PL-TR-91-2187

②

IMPROVED MODELS OF THE INNER AND OUTER RADIATION BELTS

K. A. Pfitzer

McDonnell Douglas Space Systems Corp
5301 Bolsa Avenue
Huntington Beach, CA 92647

July 1991

Scientific Report No. 1

Approved for public release; distribution unlimited

PHILLIPS LABORATORY
AIR FORCE SYSTEMS COMMAND
HANSCOM AIR FORCE BASE, MASSACHUSETTES 01731-5000

This technical report has been reviewed and is approved for publication.

Kevin J Kern

KEVIN J. KERNS
Contract Manager

E G Mullen

E.G. MULLEN
Branch Chief

Rita C Sagalyn

RITA C. SAGALYN
Division Director

This document has been reviewed by the ESD Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS).

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
July 1991

3. REPORT TYPE AND DATES COVERED
Scientific No. 1

4. TITLE AND SUBTITLE
Improved Models of the Inner and Outer Radiation Belts

5. FUNDING NUMBERS
PE 62101F
PR 7601 TA 22 WU MD
Contract F19628-90-C-0099

6. AUTHOR(S)
K. A. Pfitzer

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
McDonnell Douglas Space Systems Corporation
5301 Bolsa Avenue
Huntington Beach, CA 92647

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Phillips Laboratory
Hanscom AFB, MA 01731-5000

Contract Manager: Lt Kevin Kerns/PHP

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
PL-TR-91-2187

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release;
Distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

A pitch angle dependent invariant routine has been developed. This routine written in FORTRAN calculates the first and second invariant for an arbitrary number of pitch angles at any satellite location within the magnetosphere providing the observation is on a closed field line. The invariant routine utilizes a new fast version of the IGRF internal magnetic field and the Olson-Pfitzer 1977 external magnetic field routine. The internal field routine utilizes term dropping at large distances as well as improved coding techniques to obtain speed advantages of 1.5 to 35 over the standard internal field routine. The invariant routine is highly optimized and calculates all of the pitch angle dependent invariant utilizing only a single field line integral. The invariant routine also determines the actual minimum B value along the line of force and where in local time the field line crosses the magnetic equator. Hiton's expansion of L is used to assign an L value to each of the invariant calculations.

14. SUBJECT TERMS
Magnetic field model, IGRF, Integral invariant, B-L coordinate system.

15. NUMBER OF PAGES
82

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT
Unclassified

18. SECURITY CLASSIFICATION OF THIS PAGE
Unclassified

19. SECURITY CLASSIFICATION OF ABSTRACT
Unclassified

20. LIMITATION OF ABSTRACT
SAR

Table of Contents

1.0 Introduction

Considerable progress has been made in the understanding of magnetospheric processes in the last 25 years. During this time much of the effort has been focused on understanding processes operating in the tail of the magnetosphere and near the magnetospheric boundaries. The inner magnetosphere has not been extensively investigated in the last 10 to 20 years. The Combined Release and Radiation Effects Satellite (CRRES) Program is designed to make a substantial contribution to understanding this region of space.

In this effort McDonnell Douglas Space Systems Company (MDSSC) will introduce novel new modelling approaches and will satisfy one of main goals of the CRRES mission, the development of new static and dynamic radiation belt models. Such models are an important tool for engineers for the design of systems that can survive in the space environment. The present radiation belt models that are now used by both the scientific and engineering communities are the Vette radiation models developed by the National Space Sciences Data Center (NSSDC) in the late 1960's and early 70's. The data sets which were used to develop the Vette models were acquired by instruments that are quite primitive compared to todays state-of-the-art instruments. Nevertheless these older models have served the community well.

The present NSSDC developed models are organized in B, L space, a coordinate system developed by C. E. McIlwain in 1961. This coordinate system has been virtually unchanged since that time. Improvements have come only in the form of improved computational techniques. Although the B, L system has proven useful in the inner zone, its use in the outer zone has not been as successful.

This CRRES analysis effort will include the development of new tools for the organization of the data. This effort will develop novel new techniques for organizing charged particle data in the inner and outer zone. In the inner zone we will develop a model that not only takes into account the effect of the magnetic field in organizing the charged particles but also the effect of the solar cycle dependent atmosphere in shaping the low altitude region of the inner radiation belt. In the outer zone we will provide a coordinate system that can correctly represents adiabatic changes in the radiation belt and fully takes into account drift shell splitting and yet represents the entire outer zone in terms of only two parameters, the first and second invariant for each observation and pitch angle. Once the new tools developed under this effort are implemented and a best fit is made to the CRRES data, a high quality radiation belt model will be created that will be valid for all epoch within a solar cycle and for all magnetic conditions of the magnetosphere. In the inner zone, the new model will permit calculation of the fluxes during any part of the solar cycle. In the outer zone the model will help separate adiabatic changes from non adiabatic variations, allow the calculation of particle fluxes for all states of magnetospheric compression, and will help theorists explain many of the observed changes within the magnetosphere.

The analysis effort consists of two distinct phases, the first of these is the development of the required computer code for defining the new coordinate system and the second is the fitting of the data utilizing the new coordinate system. By its very nature the development of the computer code cannot be completely decoupled from the analysis of the data. The quality of the data and the various features found within the data stream will dictate the ultimate development of the model coordinate system and the final fit of the data.

2.0 A Pitch Angle Dependent Invariant Routine

During the first year of the contract almost all of the effort has been spent developing and polishing the various software tools that will be required to produce a new high precision CRRES model of the inner and outer zones. Some of this software must yet be verified using CRRES data. This technical report will describe the software that is considered fully operational and which is available for use by any of the CRRES investigators.

This annual report will concentrate on describing the software that could be verified without using the CRRES data. The major achievement during the first year was the development of a new B,L code that calculates the invariant at the satellite for many different pitch angle particles. Calculating B,L has always been a computationally expensive procedure and thus considerable effort was expended to produce a code that is *efficient and cost effective. The calculation of the second* invariant requires the calculation of a line integral that makes many calls to the magnetic field subroutine.

2.1 A Fast Version of IGRF

Present versions of B,L use only the internal model of the magnetic field. The CRRES code must use both internal and external models of the magnetic field, since it must take into account drift shell splitting in the outer magnetosphere. Thus one of the most important routines for saving computer time is the development of an internal and external magnetic field routine that optimizes computer speed. The Olson-Pfitzer 1977 tilt dependent model is such a routine. It, however, uses the Barraclough internal field routine and thus is not appropriate for the CRRES effort. The IGRF routines using the modern field coefficients were obtained from the National Space Sciences Data Center (NSSDC). These routines are, however, considerably slower than the routine contained in the original version of the 1977 tilt dependent model. The main field routine contained in the 1977 model is derived from Joe Cain's SPHRC routine. This routine gains additional speed at the expense of some memory. Instead of using indexed loops it explicitly writes out the spherical harmonic expansion terms and thus all of the overhead required to keep track of the various indices is abolished. It is this authors opinion that this version of representing the main field is inherently 50% faster than any other representation. The version used in the 1977 tilt dependent model has the Gauss normalized Barraclough coefficients built into the model. It, furthermore, has a term dropping algorithm, that drops the higher order terms as distances increases. This results in a considerable savings in computer time.

The IGRF internal field model developed for the CRRES analysis begins with Joe Cain's spherical harmonic expansion. The new IGRF routine has been given the name SPIGRF (SPeed IGRF). Since the IGRF coefficients are for a tenth order expansion, terms up the N=11 are contained in SPIGRF (the Barraclough model only had 10 terms). The first time SPIGRF is called, it calls a routine called FLDCOF. FLDCOF reads the appropriate IGRF coefficient sets, interpolates to the epoch of interest and converts the Schmitt normalized IGRF coefficients to Gauss normalized coefficients. If during a subsequent call, the date has changed by more than 0.1 year, FLDCOF is once again called to update the coefficients to the new epoch. It is not computationally efficient to update the coefficients for minor changes in time. In fact it might be appropriate for the CRRES mission to select a specific date and use coefficients for the internal magnetic field that do not change with time. This is a programmatic decision and will have little or no impact on the science unless CRRES continues to function for more than 4 or 5 years.

The term dropping algorithm that is a part of SPIGRF is a smooth algorithm. The algorithm uses predetermined altitudes where specific terms are to be discontinued. Table 1 lists the altitudes and maximum number of terms that are used by SPIGRF.

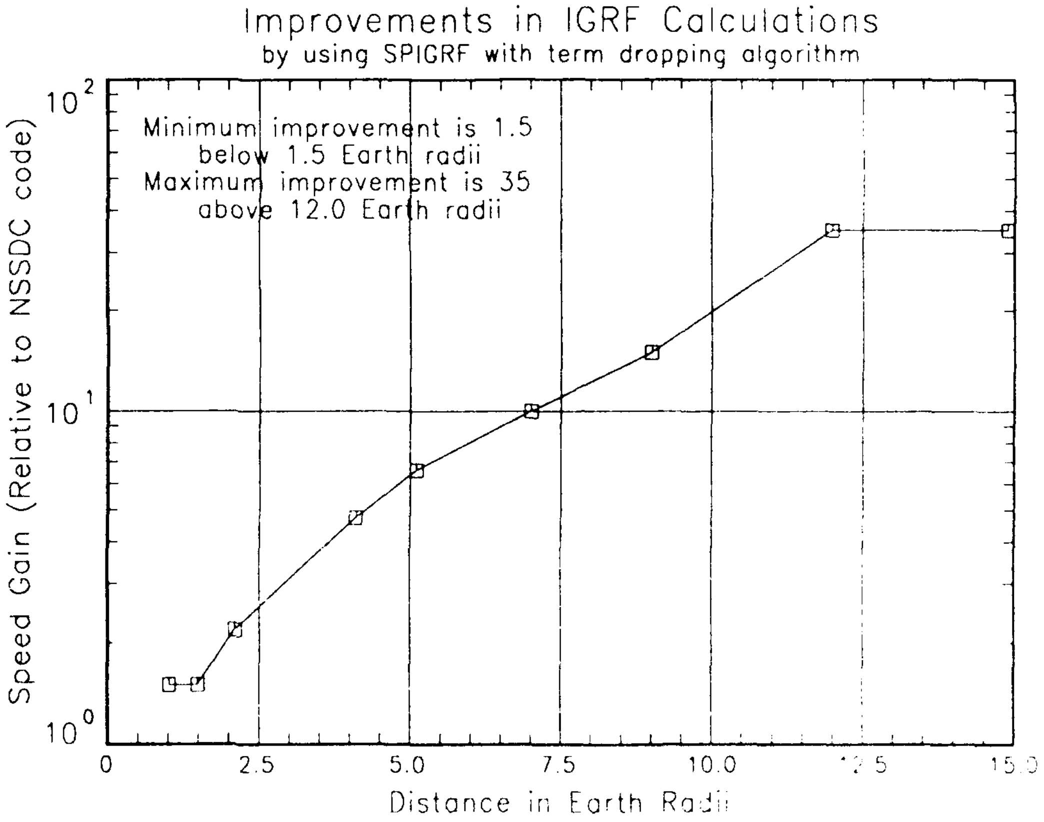
Table 1

Altitude, R_e	Number of terms
12.0	2
8.0	3
6.0	4
5.0	5
4.0	6
3.2	7
2.5	8
2.0	9
1.6	10
1.4	11

Thus for altitudes greater than 12 R_e only the dipole term is used. For altitudes less than 1.4 R_e all 11 terms are used. For altitudes between 1.4 and 1.6, the contribution of the N=11 term is linearly reduced from its full value at 1.4 to zero at 1.6. Similarly for all other intervals. The altitude values at which the term dropping takes place was experimentally determined by comparing the truncated model with the untruncated version. The truncated model differs from the untruncated model by no more than 0.1 nanotesla.

This new IGRF code is inherently 1.5 times as fast as the NSSDC code, FELDG, when used with all the terms. The term drop off algorithm significantly improves the speed without sacrificing accuracy. Figure 1 presents the speed advantage of SPIGRF when compared to the original NSSDC code. At 3.0 R_e it is 3 times as fast as FELDG, 7 times as fast at 5 R_e, 10 times as fast at 7 R_e, and 35 times as fast at 12 R_e. Since the term drop off algorithm removes the effect of a term smoothly, there are no discontinuities in the field. For a typical CRRES orbit, this version of IGRF should have an average speed advantage of 7 or 8.

Figure 1



A listing and a brief description of the calling sequence for SPIGRF is given in Appendix A.

2.2 Internal plus External Field

The new routine, SPIGRF, was combined with the remaining routines of the 1977 tilt dependent magnetic field model to produce a high speed quiet time magnetic field model that uses the IGRF coefficients. The name of the entire file that contains the tilt dependent model, SPIGRF, and a test routine was given the name BMNIGRF.

The execution speed of the new fast IGRF code, SPIGRF, plus the 1977 tilt dependent external model is faster everywhere than the old internal IGRF code, FELDG, without the external model. At 1.4 R_e or less the speed advantage is 1.2, at 2 R_e it is 1.05, at 6 R_e it is 1.6 and at 10 R_e it is 2.0. Thus calculating B,L using the external and internal field routines is faster than calculating a B,L based on the internal field alone using the older IGRF routines.

Note: The external field model presently used for the initial CRRES studies is the 1977 quiet time tilt dependent model. The dynamic model, which will be used to study the outer zone as a function of magnetospheric compression, will replace the quiet time model at the appropriate time.

Appendix B gives a listing of the BMNIGRF test routine, the 1977 tilt dependent routine, BXYZMU, and the various routines required to combine the external and internal magnetic fields. The internal field routines are described in Appendix A.

2.3 A Pitch Angle Dependent B,L Routine

In order to adequately represent directional data, it is necessary to define the first and second invariant for each directional measurement. At the present time the current B,L routines calculate the invariant for a particle that mirrors at the location of the satellite. All other particles will of course have different first and second invariant. The first invariant is simply the mirror point magnetic field of the particle and thus B_{mir} is given by

$$B_{mir} = \frac{B_{local}}{\sin^2(\alpha_{local})} \tag{1}$$

where B_{local} is the magnetic field at the location of the measurement and α_{local} is the local pitch angle of the measurement. Thus the first invariant can be easily calculated for each directional measurement at a specific location.

The second, or integral invariant, J, is given by

$$J = \int_{B_{mir}}^{B_{mir}} \sqrt{1 - \frac{B}{B_{mir}}}\, ds \tag{2}$$

This is a line integral over the bounce path of the particle and is calculated from one mirror point to the conjugate *mirror point in the other hemisphere.* If one needs to calculate the second invariant for more than one pitch angle, more than one integral must be evaluated. For a given line integral along a magnetic line of force between two conjugate mirror points, many calls must be made to the magnetic field subroutines. Since these routines contain spherical harmonic expansions or some other equally complex expansions for the internal field, the number of calls to the magnetic field routines must be minimized. If two particles have pitch angle α_1 and pitch angle α_2 and α_1 is greater than α_2, then the bounce path length of the pitch angle α_2 particle is longer than that of the α_1 particle. However, both particles will follow the same bounce path in the region of overlap. That is the line along which the integral is performed for pitch angle α_1 stops at B_{mir1} but that for pitch angle α_2 continues on through B_{mir1} to B_{mir2}. Unfortunately since B_{mir} is inside the integral sign the value of the integral along the line differs for the two particles.

Maximum computer speed is obtained by dividing the invariant calculation into two parts. The first part calculates the path along the line of force and saves all of the pertinent parameters, and the second part calculates the integral invariant for each of the pitch angles. The multiple pitch angle invariant routine, INVARM, can calculate the first and second invariant of an unspecified number of pitch angles. The angles must be greater than zero (a pitch angle of zero would give an infinite first invariant) and less than or equal to 90 degrees. The pitch angle array must be sorted from biggest to smallest (i.e. 90,80,70,...). The line integral part of INVARM steps along a line of force with a step size that is dependent on the curvature of the line of force until the first B_{max} is reached. At each step in the integration the program calculates the step size as a function of the curvature of the field line. It also approximates from the present progress of the integration the step size needed to reach B_{max}. It then chooses the smaller of the two steps. It attempts to get close to B_{max} without stepping past it on the first approach. It is important not to exceed B_{max} since the argument of the integral become imaginary if B_{max} is exceeded. The step size algorithm appears to work reasonably well and achieves an almost 100% success rate in not overstepping B_{max} on the first try. If B_{max} is exceeded the routine backs up and attempts to determine a step 'close' to B_{max} but smaller than B_{max}.

When the integration is first started, the routine first moves in the decreasing B direction in order that it can find the precise value and location of the minimum B. When minimum B is passed, the interpolation routine determines a precise value for B_{min} and also determines the magnetic longitude of minimum B. Once B_{max} has been found in one direction, the integral is re-started at the original location and that part of the line to the other mirror point is evaluated. Once the field line for the first pitch angle is found between the two mirror points, the values stored by the field line code are used to evaluate the integral for the second invariant for the first pitch angle. To calculate the invariant for additional pitch angles, the field line portion of the routine is reentered and the line integration continues from the B_{max} stopping point of the previous pitch angle. The integration continues until the field line up to but not exceeding B_{max} of the next angle is determined. The integral for the second invariant is then calculated. This continues until the invariant of all of the pitch angles are determined or until one of the mirror points, either north or south is below 1.03 R_e, or the maximum number of steps is exceeded, or until 13 R_e is exceeded. Each subsequent calculation utilizes all of the calculated values of the field strengths and step locations of the previous pitch angle, and thus the number of calls to the magnetic field line routines is minimized. For example, the computer time required to calculate the invariant for 18 pitch angles (90, 85, 80, 75,...,5) is approximate 2 to 3 times as long as the time required to calculate the invariant for the single pitch angle of 20 degrees.

The integration uses Gill's method of Runge-Kutta integration. This is a fourth order procedure and the error goes as step size to the fourth order. An internal error control parameter can be adjusted to control the errors. This parameter is set to give the "L" parameter an accuracy of approximately 0.002. The maximum number of steps is 100. Since it is a fourth order procedure, up to 400 calls to the magnetic field routines are possible. Typically on the order 10 - 15 steps are required for a single pitch angle that mirror far off the equator (i.e. a pitch angle of 20 degrees at an L of 5.0). When the invariant for 18 pitch angles are calculated an additional step may be needed at both the northern and southern conjugate points for each additional pitch angle. If successive pitch angles are very close together, the interpolation routine may be able to calculate the next invariant without the need of an additional step.

When the invariant routine calculates the second invariant it also integrates the total column density of the atmosphere between the mirror points. The density integral uses the atmospheric density function developed for the Air Force Office of Scientific Research. This function is given by

$$\text{density} = 2.7 \times 10^{-11} \exp[(120-z)/(CON^{*}\mathrm{sqrt}(z-103))] \qquad (3)$$

Where z is the altitude in kilometer and CON is an F10.7 dependent parameter (70 - 240) and is given by

$$CON= 0.99 + 0.518^{*} \mathrm{sqrt}(F10.7/55) \qquad (4)$$

Outside of 3.0 R_e or outside of a specified distance, the density function is arbitrarily set to zero, since the function has little validity above 1000 km altitude. It is, however, a smoothly decreasing function and can thus be used as an organizing parameter for atmospheric mirror depth up to 3.0 R_e.

Figures 2 - 5 are copies of the printouts for the calculation of the first and second invariant, and the 'L' parameter, and the density totals for a set of test conditions. Each page has two conditions. The top run uses the internal field only and the bottom run uses internal plus external field. All runs are started at latitude = 0, longitude =1.0 , Day of year = 1 and Universal time = 0. Figure 2 is started at an altitude of 1.5 R_e. Both the internal and internal plus external runs give the same result since the external field is unimportant in this region of space. The small variations in L between the various pitch angle are due to the inaccuracies in the integration and more importantly to the accuracy and inherent approximate definition of the L expansion (see Hilton, J. Geophys. Res. 72, 6952, 1971). Pitch angles smaller than 35 degrees have their mirror point below 200 km and the invariant is thus not be evaluated. The internal field expansion diverges for distances less than 1.0 R_e, and thus mirror points below 1.0 cannot be assigned a second invariant. A -1.0 in any of the parameters indicates that the mirror point is too low in the atmosphere to calculate the invariant and assume that the particle is not trapped. A value of 100 indicates an open field line. As the altitude increases in Figures 3, 4 and 5, differences between the top run with internal field only and the internal plus external field run become increasingly large. The bottom run in Figure 5, a run that calculates the invariant of a measurement at 7.5R_e, shows that shell splitting at 7.5 $R_e$is almost a full R_e.

The second to the last column in the printout shows the atmospheric density parameter. As the integration proceeds down the field line, the integration sums the atmospheric density producing a column number density for the amount of atmosphere a particle encounters as it bounces between the two mirror points. This number given in grams/cm^2 is intended to represent the importance of the atmosphere as a loss mechanism for the trapped particles. One observation can already be made from figure 2; 5 degree bins near the loss cone may not be an adequately fine resolution. As can be seen from figure 2, a 5 degree change in pitch angle (40 degrees to 35 degrees) causes the atmospheric column density to change by over two orders of magnitude. The next 5 degree bin mirrors below 200 km. Figure 3 gives a 7 order of magnitude change in density encountered in the last 5 degree pitch angle bin. These numbers are an indicator on the expected sharpness of the atmospheric loss cone.

The last column displayed in the printouts shows the equatorial pitch for the specified particle at the present location. It is given by

$$\alpha_{eq} = \sin^{-1}\left[\sqrt{\frac{B_{min}}{B_{local}}}\cdot\sin(\alpha_{local})\right] \tag{5}$$

where α_{eq} is the equatorial pitch angle and α_{local} is the local pitch angle. It is important to remember that the equatorial pitch angle of a particle is not an invariant. The equatorial pitch angle of a particle changes as the particle drifts around the earth. This effect is most important at large distances where the earth's field becomes more asymmetric.

INVARM, efficiently calculates the first and second invariant of numerous pitch angles at a single satellite position. It also calculates the effective L's for the given set of invariant. It, furthermore, determines the actual minimum B field along the field line, the magnetic latitude of the observation point and the magnetic longitude where the field line crosses the magnetic equator. It also provides the column density of the atmosphere between the mirror points and thus is able to estimate the amount of scattering or absorption that can take place at the specified pitch angle. INVARM provides a complete characterization of all of the pertinent magnetic parameters for any set of pitch angles anywhere within the stable trapping region.

Appendix C lists routine INVARM, a test routines, and the various subroutines and functions required for its correct operation. The magnetic field routines required to define the field are described in Appendix A and B.

3.0 Summary

The new pitch angle dependent invariant code with its ability to calculate atmospheric mirror point depth will substantially improve our ability to organize the directional charged particle data from the CRRES satellite and develop the next generation radiation belt models. Considerable effort was expended to achieve an efficient code that minimizes computer time. The speed of the IGRF internal model was considerably improved and the invariant code is highly optimized. The success of this code must now be verified by organizing the CRRES data with this code.

Figure 2

```
Lat =      .0  Long =      1.0  R =  1.5
Year =  1990.0  Day =     1.  UT =     .00  Field = INT
Blocal =    .08314  Bmin =    .08170  Mlat =     3.977  Mlong =    346.458

P. Angle    B mir      2nd Inv.    L          Density      Eq. Pitch Angle

 90.0      .08314      .01980   1.548     3.49255E-17          82.45
 85.0      .08377      .03084   1.549     7.59682E-17          80.95
 80.0      .08572      .05440   1.548     6.75144E-17          77.49
 75.0      .08911      .09770   1.547     1.13693E-16          73.25
 70.0      .09415      .15871   1.547     2.15133E-16          68.68
 65.0      .10121      .23789   1.547     4.75860E-16          63.96
 60.0      .11085      .33591   1.546     1.42643E-15          59.15
 55.0      .12390      .45367   1.545     5.81409E-15          54.30
 50.0      .14167      .59234   1.545     4.52273E-14          49.41
 45.0      .16627      .75347   1.544     7.56966E-13          44.51
 40.0      .20121      .93914   1.544     4.50108E-11          39.58
 35.0      .25270     1.15224   1.544     7.44210E-08          34.65
 30.0    -1.00000    -1.00000  -1.000    -1.00000E+00          29.71
 25.0    -1.00000    -1.00000  -1.000    -1.00000E+00          24.77
 20.0    -1.00000    -1.00000  -1.000    -1.00000E+00          19.82
 15.0    -1.00000    -1.00000  -1.000    -1.00000E+00          14.87
 10.0    -1.00000    -1.00000  -1.000    -1.00000E+00           9.91
  5.0    -1.00000    -1.00000  -1.000    -1.00000E+00           4.96
```

```
Lat =      .0  Long =      1.0  R =  1.5
Year =  1990.0  Day =     1.  UT =     .00  Field = IN+EX
Blocal =    .08314  Bmin =    .08170  Mlat =     3.977  Mlong =    346.458

P. Angle    B mir      2nd Inv.    L          Density      Eq. Pitch Angle

 90.0      .08314      .01980   1.548     3.49255E-17          82.45
 85.0      .08377      .03084   1.549     7.59682E-17          80.95
 80.0      .08572      .05443   1.548     6.75032E-17          77.49
 75.0      .08911      .09772   1.547     1.13681E-16          73.25
 70.0      .09415      .15872   1.547     2.15143E-16          68.68
 65.0      .10121      .23789   1.547     4.75860E-16          63.96
 60.0      .11085      .33591   1.546     1.42642E-15          59.15
 55.0      .12390      .45367   1.545     5.81420E-15          54.30
 50.0      .14167      .59234   1.545     4.52266E-14          49.41
 45.0      .16627      .75347   1.544     7.56985E-13          44.51
 40.0      .20121      .93914   1.544     4.50081E-11          39.58
 35.0      .25270     1.15224   1.544     7.44241E-08          34.65
 30.0    -1.00000    -1.00000  -1.000    -1.00000E+00          29.71
 25.0    -1.00000    -1.00000  -1.000    -1.00000E+00          24.77
 20.0    -1.00000    -1.00000  -1.000    -1.00000E+00          19.82
 15.0    -1.00000    -1.00000  -1.000    -1.00000E+00          14.87
 10.0    -1.00000    -1.00000  -1.000    -1.00000E+00           9.91
  5.0    -1.00000    -1.00000  -1.000    -1.00000E+00           4.96
```

Figure 3

```
Lat =      .0  Long =      1.0  R =  3.5
Year =  1990.0  Day =    1.  UT =     .00  Field = INT
Blocal =   .00671  Bmin =   .00670  Mlat =    3.977  Mlong =   347.074

P. Angle    B mir     2nd Inv.    L        Density     Eq. Pitch Angle

 90.0      .00671      .00478   3.563   0.00000E+00        87.54
 85.0      .00676      .02515   3.563   0.00000E+00        84.43
 80.0      .00692      .08661   3.564   0.00000E+00        79.71
 75.0      .00719      .18824   3.564   0.00000E+00        74.80
 70.0      .00760      .33149   3.564   0.00000E+00        69.86
 65.0      .00817      .51729   3.564   0.00000E+00        64.89
 60.0      .00895      .74715   3.565   0.00000E+00        59.91
 55.0      .01000     1.02299   3.565   0.00000E+00        54.92
 50.0      .01143     1.34728   3.565   0.00000E+00        49.94
 45.0      .01342     1.72315   3.565   3.61182E-31        44.95
 40.0      .01624     2.15459   3.566   5.97057E-30        39.96
 35.0      .02039     2.64682   3.566   1.14628E-28        34.96
 30.0      .02684     3.20675   3.567   4.36298E-27        29.97
 25.0      .03756     3.84408   3.568   4.41615E-25        24.98
 20.0      .05735     4.57326   3.570   1.96738E-22        19.98
 15.0      .10015     5.41832   3.574   1.23996E-18        14.99
 10.0      .22250     6.42566   3.581   9.83762E-12         9.99
  5.0    -1.00000    -1.00000  -1.000  -1.00000E+00         5.00
```

```
Lat =      .0  Long =      1.0  R =  3.5
Year =  1990.0  Day =    1.  UT =     .00  Field = IN+EX
Blocal =   .00629  Bmin =   .00628  Mlat =    3.977  Mlong =   347.076

P. Angle    B mir     2nd Inv.    L        Density     Eq. Pitch Angle

 90.0      .00629      .00323   3.641   0.00000E+00        87.86
 85.0      .00633      .02177   3.640   0.00000E+00        84.56
 80.0      .00648      .07700   3.637   0.00000E+00        79.78
 75.0      .00674      .16928   3.632   0.00000E+00        74.85
 70.0      .00712      .29965   3.625   0.00000E+00        69.89
 65.0      .00765      .46972   3.617   0.00000E+00        64.91
 60.0      .00838      .68166   3.607   0.00000E+00        59.93
 55.0      .00937      .93820   3.597   0.00000E+00        54.94
 50.0      .01071     1.24271   3.587   0.00000E+00        49.95
 45.0      .01257     1.59937   3.577   7.74855E-32        44.96
 40.0      .01521     2.01326   3.567   3.16436E-30        39.97
 35.0      .01911     2.49064   3.559   5.99803E-29        34.97
 30.0      .02514     3.03944   3.552   2.16502E-27        29.98
 25.0      .03520     3.67102   3.547   2.05865E-25        24.98
 20.0      .05374     4.39990   3.545   9.01949E-23        19.99
 15.0      .09384     5.24888   3.545   5.15603E-19        14.99
 10.0      .20847     6.26971   3.550   2.36695E-12         9.99
  5.0    -1.00000    -1.00000  -1.000  -1.00000E+00         5.00
```

Figure 4

```
Lat =       .0  Long =       1.0   R =  5.5
Year =  1990.0  Day =     1.  UT =      .00  Field = INT
Blocal =   .00176  Bmin =   .00175  Mlat =     3.977  Mlong =    347.274

P. Angle    B mir       2nd Inv.    L         Density      Eq. Pitch Angle

 90.0      .00176        .02370   5.570    0.00000E+00          85.61
 85.0      .00178        .05570   5.571    0.00000E+00          83.35
 80.0      .00182        .15174   5.571    0.00000E+00          79.09
 75.0      .00189        .31037   5.572    0.00000E+00          74.39
 70.0      .00200        .53408   5.572    0.00000E+00          69.54
 65.0      .00215        .82419   5.572    0.00000E+00          64.64
 60.0      .00235       1.18302   5.572    0.00000E+00          59.71
 55.0      .00263       1.61355   5.572    0.00000E+00          54.76
 50.0      .00300       2.11971   5.573    0.00000E+00          49.80
 45.0      .00353       2.70611   5.573    0.00000E+00          44.83
 40.0      .00427       3.37914   5.573    0.00000E+00          39.86
 35.0      .00536       4.14658   5.573    0.00000E+00          34.88
 30.0      .00705       5.01889   5.574    0.00000E+00          29.90
 25.0      .00987       6.01042   5.575    0.00000E+00          24.92
 20.0      .01507       7.14198   5.575    0.00000E+00          19.94
 15.0      .02632       8.44591   5.575    3.17060E-28          14.96
 10.0      .05847       9.99589   5.578    4.08695E-23           9.97
  5.0      .23210      11.96305   5.591    2.22143E-13           4.99
```

```
Lat =       .0  Long =       1.0   R =  5.5
Year =  1990.0  Day =     1.  UT =      .00  Field = IN+EX
Blocal =   .00150  Bmin =   .00149  Mlat =     3.977  Mlong =    347.279

P. Angle    B mir       2nd Inv.    L         Density      Eq. Pitch Angle

 90.0      .00150        .00888   5.875    0.00000E+00          86.92
 85.0      .00151        .03287   5.871    0.00000E+00          84.13
 80.0      .00154        .10480   5.859    0.00000E+00          79.54
 75.0      .00160        .22568   5.838    0.00000E+00          74.69
 70.0      .00170        .39743   5.810    0.00000E+00          69.77
 65.0      .00182        .62326   5.776    0.00000E+00          64.82
 60.0      .00200        .90760   5.736    0.00000E+00          59.86
 55.0      .00223       1.25613   5.693    0.00000E+00          54.88
 50.0      .00255       1.67608   5.647    0.00000E+00          49.90
 45.0      .00299       2.17641   5.602    0.00000E+00          44.92
 40.0      .00362       2.76782   5.558    0.00000E+00          39.93
 35.0      .00455       3.46351   5.518    0.00000E+00          34.94
 30.0      .00599       4.27886   5.484    0.00000E+00          29.95
 25.0      .00838       5.23284   5.455    0.00000E+00          24.96
 20.0      .01280       6.34978   5.434    0.00000E+00          19.97
 15.0      .02235       7.66456   5.418    4.39264E-29          14.98
 10.0      .04965       9.24901   5.408    8.57689E-24           9.99
  5.0      .19708      11.29810   5.417    2.64089E-14           4.99
```

Figure 5

```
Lat =      .0  Long =      1.0  R =  7.5
Year =  1990.0  Day =     1.  UT =     .00  Field = INT
Blocal =   .00070  Bmin =   .00070  Mlat =    3.977  Mlong =   347.369

P. Angle    B mir      2nd Inv.    L         Density      Eq. Pitch Angle

 90.0      .00070       .04747   7.576    0.00000E+00         84.68
 85.0      .00071       .09120   7.577    0.00000E+00         82.71
 80.0      .00072       .22136   7.578    0.00000E+00         78.69
 75.0      .00075       .43727   7.578    0.00000E+00         74.11
 70.0      .00080       .74148   7.579    0.00000E+00         69.33
 65.0      .00086      1.13600   7.579    0.00000E+00         64.48
 60.0      .00094      1.61743   7.576    0.00000E+00         59.58
 55.0      .00105      2.20718   7.579    0.00000E+00         54.65
 50.0      .00120      2.88932   7.576    0.00000E+00         49.71
 45.0      .00141      3.68936   7.578    0.00000E+00         44.75
 40.0      .00170      4.60022   7.577    0.00000E+00         39.79
 35.0      .00214      5.64240   7.577    0.00000E+00         34.83
 30.0      .00281      6.82682   7.577    0.00000E+00         29.86
 25.0      .00393      8.17300   7.577    0.00000E+00         24.89
 20.0      .00601      9.70984   7.578    0.00000E+00         19.91
 15.0      .01049     11.48335   7.578    0.00000E+00         14.93
 10.0      .02330     13.59271   7.583    1.81184E-29          9.96
  5.0      .09250     16.22668   7.587    1.54025E-20          4.98
```

```
Lat =      .0  Long =      1.0  R =  7.5
Year =  1990.0  Day =     1.  UT =     .00  Field = IN+EX
Blocal =   .00056  Bmin =   .00056  Mlat =    3.977  Mlong =   347.516

P. Angle    B mir      2nd Inv.    L         Density      Eq. Pitch Angle

 90.0      .00056       .00053   8.130    0.00000E+00         89.31
 85.0      .00057       .02219   8.119    0.00000E+00         84.95
 80.0      .00058       .09740   8.091    0.00000E+00         79.98
 75.0      .00060       .22835   8.047    0.00000E+00         74.98
 70.0      .00064       .41436   7.985    0.00000E+00         69.99
 65.0      .00069       .67362   7.915    0.00000E+00         64.99
 60.0      .00075      1.00715   7.834    0.00000E+00         59.99
 55.0      .00084      1.41840   7.744    0.00000E+00         54.99
 50.0      .00096      1.93944   7.657    0.00000E+00         50.00
 45.0      .00113      2.56532   7.568    0.00000E+00         45.00
 40.0      .00136      3.33051   7.489    0.00000E+00         40.00
 35.0      .00171      4.25138   7.420    0.00000E+00         35.00
 30.0      .00226      5.35211   7.365    0.00000E+00         30.00
 25.0      .00316      6.66066   7.326    0.00000E+00         25.00
 20.0      .00482      8.21100   7.301    0.00000E+00         20.00
 15.0      .00842     10.04846   7.289    0.00000E+00         15.00
 10.0      .01870     12.27038   7.290    1.86371E-30         10.00
  5.0      .07423     14.73075   7.168    7.07675E-22          5.00
```

Appendix A

Internal Magnetic Field Subroutines

Subroutine SPIGRF is a fast version of the IGRF internal field subroutine. Instead of using DO loops to expand the spherical harmonic coefficients, it writes out the expansion according to a routine developed by J. Cain. This version of the routine developed for the CRRES program is not designed for stand alone use and is designed to be a part of a total magnetic field program that includes the internal as well as the external magnetic field. The calling arguments are thus passed in a common block and the geomagnetic latitude and longitude are passed via their sines and cosines. This techniques saves computer time. A stand-alone version would be much easier to use if the calling variables were transmitted as standard subroutine arguments. If an easy to use stand alone version is needed, a simple change to the first few lines of the code can produce a very efficient stand-alone internal field code.

The method of coding the magnetic field in SPIGRF is inherently faster than a DO loop version. Furthermore, the addition of a term dropping algorithm increases the speed as much as a factor of 35. The variable CONA dimensioned 11 contains the altitudes at which successive terms are to be dropped. A linear interpolation between these distance values drops the terms off smoothly. The smooth feature is important since it prevents discontinuities in the magnetic field from disrupting the integration steps and the interpolation algorithms in the field line tracing program.

A.1 Calling Sequence

The transfer of information between SPIGRF and the calling routine is performed via labeled COMMON GCOM.

INPUT values

YEARI	Contains the year for which the coefficients are to be determined. The supplied coefficients are valid from 1945 to the present. The 1945 coefficients are used for years earlier than 1945. Predicting far into the future is hazardous since the time derivative terms do not have long term validity. If YEARI is changed by .1 years since the last call a new updated set of coefficients is calculated. It is suggested that YEARI be used to set the desired epoch and then left constant.
NMAXN	Contains the number of terms desired. If this is left 0 then the full 11 term expansion of IGRF is used. If NMAXN is between 2 and 10, then the maximum number of terms is set to that number. Term dropping still takes place for larger distances.
ST	Sine of the geographic **co-latitude**
CT	Cosine of the geographic **co-latitude**

SPH	Sine of the geographic longitude.
CPH	Cosine of the geographic longitude
AOR	6371.2/R, where R is the distance from the center of the Earth in km

OUTPUT values

BR	The radial component of the magnetic field in gauss (ie. nanotesla)
BT	Theta component (south pointing) component
BP	Phi component (East)

Each time the field coefficients are updated, the value of the new dipole moment is stored in labeled COMMON /MOMENT/ XM. It is thus available for use by any routine that needs it, such as the L value program.

NOTE: SPIGRF uses a true spherical coordinate system with the z axis along the geographic north pole, the x axis through the longitude of Greenwich. R, Theta and Phi are the true spherical polar coordinates.

The routine will read several of the IGRF Coefficient sets. The coefficient sets are listed at the end of this appendix. Subroutine FLDCOF sets the FORTRAN logical unit for reading the coefficients to 11. The actual read statement for reading the coefficients are found in subroutine GETGAU.

```
      SUBROUTINE SPIGRF
C
C      VERSION 4/91
C      WRITTEN BY K.A. PFITZER  (714) 896-3231
C      SPIGRF IS A MODIFIED VERSION OF J.C. CAIN'S 14 TERM FAST SPHRC
C      ROUTINE.
C      IT HAS BEEN SHORTENED TO 11 TERMS FOR CONSISTENCY WITH THE IGRF
C      COEFFICIENT SET.
C      IT HAS A TRUNCATION FOR LARGE R - THE TRUNCATION BETWEEN TERMS
C      IS SMOOTH AND MAINTAINS AN ACCURACY OVER THE NON-TRUNCATED VERSION
C      OF BETTER THAN .1 NANOTESLA.
C      DEPENDING ON ALTITUDE THIS VERSION RUNS FROM 1.5 TO 35.0 TIMES AS FAST
C      AS THE STANDARD SCHMITT NORMLIZED IGRF ROUTINES
C      THE SUPPORT ROUTINES READ THE STANDARD IGRF COEFFICIENTS AND
C      CONVERT THEM TO GAUSS NORMALIZED FOR USE BY THIS ROUTINE
C
C
C      The first time the routine is called, the routine calls routine
C      call routine FLDCOF to obtain the correct IGRF coefficients.  If
C      the date changes by more than .1 year the coefficients are updated,
C      new coefficients are obtained if required.
C
C      INPUT -- COMMON BLOCK GCOM
C         YEARI   IS THE YEAR, IF YEARI CHANGES, THE COEFFICIENTS ARE
C                 UPDATED.
C         ST      SINE OF THE GEOGRAPHIC CO-LATITUDE.
C         CT      COSINE OF THE GEOGRAPHIC CO-LATITUDE.
C         SPH     SINE OF THE GEOGRAPHIC LONGITUDE.
C         CPH     COSINE OF THE GEOGRAPHIC LONGITUDE.
C         AOR     6371.2/R, WHERE R IS THE GEOCENTRIC DISTANCE IN KM FROM
C                 THE CENTER OF THE EARTH.
C         NMAXN   MAXIMUM NUMBER OF TERMS TO BE USED  (MUST BE LESS OR
C                 EQUAL TO 11). THIS ROUTINE PRESETS IT TO 11
C                 NMAXN OF 11 CORRESPONDS TO THE 10TH ORDER IGRF MODELS
C                 IF NMAXN IS >2 AND <11, NMAXN TERMS ARE USED, ELSE THE
C                 NUMBER OF TERMS USED IS 11 OR THE MAXIMUM TERMS IN THE
C                 IGRF DATA SET.
C      OUTPUT -- COMMON BLOCK GCOM
C         BR      RADIAL COMPONENT OF FIELD IN GAUSS.
C         BT      THETA COMPONENT (SOUTH POINTING) COMPONENT.
C         BP      PHI COMPONENT (EAST)
C
       DIMENSION G(11,11),CONST(11,11),FM(11),FN(11)
       COMMON /MODEL/G
       COMMON /GCOM/ ST,CT,SPH,CPH,AOR,BT,BP,BR,NMAXN,YEARI
       COMMON /MOMENT/XM
       DIMENSION CONA(11)
       DATA YRLAST /-12345./
       DATA IFIRST/0/
       DATA CONA/0.,12.0,8.0,6.0,5.0,4.0,3.2,2.5,2.0,1.6,1.4/
C
C      SET UP INITIAL CONSTANTS DURING FIRST CALL
       IF(IFIRST.NE.0) GO TO 199
       IFIRST=1
       FM(1)=0
       DO 6 N=2,11
       FM(N)=N-1
       FN(N)=N
       DO 6 M=1,N
 6     CONST(N,M)=FLOAT((N-2)**2-(M-1)**2)/FLOAT((2*N-3)*(2*N-5))
C
C      SET UP THE COEFFICIENTS
C      IF YEARI HAS CHANGED BY MORE THAN .1 YEAR UPDATE THE COEFFICIENTS
C
```

```
199   IF(ABS(YRLAST-YEARI).LT.0.1) GO TO 230
      CALL FLDCOF(YEARI,DIMO,MAXN)
      XM=DIMO/1.0E5
      YRLAST=YEARI
C
230   NMAX=MAXN
      IF(NMAXN.GE.2.AND.NMAXN.LT.MAXN)NMAX=NMAXN
      AR=AOR*AOR*AOR                                                    1-00006
      C2=G(2,2)*CPH+G(1,2)*SPH                                          1-00007
      BR=-(AR+AR)*(G(2,1)*CT+C2*ST)                                     1-00008
      BT=AR*(C2*CT-G(2,1)*ST)                                           1-00009
      BP=AR*(G(1,2)*CPH-G(2,2)*SPH)                                     1-00010
      IF (NMAX.LE.2) RETURN                                             1-00011
      R=1./AOR
      IF(R.GT.CONA(2)) RETURN
      CON=0.
      SP2=SPH                                                           1-00012
      CP2=CPH                                                           1-00013
      P21=CT                                                            1-00014
      P22=ST                                                            1-00015
      DP21=-ST                                                          1-00016
      DP22=CT                                                           1-00017
      N=3
      SP3=(SP2+SP2)*CP2                                                 1-00019
      CP3=(CP2+SP2)*(CP2-SP2)                                           1-00020
      P31=CT*P21-CONST(3,1)                                             1-00021
      P32=CT*P22                                                        1-00022
      P33=ST*P22                                                        1-00023
      DP31=-P32-P32                                                     1-00024
      DP32=CT*DP22-P33                                                  1-00025
      DP33=-DP31                                                        1-00026
      C2=G(3,2)*CP2+G(1,3)*SP2                                          1-00027
      C3=G(3,3)*CP3+G(2,3)*SP3                                          1-00028
      AR=AOR*AR                                                         1-00029
      XR=BR-FN(3)*AR*(G(3,1)*P31+C2*P32+C3*P33)                         1-00030
      XT=BT+AR*(G(3,1)*DP31+C2*DP32+C3*DP33)                            1-00031
      XP=BP-AR*(FM(2)*(G(3,2)*SP2-G(1,3)*CP2)*P21+FM(3)*(G(3,3)*SP3-G(2,1-00032
     +3)*CP3)*P22)                                                      1-00033
      BP=BP*ST                                                          1-00035
      XP=XP*ST                                                          1-00035
      IF(NMAX.LE.3) GO TO 21
      IF(R.GT.CONA(3)) GO TO 20
      N=4
      SP4=SPH*CP3+CPH*SP3                                               1-00037
      CP4=CPH*CP3-SPH*SP3                                               1-00038
      P41=CT*P31-CONST(4,1)*P21                                         1-00039
      DP41=CT*DP31-ST*P31-CONST(4,1)*DP21                               1-00040
      P42=CT*P32-CONST(4,2)*P22                                         1-00041
      DP42=CT*DP32-ST*P32-CONST(4,2)*DP22                               1-00042
      P43=CT*P33                                                        1-00043
      DP43=CT*DP33-ST*P33                                               1-00044
      P44=ST*P33                                                        1-00045
      DP44=FM(4)*P43                                                    1-00046
      C2=G(4,2)*CP2+G(1,4)*SP2                                          1-00047
      C3=G(4,3)*CP3+G(2,4)*SP3                                          1-00048
      C4=G(4,4)*CP4+G(3,4)*SP4                                          1-00049
      AR=AOR*AR                                                         1-00050
      BR=XR-FN(4)*AR*(G(4,1)*P41+C2*P42+C3*P43+C4*P44)                  1-00051
      BT=XT+AR*(G(4,1)*DP41+C2*DP42+C3*DP43+C4*DP44)                    1-00052
      BP=XP-AR*(FM(2)*(G(4,2)*SP2-G(1,4)*CP2)*P42+FM(3)*(G(4,3)*SP3-G(2,1-00053
     +4)*CP3)*P43+FM(4)*(G(4,4)*SP4-G(3,4)*CP4)*P44)                    1-00054
      IF(NMAX.LE.4) GO TO 11
      IF(R.GT.CONA(4)) GO TO 10
      N=5
```

```
      SP5=(SP3+SP3)*CP3                                                 1-00057
      CP5=(CP3+SP3)*(CP3-SP3)                                           1-00058
      P51=CT*P41-CONST(5,1)*P31                                         1-00059
      DP51=CT*DP41-ST*P41-CONST(5,1)*DP31                               1-00060
      P52=CT*P42-CONST(5,2)*P32                                         1-00061
      DP52=CT*DP42-ST*P42-CONST(5,2)*DP32                               1-00062
      P53=CT*P43-CONST(5,3)*P33                                         1-00063
      DP53=CT*DP43-ST*P43-CONST(5,3)*DP33                               1-00064
      P54=CT*P44                                                        1-00065
      DP54=CT*DP44-ST*P44                                               1-00066
      P55=ST*P44                                                        1-00067
      DP55=FM(5)*P54                                                    1-00068
      C2=G(5,2)*CP2+G(1,5)*SP2                                          1-00069
      C3=G(5,3)*CP3+G(2,5)*SP3                                          1-00070
      C4=G(5,4)*CP4+G(3,5)*SP4                                          1-00071
      C5=G(5,5)*CP5+G(4,5)*SP5                                          1-00072
      AR=AOR*AR                                                         1-00073
      XR=BR-FN(5)*AR*(G(5,1)*P51+C2*P52+C3*P53+C4*P54+C5*P55)           1-00074
      XT=BT+AR*(G(5,1)*DP51+C2*DP52+C3*DP53+C4*DP54+C5*DP55)            1-00075
      XP=BP-AR*(FM(2)*(G(5,2)*SP2-G(1,5)*CP2)*P52+FM(3)*(G(5,3)*SP3-G(2,1-00076
     +5)*CP3)*P53+FM(4)*(G(5,4)*SP4-G(3,5)*CP4)*P54+FM(5)*(G(5,5)*SP5-G(1-00077
     +4,5)*CP5)*P55)                                                    1-00078
      IF(NMAX.LE.5) GO TO 21
      IF(R.GT.CONA(5)) GO TO 20
      N=6
      SP6=SPH*CP5+CPH*SP5                                               1-00081
      CP6=CPH*CP5-SPH*SP5                                               1-00082
      P61=CT*P51-CONST(6,1)*P41                                         1-00083
      DP61=CT*DP51-ST*P51-CONST(6,1)*DP41                               1-00084
      P62=CT*P52-CONST(6,2)*P42                                         1-00085
      DP62=CT*DP52-ST*P52-CONST(6,2)*DP42                               1-00086
      P63=CT*P53-CONST(6,3)*P43                                         1-00087
      DP63=CT*DP53-ST*P53-CONST(6,3)*DP43                               1-00088
      P64=CT*P54-CONST(6,4)*P44                                         1-00089
      DP64=CT*DP54-ST*P54-CONST(6,4)*DP44                               1-00090
      P65=CT*P55                                                        1-00091
      DP65=CT*DP55-ST*P55                                               1-00092
      P66=ST*P55                                                        1-00093
      DP66=FM(6)*P65                                                    1-00094
      C2=G(6,2)*CP2+G(1,6)*SP2                                          1-00095
      C3=G(6,3)*CP3+G(2,6)*SP3                                          1-00096
      C4=G(6,4)*CP4+G(3,6)*SP4                                          1-00097
      C5=G(6,5)*CP5+G(4,6)*SP5                                          1-00098
      C6=G(6,6)*CP6+G(5,6)*SP6                                          1-00099
      AR=AOR*AR                                                         1-00100
      BR=XR-FN(6)*AR*(G(6,1)*P61+C2*P62+C3*P63+C4*P64+C5*P65+C6*P66)    1-00101
      BT=XT+AR*(G(6,1)*DP61+C2*DP62+C3*DP63+C4*DP64+C5*DP65+C6*DP66)    1-00102
      BP=XP-AR*(FM(2)*(G(6,2)*SP2-G(1,6)*CP2)*P62+FM(3)*(G(6,3)*SP3-G(2,1-00103
     +6)*CP3)*P63+FM(4)*(G(6,4)*SP4-G(3,6)*CP4)*P64+FM(5)*(G(6,5)*SP5-G(1-00104
     +4,6)*CP5)*P65+FM(6)*(G(6,6)*SP6-G(5,6)*CP6)*P66)                  1-00105
      IF(NMAX.LE.6) GO TO 11
      IF(R.GT.CONA(6)) GO TO 10
      N=7
      SP7=(SP4+SP4)*CP4                                                 1-00108
      CP7=(CP4+SP4)*(CP4-SP4)                                           1-00109
      P71=CT*P61-CONST(7,1)*P51                                         1-00110
      DP71=CT*DP61-ST*P61-CONST(7,1)*DP51                               1-00111
      P72=CT*P62-CONST(7,2)*P52                                         1-00112
      DP72=CT*DP62-ST*P62-CONST(7,2)*DP52                               1-00113
      P73=CT*P63-CONST(7,3)*P53                                         1-00114
      DP73=CT*DP63-ST*P63-CONST(7,3)*DP53                               1-00115
      P74=CT*P64-CONST(7,4)*P54                                         1-00116
      DP74=CT*DP64-ST*P64-CONST(7,4)*DP54                               1-00117
      P75=CT*P65-CONST(7,5)*P55                                         1-00118
```

```
      DP75=CT*DP65-ST*P65-CONST(7,5)*DP55                               1-00119
      P76=CT*P66                                                        1-00120
      DP76=CT*DP66-ST*P66                                               1-00121
      P77=ST*P66                                                        1-00122
      DP77=FM(7)*P76                                                    1-00123
      C2=G(7,2)*CP2+G(1,7)*SP2                                          1-00124
      C3=G(7,3)*CP3+G(2,7)*SP3                                          1-00125
      C4=G(7,4)*CP4+G(3,7)*SP4                                          1-00126
      C5=G(7,5)*CP5+G(4,7)*SP5                                          1-00127
      C6=G(7,6)*CP6+G(5,7)*SP6                                          1-00128
      C7=G(7,7)*CP7+G(6,7)*SP7                                          1-00129
      AR=AOR*AR                                                         1-00130
      XR=BR-FN(7)*AR*(G(7,1)*P71+C2*P72+C3*P73+C4*P74+C5*P75+C6*P76+C7*P1-00131
     +77)                                                               1-00132
      XT=BT+AR*(G(7,1)*DP71+C2*DP72+C3*DP73+C4*DP74+C5*DP75+C6*DP76+C7*D1-00133
     +P77)                                                              1-00134
      XP=BP-AR*(FM(2)*(G(7,2)*SP2-G(1,7)*CP2)*P72+FM(3)*(G(7,3)*SP3-G(2,1-00135
     +7)*CP3)*P73+FM(4)*(G(7,4)*SP4-G(3,7)*CP4)*P74+FM(5)*(G(7,5)*SP5-G(1-00136
     +4,7)*CP5)*P75+FM(6)*(G(7,6)*SP6-G(5,7)*CP6)*P76+FM(7)*(G(7,7)*SP7-1-00137
     +G(6,7)*CP7)*P77)                                                  1-00138
      IF(NMAX.LE.7) GO TO 21
      IF(R.GT.CONA(7)) GO TO 20
      N=8
      SP8=SPH*CP7+CPH*SP7                                               1-00141
      CP8=CPH*CP7-SPH*SP7                                               1-00142
      P81=CT*P71-CONST(8,1)*P61                                         1-00143
      DP81=CT*DP71-ST*P71-CONST(8,1)*DP61                               1-00144
      P82=CT*P72-CONST(8,2)*P62                                         1-00145
      DP82=CT*DP72-ST*P72-CONST(8,2)*DP62                               1-00146
      P83=CT*P73-CONST(8,3)*P63                                         1-00147
      DP83=CT*DP73-ST*P73-CONST(8,3)*DP63                               1-00148
      P84=CT*P74-CONST(8,4)*P64                                         1-00149
      DP84=CT*DP74-ST*P74-CONST(8,4)*DP64                               1-00150
      P85=CT*P75-CONST(8,5)*P65                                         1-00151
      DP85=CT*DP75-ST*P75-CONST(8,5)*DP65                               1-00152
      P86=CT*P76-CONST(8,6)*P66                                         1-00153
      DP86=CT*DP76-ST*P76-CONST(8,6)*DP66                               1-00154
      P87=CT*P77                                                        1-00155
      DP87=CT*DP77-ST*P77                                               1-00156
      P88=ST*P77                                                        1-00157
      DP88=FM(8)*P87                                                    1-00158
      C2=G(8,2)*CP2+G(1,8)*SP2                                          1-00159
      C3=G(8,3)*CP3+G(2,8)*SP3                                          1-00160
      C4=G(8,4)*CP4+G(3,8)*SP4                                          1-00161
      C5=G(8,5)*CP5+G(4,8)*SP5                                          1-00162
      C6=G(8,6)*CP6+G(5,8)*SP6                                          1-00163
      C7=G(8,7)*CP7+G(6,8)*SP7                                          1-00164
      C8=G(8,8)*CP8+G(7,8)*SP8                                          1-00165
      AR=AOR*AR                                                         1-00166
      BR=XR-FN(8)*AR*(G(8,1)*P81+C2*P82+C3*P83+C4*P84+C5*P85+C6*P86+C7*P1-00167
     +87+C8*P88)                                                        1-00168
      BT=XT+AR*(G(8,1)*DP81+C2*DP82+C3*DP83+C4*DP84+C5*DP85+C6*DP86+C7*D1-00169
     +P87+C8*DP88)                                                      1-00170
      BP=XP-AR*(FM(2)*(G(8,2)*SP2-G(1,8)*CP2)*P82+FM(3)*(G(8,3)*SP3-G(2,1-00171
     +8)*CP3)*P83+FM(4)*(G(8,4)*SP4-G(3,8)*CP4)*P84+FM(5)*(G(8,5)*SP5-G(1-00172
     +4,8)*CP5)*P85+FM(6)*(G(8,6)*SP6-G(5,8)*CP6)*P86+FM(7)*(G(8,7)*SP7-1-00173
     +G(6,8)*CP7)*P87+FM(8)*(G(8,8)*SP8-G(7,8)*CP8)*P88)                1-00174
      IF(NMAX.LE.8) GO TO 11
      IF(R.GT.CONA(8)) GO TO 10
      N=9
      SP9=(SP5+SP5)*CP5                                                 1-00177
      CP9=(CP5+SP5)*(CP5-SP5)                                           1-00178
      P91=CT*P81-CONST(9,1)*P71                                         1-00179
      DP91=CT*DP81-ST*P81-CONST(9,1)*DP71                               1-00180
```

```
      P92=CT*P82-CONST(9,2)*P72                                         1-00181
      DP92=CT*DP82-ST*P82-CONST(9,2)*DP72                               1-00182
      P93=CT*P83-CONST(9,3)*P73                                         1-00183
      DP93=CT*DP83-ST*P83-CONST(9,3)*DP73                               1-00184
      P94=CT*P84-CONST(9,4)*P74                                         1-00185
      DP94=CT*DP84-ST*P84-CONST(9,4)*DP74                               1-00186
      P95=CT*P85-CONST(9,5)*P75                                         1-00187
      DP95=CT*DP85-ST*P85-CONST(9,5)*DP75                               1-00188
      P96=CT*P86-CONST(9,6)*P76                                         1-00189
      DP96=CT*DP86-ST*P86-CONST(9,6)*DP76                               1-00190
      P97=CT*P87-CONST(9,7)*P77                                         1-00191
      DP97=CT*DP87-ST*P87-CONST(9,7)*DP77                               1-00192
      P98=CT*P88                                                        1-00193
      DP98=CT*DP88-ST*P88                                               1-00194
      P99=ST*P88                                                        1-00195
      DP99=FM(9)*P98                                                    1-00196
      C2=G(9,2)*CP2+G(1,9)*SP2                                          1-00197
      C3=G(9,3)*CP3+G(2,9)*SP3                                          1-00198
      C4=G(9,4)*CP4+G(3,9)*SP4                                          1-00199
      C5=G(9,5)*CP5+G(4,9)*SP5                                          1-00200
      C6=G(9,6)*CP6+G(5,9)*SP6                                          1-00201
      C7=G(9,7)*CP7+G(6,9)*SP7                                          1-00202
      C8=G(9,8)*CP8+G(7,9)*SP8                                          1-00203
      C9=G(9,9)*CP9+G(8,9)*SP9                                          1-00204
      AR=AOR*AR                                                         1-00205
      XR=BR-FN(9)*AR*(G(9,1)*P91+C2*P92+C3*P93+C4*P94+C5*P95+C6*P96+C7*P1-00206
     +97+C8*P98+C9*P99)                                                 1-00207
      XT=BT+AR*(G(9,1)*DP91+C2*DP92+C3*DP93+C4*DP94+C5*DP95+C6*DP96+C7*D1-00208
     +P97+C8*DP98+C9*DP99)                                              1-00209
      XP=BP-AR*(FM(2)*(G(9,2)*SP2-G(1,9)*CP2)*P92+FM(3)*(G(9,3)*SP3-G(2,1-00210
     +9)*CP3)*P93+FM(4)*(G(9,4)*SP4-G(3,9)*CP4)*P94+FM(5)*(G(9,5)*SP5-G(1-00211
     +4,9)*CP5)*P95+FM(6)*(G(9,6)*SP6-G(5,9)*CP6)*P96+FM(7)*(G(9,7)*SP7-1-00212
     +G(6,9)*CP7)*P97+FM(8)*(G(9,8)*SP8-G(7,9)*CP8)*P98+FM(9)*(G(9,9)*SP1-00213
     +9-G(8,9)*CP9)*P99)                                                1-00214
      IF(NMAX.LE.9) GO TO 21
      IF(R.GT.CONA(9)) GO TO 20
      N=10
      SP10=SPH*CP9+CPH*SP9                                              1-00217
      CP10=CPH*CP9-SPH*SP9                                              1-00218
      P101=CT*P91-CONST(10,1)*P81                                       1-00219
      DP101=CT*DP91-ST*P91-CONST(10,1)*DP81                             1-00220
      P102=CT*P92-CONST(10,2)*P82                                       1-00221
      DP102=CT*DP92-ST*P92-CONST(10,2)*DP82                             1-00222
      P103=CT*P93-CONST(10,3)*P83                                       1-00223
      DP103=CT*DP93-ST*P93-CONST(10,3)*DP83                             1-00224
      P104=CT*P94-CONST(10,4)*P84                                       1-00225
      DP104=CT*DP94-ST*P94-CONST(10,4)*DP84                             1-00226
      P105=CT*P95-CONST(10,5)*P85                                       1-00227
      DP105=CT*DP95-ST*P95-CONST(10,5)*DP85                             1-00228
      P106=CT*P96-CONST(10,6)*P86                                       1-00229
      DP106=CT*DP96-ST*P96-CONST(10,6)*DP86                             1-00230
      P107=CT*P97-CONST(10,7)*P87                                       1-00231
      DP107=CT*DP97-ST*P97-CONST(10,7)*DP87                             1-00232
      P108=CT*P98-CONST(10,8)*P88                                       1-00233
      DP108=CT*DP98-ST*P98-CONST(10,8)*DP88                             1-00234
      P109=CT*P99                                                       1-00235
      DP109=CT*DP99-ST*P99                                              1-00236
      P1010=ST*P99                                                      1-00237
      DP1010=FM(10)*P109                                                1-00238
      C2=G(10,2)*CP2+G(1,10)*SP2                                        1-00239
      C3=G(10,3)*CP3+G(2,10)*SP3                                        1-00240
      C4=G(10,4)*CP4+G(3,10)*SP4                                        1-00241
      C5=G(10,5)*CP5+G(4,10)*SP5                                        1-00242
      C6=G(10,6)*CP6+G(5,10)*SP6                                        1-00243
```

```
 C7=G(10,7)*CP7+G(6,10)*SP7                                        1-00244
 C8=G(10,8)*CP8+G(7,10)*SP8                                        1-00245
 C9=G(10,9)*CP9+G(8,10)*SP9                                        1-00246
 C10=G(10,10)*CP10+G(9,10)*SP10                                    1-00247
 AR=AOR*AR                                                         1-00248
 BR=XR-FN(10)*AR*(G(10,1)*P101+C2*P102+C3*P103+C4*P104+C5*P105+C6*P1-00249
+106+C7*P107+C8*P108+C9*P109+C10*P1010)                            1-00250
 BT=XT+AR*(G(10,1)*DP101+C2*DP102+C3*DP103+C4*DP104+C5*DP105+C6*DP11-00251
+06+C7*DP107+C8*DP108+C9*DP109+C10*DP1010)                         1-00252
 BP=XP-AR*(FM(2)*(G(10,2)*SP2-G(1,10)*CP2)*P102+FM(3)*(G(10,3)*SP3-1-00253
+G(2,10)*CP3)*P103+FM(4)*(G(10,4)*SP4-G(3,10)*CP4)*P104+FM(5)*(G(101-00254
+,5)*SP5-G(4,10)*CP5)*P105+FM(6)*(G(10,6)*SP6-G(5,10)*CP6)*P106+FM(1-00255
+7)*(G(10,7)*SP7-G(6,10)*CP7)*P107+FM(8)*(G(10,8)*SP8-G(7,10)*CP8)*1-00256
+P108+FM(9)*(G(10,9)*SP9-G(8,10)*CP9)*P109+FM(10)*(G(10,10)*SP10-G(1-00257
+9,10)*CP10)*P1010)                                                1-00258
 IF(NMAX.LE.10) GO TO 11
 IF(R.GT.CONA(10)) GO TO 10
 N=11                                                              1-00260
 SP11=(SP6+SP6)*CP6                                                1-00261
 CP11=(CP6+SP6)*(CP6-SP6)                                          1-00262
 P111=CT*P101-CONST(11,1)*P91                                      1-00263
 DP111=CT*DP101-ST*P101-CONST(11,1)*DP91                           1-00264
 P112=CT*P102-CONST(11,2)*P92                                      1-00265
 DP112=CT*DP102-ST*P102-CONST(11,2)*DP92                           1-00266
 P113=CT*P103-CONST(11,3)*P93                                      1-00267
 DP113=CT*DP103-ST*P103-CONST(11,3)*DP93                           1-00268
 P114=CT*P104-CONST(11,4)*P94                                      1-00269
 DP114=CT*DP104-ST*P104-CONST(11,4)*DP94                           1-00270
 P115=CT*P105-CONST(11,5)*P95                                      1-00271
 DP115=CT*DP105-ST*P105-CONST(11,5)*DP95                           1-00272
 P116=CT*P106-CONST(11,6)*P96                                      1-00273
 DP116=CT*DP106-ST*P106-CONST(11,6)*DP96                           1-00274
 P117=CT*P107-CONST(11,7)*P97                                      1-00275
 DP117=CT*DP107-ST*P107-CONST(11,7)*DP97                           1-00276
 P118=CT*P108-CONST(11,8)*P98                                      1-00277
 DP118=CT*DP108-ST*P108-CONST(11,8)*DP98                           1-00278
 P119=CT*P109-CONST(11,9)*P99                                      1-00279
 DP119=CT*DP109-ST*P109-CONST(11,9)*DP99                           1-00280
 P1110=CT*P1010                                                    1-00281
 DP1110=CT*DP1010-ST*P1010                                         1-00282
 P1111=ST*P1010                                                    1-00283
 DP1111=FM(11)*P1110                                               1-00284
 C2=G(11,2)*CP2+G(1,11)*SP2                                        1-00285
 C3=G(11,3)*CP3+G(2,11)*SP3                                        1-00286
 C4=G(11,4)*CP4+G(3,11)*SP4                                        1-00287
 C5=G(11,5)*CP5+G(4,11)*SP5                                        1-00288
 C6=G(11,6)*CP6+G(5,11)*SP6                                        1-00289
 C7=G(11,7)*CP7+G(6,11)*SP7                                        1-00290
 C8=G(11,8)*CP8+G(7,11)*SP8                                        1-00291
 C9=G(11,9)*CP9+G(8,11)*SP9                                        1-00292
 C10=G(11,10)*CP10+G(9,11)*SP10                                    1-00293
 C11=G(11,11)*CP11+G(10,11)*SP11                                   1-00294
 AR=AOR*AR                                                         1-00295
 BR=BR-FN(11)*AR*(G(11,1)*P111+C2*P112+C3*P113+C4*P114+C5*P115+C6*P1-00296
+116+C7*P117+C8*P118+C9*P119+C10*P1110+C11*P1111)                  1-00297
 BT=BT+AR*(G(11,1)*DP111+C2*DP112+C3*DP113+C4*DP114+C5*DP115+C6*DP11-00298
+16+C7*DP117+C8*DP118+C9*DP119+C10*DP1110+C11*DP1111)              1-00299
 BP=BP-AR*(FM(2)*(G(11,2)*SP2-G(1,11)*CP2)*P112+FM(3)*(G(11,3)*SP3-1-00300
+G(2,11)*CP3)*P113+FM(4)*(G(11,4)*SP4-G(3,11)*CP4)*P114+FM(5)*(G(111-00301
+,5)*SP5-G(4,11)*CP5)*P115+FM(6)*(G(11,6)*SP6-G(5,11)*CP6)*P116+FM(1-00302
+7)*(G(11,7)*SP7-G(6,11)*CP7)*P117+FM(8)*(G(11,8)*SP8-G(7,11)*CP8)*1-00303
+P118+FM(9)*(G(11,9)*SP9-G(8,11)*CP9)*P119+FM(10)*(G(11,10)*SP10-G(1-00304
+9,11)*CP10)*P1110+FM(11)*(G(11,11)*SP11-G(10,11)*CP11)*P1111)     1-00305
 BP=BP/ST
```

```
      IF (NMAX.LE.11) RETURN                                         1-00306
      WRITE (*,2) NMAX                                               1-00476
      RETURN                                                         1-00477
C
 2    FORMAT(57H0 ERROR, THIS MODEL ONLY FOR NMAX=<11, CALL WAS FOR NMAX
     *=,I5)
C     MAKE A SMOOTH FIT BETWEEN TRUNCATED TERMS.
 10   CON=(R-CONA(N))/(CONA(N-1)-CONA(N))
 11   BR=BR+(XR-BR)*CON
      BT=BT+(XT-BT)*CON
      BP=(BP+(XP-BP)*CON)/ST
      RETURN
 20   CON=(R-CONA(N))/(CONA(N-1)-CONA(N))
 21   BR=XR+(BR-XR)*CON
      BT=XT+(BT-XT)*CON
      BP=(XP+(BP-XP)*CON)/ST
      RETURN
      END                                                            1-00480
C
```

```
      SUBROUTINE FLDCOF(YEAR,DIMO,NMAXI)
C-----------------------------------------------------------------------
C  DETERMINES COEFFICIENTS AND DIPOL MOMENT FROM IGRF MODELS
C
C     INPUT:  YEAR    DECIMAL YEAR FOR WHICH GEOMAGNETIC FIELD IS TO
C                     BE CALCULATED
C     OUTPUT: DIMO    GEOMAGNETIC DIPOL MOMENT IN GAUSS (NORMALIZED
C                     TO EARTH'S RADIUS) AT THE TIME (YEAR)
C  THIS ROUNTINE WAS INITIALLY WRITTEN BY
C  D. BILITZA, NSSDC, GSFC, CODE 633, GREENBELT, MD 20771,
C     (301)286-9536   NOV 1987.
C  MODIFIED BY K. A. PFITZER MDSSC TO WORK WITH GAUSS NORMALIZED COEFF.
C-----------------------------------------------------------------------
      CHARACTER*11        FILMOD, FIL1, FIL2
      DIMENSION     GH1(11,11), GH2(11,11),
     1                    DTEMOD(10),FILMOD(10)
      DOUBLE PRECISION    X,F0,F
      COMMON/MODEL/       G(11,11)
      COMMON/GENER/       UMR,ERAD,AQUAD,BQUAD
      DATA        FILMOD /'dgrf45.dat', 'dgrf50.dat',
     1            'dgrf55.dat', 'dgrf60.dat', 'dgrf65.dat',
     2            'dgrf70.dat', 'dgrf75.dat', 'dgrf80.dat',
     3            'igrf85.dat', 'igrf85s.dat'/
      DATA        DTEMOD / 1945., 1950., 1955., 1960.,
     1            1965., 1970., 1975., 1980., 1985., 1990./
      DATA        LOLD/0/
C
      IU = 11
C-- DETERMINE IGRF-YEARS FOR INPUT-YEAR
      TIME = YEAR
      IYEA = INT(YEAR/5.)*5
      L = (IYEA - 1945)/5 + 1
C
      IF (L.NE.LOLD) THEN
      LOLD=L
      IF(L.LT.1) L=1
      IF(L.GT.9) L=9
      DTE1 = DTEMOD(L)
      FIL1 = FILMOD(L)
      DTE2 = DTEMOD(L+1)
      FIL2 = FILMOD(L+1)
C-- GET IGRF COEFFICIENTS FOR THE BOUNDARY YEARS
      CALL GETGAU (IU, FIL1, NMAX1, ERAD, GH1, IER)
      IF (IER .NE. 0) THEN
          WRITE (*,101) IU,FIL1,NMAX1,ERAD,IER
101       FORMAT (//'     Error in subroutine FELDCOF'/
     1        ' IU, FIL1, NMAX1, ERAD, IER:'/
     2        I10,A11,I10,1PE12.3,I10)
          STOP
      ENDIF
      CALL GETGAU (IU, FIL2, NMAX2, ERAD, GH2, IER)
          IF (IER .NE. 0) THEN
          WRITE (*,102) IU,FIL2,NMAX2,ERAD,IER
102       FORMAT (//'     Error in subroutine FELDCOF'/
     1        ' IU, FIL2, NMAX2, ERAD, IER:'/
     2        I10,A11,I10,1PE12.3,I10)
          STOP
      ENDIF
      ENDIF
C-- DETERMINE IGRF COEFFICIENTS FOR YEAR
      IF (L .LE. 8) THEN
          CALL CINTPP (YEAR, DTE1, NMAX1, GH1, DTE2,
     1        NMAX2, GH2, NMAXI, G)
      ELSE
```

```
          CALL EXTRAP (YEAR, DTE1, NMAX1, GH1, NMAX2,
     1        GH2, NMAXI, G)
      ENDIF
C-- DETERMINE MAGNETIC DIPOL MOMENT
      F0=G(2,1)**2+G(2,2)**2+G(1,2)**2
      DIMO=SQRT(F0)
      RETURN
      END
```

```
      SUBROUTINE GETGAU (IU, FSPEC, NMAX, ERAD, G, IER)
C ==============================================================
C
C     Reads spherical harmonic coefficients from the specified
C     file into an array and converts the coefficients to Gauss
C     normalized coefficients.
C
C     Input:
C         IU        - Logical unit number
C         FSPEC     - File specification
C
C     Output:
C         NMAX      - Maximum degree and order of model
C         ERAD      - Earth's radius associated with the spherical
C                     harmonic coefficients, in the same units as
C                     elevation
C         GH        - Gauss quasi-normal internal spherical
C                     harmonic coefficients
C         IER       - Error number: =  0, no error
C                                   = -2, records out of order
C                                   = FORTRAN run-time error number
C
C ==============================================================

      CHARACTER    FSPEC*(*)
      DIMENSION    G(11,11)

C --------------------------------------------------------------
C     Open coefficient file. Read past first header record.
C     Read degree and order of model and Earth's radius.
C --------------------------------------------------------------
      OPEN (IU, FILE=FSPEC, STATUS='OLD', IOSTAT=IER, ERR=999)
C     1   READONLY)

      DO 10 I=1,11
      DO 10 J=1,11
10    G(I,J)=0.

      READ (IU, *, IOSTAT=IER, ERR=999)
      READ (IU, *, IOSTAT=IER, ERR=999) MAXN, ERAD

      IF(MAXN.GT.10)MAXN=10
      DO 30 NN=1,MAXN
      DO 20 MM=0,NN
      READ (IU, *, IOSTAT=IER, ERR=999) LN, LM, GNM, HNM
      IF(NN.NE.LN.OR.MM.NE.LM)THEN
         IER=-2
         GOTO 999
      ENDIF

      N=LN+1
      M=LM+1
      G(N,M)=GNM
      IF(LM.EQ.0) goto 20
      G(LM,N)=HNM
20    CONTINUE
30    CONTINUE
      NMAX=MAXN+1

C    Convert to Gauss normalized
      DO 55 N=1,NMAX
      DO 55 M=1,NMAX
      CALL CONVRT  (G(n,m),n,m,1)
```

```
55      CONTINUE

999     CLOSE (IU)
        RETURN
        END
```

```
      SUBROUTINE CONVRT(G,I,L,K)
      DIMENSION S(11,11)
      LOGICAL NEXT
      DATA NEXT/.FALSE./
      IF (NEXT) GOTO 2
      NEXT=.TRUE.
      S(1,1)=-1.
      DO 1 N=2,11
      S(N,1)=S(N-1,1)*FLOAT(2*N-3)/FLOAT(N-1)
      S(1,N)=0.
      J=2
      DO 1 M=2,N
      S(N,M)=S(N,M-1)*SQRT((FLOAT(N-M+1)*J)/FLOAT(N+M-2))
      S(M-1,N)=S(N,M)
1     J=1
2     IF(K.GT.1) GOTO 3
      G=G*S(I,L)
      RETURN
3     G=G/S(I,L)
      RETURN
      END
C
C
```

```
      SUBROUTINE CINTRP (DATE, DTE1, NMAX1, GH1, DTE2,
     1                   NMAX2, GH2, NMAX, GH)
C ================================================================
C
C     Interpolates linearly, in time, between two spherical
C     harmonic models.
C
C     Input:
C         DATE    - Date of resulting model (in decimal year)
C         DTE1    - Date of earlier model
C         NMAX1   - Maximum degree and order of earlier model
C         GH1     - Gauss quasi-normal internal spherical
C                   harmonic coefficients of earlier model
C         DTE2    - Date of later model
C         NMAX2   - Maximum degree and order of later model
C         GH2     - Gauss quasi-normal internal spherical
C                   harmonic coefficients of later model
C
C     Output:
C         GH      - Coefficients of resulting model
C         NMAX    - Maximum degree and order of resulting model
C
C ================================================================

      DIMENSION   GH1(11,11), GH2(11,11), GH(11,11)

      NMAX=MAX0(NMAX1,NMAX2)
      FACTOR=(DATE-DTE1)/(DTE2-DTE1)
      DO 234 J=  1,11
      DO 234 I = 1, 11
234   GH(I,J) = GH1(I,J) + FACTOR * (GH2(I,J) - GH1(I,J))

      RETURN
      END
C
C
```

```
      SUBROUTINE EXTRAP (DATE, DTE1, NMAX1, GH1, NMAX2,
     1                   GH2, NMAX, GH)

C ===============================================================
C
C     Extrapolates linearly a spherical harmonic model with a
C     rate-of-change model.
C
C     Input:
C         DATE     - Date of resulting model (in decimal year)
C         DTE1     - Date of base model
C         NMAX1    - Maximum degree and order of base model
C         GH1      - Gauss quasi-normal internal spherical
C                    harmonic coefficients of base model
C         NMAX2    - Maximum degree and order of rate-of-change
C                    model
C         GH2      - Gauss quasi-normal internal spherical
C                    harmonic coefficients of rate-of-change model
C
C     Output:
C         GH       - Coefficients of resulting model
C         NMAX     - Maximum degree and order of resulting model
C
C ===============================================================

      DIMENSION   GH1(11,11), GH2(11,11), GH(11,11)

      NMAX=MAX0(NMAX1,NMAX2)
      FACTOR = (DATE - DTE1)

      DO 567 J=1,11
      DO 567 I = 1,11
567   GH(I,J) = GH1(I,J) + FACTOR * GH2(I,J)

      RETURN
      END
```

```
dgrf45
10  6371.2 1945.0
  1  0  -30594.        0.
  1  1   -2285.     5810.
  2  0   -1244.        0.
  2  1    2990.    -1702.
  2  2    1578.      477.
  3  0    1282.        0.
  3  1   -1834.     -499.
  3  2    1255.      186.
  3  3     913.      -11.
  4  0     944.        0.
  4  1     776.      144.
  4  2     544.     -276.
  4  3    -421.      -55.
  4  4     304.     -178.
  5  0    -253.        0.
  5  1     346.      -12.
  5  2     194.       95.
  5  3     -20.      -67.
  5  4    -142.     -119.
  5  5     -82.       82.
  6  0      59.        0.
  6  1      57.        6.
  6  2       6.      100.
  6  3    -246.       16.
  6  4     -25.       -9.
  6  5      21.      -16.
  6  6    -104.      -39.
  7  0      70.        0.
  7  1     -40.      -45.
  7  2       0.      -18.
  7  3       0.        2.
  7  4     -29.        6.
  7  5     -10.       28.
  7  6      15.      -17.
  7  7      29.      -22.
  8  0      13.        0.
  8  1       7.       12.
  8  2      -8.      -21.
  8  3      -5.      -12.
  8  4       9.       -7.
  8  5       7.        2.
  8  6     -10.       18.
  8  7       7.        3.
  8  8       2.      -11.
  9  0       5.        0.
  9  1     -21.      -27.
  9  2       1.       17.
  9  3     -11.       29.
  9  4       3.       -9.
  9  5      16.        4.
  9  6      -3.        9.
  9  7      -4.        6.
  9  8      -3.        1.
  9  9      -4.        8.
 10  0      -3.        0.
 10  1      11.        5.
 10  2       1.        1.
 10  3       2.      -20.
 10  4      -5.       -1.
 10  5      -1.       -6.
 10  6       8.        6.
 10  7      -1.       -4.
 10  8      -3.       -2.
 10  9       5.        0.
 10 10      -2.       -2.
```

```
dgrf50
10  6371.2 1950.0
  1  0  -30554.        0.
  1  1   -2250.     5815.
  2  0   -1341.        0.
  2  1    2998.    -1810.
  2  2    1576.      381.
  3  0    1297.        0.
  3  1   -1889.     -476.
  3  2    1274.      206.
  3  3     896.      -46.
  4  0     954.        0.
  4  1     792.      136.
  4  2     528.     -278.
  4  3    -408.      -37.
  4  4     303.     -210.
  5  0    -240.        0.
  5  1     349.        3.
  5  2     211.      103.
  5  3     -20.      -87.
  5  4    -147.     -122.
  5  5     -76.       80.
  6  0      54.        0.
  6  1      57.       -1.
  6  2       4.       99.
  6  3    -247.       33.
  6  4     -16.      -12.
  6  5      12.      -12.
  6  6    -105.      -30.
  7  0      65.        0.
  7  1     -55.      -35.
  7  2       2.      -17.
  7  3       1.        0.
  7  4     -40.       10.
  7  5      -7.       36.
  7  6       5.      -18.
  7  7      19.      -16.
  8  0      22.        0.
  8  1      15.        5.
  8  2      -4.      -22.
  8  3      -1.        0.
  8  4      11.      -21.
  8  5      15.       -8.
  8  6     -13.       17.
  8  7       5.       -4.
  8  8      -1.      -17.
  9  0       3.        0.
  9  1      -7.      -24.
  9  2      -1.       19.
  9  3     -25.       12.
  9  4      10.        2.
  9  5       5.        2.
  9  6      -5.        8.
  9  7      -2.        8.
  9  8       3.      -11.
  9  9       8.       -7.
 10  0      -8.        0.
 10  1       4.       13.
 10  2      -1.       -2.
 10  3      13.      -10.
 10  4      -4.        2.
 10  5       4.       -3.
 10  6      12.        6.
 10  7       3.       -3.
 10  8       2.        6.
 10  9      10.       11.
 10 10       3.        8.
```

```
dgrf55
10  6371.2 1955.0
  1  0  -30500.        0.
  1  1   -2215.     5820.
  2  0   -1440.        0.
  2  1    3003.    -1898.
  2  2    1581.      291.
  3  0    1302.        0.
  3  1   -1944.     -462.
  3  2    1288.      216.
  3  3     882.      -83.
  4  0     958.        0.
  4  1     796.      133.
  4  2     510.     -274.
  4  3    -397.      -23.
  4  4     290.     -230.
  5  0    -229.        0.
  5  1     360.       15.
  5  2     230.      110.
  5  3     -23.      -98.
  5  4    -152.     -121.
  5  5     -69.       78.
  6  0      47.        0.
  6  1      57.       -9.
  6  2       3.       96.
  6  3    -247.       48.
  6  4      -8.      -16.
  6  5       7.      -12.
  6  6    -107.      -24.
  7  0      65.        0.
  7  1     -56.      -50.
  7  2       2.      -24.
  7  3      10.       -4.
  7  4     -32.        8.
  7  5     -11.       28.
  7  6       9.      -20.
  7  7      18.      -18.
  8  0      11.        0.
  8  1       9.       10.
  8  2      -6.      -15.
  8  3     -14.        5.
  8  4       6.      -23.
  8  5      10.        3.
  8  6      -7.       23.
  8  7       6.       -4.
  8  8       9.      -13.
  9  0       4.        0.
  9  1       9.      -11.
  9  2      -4.       12.
  9  3      -5.        7.
  9  4       2.        6.
  9  5       4.       -2.
  9  6       1.       10.
  9  7       2.        7.
  9  8       2.       -6.
  9  9       5.        5.
 10  0      -3.        0.
 10  1      -5.       -4.
 10  2      -1.        0.
 10  3       2.       -8.
 10  4      -3.       -2.
 10  5       7.       -4.
 10  6       4.        1.
 10  7      -2.       -3.
 10  8       6.        7.
 10  9      -2.       -1.
 10 10       0.       -3.
```

```
dgrf60
10  6371.2 1960.0
  1  0  -30421.       0.
  1  1   -2169.    5791.
  2  0   -1555.       0.
  2  1    3002.   -1967.
  2  2    1590.     206.
  3  0    1302.       0.
  3  1   -1992.    -414.
  3  2    1289.     224.
  3  3     878.    -130.
  4  0     957.       0.
  4  1     800.     135.
  4  2     504.    -278.
  4  3    -394.       3.
  4  4     269.    -255.
  5  0    -222.       0.
  5  1     362.      16.
  5  2     242.     125.
  5  3     -26.    -117.
  5  4    -156.    -114.
  5  5     -63.      81.
  6  0      46.       0.
  6  1      58.     -10.
  6  2       1.      99.
  6  3    -237.      60.
  6  4      -1.     -20.
  6  5      -2.     -11.
  6  6    -113.     -17.
  7  0      67.       0.
  7  1     -56.     -55.
  7  2       5.     -28.
  7  3      15.      -6.
  7  4     -32.       7.
  7  5      -7.      23.
  7  6      17.     -18.
  7  7       8.     -17.
  8  0      15.       0.
  8  1       6.      11.
  8  2      -4.     -14.
  8  3     -11.       7.
  8  4       2.     -18.
  8  5      10.       4.
  8  6      -5.      23.
  8  7      10.       1.
  8  8       8.     -20.
  9  0       4.       0.
  9  1       6.     -18.
  9  2       0.      12.
  9  3      -9.       2.
  9  4       1.       0.
  9  5       4.      -3.
  9  6      -1.       9.
  9  7      -2.       8.
  9  8       3.       0.
  9  9      -1.       1.
 10  0       1.       0.
 10  1      -3.       4.
 10  2       4.       1.
 10  3       0.       3.
 10  4      -1.       2.
 10  5       4.      -5.
 10  6       6.       1.
 10  7       1.      -1.
 10  8      -1.       6.
 10  9       2.       0.
 10 10       0.      -7.
```

```
dgrf65
10  6371.2 1965.0
  1  0  -30334.       0.
  1  1   -2119.    5776.
  2  0   -1662.       0.
  2  1    2997.   -2016.
  2  2    1594.     114.
  3  0    1297.       0.
  3  1   -2038.    -404.
  3  2    1292.     240.
  3  3     856.    -165.
  4  0     957.       0.
  4  1     804.     148.
  4  2     479.    -269.
  4  3    -390.      13.
  4  4     252.    -269.
  5  0    -219.       0.
  5  1     358.      19.
  5  2     254.     128.
  5  3     -31.    -126.
  5  4    -157.     -97.
  5  5     -62.      81.
  6  0      45.       0.
  6  1      61.     -11.
  6  2       8.     100.
  6  3    -228.      68.
  6  4       4.     -32.
  6  5       1.      -8.
  6  6    -111.      -7.
  7  0      75.       0.
  7  1     -57.     -61.
  7  2       4.     -27.
  7  3      13.      -2.
  7  4     -26.       6.
  7  5      -6.      26.
  7  6      13.     -23.
  7  7       1.     -12.
  8  0      13.       0.
  8  1       5.       7.
  8  2      -4.     -12.
  8  3     -14.       9.
  8  4       0.     -16.
  8  5       8.       4.
  8  6      -1.      24.
  8  7      11.      -3.
  8  8       4.     -17.
  9  0       8.       0.
  9  1      10.     -22.
  9  2       2.      15.
  9  3     -13.       7.
  9  4      10.      -4.
  9  5      -1.      -5.
  9  6      -1.      10.
  9  7       5.      10.
  9  8       1.      -4.
  9  9      -2.       1.
 10  0      -2.       0.
 10  1      -3.       2.
 10  2       2.       1.
 10  3      -5.       2.
 10  4      -2.       6.
 10  5       4.      -4.
 10  6       4.       0.
 10  7       0.      -2.
 10  8       2.       3.
 10  9       2.       0.
 10 10       0.      -6.
```

```
dgrf70
10  6371.2 1970.0
  1  0  -30220.       0.
  1  1   -2068.    5737.
  2  0   -1781.       0.
  2  1    3000.   -2047.
  2  2    1611.      25.
  3  0    1287.       0.
  3  1   -2091.    -366.
  3  2    1278.     251.
  3  3     838.    -196.
  4  0     952.       0.
  4  1     800.     167.
  4  2     461.    -266.
  4  3    -395.      26.
  4  4     234.    -279.
  5  0    -216.       0.
  5  1     359.      26.
  5  2     262.     139.
  5  3     -42.    -139.
  5  4    -160.     -91.
  5  5     -56.      83.
  6  0      43.       0.
  6  1      64.     -12.
  6  2      15.     100.
  6  3    -212.      72.
  6  4       2.     -37.
  6  5       3.      -6.
  6  6    -112.       1.
  7  0      72.       0.
  7  1     -57.     -70.
  7  2       1.     -27.
  7  3      14.      -4.
  7  4     -22.       8.
  7  5      -2.      23.
  7  6      13.     -23.
  7  7      -2.     -11.
  8  0      14.       0.
  8  1       6.       7.
  8  2      -2.     -15.
  8  3     -13.       6.
  8  4      -3.     -17.
  8  5       5.       6.
  8  6       0.      21.
  8  7      11.      -6.
  8  8       3.     -16.
  9  0       8.       0.
  9  1      10.     -21.
  9  2       2.      16.
  9  3     -12.       6.
  9  4      10.      -4.
  9  5      -1.      -5.
  9  6       0.      10.
  9  7       3.      11.
  9  8       1.      -2.
  9  9      -1.       1.
 10  0      -3.       0.
 10  1      -3.       1.
 10  2       2.       1.
 10  3      -5.       3.
 10  4      -1.       4.
 10  5       6.      -4.
 10  6       4.       0.
 10  7       1.      -1.
 10  8       0.       3.
 10  9       3.       1.
 10 10      -1.      -4.
```

dgrf75

```
10  6371.2 1975.0
  1  0  -30100.        0.
  1  1   -2013.     5675.
  2  0   -1902.        0.
  2  1    3010.    -2067.
  2  2    1632.      -68.
  3  0    1276.        0.
  3  1   -2144.     -333.
  3  2    1260.      262.
  3  3     830.     -223.
  4  0     946.        0.
  4  1     791.      191.
  4  2     438.     -265.
  4  3    -405.       39.
  4  4     216.     -288.
  5  0    -218.        0.
  5  1     356.       31.
  5  2     264.      148.
  5  3     -59.     -152.
  5  4    -159.      -83
  5  5     -49.       88.
  6  0      45.        0.
  6  1      66.      -13.
  6  2      28.       99.
  6  3    -198.       75.
  6  4       1.      -41.
  6  5       6.       -4.
  6  6    -111.       11.
  7  0      71.        0.
  7  1     -56.      -77.
  7  2       1.      -26.
  7  3      16.       -5.
  7  4     -14.       10.
  7  5       0.       22.
  7  6      12.      -23.
  7  7      -5.      -12.
  8  0      14.        0.
  8  1       6.        6.
  8  2      -1.      -16.
  8  3     -12.        4.
  8  4      -8.      -19.
  8  5       4.        6.
  8  6       0.       18.
  8  7      10.      -10.
  8  8       1.      -17.
  9  0       7.        0.
  9  1      10.      -21.
  9  2       2.       16.
  9  3     -12.        7.
  9  4      10.       -4.
  9  5      -1.       -5.
  9  6      -1.       10.
  9  7       4.       11.
  9  8       1.       -3.
  9  9      -2.        1.
 10  0      -3.        0.
 10  1      -3.        1.
 10  2       2.        1.
 10  3      -5.        3.
 10  4      -2.        4.
 10  5       5.       -4.
 10  6       4.       -1.
 10  7       1.       -1.
 10  8       0.        3.
 10  9       3.        1.
 10 10      -1.       -5.
```

dgrf80

```
10  6371.2 1980.0
  1  0  -29992.        0.
  1  1   -1956.     5604.
  2  0   -1997.        0.
  2  1    3027.    -2129.
  2  2    1663.     -200.
  3  0    1281.        0.
  3  1   -2180.     -336.
  3  2    1251.      271.
  3  3     833.     -252.
  4  0     938.        0.
  4  1     782.      212.
  4  2     398.     -257.
  4  3    -419.       53.
  4  4     199.     -297.
  5  0    -218.        0.
  5  1     357.       46.
  5  2     261.      150.
  5  3     -74.     -151.
  5  4    -162.      -78.
  5  5     -48.       92.
  6  0      48.        0.
  6  1      66.      -15.
  6  2      42.       93.
  6  3    -192.       71.
  6  4       4.      -43.
  6  5      14.       -2.
  6  6    -108.       17.
  7  0      72.        0.
  7  1     -59.      -82.
  7  2       2.      -27.
  7  3      21.       -5.
  7  4     -12.       16.
  7  5       1.       18.
  7  6      11.      -23.
  7  7      -2.      -10.
  8  0      18.        0.
  8  1       6.        7.
  8  2       0.      -18.
  8  3     -11.        4.
  8  4      -7.      -22.
  8  5       4.        9.
  8  6       3.       16.
  8  7       6.      -13.
  8  8      -1.      -15.
  9  0       5.        0.
  9  1      10.      -21.
  9  2       1.       16.
  9  3     -12.        9.
  9  4       9.       -5.
  9  5      -3.       -6.
  9  6      -1.        9
  9  7       7.       10.
  9  8       2.       -6.
  9  9      -5.        2.
 10  0      -4.        0.
 10  1      -4.        1.
 10  2       2.        0.
 10  3      -5.        3.
 10  4      -2.        6.
 10  5       5.       -4.
 10  6       3.        0.
 10  7       1.       -1.
 10  8       2.        4.
 10  9       3.        0.
 10 10       0.       -6.
```

igrf85

```
10  6371.2 1985.0
  1  0  -29877.        0.
  1  1   -1903.     5497.
  2  0   -2073.        0.
  2  1    3045.    -2191.
  2  2    1691.     -309.
  3  0    1300.        0.
  3  1   -2208.     -312.
  3  2    1244.      284.
  3  3     835.     -296.
  4  0     937.        0.
  4  1     780.      233.
  4  2     363.     -250.
  4  3    -426.       68.
  4  4     169.     -298.
  5  0    -215.        0.
  5  1     356.       47.
  5  2     253.      148.
  5  3     -94.     -155.
  5  4    -161.      -75.
  5  5     -48.       95.
  6  0      52.        0.
  6  1      65.      -16.
  6  2      50.       90.
  6  3    -186.       69.
  6  4       4.      -50.
  6  5      17.       -4.
  6  6    -102.       20.
  7  0      75.        0.
  7  1     -61.      -82.
  7  2       2.      -26.
  7  3      24.       -1.
  7  4      -6.       23.
  7  5       4.       17.
  7  6       9.      -21.
  7  7       0.       -6.
  8  0      21.        0.
  8  1       6.        7.
  8  2       0.      -21.
  8  3     -11.        5.
  8  4      -9.      -25.
  8  5       2.       11.
  8  6       4.       12.
  8  7       4.      -16.
  8  8      -6.      -10.
  9  0       5.        0.
  9  1      10.      -21.
  9  2       1.       16.
  9  3     -12.        9.
  9  4       9.       -5.
  9  5      -3.       -6.
  9  6      -1.        9.
  9  7       7.       10.
  9  8       2.       -6.
  9  9      -5.        2.
 10  0      -4.        0.
 10  1      -4.        1.
 10  2       2.        0.
 10  3      -5.        3.
 10  4      -2.        6.
 10  5       5.       -4.
 10  6       3.        0.
 10  7       1.       -1.
 10  8       2.        4.
 10  9       3.        0.
 10 10       0.       -6.
```

igrf85s

```
 8  [illegible] 1.[illegible] 1990.0
 1  0   23.2          [illegible]
 1  1   10.0          -[illegible]
 2  0  -13.[illegible]   [illegible]
 2  1   [illegible]   -[illegible]
 2  2   [illegible]   [illegible]
 3  0   [illegible]   [illegible]
 3  1   -[illegible]  [illegible]
 3  2   -[illegible]  [illegible]
 3  3   [illegible]   -[illegible]
 4  0    0.1          [illegible]
 4  1   -[illegible]  [illegible]
 4  2   -[illegible]  [illegible]
 4  3   -[illegible]  [illegible]
 4  4   -[illegible]  [illegible]
 5  0   [illegible]   [illegible]
 5  1   [illegible]   [illegible]
 5  2   -[illegible]  -[illegible]
 5  3   -3.2          -[illegible]
 5  4    0.1          [illegible]
 5  5   -0.1          [illegible]
 6  0    1.4           0.0
 6  1   -0.3          -0.[illegible]
 6  2   [illegible]   -[illegible]
 6  3   [illegible]   -[illegible]
 6  4   [illegible]   -[illegible]
 6  5   [illegible]   -[illegible]
 6  6   [illegible]   -[illegible]
 7  0    0.2          [illegible]
 7  1   -[illegible]  [illegible]
 7  2   -[illegible]  [illegible]
 7  3    0.8          [illegible]
 7  4   [illegible]   [illegible]
 7  5   [illegible]   [illegible]
 7  6   -0.5          [illegible]
 7  7   -[illegible]  [illegible]
 8  0   [illegible]   [illegible]
 8  1    0.0          [illegible]
 8  2   [illegible]   [illegible]
 8  3    0.4          [illegible]
 8  4   -0.3          -[illegible]
 8  5   -0.3          [illegible]
 8  6   [illegible]   -[illegible]
 8  7   -0.5          -[illegible]
 8  8   -0.[illegible]   [illegible]
```

Appendix B

External Magnetic Field Routines

These listings are the 1977 Olson-Pfitzer tilt dependent routine including the routine that combines the internal and the external field. These routines are included here for completeness. They were not developed or changed as a part of this effort. They are, however, necessary in order for the remaining software to function properly.

```
C     BMNIGRF -- A TEST ROUTINE TO CHECK THE OPERATION OF THE TILT
C     DEPENDENT MODEL AND IT COMBINATION WITH THE IGRF MAIN FIELD
C
      DIMENSION X(3),B(3)
      COMMON/BXYZCM/YEAR,DAYYR,UT,KODE,JSW

C
C     SET UP THE YEAR FOR THE MAIN FIELD ROUTINE
C
      YEAR=1985.
C
C     SET THE SWITCH TO USE EXTERNAL PLUS INTERNAL FIELD
C
      JSW=1
C
C     SET THE SWITCH TO USE INPUT AND OUTPUT IN CARTESIAN COORDS
C
      KODE=1
C
C     SET UP A CARTESIAN COORDINATE TEST LOOP
C
C     SET UP DATE AND TIME
      DO 200 ID=1,2
      DAYYR=90*ID
      DO 190 IUT=1,3
      UT=IUT*6-6
C
C     PRINT PAGE HEADER
      WRITE (6,110)
 110  FORMAT(77H1DAYOFYR   UT      X       Y       Z        BX        BY
     *        BZ       BMAG,/)
C     SET UP POSITION IN CARTESIAN COORDS
      DO 180 IZ=1,3
      X(3)=3*IZ-6
      DO 170 IY=1,3
      X(2)=3*IY-6
      DO 160 IX=1,6
      X(1)=4*IX-14
C
C     GET THE MAGNETIC FIELD VALUES
      CALL BMNEXT(X,B,BMAG)
      WRITE (6,120) DAYYR,UT,X,B,BMAG
 120  FORMAT(F6.0,4F8.2,4F10.5)
 160  CONTINUE
 170  CONTINUE
 180  CONTINUE
 190  CONTINUE
 200  CONTINUE
C
C     SET UP FOR SPHERICAL COORDINATES
C
1000      KODE=2
C
C     SET DATE AND TIME
      DO 300 ID=1,2
      DAYYR=90*ID
      DO 290 IUT=1,3
      UT=IUT*6-6
C
C     PRINT PAGE HEADER
      WRITE (6,210)
 210  FORMAT(77H1DAYOFYR   UT      R     THETA    PHI       BR       BTHE
     *TA     BPHI      BMAG,/)
C
```

```
C      SET UP POSITIONS IN SPHERICAL COORDS
       DO 280 IR=1,3
       X(1)=IR*3
       DO 270 IT=1,3
       X(2)=IT*45
       DO 260 IP=1,6
       X(3)=(IP-1)*60
C
C      GET THE MAGNETIC FIELD
       CALL BMNEXT(X,B,BMAG)
       WRITE(6,120) DAYYR,UT,X,B,BMAG
 260   CONTINUE
 270   CONTINUE
 280   CONTINUE
 290   CONTINUE
 300   CONTINUE
       END
```

```
      SUBROUTINE BMNEXT(XX,B,BMAG)
C
C     PURPOSE
C        TO DETERMINE THE MAIN MAGNETIC FIELD PLUS THE EXTERNAL
C        FIELD
C
C     METHOD
C        DETERMINES THE VECTOR MAGNETIC FIELD IN GEOGRAPHIC
C        COORDINATES USING A SPHERICAL COORDINATE EXPANSION OF THE
C        EARTHS INTERNAL FIELD AND A CARTESIAN COORDINATE EXPANSION
C        OF THE BOUNDARY, TAIL AND RING CURRENT FIELDS IN SOLAR
C        MAGNETIC COORDINATES
C
C     INPUT -- ARGUMENT LIST
C        XX      A REAL ARRAY CONTAINING THE POSITION IN GEOGRAPHIC
C                COORDINATES
C                IF KODE = 1
C                XX(1)=X, XX(2)=Y, XX(3)=Z, WHERE X, Y, Z ARE IN EARTH
C                RADII.  THE DIRECTION OF Z IS ALONG THE EARTHS ROTATION
C                AXIS TOWARDS THE GEOGRAPHIC NORTH POLE.  THE DIRECTION
C                OF X IS TO THE GREENWHICH MERIDIAN IN THE EQUATORIAL
C                PLANE.  THE Y AXIS IS IN THE EQUATORIAL PLANE NORMAL
C                TO X AND Z IN A RIGHT HANDED SENSE.
C                IF KODE = 2
C                XX(1)=R, GEOCENTRIC RADIUS IN EARTH RADII,
C                XX(2)=THETAG, COLATITUDE IN DEGREES,
C                XX(3)=PHIG, LONGITUDE IN DEGREES
C
C     INPUT -- COMMON BLOCK BXYZCM
C        UT      THE CURRENT UNIVERSAL TIME IN HOURS
C        KODE    A FLOW CONTROL VARIABLE.  KODE EQUAL TO ONE MEANS THAT
C                INPUT AND OUTPUT ARE IN CARTESIAN COORDINATES.  KODE
C                EQUAL TO TWO MEANS THAT INPUT AND OUTPUT ARE SPHERICAL
C                COORDINATES.
C        DAYYR   THE NUMBER OF THE DAY OF YEAR
C        JSW     A FLOW CONTROL VARIABLE. IF JSW IS LESS THAN ZERO, THE
C                THE FIELD IS COMPUTED USING THE INTERNAL FIELD ONLY.
C                IF JSW IS GREATER THAN OR EQUAL TO ZERO THE FIELD
C                WILL BE COMPUTED USING THE INTERNAL PLUS EXTERNAL
C                FIELD.
C        YEAR    THE YEAR USED BY THE INTERNAL MAGNETIC FIELD ROUTINE
C                TO TAKE INTO ACCOUNT THE SECULAR VARIATIONS
C                (E.G. JULY 15, 1964 = 1964.54)
C                NOTE**** YEAR SHOULD BE CHANGED ONLY EVERY FEW DAYS OR
C                MONTHS.  NEW FIELD COEFFICIENTS MUST BE COMPUTED FOR
C                EVERY CHANGE IN YEAR.  THIS COULD CAUSE A LARGE INCREASE
C                IN COMPUTER TIME.  THE EARTHS FIELD CHANGES ONLY ABOUT
C                .001 GAUSS/YEAR AT THE EARTHS SURFACE.
C
C     OUTPUT -- ARGUMENT LIST
C        B       A REAL ARRAY CONTAINING THE COMPONENTS OF THE MAGNETIC
C                FIELD IN GAUSS AT THE CURRENT POSITION AND TIME
C                IF KODE = 1
C                B(1)=BX, B(2)=BY, B(3)=BZ  THE CARTESIAN COMPONENTS
C                OF THE MAGNETIC FIELD IN GEOGRAPHIC COORDINATES
C                IF KODE = 2
C                B(1)=BR, RADIAL COMPONENT OF THE FIELD, POSITIVE IN THE
C                DIRECTION OF INCREASING RADIUS.
C                B(2)=BTHETA, COMPONENT IN LATITUDE, POSITIVE IN THE
C                DIRECTION OF INCREASING COLATITUDE
C                B(3)=BPHI, COMPONENT IN LONGITUDE, POSITIVE IN THE
C                DIRECTION OF INCREASING LONGITUDE.
C        BMAG    THE MAGNITUDE OF THE MAGNETIC FIELD VECTOR IN UNITS OF
C                GAUSS.
```

```
C
C     OUTPUT -- COMMON BLOCK BXYZCM
C        XMLAT  THE MAGNETIC LATITUDE AT THE CURRENT POSITION IN RADIANS
C
C     SUBROUTINE CONSTANTS
C        PICON  THE NUMBER OF DEGREES PER RADIAN
C        SIN D  THE SINE OF THE COLATITUDE OF THE DIPOLE AXIS
C        COS D  THE COSINE OF THE COLATITUDE OF THE DIPOLE AXIS
C        C69    COSINE OF 69
C        S69    SINE OF 69
C
C     CALLING SUBROUTINES
C        SUBROUTINE INVARM
C
C     SUBROUTINES REQUIRED
C        SUBROUTINE BXYZMU
C        SUBROUTINE ANGLE
C        SUBROUTINE SPIGRF
C
C     VARIABLES
C        AOR    INVERSE OF RADIUS VECTOR (AOR=1./R)
C        BGMX,BGMY,BGMZ  INTERMEDIATE VALUES OF THE MAGNETIC FIELD
C               VECTOR DURING COORDINATE TRANSFORMATION
C        BMX,BMY,BMZ  EXTERNAL MAGNETIC FIELD IN GEOMAGNETIC COORDINATES
C        BP,BR,BT  COMPONENTS OF INTERNAL FIELD IN SPHERICAL COORDINATES
C               BP IS LONGITUDINAL COMPONENT
C               BR IS RADIAL COMPONENT
C               BT IS LATITUDINAL COMPONENT
C        BX,BY,BZ  CARTESIAN COMPONENT OF EXTERNAL FIELD IN GEOGRAPHIC
C               COORDINATES
C        CP     COSINE OF COLATITUDE
C        CPS    COSINE OF HOUR ANGLE TO GET FROM SOLAR MAGNETIC TO
C               GEOMAGNETIC COORDINATES
C        CT     COSINE OF GEOGRAPHIC LONGITUDE
C        DAYLST LAST DAY FOR WHICH TILT AND HOUR ANGLE WERE UPDATED
C        NMAX   MAXIMUM NUMBER OF TERMS USED BY INTERNAL FIELD ROUTINE
C               SET UP BY INTERNAL FIELD ROUTINE
C        PHIG   GEOGRAPHIC COLATITUDE
C        R      RADIUS VECTOR TO POSITION POINT
C        R2     R**2
C        SP     SINE OF COLATITUDE
C        SPS    SINE OF HOUR ANGLE TO GET FROM SOLAR MAGNETIC TO
C               GEOMAGNETIC COORDINATES
C        ST     SINE OF LONGITUDE
C        THETAG GEOGRAPHIC LONGITUDE
C        TILT   TILT OF THE DIPOLE AXIS
C        UTLST  LAST UNIVERSAL TIME FOR WHICH TILT AND HOUR ANGLE WERE
C               UPDATED
C        X      A REAL ARRAY HOLDING THE POSITION VECTOR IN SOLAR
C               MAGNETIC COORDINATES
C        XP,YP,ZP  POSITION VECTOR IN GEOMAGNETIC COORDINTES
C        XPP,YPP INTERMEDIATE POSITION COMPONENT DURING COORDINATE
C               TRANSFORMATION
C        YEARI  TRANSMITS THE YEAR TO THE INTERNAL FIELD ROUTINE
C
C     VERSION 10/25/77
C     FOR MORE INFORMATION CALL OR WRITE K. A. PFITZER AT MCDONNELL
C     DOUGLAS ASTRONAUTICS CO. 5301 BOLSA AVE, HUNTINGTON BEACH CALIF.
C     PHONE (714) 896-3231.
C
      DIMENSION X(3),B(3),XX(3)
      COMMON/BXYZCM/YEAR,DAYYR,UT,KODE,JSW
      COMMON /GCOM/ ST,CT,SP,CP,AOR,BT,BP,BR,NMAX,YEARI
      DATA PICON/57.29577951/,SIND,COSD/.2027872954,.9792228106/,
```

```
     *S69,C69/.9335804265,.3583679495/,UTLST,DAYLST/2*123456./
C
C     UPDATE THE ROTATION HOUR ANGLE AND TILT ANGLE IF THE UNIVERSAL
C     TIME OR DAY OF YEAR HAS CHANGED SINCE THE LAST CALL
C
      IF(UT.EQ.UTLST.AND.DAYYR.EQ.DAYLST) GO TO 1
      UTLST=UT
      DAYLST=DAYYR
      CALL ANGLE (TILT,SPS,CPS)
 1    IF(KODE.GT.1) GO TO 3
C
C     DETERMINE THE SPHERICAL COORDINATES OF POSITION IF CARTESIAN
C     COORDINATES WERE ENTERED
C
      X(1)=XX(1)
      X(2)=XX(2)
      X(3)=XX(3)
      R2=X(1)**2+X(2)**2
      R=SQRT(X(3)**2+R2)
      R2=SQRT(R2)
      CT=X(3)/R
      ST-R2/R
      CP=X(1)/R2
      SP=X(2)/R2
      GO TO 5
C
C     DETERMINE THE CARTESIAN COORDINATES OF POSITION IF SPHERICAL
C     COORDINATES WERE ENTERED
C
 3    R=XX(1)
      THETAG=XX(2)/PICON
      PHIG=XX(3)/PICON
      CT=COS(THETAG)
      ST=SIN(THETAG)
      CP=COS(PHIG)
      SP=SIN(PHIG)
      X(1)=R*ST*CP
      X(2)=R*ST*SP
      X(3)=R*CT
 5    BX=0.
      BY=0.
      BZ=0.
C
C     IF THE EXTERNAL MAGNETIC FIELD IS TO BE USED IN THE COMPUTATION,
C     COMPUTE THE SOLAR MAGNETIC COORDINATES
C
      IF(JSW.LT.0) GO TO 9
C
C     FIRST ROTATION IS ABOUT THE Z-AXIS TROUGH AN ANGLE OF 291 DEGREES
C     (THE LONGITUDE OF THE MAGNETIC NORTH POLE)
C
      XPP=X(1)*C69-X(2)*S69
      YPP= X(1)*S69+X(2)*C69
C
C     SECOND ROTATION IS ABOUT THE NEW Y-AXIS THROUGH AN ANGLE OF 11.7
C     DEGREES (THE COLATITUDE OF THE MAGNETIC NORTH POLE)
C
      ZP=XPP*SIND+X(3)*COSD
      XP=XPP*COSD-X(3)*SIND
      YP=YPP
C
C     ROTATION IS ABOUT THE MAGNETIC Z-AXIS THROUGH THE HOUR ANGLE OF
C     THE SUN FROM THE PRIME MAGNETIC MERIDIAN (NEGATIVE ROTATION)
C
```

```
      X(1)=XP*CPS-YP*SPS
      X(2)=XP*SPS+YP*CPS
      X(3)=ZP
C
C     DETERMINE THE EXTERNAL MAGNETIC FIELD USING A TILT DEPENDENT
C     MAGNETIC FIELD
C
      CALL BXYZMU(X,B,TILT)
C
C     THE CARTESIAN COMPONENTS OF THE FIELD ARE IN SOLAR MAGNETIC
C     COORDINATES. THE COMPONENTS ARE NEEDED IN THE GEOGRAPHIC
C     COORDINATE SYSTEM
C
C     FIRST ROTATION IS ABOUT THE MAGNETIC Z-AXIS THROUGH THE HOUR
C     ANGLE OF THE SUN TO THE PRIME MAGNETIC MERIDIAN
C     (POSITIVE ROTATION) PUTS RESULTS INTO GEOMAGNETIC COORDINATES
C
      BMX=B(1)*CPS+B(2)*SPS
      BMY=-B(1)*SPS+B(2)*CPS
      BMZ=B(3)
C
C     SECOND ROTATION IS ABOUT THE MAGNETIC Y-AXIS THOUGH -11.7 DEGREES
C     COLATITUDE
C
      BGMX=BMX*COSD+BMZ*SIND
      BGMY=BMY
      BGMZ=-BMX*SIND+BMZ*COSD
C
C     THIRD ROTATION IS ABOUT THE NEW Z-AXIS THROUGH -291 DEGREES
C
      BX=BGMX*C69+BGMY*S69
      BY=-BGMX*S69+BGMY*C69
      BZ=BGMZ
C
C     DETERMINE THE MAIN FIELD
C
 9    CONTINUE
      AOR=1./R
      YEARI=YEAR
      CALL SPIGRF
      IF(KODE.GT.1) GO TO 10
C
C     IF THE OUTPUT IS TO BE IN CARTESIAN GEOGRAPHIC COORDINATES CONVERT
C     THE MAIN MAGNETIC FIELD AND ADD
C
      B(1)=(BX+CP*(ST*BR+CT*BT)-SP*BP)*0.00001
      B(2)=(BY+SP*(ST*BR+CT*BT)+CP*BP)*0.00001
      B(3)=(BZ+CT*BR-ST*BT)*0.00001
      GO TO 20
C
C     IF OUTPUT IS TO BE IN SPHERICAL GEOGRAPHIC CONVERT THE EXTERNAL
C     FIELD AND ADD
C
 10   B(1)=(BR+(BX*CP+BY*SP)*ST+BZ*CT)*0.00001
      B(2)=(BT+(BX*CP+BY*SP)*CT-BZ*ST)*0.00001
      B(3)=(BP+BY*CP-BX*SP)*0.00001
C
C     DETERMINE THE MAGNITUDE OF THE FIELD VECTOR
C
 20   BMAG=SQRT(B(1)**2+B(2)**2+B(3)**2)
      RETURN
      END
```

```
      SUBROUTINE ANGLE(TILT,SINPHE,COSPHE)
C
C     PURPOSE
C        THIS ROUTINE CALCULATES THE ANGLE BETWEEN THE MAGNETIC DIPOLE
C        AXIS AND THE SUN-EARTH LINE AS WELL AS THE ROTATION SINES
C        AND COSINES TO CONVERT FROM GEOMAGNETIC TO SOLAR MAGNETIC
C        COORDINATES
C
C     METHOD
C        MAGNETIC COORDINATES HAVE THEIR ORIGIN AT THE CENTER OF THE
C        EARTH WITH THE Z AXIS ALLIGNED THROUGH THE GEOMAGNETIC NORTH
C        POLE.  IN GEOMAGNETIC COORDINATES THE X AXIS IS IN THE
C        PLANE PASSING THROUGH THE DIPOLE AXIS AND THE GEOGRAPHIC
C        AXIS (ABOUT 69 DEGREES WEST LONG.).  IN SOLAR MAGNETIC
C        COORDINATES X AXIS LIES IN THE PLANE CONTAINING THE SUN
C        EARTH LINE AND THE Z AXIS (POSITIVE X AXIS HAS A LARGE
C        COMPONENT IN THE SOLAR DIRECTION). THE Y AXIS IS ORTHOGONAL
C        TO THE X AND Z AXIS SUCH THAT X, Y AND Z FORM A RIGHT
C        HANDED SYSTEM.  THE ECCLIPTIC COORDINATE SYSTEM HAS ITS
C        Z AXIS ALONG THE ECCLIPTIC NORTH POLE (THROUGH THE CENTER
C        OF THE EARTH AND PERPENDICULAR TO THE EARTHS ORBITAL PLANE)
C        THE X AXIS POINTS TOWARD THE SUN AND Y FORMS A RIGHT HANDED
C        COORDINATE SYSTEM.  IN THIS ROUTINE IN ORDER TO REDUCE
C        COMPUTER TIME THE APPROXIMATION OF A CIRCULAR EARTH ORBIT
C        AROUND THE SUN IS MADE.
C
C     INPUT -- COMMON BLOCK BXYZCM
C        DAYYR  IS THE DAY OF YEAR (1.-366.).  IT MUST BE A WHOLE
C               NUMBER.  DAY 1 IS JANUARY 1.
C        UT     THE UNIVERSAL TIME IN HOURS (0.0000-24.00000)
C
C     OUTPUT -- PARAMETER LIST
C        TILT   THE TILT OF THE DIPOLE AXIS IN DEGREES.
C               TILT = 90. - PSI, WHERE PSI IS THE ANGLE BETWEEN
C               THE MAGNETIC DIPOLE AXIS AND THE SOLAR DIRECTION.
C        SINPHE THE SINE OF THE ROTATION ANGLE ABOUT THE MAGNETIC
C               Z AXIS TO CONVERT FROM GEOMAGNETIC TO SOLAR MAGNETIC
C               COORDINATES
C        COSPHE THE COSINE OF THE ROTATION ANGLE ABOUT THE MAGNETIC
C               Z AXIS TO CONVERT FROM GEOMAGNETIC TO SOLAR MAGNETIC
C               COORDINATES
C
C     CONSTANTS
C        PI2    PI / 2.
C        CON    180. / PI CONVERTS RADIANS TO DEGREES
C        SALF   SINE (11.7)  INCLINATION OF MAGNETIC Z TO GEOGRAPHIC Z
C        CALF   COSINE (11.7)
C        SGAM   SIN (23.5) INCLINATION OF ROTATION AXIS TO ECLIPTIC Z
C        CGAM   COSINE (23.5)
C        SASG   SALF * SGAM
C        SACG   SALF * CGAM
C        CASG   CALF * SGAM
C        CACG   CALF * CGAM
C        W      EARTHS ANGULAR ROTATION FREQUENCY CORRECTED FOR ITS
C               ONCE A YEAR ROTATION ABOUT THE SUN (UNITS ARE 1/HOURS)
C
C     CALLING SUBROUTINES
C        SUBROUTINE BMNEXT
C
C     VARIABLES
C        WT      INSTANTANEOUS ROTATION ANGLE AT THE SPECIFIED UNIVERSAL
C                TIME AND DAY OF YEAR
C        CWT    WT/365.256
C        XX,XY,XZ COMPONENTS OF THE GEOMAGNEITIC X AXIS IN ECCLIPTIC
```

```
C               COORDINATES
C          ZX,ZY,ZZ COMPONENTS OF THE DIPOLE AXIS IN ECCLIPTIC COORDINATES
C          OSP,SSMLT,CSMLT,SBWT,CBWT,SMLSWT,SMLCWT,CMLSWT,CMLCWT ARE
C               SINES AND COSINES AND THEIR PRODUCTS AND ARE SET UP
C               TO MINIMIZE COMPUTER TIME
C
      COMMON/BXYZCM/YEAR,DAYYR,UT,KODE,JSW
      DATA  IFIRST/0/
C
C     THE FIRST TIME TROUGH THE SUBROUTINE SET UP THE FIXED CONSTANTS
      IF(IFIRST.NE.0) GO TO 10
      IFIRST=1
      PI2=ATAN2(0.,-1.)/2.
      CON=90./PI2
      SALF=SIN(11.7/CON)
      CALF=COS(11.7/CON)
      SGAM=SIN(23.5/CON)
      CGAM=COS(23.5/CON)
      SASG=SALF*SGAM
      SACG=SALF*CGAM
      CASG=CALF*SGAM
      CACG=CALF*CGAM
      W=PI2/6.*(1.+1./365.256)
C
C     MAIN ENTRY POINT.  SET UP THE THE SINES AND COSINES REQUIRED
C     BY THE TRANSFORMATIONS.
 10   WT=(UT-16.6+(DAYYR-172.)*24.)*W
      CWT=-WT/365.256
      SSMLT=SIN(WT)
      CSMLT=COS(WT)
      SBWT=SIN(CWT)
      CBWT=COS(CWT)
      SMLSWT=SSMLT*SBWT
      SMLCWT=SSMLT*CBWT
      CMLSWT=CSMLT*SBWT
      CMLCWT=CSMLT*CBWT
C
C     DETERMINE THE COMPONENTS OF THE DIPOLE AXIS IN ECCLIPTIC
C     COORDINATES
      ZX=CASG*CBWT+SACG*CMLCWT-SALF*SMLSWT
      ZY=CASG*SBWT+SACG*CMLSWT+SALF*SMLCWT
      ZZ=CACG-SASG*CSMLT
C
C     CALCULATE THE TILT ANGLE
      PSI=ACOS(ZX)
      OSP=1./(SIN(PSI))
      TILT=CON*(PI2-PSI)
C
C     DETEMINE THE COMPONENTS OF THE GEOMAGNETIC X AXIS IN ECCLIPTIC
C     COORDINATES
      XX=CACG*CMLCWT-SASG*CBWT-CALF*SMLSWT
      XY=CACG*CMLSWT-SASG*SBWT+CALF*SMLCWT
      XZ=-CASG*CSMLT-SACG
C
C     OBTAIN THE ROTATION SINES AND COSINES
      SINPHE=(XY*ZZ-XZ*ZY)*OSP
      COSPHE=(XX*(ZZ*ZZ+ZY*ZY)-ZX*(XY*ZY+XZ*ZZ))*OSP
      RETURN
      END
```

```
      SUBROUTINE BXYZMU(XX,BF,TILT)
C
C  VERSION 11/01/76
C
C     PURPOSE
C       TO CALCULATE THE CONTRIBUTION TO THE EARTHS MAGNETIC FIELD BY
C       SOURCES EXTERNAL TO THE EARTH.  NO INTERNAL FIELD IS INCLUDED
C       IN THIS ROUTINE.
C
C     METHOD
C        THE ROUTINE INCLUDES THE FIELD CONTRIBUTIONS FROM THE
C        MAGNETOPAUSE CURRENTS, AND CURRENTS DISTRIBUTED THROUGHOUT
C        THE MAGNETOSPHERE (THE TAIL AND RING CURRENTS).  IT IS VALID
C        FOR ALL TILTS OF THE EARTHS DIPOLE AXIS AND IS VALID DURING
C        QUIET MAGNETIC CONDITIONS.
C        A GENERALIZED ORTHONORMAL LEAST SQUARES PROGRAM WAS USED
C        TO FIT THE COEFFICIENTS OF A POWER SERIES (INCLUDING
C        EXPONENTIAL TERMS) THROUGH FOURTH ORDER IN SPACE AND
C        THIRD ORDER IN TILT.  THIS EXPANSION HAS BEEN OPTIMIZED
C        FOR THE NEAR EARTH REGION AND IS VALID TO 15 EARTH RADII.
C        OUTSIDE OF THIS REGION THE FIELD DIVERGES RAPIDLY AND A
C        TEMPLATE SETS THE FIELD TO ZERO.  IN ORDER TO IMPROVE
C        COMPUTATIONAL SPEED THE FIELD IS SET TO ZERO BELOW 2 EARTH
C        RADII.  (IN THIS REGION THE EARTHS INTERNAL FIELD DOMINATES
C        AND THE VARIATIONS EXCPRESSED BY THIS EXPANSION IS NOT
C        SUFFICIENTLY ACCURATE THE PREDICT VARIATIONS ON THE EARTHS
C        SURFACE)
C
C        THE POWER SERIES REPRESENTING THE MAGNETIC FIELD IS
C        BX=SUM OVER I,J,K OF  ( A(I,J,K)*X**(I-1)*Y**(2*J-2)*Z**(K-1)
C               + B(I,J,K)*X**(I-1)*Y**(2*J-2)*Z**(K-1)*EXP(-.06*R**2))
C            I GOES FROM 1 TO 5,J FROM 1 TO 3, K FROM 1 TO 5
C            THE SUM OF I + 2*J + K IS LESS THAN OR EQUAL TO 9
C        BY=SUM OVER I,J,K OF  ( C(I,J,K)*X**(I-1)*Y**(2*J-1)*Z**(K-1)
C               + D(I,J,K)*X**(I-1)*Y**(2*J-1)*Z**(K-1)*EXP(-.06*R**2))
C            I GOES FROM 1 TO 5, J FROM 1 TO 3, K FROM 1 TO 5
C            THE SUM OF I + 2*J+1 + K IS LESS THAN OR EQUAL TO 9
C        BZ=SUM OVER I,J,K OF  ( E(I,J,K)*X**(I-1)*Y**(2*J-2)*Z**(K-1)
C               + F(I,J,K)*X**(I-1)*Y**(2*J-2)*Z**(K-1)*EXP(-.06*R**2))
C            I GOES FROM 1 TO 5, J FROM 1 TO 3, K FROM 1 TO 5
C            THE SUM OF I + 2*J + K IS LESS THAN OR EQUAL TO 9
C        THE COEFFICIENTS A-F ARE DEPENDENT ONLY ON POSITION AND
C        ARE RECALCULATED EACH TIME THE TILT OF THE DIPOLE IS CHANGED.
C        THE COEEFICIENTS A-F ARE DETERMINED FROM THE TILT DEPENDENT
C        CONSTANTS AA-FF BY THE FOLLOWING EXPRESSIONS
C        A(I,J,K)=AA(I,J,K,1)*TILT**(K-1-(K-1)/2*2)
C                  +AA(I,J,K,2)*TILT**(K+1-(K-1)/2*2)
C        B(I,J,K)=BB......
C        C(I,J,K)=CC......
C        D(I,J,K)=DD......
C        E(I,J,K)=EE(I,J,K,1)*TILT**(K-(K)/2*2)
C                  +EE(I,J,K,2)*TILT**(K+2-(K)/2*2)
C        F(I,J,K)=FF......
C
C     INPUT -- CALLING SEQUENCE
C        XX      A REAL ARRAY GIVING THE POSITION WHERE THE MAGNETIC
C               FIELD IS TO BE EVALUATED.  XX(1), XX(2), XX(3) ARE
C               RESPECTIVELY THE X, Y, AND Z SOLAR MAGNETIC
C               COORDINATES IN EARTH RADII. Z IS ALONG THE EARTHS
C               NORTH DIPOLE AXIS, X IS PERPENDICULAR TO Z AND IN THE
C               PLANE CONTAINING THE Z AXIS AND THE SUN-EARTH LINE,
C               Y IS PERPENDICULAR TO X AND Z FORMING A RIGHT HANDED
C               COORDINATE SYSTEM. X IS POSITIVE IN THE SOLAR DIRECTION.
C        TILT    IS THE TILT OF THE DIPOLE AXIS IN DEGREES.  IT IS
```

```
C               THE COMPLEMENT OF THE ANGLE BETWEEEN THE NORTH DIPOLE
C               AXIS AND THE SOLAR DIRECTION (PSI). TILT=90-PSI.
C
C     OUTPUT -- CALLING SEQUENCE
C        BF     A REAL ARRAY CONTAING THE X, Y, AND Z COMPONENTS OF
C               THE MAGNETOSPHERIC MAGNETIC FIELD IN GAMMA. BF(1),
C               BF(2) AND BF(3) ARE THE BX, BY, BZ COMPONENTS.
C
C     CONSTANTS
C        AA,BB,CC,DD,EE,FF ARE REAL ARRAYS CONTAING THE TILT DEPENDED
C               COEFFICIENTS.
C               AA(I,J,K,L) ARE STORED SUCH THAT L VARIES MOST RAPIDLY
C               FOLLOWED IN ORDER BY K, J AND I.  I VARIES THE SLOWEST.
C               THE ARRAY IS CLOSE PACKED AND ALL COEFFICIENTS THAT
C               ARE ZERO BECAUSE OF SYMMETRY OR BECAUSE THE CROSS TERM
C               POWER IS TOO LARGE ARE DELETED.
C
C     VARIABLES
C        A,B,C,D,E,F  THE TILT INDEPENDENT COEFFICIENTS.  THEIR USE
C               IS DESCRIBED UNDER METHOD.
C        ITA    A REAL ARRAY WHICH CONTAINS THE SYMMETRY OF THE TILT
C               DEPENDENCE FOR EACH OF THE A AND B COEFFICIENTS
C               ITA(1) HAS THE SYMMETRY INFORMATION FOR A(1,1,1,1)
C                      AND A(1,1,1,2)
C               ITA(2) HAS THE SYMMETRY INFORMATION FOR A(1,1,2,1)
C                      AND A(1,1,2,2)  ETC.
C               IF ITA = 1 TILT SYMMETRY IS EVEN WITH RESPECT TO Z SYM.
C               IF ITA = 2 TILT SYMMETRY IS ODD WITH RESPECT TO Z SYM.
C        ITB    SYMMETRY POINTER FOR C AND D ARRAYS
C        ITC    SYMMETRY POINTER FOR E AND F ARRAYS
C        X      X COMPONENT OF POSITION
C        Y      Y COMPONENT OF POSITION
C        Z      Z COMPONENT OF POSITION
C        Y2     Y**2
C        Z2     Z**2
C        R2     X**2 + Y**2 + Z**2
C        R      SQRT(R2)
C        I      DO LOOP VARIABLE.  IN THE FIELD EXPANSION LOOP IT
C               REPRESENTS THE POWER TO WHICH X IS CURRENTLY RAISED
C               I.E. X**(I-1)
C        J      DO LOOP VARIABLE.  ALSO Y**(2*J-2)
C        K      DO LOOP VARIABLE.  ALSO Z**(K-1)
C        XB     X**(I-1)
C        YEXB   X**(I-1)*Y**(2*J-2)
C        ZEYEXB X**(I-1)*Y**(2*J-2)*Z**(K-1)
C        IJK    I + 2*J + K
C        II     POINTS TO THE ARRAY LOCATION WHERE THE CURRENT POWER
C               SERIES COEFFICIENT FOR BX IS LOCATED
C        JJ     BY COEFFICIENT LOCATION POINTER
C        KK     BZ COEFFICIENT LOCATION POINTER
C        BX,BY,BZ ARE USED TO CONSTRUCT THE MAGNETIC FIELD WITHIN THE
C               POWER SERIES LOOP.
C        EXPR   EXP(-.06*R2)
C        TILTL  HOLDS THE LAST VALUE OF THE TILT FOR WHICH THE TILT
C               INDEPENDENT COEFFICIENTS A-F WERE CALCULATED
C        TT     A REAL ARRAY HOLDING THE POWERS OF THE TILT.
C               TT(1)=TILT**0, TT(2)=TILT**1, ETC.
C        CON    =0 FOR R LESS THAN 2
C               =1 FOR R GREATER THAN 2.5
C               GOES FROM 0 TO 1 IN THE REGION 2 TO 2.5
C
C     FOR MORE INFORMATION CALL OR WRITE K. A. PFITZER OR W. P. OLSON
C     AT MCDONNEL DOUGLAS ASTRONAUTICS CO. 5301 BOLSA AVE, HUNTINGTON
C     CALIF., PHONE (714) 896-3231.
```

```
C
      DIMENSION BF(3),XX(3),AA(64),BB(64),CC(44),DD(44),EE(64),FF(64),
     *A(32),B(32),C(22),D(22),E(32),F(32),TT(4),ITA(32),ITB(22),ITC(32)
      DATA (ITA(I),I=1,32) /2,1,2,1,2,2,1,2,1,2,1,2,1,2,1,2,2,1,2,2,2,1,
     *2,1,2,1,2,1,2,2,2,1/
      DATA (ITB(I),I=1,22) /2,1,2,1,2,2,1,2,2,2,1,2,1,2,1,2,1,2,2,2,1,2/
      DATA (ITC(I),I=1,32) /1,2,1,2,1,1,2,1,2,1,2,1,2,1,2,1,1,2,1,1,1,2,
     *1,2,1,2,1,2,1,1,1,2/
      DATA (AA(I),I=1,64)/-2.26836E-02,-1.01863E-04, 3.42986E+00,        TOTAL
     *-3.12195E-04, 9.50629E-03,-2.91512E-06,-1.57317E-03, 8.62856E-08, TOTAL
     *-4.26478E-05, 1.62924E-08,-1.27549E-04, 1.90732E-06,-1.65983E-02, TOTAL
     * 8.46680E-09,-5.55850E-05, 1.37404E-08, 9.91815E-05, 1.59296E-08, TOTAL
     * 4.52864E-07,-7.17669E-09, 4.98627E-05, 3.33662E-10,-5.97620E-02, TOTAL
     * 1.60669E-05,-2.29457E-01,-1.43777E-04, 1.09403E-03,-9.15606E-07, TOTAL
     * 1.60658E-03,-4.01198E-07,-3.15064E-06, 2.03125E-09, 4.92887E-04, TOTAL
     *-1.80676E-07,-1.12022E-03, 5.98568E-07,-5.90009E-06, 5.16504E-09, TOTAL
     *-1.48737E-06, 4.83477E-10,-7.44379E-04, 3.82472E-06, 7.41737E-04, TOTAL
     *-1.31468E-05,-1.24729E-04, 1.92930E-08,-1.91764E-04,-5.30371E-08, TOTAL
     * 1.38186E-05,-2.81594E-08, 7.46386E-06, 2.64404E-08, 2.45049E-04, TOTAL
     *-1.81802E-07,-1.00278E-03, 1.98742E-06,-1.16425E-05, 1.17556E-08, TOTAL
     *-2.46079E-06,-3.45831E-10, 1.02440E-05,-1.90716E-08,-4.00855E-05, TOTAL
     * 1.25818E-07/
      DATA (BB(I),I=1,64)/ 9.47753E-02, 1.45981E-04,-1.82933E+00,        TOTAL
     * 5.54882E-04, 5.03665E-03,-2.07698E-06, 1.10959E-01,-3.45837E-05, TOTAL
     *-4.40075E-05, 5.06464E-07,-1.20112E-03, 3.64911E-06, 1.49849E-01, TOTAL
     *-7.44929E-05, 2.46382E-04, 9.65870E-07,-9.54881E-04, 2.43647E-07, TOTAL
     * 3.06520E-04, 3.07836E-07, 6.48301E-03, 1.26251E-06,-7.09548E-03, TOTAL
     *-1.55596E-05, 3.06465E+00,-7.84893E-05, 4.95145E-03, 3.71921E-06, TOTAL
     *-1.52002E-01, 6.81988E-06,-8.55686E-05,-9.01230E-08,-3.71458E-04, TOTAL
     * 1.30476E-07,-1.82971E-01, 1.51390E-05,-1.45912E-04,-2.22778E-07, TOTAL
     * 6.49278E-05,-3.72758E-08,-1.59932E-03, 8.04921E-06, 5.38012E-01, TOTAL
     *-1.43182E-04, 1.50000E-04, 5.88020E-07,-1.59000E-02, 1.60744E-06, TOTAL
     * 3.17837E-04, 1.78959E-07,-8.93794E-03, 6.37549E-06, 1.27887E-03, TOTAL
     *-2.45878E-07,-1.93210E-01, 6.91233E-06,-2.80637E-04,-2.57073E-07, TOTAL
     * 5.78343E-05, 4.52128E-10, 1.89621E-04,-4.84911E-08,-1.50058E-02, TOTAL
     * 6.21772E-06/
      DATA (CC(I),I=1,44)/-1.88177E-02,-1.92493E-06,-2.89064E-01,        TOTAL
     *-8.49439E-05,-4.76380E-04,-4.52998E-08, 1.61086E-03, 3.18728E-07, TOTAL
     * 1.29159E-06, 5.52259E-10, 3.95543E-05, 5.61209E-08, 1.38287E-03, TOTAL
     * 5.74237E-07, 1.86489E-06, 7.10175E-10, 1.45243E-07,-2.97591E-10, TOTAL
     *-2.43029E-03,-6.70000E-07,-2.30624E-02,-6.22193E-06,-2.40815E-05, TOTAL
     * 2.01689E-08, 1.76721E-04, 3.78689E-08, 9.88496E-06, 7.33820E-09, TOTAL
     * 7.32126E-05, 8.43986E-08, 8.82449E-06,-6.11708E-08, 1.78881E-04, TOTAL
     * 8.62589E-07, 3.43724E-06, 2.53783E-09,-2.04239E-07, 8.16641E-10, TOTAL
     * 1.68075E-05, 7.62815E-09, 2.26026E-04, 3.66341E-08, 3.44637E-07, TOTAL
     * 2.25531E-10/
      DATA (DD(I),I=1,44)/ 2.50143E-03, 1.01200E-06, 3.23821E+00,        TOTAL
     * 1.08589E-05,-3.39199E-05,-5.27052E-07,-9.46161E-02,-1.95413E-09, TOTAL
     *-4.23614E-06, 1.43153E-08,-2.62948E-04, 1.05138E-07,-2.15784E-01, TOTAL
     *-2.20717E-07,-2.65687E-05, 1.26370E-08, 5.88917E-07,-1.13658E-08, TOTAL
     * 1.64385E-03, 1.44263E-06,-1.66045E-01,-1.46096E-05, 1.22811E-04, TOTAL
     * 3.43922E-08, 9.66760E-05,-6.32150E-07,-4.97400E-05,-2.78578E-08, TOTAL
     * 1.77366E-02, 2.05401E-07,-1.91756E-03,-9.49392E-07,-1.99488E-01, TOTAL
     *-2.07170E-06,-5.40443E-05, 1.59289E-08, 7.30914E-05, 3.38786E-08, TOTAL
     *-1.59537E-04,-1.65504E-07, 1.90940E-02, 2.03238E-06, 1.01148E-04, TOTAL
     * 5.20815E-08/
      DATA (EE(I),I=1,64)/-2.77924E+01,-1.01457E-03, 9.21436E-02,        TOTAL
     *-8.52177E-06, 5.19106E-01, 8.28881E-05,-5.59651E-04, 1.16736E-07, TOTAL
     *-2.11206E-03,-5.35469E-07, 4.41990E-01,-1.33679E-05,-7.18642E-04, TOTAL
     * 6.17358E-08,-3.51990E-03,-5.29070E-07, 1.88443E-06,-6.60696E-10, TOTAL
     *-1.34708E-03, 1.02160E-07, 1.58219E-06, 2.05040E-10, 1.18039E+00, TOTAL
     * 1.58903E-04, 1.86944E-02,-4.46477E-06, 5.49869E-02, 4.94690E-06, TOTAL
     *-1.18335E-04, 6.95684E-09,-2.73839E-04,-9.17883E-08, 2.79126E-02, TOTAL
     *-1.02567E-06, 1.25427E-04, 3.07143E-08,-5.31826E-04,-2.98476E-08, TOTAL
```

```
     *-4.89899E-05, 4.91480E-08, 3.85563E-01, 4.16966E-05, 6.74744E-04, TOTAL
     *-2.08736E-07,-3.42654E-03,-3.13957E-06,-6.31361E-06,-2.92981E-09, TOTAL
     *-2.63883E-03,-1.32235E-07,-6.19406E-06, 3.54334E-09, 6.65986E-03, TOTAL
     *-5.81949E-06,-1.88809E-04, 3.62055E-08,-4.64380E-04,-2.21159E-07, TOTAL
     *-1.77496E-04, 4.95560E-08,-3.18867E-04,-3.17697E-07,-1.05815E-05, TOTAL
     * 2.22220E-09/
      DATA (FF(I),I=1,64)/-5.07092E+00, 4.71960E-03,-3.79851E-03,       TOTAL
     *-3.67309E-06,-6.02439E-01, 1.08490E-04, 5.09287E-04, 5.62210E-07, TOTAL
     * 7.05718E-02, 5.13160E-06,-2.85571E+00,-4.31728E-05, 1.03185E-03, TOTAL
     * 1.05332E-07, 1.04106E-02, 1.60749E-05, 4.18031E-05, 3.32759E-08, TOTAL
     * 1.20113E-01, 1.40486E-05,-3.37993E-05, 5.48340E-09, 9.10815E-02, TOTAL
     *-4.00608E-04, 3.75393E-03,-4.69939E-07,-2.48561E-02, 1.31836E-04, TOTAL
     *-2.67755E-04,-7.60285E-08, 3.04443E-03,-3.28956E-06, 5.82367E-01, TOTAL
     * 5.39496E-06,-6.15261E-04, 4.05316E-08, 1.13546E-02,-4.26493E-06, TOTAL
     *-2.72007E-02, 5.72523E-08,-2.98576E+00, 3.07325E-05, 1.51645E-03, TOTAL
     * 1.25098E-06, 4.07213E-02, 1.05964E-05, 1.04232E-04, 1.77381E-08, TOTAL
     * 1.92781E-01, 2.15734E-05,-1.65741E-05,-1.88683E-09, 2.44803E-01, TOTAL
     * 1.51316E-05,-3.01157E-04, 8.47006E-08, 1.86971E-02,-6.94074E-06, TOTAL
     * 9.13198E-03,-2.38052E-07, 1.28552E-01, 6.92595E-06,-8.36792E-05, TOTAL
     *-6.10021E-08/
      DATA TILTL/99./
C
C     SET UP SOME OF THE INITIAL POSITION VARIABLES
      X=XX(1)
      Y=XX(2)
      Z=XX(3)
      Y2=Y**2
      Z2=Z**2
      R2=X**2+Y2+Z2
C
C     SET MAGNETIC FIELD VARIABLES TO ZERO
      BX=0.
      BY=0.
      BZ=0.
C
C     CHECK TO SEE IF POSITION IS WITHIN REGION OF VALIDITY
      CON=1.
C     IF DISTANCE TOO LARGE TAKE ERROR EXIT
      IF(R2.GT.225.) GO TO 50
C     IF DISTANCE TOO SMALL SET FIELD TO ZERO AND EXIT
      IF(R2.LT.4.)GO TO 40
      IF(R2.LT.6.25) CON=CON*(R2-4.0)/2.25
C
C     IF TILT HAS NOT CHANGED, GO DRECTLY TO FIELD CALCULATION
      IF(TILTL.EQ.TILT)GO TO 6
C     SET UP POWERS OF TILT
      TILTL=TILT
      TT(1)=1
      TT(2)=TILTL
      TT(3)=TILTL**2
      TT(4)=TILT*TT(3)
C
C     SET UP THE X AND Z COMPONENT TILT INDEPENDENT COEFFICIENTS
      DO  1 I=1,32
      J=(I-1)*2+1
      K=ITA(I)
      A(I)=AA(J)*TT(K)+AA(J+1)*TT(K+2)
      B(I)=BB(J)*TT(K)+BB(J+1)*TT(K+2)
      K=ITC(I)
      E(I)=EE(J)*TT(K)+EE(J+1)*TT(K+2)
      F(I)=FF(J)*TT(K)+FF(J+1)*TT(K+2)
 1    CONTINUE
C
C     SET UP THE Y COMPONENT TILT INDEPENDENT COEFFICIENTS
```

```
      DO  2 I=1,22
      J=(I-1)*2+1
      K=ITB(I)
      C(I)=CC(J)*TT(K)+CC(J+1)*TT(K+2)
      D(I)=DD(J)*TT(K)+DD(J+1)*TT(K+2)
 2    CONTINUE
 6    EXPR=EXP(-0.06*R2)
C
C     INITIALIZE THE POINTERS
      II=1
      JJ=1
      KK=1
      XB=1.
C
C     BEGIN SUM OVER X
      DO 30 I=1,5
      YEXB=XB
C
C     BEGIN SUM OVER Y
      DO 20 J=1,3
      IF(I+2*J.GT. 8) GO TO 25
      ZEYEXB=YEXB
      IJK=I+2*J+1
      K=1
C
C     Z LOOP STARTS HERE
 10   BZ=BZ+(E(KK)+F(KK)*EXPR)*ZEYEXB
      KK=KK+1
      BX=BX+(A(II)+B(II)*EXPR )*ZEYEXB
      II=II+1
      IF(IJK .GT. 8) GO TO 15
      BY=BY+(C(JJ)+D(JJ)*EXPR )*ZEYEXB*Y
      JJ=JJ+1
      ZEYEXB=ZEYEXB*Z
      IJK=IJK+1
      K=K+1
      IF(IJK.LE.9.AND.K.LE.5) GO TO 10
 15   YEXB=YEXB*Y2
 20   CONTINUE
 25   XB=XB*X
 30   CONTINUE
C
C     SET UP THE OUTPUT ARRAY, MULTIPLY BY CON. CON IS NORMALY ONE
C     BUT INSIDE OF R=2.5 IT GOES TO ZERO. INSIDE R=2 IT IS ZERO.
 40   BF(1)=BX*CON
      BF(2)=BY*CON
      BF(3)=BZ*CON
      RETURN
C
C     ERROR EXIT IF OUTSIDE OF R = 15.
 50   WRITE(6,60) XX
 60   FORMAT(4H X= ,E10.3,4H Y= ,E10.3,4H Z= ,E10.3,76H  IS OUTSIDE THE
     *VALID REGION--POWER SERIES DIVERGES BFIELD IS SET TO ZERO  )
      GO TO 40
      END
```

Appendix C

Subroutine INVARM

This appendix presents a listing of subroutine INVARM, the central routine for developing the new radiation belt models. The operation and functions performed by this routine are spelled out in section 2.3. The routine is preceded by a test routine that presents a sample of its capabilities and also provides a means for assessing its operation.

The test routine varies the distance, latitude, longitude, and universal time and asks the INVARM program to calculate the invariant for 18 different pitch angles at each location.

All input and out variables are passed via the arguments of the subroutine call. It is possible to limit the number of terms used by the internal field routine by setting NMAX to a value between 2 and 10. This must be done via labeled COMMON /GCOM/. The internal field routine checks the value of this variable. If it is set to zero as it normally is when labeled COMMON is preset by the loader or set to any value other than 2 through 10, then the internal field routine uses the maximum coefficients defined for the IGRF field. For the coefficients supplied with this program an NMAX of 11 is used. Labeled COMMON /MOMENT/ contains the dipole moment of the main field as calculated from the coefficient set that is in use. This may be used by any routine that requires it.

The calling argument for the routine are

Input variables

XLAT	The geographic latitude measured in degrees from the geographic equator, plus is north and minus is south.
XLONG	The geographic east longitude measured from Greenwich England (0-360)
R	The radial distance from the center of the earth in unit of earth radii. One radius is 6371.2 km.
YR	The year of the calculation. This variable is used by the internal magnetic field routine to calculate the epoch of the magnetic field. Whenever this value changes by 0.1 years the coefficient set for the internal field is updated. It is suggested that this value not be changed unless the drift of the internal field during the calculations is important to the analysis. The coefficients are valid from 1945 to the present. Dates earlier than 1945, will cause the routine to use the 1945 coefficient set. Use caution in predicting the field far into the future. Historically, predicting the field into the future has not been very successful.
DAY	The day of the year. January 1 is DAY = 1. This is a floating point variable but it should be limited to whole numbers.

TIME	The universal time in hours. This is a floating point number should represent the correct universal time to the required precision.
JSWITCH	An integer variable that controls whether the external field is included in the calculation. JSWITCH negative uses the internal field only, JSWITCH = 0 or positive uses the internal plus external magnetic field.
NUMANG	An integer variable that specifies the number of pitch angles for which the invariant must be calculated.
PANGLE	A single variable or an array that contains the pitch angles for which the routine is to calculate the invariant. The dimension of PANGLE must be equal to or greater than NUMANG. If NUMANG is 1, then PANGLE may be a simple undimensioned variable.

Output parameters

EL	A simple variable or an array dimensioned to at least NUMANG that will contain the L value for the specified location and pitch angle. If no L could be calculated EL is set to -1.0, if the mirror point is below 1.03 R_e or El is set to 100 if the field line is open or the maximum number of steps is reached by the routine.
BLOCAL	The value of the magnetic field at the observation point.
BMIN	The minimum value of the magnetic field along the particle line of force.
XMLONG	The magnetic longitude of the minimum B point on the magnetic line of force. 0 degrees is local midnight.
XMLAT	The magnetic latitude of the observation point.
BMAXAN	The Mirror point magnetic field for each of the pitch angles. BMAXAN can either be a simple variable or and array dimensioned at least to order NUMANG.
XJ	The value of the second invariant for each pitch angle. XJ can either be a simple variable or and array dimensioned at least to order NUMANG.
DENSTY	The column density of the atmosphere along the particle's bounce path in gm/cm^2. DENSTY can either be a simple variable or and array dimensioned at least to order NUMANG.

```
      COMMON/GCOM/ ST,CT,SP,CP,AOR,BT,BP,BR,NMAX,YEARI
      DIMENSION BMAXAN(20),XJ(20),ANGLE(20),EL(20),DENSTY(20),ALPHEQ(20)
      common/temp/nlast,n2last
C
C     THIS PROGRAM PROVIDES A TEST RUN OF THE L VALUE SUBROUTINES
C
      CHARACTER*6 IAR(3)
      DATA IAR/6H INT   ,6H IN+EX,6H L AVE/
      DO 500 I=1,18
      ANGLE(I)=90-(I-1)*5

500   CONTINUE
      NUMANG=18
      IDSWIT=1
      LN=100
      YEAR=1990
      DA=1
      DO 5 IU=1,1,12
      UT=IU-1
      LN=100
      DO 4 IL=1,31,30
      FLAT=IL-1
      DO 3 ILG=1,181,180
      XLONG=ILG
      DO 2 IR=2,8,2
      R=IR-.5
      DO 1 IC=1,2
      call gettim(ihr,imin,isec,i100)
      btime=float(imin*60+isec)+float(i100)/100.
      CALL INVARM(FLAT,XLONG,R,YEAR,DA,UT,IC-2,ANGLE,NUMANG,
     *EL,BLOC,BM,XMLONG,XMLAT,BMAXAN,XJ,DENSTY)
      call gettim(jhr,jmin,jsec,j100)
      time=float(jmin*60+jsec)+float(j100)/100.-btime
      IF(IC.EQ.1)WRITE(*,103)
103   FORMAT(1H1)
      WRITE(*,101)FLAT,XLONG,R,YEAR,DA,UT,IAR(IC),BLOC,BM,XMLAT,XMLONG
101   FORMAT(//,' Lat = ',f6.1,'  Long = ',f7.1,'  R = ',f4.1,
     */,' Year = ',f7.1,'  Day = ',f5.0,'  UT = ',f6.2,'  Field =',A6,
     */,' Blocal = ',F8.5,'  Bmin = ',F8.5,'  Mlat = ',f8.3,
     *'  Mlong = ',f9.3)
      WRITE(*,102)
102   FORMAT(/,' P. Angle   B mir     2nd Inv.    L        Density',
     *'      Eq. Pitch Angle',/)
      LN=0
      DO 50 I=1,NUMANG
50    ALPHEQ(I)=ASIN(SQRT(BM/BLOC)*SIN(ANGLE(I)*.01745329))/.01745329
      write(*,100)(angle(i),bmaxan(i),xj(i),el(i),densty(i),
     *alpheq(i),i=1,numang)
10    write(*,*)nlast,n2last,time
 100  format(0Pf6.1,2f11.5,f8.3,1PE15.5,0PF13.2)
 1    CONTINUE
 2    CONTINUE
 3    CONTINUE
 4    CONTINUE
 5    CONTINUE
      END
```

```
      SUBROUTINE INVARM(XLAT,XLONG,R,YR,DAY,TIME,JSWTCH,PANGLE,
     *NUMANG,EL,BLOCAL,BMIN,XMLONG,XMLAT,BMAXAN,XJ,DENSTY)
C
C     PURPOSE
C        CALCULATE THE VARIOUS MAGNETIC COORDINATES OF THE PARTICLES'
C        DRIFT SHELL.  CALCULATE THE 1ST AND 2ND ADIABATIC INVARIANTS AND
C        THE L PARAMETER FOR A NUMBER OF PITCH ANGLES AT THE SPECIFIED
C        LOCATION.  ALSO DETERMINE THE LOCAL MAGNETIC FIELD, THE MAGNETIC
C        LATITUDE, THE MINIMUM MAGNETIC FIELD ON THE FIELD LINE AND THE
C        MAGNETIC LONGITUDE AT THE FIELD MINIMUM.
C
C     INPUT -- ARGUMENT LIST
C        XLAT   GEOCENTRIC GEOGRAPHIC LATITUDE IN DEGREES (+ IS NORTH)
C        XLONG  GEOCENTRIC GEOGRAPHIC LONGITUDE EAST OF GREENWHICH IN
C               DEGREES
C        R      GEOCENTRIC DISTANCE FROM THE EARTHS CENTER IN UNITS
C               EARTH RADII, RE.  RE=6371.2 KM
C        YR     THE YEAR - USED BY THE INTERNAL MAGNETIC FIELD ROUTINE
C               TO TAKE INTO ACCOUNT THE SECULAR VARIATIONS
C               (E.G. JULY 15,1964 = 1964.54)
C               NOTE*** YR SHOULD BE CHANGED ONLY EVERY FEW DAYS OR
C               MONTHS. NEW FIELD COEFFICIENTS MUST BE COMPUTED FOR
C               EVERY CHANGE IN YR, THIS COULD CAUSE A LARGE INCREASE IN
C               COMPUTER TIME. THE EARTHS FIELD CHANGES ONLY ABOUT
C               .001 GAUSS/YEAR AT THE EARTHS SURFACE.
C               IF YR IS CHANGED BY MORE THAN .1 YEAR NEW FIELD COEFFS.
C               ARE COMPUTED
C        DAY    THE DAY OF YEAR (1.-366.).  THE DAY IS USED BY THE
C               MAGNETIC FIELD ROUTINE TO CALCULATE THE TILT OF THE
C               DIPOLE AXIS FOR THE EXTERNAL FIELD ROUTINE
C               DAY MUST BE A WHOLE NUMBER AND DAY 1 IS JANUARY 1
C        TIME   UNIVERSAL TIME IN HOURS (0.000-24.0000)
C        JSWTCH A FLOW CONTROL VARIABLE
C               JSWTCH =-1 OR NAEGATIVE, COMPUTE L USING INTERNAL
C                          FIELD ONLY
C                      = 0 OR POSITIVE, COMPUTE L USING INTERNAL PLUS
C                          EXTERNAL FIELD
C        PANGLE A SINGLE PITCH ANGLE OR AN ARRAY OF PITCH ANGLES FOR THE
C               INVARIANTS WILL BE CALCULATED.  THE PITCH ANGLE MUST BE
C               .LE. 90 AND GT.0 AND THE ARRAY MUST BE ORDERED IN
C               DESCENDING ORDER (90, 80, 70,...)
C        NUMANG THE NUMBER OF ELEMENT IN THE PANGLE ARRAY
C     OUTPUT PARAMETERS
C        EL     A SINGLE VARIABLE OR AN ARRAY OF DIMENSION NUMANG.  THIS
C               RETURNS THE L VALUE CALCLUATED FROM THE INVARIANT.
C               VARIABLE JSWTCH
C               *****NOTE****
C               SINCE THIS ROUTINE USES AN ACTUAL MAGNETOSPHERIC
C               MAGNETIC FIELD, THE FIELD LINES ARE NOT ALL CLOSED.
C               THUS L IS DEFINED ONLY IN THE INNER MAGNETOSPHERE (IN
C               THE REGION OF CLOSED DRIFT SHELLS).  AN ATTEMPT
C               TO CALCULATE L OUTSIDE OF THIS REGION WILL SET EL TO
C               100 (EL=100), SET BMIN TO THE LOCAL FIELD VALUE
C               AND SET XMLONG TO ZERO UNLESS MINIMUM B WAS PASSED PRIOR
C               TO THE DETECTION OF THE ERROR
C               IF THE MIRROR POINT FOR A GIVEN PITCH ANGLE IS BELOW 200KM
C               THEN EL IS SET TO -1
C        BLOCAL THE VALUE OF THE MAGNETIC FIELD AT THE INPUT POSITION
C               (IN GAUSS)
C        BMIN   THE MINIMUM VALUE OF B ALONG THE FIELD LINE IN GAUSS
C        XMLONG THE MAGNETIC LONGITUDE OF THE MAGNETIC FIELD MINIMUM
C               MEASURED EAST OF THE PRIME MAGNETIC MERIDIAN
C               (IN DEGREES).
C               A PRESET CONSTANT IN SUBROUTINE MGLONG ALLOWS THE USER
```

```
C               TO SELECT EITHER A CENTERED DIPOLE MAGNETIC COORDINATE
C               SYSTEM WITH ZERO AT 69 DEG W. GEOGRAPHIC, OR AN OFFSET
C               DIPOLE COORDINATE SYSTEM WITH ZERO THROUGH GREENWHICH.
C        XMLAT  THE MAGNETIC LATITUDE IN DEGREES OF THE CURRENT POSITION
C        BMAXAN A SINGLE VARIABLE OR AN ARRAY OF AT LEAST DIMENSION NUMANG
C               THAT WILL HOLD THE MIRROR POINT MAGNETIC FIELD FOR THE
C               VARIOUS PITCH ANGLE.
C        XJ     A SINGLE VARIABLE OR AN ARRAY OF AT LEAST DIMENSION NUMANG
C               THAT WILL HOLD THE VALUES OF THE SECOND INTEGRAL INVARIANT
C               FOR EACH PITCH ANGLE
C
C     CONSTANTS
C        ERR    = 0.0005  SCALES THE ERROR LIMITS FOR THE INTEGRATION
C               THE ERROR IN L IS APPROXIMATELY L*ERR
C
C     SUBROUTINES REQUIRED
C        SUBROUTINE STEPSZ
C        SUBROUTINE BMNEXT
C        SUBROUTINE HILTEL
C        SUBROUTINE INTERP
C        SUBROUTINE INTGRT
C        SUBROUTINE INVR
C        SUBROUTINE MGLONG
C
C     VARIABLES (PARTIAL LIST TO HELP UNDERSTAND THE CODE)
C        BINTL  A REAL ARRAY THAT SAVES THE VALUE OF THE MAGNETIC FIELD
C               AT THE INPUT POSITION
C        B      A REAL ARRAY THAT HOLDS THE INSTANTANEOUS MAGNETIC FIELD
C               VECTOR AT EACH INTEGRATION STEP
C        BB     A 2 DIMENSIONED REAL ARRAY THAT HOLDS ALL OF THE
C               MAGNETIC FIELD MAGNITUDES CALCULATED AT ALL OF THE
C               INTEGRATION STEPS
C        BL     A REAL ARRAY THAT SAVES THE MAGNETIC FIELD VECTOR
C               FROM THE PREVIOUS INTEGRATION STEP
C        BMAX   THE MAGNETIC FIELD AT THE PARTICLE MIRROR POINT
C        COSINE THE COSINE OF THE GEOGRAPHIC LATITUDE
C        DAYYR  THE DAY OF THE YEAR
C        DDS    THE ESTIMATED STEP SIZE NECESSARY TO COMPLETE THE
C               INTEGRATION.  IF NO ESTIMATE IS YET POSSIBLE IT IS
C               SET TO 100.
C        DEL    SCALES THE INTEGRATION STEP SIZE.  IT IS PROPORTIONAL
C               TO THE FOURTH ROOT OF THE ERROR LIMITS.  IF IT IS
C               POSITIVE INTEGRATION WILL BE PARALLEL TO THE FIELD, IF
C               NEGATIVE IT IS ANTIPARALLEL
C        DS     THE CURRENT VALUE OF THE INTEGRATION STEP SIZE IN
C               EARTH RADII. POSITIVE IS FOR PARALLEL TO FIELD,
C               NEGATIVE FOR ANTIPARALLEL
C        IERFLG AN ERROR FLAG SET BY SUBROUTINE INTGRT.  IF NON-ZERO
C               THE INTEGRATION HAS GONE BEYOND THE SET LIMITS AND
C               MUST BE TERMINATED
C        JSW    A FLOW CONTROL PARAMETER USED BY THE MAGNETIC FIELD
C               SUBROUTINE
C        KODE   SET EQUAL TO ONE TO INDICATE TO SUBROUTINE BMNEXT
C               THAT CARESIAN COORDINATES ARE TO BE USED
C        KS     A VARIABLE TRANSMITTED TO SUBROUTINE INTERP.  IT IS
C               USED TO DETERMINE WHICH SOLUTION APPLIES TO THE
C               PARTICULAR INTERPOLATION
C        MINFLG INITIALLY SET TO ZERO.  IT IS SET TO ONE WHEN THE FIELD
C               MINIMUM HAS BEEN PASSED
C        N      THE CURRENT INTEGRATION STEP NUMBER
C        PICON  PI / 180.
C        Q, QL  REAL ARRAYS CONTAINING THE CURRENT AND PREVIOUS ERROR
C               ESTIMATES.  USED BY GILLS METHOD INTEGRATION ROUTINE TO
C               CONTROL ROUND OFF ERROR
```

```
C        RMIN   THE VECTOR POSITION TO THE MAGNETIC MINNIMUM
C        RMAG   THE MAGNITUDE OF THE DISTANCE TO BMIN
C        SER    ERROR CONTROL VARIABLE.  THE INTEGRATION STOPS IF
C               THE CURRENT POSITION POINT IS WITHIN DISTANCE SER OF
C               BMAX
C        SF     OUTPUT OF THE INTERPOLATION SUBROUTINE INTERP.  IT
C               INDICATES THE SCALAR DISTANCE ALONG THE FIELD WHERE
C               B IS EQUAL TO BMAX
C        SXJ    A REAL ARRAY WHICH SAVES THE INTEGRATION STEP VALUES
C               OF THE SECOND ADIABATIC INVARIANT
C        UT     UNIVERSAL TIME
C        XDS    TEMORARY VALUE USED FOR OBTAINING DISTANCE TO COMPLETION
C               OF INTEGRATION
C        XJ     FINAL VALUE OF THE SECOND ADIABATIC INVARIANT
C        XL     A REAL ARRAY HOLDING THE PREVIOUS VALUE OF THE POSITION
C               VECTOR
C        XSV    A 3 DIMENSIONED REAL ARRAY HOLDING ALL OF THE POSITION
C               VECTORS ALONG THE INTEGRATION PATH
C        XXJ    INTERPOLATED VALUE OF THE SECOND ADIABATIC INVARIANT
C        YEAR   THE YEAR
C        ZP     THE Z COMPONENT OF THE POSITION VECTOR IN CENTERED
C               DIPOLE COORDINATES
C
C     VERSION 6/91
C     FOR MORE INFORMATION CALL OR WRITE K. A. PFITZER AT MCDONNELL
C     DOUGLAS ASTRONAUTICS CO. 5301 BOLSA AVE, HUNTINGTON BEACH CALIF.
C     PHONE (714) 896-3231.
C
      COMMON/BXYZCM/YEAR,DAYYR,UT,KODE,JSW
      COMMON /INTPAR/DS,DEL,N,IERFLG,XL(3),XSV(100,3,4),
     *RSV(100),RMIN(3),RMAG,IDSW,
     *QL(3),Q(3),BL(3),SXJ(100),DDS
      common/temp/nlast,n2last
      DIMENSION BB(100,4),BB2(100,4),B(3),B2(3),X(3),X2(3),S(100),
     *S2(100),DEN(100),DEN2(100)
      DIMENSION EL(*),PANGLE(*),BMAXAN(*),XJ(*),BLL(3),BLL2(3)
      DIMENSION XX(3),BINTL(3),DENSTY(*)
      DATA PICON/.01745329252/
      DATA ERR/.0005/
      DATA CONI/.95/
C
C     OBTAIN THE CARTESIAN COMPONENTS OF THE POSITION VECTOR
C
C     CHECK THE PITCH ANGLES, THEY MUST BE BETWEEN 90 AND 0 AND THEY MUST
C     BE IN DESCENDING ORDER (IE. 90,85,80,....
C
      NMANG=NUMANG
      CALL CHECK(PANGLE,NMANG,IER)
      IF (IER.GT.0) THEN
      WRITE(*,*)'Pitch angle error, must be monotonic and >0 & <=90'
        DO 5 I=1,NMANG
          XJ(I)=-1
          BMAXAN(I)=-1
          EL(I)=-1
          DENSTY(I)=-1
5       CONTINUE
        RETURN
      ENDIF
C
      COSINE=COS(XLAT*PICON)
      XX(1)=R*COSINE*COS(XLONG*PICON)
      XX(2)=R*COSINE*SIN(XLONG*PICON)
      XX(3)=R*SIN(XLAT*PICON)
C
```

```
C     ROTATE TO DIPOLE COORDINATES (FIRST ROTATE ABOUT Z 291 DEGREES
C     THEN ABOUT THE NEW Y 11.7 DEGREES TO THE DIPOLE AXIS)
C
      ZP=(XX(1)*.3583679495-XX(2)*.9335804265)*.2027872954
     *+XX(3)*.9792228106
C
C     EVALUATE THE MAGNETIC LATITUDE
      XMLAT=90.-ACOS(ZP/R)/PICON
C
C     SET THE MAGNETIC LONGITUDE TO ZERO.  IF MINIMUM B IS REACHED
C     PRIOR TO AN ERROR BEING DETECTED XMLONG IS UPDATED TO REFLECT
C     MAGNETIC LONGITUDE AT MINIMUM B
      XMLONG=0.
C
C     SET UP THE COMMON BLOCK INPUT VARIABLES FOR THE MAGNETIC FIELD
C     SUBROUTINE
      YEAR=YR
      UT=TIME
      DAYYR=DAY
      JSW=JSWTCH
      KODE=1
      IBEFLG=0
C
C     EVALUATE THE MAGNETIC FIELD AT THE STARTING POINT
      CALL BMNEXT(XX,B,BB(2,1))
      BLOCAL = BB(2,1)
      BB2(2,1)=BB(2,1)
C
C     SAVE THE INITIAL POSITION AND MAGNETIC FIELD VECTORS
      DO 10 I=1,3
        BINTL(I)=B(I)
        B2(I)=B(I)
        XL(I)=XX(I)
        X2(I)=XX(I)
        XSV(2,I,1)=XX(I)
 10   CONTINUE
      RSV(2)=R
C
C     EXIT THE ROUTINE IF POSITION IS OVER THE POLAR CAP OR DISTANCE
C     IS TOO LARGE OR MAGNETIC FIELD IS TOO WEAK
      IF(ABS(XMLAT).GT.75..OR.R.GT.12..OR.BB(2,1).LT..00025) THEN
        NMANG=0
        BMAXAN(1)=100.
        GOTO 218
      ENDIF
C
C     SET UP THE INITIAL VALUES FOR THE VARIABLES
      NLAST=2
      N2LAST=2
      S(2)=0.
      S2(2)=0.
      DEN(2)=0
      DEN2(2)=0
      DDS=100.
      MINFLG=0
C
C     SET BMIN TO LOCAL FIELD VALUE.  IF MINIMUM B IS REACHED PRIOR
C     TO ERROR DETECTION BMIN IS UPDATED TO MINIMUM B.
      BMIN=BB(2,1)
C
C     SET UP THE ERROR LIMITS FOR THE INTEGRATION
      SER=SQRT(ERR)
C     STEP SIZE GOES AS ERROR TO THE .25 POWER
      DEL=-2.5*ERR**.25
```

```
      DS=SER
C
C     STEP ONCE IN THE INCREASING FIELD DIRECTION AND SET STEP
C     PARAMETERS TO INTEGRATE IN THE DECREASING FIELD DRECTION
      IFLAG=0
      IF(XMLAT.GT.0.)GO TO 30
 20   DEL=-DEL
      DS=-DS
 30   N=2
      DO 31 I=1,3
        Q(I)=0
        X(I)=XL(I)
 31   CONTINUE
      CALL INTGRT(X,B,BB,S,DEN)
      IFLAG=IFLAG+1
      IF((BB(3,1).LT.BB(2,1)).AND.(IFLAG.LE.1)) GOTO 20
      S(1)=S(3)
      DEN(1)=DEN(3)
      BB(1,1)=BB(3,1)
      DO 34 I=1,3
        XL(I)=X(I)
        BL(I)=B(I)
        Q(I)=0
        X(I)=XX(I)
        B(I)=BINTL(I)
 34   CONTINUE
      RSV(1)=SQRT(XL(1)**2+XL(2)**2+XL(3)**3)
      DELSV=DEL
C
C     BEGIN THE FIELD LINE INTEGRATION.  THE INTEGRATION USES A VARIABLE
C     STEPSIZE WHICH IS DEPENDENT ON THE CURVATURE OF THE FIELD LINE
C     AND ON THE DISTANCE EACH POINT IS FROM EARTH CENTER (A MEASURE
C     OF THE MAGNETIC FIELD STRENGTH).  THE INITIAL INTEGRATION IS A
C     LINE INTEGRAL OF THE MAGNETIC FIELD UNIT VECTOR.  THIS INTEGRATION
C     LOOP ALSO SAVES ALL OF THE VARIABLES WHICH ARE LATER NEEDED TO
C     EVALUATE THE SECOND INTEGRAL INVARIANT.
C
      DO 216 IA=1,NMANG
      DEL=DELSV
      DDS=100
      BMAX=BB(2,1)/SIN(PANGLE(IA)*PICON)**2
      N=NLAST
      IF (IA.NE.1)THEN
        DO 41 I=1,3
          BL(I)=BLL(I)
 41     CONTINUE
C       DS=S(N)-S(N-1)
        CALL INTERP(BB(N-2,1),S(N-2),BMAX,SF,3)
        IF(ABS(SF).GT.ABS(S(N))) THEN
          XDS=SF-S(N)
          IF(ABS(XDS).LE.SER)GOTO 100
          DDS=CONI*XDS
        ENDIF
      ENDIF
C
 40   CALL STEPSZ(X,B,BB)
 45   CALL INTGRT(X,B,BB,S,DEN)
C
C     IF FIELD IS STILL DECREASING RELOOP
      IF(BB(N,1).LT.BB(N-1,1)) GO TO 40
C
C     IF MINIMUM VALUES HAVE BEEN CALCULATED, JUMP OVER MINIMUM ROUTINES
C     WHEN THE CURRENT VALUE OF B EXCEEDS THE LAST, FIND THE
C     INTERPOLATED MINIMUM MAGNETIC FIELD VALUE AND USE THIS VALUE TO
```

```
C     UPDATE THE VALUE OF BMAX TO REFLECT THE AVERAGE DRIFT SHELL (IF
C     AVERAGE SHELLS ARE REQUIRED)
C     USE THE DISTANCE, SF, TO THE FIELD MINIMUM TO DETERMINE THE
C     MAGNETIC LONGITUDE OF THE FIELD MINIMUM
C
      IF(MINFLG.NE.0) GO TO 50
      CALL INTERP(BB(N-2,1),S(N-2),BMIN,SF,-1)
      CALL MGLONG(XSV(N-2,1,1),S(N-2),SF,XMLONG,RMIN(1),RMAG)
      MINFLG=1
C
C     CONTINUE STEPPING ALONG THE FIELD LINE AS LONG AS B IS LESS THAN
C     BMAX AND THE INTEGRATION IS MORE THAN A DISTANCE SER FROM BMAX
C     IF BMAX HAS BEEN EXCEEDED, EXIT TO INTERPOLATION SCHEME
C
 50   IF(BB(N,1).GE.BMAX) GO TO 70
C     IF WE ARE OUTSIDE OF VALID REGION EXIT
      IF(IERFLG.NE.0)THEN
        NMANG=IA-1
        IF(IERFLG.GT.0)THEN
          BMAXAN(IA)=100
        ELSE
          BMAXAN(IA)=-1
        ENDIF
        GOTO 218
      ENDIF

      CALL INTERP (BB(N-2,1),S(N-2),BMAX,SF,3)
      DDS=100.
C
C     IF S DOES NOT INCREASE MONOTONICALLY, IGNORE INTERPOLATION
C     AND RELOOP
      IF(ABS(SF).LE.ABS(S(N))) GO TO 40
      XDS=SF-S(N)
C
C     IF WITHIN SER OF BMAX STOP INTEGRATION GO GET VALUE OF INVARIANT
      IF(ABS(XDS).LT.SER) GO TO 100
      DDS=CONI*XDS
C     RELOOP
      GO TO 40
C
C
C     THE FUNCTION SQRT(1-B/BMAX) DOES NOT EXIST FOR B GREATER THAN BMAX
C     IF PREVIOUS STEP IS NOT WITHIN SER OF BMAX INTERPOLATE TO FIND
C     A STEP SIZE THAT WILL GET CLOSE TO BUT NOT EXCEED BMAX
C
 70   CALL INTERP(BB(N-2,1),S(N-2),BMAX,SF,3)
C
      IF(ABS(SF-S(N)).LT.SER) THEN
        CALL INTERP(BB(N-2,1),RSV(N-2),BMAX,RS,3)
        IF (RS.LT.1.03)THEN
          NMANG=IA-1
          BMAXAN(IA)=-1
          GOTO 218
        ENDIF
      ENDIF

C     SET UP THE STEP SIZE AND RESET INTEGRATION VALUES TO THE PREVIOUS
C     STEP
C
      N=N-1
      XDS=DS
      IF(ABS(SF).GT.ABS(S(N))) XDS=0.9*(SF-S(N))
C     IF THE STEP SIZE IS LESS THAN SER, THE PREVIOUS STEP IS CLOSE
C     ENOUGH EXIT TO INVARIANT CALCULATION
```

```
      IF(ABS(XDS).LT.SER) GOTO 100
      IF(ABS(XDS).GE.ABS(DS))THEN
        DS=DS/2
      ELSE
        DS=XDS
      ENDIF
      DO 80 I=1,3
        X(I)=XL(I)
        Q(I)=QL(I)
        B(I)=BL(I)
 80   CONTINUE
 85   CALL INTGRT(X,B,BB,S,DEN)
C
C     IF LAST STEP IS STILL PAST BMAX TRY THE INTERPOLATION SCHEME AGAIN
 90   IF(BB(N,1).GT.BMAX) GO TO 70
C
C     INTERPOLATE TO SEE IF THE INTERPOLATION STEP IS CLOSE ENOUGH
C     TO BMAX. IF IT IS NOT, INTERPOLATE AGAIN AND TRY TO COME CLOSER
C
      CALL INTERP(BB(N-2,1),S(N-2),BMAX,SF,3)
C
C     IF WE ARE CLOSE ENOUGH EXIT THE INTEGRATION LOOP
      IF(ABS(SF-S(N)).LT.SER) THEN
        CALL INTERP(BB(N-2,1),RSV(N-2),BMAX,RS,3)
        IF (RS.LT.1.03)THEN
          NMANG=IA-1
          BMAXAN(IA)=-1
          GOTO 218
        ELSE
          GOTO 100
        ENDIF
      ENDIF

      DS=DS/2
      IF(ABS(SF).GT.ABS(S(N))) DS=CONI*(SF-S(N))
      CALL INTGRT(X,B,BB,S,DEN)
      GO TO 90
C
C     THE FIELD MAXIMUM HAS NOW BEEN REACHED.  THE STORED VALUES
C     OF THE MAGNETIC FIELD AND THE PATH LENGTH VALUES CAN NOW BE
C     USED TO EVALUATE THE SECOND INVARIANT.
C
 100  IF(N.LT.3) THEN
        XJ(IA)=0
        BMAXAN(IA)=BMAX
        GOTO 216
      ELSEIF(N.EQ.3)THEN
        DS=.5*(S(N-1)-S(N))
        CALL INTGRT(X,B,BB,S,DEN)
        KS=2
      ELSE
        KS=3
      ENDIF
      NLAST=N
      DO 109 I=1,3
        BLL(I)=BL(I)
109   CONTINUE
C
C     CALL THE ROUTINE WHICH DETERMINES THE SECOND INVARIANT FROM
C     FROM THE STORED VALUES
C
 110  CALL INVR(BMAX,BB,S)
C
C     INTERPOLATE TO GET THE BEST FIT
```

```
C
      CALL INTERP(BB(N-2,1),SXJ(N-2),BMAX,XXJ,KS)
      CALL INTERP(BB(N-2,1),DEN(N-2),BMAX,XDN,KS)
C
C     SAVE THE  VALUES OF THE FIRST AND SECOND INVARIANT
      XJ(IA)=ABS(XXJ)
      DENSTY(IA)=ABS(XDN)
      BMAXAN(IA)=BMAX
C
C
C     THE INTEGRAL HAS NOW BEEN EVALUATED FROM THE STARTING POINT
C     THROUGH THE MINIMUM VALUE OF B TO BMAX.
C
C     WE MUST INTEGRATE THE REST OF THE LINE --- TURN THE STARTING
C     POINTS AROUND AND RESET THE INITIAL VALUES AND INTEGRATE TO THE
C     OTHER BMAX
C
      DEL=-DELSV
      IF(IA.EQ.1)THEN
        N=2
        BB2(1,1)=BB(3,1)
        SXJ(1)=SXJ(3)
        S2(1)=S(3)
        DEN2(1)=DEN(3)
        DS=S(2)-S(3)
      ELSE
        N=N2LAST
        DO 117 I=1,3
          BL(I)=BLL2(I)
117     CONTINUE
        DS=S2(N)-S2(N-1)
        CALL INTERP(BB2(N-2,1),S2(N-2),BMAX,SF,3)
        IF(ABS(SF).GT.ABS(S2(N))) THEN
          XDS=SF-S2(N)
          IF(ABS(XDS).LE.SER)GOTO 200
          DDS=CONI*XDS
        ENDIF
      ENDIF
      CALL STEPSZ(X2,B2,BB2)

      IF(ABS(BB(2,1)-BMAX)/BMAX.LT.ERR.OR.IBEFLG.NE.0) GO TO 216
      CALL INTERP(BB(2,1),S(2),BMAX,SF,1)
      IF(ABS(SF).LT.SER) THEN
        CALL INTERP(BB(2,1),SXJ(2),BMAX,XXJ,1)
        GOTO 215
      ENDIF
C
      IF(ABS(SF).LT.ABS(DS))DS=.7*SF
      DO 120 I=1,3
        Q(I)=0.
 120  CONTINUE
      N=N2LAST

      GO TO 140
C
C     BEGIN INTEGRATING THE SECOND PART
 130  CALL STEPSZ(X2,B2,BB2)
 140  CONTINUE
      CALL INTGRT(X2,B2,BB2,S2,DEN2)
      DDS=100.
C
C     STOP INTEGRATION IF BMAX HAS BEEN PASSED
      IF(BB2(N,1).GE.BMAX) GO TO 150
      IF(IERFLG.NE.0)THEN
```

```
        IF(IERFLG.GT.0)THEN
          BMAXAN(IA)=100
        ELSE
          BMAXAN(IA)=-1
        ENDIF
         NMANG=IA-1
         GOTO 218
      ENDIF

      CALL INTERP (BB2(N-2,1),S2(N-2),BMAX,SF,3)
C
C     IGNORE INTERPOLATION IF RESULT IS NOT MONOTONIC
      DDS=100
      IF(ABS(SF).LE.ABS(S2(N))) GOTO 130

      XDS=SF-S2(N)
C
C     STOP INTEGRATION IF WITHIN SER OF BMAX
      IF(ABS(XDS).LT.SER) GO TO 200
      DDS=CONI*XDS
      GO TO 130
C
C     BMAX HAS BEEN PASSED, BEGIN INTERPOLATION SCHEME TO FIND A POINT
C     CLOSE TO BMAX BUT LESS THAN IT.
C
 150  CALL INTERP(BB2(N-2,1),S2(N-2),BMAX,SF,3)
      IF(ABS(SF-S2(N)).LE.SER) THEN
        CALL INTERP(BB2(N-2,1),RSV(N-2),BMAX,RS,3)
        IF (RS.LT.1.03)THEN
           NMANG=IA-1
           BMAXAN(IA)=-1
           GOTO 218
        ENDIF
      ENDIF

      N=N-1
      XDS=DS
      IF(ABS(SF).GT.ABS(S2(N))) XDS=0.9*(SF-S2(N))
      IF(ABS(XDS).LT.SER) GOTO 200
      IF(ABS(XDS).GE.ABS(DS))THEN
        DS=DS/2
      ELSE
        DS=XDS
      ENDIF
      DO 160 I=1,3
        X2(I)=XL(I)
        Q(I)=QL(I)
        B2(I)=BL(I)
 160  CONTINUE
      CALL INTGRT(X2,B2,BB2,S2,DEN2)
 170  IF(BB2(N,1).GT.BMAX) GO TO 150
C
C     IF THE POINT IS LESS THAN BMAX MAKE SURE IT IS CLOSE ENOUGH. IF
C     NOT, TRY TO GET CLOSER
C
      CALL INTERP(BB2(N-2,1),S2(N-2),BMAX,SF,3)
      IF(ABS(SF-S2(N)).LT.SER) THEN
        CALL INTERP(BB2(N-2,1),RSV(N-2),BMAX,RS,3)
        IF (RS.LT.1.03)THEN
          NMANG=IA-1
          BMAXAN(IA)=-1
          GOTO 218
        ELSE
          GOTO 200
```

```
        ENDIF
      ENDIF
      DS=DS/2
      IF(ABS(SF).GT.ABS(S2(N))) DS=CONI*(SF-S2(N))
      CALL INTGRT(X2,B2,BB2,S2,DEN2)
      GO TO 170
C
C     INTEGRAL IS COMPLETE USE STORED VALUES TO GET INVARIANT
 200  CALL INVR(BMAX,BB2,S2)
      CALL INTERP(BB2(N-2,1),SXJ(N-2),BMAX,XXJ,3)
      CALL INTERP(BB2(N-2,1),DEN2(N-2),BMAX,XDN,3)
      N2LAST=N
      DO 205 I=1,3
        BLL2(I)=BL(I)
 205  CONTINUE
C
C     ADD IN REMAINING CONTRIBUTION OF SECOND INVARIANT
 215  XJ(IA)=XJ(IA)+ABS(XXJ)
      DENSTY(IA)=DENSTY(IA)+ABS(XDN)
C
 216  CONTINUE
C
C
C     WE ARE DONE WITH ALL THE INTEGRALS - SET UP ANY ERROR VALUES
C
218   IF(NMANG.LT.NUMANG) THEN
        DO 219 I=NMANG+1,NUMANG
          XJ(I)=BMAXAN(NMANG+1)
          BMAXAN(I)=BMAXAN(NMANG+1)
          DENSTY(I)=BMAXAN(NMANG+1)
219     CONTINUE
      ENDIF
C
C     IF INVARIANT EXIST CALCULATE L'S
C
      DO 220 I=1,NUMANG
        IF(BMAXAN(I).GT.0.AND.BMAXAN(I).NE.100)THEN
          CALL HILTEL(BMAXAN(I),XJ(I),EL(I))
        ELSEIF(BMAXAN(I).LT.0) THEN
            EL(I)=-1
        ELSE
            EL(I)=100
        ENDIF
220   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CHECK(PANGLE,NUMANG,IER)
      DIMENSION PANGLE(*)
C       CHECK TO SEE IF THE PITCH ANGLES ARE BETWEEN 0 AND 90 AND THE THE
C       PITCH ANGLE ARRAY IS MONOTONICALLY DECREASING
      IER=0
      IF (PANGLE(1).GT.90..OR.PANGLE(1).LE.0) IER=1
      IF (NUMANG.GT.1) THEN
         DO 10 I=2,NUMANG
         IF(PANGLE(I).GT.PANGLE(I-1)) IER=1
         IF(PANGLE(I).GT.90..OR.PANGLE(I).LE.0) IER=1
 10      CONTINUE
      ENDIF
      RETURN
      END
```

```
      FUNCTION ADENS(X)
C
C     DETERMINE THE AVERAGE ATMOSPHERIC DENSITY AT A GIVEN ALTITUDE IN
C     GM/(CM2*Re)/3
C
      DIMENSION X(3)
C     NOMINAL VALUE OF F10.7
      DATA F107/114./
C     CONSTANT THAT CONVERTS TO CENTIMETERS AND APPLIES THE DIVIDE BY
C     THREE FROM GILL'S METHOD
C     5.7339E-3=6.371E8*2.7E-11/3
      R=SQRT(X(1)**2+X(2)**2+X(3)**2)
      IF (R.GT.3) THEN
        ADENS=0
      ELSE
        A=0.99+.518*SQRT(F107/55)
        R=(R-1)*6371
        IF (R.LT.110) THEN
          ADENS=0
        ELSE
          CON=(120-R)/(A*SQRT(R-103))
          ADENS=5.7339E-3*EXP(CON)
        ENDIF
      ENDIF
      RETURN
      END
```

```
      SUBROUTINE INVR(BMAX,BB,S)
C
C     PURPOSE
C        TO CALCULATE THE VALUE OF THE SECOND INVARIANT
C
C     METHOD
C        USE THE VALUES STORED IN THE S AND BB ARRAYS TO EVALUATE THE
C        INTEGRAL  SQRT(1-BB/BMAX) ALONG THE FIELD LINE.  USE THE
C        SAME INTERGRATION METHOD (GILLS METHOD) USED IN INTEGRATING
C        THE FIELD LINE
C
C     INPUT -- COMMON BLOCK INTPAR
C        N      THE NUMBER OF INTEGRATION STEPS
C        BMAX   THE VALUE OF THE MAXIMUM MAGNETIC FIELD (THE POINT
C               WHERE THE PARTICLE HAS ITS MIRROR POINT)
C        BB     A REAL 2 DIMENSIONED ARRAY CONTAINING ALL OF THE
C               MAGNETIC FIELD MAGNITUDES CALCULATED IN THE FIELD LINE
C               INTEGRATION
C        S      AN ARRAY THAT HOLDS THE TOTAL INTEGRATED PATH LENGHT ALONG
C               FIELD LINE
C     OUTPUT -- COMMON BLOCK INTPAR
C        SXJ    THE VALUES OF THE SECOND INVARIANT INTEGRATION AT
C               EACH INTEGRATION STEP.  SXJ(N) CONTAINS THE BEST
C               APPROXIMATION TO THE VALUE OF THE SECOND INVARIANT.
C               THE SAVING OF THE STEPS PERMITS THE USE OF INTERPOLATION
C               SCHEMES TO OBTAIN A MORE ACCURATE VALUE OF THE INVARIANT
C
C     CALLING SUBROUTINES
C        SUBROUTINE INVARM
C
C     CONSTANTS
C        P29    1.0-SQRT(.5)
C        OP7    1.0+SQRT(.5)
C
      COMMON /INTPAR/DS,DEL,N,IERFLG,XL(3),XSV(100,3,4),
     *RSV(100),RMIN(3),RMAG,IDSW,
     *QL(3),Q(3),BL(3),SXJ(100),DDS
      DIMENSION BB(100,4),S(100)

      DIMENSION CON(4)
      DATA (CON(I),I=1,4)/.5,.29289322,1.70710678,.5/
      SXJ(2)=0.
C
C     START THE INTEGRATION LOOP.
C     THIS IS GILLS METHOD MADE SIMPLE IF ALL THE POINTS ARE GIVEN
C     CUMULATIVE ROUND OFF ERROR CONTROL IS NOT IMPLEMENTED
      NN=N-1
      DO 210 K=2,NN
        TEMP1=0
        DO 110 I=1,4
          IF(BB(K,I).GE.BMAX) GO TO 110
          ROOT=SQRT(1.-BB(K,I)/BMAX)
          TEMP1=TEMP1+CON(I)*ROOT
 110    CONTINUE
        DELS=(S(K+1)-S(K))/3.
        SXJ(K+1)=SXJ(K)+TEMP1*DELS
 210  CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STEPSZ(X,B,BB)
C
C     PURPOSE
C        DETERMINE THE STEP SIZE FOR THE NEXT INTEGRATRION STEP
C
C     METHOD
C        THE STEP SIZE OF THE RUNGE KUTTA INTEGRATION IS A FUNCTION
C        OF THE ERROR LIMITS, THE CURVATURE OF THE FIELD LINE, THE
C        GRADIENT IN THE MAGNETIC FIELD, AND THE ESTIMATED DISTANCE
C        TO THE END OF THE INTEGRATION.
C
C     INPUT -- COMMON BLOCK INTPAR
C        DEL     A PARAMETER SET UP BY THE CALLING PROGRAM TO SCALE THE
C                STEP SIZE.  IT DEPENDS ON THE ERROR LIMITS OF THE
C                INTEGRATION.
C        B       A REAL ARRAY WHICH CONTAINS THE MAGNETIC FIELD VECTOR
C                AT THE CURRENT STEP
C        BL      A REAL ARRAY WHICH CONTAINS THE MAGNETIC FIELD VECTOR
C                AT THE PREVIOUS STEP
C        BB      A 2 DIMENSIONED REAL ARRAY
C                BB(N,1) IS THE MAGNETIC FIELD MAGNITUDE AT THE CURRENT
C                STEP
C                BB(N-1,1) IS THE MAGNETIC FIELD MAGNITUDE AT THE
C                PREVIOUS STEP
C        DDS     THE ESTIMATED STEP SIZE REQUIRED TO COMPLETE THE
C                INTEGRATION
C
C     INPUT/OUTPUT -- COMMON BLOCK INTPAR
C        DS      ON ENTRY TO THE ROUTINE DS CONTAINS THE SIZE OF THE
C                LAST STEP.  THE ROUTINE RESETS THE VALUE TO THE BEST
C                STEP SIZE FOR THE NEXT INTEGRATION STEP.
C
C     CALLING SUBROUTINES
C        INVARM
C
C     TEMPORARY VARIABLES
C        CURVMN THE MINIMUM ACCEPABLE CURVATURE.  THIS LIMITS THE STEP
C                SIZE IN THE VICINITY OF THE EARTH WHERE THE FIELD
C                CHANGES RAPIDLY
C        CURV    THE CURVATURE OF THE FIELD LINE
C
      COMMON /INTPAR/DS,DEL,N,IERFLG,XL(3),XSV(100,3,4),
     *RSV(100),RMIN(3),RMAG,IDSW,
     *QL(3),Q(3),BL(3),SXJ(100),DDS
      DIMENSION BB(100,4),BB2(100,4),B(3),B2(3),X(3),X2(3),S(3),S2(3)

C
C     DETERMINE THE MINIMUM CURVATURE
C
      CURVMN=1.6667/(X(1)**2+X(2)**2+X(3)**2)**(.75)
C
C     DETERMINE THE CURVATURE OF THE FIELD BY USING THE RATE OF CHANGE
C     OF THE UNIT FIELD VECTOR OVER THE LAST STEP
C
      CURV=0.
      DO 50 I=1,3
      CURV=CURV+((B(I)/BB(N,1)-BL(I)/BB(N-1,1))/DS)**2
 50   CONTINUE
      CURV=SQRT(CURV)
      CURV=AMAX1(CURV,CURVMN)
C
C     SET UP THE NEW STEP SIZE AND LIMIT THE STEP SIZE TO LESS THAN 2.8
C     EARTH RADDII TO PREVENT THE INTEGRATION FROM STEPPING OUT OF THE
C     VALID FIELD REGION
```

```
C
      DS=DEL/CURV
      DS=SIGN(AMIN1(ABS(DS),1.0),DS)
      IF(N.LE.3) DS=DS*(N*2-3)*.2
C
C     IF THE DISTANCE TO THE END OF THE INTEGRATION IS SMALLER THAN THE
C     NEW STEP SIZE, SET THE STEP SIZE TO THE SMALLER VALUE.
C
      IF(ABS(DDS).LT.ABS(DS)) DS=DDS
      RETURN
      END
```

```
      SUBROUTINE INTGRT(X,B,BB,S,DEN)
C
C     PURPOSE
C        THIS SUB MODULE PERFORMS A SINGLE RUNGE-KUTTA INTEGRATION
C        STEP AND UPDATES ALL OF THE VARIABLES IN THE INTEGRATION LOOP
C
C     METHOD
C        PERFORM A SINGLE FOURTH ORDER INTEGRATION STEP USING GILLS
C        METHOD OF INTEGRATION (REF. S. GILL CAMBRIDGE PHILOSOPHICAL
C        SOCIETY PROCEEDINGS VOL. 47, 1951)
C
C     INPUT -- COMMON BLOCK INTPAR
C        DS     THE INTEGRATION STEP SIZE IN UNITS OF EARTH RADII.
C               THE INTEGRATION MOVES THE SPACE COORDINATE A DISTANCE
C               DS ALONG THE MAGNETIC FIELD LINE. IF DS IS POSITIVE,
C               MOTION IS IN THE DIRECTION OF THE FIELD. IF DS IS
C               NEGATIVE MOTION IS ANTI-PARALLEL TO THE FIELD.
C
C     INPUT/OUTPUT -- COMMON BLOCK INTPAR
C        N      THE INTEGRATION STEP NUMBER. IT IS INCREMENTED BY
C               ONE AT THE END OF THIS ROUTINE. (NOTE N=2 IS THE
C               BEGINNING OF THE INTEGRATION)
C        X      A REAL ARRAY GIVING THE VECTOR LOCATION OF THE
C               INTEGRATION VARIABLE.
C               INPUT - THE INITIAL POSITION PRIOR TO THE INTEGRATION
C                       STEP
C               OUTPUT- THE FINAL VALUE AFTER THE INTEGRATION STEP
C        B      A REAL ARRAY CONTAINING THE VECTOR MAGNETIC FIELD
C               IN GAUSS
C               INPUT - THE VECTOR FIELD BEFOR THE INTEGRATION STEP
C               OUTPUT- THE VECTOR FIELD AFTER THE STEP
C        Q      A REAL ARRAY CONTAINING AN ERROR CONTROL VARIABLE
C               USED BY GILLS INTEGRATION METHOD
C               INPUT - ERROR FROM PREVIOUS STEP
C               OUTPUT- ERROR AFTER PRESENT STEP FOR INPUT TO SUBSEQUENT
C                       STEPS
C
C     OUTPUT -- COMMON BLOCK INPTAR
C        S      A REAL ARRAY WHICH SAVES EACH OF THE DISTANCES (SINCE
C               THE START OF THE INTEGRATION) ALONG THE MAGNETIC FIELD
C               LINE.
C               S(2)=0
C               S(N+1)=S(N)+DS ETC.
C        XSV    A REAL 3 DIMENSIONED ARRAY WHICH SAVES THE VECTOR
C               POSITION IN EARTH RADII FOR EACH OF THE INTEGRATION
C               STEPS.  XSV(N,1,1), XSV(N,2,1), XSV(N,3,1) ARE VECTOR
C               CARTESIAN POSITION COORDINATES CORESPONDING TO POSITION
C               S(N) ON THE FIELD LINE
C        BB     A REAL 2 DIMENSIONED ARRAY WHCIH SAVES THE MAGNITUDE
C               OF THE MAGNETIC FIELD FORM EACH INTEGRATION STEP.
C               BB(N,1) IS MAGNETIC FIELD VALUE AT DISTANCE S(N).
C               BB(N-1,2), BB(N-1,3), BB(N-1,4) ARE THE INTERMEDIATE
C               VALUES OF THE FIELD USED BY GILLS METHOD TO GET FROM
C               BB(N-1,1) TO BB(N,1).
C        XL     A REAL ARRAY WHICH SAVES THE INITIAL POSITION VALUES
C               PRIOR TO STARTING THE INTEGRATION STEP
C        BL     A REAL ARRAY WHICH SAVES THE VECTOR MAGNETIC FIELD
C               VALUES PRIOR TO STARTING THE INTEGRATION STEP
C        QL     A REAL ARRAY WHICH SAVES THE INITIAL VALUES OF THE
C               ERROR CONTROL VARIABLE
C        IERFLG AN ERROR CONTROL INDICATOR WHICH IS USED BY THE CALLING
C               PROGRAM TO CONTROL THE PROGRAM FLOW
C               IERFLG = 0 NO ERROR
C               IERFLG = 1 INTEGRATION IS OUTSIDE VALID FIELD LIMITS
```

```
C                         OR THE MAXIMUM STEP NUMBER (100) HAS BEEN
C                         REACHED.
C
C     CONSTANTS
C        P29     1.0-SQRT(0.5)
C        OP7     1.0+SQRT(0.5)
C
C     VARIABLES
C        P5DS    .5 * STEP SIZE
C        P29DS   (1.0-SQRT(0.5)) * STEP SIZE
C        OP7DS   (1.0-SQRT(0.5)) * STEP SIZE
C        RR,SS   REAL ARRAYS USED BY GILLS METHOD TO MINIMIZE COMPUTER
C                TIME AND MINIMIZE ROUNDOFF ERROR
C
C     CALLING SUBROUTINES
C        SUBROUTINE INVARM
C
C     SUBROUTINES REQUIRED
C        SUBROUTINE BMNEXT
      COMMON/BXYZCM/YEAR,DAYYR,UT,KODE,JSW
C
      COMMON /INTPAR/DS,DEL,N,IERFLG,XL(3),XSV(100,3,4),
     *RSV(100),RMIN(3),RMAG,IDSW,
     *QL(3),Q(3),BL(3),SXJ(100),DDS
      DIMENSION BB(100,4),B(3),X(3),S(100),DEN(100)
      DIMENSION SS(3),RR(3)
      DATA P29,OP7/.29289322,1.70710678/
      IERFLG=0
C     SAVE THE INITIAL VALUES.  THESE INITIAL VALUES MAY BE NEEDED IF
C     IF THE INTEGRATION STEP IS UNSUCCESSFUL (GOES TOO FAR) AND THE
C     STEP MUST BE REPEATED.
C
      DO 65 I=1,3
      XL(I)=X(I)
      QL(I)=Q(I)
      BL(I)=B(I)
 65   CONTINUE
C
C     SET UP THE CONSTANST NEEDED BY THE INTEGRATION LOOP
C
      P5DS=.5*DS
      P29DS=P29*DS
      OP7DS=OP7*DS
C
C     BEGIN GILLS METHOD (GILL 1951) OF FOURTH ORDER INTEGRATION
C
      TEMP2=P5DS*ADENS(X)
      DO 70 I=1,3
      SS(I)=P5DS*B(I)/BB(N,1)
      RR(I)=SS(I)-Q(I)
      X(I)=X(I)+RR(I)
      Q(I)=Q(I)+3.*RR(I)-SS(I)
      XSV(N,I,2)=X(I)
 70   CONTINUE
      TEMP2=TEMP2+P29DS*ADENS(X)
      CALL BMNEXT(X,B,BB(N,2))
      DO 71 I=1,3
      SS(I)=P29DS*B(I)/BB(N,2)
      RR(I)=SS(I)-P29*Q(I)
      X(I)=X(I)+RR(I)
      Q(I)=Q(I)+3.*RR(I)-SS(I)
      XSV(N,I,3)=X(I)
 71   CONTINUE
      TEMP2=TEMP2+OP7DS*ADENS(X)
```

```
      CALL BMNEXT(X,B,BB(N,3))
      DO 72 I=1,3
      SS(I)=OP7DS*B(I)/BB(N,3)
      RR(I)=SS(I)-OP7*Q(I)
      X(I)=X(I)+RR(I)
      Q(I)=Q(I)+3.*RR(I)-SS(I)
      XSV(N,I,4)=X(I)
 72   CONTINUE
      TEMP2=TEMP2+P5DS*ADENS(X)
      CALL BMNEXT(X,B,BB(N,4))
      DO 73 I=1,3
      SS(I)=P5DS*B(I)/BB(N,4)
      RR(I)=(SS(I)-Q(I))/3.
      X(I)=X(I)+RR(I)
      Q(I)=Q(I)+3.*RR(I)-SS(I)
      XSV(N+1,I,1)=X(I)
 73   CONTINUE
      N=N+1
C
C     SAVE THE CURRENT DISTANCE ALONG THE FIELD LINE
C
      S(N)=S(N-1)+DS
      DEN(N)=DEN(N-1)+TEMP2
C
C     OBTAIN THE CURRENT VALUES OF THE MAGNETIC FIELD
C
      CALL BMNEXT(X,B,BB(N,1))
C
C     IF N IS TOO BIG, SET ERROR FLAG
      IF(N.GE.100) IERFLG=1
C
C     IF OUTSIDE INTEGRATION LIMITS SET ERROR FLAG
      R=X(1)**2+X(2)**2+X(3)**2
      RSV(N)=SQRT(R)
C     IF EXTERNAL FIELD IS USED STAY WITHIN VALID REGION
      IF((R.GT.144..OR.BB(N,1).LT.0.00015).AND.JSW.GE.0) IERFLG=1
C     IF BELOW EARTHS SURFACE SET FLAG NEGATIVE
      IF(RSV(N).LT.1.03) IERFLG=-1
      RETURN
      END
```

```
      SUBROUTINE INTERP(BB,CC,D,E,J)
C
C     PURPOSE
C        INTERPOLATION ROUTINE
C
C     METHOD
C        GIVEN A SET OF THREE X,Y POINT PAIRS, INTERP FINDS THE SOLUTION
C        TO THE THREE LINEAR EQUATIONS EXPRESSING THE LOGARITHM OF THE
C        DEPENDENT VARIABLE Y AS A SECOND ORDER POLYNOMIAL OF THE
C        INDEPENDENT VARIABLE X. (LOG Y = A*X**2 +B*X +C)
C        USING THE BINOMIAL FORMULA, X CAN THEN BE EVALUATED AT A
C        SPECIFIED VALUE OF Y1
C        X = (-B +- SQRT(B**2-4*A*(C-LOG(Y1))))/(2*A)
C
C     INPUT -- ARGUMENT LIST
C        BB     A REAL ARRAY CONTAINING THE THREE VALUES OF THE
C               DEPENDENT VARIABLE
C        CC     A REAL ARRAY CONTAINING THE THREE CORESPONDING VALUES
C               OF THE INDEPENDENT VARIABLE
C        J      A FLOW CONTROL VARIABLE
C               IF J IS LESS THAN 0
C               FIT THE POLYNOMIAL TO CC AND BB AND FIND THE MINIMUM
C               VALUE OF THE DEPENDENT VARIABLE
C               IF J IS GRATER THAN 0
C               USE THE BINOMIAL FORMULA TO TO FIND THE VALUE OF
C               THE INDEPENDENT VARIABLE WHEN THE DEPENDENT VARIABLE
C               HAS THE VALUE D.CHOOSE THE ROOT THAT IS CLOSEST TO CC(J)
C        D      WHEN J IS GREATER THAN ZERO, D IS USED FOR INPUT.
C               IT IS THE VALUE OF THE DEPENDENT VARIABLE WHERE THE
C               SOLUTION TO THE DEPENDENT VARIABLE IS WANTED
C
C     OUTPUT -- ARGUMENT LIST
C        D      WHEN J IS LESS THAN 0, D OUTPUTS THE VALUE OF THE
C               DEPENDENT VARIABLE WHERE THE FUNCTION IS A MINIMUM
C        E      WHEN J IS LESS THAN 0, E OUTPUTS THE VALUE OF THE
C               INDEPENDENT VARIABLE WHERE THE FUNCTION IS A MINIMUM
C               WHEN J IS GREATER THAN 0, E OUPUTS THE VALUE OF THE
C               INDEPENDENT VARIABLE WHERE THE FUNCTION HAS THE VALUE D
C
C     CALLING SUBROUTINES
C        SUBROUTINE INVARM
C
C     VARIABLES
C        X2,X3,Y1,Y2,Y3,DD  ARE USED BY THE LINEAR EQUATION SOLUTION
C               TO MINIMIZE COMPUTER TIME
C        A,B,C  THE THREE POLYNOMIAL COOEFICIENTS
C        DIS    B**2-4*A*C
C        SA,SB  THE TWO ROOTS OF THE POLYNOMIAL
C
      DIMENSION BB(3),CC(3)
      REAL*8 Y1,Y2,Y3,X2,X3,DD,A,B,C,DIS
C
C     SET UP THE INITIAL VARIABLES, MOVE THE ORIGIN OF THE INDEPENDENT
C     VARIABLE TO CC(1)
C
      Y1=ALOG(BB(1))
      Y2=ALOG(BB(2))
      Y3=ALOG(BB(3))
      X2=CC(2)-CC(1)
      X3=CC(3)-CC(1)
C
C     SOLVE THE LINEAR EQUATIONS
      DD=(X3-X2)*X2*X3
      IF(DD.EQ.0)THEN
```

```
      IF(J.LT.0) THEN
          D=BB(2)
        ELSE
          E=CC(J)
        ENDIF
        RETURN
      ENDIF
      A=(X3*(Y1-Y2)+X2*(Y3-Y1))/DD
      B=(X3**2*(Y2-Y1)-X2**2*(Y3-Y1))/DD
C
C     IF J THE FLOW CONTROL VARIABLE IS LESS THAN ZERO BRANCH TO
C     MINIMUM EVALUATION ROUTINE
      IF(J.LT.0)GO TO 100
      C=Y1-DLOG(D)
      DIS=B**2-4.*A*C
C
C     IF DIS IS NEGATIVE NO SOLUTION EXIST, EXCHANGE DEPENDENT AND
C     INDEPENDENT VARIABLE ROLES AND TRY ANOTHER SOLUTION
      IF(DIS.LE.0.) GO TO 200
      DIS=DSQRT(DIS)
C
C     OBTAIN THE TWO ROOTS
      SA=(-B+DIS)/(2.*A)+CC(1)
      SB=(-B-DIS)/(2.*A)+CC(1)
      E=SA
C
C     FIND THE ROOT CLOSEST TO CC(J)
      IF(ABS(SB-CC(J)).LT.ABS(SA-CC(J)))E=SB
      RETURN
C
C     FIND THE VALUES AT THE MINIMUM
 100  X=-B/(2.*A)
      E=X+CC(1)
      XM=A*X**2+B*X+Y1
      D=EXP(XM)
      RETURN
C
C     ALTERNATE INTERPOLATION SCHEME PLACED HERE AS A SAFEGUARD
C     AGAINST A STRANGE FIELD CONFIGURATION CAUSING AN IMAGINARY
C     SOLUTION (EXCHANGE THE ROLES OF DEPENDENT AND INDEPENDENT
C     VARIABLES)
C
 200  Y1=CC(1)
      Y2=CC(2)
      Y3=CC(3)
      X2=BB(2)-BB(1)
      X3=BB(3)-BB(1)
      DD=(X3-X2)*X2*X3
      IF(DD.EQ.0)THEN
        E=CC(J)
        RETURN
      ENDIF
      A=(X3*(Y1-Y2)+X2*(Y3-Y1))/DD
      B=(X3**2*(Y2-Y1)-X2**2*(Y3-Y1))/DD
      DX=D-BB(1)
      E=(A*DX+B)*Dx+Y1
      RETURN
      END
```

```
      SUBROUTINE MGLONG(X,S,SF,XMLONG,RMIN,RMAG)
C
C     PURPOSE
C        TO DETERMINE THE MAGNETIC LONGITUDE OF THE MINIMUM B LOCATION
C        OF THE MAGNETIC FIELD LINE
C
C     METHOD
C        GIVEN A LOCUS OF POSITIONS ALONG A FIELD LINE AS A FUNCTION
C        OF THE SCALAR DISTANCE ALONG THE FIELD LINE AND GIVEN THE
C        SCALAR DISTANCE WHERE THE FIELD IS A MINIMUM, THE ROUTINE
C        FINDS THE VECTOR POSITION OF THE MINIMUM.  IT THEN TRANSFORMS
C        THIS MINIMUM TO OFFSET DIPOLE COORDINATES AND CALCULATES
C        THE MAGNETIC LONGITUDE OF THE MINIMUM
C        NOTE*******THE CONSTANT ISWTCH IS SET BY A DATA STATEMENT,
C        IF IT IS SET TO ZERO XMLONG IS CALCULATED USING A CENTERED
C        DIPOLE COORDINATE SYSTEM WITH ZERO LONGITUDE AT 69 DEGREES
C        WEST GEOGRAPHIC.  IF ISWTCH IS SET NON-ZERO, AN OFFSET DIPOLE
C        COORDINATE SYSTEM IS USED WITH XMLONG=0 GOING THROUGH
C        GREENWHICH
C
C     INPUT -- ARGUMENT LIST
C        X      A REAL 2 DIMENSIONED ARRAY CONTAINING THE LOCUS OF
C               POINTS ALONG A FIELD LINE
C               X(1,1), X(1,2) AND X(1,3) ARE THE X, Y, Z VALUES
C               (RIGHT HANDED CARTESIAN COORDINATES) AT THE FIRST
C               POINT, X(2,1), X(2,2) AND X(2,3) THE SECOND LOCATION
C               AND X(3,1), X(3,2) AND X(3,3) ARE AT THE THIRD LOCATION
C               THE FIRST DIMENSION OF X MUST BE THE SAME AS THE
C               CALLING PROGRAMS DIMENSION - IN THIS CASE IT IS 100
C        S      A REAL ARRAY CONTAINING THE SCALAR DISTANCE ALONG THE
C               FIELD LINE IN EARTH RADII. S(1) IS THE SCALAR DISTANCE
C               TO THE X(1,1), X(1,2), X(1,3) POINT FROM THE START
C               OF THE INTEGRATION, S(2) IS THE DISTANCE TO X(2,1),...
C        SF     THE SCALAR DISTANCE TO THE MAGNETIC MINIMUM
C
C     OUTPUT -- ARGUMENT LIST
C        XMLONG THE MAGNETIC LONGITUDE (IN DEGREES) OF THE MINIMUM
C               OF THE MAGNETIC LINE OF FORCE
C               IF ISWTCH IS ZERO, THE ZERO OF MAGNETIC LONGITUDE IS
C               ALONG 69 DEG WEST GEOGRAPHIC
C               IF ISWTCH IS NOT ZERO, THE ZERO OF MAGNETIC LONGITUDE
C               IS THROUGH GREENWHICH
C
C     CONSTANTS
C        DX     THE 3 VECTOR COMPONENTS OF THE LOCATION OF THE OFFSET
C               DIPOLE IN EARTH RADII (GEOGRAPHIC CARTESIAN COORDS)
C        A22-A34 TRANSFORMATION MATRIX TO OFFSET DIPOLE COORDS. FIRST
C               ROTATE ABOUT THE GEOGRAPHIC Z AXIS, TO THE MERIDIAN
C               CONTAINING THE OFFSET DIPOLE,  THEN ABOUT THE NEW Y AXIS
C               TO THE LATITUDE CONTAINING THE OFFSET DIPOLE AND THEN
C               ABOUT THE NEW Z AXIS SUCH THAT THE ZERO OF LONGITUDE
C               PASSES THROUGH GREENWHICH
C        ISWTCH A FLOW CONTROL CONSTANT
C               IF SET TO ZERO BY THE DATA STATEMENT USE CENTERED DIPOLE
C               COORDINATES
C               IF SET NON-ZERO USE OFFSET DIPOLE COORDINATES
C        SIN D  SINE OF THE COLATITUDE OF THE CENTERED DIPOLE AXIS
C        COS D  COSINE OF THE COLATITUDE OF THE CENTERED DIPOLE AXIS
C        S69    SINE OF 69 DEGREES
C        C69    COSINE OF 69 DEGREES
C
C     TEMPORARY VARIABLES
C        XF,X1,X2,Y1,Y2,Y3,A,B,DD  THESE VARIABLE ARE USED IN THE
C               INTERPOLATION LOOP TO MINIMIZE THE NUMBER OF MEMORY
```

```
C               REFERENCES AND TO MINIMIZE THE NUMBER OF MULTIPIES
C        XT     A REAL ARRAY HOLDING THE LOCATION OF THE MINIMUM AND
C               LATER THE OFFSET MINIMUM OF THE FIELD LINE
C        XP,YP  THE POSITION OF THE MINIMUM IN OFFSET MAGNETIC CORDS.
      DIMENSION X(100,3),S(100),DX(3),XT(3),RMIN(3)
      DATA DX(1),DX(2),DX(3)/0.0576,-0.0321,-0.0184/
      DATA A22,A23,A24,A32,A33,A34/0.97056,0.23948,-0.02556,
     *-0.22969,0.95232,0.20082/
      DATA SIND,COSD,S69,C69/.2027872954,.9792228106,.9335804265,
     *.3583679495/
C
C
C     ********SET UP THE FLOW CONTROL SWITCH**********
C     COORDINATE SYSTEM DEFINITION USED. (SEE METHOD)
      DATA ISWTCH/1/
C
C     BEGIN QUADRATIC INTERPOLATION
      XF=SF-S(1)
      X2=S(2)-S(1)
      X3=S(3)-S(1)
      DD=(X3-X2)*X2*X3
C
C     INTERPOLATE EACH COMPONENT SEPERATELY
      DO 10 I=1,3
      Y1=X(1,I)
      Y2=X(2,I)
      Y3=X(3,I)
      A=(X3*(Y1-Y2)+X2*(Y3-Y1))/DD
      B=(X3**2*(Y2-Y1)-X2**2*(Y3-Y1))/DD
C
C     EVALUATE THE POSITION OF THE MINIMUM
      XT(I)=(A*XF+B)*XF+Y1
      RMIN(I)=XT(I)
 10   CONTINUE
      RMAG2=XT(1)**2+XT(2)**2+XT(3)**2
      RMAG=SQRT(RMAG2)
C
C     IF ISWTCH IS ZERO GO TO CENTERED DIPOLE DEFINITION
      IF (ISWTCH.EQ.0) GO TO 30
C
C     ADD IN THE DIPOLE OFFSET
      DO 20 I=1,3
 20   XT(I)=XT(I)+DX(I)
C
C     TRANSFORM TO OFFSET DIPOLE COORDINATES AND EVALUATE THE LONGITUDE
      XP=A22*XT(1)+A23*XT(2)+A24*XT(3)
      YP=A32*XT(1)+A33*XT(2)+A34*XT(3)
      GO TO 40
C
C     TRANSFORM TO CENTERED DIPOLE COORDINATES
 30   XP=(XT(1)*C69-XT(2)*S69)*COSD-XT(3)*SIND
      YP=XT(1)*S69+XT(2)*C69
C
C     CALCULATE MAGNETIC LONGITUDE
 40   XMLONG=ATAN2(YP,XP)*57.2957795
      IF(XMLONG.LT.0.) XMLONG=XMLONG+360.
      RETURN
      END
```

```
      SUBROUTINE HILTEL (B,XI,VL)
C     PURPOSE
C        CALCULATE THE L VALUE
C        THE ORIGINAL MCILWAIN L EXPANSION GIVEN BY THE OLD
C        SUBROUTINE CARMEL HAS BEEN REPLACED BY HILTONS SIMPLER
C        EXPANSION.  DIFFERENCES BETWEEN HILTONS AND MCILWAINS
C        EXPANSION ARE TYPICALLY LESS THAN .01 PERCENT.
C
C     METHOD
C        SEE J. HILTON, J. GEOPHYS. RES. 76, 6952 (1971)
C
C     INPUT -- CALLING SEQUENCE
C        B      THE MAGNETIC FIELD AT THE PARTICLE MIRROR POINT
C        XI     THE SECOND INVARIANT EVALUATED BETWEEN MIRROR POINTS
C               EXPRESSED IN UNITS OF EARTH RADII
C
C     OUTPUT -- CALLING SEQUENCE
C        VL     THE L VALUE
C
C     THE NEXT STATEMENT CONTAINS THE ORIGINAL McILWAIN MOMENT
C     DATA XM/.311653/
C     USE THE DIPOLE MOMENT CALCULATED FROM THE CURRENT FIELD MODEL
      COMMON /MOMENT/XM
      IF(XI.GT.1.0E-36) GO TO 10
      VL=(XM/B)**(1./3.)
      RETURN
 10   X=XI*(B/XM)**(1./3.)
      V=1.+X*(1.35047+X*(.465376+.0475455*X))
      VL=(V*XM/B)**(1./3.)
C     END COMPUTE L
      RETURN
      END
```