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Adaptive Control of Space Robot System with an Attitude Controlled Base

Yangsheng Xu, Heung-Yeung Shum,
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ABSTRACT

In this report, we discuss adaptive control of a space robot system with an attitude controlled base on which the robot is attached. We at first derive the system kinematic and dynamic equations based on Lagrangian dynamics and linear momentum conservation law. Using the dynamic model developed, we discuss the problem of linear parameterization in terms of dynamic parameters, and have found that in joint space the dynamics can be linearized by a set of combined dynamic parameters, but in inertia space linear parameterization is impossible in general. Then we propose an adaptive control scheme in joint space which has been shown effective and feasible for the cases where unknown or unmodeled dynamics must be considered, such as in the tasks of transport an unknown payload, or catching a moving object. The scheme avoids the use of joint acceleration measurement, inversion of inertial matrix, high gain feedback, and considerable computation cost, and at meantime, is also applicable for the fixed-base robot system by slight modification. Since most tasks are specified in inertia space, instead of joint space, we discuss the issues associated to adaptive control in inertia space and identify two potential problems, unavailability of joint trajectory since mapping from inertia space trajectory is dynamic dependent and subject to uncertainty, and nonlinear parameterization in inertia space. We approach the problem by making use of the proposed joint space adaptive controller and updating joint trajectory by the estimated dynamic parameters and given trajectory in inertia space. In the case study of a planar system, the linear parameterization problem is investigated, the design procedure of the controller is illustrated, and the validity and effectiveness of the proposed control scheme is demonstrated.

1 Introduction

Considerable research efforts have been directed to some primary functions of robots in space applications, such as material transport [17], simple manipulation [4], basic locomotion [14], inspection and maintenance of the space station and satellites [1, 4]. The adaptive control is critical for the robot system subject to dynamic uncertainty in these tasks.

For material transport and manipulation tasks, space robots have to face uncertainty on the parameters describing the dynamic properties of the grasped load, such as moments of inertia or exact position of the mass center. In most cases, these parameters are unknown and thus they can not be specified off-line, in inverse dynamics for feedforward compensation or any model-based control scheme. In catching a moving object [15], the robot is expected to be capable of adaptation to the dynamics change at the moment of catching operation. On the other hand, most space robots are designed to be light-weighted and thus low-powered, partially due to the fact zero gravity environment. Therefore, joint friction and damping in space robots are much more significant than those in industrial robots. These effects are neither negligible nor easy to model. Adaptive control may provide a feasible solution to those system dynamics uncertainties. Adaptive control is also able to accommodate various unmodeled disturbances, such as base disturbance, microgravity effect, sensor and actuator noise due to extremes of temperature and glare, or impact effect during docking or rendezvous process.

Most of existing adaptive control algorithms have the following shortcomings which cause their applications in space robots unrealistic, the use of joint acceleration measurement, the need of inversion of inertia matrix, high gain feedback and considerable computational cost. The first two must be avoided even for fixed-base industrial robots, because of lack of joint acceleration sensor and the complexity of inversion of inertia matrix. Slotine and Li [11] have tackled these problems successfully. A high gain feedback is extremely harmful for a space robot which is usually light-weighted and low-powered. Considerable computation also needs to be avoided for allowable package of self-contained space robots.

This report focuses on the robot system where the base attitude is controlled by either thrust jets, or reaction wheels. The reaction wheels are arranged in orthogonal directions, and the number of reaction wheels can be three or two depending on different tasks, and a standard reaction wheel configuration can be found in [6]. When the attitude of the base is controlled, the orientation and position of both robot and base are no longer free, and the dynamic interaction between the base and robot results in the dynamic dependent kinematics, i.e., the kinematics is in relation to the mass property of the base and robot. Control is not only applied to robot joint angles, but also three orientations of the base.

In this report, based on linear momentum conservation law and Lagrangian dynamics, we at first formulate kinematics and dynamics equations of the space robot system with an attitude controlled base, in a systematic way. Based on the dynamic model developed, we study the linear parameterization problem, i.e., dynamics can be linearly expressed in terms of dynamic parameters, such as mass and inertia. We have found that for the space robot system with an attitude control base, the linear parameterization is valid in joint space, while is not valid in inertia space which can be viewed as Cartesian space for earth-based robots.

Using the dynamic model, we propose an adaptive control scheme in joint space. The scheme does not need to measure accelerations in joint space, and a high feedback gain is not required. The proposed method is effective and feasible for space robot applications when dynamic parameters are unknown or unmodeled dynamics effect must be considered. Since in most applications, the tasks are specified in inertia space normally, instead of joint space, we discuss the issues in relation

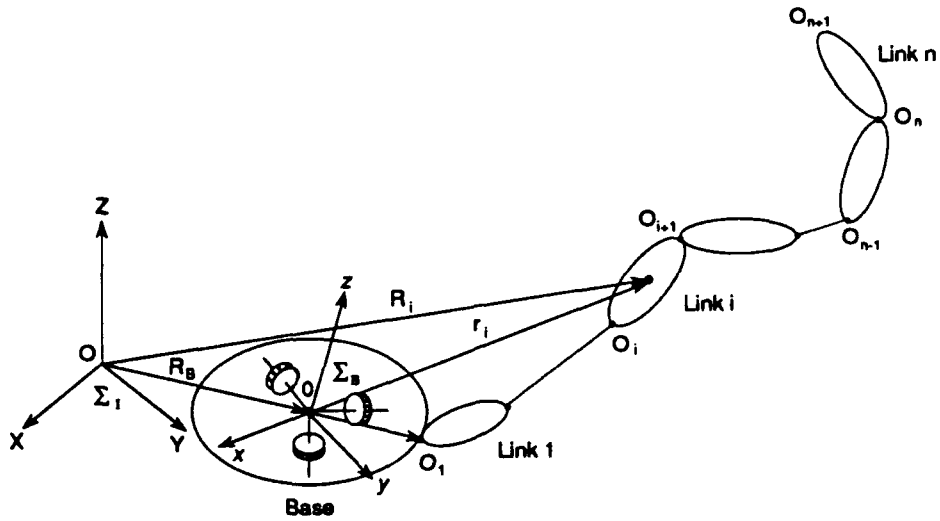

Figure 1: Space robot system with an attitude controlled base.

to implementation of adaptive control in inertia space and identify two main problems. The first problem occurs when the joint adaptive control is executed. The required joint trajectory cannot be generated by the given trajectory in inertia space due to the parameter uncertainty in the kinematic mapping which is dynamics dependent. The second problem is nonlinear parameterization in inertia space which make impossible to implement the same structured adaptive control as that in joint space. We approach this problem by making use of joint space adaptive controller and updating joint trajectory from identified kinematic mapping and the given trajectory in inertia space. This method has shown its effectiveness in simulation, and some issues associated to parameter estimation and updating time are discussed.

Finally, we study a planar robot system to investigate linear parameterization problem of robot system dynamics, and illustrate the validity and effectiveness of the proposed adaptive control schemes.

2 Kinematics of Space Robot System with An Attitude Controlled Base

In this section we discuss the kinematics of the space robot system when the orientation of the base is controlled and the translation of the base is free. The relationship between the robot hand motion in inertia space and robot joint motion is derived using linear momentum conservation law.

As shown in Figure 1, a space robot system with an attitude controlled base can be modeled as a multibody chain composed of $n+1$ rigid bodies connected by n joints, which are numbered from 1 to n. Each body is numbered from 0 to n, and the base is denoted by **B** in particular. The mass and inertia of ith body are denoted by m_i and $\mathbf{I}_i$, respectively. A joint variable vector $\mathbf{q} = (q_1, q_2, \cdots, q_n)^T$ is used to represent those joint displacements. The orientation of the base is represented by a vector $\mathbf{q}_B = (q_{B1}, q_{B2}, q_{B3})^T$.

Two coordinate frames are defined, the inertia coordinate $\sum_I$ on the orbit, and the base coordinate $\sum_B$ attached on the base body with its origin at the centroid of the base. As shown in

Figure 1, let $\mathbf{R}_i$ and $\mathbf{r}_i$ be the position vectors pointing the centroid of ith body with reference to $\sum_I$ and $\sum_B$ respectively, then

$$\mathbf{R}_i = \mathbf{r}_i + \mathbf{R}_B \tag{1}$$

where $\mathbf{R}_B$ is the position vector pointing the centroid of the base with reference to $\sum_I$. Let $\mathbf{V}_i$ and $\mathbf{\Omega}_i$ be linear and angular velocities of ith body with respect to $\sum_I$, $\mathbf{v}_i$ and ω_i with respect to $\sum_B$. Then we have

$$\begin{aligned} \mathbf{V}_i &= \mathbf{v}_i + \mathbf{V}_B + \Omega_B \times \mathbf{r}_i \\ \mathbf{\Omega}_i &= \omega_i + \Omega_B \end{aligned} \tag{2}$$

where $\mathbf{V}_B$ and $\mathbf{\Omega}_B$ are linear and angular velocities of the centroid of the base with respect to $\sum_I$, and operator '$\times$' represents outer product of R^3 vector. The velocities $\mathbf{v}_i$ and ω_i in base coordinates can be represented by

$$\mathbf{v}_i = \mathbf{J}_{Li}(\mathbf{q})\dot{\mathbf{q}} \tag{3}$$

$$\omega_i = \mathbf{J}_{Ai}(\mathbf{q})\dot{\mathbf{q}} \tag{4}$$

where $\mathbf{J}_{Li}(\mathbf{q})$ and $\mathbf{J}_{Ai}(\mathbf{q})$ are the submatrices of Jacobian of the ith body representing linear part and angular part respectively. The centroid of the total system can be determined by

$$m_c = \sum_{i=0}^{n} m_i \tag{5}$$

$$\mathbf{I}_c = \sum_{i=0}^{n} \mathbf{I}_i \tag{6}$$

$$\mathbf{r}_c = \sum_{i=0}^{n} m_i \mathbf{r}_i / m_c \tag{7}$$

$$\mathbf{J}_c = \sum_{i=1}^{n} m_i \mathbf{J}_{Li} / m_c \tag{8}$$

The linear momentum can be determined by

$$\begin{aligned} \mathbf{P} &= [\mathbf{H}_V, \mathbf{H}_{V\Omega}] \begin{bmatrix} \mathbf{V}_B \\ \mathbf{\Omega}_B \end{bmatrix} + \mathbf{H}_{Vq}\dot{\mathbf{q}} \\ &= \mathbf{H}_V \mathbf{V}_B + \mathbf{H}_{V\Omega}\mathbf{\Omega}_B + \mathbf{H}_{Vq}\dot{\mathbf{q}} \end{aligned} \tag{9}$$

where

$$\mathbf{H}_V = m_c \mathbf{U}_3 \tag{10}$$

$$\mathbf{H}_{V\Omega} = -m_c[\mathbf{r}_c \times] \tag{11}$$

$$\mathbf{H}_{Vq} = m_c \mathbf{J}_c \tag{12}$$

and $\mathbf{U}_3$ is a 3×3 unity matrix. The matrix function $[\mathbf{r}\times]$ for a vector $\mathbf{r} = [r_x, r_y, r_z]^T$ is defined as

$$[\mathbf{r}\times] = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix} \tag{13}$$

Because there is no external force applied to the system, the linear momentum is conserved. However, the angular momentum is not conserved for attitude control torques are applied. The linear momentum is zero, assuming stationary initial condition.

$$\mathbf{P} = 0 \tag{14}$$

Therefore, we may represent the base linear velocities by base angular velocities and robot joint velocities, i.e.,

$$\mathbf{V}_B = -1/m_c [\mathbf{H}_{V\Omega}, \mathbf{H}_{Vq}] \begin{bmatrix} \mathbf{\Omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} \tag{15}$$

Now we derive the relationship between the motion rate in inertia space and that in joint space. For position control tasks, we are interested in controlling three orientations of the base, and six generalized displacements of the robot end-effector simultaneously. Control actions are instead applied at n robot joints and three base attitudes. We therefore define $\mathbf{V}$ and $\dot{\theta}$ as generalized velocities in inertia space and joint space,

$$\mathbf{V} = [\mathbf{\Omega}_B, \mathbf{V}_E]^T \tag{16}$$

$$\dot{\theta} = [\mathbf{\Omega}_B, \dot{\mathbf{q}}]^T \tag{17}$$

where $\mathbf{V}_E$ is the velocity of the robot end-effector in inertia space.

$$\mathbf{V}_E = \mathbf{v}_E + \mathbf{V}_B + \Omega_B \times \mathbf{r}_E \tag{18}$$

Since the velocity of the end-effector in the base coordinates is determined by

$$\mathbf{v}_E = \mathbf{J}_E \dot{\mathbf{q}} \tag{19}$$

where $\mathbf{J}_E$ is the manipulator Jacobian with respect to the base coordinates,

$$\begin{aligned} \mathbf{V}_E &= \mathbf{J}_E \dot{\mathbf{q}} - 1/m_c [\mathbf{H}_{V\Omega}, \mathbf{H}_{Vq}] \begin{bmatrix} \mathbf{\Omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} - [\mathbf{r}_E\times]\mathbf{\Omega}_B \\ &= [-\frac{1}{m_c}\mathbf{H}_{V\Omega} - [\mathbf{r}_E\times], \mathbf{J}_E - \frac{1}{m_c}\mathbf{H}_{Vq}] \begin{bmatrix} \mathbf{\Omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} \\ &= [[(\mathbf{r}_c - \mathbf{r}_E)\times], \mathbf{J}_E - \mathbf{J}_c] \begin{bmatrix} \mathbf{\Omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} \end{aligned} \tag{20}$$

Therefore, the motion rate relationship between joint space and inertia space can be obtained by introducing a special Jacobian matrix $\mathbf{N}$ which differs from the Jacobian in fixed-base robot or the generalized Jacobian in a completely free-flying space robot system.

$$\begin{bmatrix} \mathbf{\Omega}_B \\ \mathbf{V}_E \end{bmatrix} = \mathbf{N} \begin{bmatrix} \mathbf{\Omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} \tag{21}$$

$$\mathbf{N} = \begin{bmatrix} \mathbf{U}_3 & \mathbf{O}_3 \\ \mathbf{J}_{rr} & \mathbf{J}_{rE} \end{bmatrix} \tag{22}$$

where

$$\mathbf{J}_{rr} = [(\mathbf{r}_c - \mathbf{r}_E)\times] \tag{23}$$

$$\mathbf{J}_{rE} = \mathbf{J}_E - \mathbf{J}_c \tag{24}$$

and $\mathbf{O}_3$ is a 3×3 zero matrix.

3 Dynamics of Space Robot System with An Attitude Controlled Base

In this section we discuss dynamics of the space robot system with an attitude controlled base. After formulating total kinetic energy of the system we derive the dynamics equation of the system. Then we investigate the property of linear parameterization of the system dynamics which is critical for developing the adaptive control algorithms in the following section.

The total system kinetic energy is represented by

$$\begin{aligned}
\mathbf{T} &= 1/2\mathbf{V}_B{}^T\mathbf{H}_V\mathbf{V}_B + \mathbf{V}_B{}^T[\mathbf{H}_{V\Omega}, \mathbf{H}_{Vq}]\begin{bmatrix} \boldsymbol{\Omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} \\
&\quad +1/2[\boldsymbol{\Omega}_B, \dot{\mathbf{q}}]\begin{bmatrix} \mathbf{H}_\Omega & \mathbf{H}_{\Omega q} \\ \mathbf{H}_{\Omega q}{}^T & \mathbf{H}_q \end{bmatrix}\begin{bmatrix} \boldsymbol{\Omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} \\
&= 1/(2m_c{}^2)[\boldsymbol{\Omega}_B, \dot{\mathbf{q}}]\begin{bmatrix} \mathbf{H}_{V\Omega} \\ \mathbf{H}_{Vq} \end{bmatrix}\mathbf{H}_V[\mathbf{H}_{V\Omega}, \mathbf{H}_{Vq}]\begin{bmatrix} \boldsymbol{\Omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} \\
&\quad -1/m_c[\boldsymbol{\Omega}_B, \dot{\mathbf{q}}]\begin{bmatrix} \mathbf{H}_{V\Omega} \\ \mathbf{H}_{Vq} \end{bmatrix}[\mathbf{H}_{V\Omega}, \mathbf{H}_{Vq}]\begin{bmatrix} \boldsymbol{\Omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} \\
&\quad +1/2[\boldsymbol{\Omega}_B, \dot{\mathbf{q}}]\begin{bmatrix} \mathbf{H}_\Omega & \mathbf{H}_{\Omega q} \\ \mathbf{H}_{\Omega q}{}^T & \mathbf{H}_q \end{bmatrix}\begin{bmatrix} \boldsymbol{\Omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} \\
&= 1/2[\boldsymbol{\Omega}_B, \dot{\mathbf{q}}]\begin{bmatrix} \mathbf{H}_\Omega - \mathbf{H}_{V\Omega}{}^T\mathbf{H}_{V\Omega}/m_c & \mathbf{H}_{\Omega q} - \mathbf{H}_{V\Omega}{}^T\mathbf{H}_{Vq}/m_c \\ \mathbf{H}_{\Omega q} - \mathbf{H}_{V\Omega}{}^T\mathbf{H}_{Vq}/m_c & \mathbf{H}_q - \mathbf{H}_{Vq}{}^T\mathbf{H}_{Vq}/m_c \end{bmatrix}\begin{bmatrix} \boldsymbol{\Omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} \\
&= 1/2[\boldsymbol{\Omega}_B, \dot{\mathbf{q}}]\mathbf{M}(\theta)\begin{bmatrix} \boldsymbol{\Omega}_B \\ \dot{\mathbf{q}} \end{bmatrix} \\
&= 1/2\dot{\theta}^T\mathbf{M}(\theta)\dot{\theta}
\end{aligned} \tag{25}$$

where $\mathbf{M}$ is the inertia matrix of the system, $\mathbf{H}_q$ is the robot inertia matrix in base coordinate, i.e., fixed base inertia matrix. and

$$\theta = [\mathbf{q}_B, \mathbf{q}]^T \tag{26}$$

$$\mathbf{H}_\Omega = \mathbf{I}_c + \sum_{i=1}^{n} \mathbf{D}(\mathbf{r}_i)m_i \tag{27}$$

$$\mathbf{H}_{\Omega q} = \sum_{i=1}^{n} (\mathbf{I}_i \mathbf{J}_{Ai} + m_i [\mathbf{r}_i] \mathbf{J}_{Li}) \tag{28}$$

$$\mathbf{M}(\theta) = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} \tag{29}$$

$$\mathbf{M}_{11} = \mathbf{H}_{\Omega} - \mathbf{H}_{V\Omega}^T \mathbf{H}_{V\Omega}/m_c \tag{30}$$

$$\mathbf{M}_{12} = \mathbf{H}_{\Omega q} - \mathbf{H}_{V\Omega}^T \mathbf{H}_{Vq}/m_c \tag{31}$$

$$\mathbf{M}_{21} = \mathbf{M}_{12}^T \tag{32}$$

$$\mathbf{M}_{22} = \mathbf{H}_q - \mathbf{H}_{Vq}^T \mathbf{H}_{Vq}/m_c \tag{33}$$

The property of linear parameterization to dynamic parameters is one of prerequisite conditions under which most adaptive and nonlinear dynamic control schemes are developed. It has been shown that the problem of parameterization linearity in dynamics can be reduced to the problem of parameterization linearity in inertia matrix. Therefore, in order to study whether linear parameterization is valid for the space robot system with an attitude controlled base, we need to show whether the inertia matrix $\mathbf{M}$ can be linearly represented by a set of properly chosen combinations of dynamic parameters.

Based on the previous derivation, each member of matrix $\mathbf{M}$ can be further expanded in the following forms.

$$\mathbf{M}_{11} = \mathbf{I}_c + \sum_{i=1}^{n} \mathbf{D}(\mathbf{r}_i) m_i - \frac{1}{m_c} \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} \tag{34}$$

where $s_{ij} = s_{ji}$ and is determined by

$$s_{11} = (\sum_{i=1}^{n} m_i r_{iy})^2 + (\sum_{i=1}^{n} m_i r_{iz})^2 \tag{35}$$

$$s_{22} = (\sum_{i=1}^{n} m_i r_{ix})^2 + (\sum_{i=1}^{n} m_i r_{iz})^2 \tag{36}$$

$$s_{33} = (\sum_{i=1}^{n} m_i r_{ix})^2 + (\sum_{i=1}^{n} m_i r_{iy})^2 \tag{37}$$

$$s_{12} = -(\sum_{i=1}^{n} m_i r_{ix}) \cdot (\sum_{i=1}^{n} m_i r_{iy}) \tag{38}$$

$$s_{23} = -(\sum_{i=1}^{n} m_i r_{iy}) \cdot (\sum_{i=1}^{n} m_i r_{iz}) \tag{39}$$

$$s_{13} = -(\sum_{i=1}^{n} m_i r_{ix}) \cdot (\sum_{i=1}^{n} m_i r_{iz}) \tag{40}$$

and

$$\mathbf{D}(\mathbf{r}) = [\mathbf{r}\times]^T[\mathbf{r}\times] = \begin{bmatrix} r_y^2 + r_z^2 & -r_x r_y & -r_x r_z \\ -r_x r_y & r_x^2 + r_z^2 & -r_y r_z \\ -r_x r_z & -r_y r_z & r_x^2 + r_y^2 \end{bmatrix} \tag{41}$$

Thus,

$$\mathbf{M}_{11} = \mathbf{I}_c + \sum_{i=1}^{n} \mathbf{D}(\mathbf{r}_i) m_i - \sum_{i=1}^{n}\sum_{j=1}^{n} R_{ij} m_i m_j / m_c \tag{42}$$

$$\mathbf{M}_{22} = \mathbf{H}_q - \frac{1}{m_c}(\sum_{i=1}^{n} m_i \mathbf{J}_{Li})^T (\sum_{i=1}^{n} m_i \mathbf{J}_{Li})$$

$$= \mathbf{H}_q + \sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij} m_i m_j / m_c \tag{43}$$

$$\mathbf{M}_{12} = \sum_{i=1}^{n} \mathbf{I}_i \mathbf{J}_{Ai} + \sum_{i=1}^{n} [\mathbf{r}_i] \mathbf{J}_{Li} m_i + \sum_{i=1}^{n}\sum_{j=1}^{n} S_{ij} m_i m_j / m_c \tag{44}$$

where the matrices $[\mathbf{r}_i]\mathbf{J}_{Li}$, $\mathbf{J}_{Ai}$, R_i, Q_i, S_i are only functions of geometric parameters, i.e., independent of dynamic parameters. The above formulations imply that the inertia matrix can be linearly represented by a set of combination of dynamic parameters, m_k, $\mathbf{I}_k$, $m_i m_j / m_c$, $i, j, k = 0, 1, \cdots, n$.

From the kinetic energy formulation, we can derive dynamics equation by Lagrangian dynamics.

$$\mathbf{M}\ddot{\theta} + \mathbf{B}(\theta, \dot{\theta})\dot{\theta} = \tau \tag{45}$$

where

$$\mathbf{B}(\theta, \dot{\theta})\dot{\theta} = \dot{\mathbf{M}}\dot{\theta} - \frac{\partial}{\partial \theta}(\frac{1}{2}\dot{\theta}^T \mathbf{M}\dot{\theta}) \tag{46}$$

The corresponding dynamic equation in inertia space is

$$\mathbf{H}\ddot{\mathbf{x}} + \mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} = \mathbf{F} \tag{47}$$

where

$$\mathbf{H} = \mathbf{N}^{-T}\mathbf{M}\mathbf{N}^{-1} \tag{48}$$

$$\mathbf{C} = \mathbf{N}^{-T}\mathbf{B}\mathbf{N}^{-1} - \mathbf{H}\dot{\mathbf{N}}\mathbf{N}^{-1} \tag{49}$$

$\mathbf{N}$ is a generalized Jacobian matrix and is dynamics depenedent for the space robot system. The inertia space dynamic equation can be linearly expressed in terms of dynamic parameters if and only if the inertia matrix $\mathbf{H}$ can be linearly parameterized since

$$\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} = \dot{\mathbf{H}}\dot{\mathbf{x}} - \frac{\partial}{\partial \mathbf{x}}(\frac{1}{2}\dot{\mathbf{x}}^T \mathbf{H}\dot{\mathbf{x}}) \tag{50}$$

We suppose $\mathbf{N}^{-1}$ exists, and

$$\mathbf{N}^{-1} = \frac{\mathbf{N}^*}{det(\mathbf{N})} \tag{51}$$

where $\mathbf{N}^*$ and $det(\mathbf{N})$ are the adjoint and determinant of the matrix $\mathbf{N}$, then

$$\mathbf{H} = \frac{\mathbf{N}^{*T}\mathbf{M}\mathbf{N}^*}{[det(\mathbf{N})]^2} \tag{52}$$

In the above equation, the generalized Jacobian matrix $[det(\mathrm{N})]^2$ appears as the denominator. From derivation procedure of the N in the last section, it is clear that the N is time-varying and highly coupled by dynamic parameters, i.e., mass/inertia. For such a complicated nonlinear, time-varying function combining with dynamic parameters and time-varying joint angles, it is impossible that every element of $\mathrm{N}^{*\mathrm{T}}\mathrm{M}\mathrm{N}^*$ has the common factor $[det(\mathrm{N})]^2$ at every instant.

Even if the above statement is true, there is still possibility to linearly parameterize H, provided that the numerator can be linearly parameterized and the denominator can be expressed as a product of two scalar functions with only one containing dynamic parameters, i.e.,

$$det(\mathrm{N}) = f_1(m_i, I_i) f_2(\theta_i) \tag{53}$$

where f_1 is a function of dynamic parameters which are unknown but constant, f_2 is a function independent of any dynamic parameters. This, unfortunately, is impossible in general due to high coupling between dynamic parameters and joint variables. For example, two DOF generalized Jacobian may contain the following simple terms

$$det(\mathrm{N}) = m_1 sin(\theta_1) + m_2 cos(\theta_2) \tag{54}$$

Even for such a simple form, $det(\mathrm{N})$ cannot be decomposed as a product of two functions with one containing m_1 and m_2 only, and nor can $[det(\mathrm{N})]^2$.

This may raise a question why for a fixed-base robot the similar structured adaptive control can be implemented in Cartesian space. This is because that the Jacobian in the fixed-base robots is only kinematic dependent, i.e., a function of geometric parameters and joint angles. Since the dynamic interaction between the base and the robot, the generalized Jacobian for a space robot with an attitude control base is dynamics dependent, i.e., not only a function of geometric parameters and joint angles, but also a function of the dynamic parameters. It is to these parameters that we aim at adapting in our problem. Therefore, the inertia matrix for the fixed-base robot can be linearly parameterized for dynamic parameters in Cartesian space, while for a space robot it is impossible in inertia space.

Generally speaking, for a space robot with attitude controlled base, dynamics can be linearly parameterized in terms of dynamic parameters in joint space, but it cannot in inertia space.

4 Adaptive Control Scheme

In this section, based on the dynamic model developed and the property of parameterization linearity in Section 3, we discuss the adaptive control strategy for space robot system with an attitude controlled base.

At early state, adaptive control approaches for conventional fixed-base robot manipulators are based on unrealistic assumptions or approximations on local linearization, time-invariant and decoupled dynamics [3, 5]. These assumptions or approximations are relaxed after some results developed in the context of parameter estimation [8]. Based on the possibility of selecting a proper set of equivalent parameters such that the manipulator dynamics depends linearly on these parameters, research on adaptive robot control can now take full consideration of the nonlinear, time-varying and coupled robot dynamics. As stated in [7], all three kinds of adaptive controllers in use, direct [2, 11], indirect [10], composite adaptive controllers [12], rely on the possibility of linear parameterization of manipulator dynamics.

From previous discussion, we have learned that the dynamics of the space robot system in joint space is linear in terms of a set of combinations of dynamic parameters. Therefore, this set of new

combined parameters can be used in the design of our adaptive controller. This leads us to propose an adaptive control algorithm in joint space. Since a unique solution may be found from inverse kinematics of the robot system *with the attitude controlled base*, adaptive control algorithm in joint space is feasible. However, this is not true for a complete free-flying space robot system.

Let's recall the dynamic equation in joint space

$$\mathbf{M}\ddot{\theta} + \mathbf{B}(\theta, \dot{\theta})\dot{\theta} = \tau \tag{55}$$

We define a composite error $\mathbf{s}$

$$\mathbf{s} = \dot{\mathbf{e}}_p + \zeta \mathbf{e}_p \tag{56}$$

$$\mathbf{e}_p = \theta_d - \theta \tag{57}$$

$$\dot{\mathbf{e}}_p = \dot{\theta}_d - \dot{\theta} \tag{58}$$

and we also define modified joint velocity

$$\theta' = \dot{\theta} + \mathbf{s} \tag{59}$$

and modified joint acceleration,

$$\theta'' = \frac{d}{dt}\theta' + \mathbf{s} \tag{60}$$

i.e.,

$$\begin{aligned} \theta'' &= \ddot{\theta} + \dot{\mathbf{s}} + \mathbf{s} = (\ddot{\theta}_d - \ddot{\mathbf{e}}_p) + (\ddot{\mathbf{e}}_p + \zeta\dot{\mathbf{e}}_p) + (\dot{\mathbf{e}}_p + \zeta \mathbf{e}_p) \\ &= \ddot{\theta}_d + (\zeta + 1)\dot{\mathbf{e}}_p + \zeta \mathbf{e}_p = \ddot{\theta}_d + \mathbf{s} + \zeta\dot{\mathbf{e}}_p \end{aligned} \tag{61}$$

If we apply the following control law in joint space,

$$\tau = \hat{\mathbf{M}}\theta'' + \hat{\mathbf{B}}\theta' \tag{62}$$

then

$$\mathbf{M}\ddot{\theta} + \mathbf{B}(\theta, \dot{\theta})\dot{\theta} = \hat{\mathbf{M}}\theta'' + \hat{\mathbf{B}}\theta' \tag{63}$$

i.e.,

$$\mathbf{M}\ddot{\theta} = -\mathbf{B}(\theta, \dot{\theta})\dot{\theta} + \hat{\mathbf{M}}\theta'' + \hat{\mathbf{B}}\theta' \tag{64}$$

Defining $\tilde{\mathbf{M}} = \hat{\mathbf{M}} - \mathbf{M}$, $\tilde{\mathbf{B}} = \hat{\mathbf{B}} - \mathbf{B}$, we have

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{e}}_p &= \mathbf{M}\ddot{\theta}_d - \mathbf{M}\ddot{\theta} \\ &= \mathbf{M}[\theta'' - \mathbf{s} - \zeta\dot{\mathbf{e}}_p] - [-\mathbf{B}\dot{\theta} + \hat{\mathbf{M}}\theta'' + \hat{\mathbf{B}}\theta'] \\ &= \mathbf{M}[\theta'' - \mathbf{s} - \zeta\dot{\mathbf{e}}_p] - [-\mathbf{B}(\theta' - \mathbf{s}) + \hat{\mathbf{M}}\theta'' + \hat{\mathbf{B}}\theta'] \\ &= -\tilde{\mathbf{M}}\theta'' - \tilde{\mathbf{B}}\theta' - (\mathbf{M} + \mathbf{B})\mathbf{s} - \mathbf{M}\zeta\dot{\mathbf{e}}_p \\ &= -\mathbf{Y}(\theta, \dot{\theta}, \theta_d, \dot{\theta}_d, \ddot{\theta}_d)\tilde{\mathbf{a}} - (\mathbf{M} + \mathbf{B})\mathbf{s} - \mathbf{M}\zeta\dot{\mathbf{e}}_p \end{aligned}$$

where

$$\mathbf{Y}\tilde{\mathbf{a}} = \tilde{\mathbf{M}}\theta'' + \tilde{\mathbf{B}}\theta' \tag{65}$$

$$\tilde{\mathbf{a}} = \hat{\mathbf{a}} - \mathbf{a} \tag{66}$$

and $\hat{\mathbf{a}}$ is estimation of the unknown dynamic parameters of the space robot system including the robot, the base, and probably the payload which is being manipulated.

We now design our adaptive control algorithm using Lyapunov function candidate

$$V = 1/2\mathbf{s}^T\mathbf{M}\mathbf{s} + 1/2\tilde{\mathbf{a}}^T\Gamma\tilde{\mathbf{a}} \tag{67}$$

where the matrix Γ is diagonal and positive definite. This yields

$$\begin{aligned}\dot{V} &= 1/2\mathbf{s}^T\dot{\mathbf{M}}\mathbf{s} + \mathbf{s}^T\mathbf{M}\dot{\mathbf{s}} + \tilde{\mathbf{a}}^T\Gamma\dot{\tilde{\mathbf{a}}} \\ &= 1/2\mathbf{s}^T\dot{\mathbf{M}}\mathbf{s} + \mathbf{s}^T\mathbf{M}(\ddot{\mathbf{e}}_p + \zeta\dot{\mathbf{e}}_p) + \tilde{\mathbf{a}}^T\Gamma\dot{\tilde{\mathbf{a}}} \\ &= -\mathbf{s}^T\mathbf{Y}\tilde{\mathbf{a}} - \mathbf{s}^T(\mathbf{M}+\mathbf{B})\mathbf{s} + 1/2\mathbf{s}^T\dot{\mathbf{M}}\mathbf{s} + \tilde{\mathbf{a}}^T\Gamma\dot{\tilde{\mathbf{a}}} \\ &= -\mathbf{s}^T\mathbf{M}\mathbf{s} + 1/2\mathbf{s}^T(\dot{\mathbf{M}} - 2\mathbf{B})\mathbf{s} + \tilde{\mathbf{a}}^T(\Gamma\dot{\tilde{\mathbf{a}}} - \mathbf{Y}^T\mathbf{s})\end{aligned}$$

If we use adaptation law

$$\dot{\tilde{\mathbf{a}}} = \Gamma^{-1}\mathbf{Y}^T\mathbf{s} \tag{68}$$

then

$$\dot{V} = -\mathbf{s}^T\mathbf{M}\mathbf{s} \leq 0 \tag{69}$$

due to the fact that the matrix $\dot{\mathbf{M}} - 2\mathbf{B}$ is skew-symmetric, and $\mathbf{M}$ is positive definite. Therefore, the system is stable in the sense of Lyapunov, because V is a positive, nonincreasing function bounded below by zero. $\mathbf{s}(t)$ and $\tilde{\mathbf{a}}(t)$ are then bounded, and $\mathbf{s}(t)$ is a so-called square integrable or L_2 function [13]. Provided that the function $\mathbf{Y}$ is bounded, this is sufficient for the purpose of control because $\mathbf{s}(t)$ converge to zero as the L_2 function must converge to zero as $t \to \infty$. The parameter estimation error $\tilde{\mathbf{a}}(t)$ will converge to zero only if persistent excited input is utilized.

The output error

$$\mathbf{s} = \dot{\mathbf{e}}_p + \zeta\mathbf{e}_p \tag{70}$$

converges to zero, which in turn implies that $\mathbf{e}_p \to 0$ as $t \to \infty$ since ζ is positive. We can now readily conclude our adaptive control algorithm in Theorem 1.

Theorem 1 *For the dynamic system (55), the adaptive control law defined by (62) and (68) is globally stable and guarantees zero steady state error in joint space.*

The composite error $\mathbf{s}$ is of PD type structure which is the same as the composite error defined by Slotine and Li [11]. In general the PD structure control adds damping to the system but the steady-state response is not affected. The PI structure adds damping and improve the steady-state error at the same time, but rising time and settling time are penalized. To improve the system steady-state error, in the proposed adaptive control algorithm, the PID type $\mathbf{s}$ can also be used. Since when the PID type $\mathbf{s}$ is used, the order and type of the system is increased by one, the steady-state error is decreased, and thus the system is more robust to parameters uncertainties which usually cause a significant steady-state error. Moreover, the PID type $\mathbf{s}$ allows two parameters, instead of one, to be adjustable in order to achieve a desired system performance. In what follows, we discuss the stability of the control scheme when the PID type $\mathbf{s}$ is employed.

Define

$$\mathbf{s} = \dot{\mathbf{e}}_p + \zeta_1\mathbf{e}_p + \zeta_2\int_{t_0}^{t}\mathbf{e}_p d\tau \tag{71}$$

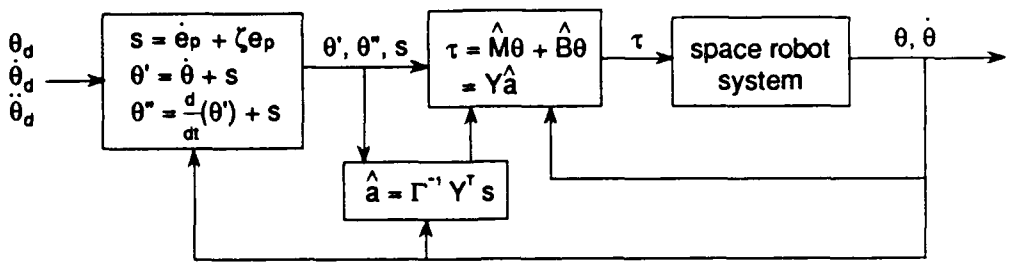

Figure 2: Block diagram of adaptive controller in joint space.

and the gains ζ_1 and ζ_2 can be selected such that the eigenvalues of the tracking error equation

$$\ddot{\mathbf{e}}_p + \zeta_1 \dot{\mathbf{e}}_p + \zeta_2 \mathbf{e}_p = 0 \tag{72}$$

have negative real parts. This ensures the global stability of the system when $\mathbf{s}$ converges to zero.

Using the PID type $\mathbf{s}$ and the same definitions of θ' and θ'', we can derive that

$$\mathbf{M}\ddot{\mathbf{e}}_p = \mathbf{M}\ddot{\theta}_d - \mathbf{M}\ddot{\theta}$$

$$= \mathbf{M}[\theta'' - \mathbf{s} - \zeta_1 \dot{\mathbf{e}}_p - \zeta_2 \mathbf{e}_p] - [-\mathbf{B}\dot{\theta} + \hat{\mathbf{M}}\mathbf{q}'' + \hat{\mathbf{B}}\theta']$$
$$= -\mathbf{Y}(\theta, \dot{\theta}, \dot{\theta}_d, \ddot{\theta}_d)\tilde{\mathbf{a}} - (\mathbf{M} + \mathbf{B})\mathbf{s} - \mathbf{M}\zeta_1 \dot{\mathbf{e}}_p - \mathbf{M}\zeta_2 \mathbf{e}_p$$

where

$$\mathbf{Y}\tilde{\mathbf{a}} = \tilde{\mathbf{M}}\theta'' + \tilde{\mathbf{B}}\theta' \tag{73}$$

$$\tilde{\mathbf{a}} = \hat{\mathbf{a}} - \mathbf{a} \tag{74}$$

When the same type of Lyapunov function is used

$$V = 1/2\mathbf{s}^T\mathbf{M}\mathbf{s} + 1/2\tilde{\mathbf{a}}^T\Gamma\tilde{\mathbf{a}} \tag{75}$$

then,

$$\dot{V} = 1/2\mathbf{s}^T\dot{\mathbf{M}}\mathbf{s} + \mathbf{s}^T\mathbf{M}\dot{\mathbf{s}} + \tilde{\mathbf{a}}^T\Gamma\dot{\tilde{\mathbf{a}}}$$
$$= 1/2\mathbf{s}^T\dot{\mathbf{M}}\mathbf{s} + \mathbf{s}^T\mathbf{M}(\ddot{\mathbf{e}}_p + \zeta_1\dot{\mathbf{e}}_p + \zeta_2\mathbf{e}_p) + \tilde{\mathbf{a}}^T\Gamma\dot{\tilde{\mathbf{a}}}$$
$$= -\mathbf{s}^T\mathbf{M}\mathbf{s} + 1/2\mathbf{s}^T(\dot{\mathbf{M}} - 2\mathbf{B})\mathbf{s} + \tilde{\mathbf{a}}^T(\Gamma\dot{\tilde{\mathbf{a}}} - \mathbf{Y}^T\mathbf{s})$$

If adaptation law

$$\dot{\tilde{\mathbf{a}}} = \Gamma^{-1}\mathbf{Y}^T\mathbf{s} \tag{76}$$

is used, then

$$\dot{V} = -\mathbf{s}^T\mathbf{M}\mathbf{s} \leq 0 \tag{77}$$

for all $\mathbf{s}$ due to the fact that the matrix $\dot{\mathbf{M}} - 2\mathbf{B}$ is skew-symmetric, and ζ_1, $\zeta_2 > 0$, and $\mathbf{M}$ is positive definite.

A block diagram of the proposed control algorithm with PD type $\mathbf{s}$ is shown in Figure 2. Our adaptive controller is conceptually simple and easy to implement. This approach does not require the use of joint accelerations and inversion of inertia matrix. Its computational cost is low because

it can be implemented through the use of Newton-Euler recursive formulation. It can be seen from Equation (62) which has the same structure as computed torque method, that the control law can be computed efficiently using a Newton-Euler formulation once $\hat{\mathbf{a}}$ have been specified. A high gain feedback is not a must for the system stability. This adaptive approach can also be applied to industrial robot control by a slight modification.

5 Adaptive Control in Inertia Space

In this section, we extend our joint space adaptive control approaches to the problems where control variables are specified in inertia space.

Conceptually, for most applications, the desired robot hand trajectory (i.e., position, velocity and acceleration) must be specified in inertia space. For example, let's consider catching a moving object by a space robot. The desired trajectory after catching depends upon the tasks and the motion trajectory of the object before catching, and thus must be specified in inertia space. In other words, as in the case of fixed-base robot tasks are normally specified in Cartesian space, tasks in space applications are unlikely to be specified in joint space. Fortunately, the mapping from robot hand position in inertia space to displacements in joint space can be uniquely determined for space robot system with an attitude controlled base, which differs from the case of a completely free-flying space robot system. This unique kinetic relationship has been first studied by Longman et al. [9], and also is illustrated by a planar example in our case study.

However, the unique kinematics relationship can only be determined when dynamic parameters are given, for this relationship is indeed dynamic dependent. When some dynamic parameters are unknown, which is indeed the reason why we come to adaptive control, the mapping cannot be determined! Therefore, the primary difficulty of extending our approach from joint space to inertia space is that the desired trajectory in inertia space cannot be transformed to the desired trajectory in joint space because some dynamic parameters are unknown. In previous discussion, we have utilized a desired trajectory in joint space, as other researchers have done [16], without giving any explanation about how the trajectory is generated. The problem is not significant if the objective is to identify dynamic parameters, but is important if the objective is to control the system.

The problem can be resolved if the same structured adaptive control scheme can be implemented in inertia space. This, however, is not feasible because the proposed adaptive control scheme in joint space requires that the dynamic model must be linearly parameterized. Therefore, the same type of the control scheme cannot be developed in inertia space. As has been known, the dynamic related generalized Jacobian of space robot makes it impossible to suitably choose a set of dynamic parameters such that the inertia space system dynamics can be linearized. That is why the same structured adaptive controller in joint space is not feasible for adaptive control in inertia space.

We approach the problem in the following way. First, given trajectory in inertia space, we use an initial estimation of dynamic parameters to compute initial joint trajectory. Then the initial joint trajectory and dynamic parameters are used in the proposed joint space adaptive control algorithms. After a certain period of time we update the system dynamic parameters by using new estimated ones in the outer loop of our controller. We can then specify more precise joint space trajectory based on these new parameters and the inertia space trajectory. Since the inertia space trajectory is uniquely determined by the joint space trajectory and dynamic parameters, it can be shown from the Jacobian relationship that position errors in inertia space converges to a given boundary if position errors in joint space and parameter errors are bounded, provided that the robot is not in its singularity configuration. The control scheme is illustrated in Figure 3.

It is worthwhile to discuss two issues in the implementation of the proposed control scheme.

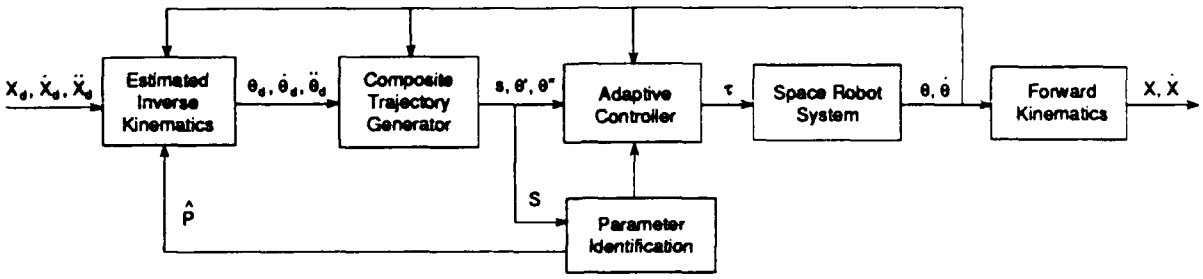

Figure 3: Block diagram of adaptive control scheme in inertia space.

First, to accurately estimate unknown parameters, a persistent excitation (PE) trajectory is required to drive the robot joints. PE trajectories in joint space and in inertia space are not equivalent because the spectrum of trajectory signal in inertia space is different from the spectrum of the same signal in joint space due to nonlinear kinematic transformation. Therefore, it is of importance to carefully choose initial trajectory in inertia space such that the same trajectory in joint space is PE. If the PE condition is not satisfied, parameter identification error occurs, although the joint space position errors may still converge.

Second, the updating time for inverse kinematics using the estimated parameters in outer loop of our controller must be slow enough to maintain the system stable. The outer loop, as shown in Figure 3, is used to update the inverse kinematics and therefore the desired joint trajectory which is used in joint space adaptive controller. A fast updation, especially using incorrect parameters $\hat{p}_i$, may not guarantee the convergence of parameter errors. In the simulation, the updating time for inverse kinematics is set to 10 seconds. Simulation results have shown that position errors in inertia space converge to zero as errors in joint space converge to zero and estimated parameters converge to their true values.

In fact, if the updating time for inverse kinematics is long enough, we can also view the control scheme as a two-phase approach, parameter identification phase and control phase. That is, to estimate dynamic parameters in joint space using the joint space trajectory transformed by the given inertia space trajectory and initial guess of parameters, then to control the system in inertia space, once the dynamic parameters has been correctly identified. If the dynamic parameters are estimated ideally, the control phase may also be executed using dynamic control algorithm.

6 Simulation Study

In previous discussion, we studied kinematics and dynamics, and presented an adaptive algorithm in joint space for a general multiple-degrees-of-freedom space robot system with an attitude controlled base. In this section, we conduct a case study to show the computation of the proposed algorithms and their feasibility in robot motion control. Though the following discussion is confined to adaptation to mass variation only, our algorithm is also applicable to other parameter adaptation, provided that a set of combinations of those parameters can be chosed such that the dynamics can be linearly expressed in terms of the parameters interested.

A two-DOF revolute manipulator with link length given by l_1 and l_2 (l_1=l_2=l) is considered as a lumped-parameter model with point mass m_1 and m_2 at the end of each link. For simplicity, we assume that the base attitude can be successfully controlled so that we need only consider the

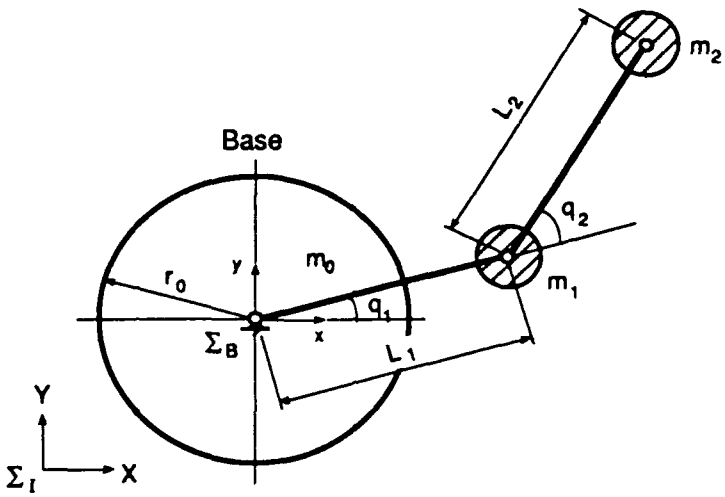

Figure 4: A planar space robot system model.

control of the robot itself. However, it must be pointed out that our adaptive control algorithm can be applied to control the robot motion and the base orientation simultaneously, albeit more complicated. The system model for simulation study is shown in Figure 4.

At initialization, m_c and $\mathbf{R}_c$ are computed, and they remain unchanged unless a load is added.

$$m_c = m_0 + m_1 + m_2 \tag{78}$$

$$m_c\mathbf{R}_c = m_0\mathbf{R}_0 + m_1\mathbf{R}_1 + m_2\mathbf{R}_2 \tag{79}$$

$$\mathbf{R}_1 = \mathbf{R}_0 + \mathbf{r}_1 \tag{80}$$

$$\mathbf{R}_2 = \mathbf{R}_0 + \mathbf{r}_2 \tag{81}$$

When the robot is in motion,

$$\mathbf{R}_0 = \mathbf{R}_c - \frac{m_1}{m_c}\mathbf{r}_1 - \frac{m_2}{m_c}\mathbf{r}_2 \tag{82}$$

$$\mathbf{R}_E = \mathbf{R}_2 \tag{83}$$

The generalized Jacobian is

$$\mathbf{N} = \mathbf{J}_E - \mathbf{J}_c \tag{84}$$

and

$$\mathbf{J}_E = \mathbf{J}_2 \tag{85}$$

$$\mathbf{J}_c = \frac{m_1}{m_c}\mathbf{J}_1 + \frac{m_2}{m_c}\mathbf{J}_2 \tag{86}$$

$$= \frac{l}{m_c}\begin{bmatrix} -(m_1+m_2)s_1 - m_2 s_{12} & -m_2 s_{12} \\ (m_1+m_2)c_1 + m_2 c_{12} & m_2 c_{12} \end{bmatrix} \tag{87}$$

where s and c stand for sine and cosine, e.g., $s_1 = sin(q_1)$, $c_{12} = cos(q_1 + q_2)$. The system dynamics has the following form,

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{B}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} = \tau \tag{88}$$

where

$$\mathbf{M} = \mathbf{M}_q - \mathbf{M}_2 \tag{89}$$

$$\mathbf{M}_q = l^2 \begin{bmatrix} m_1 + 2m_2(1+c_2) & m_2(1+c_2) \\ m_2(1+c_2) & m_2 \end{bmatrix} \tag{90}$$

$$\mathbf{M}_2 = m_c \mathbf{J}_c^T \mathbf{J}_c \tag{91}$$

$$= \frac{l^2}{m_c} \begin{bmatrix} m_1^2 + 2m_2(m_1+m_2)(1+c_2) & m_2^2(1+c_2) + m_1 m_2 c_2 \\ m_2^2(1+c_2) + m_1 m_2 c_2 & m_2^2 \end{bmatrix} \tag{92}$$

therefore,

$$\mathbf{M} = \frac{l^2}{m_c} \begin{bmatrix} m_0 m_1 + m_1 m_2 + 2m_0 m_2(1+c_2) & m_1 m_2 + m_0 m_2(1+c_2) \\ m_1 m_2 + m_0 m_2(1+c_2) & m_1 m_2 + m_2 m_0 \end{bmatrix} \tag{93}$$

$$= p_1 \mathbf{R}_1 + p_2 \mathbf{R}_2 + p_3 \mathbf{R}_3 \tag{94}$$

where

$$p_1 = \frac{m_0 m_1}{m_c} \tag{95}$$

$$p_2 = \frac{m_1 m_2}{m_c} \tag{96}$$

$$p_3 = \frac{m_0 m_2}{m_c} \tag{97}$$

$$\mathbf{R}_1 = \begin{bmatrix} l^2 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{R}_2 = \begin{bmatrix} l^2 & l^2 \\ l^2 & l^2 \end{bmatrix} \mathbf{R}_3 = \begin{bmatrix} 2(1+c_2)l^2 & (1+c_2)l^2 \\ (1+c_2)l^2 & l^2 \end{bmatrix} \tag{98}$$

It is noted that $\mathbf{M}$ is linear in terms of combined dynamic parameters p_1, p_2 and p_3. This is an example to show that dynamics of the space robot system with an attitude controlled base can be linearly parameterized in joint space. We also note that m_0, m_1 and m_2 can be uniquely determined by p_1, p_2 and p_3,

$$m_1 = p_1 p_2 (\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}) \tag{99}$$

$$m_2 = p_2 p_3 (\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}) \tag{100}$$

$$m_0 = p_1 p_3 (\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}) \tag{101}$$

The matrix $\mathbf{B}$ is determined by

$$\mathbf{B} = \frac{m_0 m_2}{m_c} \begin{bmatrix} -2l^2 s_2 \dot{q}_2 & -l^2 s_2 \dot{q}_2 \\ l^2 s_2 \dot{q}_1 & 0 \end{bmatrix} = p_3 \mathbf{R}_4 \tag{102}$$

where

$$\mathbf{R}_4 = \begin{bmatrix} -2l^2 s_2 \dot{q}_2 & -l^2 s_2 \dot{q}_2 \\ l^2 s_2 \dot{q}_1 & 0 \end{bmatrix} \tag{103}$$

Our adaptive control law is

$$\tau = \hat{\mathbf{M}} \mathbf{q}'' + \hat{\mathbf{B}} \mathbf{q}' = \mathbf{Y} \hat{\mathbf{a}} \tag{104}$$

$$\mathbf{Y} = \begin{bmatrix} \mathbf{R}_1 \mathbf{q}'' & \mathbf{R}_2 \mathbf{q}'' & \mathbf{R}_3 \mathbf{q}'' + \mathbf{R}_4 \mathbf{q}' \end{bmatrix} \tag{105}$$

$$\hat{\mathbf{a}} = \begin{bmatrix} \hat{p}_1 \\ \hat{p}_2 \\ \hat{p}_3 \end{bmatrix} \tag{106}$$

with the following adaptation law

$$\dot{\hat{\mathbf{a}}} = \begin{bmatrix} \gamma_1 \mathbf{s}^T \mathbf{R}_1 \mathbf{q}'' \\ \gamma_2 \mathbf{s}^T \mathbf{R}_2 \mathbf{q}'' \\ \gamma_3 \mathbf{s}^T (\mathbf{R}_3 \mathbf{q}'' + \mathbf{R}_4 \mathbf{q}') \end{bmatrix} \tag{107}$$

To study the proposed adaptive algorithms, we use the following common set of conditions:

$$q_{1d} = \frac{\pi}{180}(54 + 6(sin(t) + cos(4t))) \tag{108}$$

$$q_{2d} = \frac{\pi}{180}(-126 + 6(sin(2t) + cos(6t))) \tag{109}$$

$$\zeta = 10 \tag{110}$$

In the first case we used the following mass parameters, $m_0 = 41kg$, $m_1 = 5kg$, $m_2 = 4kg$, and the initial guess of all three parameters is set to 50% of their true values. It can be found from Figure 5 that joint errors converge to zero and all parameters converge to their true values 4.1, 0.4, and 3.28 (with small relative errors 1.2%, 2.1%, 2.5%, respectively) after a transient period (approximately 10 seconds). The results showed the validity and efficiency of the adaptive algorithm proposed.

We then compare the performance of adaptive controller and dynamic controller without adaptation when there is uncertainty in dynamic parameters. In order to make the dynamic control more favorable, we use 80% of true values as initial estimates of those dynamic parameters. The dynamic control algorithm is based on PD type structure in joint space without consideration of parameter uncertainty. Figure 6 gives plots of the variations of two joint position errors by using adaptive control and dynamic control. The adaptive control performance is distinctly superior to the dynamic control response.

To study the effect of mass ratio of the base with respect to the robot, we performed simulation when the base mass is sufficiently large compared to that of robot. Figure 7 gives the simulation results when the base mass is 50000kg. The results have shown that the performance is not sensitive to the mass ratio, which also shows that the proposed control algorithm is applicable to fixed-base robots.

Figure 8 shows identification of combined parameters p_1, p_2, and p_3, and the resultant mass m_1, m_2, and m_0, in the above case. From Figure 8 we found that estimation of all parameters m_1, m_2, and m_0 are very close to their true values. This demonstrated that identification of combined dynamic parameters is equivalent to the identification of dynamic parameters m_1, m_2, and m_0, as we have discussed previously. It is interesting to note that in Figure 8 the estimation of nonlinear dynamic parameters p_1, and p_3 converged to m_1 and m_2 due to the fact that the base mass is almost infinite.

In order to compare two different adaptive control algorithms, PD type and PID type, various cases have been tested. For a persistent excitation (PE) trajectory, both algorithms presented almost identical performance. For a non-PE trajectory, such as

$$q_{1d} = \frac{\pi}{180}(60 - t + 0.05t^2) \tag{111}$$

$$q_{2d} = \frac{\pi}{180}(-120 - t + 0.05t^2) \tag{112}$$

the steady state performance is improved significantly using PID type adaptive controller, as shown in Figure 9.

For inertia space adaptive controller, an initial guess of the updating parameters is set to 80% of the true value. The inertia space trajectory and joint space trajectory employed in the simulation are shown in Figure 10. We used 10 seconds as updating time for inverse kinematics. The effectiveness of this adaptive scheme has been verified by the tracking errors shown in Figure 11. It is found that position errors in inertia space converge to zero as errors in joint space converge to zero and estimated parameters converge to their true values.

7 Conclusions

In this report, we have discussed adaptive control of a space robot system with an attitude controlled base on which the robot is attached. Adaptive control is critical for various robotic applications in space, such as material transport and light manipulation, in which robots have to face uncertainty on the dynamic parameters of the load or the structure. Based on Lagrangian dynamics and linear momentum conservation law, we derived system dynamic equations. Then we showed that the system dynamics in joint space can be linearly parameterized, i.e., the dynamics can be linearized in joint space by a set of combined dynamic parameters, while the same conclusion is not true in inertia space.

An adaptive control scheme in joint space is proposed to cope with dynamic uncertainties based on the dynamic model developed. The scheme is effective and feasible for space robot applications by eliminating the use of joint acceleration measurement, inversion of inertial matrix, high gain feedback, and considerable computation cost. At meantime, the scheme is also applicable for the fixed-base robot system by slight modification.

Considering that the tasks in space are specified in inertia space in most applications, we discussed the issues of adaptive control of the robot for the tasks that must be filfull in inertia space. Two main problems have been identified. If the joint adaptive control is implemented, the desired joint trajectory cannot be generated from the given inertia space trajectory since kinematic mapping is dynamics dependent, and thus is subjected to uncertainty in parameters. Moreover, the same structured adaptive control as in joint space is not feasible for inertia space due to nonlinear parameterization in inertia space. We approached this problem by making use of the proposed joint space adaptive controller while updating joint trajectory by using the estimated dynamic parameters and the given trajectory in inertia space. This method has shown its effectiveness in simulation. Parameter estimation and updating time are discussed.

Finally, a planar system is studied numerically to investigate the linear parameterization problem and illustrate the procedure to design the controller. The results demonstrated validity and effectiveness of the proposed adaptive control schemes in both joint and inertia space descriptions.

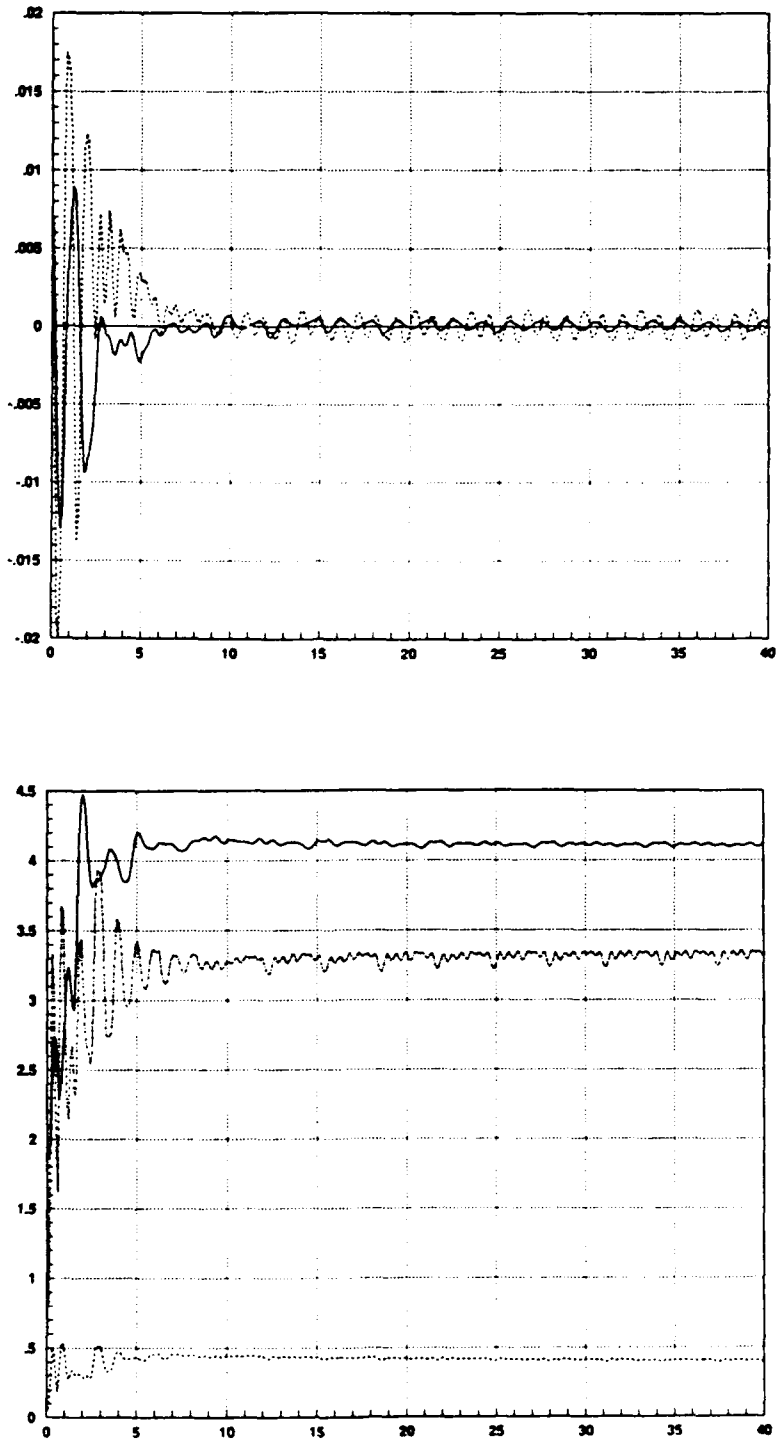

Figure 5 Tracking errors and parameter estimations using joint space adaptive control

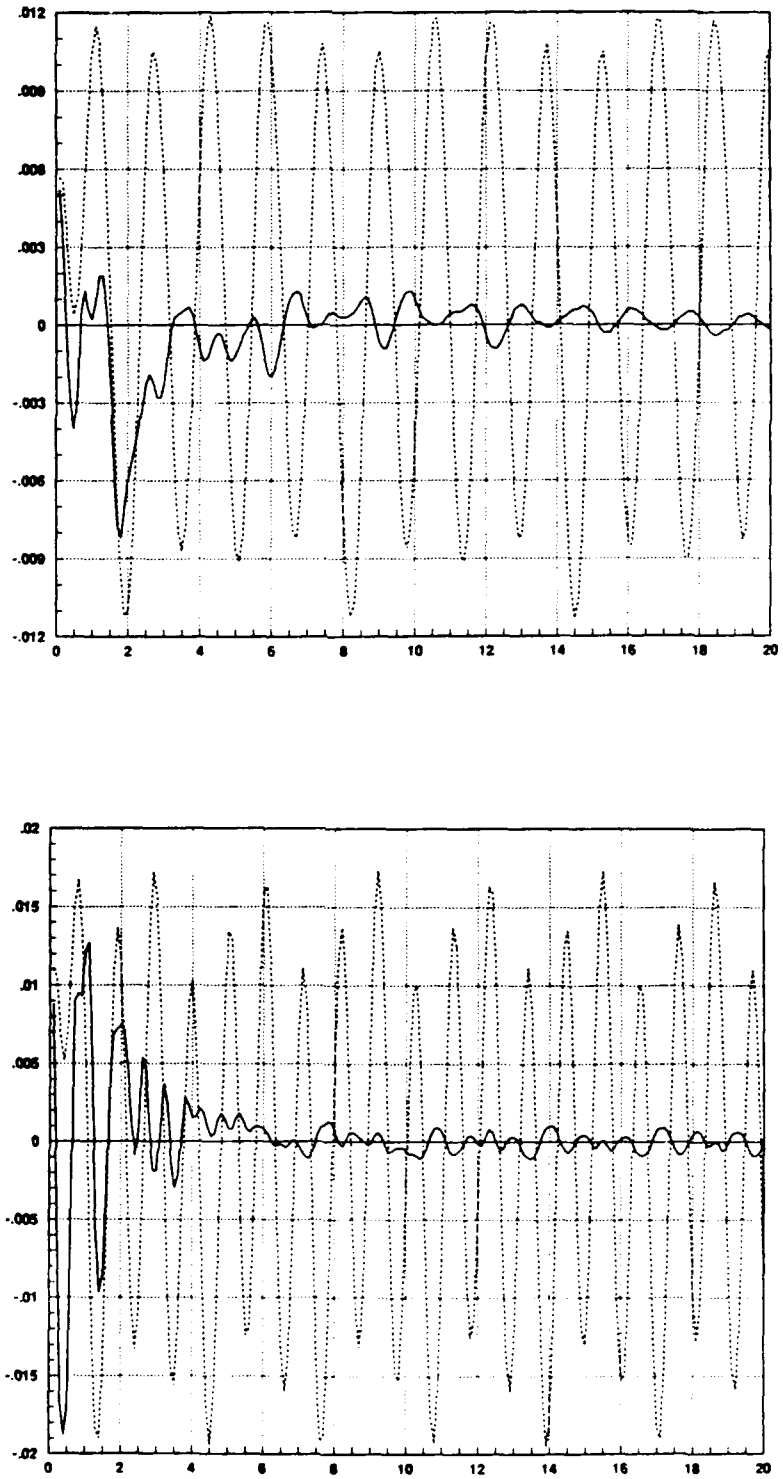

Figure 6 Comparison between adaptive control and dynamic control

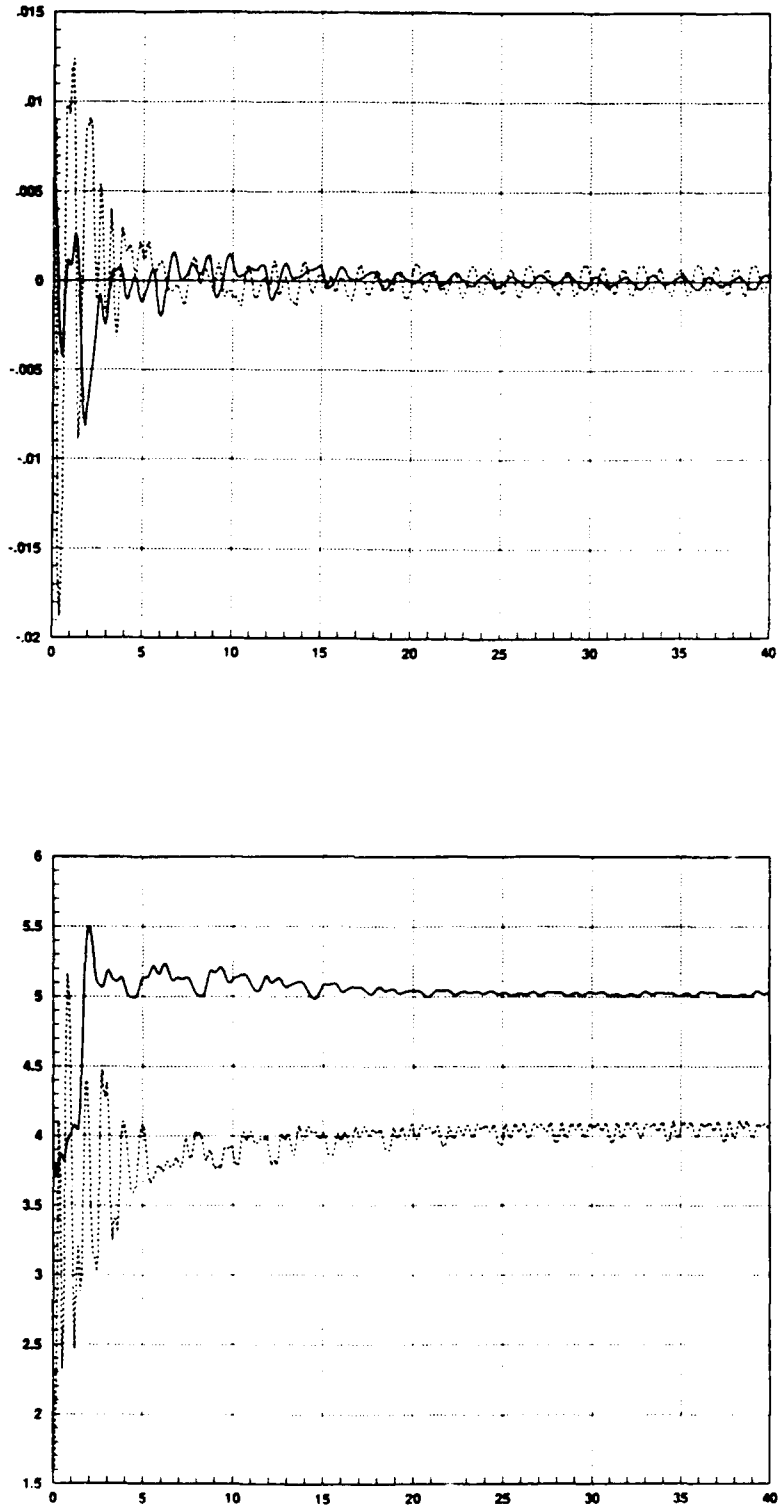

Figure 7 Example of adaptive control for fixed-based robot

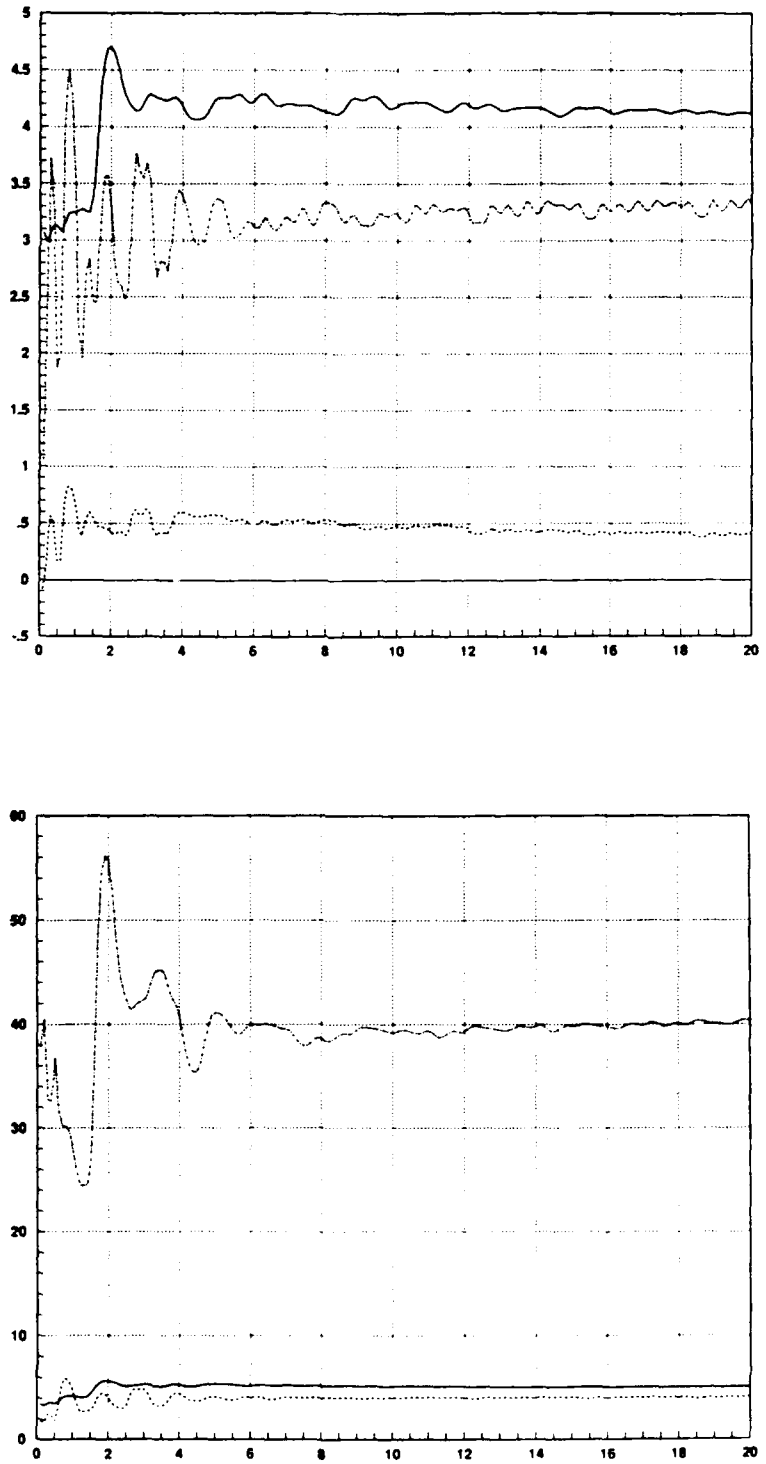

Figure 8 Illustration of combined dynamic parameter identification

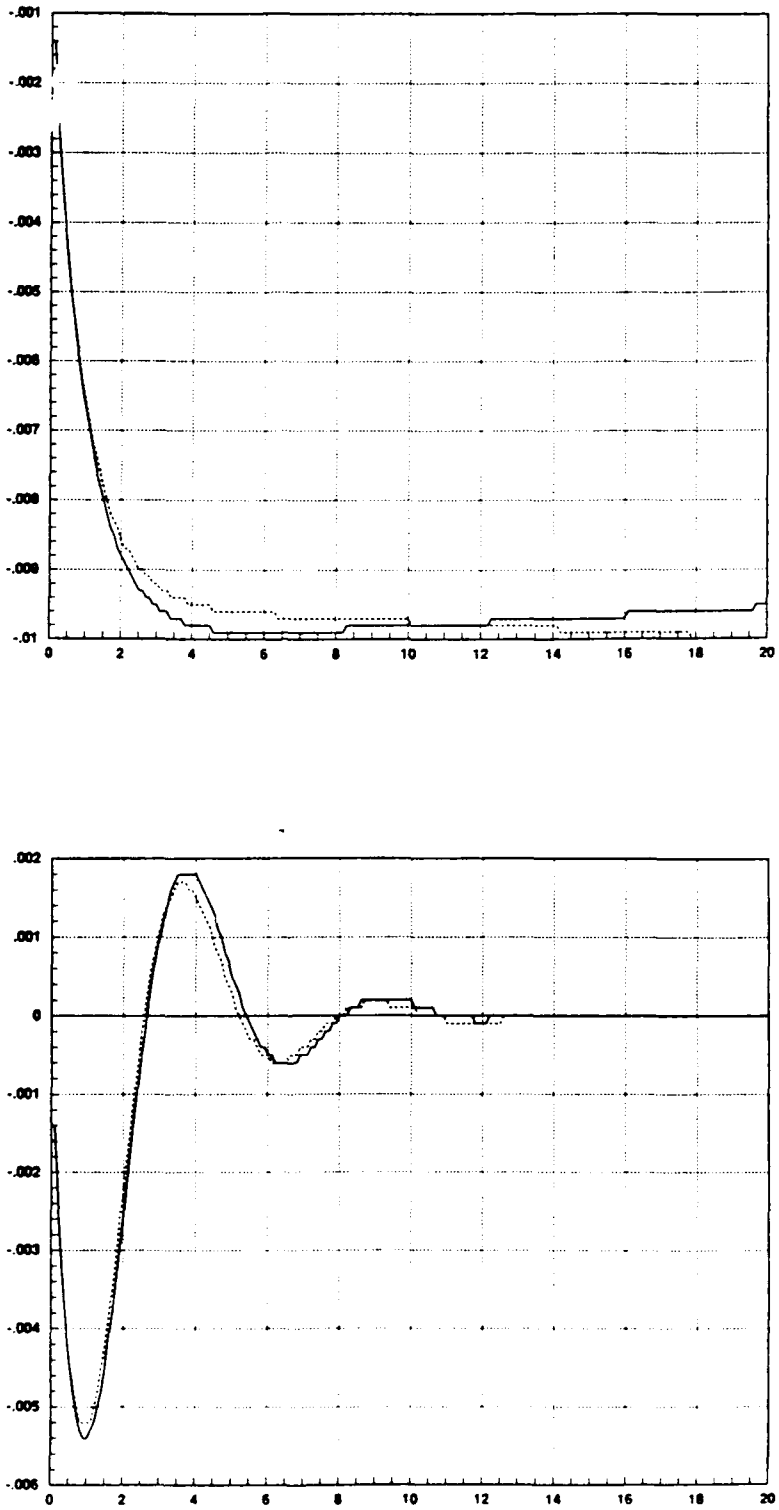

Figure 9 Comparison between PD type and PID type adaptive control schemes

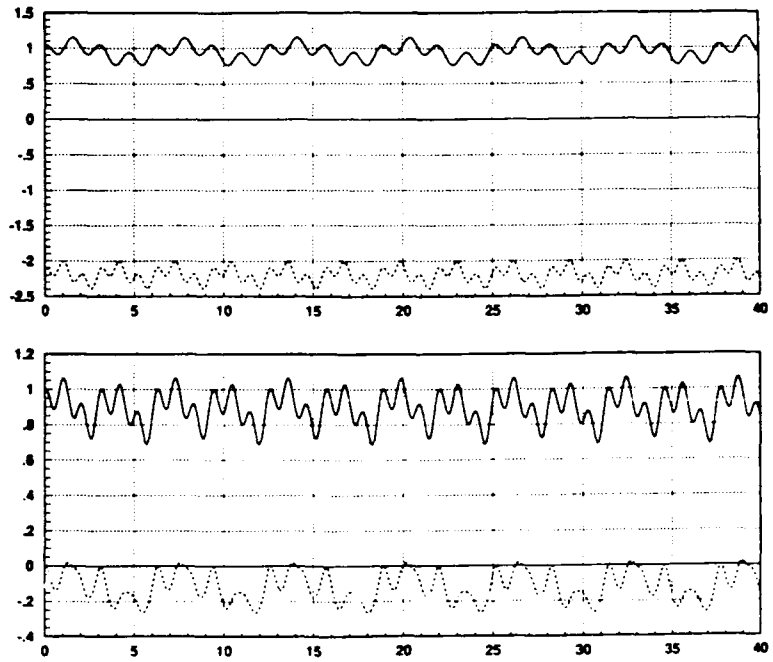

Figure 10 Trajectories in joint space and in inertia space using inertia space adaptive controller

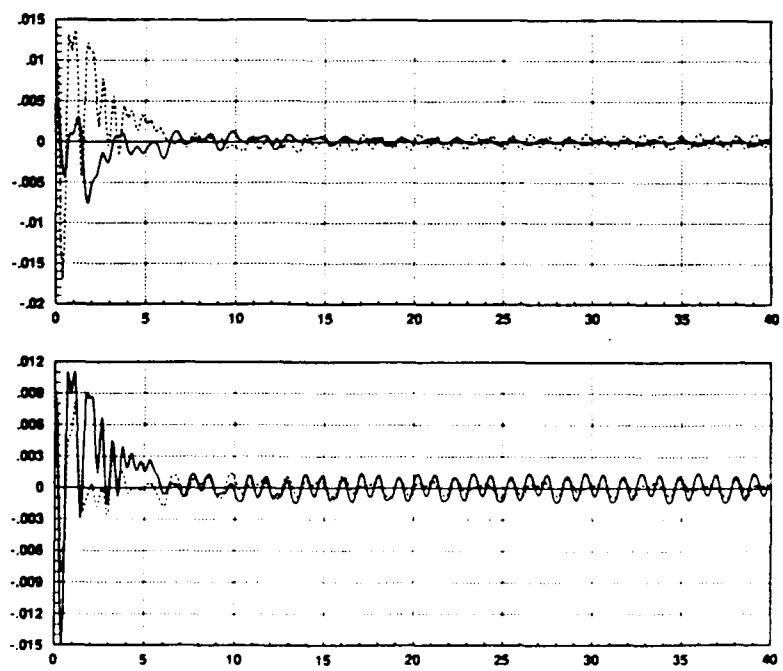

Figure 11 Tracking errors in joint space and in inertia space using inertia space adaptive controller

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