturbation of attractors of differential equations*, J. Differential Equations, vol 92, 1991, pp. 100-124.

Summary: In this paper we study small C^1 -perturbations of a differential equation that has a hyperbolic attractor $\mathcal{K}$. We show that if $\mathcal{K}$ has a suitable Lipschitz property and if the perturbation is small enough, then there is a homeomorphism $h : \mathcal{K} \rightarrow \mathcal{K}^Y$, where $\mathcal{K}^Y$ is a hyperbolic attractor for the perturbed equation. Examples are included.

14. George R. Sell and Mario Taboada, *Attractors for the Kuramoto-Sivashinsky equation in two dimensions*, J. Nonlinear Analysis, TMA, 1991, to appear.

15. George R. Sell and Yuncheng You, *Inertial manifolds: The non-self adjoint case*, J. Differential Equations, to appear.

Summary: In contrast with the existing theories of inertial manifolds, which are based on the self-adjoint assumption of the principal differential operator, in this paper we show that for general dissipative evolutionary systems described by semi-linear parabolic equations with principal differential operator being sectorial and having compact resolvent, there exists an inertial manifold provided that certain gap conditions hold. We also show that by using an elliptic regularization, this theory can be extended to a class of KdV equations, where the principal differential operator is not sectorial.

16. Robert J. Sacker and George R. Sell, *Dichotomies in linear evolutionary equations in Banach spaces* IMA Preprint No. 838, August, 1991, submitted for publication.

Summary: In this paper we present a characterization for the existence of an exponential dichotomy for a linear evolutionary system on a Banach space. The theory we present here applies to general time varying linear equations in Banach spaces. As a result it gives a description of the behavior of the nonlinear dynamics generated by certain nonlinear evolutionary equations in the vicinity of a compact invariant set. In the case of dissipative systems, our theory applies to the study of the flow in the vicinity of the global attractor. The theory formulated here holds for linear evolutionary systems which are uniformly α -contracting and applies to the study of the linearization of nonlinear equations of the following type: (a) parabolic PDEs, including systems of reaction diffusion equations and the Navier-Stokes equations, (b) hyperbolic PDEs, including the nonlinear wave equation and the nonlinear Schrödinger equation with dissipation, (c) retarded differential equations, and (d) certain neutral differential delay equations.

17. Shui-Nee Chow, Kening Lu, and George R. Sell, *Smoothness of inertial manifolds*, IMA Preprint, 1990.

18. George R. Sell, *An optimality condition for approximate inertial manifolds* Dynamical Theories of Turbulence, IMA Proceedings, to appear.

Summary: There are three objectives in this paper. First we present a general theory of approximate inertial manifolds (AIMs) for nonlinear dissipative dynamical systems on infinite dimensional Hilbert spaces. The goal of this theory is to prove the Basic Theorem

of Approximation Dynamics, wherein we show that there is a fundamental connection between the order of the approximating manifold and the amount of long-time dynamical information which is preserved by the approximation. The second objective is to present a new general method for the construction of an AIM. Thirdly we show that this construction applies to the Navier-Stokes equations on any bounded region in 2D (and on certain thin 3D regions) as well as to reaction diffusion equations in any space dimension. All these equations have good AIMs which preserve the essential dynamics of the global attractor.

19. Yin Yan, *Dimensions of attractors for discretizations for Navier-Stokes Equations*, AHPCRC Preprint 91-01, J. Dynamics and Differential Equations, to appear.

Summary: In this paper, we discretize the 2D Navier-Stokes equations with periodic boundary conditions by the finite difference method. We prove that with a shift for discretization the global solutions exist. After proving some discrete interpolated Sobolev inequalities in the sense of finite differences, we prove the existence of the global attractors of our discretized model, and we estimate the upper bounds for the Hausdorff and the fractal dimensions of the attractors, which are independent of the mesh size. These bounds are considerably close to those of continuous version.

20. Yin Yan, *Attractors and error estimates for discretizations of incompressible Navier-Stokes Equation*, AHPCRC Preprint, submitted for publication.

Summary: By imbedding sets of nodal values to function spaces, we apply variational arguments to finite difference approximations to the 2D incompressible Navier-Stokes equations. In addition to proving error estimates for trajectories, we prove time-free error estimates to a tolerance for attractors. An argument of applying our techniques to finite element approximations is also given.

21. Minkyu Kwak, *Finite dimensional description of convective reaction-diffusion equations*, AHPCRC Preprint 91-29, J. Dynamics and Differential Equations, to appear.

Summary: We are concerned with the asymptotic dynamics of a certain type of semi-linear parabolic equation namely, $u_t = uxx + (f(u))x + g(h) + h(x)$ on the interval $[0, L]$. Under the general condition we prove that this equation admits a dissipative dynamical system and it possesses the global attractor. But for large $L > 0$, we do not know whether there exists an inertial manifold or not. Here we introduce a nonlinear change of variables so that we transform the above equation to reaction diffusion system which possess the exactly same asymptotic dynamics. We then prove the existence of inertial manifold for the transformed equation, thereby we find the ordinary differential equation which describe completely the long-time dynamics of the original equation.

22. Minkyu Kwak, *Finite dimensional inerital forms for the 2D Navier-Stokes equations*, AHPCRC Preprint 91-30, submitted for publication.

Summary: In this paper we explain how the long time dynamics of 2D Navier-Stokes (N-S) equations with periodic boundary conditions on a suitable bounded region $1/2$ in R^2 can be described completely by a finite dimensional system of ordinary differential equations. Our approach is to imbed the 2D N-S equations into a reaction diffusion system which possesses the exactly same asymptotic dynamics. We then prove the existence of inertial manifold for the transformed equations and we interpret the dynamics of N-S equations via the inertial form of the transformed equations.

23. Mitchell Luskin and George R. Sell, *The construction of inertial manifolds for reaction diffusion equations by elliptic regularization*, IMA Preprint.

Summary: We demonstrate that the method of elliptic regularization developed in Fabes, Luskin, and Sell (1991) can be used to construct invariant manifolds for reaction diffusion equations.

24. Mitchell Luskin and George R. Sell, *Parabolic regularization and inertial manifolds*, in preparation.

Preliminary Summary: In many cases an inertial manifold $\mathfrak{M}$ for an infinite dimensional nonlinear evolutionary equation

$$p' = G_1(p, q), \quad q' + Aq = G_2(p, q)$$

can be represented as the graph of a function $\Phi : H^p \rightarrow H^q$, where H^p is a finite dimensional space and H^q is infinite dimensional. One method for proving the existence of Φ is the Lyapunov-Perron method, wherein Φ is a fixed point of the integral operator

$$T\Phi(p_0) = \int_{-\infty}^0 e^{As} G_2(p(s), \Phi(p(s))) ds,$$

and $p(t)$ is the solution of $p' = G_1(p, \Phi(p))$ satisfying $p(0) = p_0$. A second method for proving the existence of Φ is the Sacker method of elliptic regularization where Φ is realized as the weak limit (as $\varepsilon \rightarrow 0^+$) of solutions Φ_ε of the nonlinear elliptic equation

$$-\varepsilon \Delta \Phi + D\Phi G_1(p, \Phi) = G_2(p, \Phi) - A\Phi,$$

with suitable boundary conditions. In this paper we present a theory which combines these two approaches. As a result, we can show that the weak limit of Φ_ε is a smooth C^1 -function and that the error term satisfies $\|\Phi - \Phi_\varepsilon\|_{0,\infty} \leq L_1 \varepsilon^{1/2}$ for some constant L_1 . Moreover, we show that Φ_ε converges to Φ in the uniform C^1 topology on H^p . We also derive two results which describe the exponential attraction properties of the inertial manifold $\mathfrak{M} = \text{Graph } \Phi$ and the approximate inertial manifolds $\mathfrak{M}_\varepsilon = \text{Graph } \Phi_\varepsilon$, for $\varepsilon > 0$.

INVITED LECTURES

George R. Sell

1. Howard Conference on Semigroups, PDEs and Attractors. August, 1987. *Melnikov Transform and Bernoulli Bundles for Almost Periodic Perturbations.*
2. Equadiff 87 Conference, Xanthi, Greece. August, 1987. *Inertial Manifolds: Existence and Approximation Theories.*
3. Mathematics Institute, University of Heidelberg. September, 1987. *Inertial Manifolds for Gradient Systems.*
4. Czechoslovak Summer School on Dynamics and Differential Equations. September, 1987. *The Principle of Spatial Averaging and Inertial Manifolds.*
5. Luminy Conference on Infinite Dimensional Dynamical Systems, Marseilles, France. September, 1987. *Inertial Manifolds: Existence and Approximation Theories.*
6. Applied Mathematics Seminar, Ecole Polytechnique, Palaiseau, France. September, 1987. *Poincaré-Bendixson Theory for Differential Delay Equations.*
7. Applied and Computational Mathematics Program (DARPA) Conference, Washington, DC. October, 1987. *Inertial Manifolds: Existence and Approximation Theories.*
8. Earth Sciences Center Seminar, Penn State University. December, 1987. *Homoclinic Orbits: A Source of Chaos.*
9. Dynamics Days, University of Houston. January, 1988. *Inertial Manifolds: Approximation Theories.*
10. Special Session on Nonlinear Differential-Delay Equations, AMS Annual Meeting, Atlanta. January, 1988. *Exponential Dichotomies for Linear Differential Equations in Banach Spaces.*
11. Twin Cities Urban Math Collaborative. January, 1988. *The Mad Dogs of Calais.*
12. Summer School on Differential Equations and Dynamical Systems (A conference in honor of Jack Hale's 60th birthday), Campinas, Brazil, February, 1988. *Approximation Theories for Inertial Manifolds.*
13. Conference on Differential Equations and Population Dynamics, Memorial Conference for Geoffrey Butler, University of Alberta, Edmonton, Canada, June, 1988. *Approximation Theories for Inertial Manifolds.*
14. IMA Program on Signal Processing, July, 1988. *Introduction to Inertial Manifolds.*
15. Colloquium Lecture, Courant Math. Inst., New York, October, 1988. *Elliptic Regularization and Inertial Manifolds.*
16. PDE Seminar Lecture, University of Minnesota, October, 1988. *Global Regularity of Solutions of the 3D Navier Stokes Equations.*
17. Chemical Engineering and Material Science Department Colloquium, University of Minnesota, November, 1988. *Inertial Manifolds for Chemical Reaction Dynamics.*

18. Seminar Lecture, Indiana University, November, 1988. *Global Regularity of Solutions of the 3D Navier Stokes Equations.*
19. AMS Special Session on Differential Equations, Claremont, California, November, 1988. *Melnikov Transforms and Bernoulli Bundles for Almost Periodic Perturbations.*
20. Conference on Differential Equations, University of Southern California, November, 1988. *Inertial Manifolds and a Theorem of Sacker.*
21. Seminar Lecture, Institute for Advanced Study, Princeton, December, 1988. *The Principle of Spatial Averaging and Inertial Manifolds.*
22. Seminar Lecture, Institute for Advanced Study, Princeton, December, 1988. *Global Regularity of Solutions of the 3D Navier Stokes Equations.*
23. Seminar Lecture, Université de Paris-Sud, Orsay, France, December, 1988. *Melnikov Transforms and Bernoulli Bundles for Almost Periodic Perturbations.*
24. Colloquium Lecture, Department of Applied Mathematics, Ecole Polytechnique, Palaiseau, France, December, 1988. *Elliptic Regularization and Inertial Manifolds.*
25. MAA Invited Address, Annual AMS-MAA Meetings, Phoenix, January, 1989. *Inertial Manifolds.*
26. Los Alamos, Center for Nonlinear Phenomena, February, 1989. *Approximation Dynamics for Dissipative Systems.*
27. Colloquium Lecture, University of New Mexico, February, 1989. *Approximation Dynamics for Dissipative Systems.*
28. Seminar Lecture, University of New Mexico, February, 1989. *Elliptic Regularization and Inertial Manifolds.*
29. Control Theory Sciences Colloquium Lecture, University of Minnesota, February, 1989. *Approximation Dynamics for Dissipative Systems.*
30. Mathematics Colloquium Lecture, Cornell University, March, 1989. *Approximation Dynamics for the Navier-Stokes Equations.*
31. Mathematics Colloquium Lecture, Georgia Institute of Technology, April, 1989. *Approximation Dynamics for the Navier-Stokes Equations.*
32. Seminar Lecture, Georgia Institute of Technology, May, 1989. *Elliptic Regularization and Inertial Manifolds.*
33. Mathematics Colloquium Lecture, University of Arizona, May, 1989. *Approximation Dynamics for the Navier-Stokes Equations.*
34. Seminar Lecture, Moscow State University, June 1989. *Approximation Dynamics for the Navier-Stokes equations and Related Dynamical Systems.*
35. Seminar Lecture, Moscow State University, June 1989. *Global Regularity of Solutions of the Navier-Stokes Equations on Thin Three Dimensional Domains.*
36. Seminar Lecture, Steklov Mathematics Institute, Moscow, June 1989. *The Principle of Spatial Averaging and Inertial Manifolds.*

37. Seminar Lecture, Steklov Mathematics Institute, Leningrad, June 1989. *Global Regularity of Solutions of the Navier-Stokes Equations on Thin Three Dimensional Domains.*
38. Seminar Lecture, Leningrad State University, June 1989. *Approximation Dynamics: Hyperbolic Sets and the Pliss Reduction Principle.*
39. Invited Lecture, DARPA Summer School Conference on Mathematical Models and Manufacturing Science, July 1989. *Inertial Manifolds for Problems in Chemical Engineering.*
40. Invited Address, U.S. - Japan Conference on Dynamical Systems. July 1989. *Global Regularity of Solutions of the Navier-Stokes Equations on Thin Three Dimensional Domains.*
41. Seminar Lecture, University of Tokyo, July 1989. *Approximation Dynamics for the Navier-Stokes equations and Related Dynamical Systems.*
42. Colloquium Lecture, Institute for Mathematics and its Applications, October, 1989. *Introduction to Inertial Manifolds.*
43. Invited Lecture, Conference on Differential Equations, Conference in Honor of Kenneth Cooke's 65th Birthday, Claremont University, February, 1990. *Global Regularity of Solutions of the Navier-Stokes Equations on Thin Three Dimensional Domains.*
44. Invited Lecture, AMS-SIAM Minisymposium on Low Dimensional Structures in Dynamics, Albuquerque, March, 1990. *Approximation Dynamics for the Navier-Stokes equations and Related Dynamical Systems.*
45. Invited Lecture, Southeastern Conference on Differential Equations, Birmingham, Alabama, March, 1990. *Approximation Dynamics for the Navier-Stokes equations and Related Dynamical Systems.*
46. Invited Lecture, SIAM Conference on Dynamical Systems, Orlando, April, 1990. *Global Regularity of Solutions of the Navier-Stokes Equations on Thin Three Dimensional Domains.*
47. Colloquium Lecture, Mathematics Department, University of Wisconsin, Milwaukee, April, 1990. *Approximation Dynamics for the Navier-Stokes equations and Related Dynamical Systems.*
48. Invited Lecture, Conference on Dynamical Systems Theories of Turbulence, Institute for Mathematics and its Applications, May, 1990. *An optimality condition for approximate inertial manifolds.*
49. Colloquium Lecture, Mathematics Institute, Ludwig Maximilian University, Munich, Germany, July, 1990. *Global Attractors for the Navier-Stokes Equations on Thin Three Dimensional Domains.*
50. Invited Lecture Series, Summer Institute on Infinite Dimensional Dynamical Systems, Blaubauren, Germany, July, 1990. *Approximation Methods for Infinite Dimensional*

Dynamical Systems.

51. Colloquium Lecture, Weierstrass Mathematics Institute, East German Academy of Sciences, Berlin, July, 1990. *Global Attractors for the Navier-Stokes Equations on Thin Three Dimensional Domains.*

52. Invited Lecture, International Conference on Functional Differential Equations, Kyoto, August, 1990. *Global Attractors for the Navier-Stokes Equations on Thin Three Dimensional Domains.*

53. Invited Lecture, Conference on Differential Equations, Conference in Honor of Henry Antosiewicz's 65th Birthday, University of Southern California, Los Angeles, September, 1990. *Perturbations of Attractors of Differential Equations*

54. Colloquium Lecture, Mathematics Department, Memphis State University, Memphis, October, 1990. *Approximation Dynamics for the Navier-Stokes equations and Related Dynamical Systems.*

55. Colloquium Lecture, Leningrad State University, Leningrad, October, 1990. *Approximation Dynamics for the Navier-Stokes equations and Related Dynamical Systems.*

56. Colloquium Lecture, Mathematics Department, Virginia Polytechnical Institute, Blacksburg, November, 1990. *Approximation Dynamics for the Navier-Stokes equations and Related Dynamical Systems.*

57. Seminar Lecture, Mathematics Department, Universita di Roma, II, March, 1991. *Global Attractors for the Navier-Stokes Equations on Thin Three Dimensional Domains.*

58. Invited Lecture, Oberwolfach Conference on Invariant Manifolds for Differential Equations, March, 1991. *New Developments in Navier-Stokes Dynamics.*

59. Colloquium Lecture, Mathematics Department, Marquette University, April, 1991. *Global Attractors for the Navier-Stokes Equations on Thin Three Dimensional Domains.*

60. Invited Lecture, International Conference on Turbulence, Arizona State University, Tempe, May, 1991. *Global Attractors for the Navier-Stokes Equations on Thin Three Dimensional Domains.*

61. Colloquium Lecture, Mathematics Department, Michigan State University, May, 1991. *Global Attractors for the Navier-Stokes Equations on Thin Three Dimensional Domains.*

Mitchell Luskin :

1. Heriot-Watt University, Edinburgh, Scotland. August, 1987. *Minimum Energy Configurations for Liquid Crystals: Computational Methods and Results.*

2. Oberwolfach Mathematics Institute, West Germany. August, 1987. *Minimum Energy Configurations for Liquid Crystals: Computational Methods and Results.*

3. University of Chicago, Conference on Advances in Computational Modeling and Numerical Analysis, September, 1987. *Numerical Results for Liquid Crystals.*

4. University of Wyoming, Mathematics Colloquium. October, 1987. *Defects and Transitions in Liquid Crystals*.
5. Carnegie-Mellon University and University of Pittsburgh, Applied Mathematics Colloquium. November, 1987. *Defects and Transitions in Liquid Crystals*.
6. University of Pavia, Italy. January, 1988. *Defects and Transitions in Liquid Crystals*.
7. University of Nice, Conference on Partial Differential Equations and Continuum Models of Phase Transitions, January, 1988. *Defects and Transitions in Liquid Crystals*.
8. Caltech, Applied Mathematics Colloquium, February, 1988. *Defects and Transitions in Liquid Crystals*.
9. UCLA, Applied Mathematics Colloquium, February, 1988. *Defects and Transitions in Liquid Crystals*.
10. Conference on Group Theoretic and Analytic Methods in Continuum Mechanics, Cornell University, June, 1988. *Computational results for phase transitions in shape-memory materials*.
11. Caltech Seminar, July 1988. *Computational results for shape memory materials*
12. Virginia Polytechnic Institute, Workshop on Smart Materials, Structures, and Mathematical Issues, September, 1988. *Computational results for phase transitions in shape memory materials*.
13. Indiana University, Institute for Applied Mathematics and Scientific Computing, Seminar Lecture, September, 1988. *Construction of invariant manifolds by elliptic regularization*.
14. Technion - Israel Institute of Technology, March, 1989. *Numerical results for some phase transitions in crystals*.
15. Tel Aviv University, Seminar in Applied Mathematics, March, 1989. *Numerical results for some phase transitions in crystals*.
16. Weizmann Institute of Science, March, 1989. *Numerical results for some phase transitions in crystals*.
17. Minisymposium lecture, Annual SIAM Meeting, San Diego, July, 1989. *The computation of microstructure for crystals*.
18. UCLA, October, 1989.
19. Istituto Applicato Calcolo, Rome, Italy, December, 1989.
20. Trento, Italy, Meeting on "Calculus of variations, elasticity, and crystals," December, 1989.
21. Oberwolfach, West Germany, Meeting on "Theory and numerical methods for initial-value problems," December, 1989.
22. University of California, Irvine, January, 1990.
23. US-Japan Workshop on Smart Materials, Honolulu, March, 1990.

24. University of Paris XI, NATO Workshop on "Defects, Singularities, and Patters in Nematic Liquid Crystals," June, 1990.
25. Ecole Polytechnique, Palaiseau, France, June, 1990.
26. University of Metz, France, Metz Days Meeting, June, 1990.
27. Princeton University, Applied Math Seminar, November, 1990.
28. New Jersey Institute of Technology, November, 1990.
29. Mathematical Sciences Research Institute, Berkeley, California, January, 1991.
30. Americal Mathematical Society Annual Meeting, Special Session, January, 1991.
31. Laval University, Quebec, January, 1991.
32. American Association for the Advancement of Science Annual Meeting, February, 1991.
33. Kent State University, March, 1991.
34. Carnegie Mellon University, March, 1991.
35. IMA, Minneapolis, March, 1991.

ACTIVITIES OF GRADUATE STUDENTS

Michael S. Jolly completed his PhD thesis with a dissertation on "Explicit construction of an inertial manifold for a reaction diffusion equation" in September, 1987. Sell was his thesis advisor. Jolly has been a frequent visitor on this project helping with the efforts on approximating inertial manifolds.

Mario Taboada completed his PhD thesis under the direction of Sell in July 1989. The main objective of his research is to extend the dynamical theory of the Kuramoto-Sivashinsky equation to 2-space dimensions. He has now succeeded in showing that for thin domains the 2D problem has an attractor with a large basin of attraction.

Yin Yan did his thesis research under the direction of George Sell, and he recieved his PhD degree in June, 1990. During the period from June, 1990 to May, 1991, Yan served as a postdoctoral fellow on this project and he assisted in the simulation of the Kolmogorov flow. His PhD thesis project was to study the attractors for discretizations of a number of nonlinear partial differential equations, including the nonlinear Schrödinger equation with weak damping, the sine Gordon equation with weak damping and the 2D Navier-Stokes equations. He derived estimates of the Hausdorff dimension of the global attractor as a function of the physical parameters for these systems. The hope is to show that the estimates for the discretized system is the same as that for the continuous systems. See References numbered 19 and 20.

Ling Ma did research under the joint direction of Luskin and Sell. Ma is working to prove rates of convergence for several iterative methods for the approximation of inertial manifolds which are suggested by the analysis in the paper *Construction of inertial manifolds by elliptic regularization* by Fabes, Luskin, and Sell. These rates of convergence will

then be used to compare the efficiency and to optimize the rate of convergence for the iterative methods. Numerical experiments will be utilized to suggest research directions for the analysis. Ma finished his PhD thesis on a related topic under the direction of Luskin in June, 1991.

Minkyu Kwak did his PhD thesis on the dynamics of the 2D Navier-Stokes equations. His thesis represents a major breakthrough. It had been unknown whether the long-time dynamics of the 2D Navier-Stokes equation could be completely described by the dynamics of a finite system of ODEs. By using a very ingenious nonlinear change of variables, Dr. Minkyu Kwak, a recent Ph.D student of George Sell, solved this problem. His approach was to use the nonlinear change of variables to imbed the Navier-Stokes equations into a system of reaction diffusion equations. This imbedding, which is now being called the Kwak Transformation by some experts, is chosen so that it preserves all the dynamics of the Navier-Stokes equation. More importantly, the system of reaction diffusion equations has only algebraic nonlinearities (i.e., no derivative terms), and consequently, it has an inertial manifold. As a result, the dynamics of the associated inertial form then completely describes the long-time dynamics of the original 2D Navier-Stokes equations.

The work of Kwak has already attracted wide spread interest. For example, his paper was recently presented at an International Conference on Turbulence Modeling at Arizona State University, and it was generally felt to be the most important new result presented at this meeting. Kwak received his PhD in June, 1991.

VISITING SCIENTISTS

C. Foias, from Indiana University, was in residence for about one week. Two joint papers with G. Sell (and others) were completed at that time. These results are reported above.

M. Marion, from the Université de Paris-Sud (Orsay), visited the University of Minnesota for the month of April, 1988. She and G. Sell began a project on the existence of inertial manifolds for systems of reaction diffusion equations in space dimension 3. The objective of this project is to use the Principle of Spatial Averaging for systems of equations, as opposed to a single equation, thereby extending the theory presented in Mallet-Paret and Sell (1988). This work continued with return visits by Marion to Minnesota in November, 1989 and April, 1991.

Geneviève Raugel from Ecole Polytechnique in Paris visited the University of Minnesota in July 1988. The purpose of this visit was to do some joint work with George Sell on the existence of attractors for the 3D Navier-Stokes equations. A first step in this project is to study the regularity of solutions of the 3-dimensional setting. The regularity of solutions in 3D is a nontrivial issue, which has been an open problem since the publication of the first papers of Leray, over 50 years ago. Raugel and Sell are trying to use methods of dynamical systems to address this problem.