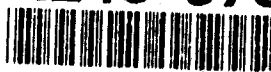


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Many important Hamiltonian systems have periodic solutions that are associated with symmetries of the equations. While it is well known that stationary solutions of a Hamiltonian system can be characterized as extremals of the potential energy, it is less widely appreciated that symmetry-related periodic solutions, or relative equilibria, can also be given a variational characterization, typically involving constraints. This variational characterization is important because if a periodic solution is associated with a constrained minimizer (in some sense), as opposed to merely being a stationary point, then a stability result is very often available. We are therefore left with the problem of characterizing those extremals of a constrained variational principle that are actually constrained local minima. It is shown how to apply the new results in the special context of Hamiltonian mechanics, and various stability and instability theorems are described. The machinery developed here can be viewed as an alternative to the energy-casimir and energy-momentum methods with the benefit that the necessary tests can be concretely and rigorously applied in several complex examples of physical importance.

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**FINAL REPORT ON RESEARCH
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This report describes papers that have been accepted or submitted in the period June 1989 through October 1991. This work was supported by the grant AFOSR-89-0376 (with Maddocks as PI) as well as by the AFOSR URI Grant 87-0073 through the Systems Research Center, University of Maryland (with Maddocks as co-PI). Completed research results that will be published in manuscripts that are currently in preparation are also described here. Preprints are either enclosed or have been forwarded previously. References of the form [A.13] etc. are to the attached *curriculum vitae*. Manuscripts that have appeared or have been accepted have the prefix [A.], manuscripts that have been submitted have the prefix [B.], and manuscripts in preparation have the prefix [C.].

Article [A.13] "**On second-order conditions in constrained variational principles**", is to appear in the Journal of Optimization Theory and Applications. In this work I combine results from [A.9] with a multi-parameter version of the results appearing in [A.6] to obtain a comprehensive approach to the analysis of the second-order tests that determine which extremals of multiply constrained isoperimetric variational principles are actually constrained minima. The results are couched in terms of the shape, more specifically the singularities and curvatures, of the bifurcation hyper-surface that is obtained when a modified Lagrangian (evaluated on solutions) is plotted above the Lagrange multipliers associated with the constraints.

Article [A.11] "**On the Stability of Relative Equilibria**" has appeared in the IMA Journal of Applied Math. (1991) 46 pp. 71-99. Many important Hamiltonian systems have periodic solutions that are associated with symmetries of the equations. While it is well known that stationary solutions of a Hamiltonian system can be characterized as extremals of the potential energy, it is less widely appreciated that symmetry-related periodic solutions, or relative equilibria, can also be given a variational characterization, typically involving constraints. This variational characterization is important because if a periodic solution is associated with a constrained minimizer (in some sense), as opposed to merely being a stationary point, then a stability result is very often available. We are therefore left with the problem of characterizing those extremals of a constrained variational principle that are actually constrained local minima. ~~In this paper I show how to apply the new results described in [A.13] in the special context of Hamiltonian mechanics,~~ and various stability and instability theorems are described. The machinery developed here can be viewed as an alternative to the energy-casimir and energy-momentum methods with the benefit that the necessary tests can be concretely and rigorously applied in several complex examples of physical importance [see [A.12], [B.1], [B.2], [F.2]].

-J-

Article [B.1] (with R.L. Sachs) "**On the stability of KdV multi-solitons**" has been submitted to Comm. Pure and Applied Math. In this work we use the approach of [A.11] and [A.13] to examine stability of the n -soliton of the Korteweg-de Vries equation. We apply the theory to obtain a stability result by showing that in the classic variational characterization of multi-solitons of KdV due to Kruskal and Lax, the multi-soliton actually realizes a constrained minimum (as opposed to any other type of critical point). This result extends the seminal work of Benjamin which considered the single soliton case $n = 1$. Our

methods promise to extend to various other classic partial differential equations with multi-solitons, as well as to other physically interesting systems such as the Toda lattice.

Article [A.12] (with L-S. Wang and P.K. Krishnaprasad) "**Hamiltonian dynamics of a rigid body in a central gravitational field**", has appeared in *Celestial Mechanics and Dynamical Astronomy* **50** (1991) pp. 349-386. In this work we show that the dynamics of a satellite in an inverse square field can be cast as a noncanonical Hamiltonian system, and we show how to make consistent approximation of the potential that arises for satellites of finite size, i.e. the satellite is not treated as a point mass. It is shown that the families of relative equilibria in the models arising at different levels of approximation can have markedly different properties. Stability and instability properties for various relative equilibria are examined using ideas described in [A.11].

Article [B.2] (with L-S. Wang and P.K. Krishnaprasad) "**Steady rigid-body motions in a central gravitational field**" has been submitted to the *J. of Astronautical Sciences*. This paper continues the analysis of [A.12]. Here we prove the existence of non-great circle relative equilibria, which are families of relative equilibria that are qualitatively different from those previously known. These non-great circle relative equilibria arise for the exact potential of a finite size satellite, and their special properties are destroyed in the classic approximate analysis. We also show through numerical studies of the problem in the parameter regime pertinent to artificial satellites that differences between solutions to the approximate and exact models can be of physical significance. These studies lead to extremely stiff, numerically ill-conditioned systems, and associated difficulties which were resolved by performing computations on a CRAY super-computer to obtain additional accuracy (24 significant digits).

Article [F.2] (with L-S. Wang) "**Stability of steady motions of a rigid body with gyrostats**" is in preparation. In this paper we extend the theory of [A.11] and [A.13] to a rigid body (in the absence of gravitational forces) with momentum wheels or rotors. This system is frequently adopted as a model for artificial (i.e. man-made) space satellites, with the momentum wheels being used as the controls for orientation. There are three types of momentum wheels possible. Fixed speed wheels, freely spinning wheels, and damped wheels, and we compare and contrast the stability of the permanent rotations in the presence of the various types of rotors. The case of damped wheels is particularly interesting because it presents a nontrivial, practically important but still tractable example of a system with dissipative perturbations of a conservative Hamiltonian. Our analysis exemplifies the general principle that the only stable motions in a Hamiltonian system subject to dissipative perturbations are those that are minima of an appropriate constrained variational principle.

Article [A.14] (with R. Malek-Madani) "**Steady-state shear-bands in thermo-plasticity, Part I: vanishing yield stress**", has been accepted in the *Int. J. of Solids and Structures*. This paper applies the variational bifurcation theory developed in [A.6] to consider the problem of shear-band formation in a thermo-plastic material where the constitutive law is dependent upon temperature and strain rate only. The existence of solutions and the qualitative form of the bifurcation diagram is obtained via a phase-plane analysis. Then stability properties of the solutions immediately follow from my previous work on stability exchange [A.6].

Article [F.1] (with R. Malek-Madani) **"Steady-state shear-bands in thermoplasticity, Part II: non-vanishing yield stress"**, is in preparation. This work extends the analysis of [B.1] to the case where the constitutive law has a nonzero value as strain rate is decreased to zero. This behaviour is often adopted as a simple model for a vestigial elastic yield stress. Having such a discontinuous constitutive law gives rise to markedly different phenomena from those found in [B.1]. In particular we obtain the ad hoc slip-line model of plasticity as a limit within our partial differential equation model.

Article [B.3] (with J.C. Alexander) **"Bounds on the friction-dominated motion of a pushed object"** has been submitted to the Int. J. of Robotics Research. In this work we used the variational principle introduced in [A.10] that is associated with minimal frictional dissipation to analyze a problem that arises in some robot manipulation problems, namely the friction dominated motion of an object being pushed over a rough plane. We have shown that in certain physically plausible circumstances the problem for an arbitrary pressure distribution can be reduced to the case of singular pressure distributions with two-point support in the boundary of the work piece, and used this reduction to obtain calculable bounds on all possible motions.

Article [D.1] (with J.C. Alexander) **"On the kinematics of wheeled mobile robots"** is a reprint of the paper [A.10] in a book format.

Article [B.4] (with D. Yakobson) **"On the dynamics of chains"** has been submitted to the SIAM J. Applied Math. The method of local Lie symmetry groups is used to systematically find all similarity solutions of the system of nonlinear partial differential equations that govern the quasi-static motion of a heavy chain being dragged over a rough table-top. The similarity solutions that we find include the shape of a steadily rotating segment with vanishing tension at one end, and a moving segment propagating into stationary chain. The problem presents interesting mathematical challenges because of the unusual form of the nonlinearity. The exact form of this nonlinearity is ubiquitous throughout models involving Coulomb friction and the chain is of some interest because it is the simplest example of a flexible body moving under the effects of friction.

Article [F.3] (with S. Alvarez) **"Minimum time trajectories of robot carts"** is in preparation. In this work we consider the optimal control problem of generating minimal time trajectories of wheeled robots subject to the constraint of finite available friction at the wheels. The Pontryagin Maximum Principle is applied to characterize optimal controls, namely optimal steering and driving of wheels, and optimal trajectories are explicitly constructed for various particular robot designs and boundary conditions. More generally the generation of optimal time trajectories is reduced to the solution of a two-point boundary value problem which can be solved numerically by shooting.

Article [F.4] (with D. Dichmann) **"Nonlinear Stability of Solitary Waves in an Elastica"** In this work we show that the exact inertial dynamics associated with the finite deformation of an elastic rod can be written as a constrained Hamiltonian system. The system is unusual in that involves both partial differential and nonlocal terms. We use the Hamiltonian structure to characterize solitary waves, finding a previously unknown conserved quantity for rod dynamics, and prove that solitary waves with speed below a certain critical number are stable.