NASA
Technical Memorandum 105150

AVSCOM
Technical Report 91-C-018

How to Determine Spiral Bevel Gear Tooth Geometry for Finite Element Analysis

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Prepared for the
International Conference on Motion and Power Transmissions
sponsored by the Japan Society of Mechanical Engineers with the
participation of ASME, I. Mech. E., VDI, I.E.T., CSME and other Societies
Hiroshima, Japan, November 24-26, 1991

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US ARMY
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ABSTRACT

An analytical method has been developed to determine gear tooth surface coordinates of face-milled spiral bevel gears. The method combines the basic gear design parameters with the kinematical aspects for spiral bevel gear manufacturing. A computer program was developed to calculate the surface coordinates. From this data a three-dimensional model for finite element analysis can be determined. Development of the modeling method and an example case are presented.

INTRODUCTION

Spiral bevel gears are currently used in all helicopter power transmission systems. This type of gear is required to turn the corner from a horizontal engine to the vertical rotor shaft. These gears carry large loads and operate at high rotational speeds. Recent research has focused on understanding many aspects of spiral bevel gear operation, including gear geometry [1-12], gear dynamics [13-15], lubrication [16], stress analysis and measurement [17-21], misalignment [22,23], and coordinate measurements [24,25], as well as other areas.

Research in gear geometry has concentrated on understanding the meshing action of spiral bevel gears [8-11]. This meshing action often results in much vibration and noise due to an inherent lack of conjugation. Vibration studies [26] have shown that in the frequency spectrum of an entire helicopter transmission, the highest response can be that from the spiral bevel gear mesh. Therefore if noise reduction techniques are to be implemented effectively, the meshing action of spiral bevel gears must be understood.

Also, investigators [18,19] have found that typical design stress indices for spiral bevel gears can be significantly different from those measured experimentally. In addition to making the design process one of trial and error (forcing one to rely on past experience), this inconsistency makes extrapolating over a wide range of sizes difficult, and an overly conservative design can result.

The objective of the research reported herein was to develop a method for calculating spiral bevel gear-tooth surface coordinates and a three-dimensional model for finite element analysis. Accomplishment of this task requires a basic understanding of the gear manufacturing process, which is described herein by use of differential geometry techniques [1]. Both the manufacturing machine settings and the basic gear design data were used in a numerical analysis procedure that yielded the tooth surface coordinates. After the tooth surfaces (drive and coast sides) were described, a three-dimensional model for the tooth was assembled. A computer program was developed to automate the calculation of the tooth surface coordinates, and hence, the coordinate for the gear-tooth three-dimensional finite element model. The basic development of the analytical model is explained, and an example of the finite element method is presented.

DETERMINATION OF TOOTH SURFACE COORDINATES

The spiral gear machining process described in this paper is that of the face-milled type. Spiral bevel gears manufactured in this way are used extensively in aerospace power transmissions (i.e., helicopter main/tail rotor transmissions) to transmit power between horizontal gas turbine engines and the vertical rotor shaft. Because spiral bevel gears can accommodate various shaft orientations, they allow greater freedom for overall aircraft layout.

In the following sections the method of determining gear-tooth surface coordinates will be described. The manufacturing process must first be understood and then analytically described. Equations must be developed that relate machine and workpiece motions and settings with the basic gear design data. The simultaneous solution of these equations must be done numerically since no closed-form solution exists. A description of this procedure follows.

Gear Manufacture

Spiral bevel gears are manufactured on a machine like the one shown in Fig. 1. This machine cuts away the material between the concave and convex tooth surfaces of adjacent teeth simultaneously. The machining process is better illustrated in Fig. 2. The head cutter (holding the cutting blades or the grinding wheel) rotates about its own axis at the proper cutting speed,

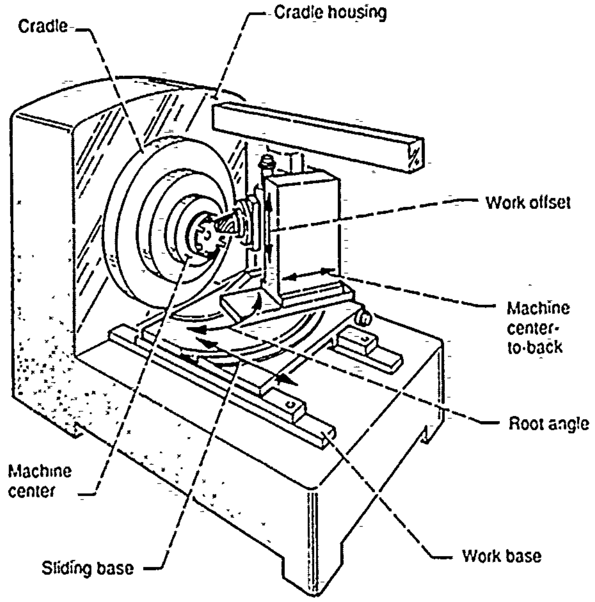

Figure 1.—Machine used for spiral bevel gear tooth surface generation.

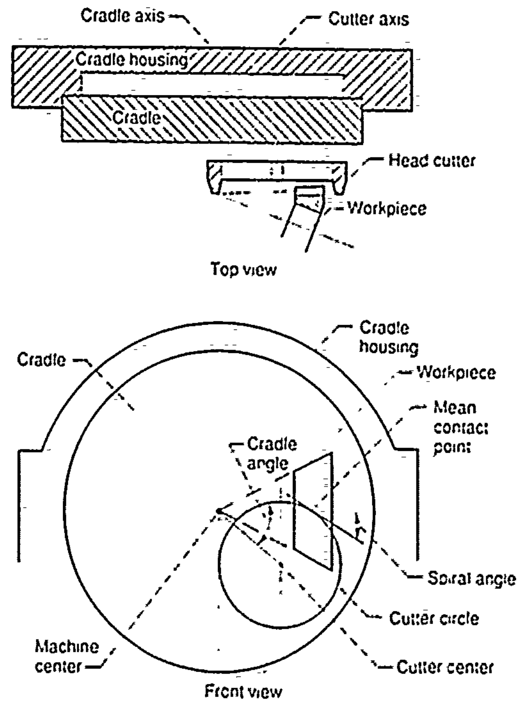

Figure 2.—Orientation of workpiece to generation machinery.

about its own axis at the proper cutting speed, independent of the cradle or workpiece rotation. The head cutter is connected to the cradle through an eccentric that allows adjustment of the axial distance between the cutter center and cradle (machine) center, and adjustment of the angular position between the two axes to provide the desired mean spiral angle. The cradle and workpiece are connected through a system of gears and shafts, which controls the ratio of rotational motion between the two (ratio of roll). For cutting, the ratio is constant, but for grinding, it is a variable.

Computer numerical controlled (CNC) versions of the cutting and grinding manufacturing processes are currently being developed. The basic kinematics, however, are still maintained for the generation process; this is accomplished by the CNC machinery duplicating the generating motion through point-to-point control of the machining surface and location of the workpiece.

Coordinate Transformations

The surface of a generated gear is an envelope to the family of surfaces of the head cutter. In simple terms this means that the points on the generated tooth surface are points of tangency to the cutter surface during manufacture. The conditions necessary for envelope existence are given kinematically by the equation of meshing. This equation can be stated as follows: the normal of the generating surface must be perpendicular to the relative velocity between the cutter and the gear-tooth surface at the point in question [1].

The coordinate transformation procedure that will now be described is required to locate any point from the head cutter into a coordinate system rigidly attached to the gear being manufactured. Homogeneous coordinates are used to allow rotation and translation of vectors simply by multiplying the matrix transformations. The method used for the coordinate transformation can be found in Refs. 1, 5, 8 to 11, and 27.

To begin the process we start with the head cutter (Fig. (3)). In this report it is assumed that the cutters are straight-sided. The parameters u and θ determine the location of a

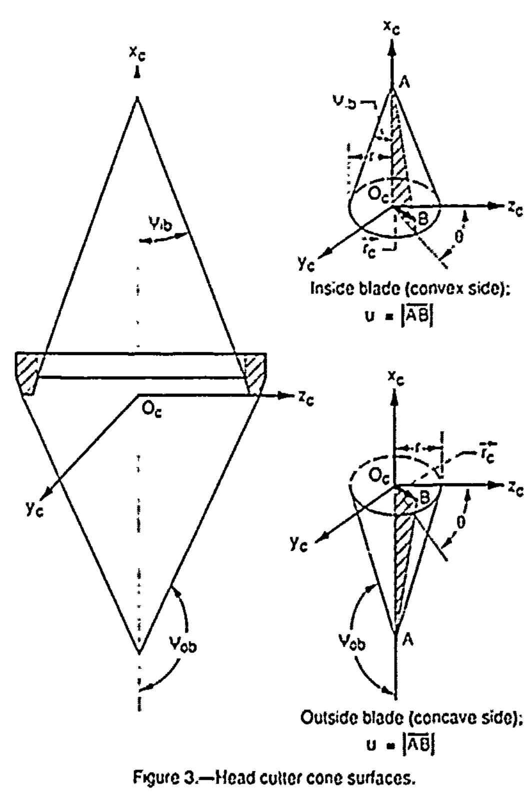

Figure 3.—Head cutter cone surfaces.

current point in the cutter coordinate system S_c. Angles ψ_{ib} and ψ_{ob} are the blades that cut the convex and concave sides of the gear tooth respectively. Thus $\bar{r}_c$ is given by the following equation:

$$r_c = \begin{bmatrix} r\cot\psi - u\cos\psi \\ u\sin\psi\sin\theta \\ u\sin\psi\cos\theta \\ 1 \end{bmatrix} \qquad (1)$$

Once we have $\bar{r}_c$ we then transform from one coordinate system to the next for the coordinate systems as shown in Figs. (4) and (5) [27].

Using matrix transformations we can determine the coordinates in S_w of a point on the generating surface by:

$$r_w = \left[M_{wa}\{f(\phi_c)\}\right]\left[M_{ap}\right]\left[M_{pm}\right]\left[M_{ms}(\phi_c)\right]\left[M_{sc}\right]r_c(u,\theta) \qquad (2)$$

Here $[M_{ij}]$ describes the required homogeneous coordinate transformation from system "j" to system "i". Therefore Eq. (2) describes the location of a point in the gear fixed coordinate system based on machine settings L_m, E_m, q, s, r, ψ, and gear design information μ and δ. At this point the machine settings and the gear design values are known. Parameters u, θ, and ϕ_c are the unknown variables that are solved for numerically.

<u>Tooth Surface Coordinate Solution Procedure</u>

In order to solve for the coordinates of a spiral bevel gear-tooth surface, the following items must be used simultaneously: the transformation process, the equation of meshing, and the basic gear design information. The transformation process described previously is used to determine the location of a point on the head cutter in coordinate system S_w. Since there are three unknown quantities (u, θ, and ϕ_c), three equations relating them must be developed.

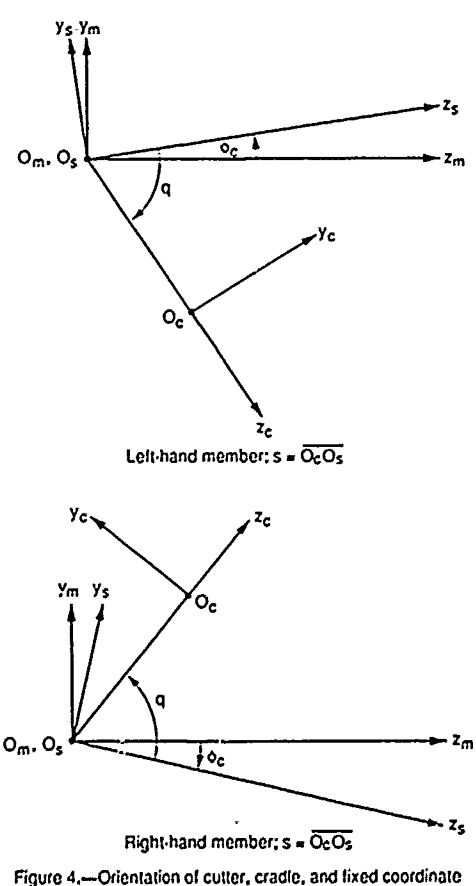

Figure 4.—Orientation of cutter, cradle, and fixed coordinate systems, S_c, S_s, and S_m respectively.

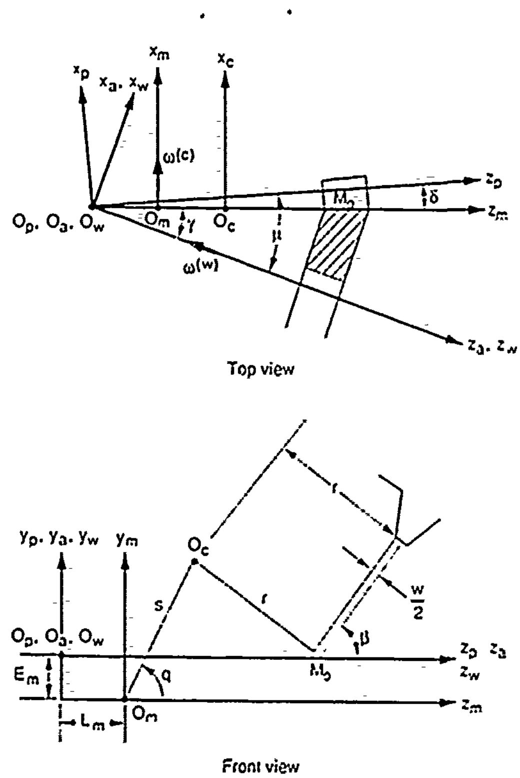

Figure 5.—Top and front view of right hand gear surface generation (σ_c = 0 shown here).

Values for u, θ, and ϕ_c are used to satisfy the equation of meshing given by Refs. 1 and 9.

$$\vec{n} \cdot \vec{V} = 0 \qquad (3)$$

where $\vec{n}$ is the normal vector to the cutter and workpiece surfaces at the specified location of interest, and $\vec{V}$ is the relative velocity between the cutter and workpiece surfaces at the specified location.

Gear design information is then used to establish an allowable range of values of the radial ($\bar{r}$) and axial ($\bar{z}$) positions that are known to exist on the gear being generated. This is shown in Fig. 6.

First the equation of meshing must be satisfied. This is given as [9]:

$$\left(u - r\cot\psi\cos\psi\right)\cos\gamma\sin\left(\theta \mp q \pm \phi_c\right)$$

$$+ s\left[\left((\phi_c/\phi_w) - \sin\gamma\right)\cos\psi\sin\theta \mp \cos\gamma\sin\psi\sin\left(q - \phi_c\right)\right]$$

$$\pm E_m\left(\cos\gamma\sin\psi + \sin\gamma\cos\psi\cos\left(\theta \mp q \pm \phi_c\right)\right)$$

$$- L_m\left(\sin\gamma\cos\psi\sin\left(\theta \mp q \pm \phi_c\right)\right) = 0$$

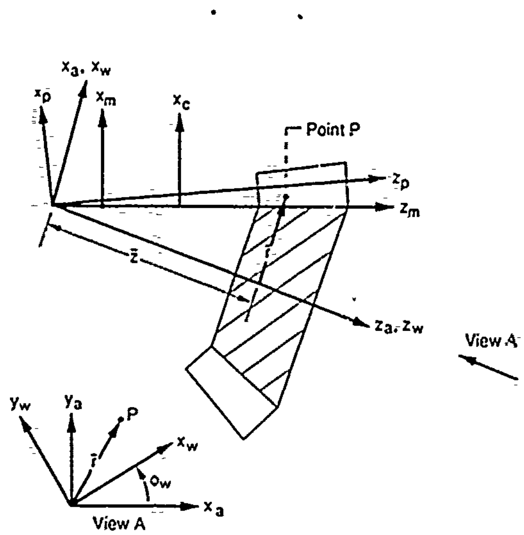

Figure 6.—Orientation of gear to be generated with assumed positions $\bar{r}$ and $\bar{z}$.

The upper and lower signs preceding the above terms pertain to left and right hand gears respectively.

The axial position must match the value found from transforming the cutter coordinates S_c to workpiece coordinates S_w. This is satisfied by the following (Fig. 6):

$$z_w - \bar{z} = 0 \tag{4}$$

Finally the radial location from the work axis of rotation must be satisfied. This is accomplished by using the magnitude of the location in question in the x_w - y_w plane (Fig. 6):

$$\bar{r} - \left(x_w^2 + y_w^2\right)^{0.5} = 0 \tag{5}$$

Now a system of three equations (Eqs. (3, to (5)) is solved simultaneously for the three parameters u, θ, and ϕ_c, for a given gear design with a set of machine tool settings. These are nonlinear algebraic equations that can be solved numerically with commercially available mathematical subroutines. These equations are then solved simultaneously for each location of interest along the tooth flank, as shown in Fig. 7. From the surface grids, the active profile (working depth) occupied by a single tooth is defined.

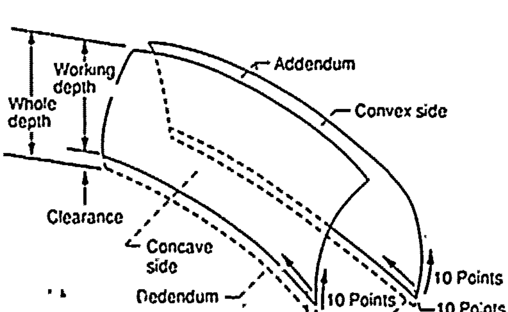

Figure 7.—Calculation points (100 each side) for concave and convex sides of tooth surface.

In summary the procedure is as follows. Known locations $\bar{r}$ and $\bar{z}$ on the active profile are used along with the equation of meshing to determine unknown parameters u, θ, and ϕ_c. The parameter values, machine tool settings, and gear design values are used in the coordinate transformation shown earlier to find the radial and axial positions in the gear fixed coordinate system S_w. This procedure results in the solution of three simultaneous nonlinear algebraic equations that are solved numerically.

APPLICATION OF SOLUTION TECHNIQUE

An application of the techniques previously discussed will now be presented. The component to be modeled was from the NASA Lewis Spiral Bevel Gear Test Facility. A photograph of the spiral bevel gear mesh is shown in Fig. 8, and the design data for the pinion member are shown in Table 1.

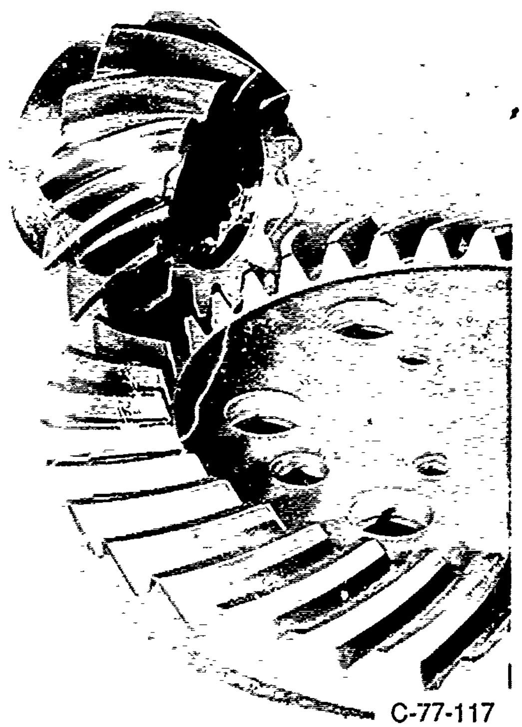

Figure 8.—Photograph of NASA spiral bevel gear test rig components.

Surface Coordinate Calculation

Using Figs. 7 and 9 as references, we will describe the calculation procedure for surface coordinates. First, the concave side of the tooth is completely defined before moving to the convex side. These points are calculated by starting at the toe end and at the lowest point of active profile. Nine steps of equal distance are used from the beginning of the active profile to the face angle (addendum) of the gear tooth, and then back to the next axial position (Fig. 9). The procedure is repeated until the concave side is completely described. Then the same procedure is followed for the convex side.

Example Model and Results

From the one-tooth model described earlier the analysis techniques can be demonstrated. The

TABLE I. - EXAMPLE CASE FOR SURFACE COORDINATE GENERATION

Gear design data	
Number of teeth (pinion, gear)	12, 36
Dedendum angle, δ, deg	1.0
Addendum angle, deg	3.883
Pitch angle, μ, deg	18.433
Shaft angle, deg	90.0
Mean spiral angle, deg	35.0
Face width, mm, (in.)	25.4 (1.0)
Mean cone distance, mm (in.)	81.05 (3.191)
Inside radius of gear blank, mm (in.)	15.3 (0.6094)
Top land thickness, mm (in.)	2.032 (0.080)
Clearance mm (in.)	0.762 (0.030)

Generation machine settings		
	Concave	Convex
Radius of cutter, r, mm (in.)	75.222 (2.9615)	78.1329 (3.0751)
Blade angle, ψ, deg	161.358	24.932
Vector sum, L_a, mm, in.	1.0363 (0.0408)	-1.4249 (-0.0561)
Machine offset, E_m, mm (in.)	3.9802 (0.1567)	-4.4856 (-0.1766)
Cradle to cutter distance, s, mm (in.)	74.839 (2.9646)	71.247 (2.3050)
Cradle angle, q, deg	64.01	53.82

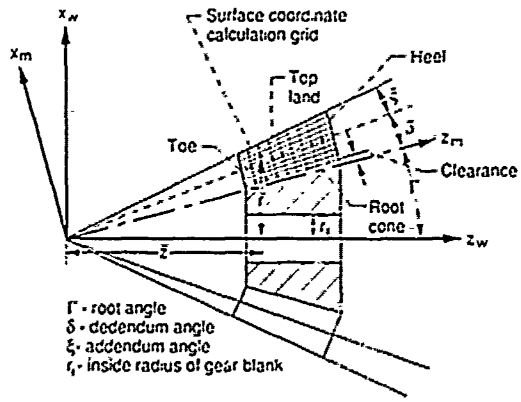

Figure 9 —Cross section of calculation grid.

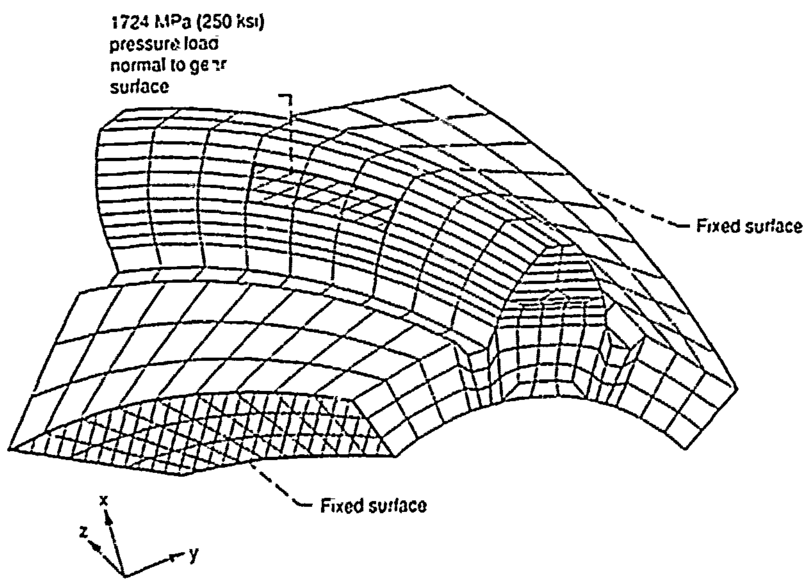

Figure 10.—Boundary conditions for the constant fillet/root radius model for the example application.

model shown in Fig. 10 is that for a constant fillet and root radius (also called full fillet) model. The fillet and root radius on the convex side has been added along with the tooth section (without the tooth) to make the model symmetric about the tooth centerline. Figure 10 shows a hidden line plot of the finite element mesh with eight-noded isoperimetric three-dimensional solid continuum elements. This model has 765 elements and 1120 nodes. The boundary conditions are also shown in Fig. 10. A 1724-MPa (250 ksi) constant pressure load was applied normal to the tooth surface of nine elements, and the two edge surfaces of the gear rim had all degrees of freedom constrained.

The results were calculated by MSC/NASTRAN and were subsequently displayed by PATRAN. Figure 11 shows the major principle stresses for the boundary conditions shown in Fig. 10.

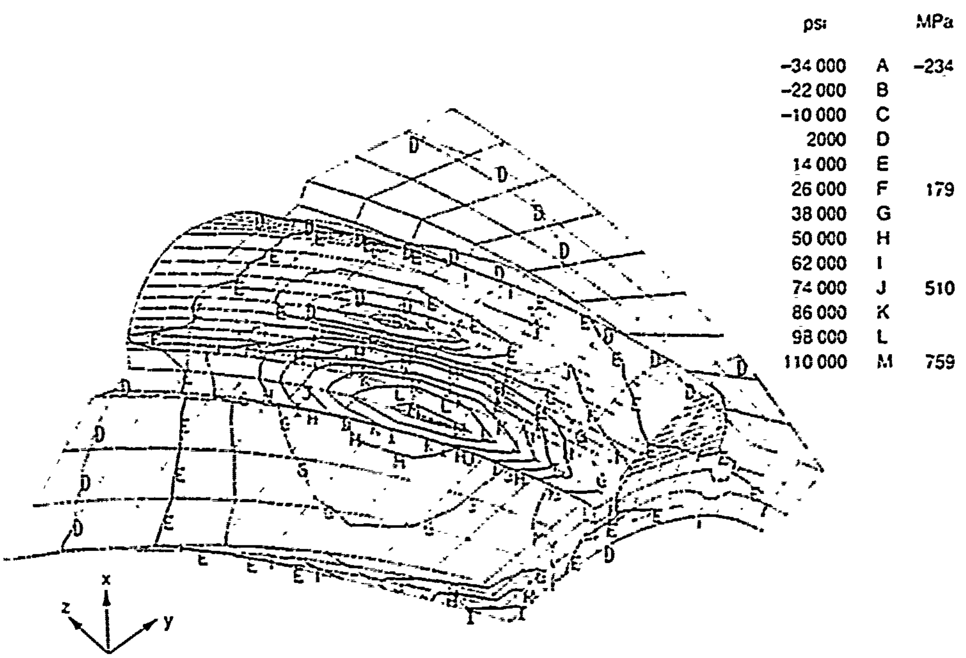

Figure 11.—Major principle stress for the boundary conditions specified in figure 10.

SUMMARY OF RESULTS

A method has been presented that uses differential geometry techniques to calculate the surface coordinates of face-milled spiral bevel gear teeth. The coordinates must be solved for numerically by a simultaneous solution of nonlinear algebraic equations. These equations relate the kinematics of manufacture to the gear design parameters. Coordinates for a grid of points are determined for both the concave and convex sides of the gear tooth. These coordinates are then combined to form the enclosed surface of one gear tooth. A computer program, was developed to solve for the gear-tooth surface coordinates and provide input to a three-dimensional geometric modeling program, (i.e., PATRAN). This enables an analysis by the finite element method. An example of the technique was presented.

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Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-105150 AVSCOM TR 91-C-018		

4. Title and Subtitle	5. Report Date
How to Determine Spiral Bevel Gear Tooth Geometry for Finite Element Analysis	
	6. Performing Organization Code

7. Author(s)	8. Performing Organization Report No.
Robert F. Handschuh and Faydor L. Litvin	E-6433
	10. Work Unit No.
	505-63-36 1L162211A47A

9. Performing Organization Name and Address	11. Contract or Grant No.
NASA Lewis Research Center Cleveland, Ohio 44135-3191 and Propulsion Directorate U.S. Army Aviation Systems Command Cleveland, Ohio 44135-3191	

12. Sponsoring Agency Name and Address	13. Type of Report and Period Covered
National Aeronautics and Space Administration Washington, D.C. 20546-0001 and U.S. Army Aviation Systems Command St. Louis, Mo. 63120-1798	Technical Memorandum
	14. Sponsoring Agency Code

15. Supplementary Notes

Prepared for the International Conference on Motion and Power Transmissions sponsored by the Japan Society of Mechanical Engineers with the participation of ASME, I. Mech. E., VDI, I.E.T., CSME and other societies, Hiroshima, Japan, November 24 - 26, 1991. Robert F. Handschuh, Propulsion Directorate, U.S. Army Aviation Systems Command; Faydor L. Litvin, University of Illinois at Chicago, Chicago, Illinois. Responsible person, Robert F. Handschuh, (216) 433-3969.

16. Abstract

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17. Key Words (Suggested by Author(s))	18. Distribution Statement
Gears Gear teeth Mechanical drives Finite element analysis	Unclassified - Unlimited Subject Category 37

19. Security Classif. (of the report)	20. Security Classif. (of this page)	21. No. of pages	22. Price*
Unclassified	Unclassified		A

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