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PLANE WAVE SCATTERING FROM CIRCULAR AND HOLLOW CIRCULAR SLIT CYLINDERS USING THE ON-SURFACE RADIATION CONDITION (OSRC) APPROACH

by

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Plane Wave Scattering from Circular and Hollow Circular Slit Cylinders using the On Surface Radiation Condition Approach

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ABSTRACT

A new formulation of electromagnetic wave scattering by convex, two dimensional conducting bodies was introduced by Kriegsman et al in ref.[1]. This formulation, called the On Surface Radiation Condition (OSRC) approach is based upon an expansion of the radiation condition applied directly on the surface of the scatterer. Past approaches involved applying a radiation condition at some distance from the scatterer in order to achieve a nearly reflection free truncation of a finite difference time domain lattice. Application of a suitable radiation condition directly on the surface of the convex conducting scatterer can lead to substantial simplification of the frequency domain integral equation for the scattered field, which is reduced to just a line integral. In this thesis, the application of the second order radiation boundary condition of ref.[1] to concave and re-entrant two dimensional bodies is considered. The specific shapes considered are slit circular cylinders and thick semi-circular cylinders. The formulation is done for the more challenging case of Transverse Electric (TE) polarization. The bodies considered here are two dimensional in nature and are perfectly conducting. Results are presented for the surface current distribution and scattering width of these cylinders and comparison is made with known results. It was found that the OSRC approach may not be applicable to concave structures and also for higher values of kb for convex bodies.

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I. INTRODUCTION

The On-Surface Radiation Condition (OSRC) approach presented here is a medium frequency technique for modeling electromagnetic scattering from complex bodies. It is based upon an expansion of the radiation condition applied directly on the surface of the scatterer. Past techniques involved applying a radiation condition at some distance from the scatterer in order to achieve a nearly reflection free truncation of a finite time domain lattice. Application of a suitable radiation condition directly on the surface of the scatterer can lead to substantial simplification of the frequency domain integral equation for the scattered field, which is reduced to just a line integral. The OSRC technique converts the usual surface integral equation for the scattering problem into either an integration of unknown quantities or a simple ordinary differential equation for convex two-dimensional objects. For the Transverse Electric (TE) case, the problem reduces to solving an ordinary differential equation around the scatterer contour and for the Transverse Magnetic (TM) case, the solution is known explicitly. It is currently applicable to convex conducting cylinders of arbitrary cross sections for both TE and TM cases. The OSRC approach was motivated, as mentioned in ref. [1], by numerical experiments conducted over the past

twenty years aimed at simulating scalar or vector wave propagation and scattering using a finite difference time domain (FD-TD) model of the governing wave equation.

In the subsequent sections of this thesis, we will use the OSRC theory to study scattering by two dimensional, convex and concave shaped conducting scatterers for the TE case. The specific shapes considered are the slit circular cylinder and the thick semi-circular cylinder. More emphasis is placed on the TE case using the second order boundary condition. Results and comparisons are shown for the TE case. Comparisons of current distributions for circular cylinders with nomalized circumferences (kb) of 5 and 10 with ref.[1] are made. Also comparisons of bistatic cross sections for circular cylinders are made for several values of Kb. For the thick semi-circular cylinder and the slit cylinders, comparison is shown for the induced current density and the normalized scattering width.

II. SCATTERING FROM CIRCULAR CYLINDERS: OSRC APPROACH

In this chapter we will first apply the OSRC technique to two dimensional, perfectly conducting and convex shaped cylinders. Next we will apply the second order boundary conditions to circular cylinders with radius r_0 and obtain a second order differential equation for the surface scattered field. The differential equation is solved by the Fourier series method. Finally expressions are obtained for the surface current distribution and the scattered fields.

A. GENERAL FORMULATION FOR THE TE POLARIZATION CASE

This formulation is based on the second order radiation conditions mentioned in ref.[1]. For the TE polarization case, the problem reduces to solving an ordinary differential equation around the scatterer surface contour.

Consider a plane electromagnetic wave illuminating a two dimensional, perfectly conducting, convex shaped cylinder for the TE polarization case. The incident wave, propagating at an angle α with respect to the -x axis (see Fig. 1), is given by

$$H_{inc} = U_{inc} e^{j\omega t} \hat{z} \; ; \quad U_{inc} = e^{-jk(x\cos\alpha - y\sin\alpha)} \tag{1}$$

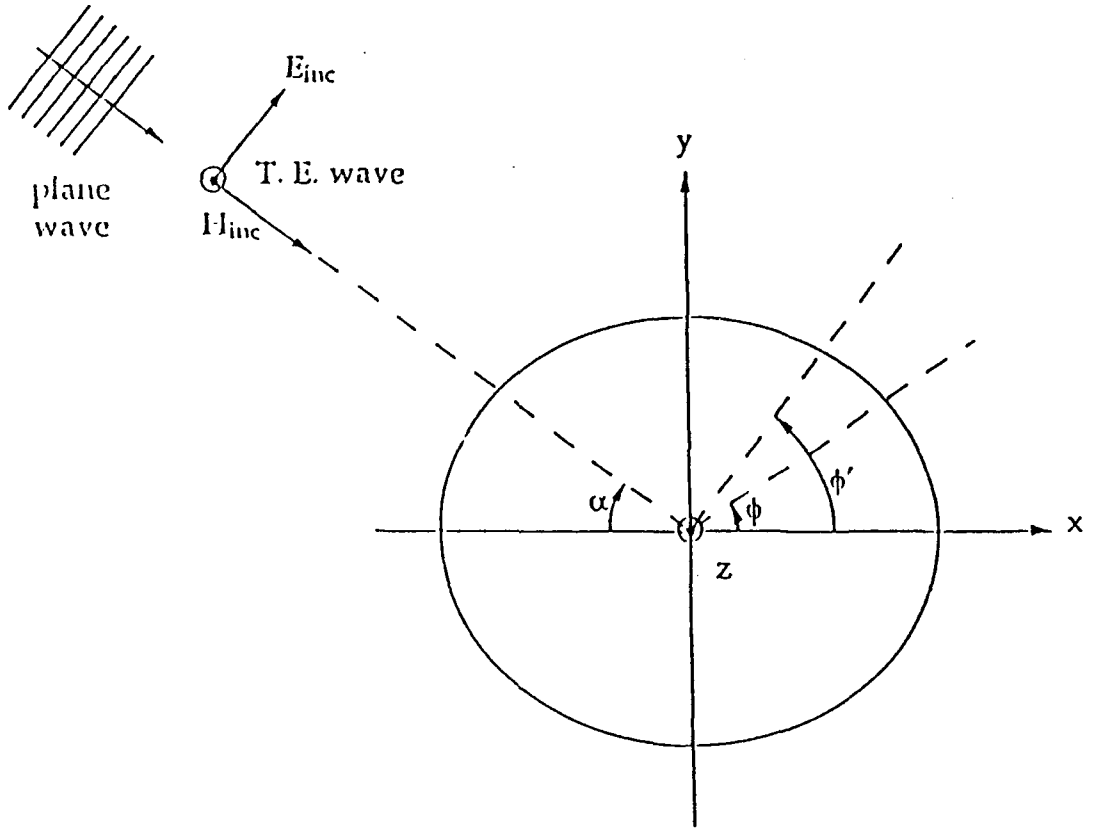

Fig.1 TE incident field

where $\hat{z}$ is the unit vector parallel to the cylinder axis; ω is the frequency of the incident wave ($k= \omega/c$); c is the is the speed of light in free space; x and y are the corresponding cartesian coordinates in the plane orthogonal to $\hat{z}$.

Since the problem is two dimensional, the scattered magnetic field (see Fig.2) is assumed to be

$$H_z^S = U_s(\bar{r}) e^{j\omega t} \hat{z} \tag{2}$$

The scattered field at any distance r is known from the values of the field $U_S(\bar{r}')$ and its normal derivative $\partial U_S(\bar{r}')/\partial v'$ on the surface of the scatterer as (ref.[6])

$$U_S(\bar{r}) = \int_C [G(\bar{r}|\bar{r}') \frac{\partial U_S}{\partial v'} - U_S(\bar{r}') \frac{\partial G(\bar{r}|\bar{r}')}{\partial v'}] ds' \tag{3}$$

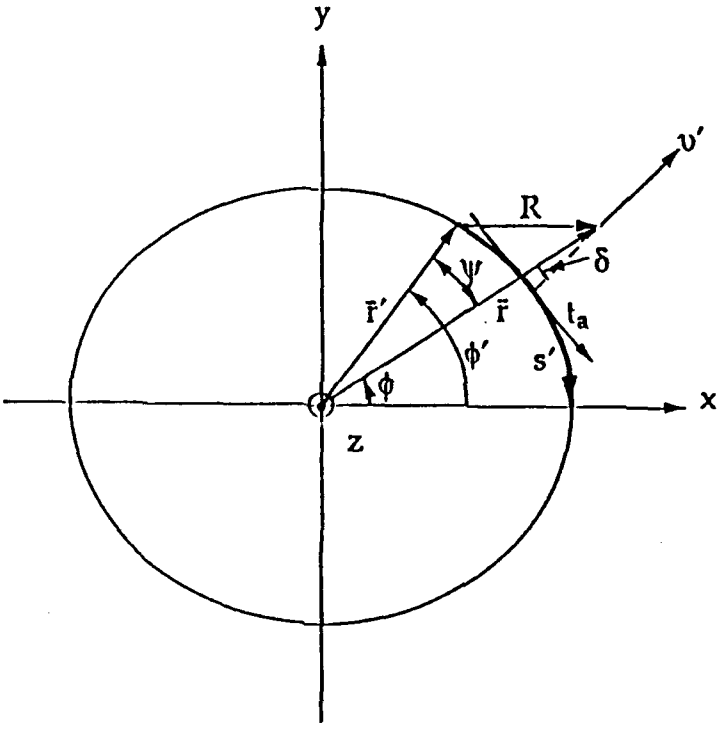

Fig.2 Geometry of the problem

where C represents the boundary of the cylinder's cross section; $\partial/\partial v'$ denotes an outward normal derivative on C; G is the two dimensional free space Green's function,

$$G(\bar{r}/\bar{r}') = \frac{1}{4j} H_o^{(2)}(kR) \tag{4}$$

$$R = |\bar{r} - \bar{r}'| = \sqrt{(x-x')^2 + (y-y')^2} \tag{5}$$

$\bar{r}$ and $\bar{r}'$ are co-ordinates of the field and source points, i.e. $\bar{r}=(x,y)$ and $\bar{r}'=(x',y')$, and s' is the arc length parameter.

For the TE case, vanishing of the total tangential electric field on the surface of the cylinder produces

$$\frac{\partial U_s}{\partial v'} = -\frac{\partial U_{inc}}{\partial v'} \ .$$

The objective is to determine U_s on the surface of the scatterer. For later purposes, it is necessary to obtain an expression for the far-zone fields. Using the asymptotic form of the Hankel function it can be shown that the far-zone scattered field is

$$U_s \sim B_o \frac{e^{-jkr}}{\sqrt{r}} \tag{7}$$

with

$$B_o = \frac{e^{j\pi/4}}{\sqrt{8k\pi}} \int_c \left[\frac{\partial U_s}{\partial v'} + jk\cos\delta U_{inc}\right] e^{jkr'\cos\psi} ds' \ , \tag{8}$$

where

$$\cos\delta = \hat{v}'.\hat{f} \quad \text{and} \quad \cos\psi = (\hat{r}'.\hat{f}) \ .$$

The true fields satisfy Sommerfeld's radiation condition (ref.[6]) on an infinitely large outer cylinder. However, in the OSRC technique, a higher order boundary condition such as

B_2 (defined below) is applied directly on the surface of the object. Implicit in this approximation is the fact that only outgoing waves are assumed to exist on the surface. Thus, the approximation is expected to have serious drawbacks in geometries where significant stored energy can exist such as in re-entrant and slit cylinders. In the sections to follow, we will be dealing precisely with these situations.

The form of second order boundary condition B_2 is given from (ref.[1]) with -jk in place of jk as

$$[-jk-\frac{\zeta(s')}{2}+\frac{\zeta^2(s')}{8[jk+\zeta(s')]}]+\frac{1}{2[jk+\zeta(s')]}\frac{\partial^2}{\partial s'^2}-\frac{\partial}{\partial v'} \quad , \qquad (9)$$

where $\zeta(s')$ is the curvature of the surface at **s'**.

The condition $B_2U_s = 0$ is applied to circularly shaped cylinders in the next section.

B. APPLICATION TO CIRCULAR CYLINDERS

At the surface of the perfect electric conductor, the boundary conditions on the total fields state that

$$\hat{n}\times\vec{H}=\vec{J}_s \quad ; \qquad \hat{n}\times\vec{E}=0 \qquad (10)$$

where J_s is the total surface current. For the TE case the scattered fields $(\vec{E}^S,\vec{H}^S)$ are

$$H_z^s = U_S \quad \text{and} \quad E_\phi^s = -\frac{1}{j\omega\mu_o}\frac{\partial U_S}{\partial v'} \ . \tag{11}$$

Hence we have $\partial(U_{inc}+U_s)/\partial v'=0$ and $J_\phi=-(U_s+U_{inc})$ on the surface, where v' is positive for convex surfaces and negative for concave surfaces.

For $\alpha=0$ in (1), the normal derivative of U_s can be rewritten as

$$\frac{\partial U_S}{\partial v'} = -\frac{\partial U_{inc}}{(\pm)\partial r'} = (\pm) - \frac{\partial}{\partial r'}\left(e^{-jkr'\cos\phi'}\right) = \pm jk\cos\phi' U_{inc} \ . \tag{12}$$

Substituting the expression for J_ϕ and (12) in (9), setting the curvature $\zeta(s')$ equal to $1/r_o$, and replacing $\frac{\partial^2}{\partial s'^2}$ by $\frac{1}{r'^2}\frac{\partial^2}{\partial\phi'^2}$

gives

$$C_2\frac{\partial^2 J_\phi}{\partial\phi'^2} - J_\phi = U_{inc}(1-C_1\cos\phi) - C_2\frac{\partial^2 U_{inc}}{\partial\phi'^2} \ , \tag{13}$$

where

$$C_1 = \pm\frac{8k^2r_o^2-8jkr_o}{8k^2r_o^2-12jkr_o-3} \quad \text{and} \quad C_2 = -\frac{4}{8k^2r_o^2-12jkr_o-3} \ . \tag{14}$$

In C_1, the upper sign is valid for convex surfaces and the lower one for concave surfaces. The above equation is valid on the surface $r'=r_o$ with J_ϕ being 2π periodic. Equation (13) is

linear with constant coefficients and can be solved using standard analytical or numerical methods. Analytical methods such as the method of undetermined coefficients and variation of parameters yielded complicated integrals for the solution of the second order non-homogeneous differential equation (13). Therefore (13) was solved using a Fourier series expansion.

1. SOLUTION OF THE DIFFERENTIAL EQUATION FOR THE SURFACE SCATTERED FIELD

Rewriting (11) in terms of U_S gives

$$C_2 \frac{\partial^2 U_S}{\partial \phi'^2} - U_S = C_1 \cos\phi' U_{inc} \ . \tag{15}$$

The incident field on the surface of the cylinder is

$$U_{inc}|_{r'=r_o} = e^{-jkr'\cos\phi'} \tag{16}$$

which may be expressed in a Fourier series as (ref.[6])

$$U_{inc}|_{r'=r_o} = \sum_{-\infty}^{\infty} j^{-n} J_n(kr_o)\, e^{jn\phi'} \ . \tag{17}$$

We will assume a complex exponential form for the Fourier series representation of U_S,

$$U_S = \sum_{n=-\infty}^{\infty} A_n e^{jn\phi'} \quad . \tag{18}$$

The coefficient A_n will be solved by substituting into equation (15). For U_s on the circular portion of the cylinder, we have

$$\frac{\partial U_S}{\partial \phi'} = j \sum_{n=-\infty}^{\infty} n A_n e^{jn\phi'} \tag{19}$$

and

$$\frac{\partial^2 U_S}{\partial \phi'^2} = - \sum_{n=-\infty}^{\infty} n^2 A_n e^{jn\phi'} \quad . \tag{20}$$

Substituting these into the Left Hand Side (LHS) of (15), we get

$$LHS = - \sum_{n=-\infty}^{\infty} (C_2 n^2 + 1) A_n e^{jn\phi'} \quad . \tag{21}$$

Now expanding the Right Hand Side (RHS) of (15),

$$C_1 U_{inc} \cos\phi' = C_1 \sum_{-\infty}^{\infty} j^{-n} J_n(kr_o)\, e^{jn\phi'} \frac{1}{2} (e^{j\phi'} + e^{-j\phi'})$$

$$= \frac{C_1}{2} \left[\sum_{-\infty}^{\infty} j^{-n} J_n(kr_o)\, e^{j(n+1)\phi'} + \sum_{-\infty}^{\infty} j^{-n} J_n(kr_o)\, e^{j(n-1)\phi'} \right] \quad . \tag{22}$$

We now let n+1=m → n=m-1 in the first series and n-1=m → n=m+1 in the second to obtain

$$RHS = -C_1 \sum_{-\infty}^{\infty} j^{-(m+1)} e^{jm\phi} J_m'(kr_o) . \qquad (23)$$

In obtaining (23), use was made of the expression $2J_m'(Kr_o) = J_{m-1}(Kr_o) - J_{m+1}(Kr_o)$. Equating the coefficients of (21) and (23) yields the following expression for the Fourier series coefficient A_n,

$$A_n(r_o) = \frac{C_1 j^{-(n+1)} J_n'(kr_o)}{1 + C_2 n^2} . \qquad (24)$$

We note that the coefficients are dependent on the radius r_o of the cylindrical surface. For numerical calculations, the series in (18) needs to be truncated. For determining the truncation value of n, we determine A_n for large n. For large values of the order n, we have from ref.[10]

$$J_n(kr_o) \sim \frac{1}{\sqrt{2\pi n}} \left(\frac{ekr_o}{2n}\right)^n . \qquad (25)$$

Also using the recurrence relation

$$J_n'(kr_o) = \frac{1}{2} \left[J_{n-1}(kr_o) - J_{n+1}(kr_o) \right] , \qquad (26)$$

we obtain for $\frac{ekr_o}{2n} < 1$

$$J_n'(kr_o) \sim \frac{1}{2\sqrt{2\pi n}} \left(\frac{ekr_o}{2n}\right)^{n-1} . \qquad (27)$$

Therefore for large n, A_n can be expressed as

$$A_n \sim \frac{C_1}{C_2} j^{-(n+1)} \frac{1}{2\sqrt{2\pi}} \left(\frac{ekr_o}{2}\right)^{n-1} \left(\frac{1}{n}\right)^{\frac{5}{2}} . \qquad (28)$$

Hence A_n decays very rapidly, for example

$$\frac{A_{10}}{A_1} \sim 10^{-11} .$$

2. SURFACE CURRENT DISTRIBUTION

Having found the surface magnetic field, it is a simple task to determine the surface current. The total electric surface current J_ϕ for the circular cylinder is given by

$$J_\phi = -(U_s + U_{inc}) . \qquad (29)$$

Substituting (17) and (18) in (29) gives

$$J_\phi = -\sum_{-\infty}^{\infty} [A_n + j^{-n} J_n(kr_o)] e^{jn\phi} . \qquad (30)$$

After expanding and simplifying, (30) reduces to

$$J_\phi = -\sum_0^\infty \epsilon_n \left[\frac{C_1 j^{-(n+1)} J_n'(kr_o)}{1+C_2 n^2} + j^{-n} J_n(kr_o) \right] \cos n\phi' \ , \qquad (31)$$

where $\epsilon_n = 1$ for n equal to zero and $\epsilon_n = 2$ for n not equal to zero, and C_1 and C_2 are given in (14). However for C_1 the positive sign is taken since the surface is convex. The magnitude of the normalized surface current can be expressed as

$$\left| \frac{J_\phi}{H_{inc}} \right| = \left| -\sum_0^\infty \epsilon_n j^{-n} \left[\frac{C_1 j^{-1} J_n'(kr_o)}{1+C_2 n^2} + J_n(kr_o) \right] \cos n\phi' \right| \ . \qquad (32)$$

3. FAR ZONE SCATTERED FIELDS

The far-zone scattered magnetic field H_z^s for the TE case given at (8) can also be expressed as (ref.[5] page 374)

$$H_z^s = \frac{e^{-jkr}}{\sqrt{r}} \frac{j}{4} \left(\frac{2j}{\pi k} \right)^{\frac{1}{2}} \int_C \left[e^{jk\hat{r}\cdot\bar{r}'} \frac{\partial H_z^s}{\partial \nu'} - H_z^s(\hat{r}, \nu') \, jk e^{jk\hat{r}\cdot\bar{r}'} \right] dC' \ , \qquad (33)$$

where C is a closed contour around the cylinder.

In our case, with reference to Fig.2, we have

$$e^{jk\hat{r}.\bar{r}'}=e^{jkr_o\cos\psi}=e^{jkr_o\cos(\phi'-\phi)}\ , \tag{34}$$

$$H_z^S=U_S=\sum_{-\infty}^{\infty}A_n e^{jn\phi'}\ , \tag{35}$$

$$(\hat{r}.\hat{r}')=\pm\cos(\phi'-\phi)\ , \tag{36}$$

$$\frac{\partial H_z^S}{\partial r'}=\mp k\sum_{-\infty}^{\infty}j^{-n}J_n'(kr_o)\,e^{jn\phi'}\ . \tag{37}$$

Substituting (34) through (37) in (33) and simplifying we get

$$H_z^S=-k\frac{e^{-jkr}}{\sqrt{r}}\frac{j}{4}\left(\frac{2j}{\pi k}\right)^{\frac{1}{2}}r_o\Big[\sum_{-\infty}^{\infty}j^{-n}J_n'(kr_o)\int_c e^{jkr_o\cos(\phi'-\phi)}\cdot e^{jn\phi'}d\phi'+\frac{1}{k}\sum_{-\infty}^{\infty}A_n\frac{\partial}{\partial r'}\Big|_{r'=r_o}\left(\int_c e^{jkr'\cos(\phi'-\phi)}e^{jn\phi'}d\phi'\right)\Big]\ . \tag{38}$$

In the derivation of the above equation, we made use of the fact that $jk\cos(\phi'-\phi)e^{jkr_o\cos(\phi'-\phi)}=\frac{\partial}{\partial r'}\Big|_{r'=r_o}\left(e^{jkr_o\cos(\phi'-\phi)}\right)$.

For a full cylinder we can further simplify equation (38) by solving the integral portion as

$$\int_0^{2\pi}e^{jkr_o\cos(\phi'-\phi)}e^{jn\phi'}d\phi'=(-1)^n 2\pi e^{\frac{-jn\pi}{2}}e^{jn\phi}J_n(kr_o)\ . \tag{39}$$

Substituting the above value in (38) and after some manipulation, we get

$$H_z^s = -k\frac{e^{jkr}}{\sqrt{r}}\frac{j}{4}\left(\frac{2j}{\pi k}\right)^{\frac{1}{2}} r_o\left[2\pi\epsilon_n\left[J_n'(kr_o)\,J_n(kr_o) + j^n A_n J_n'(kr_o)\right]\cos n\phi\right. . \qquad (40)$$

We list below a summary of the various quantities that were determined in this chapter.

$$\textit{Surface scattered field...}\; U_S\big|_{r'=r_o} = \frac{\sum_{-\infty}^{\infty} C_1 j^{-(n+1)} J_n'(kr_o)\, e^{jn\phi'}}{1+C_2 n^2}$$

$$\textit{Total surface field...}\; U_t\big|_{r'=r_o} = -\sum_{-\infty}^{\infty}\left[A_n + j^{-n}J_n(kr_o)\right] e^{jn\phi'}$$

$$\textit{Surface current.....}\; J_\phi = -\sum_{-\infty}^{\infty}\left[A_n + j^{-n}J_n(kr_o)\right] e^{jn\phi'}$$

$$\textit{Fourier coefficient.....}\; A_n = \frac{C_1 j^{-(n+1)} J_n'(kr_o)}{1+C_2 n^2} \sim \frac{C_1}{C_2} j^{-(n+1)} \frac{1}{2\sqrt{2\pi}} \left(\frac{eKr_o}{2n}\right)^{n-1}\left(\frac{1}{n}\right)^{\frac{5}{2}}$$

$$\textit{Far fields..}\quad H_z^s = -k\frac{e^{-jKr}}{\sqrt{r}}\frac{j}{4}\left(\frac{2j}{\pi k}\right)^{\frac{1}{2}} r_o\left[\sum_{-\infty}^{\infty} j^{-n} J_n'(kr_o)\int_C e^{jkr_o\cos(\phi'-\phi)}\right.$$
$$\left. .\, e^{jn\phi'}d\phi' + \frac{1}{k}\sum_{-\infty}^{\infty} A_n \frac{\partial}{\partial r'}\Big|_{r'=r_o}\left(\int_C e^{jkr'\cos(\phi'-\phi)} e^{jn\phi'}d\phi'\right)\right]$$

In the next chapter we will develop expressions for the case when the cylinder is comprised of more than one circular surface.

III. SCATTERING FROM CIRCULAR SLIT CYLINDERS: OSRC APPROACH

In this chapter we will consider the case of a slit cylinder with finite wall thickness. It is assumed that the thickness of the wall is electrically small so that the fields do not vary along it. Furthermore, we ignore the effect of the edges. Fig.3 shows the geometry of the slit circular cylinder. On the outer surface C_1, the body is convex, while on the inner surface C_3 it is concave. Since both outward and inward travelling waves can exist in the interior region, and since the OSRC technique assumes waves of only one kind, it is not expected to yield correct results for the geometry. We will use the results developed in the previous sections for the circular surfaces. To distinguish between the outer and inner surface we shall use superscripts (**b** for the outer radius and **a** for the inner radius) for the Fourier coefficients.

A. DETERMINATION OF SURFACE CURRENTS

The expression for the current distribution is given in (29) i.e. $J_\phi = -(U_s + U_{inc})$, where

$$U_s = \sum_{-\infty}^{\infty} A_n e^{jn\phi} \qquad (41)$$

and

$$U_{inc}=e^{-jkr'\cos\phi'} \quad . \tag{42}$$

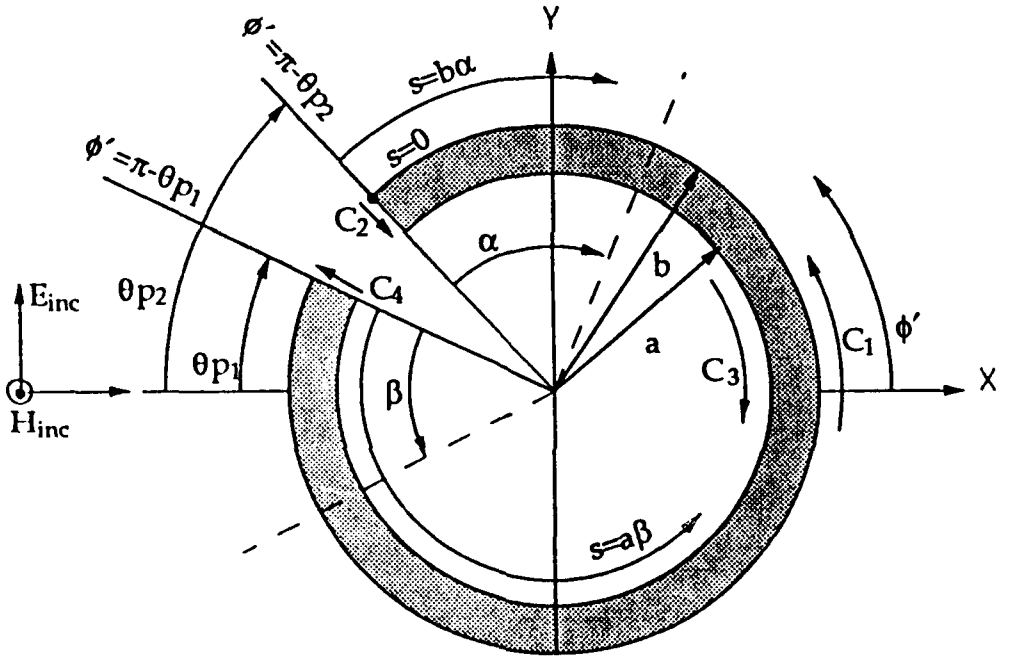

Fig.3 Circular slit cylinder

In Fig.3, θ_{p1} and θ_{p2} are the first and second slit angles from the -x axis respectively; **b** and **a** are the outer and inner radii of the cylinder respectively; **s** is the arc length and is taken as zero at $\phi'=\pi-\theta_{p2}$ at the outer contour C_1; the angles α and β are considered zero at $\phi'=\pi-\theta_{p2}$ and $\phi'=\pi-\theta_{p1}$ respectively; $s=b\alpha$ and $s=a\beta$ are the arc lengths of the outer and inner surfaces respectively.

For the outer contour C_1 we get from fig.3,

$$U_S=\sum_{-\infty}^{\infty} A_n^{(b)} e^{jn\phi'}\Big|_{\phi'=\pi-\theta_{p2}-\alpha}=\sum_{-\infty}^{\infty} A_n^{(b)} e^{jn(\pi-\theta_{p2}-\alpha)} \qquad (43)$$

and $$U_{inc}=e^{-jkbcos\phi'}\Big|_{\phi'=\pi-\theta_{p2}-\alpha}=e^{-jkbcos(\pi-\theta_{p2}-\alpha)} \ . \qquad (44)$$

Similarly for the inner contour C_2 we get

$$U_S=\sum_{-\infty}^{\infty} A_n^{(a)} e^{jn\phi'}\Big|_{\phi'=\beta+\pi-\theta_{p1}}=\sum_{-\infty}^{\infty} A_n^{(a)} e^{jn(\pi+\beta-\theta_{p1})} \qquad (45)$$

and $$U_{inc}=e^{-jkacos\phi'}\Big|_{\phi'=\beta+\pi-\theta_{p1}}=e^{-jka(\pi+\beta-\theta_{p1})} \ . \qquad (46)$$

Therefore the current distribution for the outer and inner contours can be obtained by substituting (43) and (44) for the outer surface, and (45) and (46) for the inner surface respectively in (29). After simplification the following expression is obtained for the magnitude of the normalized surface currents.

For the outer contour C_1 we have

$$\left|\frac{J_\phi}{H_{inc}}\right|=\left|-\left[\sum_{0}^{\infty}\epsilon_n A_n^{(b)}(-1)^n cosn(\alpha+\theta_{p2})+e^{jkbcos(\theta_{p2}+\alpha)}\right]\right| \ , \qquad (47)$$

where α varies from 0 to $2\pi-\overline{\theta_{p1}-\theta_{p2}}$; $C_1^{(b)}$ and $C_2^{(b)}$ in $A_n^{(b)}$ are given by the expressions from (16) as

$$C_1^{(b)}=+\frac{8k^2b^2-8jkb}{8k^2b^2-12jkb-3} \quad \text{and} \quad C_2^{(b)}=-\frac{4}{8k^2b^2-12jkb-3} \ . \qquad (48)$$

For the inner contour C_3 we have

$$\left|\frac{J_\phi}{H_{inc}}\right| = \left| -\left[\sum_0^\infty \epsilon_n A_n^{(a)} (-1)^n \cos n(\beta-\theta_{p1}) + e^{jka\cos(\beta-\theta_{p1})}\right]\right| , \qquad (49)$$

where β varies from 0 to $2\pi-\overline{\theta_{p1}-\theta_{p2}}$; $C_1^{(a)}$ and $C_2^{(a)}$ are similar except that a negative sign is taken for C_1 in (12) since the inner contour is a concave surface.

Along the linear portions C_2 and C_4, we assume that the current varies linearly between the inner and outer curved surfaces.

B. DETERMINATION OF THE FAR FIELDS

From Fig.4 the total scattered magnetic field H_z^S for the hollow circular slit cylinder with finite thickness will be

$$H_z^S = H_{zC_1}^S + H_{zC_2}^S + H_{zC_3}^S + H_{zC_4}^S , \qquad (50)$$

where C_1 and C_3 are the outer and inner contours with radii **b** and **a** respectively; $\overline{\theta_{p1}-\theta_{p2}}$ is the total slit extent.

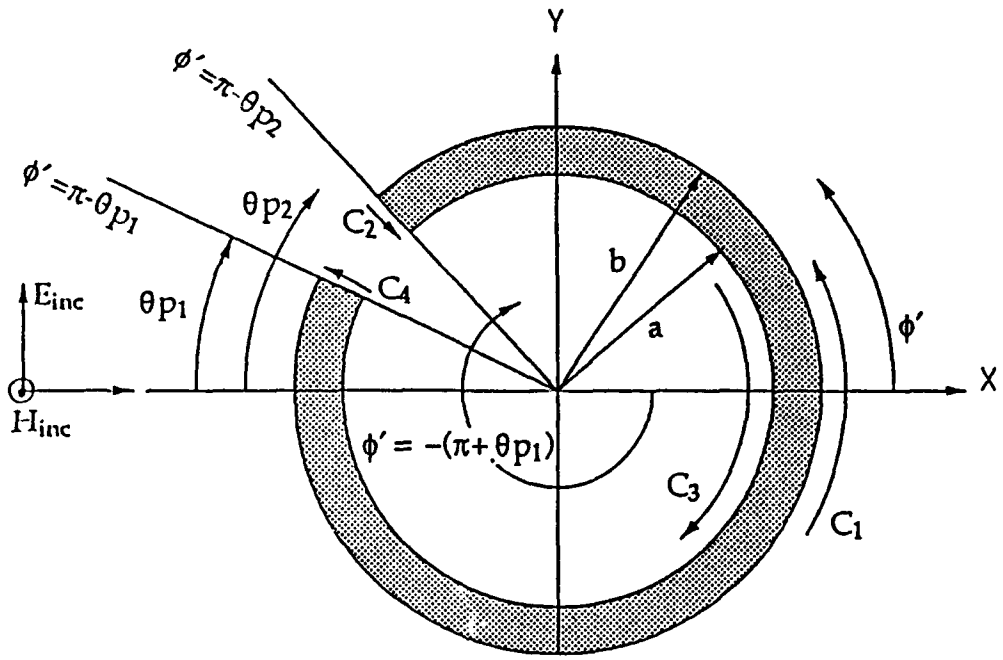

Fig.4 Slit cylinder geometry

From Fig.4 the integration limits in (50) would be

$$\int_C = \underset{r_o=b}{\int_{C_1}} + \underset{r_o=b\to a}{\int_{C_2}} + \underset{r_o=a}{\int_{C_3}} + \underset{r_o=a\to b}{\int_{C_4}} . \qquad (51)$$

For the convex contour C_1, (38) is used with $r_o=b$ and $A_n=A_n^{(b)}$ where

$$A_n^{(b)} = \frac{C_1^{(b)} j^{-(n+1)} J_n'(kb)}{1+C_2^{(b)} n^2} . \qquad (52)$$

Substituting these values in (38) we get

$$H^{s}_{z_{c_1}} = -k\frac{e^{-jkr}}{\sqrt{r}}\frac{j}{4}\left(\frac{2j}{\pi k}\right)^{\frac{1}{2}} b\Big[\sum_{-\infty}^{\infty} j^{-n} J_n'(kb)\int_{C_1} e^{jkb\cos(\phi'-\phi)}$$
$$\cdot e^{jn\phi'}d\phi' + \frac{1}{k}\sum_{-\infty}^{\infty} A_n^{(b)}\frac{\partial}{\partial r'}\Big|_{r'=b}\Big(\int_{C_1} e^{jkr'\cos(\phi'-\phi)} e^{jn\phi'}d\phi'\Big]\ . \qquad (53)$$

The integral over C_1 is evaluated in the following fashion:

$$\int_{C_1} e^{jkb\cos(\phi'-\phi)} e^{jn\phi'}d\phi'$$
$$= \int_{0}^{2\pi} e^{jkb\cos(\phi'-\phi)} e^{jn\phi'} - \int_{\pi-\theta_{p2}}^{\pi-\theta_{p1}} e^{jkb\cos(\phi'-\phi)} e^{jn\phi'}d\phi'\ . \qquad (54)$$

The first integral can be evaluated in a closed form as in the previous chapter. In the second integral we let $u=\pi-\phi' \Rightarrow d\phi'=-du$. After some manipulation we get

$$\int_{C_1} e^{jkb\cos(\phi'-\phi)} e^{jn\phi'}d\phi'$$
$$= (-1)^n\Big[2\pi j^{-n} J_n(kb)\, e^{jn\phi} - \int_{\theta_{p1}}^{\theta_{p2}} e^{-jkb\cos(u+\phi)} e^{-jn\phi}du\Big]\ . \qquad (55)$$

Substituting (55) in (53) and carrying out some manipulation we get

$$H^S_{z_{C_1}} = -k\frac{e^{-jkr}}{\sqrt{r}}\frac{j}{4}\left(\frac{2j}{\pi k}\right)^{\frac{1}{2}} b\Big[\sum_0^{\infty} 2\pi\epsilon_n \big[J_n'(kb)\,J_n(kb) + j^{\,n}A_n^{(b)} \cdot J_n'(kb)\big]\cos n\phi - \sum_0^{\infty}\epsilon_n j^{\,n}J_n'(kb)\int_{\theta_{p1}}^{\theta_{p2}} e^{-jkb\cos(u+\phi)}\cos(nu)\,du + j\sum_0^{\infty}\epsilon_n A_n^{(b)}(-1)^n\int_{\theta_{p1}}^{\theta_{p2}}\cos(u+\phi)\,e^{-jkb\cos(u+\phi)}\cos(nu)\,du\Big] \quad . \qquad (56)$$

For the inner contour C_3 with radius **a**, which has a concave surface we have

$$\frac{\partial H_z^S}{\partial \nu'} = k\sum_{-\infty}^{\infty} j^{-n}J_n'(ka)\,e^{jn\phi'} \quad ,$$

$$H_z^S = \sum_{-\infty}^{\infty} A_n(a)\,e^{jn\phi'} \quad ,$$

$$A_n^{(a)} = \frac{C_1^{(a)}\,j^{-(n+1)}J_n'(ka)}{1 + C_2^{(a)}n^2} \quad ,$$

$$(f.\,\nu') = -\cos(\phi' - \phi) \quad ,$$

and $$\int_{C_3} ds' = \int_{C_3} a\,d\phi' = -a\int_{C_1} d\phi' \quad .$$

Substituting these values in (33) gives

$$H^S_{z_{C_3}} = -k\frac{e^{-jkr}}{\sqrt{r}}\frac{j}{4}\left(\frac{2j}{\pi k}\right)^{\frac{1}{2}} a\Big[\sum_{-\infty}^{\infty} j^{-n}J_n'(kr_o)\int_{C_3} e^{jkr_o\cos(\phi'-\phi)} \cdot e^{jn\phi'}\,d\phi' + \frac{1}{k}\sum_{-\infty}^{\infty} A_n\frac{\partial}{\partial r'}\Big|_{r'=a}\left(\int_{C_3} e^{jkr'\cos(\phi'-\phi)}\,e^{jn\phi'}\,d\phi'\right)\Big] \quad . \qquad (57)$$

Following the same procedure as for the contour C_1, we can simplify the integral portion and after some manipulations we get

$$H_{z_{C_3}}^{S} = -k\frac{e^{-jkr}}{\sqrt{r}}\frac{j}{4}\left(\frac{2j}{\pi k}\right)^{\frac{1}{2}} a\left[\sum_{0}^{\infty} 2\pi\epsilon_n\left[(-1)^n J_n'(ka)\,J_n(ka) - j^{-n}A_n^{(a)}\right.\right.$$
$$\left.\cdot J_n'(ka)\right]\cos n\phi - \sum_{0}^{\infty}\epsilon_n j^n J_n'(ka)\int_{\theta_{p1}}^{\theta_{p2}} e^{-jka\cos(u+\phi)}\cos(nu)\,du$$
$$\left.+ j\sum_{0}^{\infty}\epsilon_n A_n^{(a)}(-1)^n\int_{\theta_{p1}}^{\theta_{p2}}\cos(u+\phi)\,e^{-jka\cos(u+\phi)}\cos(nu)\,du\right] . \qquad (58)$$

For solving the linear portions C_2 and C_4, we can ignore the edge effects since the OSRC approach does not currently provide edge current singularity behaviour as mentioned in ref.[1]. Therefore, for the linear portions we let U_s be equal to the average of U_s on the contours C_1 and C_3 at the position of the linear portion.

For contour C_2 we have

$$U_S|_{C_2} = \frac{U_S|_{C_1} + U_S|_{C_3}}{2}\Big|_{\phi'=\pi-\theta_{p2}} = \sum_{-\infty}^{\infty}\left[\frac{A_n^{(b)} + A_n^{(a)}}{2}\right] e^{jn\phi'}\Big|_{\phi'=\pi-\theta_{p2}} , \qquad (59)$$

$$\frac{\partial H_z^S}{\partial v'} = \frac{\partial}{\partial r'}\left(e^{-jkr'\cos\phi'}\right) = -jk\cos(\pi-\theta_{p2})\,e^{-jkr'\cos(\pi-\theta_{p2})} \qquad (60)$$

and
$$(\hat{f}\cdot\hat{\rho}') = \cos(\phi'-\phi) = \cos(\pi-\theta_{p2}-\phi) . \qquad (61)$$

Substituting these values in (33) and simplifying we get

$$H^S_{z_{C_2}} = \frac{e^{-jkr}}{\sqrt{r}} \frac{j}{4} \left(\frac{2j}{\pi k}\right)^{\frac{1}{2}} \left[\frac{\cos\theta_{p2}}{\cos\theta_{p2} - \cos(\theta_{p2}+\phi)} \left[e^{jka[\cos\theta_{p2} - \cos(\theta_{p2}+\phi)]}\right.\right.$$
$$\left. - e^{jkb[\cos\theta_{p2} - \cos(\theta_{p2}+\phi)]}\right] - \sum_{-\infty}^{\infty} \left[\frac{A_n^{(b)} + A_n^{(a)}}{2}\right] \epsilon_n (-1)^n \cos n\theta_{p2}$$
$$\left. \cdot \left[e^{-jka\cos(\theta_{p2}+\phi)} - e^{-jkb\cos(\theta_{p2}+\phi)}\right]\right] . \qquad (62)$$

Similarly for the linear portion C_4 we have from Fig.4

$$U_S|_{C_4} = \frac{U_S|_{C_1} + U_S|_{C_3}}{2}\Big|_{\phi' = \pi - \theta_{p1}} = \sum_{-\infty}^{\infty} \left[\frac{A_n^{(b)} + A_n^{(a)}}{2}\right] e^{jn\phi'}\Big|_{\phi' = \pi - \theta_{p1}} , \qquad (63)$$

$$\frac{\partial H_z^S}{\partial v'} = -jk\cos(\pi - \theta_{p1})\, e^{-jkr'\cos(\pi - \theta_{p1})} \qquad (64)$$

$$(\hat{f} \cdot \hat{v}') = \cos(\pi - \theta_{p1} - \phi) . \qquad (65)$$

Substituting values in (33) and after simplification we get

$$H^S_{z_{C_4}} = \frac{e^{-jkr}}{\sqrt{r}} \frac{j}{4} \left(\frac{2j}{\pi k}\right)^{\frac{1}{2}} \left[\frac{\cos\theta_{p1}}{\cos\theta_{p1} - \cos(\theta_{p1}+\phi)} \left[e^{jka[\cos\theta_{p1} - \cos(\theta_{p1}+\phi)]}\right.\right.$$
$$\left. - e^{jkb[\cos\theta_{p1} - \cos(\theta_{p1}+\phi)]}\right] - \sum_{-\infty}^{\infty} \left[\frac{A_n^{(b)} + A_n^{(a)}}{2}\right] \epsilon_n (-1)^n \cos n\theta_{p1}$$
$$\left. \cdot \left[e^{-jka\cos(\theta_{p1}+\phi)} - e^{-jkb\cos(\theta_{p1}+\phi)}\right]\right] . \qquad (66)$$

In summary, the total scattered magnetic field for the hollow circular slit cylinder is

$$H_z^S=\frac{e^{-jkr}}{\sqrt{r}}\frac{j}{4}\left(\frac{2j}{\pi k}\right)^{\frac{1}{2}}\left[-kb\left[\sum_0^{\infty}2\pi\epsilon_n\left[J_n(kb)\,J_n'(kb)\right.\right.\right.$$

$$\left.+j^{\,n}A_n^{(b)}J_n'(kb)\right]\cos n\phi'-\sum_0^{\infty}\epsilon_n j^{\,n}J_n'(kb)\int_{\theta_{p1}}^{\theta_{p2}}e^{-jkb\cos(u+\phi)}\cos(nu)\,du$$

$$+j\sum_0^{\infty}\epsilon_n A_n^{(b)}(-1)^n\int_{\theta_{p1}}^{\theta_{p2}}\cos(u+\phi)\cos(nu)\,e^{-jkb\cos(u+\phi)}\,du\Big]$$

$$-ka\left[\sum_0^{\infty}2\pi\epsilon_n\left[(-1)^nJ_n(ka)\,J_n'(ka)-j^{-n}A_n^{(a)}J_n'(ka)\right]\cos n\phi\right.$$

$$-\sum_0^{\infty}\epsilon_n j^{\,n}J_n'(ka)\int_{\theta_{p1}}^{\theta_{p2}}e^{jka\cos(u+\phi)}\cos(nu)\,du$$

$$+j\sum_0^{\infty}\epsilon_n A_n^{(a)}(-1)^n\int_{\theta_{p1}}^{\theta_{p2}}\cos(u+\phi)\,e^{jka\cos(u+\phi)}\cos(nu)\,du\Big]$$

$$-\left[\frac{\cos\theta_{p2}}{\cos\theta_{p2}-\cos(\theta_{p2}+\phi)}\left[e^{jka[\cos\theta_{p2}-\cos(\theta_{p2}+\phi)]}-e^{jkb[\cos\theta_{p2}-\cos(\theta_{p2}+\phi)]}\right]\right.$$

$$-\sum_0^{\infty}\epsilon_n(-1)^n\left[\frac{A_n^{(b)}+A_n^{(a)}}{2}\right]\cos n\theta_{p2}\left[e^{-jka\cos(\theta_{p2}+\phi)}-e^{-jkb\cos(\theta_{p2}+\phi)}\right]\Big]$$

$$-\left[\frac{\cos\theta_{p1}}{\cos\theta_{p1}-\cos(\theta_{p1}+\phi)}\left[e^{jka[\cos\theta_{p1}-\cos(\theta_{p1}+\phi)]}-e^{jkb[\cos\theta_{p1}-\cos(\theta_{p1}+\phi)]}\right]\right.$$

$$\left.-\sum_0^{\infty}(-1)^n\left[\frac{A_n^{(b)}+A_n^{(a)}}{2}\right]\cos n\theta_{p1}\left[e^{-jka\cos(\theta_{p1}+\phi)}-e^{-jkb\cos(\theta_{p1}+\phi)}\right]\right]\Big] \qquad (67)$$

The scattering width $\sigma_{2\text{-}D}$ is defined as (ref.[6])

$$\sigma_{2\text{-}D}=2\pi r\frac{|H_z^S|^2}{|H_z^{inc}|^2}\ . \qquad (68)$$

For the scattered field given in (49) we have

$$\sigma_{2\text{-}D}=\frac{1}{4k}\Big|-kb\Big[\sum_{0}^{\infty}2\pi\epsilon_n\left[J_n(kb)\,J_n'(kb)+j\,{}^{n}A_n^{(b)}\,J_n'(kb)\right]\cos n\phi'$$

$$-\sum_{0}^{\infty}\epsilon_n j\,{}^{n}J_n'(kb)\int_{\theta_{p1}}^{\theta_{p2}}e^{-jkbcos(u+\phi)}\cos(nu)\,du$$

$$+j\sum_{0}^{\infty}\epsilon_n A_n^{(b)}\,(-1)^n\int_{\theta_{p1}}^{\theta_{p2}}\cos(u+\phi)\cos(nu)\,e^{-jkbcos(u+\phi)}\,du\Big]$$

$$-ka\Big[\sum_{0}^{\infty}2\pi\epsilon_n\left[(-1)^n J_n(ka)\,J_n'(ka)-j\,{}^{-n}A_n^{(a)}\,J_n'(ka)\right]\cos n\phi$$

$$-\sum_{0}^{\infty}\epsilon_n j\,{}^{n}J_n'(ka)\int_{\theta_{p1}}^{\theta_{p2}}e^{jKacos(u+\phi)}\cos(nu)\,du$$

$$+j\sum_{0}^{\infty}\epsilon_n A_n^{(a)}\,(-1)^n\int_{\theta_{p1}}^{\theta_{p2}}\cos(u+\phi)\,e^{jkacos(u+\phi)}\cos(nu)\,du\Big]$$

$$-\Big[\frac{\cos\theta_{p2}}{\cos\theta_{p2}-\cos(\theta_{p2}+\phi)}\left[e^{jka[\cos\theta_{p2}-\cos(\theta_{p2}+\phi)]}-e^{jkb[\cos\theta_{p2}-\cos(\theta_{p2}+\phi)]}\right]$$

$$-\sum_{0}^{\infty}\epsilon_n(-1)^n\Big[\frac{A_n^{(b)}+A_n^{(a)}}{2}\Big]\cos n\theta_{p2}\left[e^{-jkacos(\theta_{p2}+\phi)}-e^{-jkbcos(\theta_{p2}+\phi)}\right]\Big]$$

$$-\Big[\frac{\cos\theta_{p1}}{\cos\theta_{p1}-\cos(\theta_{p1}+\phi)}\left[e^{jka[\cos\theta_{p1}-\cos(\theta_{p1}+\phi)]}-e^{jkb[\cos\theta_{p1}-\cos(\theta_{p1}+\phi)]}\right]$$

$$-\sum_{0}^{\infty}(-1)^n\Big[\frac{A_n^{(b)}+A_n^{(a)}}{2}\Big]\cos n\theta_{p1}\left[e^{-jkacos(\theta_{p1}+\phi)}-e^{-jkbcos(\theta_{p1}+\phi)}\right]\Big]\Big|^2 \quad . (69)$$

IV. NUMERICAL RESULTS AND DISCUSSION

All analyses were carried out for the TE polarization case and when the incident wave is incident from the -x axis, i.e. from ϕ=180°. In all cases **a** and **b** denote the inner and outer radii of the cylinders.

For the circular cylinder, Figs. 5 and 6 show the comparisons of the normalized surface current distribution between the present formulation (32) and of ref.[1], both using the OSRC approach for kb=5 and 10 respectively. The results are comparable, checking the validity of our formulation. It was shown in ref.[1], that the results approach the exact solution. The computer program to compute the normalized surface current distribution using the OSRC approach is given in Appendix A. Figs. 7,8 and 9 show the comparisons of the bistatic widths for the circular cylinder between the present formulation and ref.[8] for kb=1,5 and 10 respectively. Ref.[8] uses the rigorous moment method technique for determining the scattering cross sections of infinite cylinders of arbitrary geometrical cross section. The results again are in good agreement. The computer program to compute the scattering width for circular cylinders using the OSRC approach is given in Appendix B. Figs.10 and 11 show the comparisons of forward (ϕ=0°) and backward (ϕ=180°) scattering

widths respectively, versus kb, between the computed and ref.[5] (exact solution). It is seen that the results due to the OSRC approach show some deviation at higher frequencies (i.e. for large kb) in both cases. The computer program to compute $\sigma/\pi b$ versus kb is given in Appendix C.

For the hollow circular slit cylinder, Fig.12 shows the comparison of the normalized surface current distribution for a hollow semi-circular cylinder obtained using the present formulation (47) and (49) and that in ref.[7]. The problem of scattering in ref.[7] is solved by means of an integral equation. In Fig.14, the current density, normalized to unity for the magnetic field strength of the incident wave, is plotted versus perimeter **s** normalized to unity for the half perimeter. The distance variable **s** is taken equal to zero at the point on the center face nearest the source of the incident wave. Moving clockwise around the perimeter, **s** increases in value reaching the value **s**=1 at the center face furthest from the source. The results again are comparable to the exact solution. The computer program to compute (47) and (49) for the circular slit cylinder is given at Appendix D. Fig.13 shows the comparison of the scattering width. The results obtained using the OSRC approach are quite different from the exact solutions as shown in Fig.13. This is in spite of the good agreement in the magnitude of the current distribution. At this point it seemed necessary to carry out some phase comparisons for the current distribution, however

no known results were available. The computer program in Appendix E was thoroughly checked for any bugs and even tested for circular cylinders which yielded satisfactory results. It is felt that the disagreement in the scattering width due to the OSRC method is problably due to the incorrect modeling of phase of the current distribution. The OSRC technique assumes waves of only one kind (outgoing) to exist on the scattering surfaces which may not yield correct results for concave structures.

Fig.14 shows comparisons of forward scattering width (ϕ=0°) for the hollow circular slit cylinder (b=a and total slit angle of 10°) between the OSRC approach and the full circular cylinder from ref.[5] (exact solution). Fig.15 shows the comparison between the OSRC approach and ref.[5] results for the hollow circular slit cylinder. Such shapes are expected to have cross section resonances, indicating that there is interior information contained in the exterior scattering data. Here, again the results are quite different. Another peculiar result was that we were getting the same results for forward and backward scattering widths using the computer program given in Appendix F. This might be due to some unforseen error in the program.

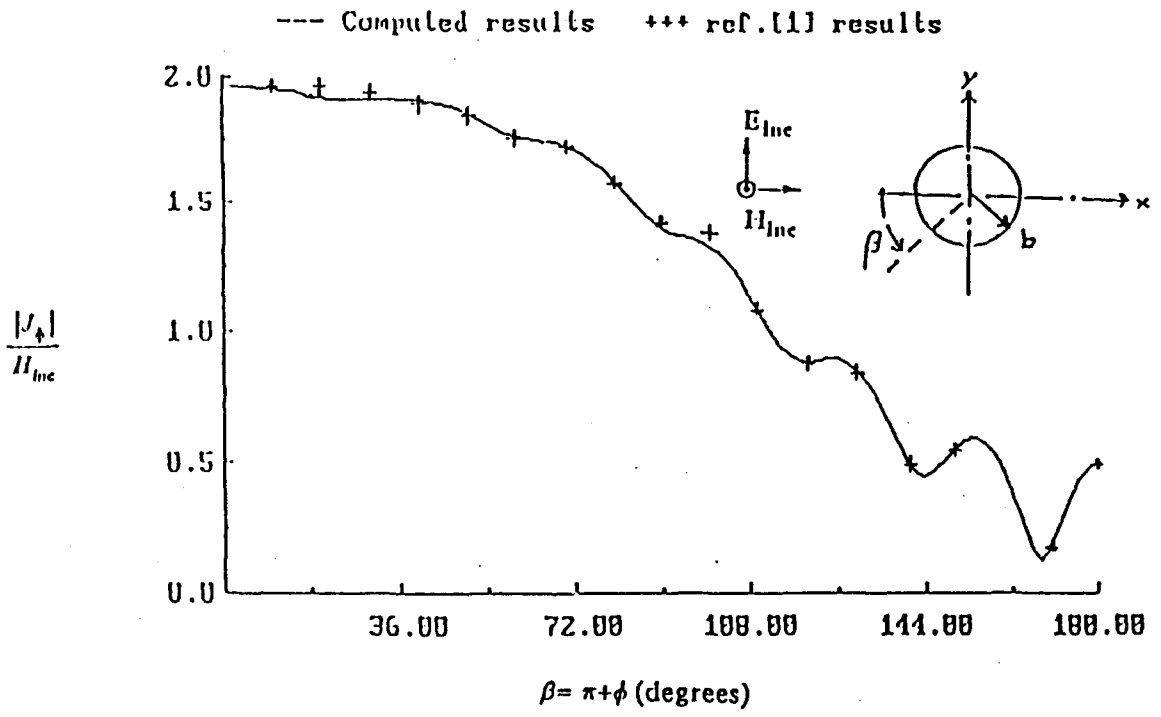

Fig.5 Comparison of surface current distribution on a PEC circular cylinder with kb=5.

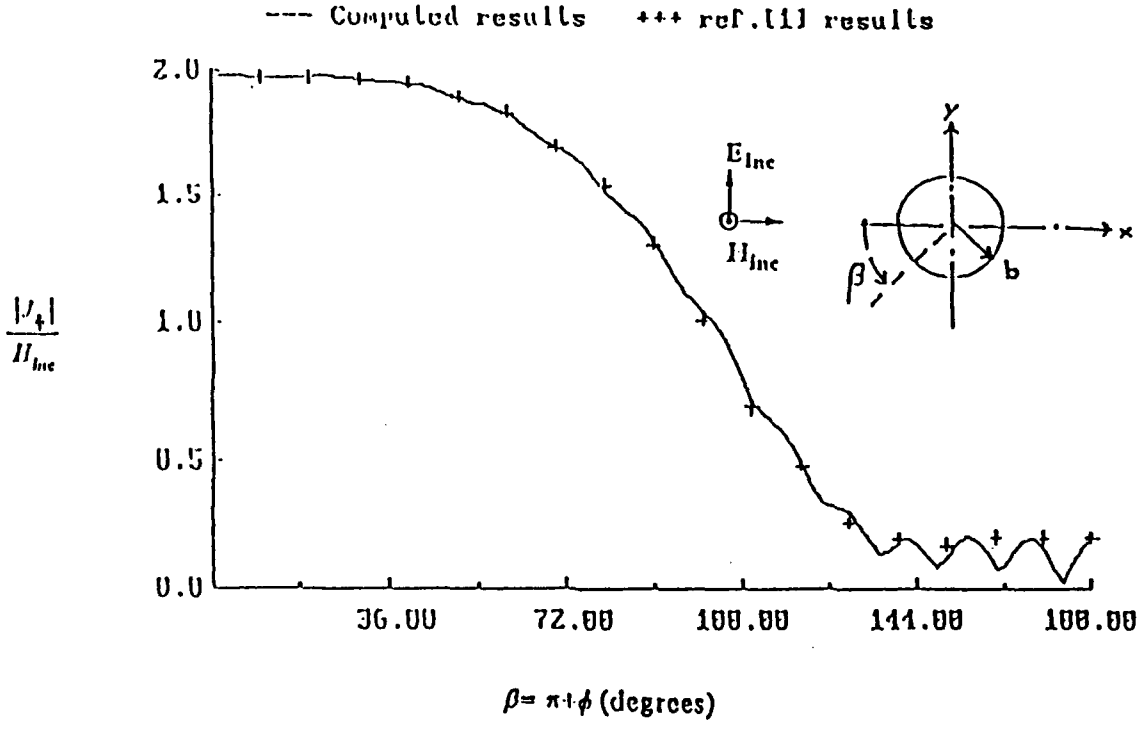

Fig.6 Comparison of surface current distribution on a PEC cylinder with kb=10

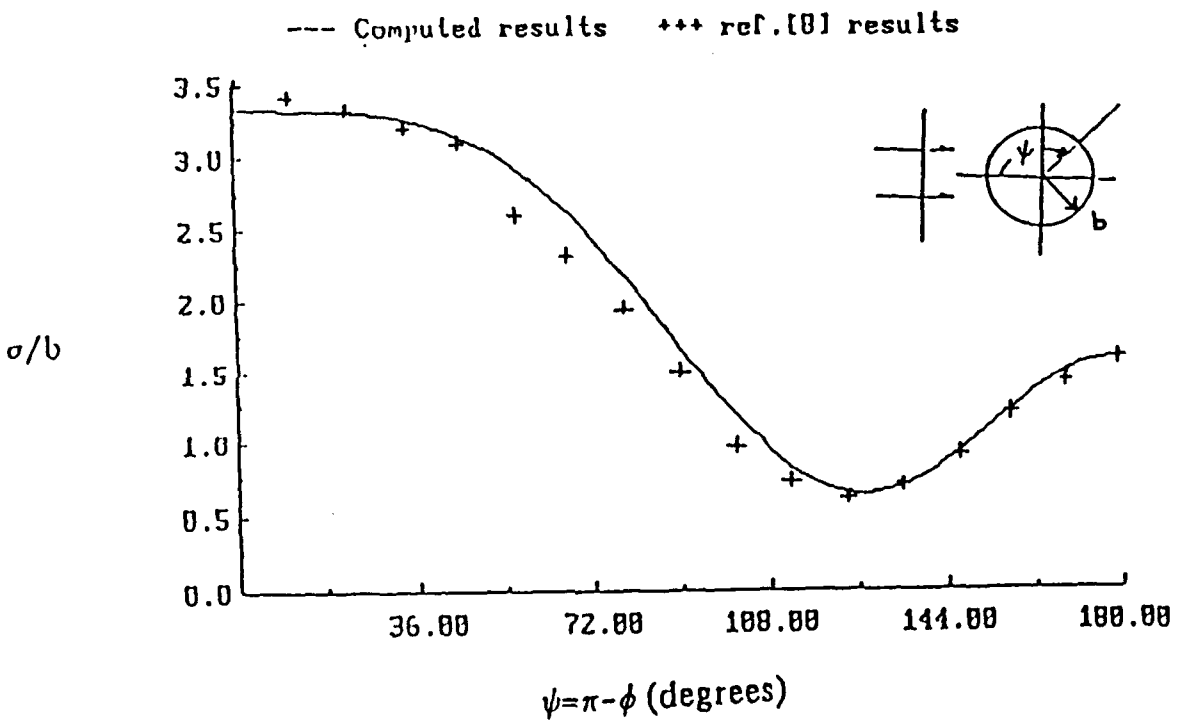

Fig.7 Comparison of bistatic scattering width for the circular cylinder with kb=1.

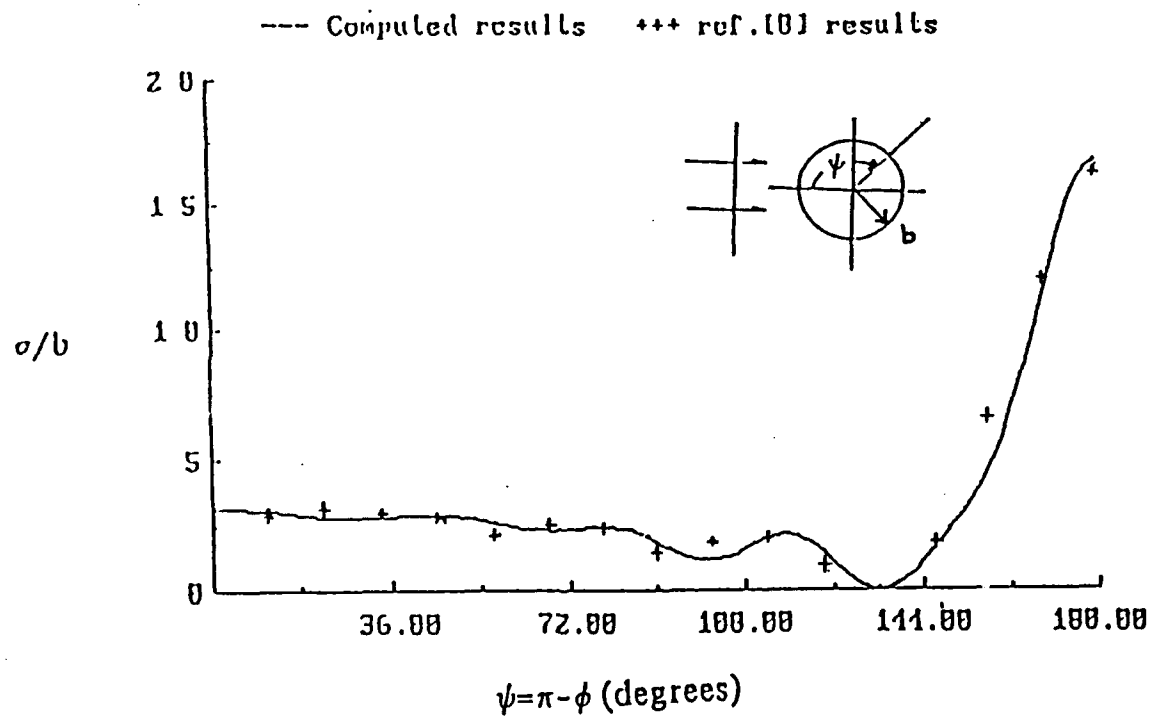

Fig.8 Comparison of bistatic scattering width for the circular cylinder with kb=5.

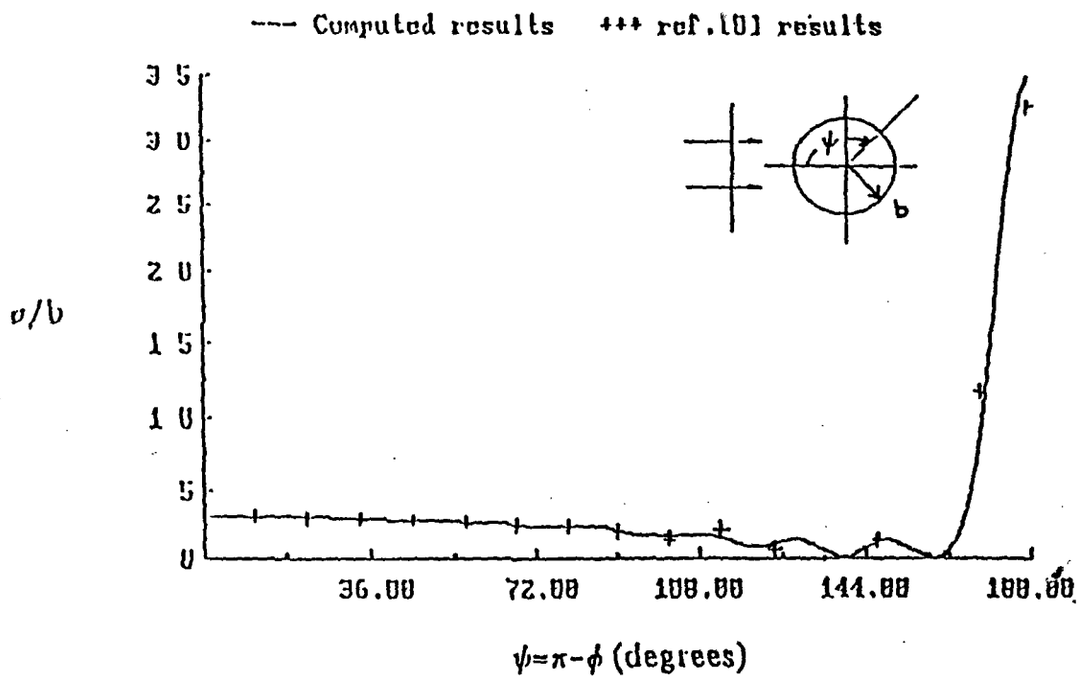

Fig.9 Comparison of bistatic scattering width for the circular cylinder with kb=10.

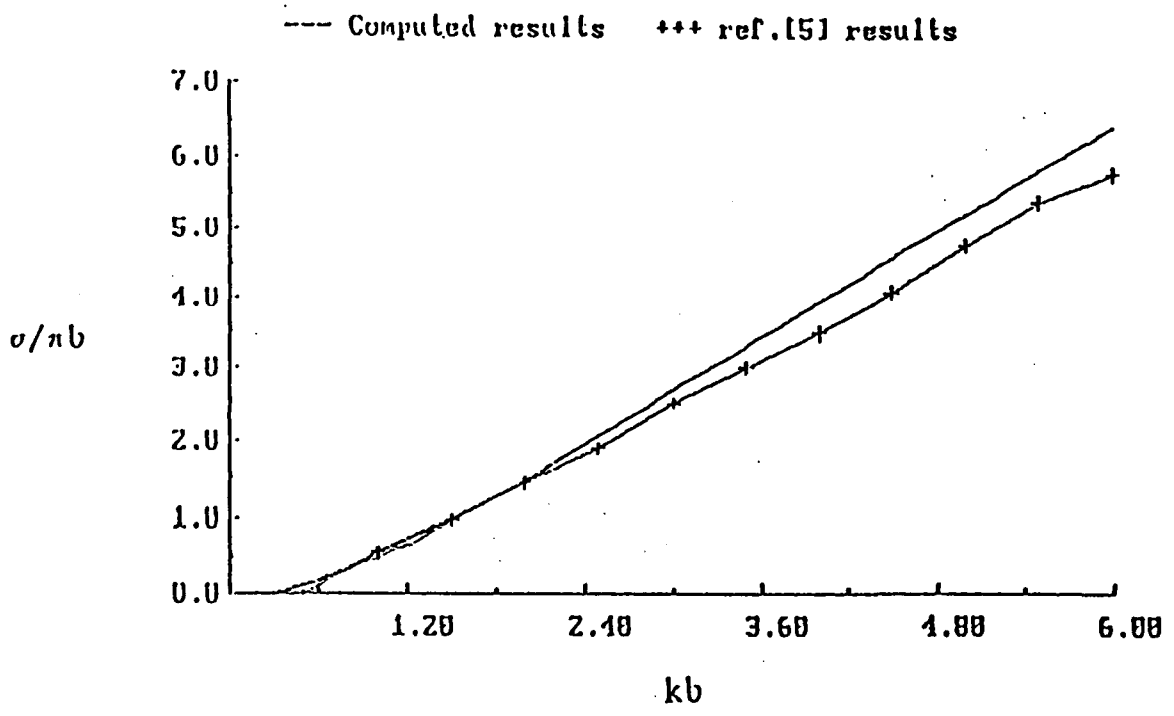

Fig.10 Comparison of forward scattering width (ϕ=0°) for the circular cylinder between the computed (OSRC approach) and ref.[5] (exact solution).

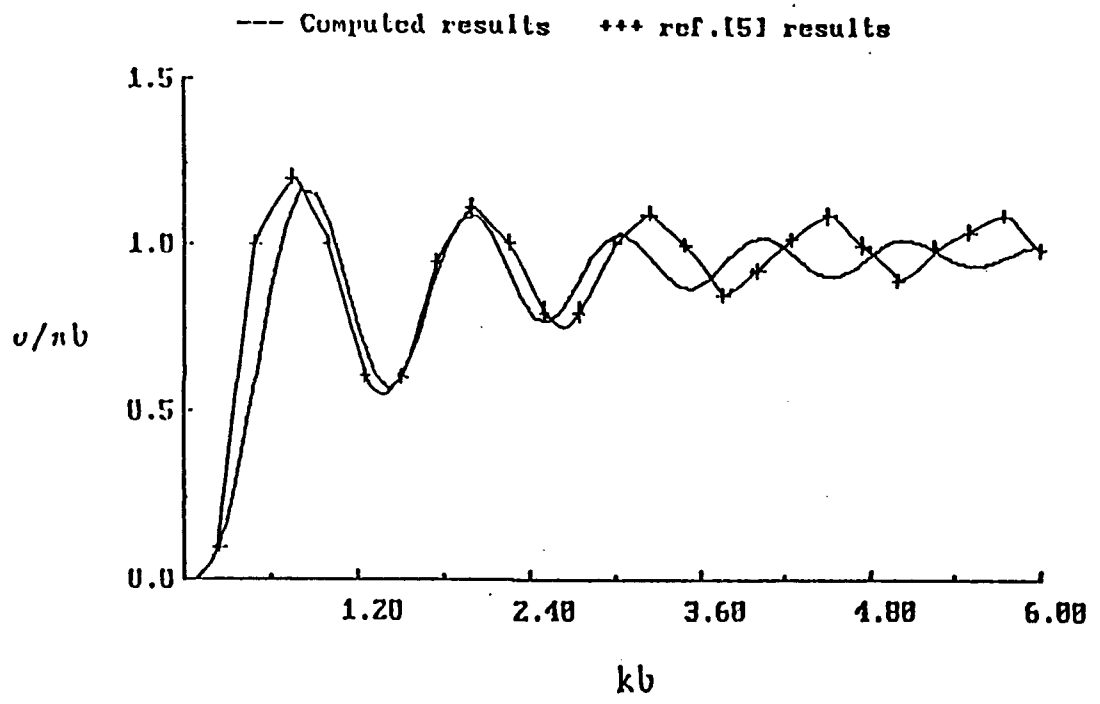

Fig.11 Comparison of backward scatting width (ϕ=180°) for the circular cylinder between the computed (OSRC approach) and ref.[5] (exact solution).

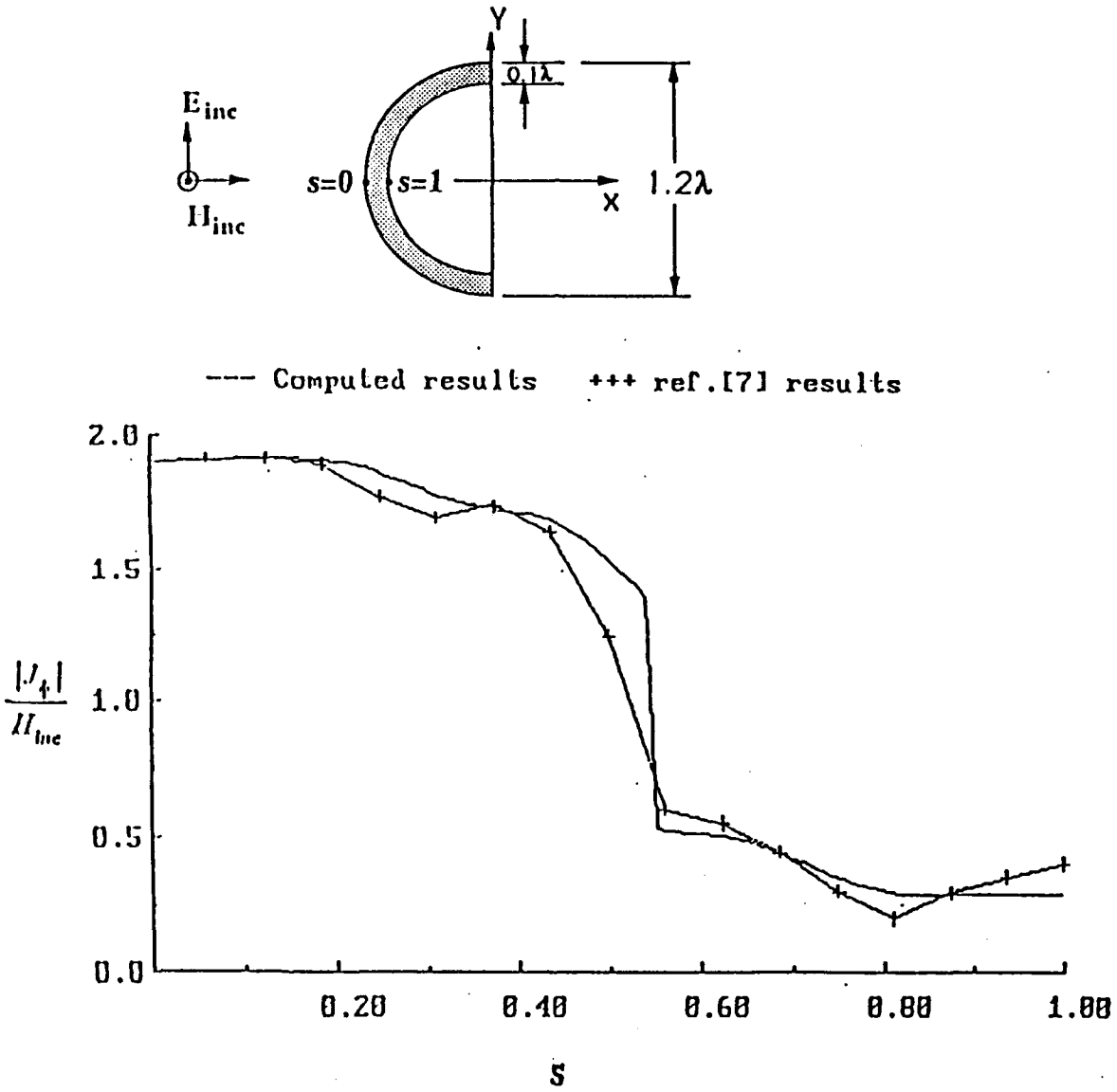

Fig.12 Comparison of surface current distribution for the hollow semi-circular slit cylinder (J_ϕ versus s).

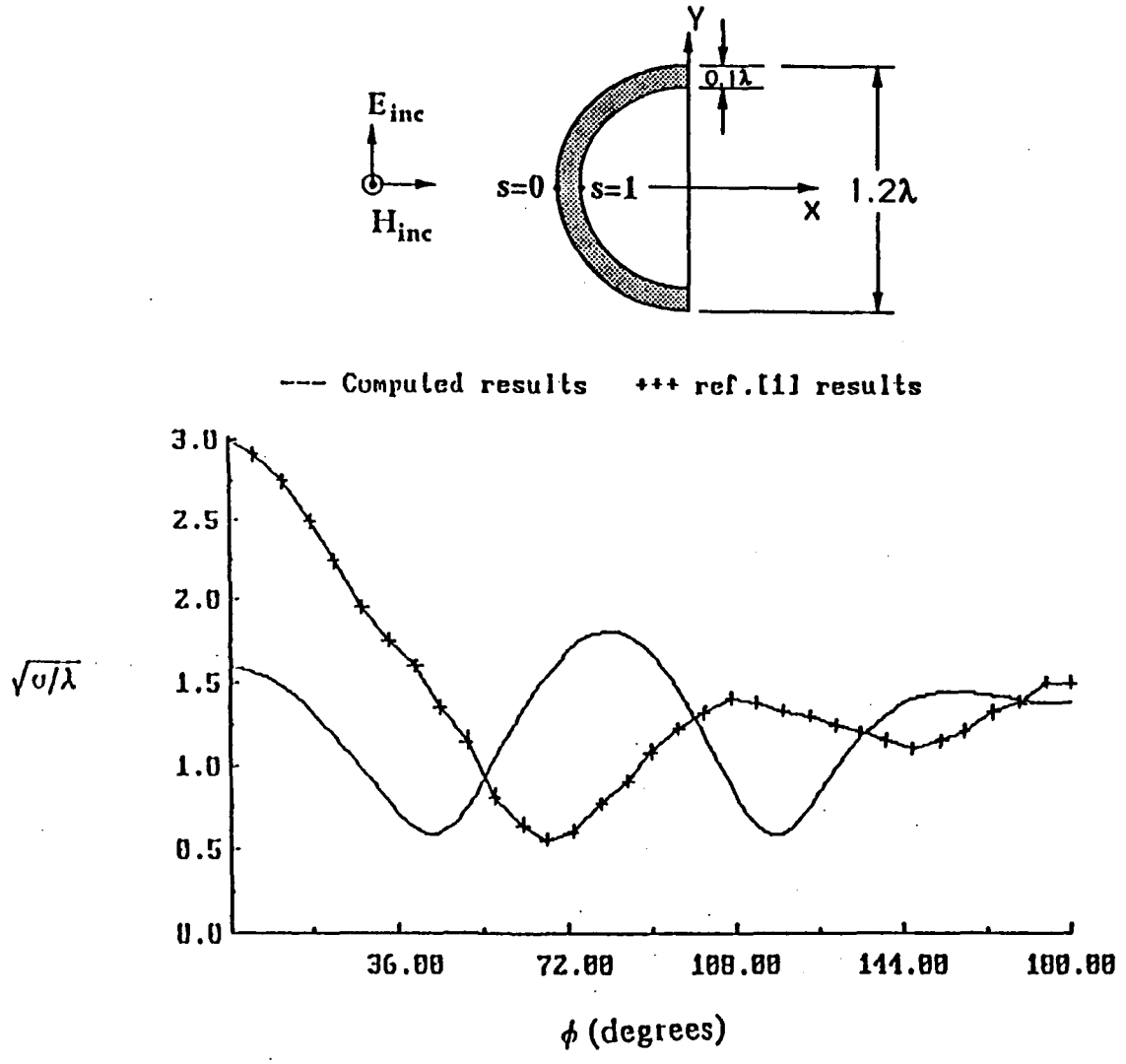

Fig.13 Comparison of scattering width for the hollow semi-circular slit cylinder.

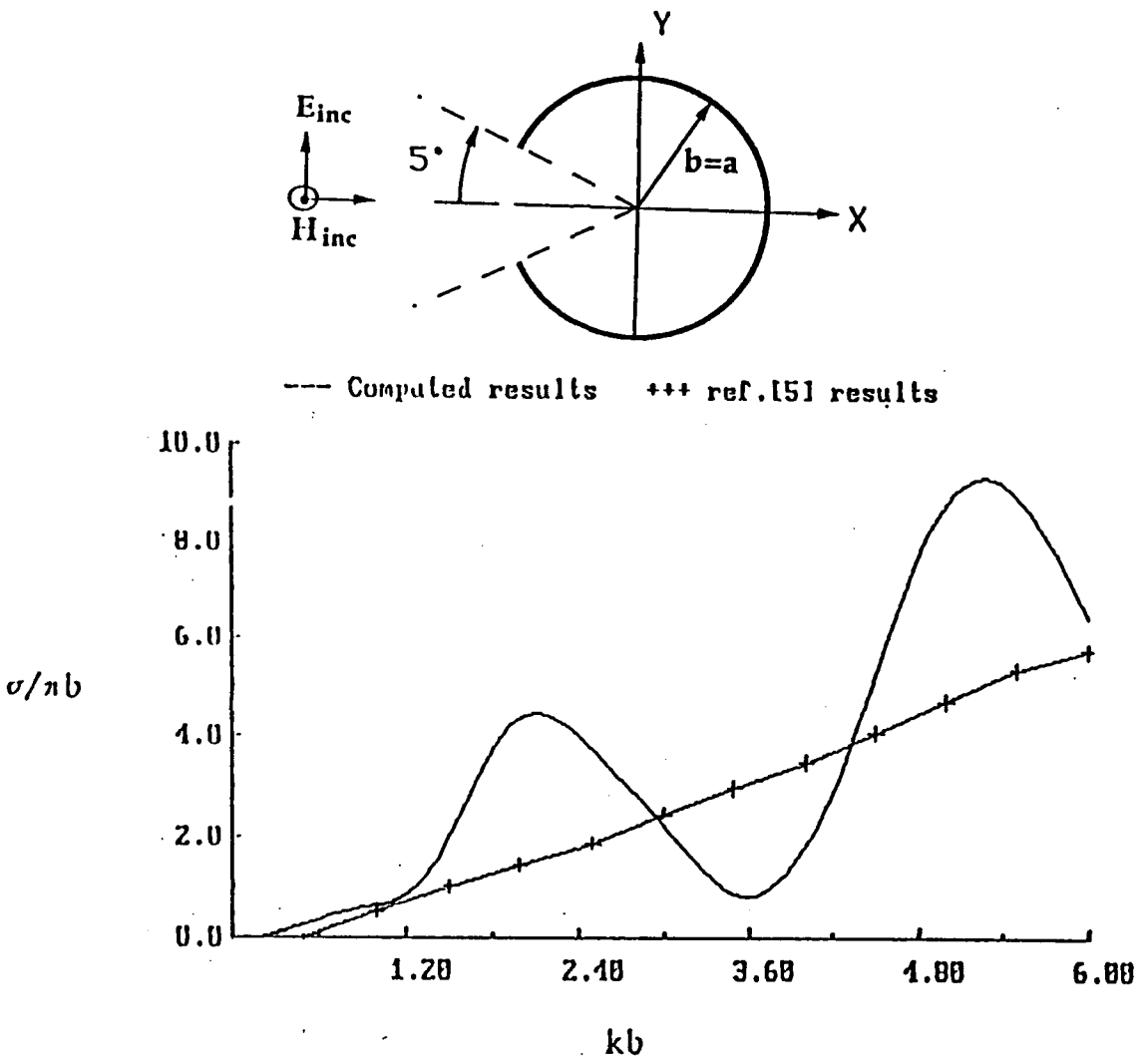

Fig.14 Comparison of forward scattering width for a hollow circular slit cylinder with b=a and total slit angle of 10° and full circular cylinder from ref.[5] .

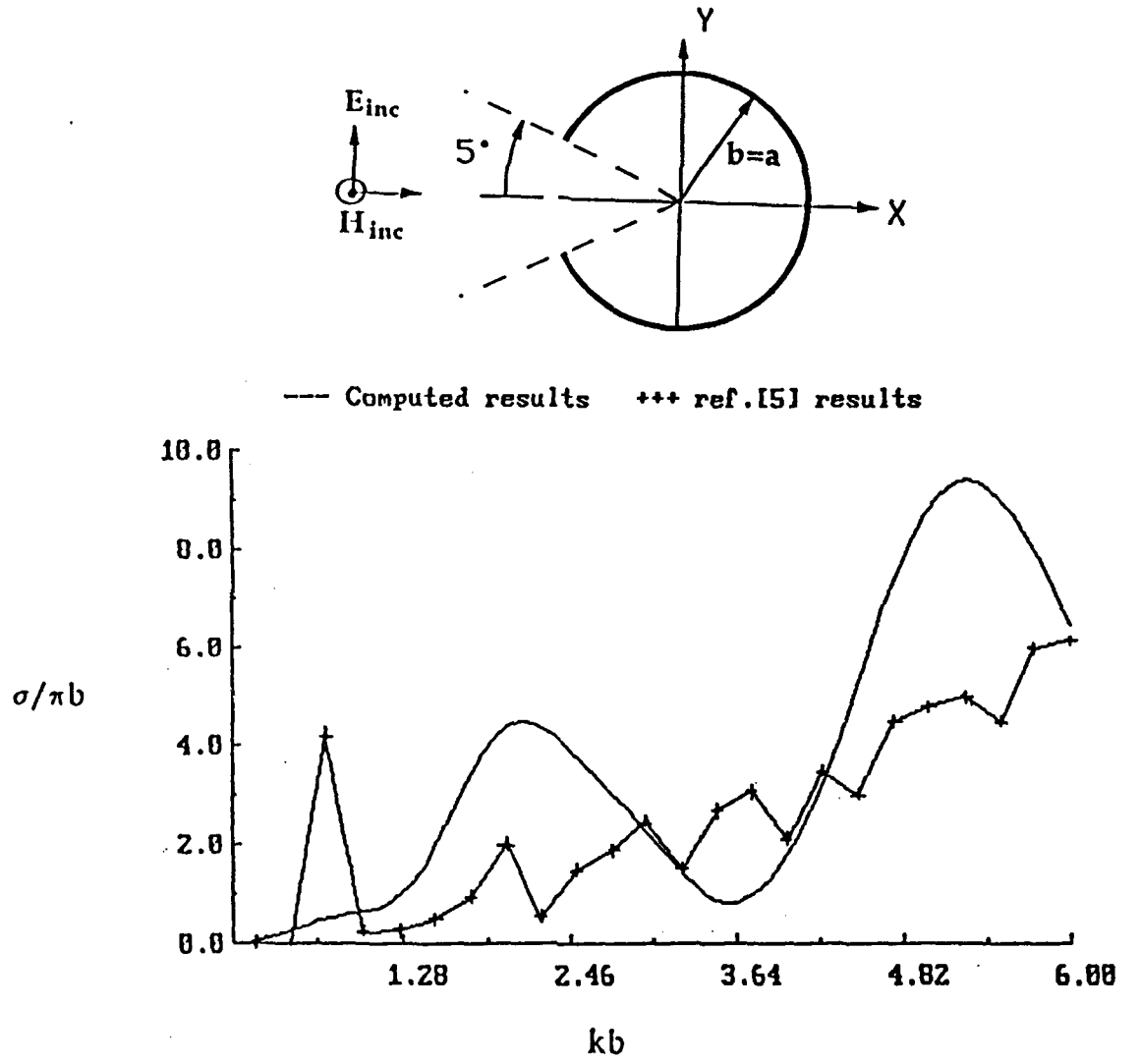

Fig.15 Comparison of forward scattering width for the hollow circular slit cylinder with b=a and slit angle of 10°.

V. CONCLUSIONS

The OSRC approach was applied to circular and hollow slit circular cylinders. For circular cylinders the normalized surface current distribution agrees very well with the exact solution. Scattering width results, with different values of kb were also comparable with the exact solution, as long as kb was not large. For the case of hollow circular slit cylinders, the OSRC normalized surface current distribution was comparable to exact solution results. However, in case of the scattering width, the slit cylinder results were quite different from the exact solution. This indicated a need to carry out phase comparisons of the OSRC surface current distribution with the exact solution.

It is possible that the OSRC approach may not be applicable to concave structures and also for higher values of kb for convex bodies. As mentioned earlier, both outward and inward travelling waves can exist in the interior region of the concave scatterer. Since the OSRC approach assumes waves of only one kind, it is not expected to be applicable to concave bodies. Further the method fails when the scattering surface has cavity shaped objects.

APPENDIX A. COMPUTER PROGRAM TO COMPUTE CURRENT DISTRIBUTION FOR CIRCULAR CYLINDERS

```
C     THIS PROGRAM COMPUTES THE SURFACE CURRENT DISTRIBUTION FOR A
C     CIRCULAR  CYLINDER FOR TRANSVERSE ELECTRIC CASE USING   OSRC
C     APPROACH.
      REAL*8 J(0:200),Y(0:200),DY(0:200),DJ(0:200),K1D,K2D
      REAL*8 J1(0:200),Y1(0:200),DY1(0:200),DJ1(0:200)
      REAL EN,NMX,P,PHI,K,K1,DTR,PI,FR,B,JPHI
      INTEGER NMAX,N
      COMPLEX CB1,CB2,i,S1,S2,S3,S4,DN(999),JP
      OPEN(3,FILE='DAT.1')
      WRITE(*,*) 'ENTER THE WAVE NUMBER K (REAL) IN 1/METERS'
      READ(*,*) K
      WRITE(*,*) 'ENTER THE RADIUS (REAL) IN METERS'
      READ(*,*) B
      WRITE(*,*) 'ENTER NMAX (INT)'
      READ(*,*) NMAX
      PI=3.1415927
      DTR=PI/180.
      i=(0,1)
      K1=K*B
      K1D=DBLE(K1)
      CB1=(8.*K1**2-8.*i*K1)/(8.*K1**2-12.*i*K1-3.)
      CB2=-4./(8.*K1**2-12.*i*K1-3.)
      CALL BES(NMAX,K1D,J,Y,DJ,DY)
C PLOTTING ROUTINE
      WRITE(3,100) 'SURFACE CURRENT DISTRIBUTION FOR CIRCULAR CYL.'
      NPHI=91
      PHI1=0.
      PHI2=180.
      WRITE(3,110) NPHI
      WRITE(3,120) PHI1
      WRITE(3,120) PHI2
      DO 33 P=0.,180.,2.
        PHI=DTR*P
        S4=(0,0)
        DO 22 N=1,NMAX+1
          NMX=FLOAT(N)-1.
          IF(NMX.EQ.0.) THEN
          EN=1.
          ELSE
          EN=2.
          ENDIF
          S1=(0,0)
          S2=(0,0)
          S3=(0,0)
          S1=CB1*i**(-1) * DJ(NMX)/(1.+CB2*NMX**2)
          S2=J(NMX)
          S3=COS(NMX*(PI+PHI))
          DN(N)=-(EN*i**(-NMX)*(S1+S2)*S3)
```

```
          S4=S4+DN(N)
22       CONTINUE
          JP=S4
          JPHI=CABS(JP)
          WRITE(3,120) JPHI
33      CONTINUE
          CLOSE(3)
100       FORMAT(A)
110       FORMAT(I5)
120       FORMAT(E12.3)
          STOP
          END
```

APPENDIX B. COMPUTER PROGRAM TO COMPUTE σ/b FOR CIRCULAR CYLINDERS

```
C     THIS PRCGRAM COMPUTES THE SCATTERING WIDTH FOR A CIRCULAR
C     CYLINDER  FOR THE  TRANSVERSE ELECTRIC POLARIZATION  CASE
C     USING THE   ON - SURFACE  RADIATION  CONDITION  APPROACH.
C     ANGLE SI = PI - PHI GOING CLOCKWISE FROM -180 DEGREES
C     ALONG THE X-AXIS.
      REAL*8 J(0:200),Y(0:200),DY(0:200),DJ(0:200),K1D
      INTEGER N,NMAX
      REAL TA,TB,K,K1,K2,TP1,TP2,FILON,FR,B,A,L,PI,P,NMX,EN,PHI
      COMPLEX DN(999),S2,S3,S4,CN1,i,CB1,CB2
      OPEN(3,FILE='DAT.2')
      WRITE(*,*) 'ENTER THE WAVE NUMBER(REAL) IN 1/METERS'
      READ(*,*) K
      WRITE(*,*) 'ENTER THE OUTER RADIUS(REAL) IN METERS'
      READ(*,*) B
      WRITE(*,*) 'ENTER NMAX(INTEGER)'
      READ(*,*) NMAX
      PI=3.1415927
      DTR=PI/180.
      i=(0,1)
      K1=K*B
      K1D=DBLE(K1)
      CB1=(8.*K1**2-8.*i*K1)/(8.*K1**2-12.*i*K1-3.)
      CB2=-4./(8.*K1**2-12.*i*K1-3.)
      CALL BES(NMAX,K1D,J,Y,DJ,DY)
C   PLOTTING ROUTINE
      WRITE(3,960)'TE CASE PEC CIR. CYLINDER SCATTERING WIDTH '
      NPHI=91
      PHI1=0.
      PHI2=180.
      WRITE(3,970) NPHI
      WRITE(3,980) PHI1
      WRITE(3,980) PHI2
      DO 50 P =0.,180.,2.
            PHI=DTR*P
C   EVALUATING THE  FUNCTION  CN1
             S2=(0,0)
            DO 800 N=1,NMAX+1
              NMX=FLOAT(N)-1.
              S3=(0,0)
              S4=(0,0)
              IF(NMX.EQ.0.) THEN
              EN=1.
              ELSE
              EN=2.
              ENDIF
              S3=J(NMX)*DJ(NMX)
              S4=i**(-1)*CB1*DJ(NMX)**2/(1+CB2*NMX**2)
              DN(N)=2.*PI*EN*(S3+S4)*COS(NMX*(PI-PHI))
              S2=S2+DN(N)
```

```
 800        CONTINUE
              CN1=S2
C    EVALUATING THE SCATTERING WIDTH DIVIDED BY CYLINDER RADIUS B
         SIGMA=1./(4.*K1)*CABS(-K1*CN1)**2
            WRITE(3,980) SIGMA
  50      CONTINUE
          CLOSE(3)
960       FORMAT(A)
970       FORMAT(I5)
980       FORMAT(E12.3)
        STOP
        END
```

APPENDIX C. COMPUTER PROGRAM TO COMPUTE $\sigma/\pi b$ versus kb FOR CIRCULAR CYLINDERS

```
C      THIS PROGRAM COMPUTES THE SCATTERING WIDTH FOR A CIRCULAR
C      CYLINDER  FOR THE  TRANSVERSE ELECTRIC POLARIZATION CASE
C      USING THE   ON - SURFACE  RADIATION  CONDITION  APPROACH.
C      (SIGMA /PI*B   Vs    KB)
       REAL*8 J(0:200),Y(0:200),DY(0:200),DJ(0:200),K1D
       INTEGER N,NMAX
       REAL TA,TB,K,K1,K2,TP1,TP2,FILON,FR,B,A,L,PI,P,NMX,EN,PHI,R
       COMPLEX DN(999),S2,S3,S4,CN1,i,CB1,CB2
       OPEN(3,FILE='DAT.3')
       WRITE(*,*) 'ENTER THE PHI ANGLE(REAL) IN DEGREES'
       READ(*,*) P
       WRITE(*,*) 'ENTER THE OUTER RADIUS(REAL) IN METERS'
       READ(*,*) B
       WRITE(*,*) 'ENTER NMAX(INTEGER)'
       READ(*,*) NMAX
       PI=3.1415927
       DTR=PI/180.
       PHI = DTR*P
       i=(0,1)
C    PLOTTING ROUTINE
       WRITE(3,960) 'TE CASE PEC CIR. CYLINDER SCATTERING WIDTH '
       NPHI=59
       PHI1=0.1
       PHI2=6.
       WRITE(3,970) NPHI
       WRITE(3,980) PHI1
       WRITE(3,980) PHI2
       DO 50 R =0.1,6.,.1
            K1=R*B
            K1D=DBLE(K1)
            CB1=(8.*K1**2-8.*i*K1)/(8.*K1**2-12.*i*K1-3.)
            CB2=-4./(8.*K1**2-12.*i*K1-3.)
            CALL BES(NMAX,K1D,J,Y,DJ,DY)
C    EVALUATING THE  FUNCTION  CN1
              S2=(0,0)
              DO 800 N=1,NMAX+1
                   NMX=FLOAT(N)-1.
                   S3=(0,0)
                   S4=(0,0)
                   IF(NMX.EQ.0.) THEN
                   EN=1.
                   ELSE
                   EN=2.
                   ENDIF
                   S3=J(NMX)*DJ(NMX)
                   S4=i**(-1.)*CB1*DJ(NMX)**2/(1.+CB2*NMX**2)
                   DN(N)=2.*PI*EN*(S3+S4)*COS(NMX*PHI)
                   S2=S2+DN(N)
 800         CONTINUE
```

```
            CN1=S2
C    EVALUATING THE SCATTERING WIDTH DIVIDED BY CYLINDER RADIUS B
        SIGMA=1./(4.*K1*PI)*CABS(-K1*CN1)**2
            WRITE(3,980) SIGMA
  50     CONTINUE
         CLOSE(3)
960      FORMAT(A)
970      FORMAT(I5)
980      FORMAT(E12.3)
       STOP
       END
```

APPENDIX D. COMPUTER PROGRAM TO COMPUTE J_s FOR SLIT CIRCULAR CYLINDERS

```
C THIS PROGRAM COMPUTES THE  SURFACE CURRENT DISTRIBUTION  FOR  A
C CIRCULAR SLIT CYLINDER FOR TRANSVERSE ELECTRIC CASE USING  OSRC
C APPROACH.

      REAL*8 J(0:200),Y(0:200),DJ(0:200),DY(0:200),K1D,K2D
      REAL*8 J1(0:200),Y1(0:200),DJ1(0:200),DY1(0:200)
      REAL EN,NMX,P,PHI,K,K1,K2,SMAX,PI,FR,A,B,JPHI,L,S,S2,SM2,R
      REAL SMAX2
      INTEGER NMAX,N
      COMPLEX CB1,CB2,CA1,CA2,i,S1,S3,S4,DN(999),JP,S5
      OPEN(3,FILE='DAT.5')
      WRITE(*,*) 'ENTER THE FREQUENCY (REAL) IN MHZ'
      READ(*,*) FR
      WRITE(*,*) 'ENTER THE FIRST SLIT ANGLE (REAL) IN DEGREES'
      READ(*,*) TA
      WRITE(*,*) 'ENTER THE SECOND SLIT ANGLE (REAL) IN DEGREES'
      READ(*,*) TB
      WRITE(*,*) 'ENTER NMAX (INT)'
      READ(*,*) NMAX
      PI=3.1415927
      i=(0,1)
      DTR=PI/180.
      K=PI*FR/150.
      L=300./FR
      B=.6*L
      A=.5*L
      K1=K*B
      K2=K*A
      K1D=DBLE(K1)
      K2D=DBLE(K2)
      CALL BES(NMAX,K1D,J,Y,DJ,DY)
      CALL BES(NMAX,K2D,J1,Y1,DJ1,DY1)
      TP1=TA/57.29578
      TP2=TB/57.29578
      CB1=(8.*K1**2-8.*i*K1)/(8.*K1**2-12.*i*K1-3.)
      CB2=-4./(8.*K1**2-12.*i*K1-3.)
      CA1=-(8.*K2**2-8.*i*K2)/(8.*K2**2-12.*i*K2-3.)
      CA2=-4./(8.*K2**2-12.*i*K2-3.)
C PLOTTING ROUTINE
      WRITE(3,100) ' CURRENT DISTRIBUTION FOR CIRCULAR SLIT CYL.'
      NPHI=95
      PHI1=0.
      PHI2=1.
      WRITE(3,110) NPHI
      WRITE(3,120) PHI1
      WRITE(3,120) PHI2
 SMAX=B*(2*PI-ABS(TP2-TP1)-PI/2)+(B-A)+A*(2*PI-ABS(TP2-TP1)-PI/2)
      DO 33 P=0.,.52,.01
          S=P*SMAX/B+PI/2
```

```
          S3=EXP(i*K1*COS(TP2+S))
          S4=(0,0)
         DO 22 N=1,NMAX+1
            NMX=FLOAT(N)-1.
            IF(NMX.EQ.0.) THEN
            EN=1.
            ELSE
            EN=2.
            ENDIF
            S1=(0,0)
            S2=0.
            S1=EN*CB1*i**(NMX-1.) * DJ(NMX)/(1.+CB2*NMX**2)
            S2=COS(NMX*(S+TP2))
            DN(N)=S1*S2
            S4=S4+DN(N)
 22       CONTINUE
          JP=-(S4+S3)
          JPHI=CABS(JP)
          WRITE(3,120) JPHI
 33       CONTINUE
C PLOTTING ROUTINE

               DO 10 R=.57,1.,.01
                S= (R-.57)*SMAX/A
         S5=EXP(i*K2*COS(TP1-S))
         S4=(0,0)
          DO 5 N=1,NMAX+1
              NMX=FLOAT(N)-1.
              IF(NMX.EQ.0.) THEN
              EN=1.
              ELSE
              EN=2.
              ENDIF
           S1=(0,0)
           S2=0.
        S1=EN*CA1*i**(NMX-1.)*DJ1(NMX)/(1.+CA2*NMX**2)
        S2=COS(NMX*(TP1-S))
        DN(N)=S1*S2
        S4=S4+DN(N)
 5      CONTINUE
        JP=-(S4+S5)
        JPHI=CABS(JP)
        WRITE(3,120) JPHI
 10     CONTINUE
        CLOSE(3)
 100    FORMAT(A)
 110    FORMAT(I5)
 120    FORMAT(E12.3)
          STOP
          END
```

APPENDIX E. COMPUTER PROGRAM FOR $\sqrt{\sigma/\lambda}$ versus ϕ FOR SLIT CIRCULAR CYLINDERS

```
C THIS PROGRAM COMPUTES THE  SCATTERING WIDTH  SQRT(SIGMA/LAMBDA)
C FOR A CIRCULAR HOLLOW CYLINDER WITH A  SLIT OF  ANY SIZE  FOR THE
C TRANSVERSE  ELECTRIC  POLARIZATION CASE USING THE  ON - SURFACE
C RADIATION CONDITION APPROACH.

      REAL*8 J(0:200),Y(0:200),DY(0:200),DJ(0:200),K1D,K2D
      REAL*8 J1(0:200),Y1(0:200),DJ1(0:200),DY1(0:200),H1
      REAL EN,NMX,R,U,UU,H,P,PI,PHI,FAC,F(99),P1
      INTEGER M,N,NT,NN,NMAX
      REAL TA,TB,K,K1,K2,TP1,TP2,FILON,FR,B,A,L
      COMPLEX P2,P3,i,S1,T1,T2,T3,T4,CB1,CB2,CA1,CA2
      COMPLEX C1,C2,C3,C4,C5,C6,C7,C8,CN2,CN3,CN5,CN6,CN7,CN8
      COMPLEX DN(999),S2,S3,S4,S5,CN1,CN4
      EXTERNAL FILON
      OPEN(16,FILE='DAT.6')
      WRITE(*,*) 'ENTER THE FREQUENCY(REAL) IN MHZ'
      READ(*,*) FR
      WRITE(*,*) ' ENTER THE FIRST SLIT ANGLE(REAL) IN DEGREES'
      READ(*,*) TA
      WRITE(*,*) 'ENTER SECOND SLIT ANGLE(REAL) IN DEGREES'
      READ(*,*) TB
      WRITE(*,*) 'ENTER NMAX(INTEGER)'
      READ(*,*)NMAX
      TP1=TA/57.29578
      TP2=TB/57.29578
      PI=3.1415927
      DTR=PI/180.
      i=(0,1)
      K=PI*FR/150.
      L=300./FR
      B=.6*L
      A=.5*L
      K1=K*B
      K1D=DBLE(K1)
      K2=K*A
      K2D=DBLE(K2)
      M=98
      H=ABS(TP2-TP1)/FLOAT(M)
      CB1=(8.*K1**2-8.*i*K1)/(8.*K1**2-12.*i*K1-3.)
      CA1=-(8.*K2**2-8.*i*K2)/(8.*K2**2-12.*i*K2-3.)
      CB2=-4./(8.*K1**2-12.*i*K1-3.)
      CA2=-4./(8.*K2**2-12.*i*K2-3.)
      CALL BES(NMAX,K1D,J,Y,DJ,DY)
      CALL BES(NMAX,K2D,J1,Y1,DJ1,DY1)

C     PLOTTING ROUTINE
      WRITE(16,960) 'TE CASE PEC SLIT CYLINDER SCATTERING WIDTH '
```

```
      NPHI=91
      PHI1=0.
      PHI2=180.
      WRITE(16,970) NPHI
      WRITE(16,980) PHI1
      WRITE(16,980) PHI2
      DO 50 P =0.,180.,2.
            PHI=DTR*P
            IF(P.EQ.0.) THEN
            T1=i*K*(A-B)*COS(TP2)
            T3=i*K*(A-B)*COS(TP1)
            ELSE

T1=COS(TP2)/(COS(TP2)-COS(TP2+PHI))*(EXP(i*K2*(COS(TP2)-COS(TP2+
     + PHI)))-EXP(i*K1*(COS(TP2)-COS(TP2+PHI))))

T3=COS(TP1)/(COS(TP1)-COS(TP1+PHI))*(EXP(i*K2*(COS(TP1)-COS(TP1+
     + PHI)))-EXP(i*K1*(COS(TP1)-COS(TP1+PHI))))

           ENDIF
T2=EXP((-i)*K2*COS(TP2+PHI))-EXP((-i)*K1*COS(TP2+PHI))

T4=EXP((-i)*K2*COS(TP1+PHI))-EXP((-i)*K1*COS(TP1+PHI))

C   EVALUATING 1ST FUNCTION F(U)

                DO 100 U =1,M+1
                     UU=TP1+(H*(U-1.))
                     F(U)=COS(K1*COS(UU+PHI))
  100           CONTINUE
                   S1=(0,0)
                   P1=0.
                       DO 200 N=1,NMAX+1
                       NMX=FLOAT(N)-1.
                   IF(NMX.EQ.0.) THEN
                   EN=1.
                   ELSE
                   EN=2.
                   ENDIF
                    P2=(0,0)
                    P1 = FILON(F,NMX,TP1,TP2,M+1,99,1)
                    WRITE(*,*) NMX,P1
                    P2=EN*i**NMX*DJ(NMX)
                    WRITE(*,*) P2
                    P3=P1*P2
                    WRITE(*,*) P3
                    S1 = S1 + P3
  200               CONTINUE
                    C1=S1

C   EVALUATING 2ND FUNCTION F(U) (CN2=C1-jC2) & CN4
```

```
          DO 250 U=1,M+1
                 UU=TP1+(H*(U-1.))
                 F(U)=SIN(K1*COS(UU+PHI))
250              CONTINUE
                 S1=(0,0)
                 S2=(0,0)
                 P1=0.
                   DO 300 N=1,NMAX+1
                      NMX=FLOAT(N)-1.
                      S3=(0,0)
                      S4=(0,0)
                      P2=(0,0)
                      IF(NMX.EQ.0.) THEN
                      EN=1.
                      ELSE
                      EN=2.
                      ENDIF
                      P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
                      S3=(-1.)**NMX*J1(NMX)*DJ1(NMX)
     S4=(-1.)**NMX*i**(-1.)*CA1*DJ1(NMX)**2/(1.+CA2*NMX**2)
                      DN(N)=2.*PI*EN*(S3-S4)*COS(NMX*PHI)
                      P2=EN*i**NMX*DJ(NMX)
                      P3=P1*P2
                      S1=S1+P3
                      S2=S2+DN(N)
300                   CONTINUE
                      CN4=S2
                      C2=S1
                      CN2=C1-i*C2

C    EVALUATING 3RD FUNCTION F(U) & CN7

          DO 350 U=1,M+1
                UU=TP1+(H*(U-1.))
                F(U)=COS(UU+PHI)*COS(K1*COS(UU+PHI))
350             CONTINUE
                S1=(0,0)
                S2=(0,0)
                P1=0.
                DO 400 N=1,NMAX+1
                NMX=FLOAT(N)-1.
                S3=(0,0)
                S4=(0,0)
                S5=(0,0)
                P2=(0,0)
                IF(NMX.EQ.0.) THEN
                EN=1.
                ELSE
                EN=2.
                ENDIF
                P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
      P2=EN*CB1*i**(-NMX-1.)*(-1.)**NMX*DJ(NMX)/(1.+CB2*NMX**2)
```

```
                S3=COS(NMX*TP2)*T2
                S4=(CB1*i**(-NMX-1.)*DJ(NMX))/(1.+CB2*NMX**2)
                S5=(CA1*i**(-NMX-1.)*DJ1(NMX))/(1.+CA2*NMX**2)
                DN(N)=EN/2.*(-1.)**NMX*(S4+S5)*S3
                P3=P1*P2
                S1=S1+P3
                S2=S2+DN(N)
  400           CONTINUE
                C3=S1
                CN7=S2

C   EVALUATING 4RTH FUNCTION (CN3=C3-jC4) & CN8

          DO 450 U=1,M+1
                UU=TP1+(H*(U-1.))
                F(U)=COS(UU+PHI)*SIN(K1*COS(UU+PHI))
 450            CONTINUE
                S1=(0,0)
                S2=(0,0)
                P1=0.
                DO 500 N=1,NMAX+1
                NMX=FLOAT(N)-1.
                S3=(0,0)
                S4=(0,0)
                S5=(0,0)
                P2=(0,0)
                IF(NMX.EQ.0.) THEN
                EN=1.
                ELSE
                EN=2.
                ENDIF
                P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
       P2=EN*CB1*i**(-NMX-1.)*(-1.)**NMX*DJ(NMX)/(1.+CB2*NMX**2)
                P3=P1*P2
                S3=COS(NMX*TP1)*T4
                S4=(CB1*i**(-NMX-1.)*DJ(NMX))/(1.+CB2*NMX**2)
                S5=(CA1*i**(-NMX-1.)*DJ1(NMX))/(1.+CA2*NMX**2)
                DN(N)=EN/2.*(-1.)**NMX*(S4+S5)*S3
                S1=S1+P3
                S2=S2+DN(N)
  500           CONTINUE
               CN8=S2
               C4=S1
               CN3=C3-i*C4

C     EVALUATING 5TH FUNCTION
               DO 550 U=1,M+1
                     UU=TP1+(H*(U-1.))
                     F(U)=COS(K2*COS(UU+PHI))
  550          CONTINUE
               S1=(0,0)
               P1=0.
```

```
                DO 600 N=1,NMAX+1
                NMX=FLOAT(N)-1.
                IF(NMX.EQ.0.) THEN
                EN=1.
                ELSE
                EN=2.
                ENDIF
                P2=(0,0)
                P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
                P2=EN*i**NMX*DJ1(NMX)
                P3=P1*P2
                S1=S1+P3
  600           CONTINUE
                C5=S1

C     EVALUATING 6TH FUNCTION (CN5=C5-jC6)
              DO 650 U=1,M+1
                   UU=TP1+(H*(U-1.))
                   F(U)=SIN(K2*COS(UU+PHI))
  650          CONTINUE
               S1=(0,0)
               P1=0.
               DO 700 N=1,NMAX+1
               NMX=FLOAT(N)-1.
               IF(NMX.EQ.0.) THEN
               EN=1.
               ELSE
               EN=2.
               ENDIF
               P2=(0,0)
               P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
               P2=EN*i**NMX*DJ1(NMX)
               P3=P1*P2
               S1=S1+P3
  700          CONTINUE
               C6=S1
               CN5=C5+i*C6

C    EVALUATING THE 7TH FUNCTION & CN1
             DO 750 U=1,M+1
                  UU=TP1+(H*(U-1.))
                 F(U)=COS(UU+PHI)*COS(K2*COS(UU+PHI))
 750         CONTINUE
             S1=(0,0)
             S2=(0,0)
             P1=0.
             DO 800 N=1,NMAX+1
             NMX=FLOAT(N)-1.
             S3=(0,0)
             S4=(0,0)
             IF(NMX.EQ.0.) THEN
             EN=1.
```

```
              ELSE
              EN=2.
              ENDIF
              P2=(0,0)
              P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
        P2=EN*CA1*i**(-NMX-1.)*(-1.)**NMX*DJ1(NMX)/(1.+CA2*NMX**2)
              S3=J(NMX)*DJ(NMX)
              S4=i**(-1)*CB1*DJ(NMX)**2/(1+CB2*NMX**2)
         DN(N)=2.*PI*EN*(S3+S4)*COS(NMX*PHI)
              P3=P1*P2
              S1=S1+P3
              S2=S2+DN(N)
 800          CONTINUE
              C7=S1
              CN1=S2

C          EVALUATING THE 8TH FUNCTION (CN6=C7-jC8)
            DO 850 U=1,M+1
                 UU=TP1+(H*(U-1.))
                 F(U)=COS(UU+PHI)*SIN(K2*COS(UU+PHI))
 850             CONTINUE
                 S1=(0,0)
                 P1=0.
                 DO 900 N=1,NMAX+1
                  NMX=FLOAT(N)-1.
                  IF(NMX.EQ.0.) THEN
                  EN=1.
                  ELSE
                  EN=2.
                  ENDIF
                  P2=(0,0)
                  P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
        P2=EN*CA1*i**(-NMX-1.)*(-1.)**NMX*DJ1(NMX)/(1.+CA2*NMX**2)
                  P3=P1*P2
                  S1=S1+P3
 900              CONTINUE
                  C8=S1
                  CN6=C7+i*C8
       SIGMA=1./(8.*PI)*CABS(-K1*(CN1-CN2+i*CN3)-K2*(CN4-CN5+i*CN6)
     +          -T1-T3+CN7+CN8)**2
                 SIG=(SIGMA)**.5
              WRITE(16,980) SIG
  50       CONTINUE
           CLOSE(16)
 960       FORMAT(A)
 970       FORMAT(I5)
 980       FORMAT(E12.3)
         STOP
         END
```

APPENDIX F. PROGRAM TO COMPUTE $\sigma/\pi b$ versus KB FOR SLIT CIRCULAR CYLINDERS

```
C  THIS PROGRAM COMPUTES THE SCATTERING WIDTH/PI*B FOR A CIRCULAR
C  CIRCULAR   HOLLOW  CYLINDER WITH A  SLIT  OF ANY  SIZE FOR THE
C  TRANSVERSE ELECTRIC POLARIZATION CASE  USING  THE ON - SURFACE
C  RADIATION  CONDITION  APPROACH .
      REAL*8 J(0:200),Y(0:200),DY(0:200),DJ(0:200),K1D,K2D
      REAL*8 J1(0:200),Y1(0:200),DJ1(0:200),DY1(0:200),H1
      REAL EN,NMX,R,U,UU,H,P,PI,PHI,F(99),P1
      INTEGER M,N,NT,NMAX
      REAL TA,TB,K,K1,K2,TP1,TP2,FILON,FR,B,A,L
      COMPLEX P2,P3,i,S1,T1,T2,T3,T4,CB1,CB2,CA1,CA2
      COMPLEX C1,C2,C3,C4,C5,C6,C7,C8,CN2,CN3,CN5,CN6,CN7,CN8
      COMPLEX DN(99),S2,S3,S4,S5,CN1,CN4
      EXTERNAL FILON
      OPEN(3,FILE='DAT.71')
      WRITE(*,*) 'ENTER THE PHI ANGLE(REAL) IN DEGREES'
      READ(*,*) P
      WRITE(*,*) 'ENTER THE OUTER RADIUS(REAL) IN METERS'
      READ(*,*) B
      WRITE(*,*) 'ENTER THE INNER RADIUS(REAL) IN METERS'
      READ(*,*) A
      WRITE(*,*) ' ENTER THE FIRST SLIT ANGLE(REAL) IN DEGREES'
      READ(*,*) TA
      WRITE(*,*) 'ENTER SECOND SLIT ANGLE(REAL) IN DEGREES'
      READ(*,*) TB
      WRITE(*,*) 'ENTER NMAX(INTEGER)'
      READ(*,*)NMAX
      TP1=TA/57.29578
      TP2=TB/57.29578
      PI=3.1415927
      DTR=PI/180.
      PHI=P*DTR
      i=(0,1)
      M=98
      H=ABS(TP2-TP1)/FLOAT(M)
C   PLOTTING ROUTINE
      WRITE(3,960) 'TE CASE PEC SLIT CYLINDER SCATTERING WIDTH '
      NPHI=59
      PHI1=0.1
      PHI2=6.
      WRITE(3,970) NPHI
      WRITE(3,980) PHI1
      WRITE(3,980) PHI2
      DO 50 R =0.1,6.,.1
            K1=R*B
            K2=R*A
            K1D=DBLE(K1)
            K2D=DBLE(K2)
            CB1=(8.*K1**2-8.*i*K1)/(8.*K1**2-12.*i*K1-3.)
            CA1=-(8.*K2**2-8.*i*K2)/(8.*K2**2-12.*i*K2-3.)
```

```
              CB2=-4./(8.*K1**2-12.*i*K1-3.)
              CA2=-4./(8.*K2**2-12.*i*K2-3.)
              CALL BES(NMAX,K1D,J,Y,DJ,DY)
              CALL BES(NMAX,K2D,J1,Y1,DJ1,DY1)
              IF(P.EQ.0.) THEN
              T1=i*K*(A-B)*COS(TP2)
              T3=i*K*(A-B)*COS(TP1)
              ELSE

T1=COS(TP2)/(COS(TP2)-COS(TP2+PHI))*(EXP(i*K2*(COS(TP2)-COS(TP2+
     + PHI)))-EXP(i*K1*(COS(TP2)-COS(TP2+PHI))))


T3=COS(TP1)/(COS(TP1)-COS(TP1+PHI))*(EXP(i*K2*(COS(TP1)-COS(TP1+
     + PHI)))-EXP(i*K1*(COS(TP1)-COS(TP1+PHI))))
              ENDIF
       T2=EXP((-i)*K2*COS(TP2+PHI))-EXP((-i)*K1*COS(TP2+PHI))

       T4=EXP((-i)*K2*COS(TP1+PHI))-EXP((-i)*K1*COS(TP1+PHI))

C    EVALUATING 1ST FUNCTION F(U)

                  DO 100 U =1,M+1
                        UU=TP1+(H*(U-1.))
                        F(U)=COS(K1*COS(UU+PHI))
  100             CONTINUE
                      S1=(0,0)
                      P1=0.
                   DO 200 N=1,NMAX+1
                         NMX=FLOAT(N)-1.
                         IF(NMX.EQ.0.) THEN
                         EN=1.
                         ELSE
                         EN=2.
                         ENDIF
                         P2=(0,0)
                         P1 = FILON(F,NMX,TP1,TP2,M+1,99,1)
                         WRITE(*,*) NMX,P1
                         P2=EN*i**NMX*J(NMX)
                         H1=J(NMX)
                         WRITE(*,*) P2
                         P3=P1*P2
                         WRITE(*,*) P3
                         S1 = S1 + P3
  200              CONTINUE
                         C1=S1

C    EVALUATING 2ND FUNCTION F(U) (CN2=C1-jC2) & CN4

              DO 250 U=1,M+1
                     UU=TP1+(H*(U-1.))
                     F(U)=SIN(K1*COS(UU+PHI))
```

```
250       CONTINUE
                  S1=(0,0)
                  S2=(0,0)
                  P1=0.
               DO 300 N=1,NMAX+1
                     NMX=FLOAT(N)-1.
                     S3=(0,0)
                     S4=(0,0)
                     P2=(0,0)
                     IF(NMX.EQ.0.) THEN
                     EN=1.
                     ELSE
                     EN=2.
                     ENDIF
                     P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
                     S3=(-1.)**NMX*J1(NMX)*DJ1(NMX)
          S4=(-1.)**NMX*i**(-1.)*CA1*DJ1(NMX)**2/(1.+CA2*NMX**2)
                     DN(N)=2.*PI*EN*(S3-S4)*COS(NMX*PHI)

                     P2=EN*i**NMX*J(NMX)
                     P3=P1*P2
                     S1=S1+P3
                     S2=S2+DN(N)
300                  CONTINUE
                     CN4=S2
                     C2=S1
                     CN2=C1-i*C2

C    EVALUATING 3RD FUNCTION F(U) & CN7

            DO 350 U=1,M+1
                  UU=TP1+(H*(U-1.))
                  F(U)=COS(UU+PHI)*COS(K1*COS(UU+PHI))
350               CONTINUE
                  S1=(0,0)
                  S2=(0,0)
                  P1=0.
                  DO 400 N=1,NMAX+1
                  NMX=FLOAT(N)-1.
                  S3=(0,0)
                  S4=(0,0)
                  S5=(0,0)
                  P2=(0,0)
                  IF(NMX.EQ.0.) THEN
                  EN=1.
                  ELSE
                  EN=2.
                  ENDIF
                  P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
        P2=EN*CB1*i**(-NMX-1.)*(-1.)**NMX*DJ(NMX)/(1.+CB2*NMX**2)
                  S3=COS(NMX*TP2)*T2
                  S4=(CB1*i**(-NMX-1.)*DJ(NMX))/(1.+CB2*NMX**2)
```

```
                S5=(CA1*i**(-NMX-1.)*DJ1(NMX))/(1.+CA2*NMX**2)
                DN(N)=EN/2.*(-1.)**NMX*(S4+S5)*S3
                P3=P1*P2
                S1=S1+P3
                S2=S2+DN(N)
 400            CONTINUE
                C3=S1
                CN7=S2

C   EVALUATING 4RTH FUNCTION (CN3=C3-jC4) & CN8

        DO 450 U=1,M+1
                UU=TP1+(H*(U-1.))
                F(U)=COS(UU+PHI)*SIN(K1*COS(UU+PHI))
 450            CONTINUE
                S1=(0,0)
                S2=(0,0)
                P1=0.
                DO 500 N=1,NMAX+1
                NMX=FLOAT(N)-1.
                S3=(0,0)
                S4=(0,0)
                S5=(0,0)
                P2=(0,0)
                IF(NMX.EQ.0.) THEN
                EN=1.
                ELSE
                EN=2.
                ENDIF
                P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
      P2=EN*CB1*i**(-NMX-1.)*(-1.)**NMX*DJ(NMX)/(1.+CB2*NMX**2)
                P3=P1*P2
                S3=COS(NMX*TP1)*T4
                S4=(CB1*i**(-NMX-1.)*DJ(NMX))/(1.+CB2*NMX**2)
                S5=(CA1*i**(-NMX-1.)*DJ1(NMX))/(1.+CA2*NMX**2)
                DN(N)=EN/2.*(-1.)**NMX*(S4+S5)*S3
                S1=S1+P3
                S2=S2+DN(N)
 500            CONTINUE
               CN8=S2
               C4=S1
               CN3=C3-i*C4

C     EVALUATING 5TH FUNCTION
               DO 550 U=1,M+1
                     UU=TP1+(H*(U-1.))
                     F(U)=COS(K2*COS(UU+PHI))
 550           CONTINUE
               S1=(0,0)
               P1=0.
               DO 600 N=1,NMAX+1
               NMX=FLOAT(N)-1.
```

```
                IF(NMX.EQ.0.) THEN
                EN=1.
                ELSE
                EN=2.
                ENDIF
                P2=(0,0)
                P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
                P2=EN*i**NMX*J1(NMX)
                P3=P1*P2
                S1=S1+P3
  600           CONTINUE
                C5=S1

C     EVALUATING 6TH FUNCTION (CN5=C5-jC6)
              DO 650 U=1,M+1
                   UU=TP1+(H*(U-1.))
                   F(U)=SIN(K2*COS(UU+PHI))
  650          CONTINUE
               S1=(0,0)
               P1=0.
               DO 700 N=1,NMAX+1
               NMX=FLOAT(N)-1.
               IF(NMX.EQ.0.) THEN
               EN=1.
               ELSE
               EN=2.
               ENDIF
               P2=(0,0)
               P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
               P2=EN*i**NMX*J1(NMX)
               P3=P1*P2
               S1=S1+P3
  700          CONTINUE
               C6=S1
               CN5=C5-i*C6

C    EVALUATING THE 7TH FUNCTION & CN1
              DO 750 U=1,M+1
                   UU=TP1+(H*(U-1.))
                   F(U)=COS(UU+PHI)*COS(K2*COS(UU+PHI))
 750          CONTINUE
              S1=(0,0)
              S2=(0,0)
              P1=0.
             DO 800 N=1,NMAX+1
              NMX=FLOAT(N)-1.
              S3=(0,0)
              S4=(0,0)
              IF(NMX.EQ.0.) THEN
              EN=1.
              ELSE
              EN=2.
```

```
            ENDIF
            P2=(0,0)
            P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
      P2=EN*CA1*i**(-NMX-1.)*(-1.)**NMX*DJ1(NMX)/(1.+CA2*NMX**2)
            S3=J(NMX)*DJ(NMX)
            S4=i**(-1)*CB1*DJ(NMX)**2/(1+CB2*NMX**2)
       DN(N)=2.*PI*EN*(S3+S4)*COS(NMX*PHI)
            P3=P1*P2
            S1=S1+P3
            S2=S2+DN(N)
 800      CONTINUE
            C7=S1
            CN1=S2

C         EVALUATING THE 8TH FUNCTION (CN6=C7-jC8)
          DO 850 U=1,M+1
               UU=TP1+(H*(U-1.))
               F(U)=COS(UU+PHI)*SIN(K2*COS(UU+PHI))
 850           CONTINUE
               S1=(0,0)
               P1=0.
            DO 900 N=1,NMAX+1
               NMX=FLOAT(N)-1.
               IF(NMX.EQ.0.) THEN
               EN=1.
               ELSE
               EN=2.
               ENDIF
               P2=(0,0)
               P1=FILON(F,NMX,TP1,TP2,M+1,99,1)
      P2=EN*CA1*i**(-NMX-1.)*(-1.)**NMX*DJ1(NMX)/(1.+CA2*NMX**2)
               P3=P1*P2
               S1=S1+P3
 900         CONTINUE
               C8=S1
               CN6=C7-i*C8

SIGMA=1./(4.*K1*PI)*CABS(-K1*(CN1-CN2+i*CN3)-K2*(CN4-CN5+i*CN6)
     +         -T1-T3+CN7+CN8)**2

          WRITE(3,980) SIGMA
  50    CONTINUE
        CLOSE(3)
960     FORMAT(A)
970     FORMAT(I5)
980     FORMAT(E12.3)
      STOP
      END
```

APPENDIX G. PROGRAM TO COMPUTE THE BESSEL FUNCTIONS

```
      SUBROUTINE BES(N,X,J,Y,DJ,DY)
C     THIS SUBROUTINE WAS OBTAINED DURING EC 4600 FROM PROFESSOR
C     MICHAEL A. MORGAN.
C      Double precision calculation of ordinary Bessel functions,
C     Jn(X)
C    and Yn(X), and their first derivative, DJ and DY, for integer

C     order "n" from n=0 to N with real argument X.
C
      REAL*8 J(0:200),Y(0:200),DJ(0:200),DY(0:200),SCALE,JTEMP2,X
      REAL*8 SCLFAC,A,B,C,D,E,F,PI,JTEMP,JTEMP1
      PI=3.14159265359D0
      IF (X.EQ.0.0D00) THEN
C     X = 0.0 BOUNDARY CASE
           IF (N.EQ.1) THEN
                J(1) = 0.0D00
                DJ(1) = 0.5D00
           ELSE
                DO 5, I = N, 2, -1
                     J(I) = 0.0D00
                     DJ(I) = 0.0D00
 5               CONTINUE
           J(1) = 0.0D00
           DJ(1) = 0.5D00
           ENDIF
           J(0) = 1.0D00
           DJ(0) = 0.0D00
           Y(N) = -1.0D-300
           DY(N) = 1.0D300
      ELSEIF (N.EQ.0) THEN
C          POLYNOMIAL EXPANSION ONLY FOR N = 0
           CALL BES0(X,J,Y,PI,DJ,DY)
      ELSE
C          RECURSION FOR ALL OTHER CASES
C          Y IS A FORWARD RECURSION
           CALL BES0(X,J,Y,PI,DJ,DY)
           Y(1) = -DY(0)
        DY(1)=Y(0) - Y(1)/X
      IF (N.EQ.1) GO TO 20
      DO 10, I = 0, N-2
      Y(I+2) = (2.0D00*(I+1)/X)*Y(I+1) - Y(I)
      DY(I+2) = Y(I+1) - ((I+2)/X)*Y(I+2)
 10    CONTINUE
C     J IS A REVERSE RECURSION BASED ON A PAIR OF BESSEL FUNCTION
C     POINTS DERIVED FROM A TRUNCATED POWER SERIES EXPANSION. THE
C     RECURSION IS THEN SCALED TO A KNOWN VALUE, J1(X).
 20    SCALE = -DJ(0)
      NSAVE = N
      IF (X.LE.N) THEN
           N = 5*N+50
```

```
            GOTO 25
         ENDIF
         N = IDNINT(N + X*X + 0.5D00)
C
 25       A = 1.0D00/DFLOAT(N+1)
         B = 1.0D00/DFLOAT(N+2)
         C = 1.0D00/DFLOAT(N+3)
         D = 1.0D00/DFLOAT(N+4)
         E = 1.0D00/DFLOAT(N+5)
         F = X/2.0D00
C
         JTEMP = 1-A*F**2+0.5D00*A*B*F**4-(1.0D00/6.0D00)*A*B*C*F**6+

+(1.0D00/24.0D00)*A*B*C*D*F**8-(1.0D00/120.0D00)*A*B*C*D*E*F**10
C
         N  = N - 1
         A = 1.0D00/DFLOAT(N+1)
         B = 1.0D00/DFLOAT(N+2)
         C = 1.0D00/DFLOAT(N+3)
         D = 1.0D00/DFLOAT(N+4)
         E = 1.0D00/DFLOAT(N+5)
         F = X/2.0D00
C
         JTEMP1 = 1-A*F**2+0.5D00*A*B*F**4-(1.0D00/6.0D00)*A*B*C*F**6+

+(1.0D00/24.0D00)*A*B*C*D*F**8-(1.0D00/120.0D00)*A*B*C*D*E*F**10
C
         DO 30, I = N+1,2,-1
         JTEMP2 = 2*((I - 1)/X)*JTEMP1 - JTEMP
         IF(DABS(JTEMP2).GE.1.0D250) THEN
              JTEMP2 = JTEMP2*1.0D-250
              JTEMP1 = JTEMP1*1.0D-250
         ENDIF
         JTEMP = JTEMP1
         JTEMP1 = JTEMP2
         IF ((I-2).LE.NSAVE) THEN
              J(I-2) = JTEMP2
         ENDIF
 30       CONTINUE
C        SCALING
         N = NSAVE
         SCLFAC = SCALE/J(1)
         DO 40, I = 0, N
         J(I) = SCLFAC*J(I)
         IF (I.EQ.0) THEN
              GOTO 40
         ENDIF
         IF (ABS(J(I)/J(I-1)).LT.1.0D-50) THEN
              J(I) = J(I)*1.0D250
         ELSEIF (ABS(J(I)/J(I-1)).GT.1.0D50) THEN
              J(I) = J(I)*1.0D-250
         ENDIF
```

```
      DJ(I) = J(I-1) - (I/X)*J(I)
 40    CONTINUE
      ENDIF
      RETURN
      END
C
      SUBROUTINE BES0(X,J,Y,PI,DJ,DY)
C     FOR ZERO ORDER BESSEL FUNCTIONS ONLY
      DIMENSION J(0:200), Y(0:200), DJ(0:200), DY(0:200)
      DOUBLE PRECISION J, Y, X, PI, DJ, DY, F0, F1, THETA0
      DOUBLE PRECISION THETA1, A
      IF (X.LE.3.0D00) THEN
      A = X/3.0D00
           J(0)    =    1.0D00    -    2.2499997D00*(A**2)    +
1.2656208D00*(A**4) -
     +0.3163866D00*(A**6)    +    0.0444479D00*(A**8)    -
0.0039444D00*(A**10) +
     +0.00021D00*(A**12)
           Y(0) = (2.0D00/PI)*DLOG(X/2.0D00)*J(0) + 0.36746691D00
+
     +.60559366D00*(A**2)    -    0.74350384D00*(A**4)    +
0.25300117D00*(A**6)
     +   -   0.04261214D00*(A**8)   +   0.00427916D00*(A**10)   -
0.00024846D00*
     + (A**12)
           DJ(0)=-X*(.5D00-0.56249985D00*(A**2)+0.21093573D00
     +*(A**4)-  0.03954289D00*(A**6)  +  0.00443319D00*(A**8)  -
0.00031761
     +D00*(A**10) + 0.00001109D00*(A**12))
                                          D   Y   (   0   )        =
(-1.0D00/X)*((2.0D00/PI)*X*DLOG(X/2D00)*(-1.0D00*
     +DJ(0))-0.6366198D00+0.2212091D00*(A**2)+2.1682709D00*(A**4)
-
     +1.3164827D00*(A**6)    +    0.3123951D00*(A**8)    -
0.0400976D00*(A**10)
     + + 0.0027873D00*(A**12))
      ELSE
      A = 3.0D00/X
           F0    =    .79788456D00    -    0.00000077D00*A    -
0.00552740D00*(A**2)
     +-0.00009512D00*(A**3)    +    0.00137237D00*(A**4)
-0.00072805D00*(A**5)
     ++0.00014476D00*(A**6)
           THETA0  =  X  -  0.78539816D00  -  0.04166397D00*A  -
0.00003954
     +D00*(A**2) + 0.00262573D00*(A**3) - 0.00054125D00*(A**4) -
     +0.00029333D00*(A**5) + 0.00013558D00*(A**6)
           J(0) = F0*DCOS(THETA0)/DSQRT(X)
           Y(0) = F0*DSIN(THETA0)/DSQRT(X)
           F1 = 0.79788456D0C + 0.00000156D00*A + 0.01659667D00*A*A
     ++0.00017105D00*(A**3) - 0.00249511D00*(A**4) + 0.00113653D00
     +*(A**5) -0.00020033D00*(A**6)
```

```
      THETA1 = X - 2.35619449D00 + .12499612D00*A + 0.00005650
     +D00*(A**2) - 0.00637879D00*(A**3) + 0.00074348D00*(A**4) +
     + 0.00079824D00*(A**5) - 0.00029166D000*(A**6)
      DJ(0) = -F1*DCOS(THETA1)/DSQRT(X)
      DY(0) = -F1*DSIN(THETA1)/DSQRT(X)
 ENDIF
 RETURN
 END
```

APPENDIX H. PROGRAM TO COMPUTE THE INTEGRATION ROUTINES

```
C                    B
C     This program computes INTEGRAL F(x) cosTx dx if KEY = 1  and
C      B              A
C     Int F (x) sin Tx dx when KEY=2 using Filon's mwthod. F(x) is
C      A
C     assumed to be given as a table of   functional values in the
C     array at M equidistant points from A to B, M odd. MAX is the
C     dimension of the array declared in the calling program.
C     Ref: P. J. Davis and P. Rabinowitz, Methods of Numerical
C     Integration, Academic Press, p. 488
C
C
      REAL FUNCTION FILON (F, T, A, B, M, MAX, KEY)
      INTEGER KEY, M, N, I, MAX
      REAL F (MAX), AL, BE, GA, SUM, TH, H, S, C, S1, C1, S2, C2
      REAL F1, F2, SU, SU1, A1, A, B, T, TH2, TH4
      N = M - 1
      H =ABS(B - A) / FLOAT (N)
      TH = T * H
      S = SIN (TH)
      C = COS (TH)
      TH2 = TH * TH
      IF (ABS (TH) .LE. 0.1) THEN
      TH4 = TH2 * TH2
      AL = 2. / 45. * TH * TH2 * (1. - TH2 / 7. + TH4 / 105.)
      BE = 2. / 3. + 2. / 15. * TH2 - 4. / 105. * TH4
      GA = 4. / 3. - 2. / 15. * TH2 + 1. / 210. * TH4
      ELSE
      AL = 1. / TH + S * C / TH2 - 2. * S * S / (TH2 * TH)
      BE = (2. + 2. * C * C  - 4. * S * C / TH) / TH2
      GA = 4. * (S / TH - C) / TH2
      END IF
      SUM = 0.0
      F1 = F (1)
      F2 = F (M)
      S1 = SIN (A * T)
      S2 = SIN (B * T)
      C1 = COS (A * T)
      C2 = COS (B * T)
      A1 = A + H
      GO TO (1, 2), KEY
    1     SU = F2 * S2 - F1 * S1
      SU1 = -0.5 * (F2 * C2 - F1 * C1)
      DO 3 I = 2, N, 2
      SUM = SUM + F (I) * COS (A1 * T)
      A1 = A1 + H
      SU1 = SU1 + F (I+1) * COS (A1 * T)
    3     A1 = A1 + H
      GO TO 4
```

```
2      IF (T .EQ. 0.) THEN
 FILON = 0.
 RETURN
 END IF
 SU = F1 * C1 - F2 * C2
 SU1 = -0.5 * (F2 * S2 - F1 * S1)
 DO 5 I = 2, N, 2
 SUM = SUM + F (I) * SIN (A1 * T)
 A1 = A1 + H
 SU1 = SU1 + F (I+1) * SIN (A1 * T)
5      A1 = A1 + H
4      FILON = H * (AL * SU + BE * SU1 + GA * SUM)
 RETURN
 END
```

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