

AD-A245 702



David Taylor Research Center

Bethesda, Maryland 20084-5000

DTRC/SHD-1374-01 November 1991

Ship Hydromechanics Department

Departmental Report

A Note on Structural Loads Analysis in the Reliability Context

by
J.F. Dalzell



92-03000



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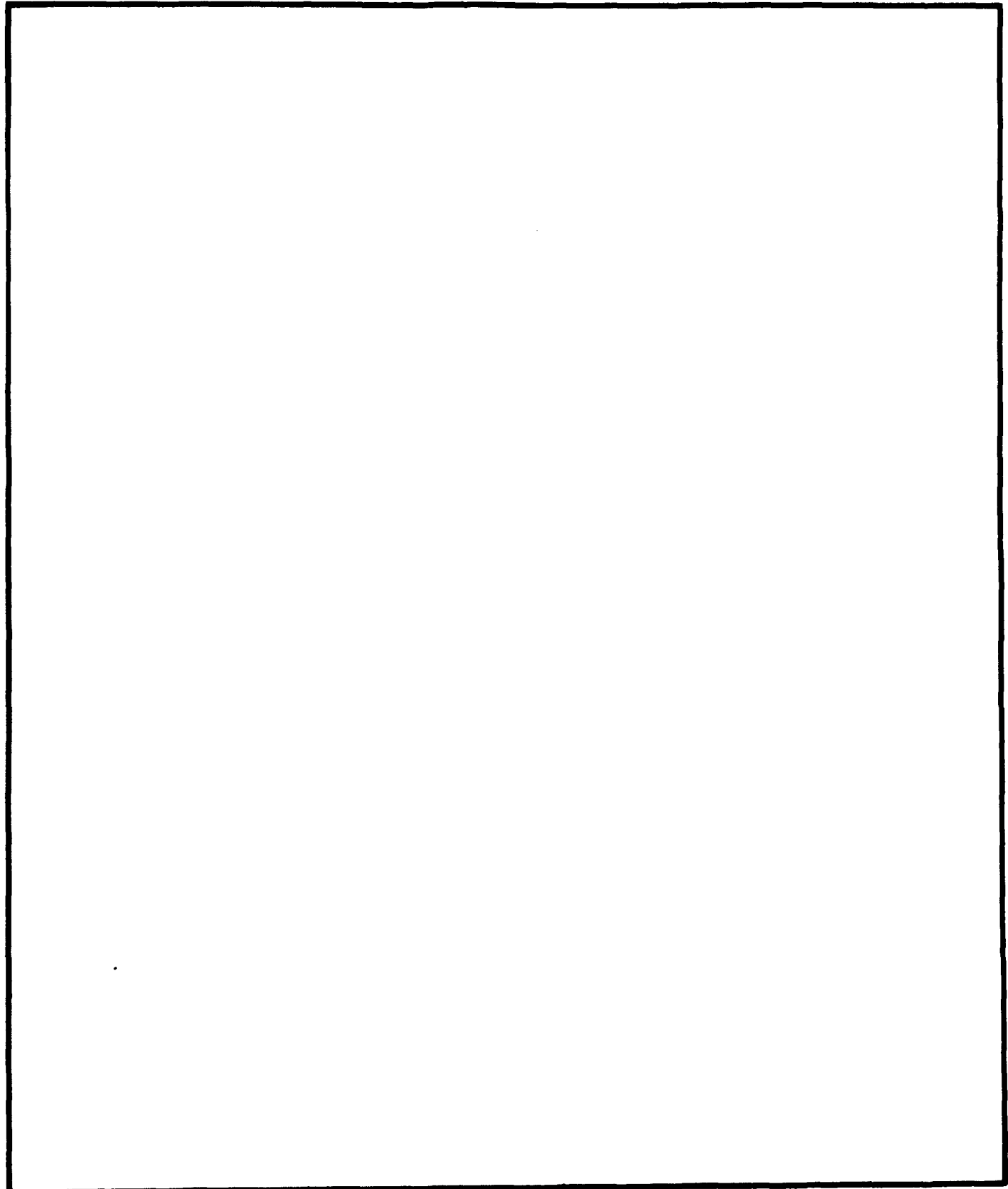
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REPORT DOCUMENTATION PAGE				Form Approved OMB No 0704-0188	
1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED			1b RESTRICTIVE MARKINGS		
2a SECURITY CLASSIFICATION AUTHORITY			3 DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release: Distribution unlimited.		
2b DECLASSIFICATION / DOWNGRADING SCHEDULE					
4 PERFORMING ORGANIZATION REPORT NUMBER(S) DTRC/SHD-1374-01			5 MONITORING ORGANIZATION REPORT NUMBER(S)		
6a NAME OF PERFORMING ORGANIZATION David Taylor Research Center Ship Hydromechanics Dept.		6b OFFICE SYMBOL (If applicable) Code 1561	7a NAME OF MONITORING ORGANIZATION		
6c ADDRESS (City, State, and ZIP Code) Bethesda, Maryland 20084-5000			7b ADDRESS (City, State, and ZIP Code)		
8a NAME OF FUNDING / SPONSORING ORGANIZATION Office of Naval Technology		8b OFFICE SYMBOL (If applicable) ONT 211	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER		
8c ADDRESS (City, State, and ZIP Code) Arlington, Virginia 22217-5000			10 SOURCE OF FUNDING NUMBERS		
			PROGRAM ELEMENT NO 62121N	PROJECT NO RH21S23	TASK NO 7
			WORK UNIT ACCESSION NO DN501138		
11 TITLE (Include Security Classification) A NOTE ON STRUCTURAL LOADS ANALYSIS IN THE RELIABILITY CONTEXT					
12 PERSONAL AUTHOR(S) J.F. DALZELL					
13a TYPE OF REPORT Final		13b TIME COVERED FROM 901001 TO 910930		14 DATE OF REPORT (Year, Month, Day) 1991 November	
15 PAGE COUNT 42					
16 SUPPLEMENTARY NOTATION DTRC Work Unit No. 1-1506-123					
17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number) Hydrodynamic Loads, Reliability Design		
FIELD	GROUP	SUB-GROUP			
19 ABSTRACT (Continue on reverse if necessary and identify by block number) In order to fully exploit reliability based structural design methods in the design of naval vessels, means to analytically predict the magnitude and distribution of wave induced and slamming loads must be developed in a form which is consistent with the requirements of both the structural designer and the reliability analyst. This requirement defined the objectives of the present exploratory development task in hydrodynamic loads. A first step in this task was to assess the framework within which the products of this task should fit in order to insure the desired consistency with the other parts of the problem. The present document summarizes an initial attempt to define such a framework. It includes a review of modern reliability methods from the point of view of the load analyst, some thoughts on a practical strategy of meeting the perceived needs with state-of-art technology, and a discussion of a few issues which, according to the literature, appear not to have been completely resolved.					
20 DISTRIBUTION / AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS			21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
22a NAME OF RESPONSIBLE INDIVIDUAL John F. Dalzell			22b TELEPHONE (Include Area Code) 301-227-1210		22c OFFICE SYMBOL Code 1561

SECURITY CLASSIFICATION OF THIS PAGE



CONTENTS

	Page
NOMENCLATURE	iv
ABSTRACT	1
ADMINISTRATIVE INFORMATION	1
INTRODUCTION	1
NATURE OF RELIABILITY BASED DESIGN	2
LOAD DEFINITION	4
THE FORM OF THE DESIRED ANSWER	6
LEVEL III ANALYSIS	6
LEVEL II ANALYSIS	6
FUNDAMENTAL LOAD SYNTHESIS	7
INPUT PARAMETERS	7
SHIP LIFETIME SYNTHESIS	9
PRACTICAL ISSUES	10
SYNTHESIS STRATEGY	13
INITIAL SEPARATION OF FUNCTION	15
STRATEGY FOR CONDITIONAL LOAD COMPUTATIONS	15
CONDITIONAL SYNTHESIS	19
COMBINATION OF DEAD AND DYNAMIC LOADS	20
COMBINATION OF SPEED AND WAVE INDUCED LOADS	22
WAVE INDUCED AND SLAMMING LOADS	25
SUMMARY	29
APPENDIX A, RELIABILITY ANALYSIS FROM A LOADS PERSPECTIVE	33
APPENDIX B, SYNTHESIS OF LIFETIME MEANS AND COVARIANCES .	39
REFERENCES	41

FIGURES

1. Hull Girder Reliability Analysis Elements	3
2. Practical lifetime loads synthesis.	12
3. Strategy for Conditional Computations.	17

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NOMENCLATURE

$\text{Cov}(\vec{D})$	Covariance matrix of the total load vector.
$\text{Cov}(\vec{X} \vec{A}_1, \dots)$	Conditional covariance matrix of vector $\vec{X}$, given vector(s) $\vec{A}_i$.
d_j	The j^{th} element of the random load vector.
$\bar{d}_j$	The mean of the j^{th} element of the random load vector.
$\vec{D}$	Total load vector, $[d_1, \dots, d_5]$.
$\vec{D}_{SW}$	Vector of dead loads.
$\vec{D}_{DY}$	Vector of dynamic loads.
$\vec{D}_U$	Speed induced load vector.
$\vec{D}_W$	Wave induced load vector.
$E[\vec{D}]$	Expectation or mean of the total load vector.
$E[\vec{X} \vec{A}_1, \dots]$	Conditional expectation or mean of vector $\vec{X}$, given vector(s) $\vec{A}_i$.
$f_D(\vec{D})$	Joint probability density of the load vector, $d_1, \dots, d_5$.
$f_{DC}(\vec{D} \vec{M}, \vec{R}, U, \mu, \vec{W})$	Conditional density of the total load vector.
$f_O(\vec{M}_i, U_j, \mu_k, \vec{W}_t)$	The joint density of operational variables.
$f_R(\vec{R}_m)$	The density of a discrete value of the flexural rigidity vector.
$f_Y(\vec{X}_Y \vec{A}_1, \dots)$	Conditional density of vector $\vec{X}_Y$, given vector(s) $\vec{A}_i$.
$\vec{M}$	Generalized mass distribution vector.
$\vec{M}_i$	Central value of generalized mass distribution vector.
$p_O(\vec{M}_i, U_j, \mu_k, \vec{W}_t)$	The joint frequency of occurrence of operational variables.
$p_R(\vec{R}_m)$	The frequency of occurrence of a discrete value of the flexural rigidity vector.
$\vec{R}$	Generalized flexural rigidity vector.
$\vec{R}_m$	Central value of generalized flexural rigidity vector.
U	Ship speed.
U_j	Central value of ship speed.
$\vec{W}$	Generalized wave condition vector.
$\vec{W}_t$	Central value of generalized wave condition vector.
$\delta\vec{M}_i, \delta U_j, \delta\mu_k, \delta\vec{W}_t, \delta\vec{R}_m$	Sub-domain ranges of generalized condition vectors.
μ	Ship heading relative to waves.
μ_k	Central value of ship heading relative to waves.
ρ_{d_i, d_j}	Correlation coefficient between d_i and d_j .
$\sigma_{d_j}^2$	Variance of the j^{th} element of the random load vector.

ABSTRACT

In order to fully exploit reliability based structural design methods in the design of naval vessels, means to analytically predict the magnitude and distribution of wave induced and slamming loads must be developed in a form which is consistent with the requirements of both the structural designer and the reliability analyst. This requirement defined the objectives of the present exploratory development task in hydrodynamic loads. A first step in this task was to assess the framework within which the products of this task should fit in order to insure the desired consistency with the other parts of the problem. The present document summarizes an initial attempt to define such a framework. It includes a review of modern reliability methods from the point of view of the load analyst, some thoughts on a practical strategy of meeting the perceived needs with state-of-art technology, and a discussion of a few issues which, according to the literature, appear not to have been completely resolved.

ADMINISTRATIVE INFORMATION

This work was supported by the Office of Naval Technology under the Surface Ship Technology Exploratory Development Block Program, ND1A, Advanced Hull Project RH21S23, Task 7, and performed under DTRC Work Unit Number 1506-123.

INTRODUCTION

The current task arose out of the broad objective of the ultimate customer (NAVSEA) to "Develop Revised Structural Design Procedures". Three related customer sub-objectives have been mentioned:

- a. Establish a reliability based structural analysis methodology.
- b. Integrate new analytic methods into an overall design methodology.
- c. Establish a basis for structural design of ships which are outside the historical data base.

The present exploratory development task in hydrodynamic loads was established in support of these overall objectives.

The key goal is the establishment of *reliability based* design procedures. From the load analyst's point of view, in order to approach this goal, the most important task is to provide practical means by which the magnitude and distribution of wave induced and slamming loads may be analytically predicted. Moreover, in addition to being reasonably affordable and timely, the resulting methods and procedures must be consistent with the requirements of both the structural designer and the reliability analyst.

A first step in the present task was to assess the framework within which the products of this task should fit in order to insure the desired consistency. The purpose of this note is to summarize the current understanding of some fundamental definitions, issues, and problems, as well as to indicate some strategies.

NATURE OF RELIABILITY BASED DESIGN

The fundamental idea in reliability based design is that in practice *neither* the loads on a structure or its resistance to load (strength) can be determined exactly. All the pertinent parameters are considered to be uncertain to a greater or lesser extent, and the evaluation of the adequacy of the structure is evaluated in terms of a probability of failure, or of the magnitude of safety indices which are related to the probability of failure.

Appendix A contains a brief review of the three "Levels" of reliability analysis from the perspective of the loads analyst. This review is based in large part upon material presented by Melton^{1*}, Mansour, *et al*,² and Faulkner and Sadden³.

It seems certain that the customer would wish to have a "Level I" reliability based design *code* rather than the involved *procedures* which are implied by the definitions of the higher level reliability analyses. However, a Level I code has to be calibrated by means of Level II analyses of "reliability indices", and a Level II analysis in turn has to be verified to some extent by at least approximate Level III analyses. Thus, at the present stage, to aid in the development of "reliability based" design, the requirements of *both* Level II and III analyses must be considered.

In Level II analysis, the means and covariances of loads and resistance must be synthesized. In Level III analysis, the joint probability density of loads and resistance must be estimated. If it is assumed, as in Appendix A, that the random variables which define the loads are statistically independent of those which define the resistances, the work of Level III reliability analysis may be divided into four main technical elements. If it is assumed that loads and resistances are at least uncorrelated, the work of Level II analysis divides into the same elements. These main elements are illustrated in Fig. 1 on page 3, and include:

- I. MODELING: The (nontrivial) definition of structural limit states and the definition of the pertinent loads and resistance variables.
- II. RESISTANCE SYNTHESIS. The synthesis of the joint density (or means and covariances) of resistance parameters. The sub-elements shown in Fig. 1 (IIa through IIc) are the ones defined by Melton¹.
- III. LOADS SYNTHESIS. The development of the joint density (or means and covariances) of load parameters. The sub-elements reflect the conventional division of loads into "dead" or static loads and dynamic loads.

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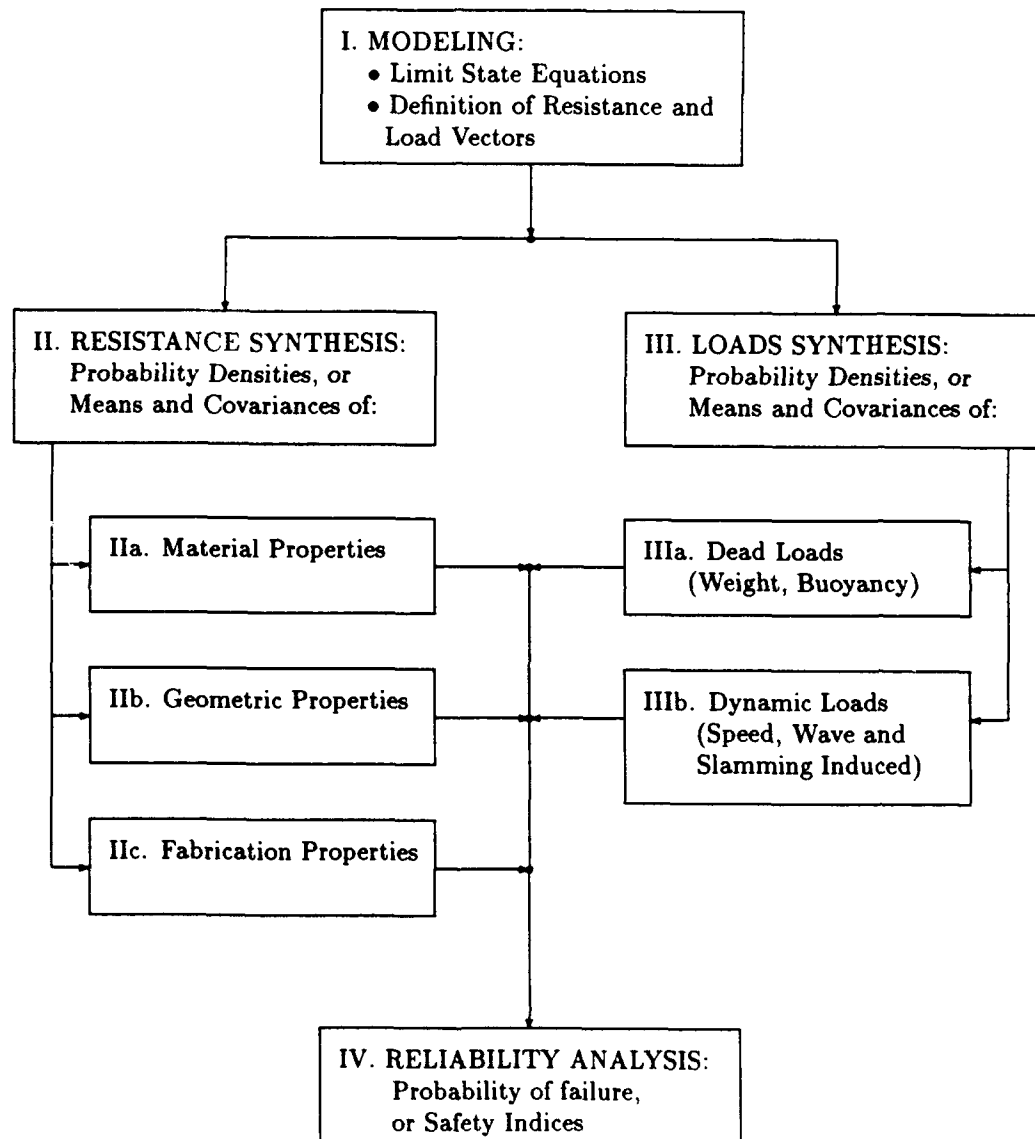


Fig. 1. Hull Girder Reliability Analysis Elements

IV. RELIABILITY ANALYSIS. The final coordination of the syntheses of Elements II and III, and the invocation of the reliability analysis machinery.

Thus in principle, the statistical independence assumption decouples the reliability problem into the same elements as are inherent in empirically based design in which loads and resistance may, up to a point, be approached separately.

LOAD DEFINITION

An explicit assumption in the detailed formulation of the present task was that the concentration is on *primary* loads on *monohulls*. Thus, by assumption, we are interested in the overall moments and shearing forces which act upon the hull girder. These we take to be;

1. Vertical bending moment,
2. Vertical shearing force,
3. Lateral bending moment,
4. Lateral shearing force and
5. Torsional bending moment,

where the order is in approximate order of importance to monohulls, and the implied coordinate system is that of the ship structure. In principle, each of the load components must be defined all along the length of the ship, and must include both the "dead" and "dynamic" components, Fig. 1. Note that the logical sixth load type, longitudinal tension and compression, has been omitted from the list on the basis that a ship has no end fixity and thus the maximum compression force is of the order of the thrust.

The basic hydrodynamic definition of "loads" involves pressures (hydrostatic and dynamic) and inertial reactions of the ship, which is assumed to be rigid. The stress analyst's definition includes, in addition, any moments and shears which arise from beam-like elastic deflections of the hull in response to dynamic loads. The stress analyst's expanded definition of loads is that implied by reliability theory, and is accordingly that which *must* be used in the present task.

Melton¹ summarizes the physical sources of loads in terminology consistent with current Naval surface ship design, and makes a number of recommendations which are essentially adopted here. In the Naval surface ship design context, the loads are classified into groups entitled: "Basic loads", "Environmental loads", "Operational environment and extreme loads", and "Loads due to the combat environment". Melton does not consider the combat loads. Each of the classifications are further subdivided, in part with terminology difficult for a hydrodynamic specialist to follow. It appears from Melton's analysis that for hull girder purposes three general non-combat primary "load" sources can be stated:

- Dead loads. These are what many have called “still water” loads; that is, the net moment and shears produced by the difference between weight and buoyancy in still water with the ship at zero speed. This load source probably has meaning only for vertical moments and shears. This component of the load can be highly variable in commercial ships, and thus may be expected to be highly variable in Naval auxiliaries. The magnitude of the variability is assumed to be low in combatants, but verification studies may not have been attempted.
- Hull Girder Loads due to the wave environment. These, the writer interprets as the moments and shears which would be produced by wave action on a rigid ship.
- Whipping or Slamming Loads. These, the writer interprets as the internal vibratory moments and shears which arise from the dynamic elastic deflection of the ship in response to transient wave excitation.

One additional vertical bending and shear load source is the speed induced ship wave pattern, and the consequent sinkage and trim of the vessel—essentially the net effect of speed induced changes in buoyancy distribution. This component of vertical load is likely to be important only for high speed ships; that is, for the fastest combatants. It is not clear whether this load is considered at all in the standard Naval ship design approach. Since it is speed dependent, it appears rational to carry this component of loads with the dynamic contributions as indicated in Fig. 1.

The general definition of loads just described appears to be that associated with primary direct stresses, Lewis⁴; that is, deals with the in-plane loads on the elements of the hull girder. This definition of loads appears to be that which has been in mind in much of the recent literature on the application of reliability analyses to ship structure, for example, Mansour^{5,6}, Faulkner and Sadden³, Stiansen, *et al.*,⁷ and Mansour, *et al.*² It appears also to be the load definition considered by Sikora, *et al.*⁸

The design scenario which is not addressed by this load definition is that where a hull is loaded in bending and shear, with simultaneous “out of plane” pressure loading. An example for a structural sub-element is given by Melton.¹ Though the inclusion of external pressure distributions in the present loads definition is not ruled out in principle, an extension of the definition to include external pressures has been deferred in order to increase the probability of success of the present task by limiting some of the less important complications.

Another “load source” mentioned in the commercially oriented literature (Stiansen, *et al.*,⁷ for example) might be noted. This “load source” involves the stresses induced by temperature differences between exposed deck structure and immersed bottom structure. The computation of the stress changes is entirely a matter of structural analysis, given temperature differentials along the ship as inputs. Thus, though the effect is sometimes termed a “thermal load”, it is not understood how it can be considered to be a “load” in the same sense as those discussed above. Within the reliability framework, it may be that the appropriate “thermal load” input would be the statistics of

air-sea temperature differences, but it is not clear where this might fit with the reliability elements noted in Fig. 1. (Evidently Stiansen, *et al.*,⁷ had similar doubts since they appear to recommend that the effect be considered as a constant deterministic bias of stresses and do not to include it in their reliability model.)

THE FORM OF THE DESIRED ANSWER

There seems no way to generalize about the form of the required loads answer in the Level I reliability context—it will have to be sorted out as serious higher level analyses are pursued. Since both of the higher levels of reliability analysis are clearly required, there are two (related) forms of loads answer required, and from Appendix A:

LEVEL III ANALYSIS

An ideal Level III analysis requires the synthesis of the joint probability density of the load vector, $f_D(d_1, \dots, d_5)$, where the elements of the load vector, $[d_1 \dots d_5]$, correspond to the bending and shear loads noted earlier. The population, or sample space of loads is defined by all the possible load vectors which could occur during the life of an infinite number of sister ships. The ideal requirement for a *joint* probability density takes care of the structural analyst's requirement to examine stresses which occur from the combination of two or more loads.

The major problem with the synthesis of probability densities is that it is philosophically impossible to "calculate" probability densities, or even to verify approximations with anything approaching 100% confidence. The low probability "tails" of the densities are important, and are the regions where the least confidence will exist. The practical result is that the synthesis requires *extrapolations* to events of very low probability, and in practice this means fitting computed or empirical data to analytical probability density functions.

The marine literature examined thus far has concentrated primarily upon the problem of vertical bending moment (d_1), and has not addressed the problem of the synthesis of *joint* densities. Thus, the synthesis of joint densities is presently a *goal* rather than a procedure for which an approach is defined.

LEVEL II ANALYSIS

An ideal Level II analysis requires the synthesis of the vector of mean values of the load components, $[\bar{d}_1 \dots \bar{d}_5]$, as well as the 5 by 5 covariance matrix. The diagonal elements of the covariance matrix are the variances of the individual loads, $\sigma_{d_i}^2$ for $i = 1 \dots 5$. The off-diagonal elements are defined by the product $(\rho_{d_i d_j} \sigma_{d_i} \sigma_{d_j})$ for $i \neq j$, where $\rho_{d_i d_j}$ is the correlation coefficient between loads d_i and d_j . The implied population, or sample space, of loads is the same as that for Level III analysis. The inclusion of correlation coefficients in the ideal requirement takes care of the structural analyst's requirement to examine stresses which occur from the combination of two or more loads.

As in the case of Level III analyses, the marine literature examined thus far has concentrated primarily upon the problem of vertical bending moment (d_1), and has not addressed the problem of the synthesis of correlation coefficients, which, in addition to the variances of the other load elements, is presently a *goal*.

In the context of the synthesis of vertical bending moments, for Level II analyses, there appear to be two main approaches to the definition of the mean and variance.

- The first follows the basic development, and defines the mean and variance as the mean and variance of the population of short term extremes which may experienced for a fixed length of exposure time (the exact exposure time appears to vary from a half hour to a day depending upon the literature source). This is the school of thought initiated by Mansour⁵ and followed in his later papers.
- The second, Faulkner and Sadden,³ defines the mean value as the largest expected extreme in a ship life, and postulates a variance relative to this mean on the basis of extreme value theory. This approach is closest to the state of art "long term distribution" concept noted in Lewis,⁴ and results in a relatively smaller "variance" presented to the reliability machinery (a possible computational advantage).

The distinction will have to be sorted out—there appears to be a considerable philosophical difference.

FUNDAMENTAL LOAD SYNTHESIS

The basics of the usual synthesis procedure involve the interrelation of groups of variables, as well as some practical matters.

INPUT PARAMETERS

The hydrodynamic technology for total loads which we have, or might have, in the immediately foreseeable future, involves, at best, the load responses to waves under some basic limitations, which are in general:

1. Constant ship geometry and mass distribution parameters,
2. "Wheel and throttle fixed" ship operation, and
3. Statistically stationary waves and response.

These limitations are part practical and part theoretical. Essentially, the only random inputs to state-of-art hydrodynamic simulations are the waves—after the averages which define a particular statistically stationary wave process are set. Under these limitations, any statistical measures of the load vector which may be derived are conditioned upon (functions of) given, *constant*, values of ship geometry, loading, heading, speed, and the parameters which define a statistically stationary wave condition.

Though it is reasonable to suppose for hydrodynamic purposes that the external hull geometry is constant over the life of a class of ships, the mass distribution may vary either systematically or randomly on a relatively long time scale—the purpose of ships is to carry whatever loads are required at a given time. The mass distribution, or certain of its integrated properties, influences the loads from all four of the physical sources mentioned earlier. Thus, for the purposes of synthesis of most ship types, the mass distribution must be carried along as at least a quasi-random parameter, and will be denoted generally by $\vec{M}$, with the understanding that it is a vector function of ship length. (With the conventional lumped mass representation, a vector, $\vec{M}$, consisting of mass, centers, and gyradii may be defined for each of the lumps which approximate the mass distribution over the length of the ship.)

For the purpose of the synthesis of the transient vibratory response of the hull to impact, it is required that the flexural and shear rigidities, and the structural damping in the lower modes of vibration be specified; that is, that the hull girder be modeled as a beam with flexural rigidities varying over the length and having certain given energy dissipation properties. For present purposes the flexural rigidity vector and the structural damping properties will be denoted by the general vector, $\vec{R}$.

For a given ship, the value of $\vec{R}$ should ideally be as constant as the external hull geometry. Flexural rigidities are functions of the geometric and material properties of the structure, that is, are subject to random errors in fabrication and material properties which, earlier, have been assumed to be statistically independent of loads. Thus, if whipping loads are to be included, a basic inconsistency in the general formulation is exposed, since the flexural rigidities of the structure are in part the object of the structural design and formally outside the loads sphere. It is not presently known what can be assumed to mitigate this inconsistency. At present it will be assumed that the flexural rigidity/damping vector must also be carried as at least a quasi-random parameter.

Ship speed, U , and heading relative to waves, μ , both influence all three of the dynamic load components. They are mostly operational parameters, which can be expected to vary widely in combatants, and which may depend in part upon the severity of the waves.

“Wave conditions” are amenable to a wide range of systems of description. At a minimum, it is normal to define significant wave height, some measure of average period, and a spectral shape (which may include directionality). For present purposes the wave conditions will be denoted by the vector, $\vec{W}$.

Thus the input parameters to the hydrodynamic simulation have been generalized as $\vec{M}$, $\vec{R}$, U , μ , and $\vec{W}$.

SHIP LIFETIME SYNTHESIS

For notational convenience, the total load vector, will be defined as $\vec{D} = [d_1, \dots, d_5]$, where it is to be understood that the load vector is a function of position along the length of the ship. For the purposes of this section, each total load component is considered to be the sum of the contributions from the four physical sources mentioned earlier.

It has been noted that the object of the loads synthesis is to synthesize the statistics of the load vector, $\vec{D}$, under the assumption that the population is defined by all the possible load vectors which could occur during the life of an infinite number of sister ships.

Noting the discussion above, the basic hydrodynamic simulation would, at most, yield the *conditional* joint probability density of total loads, for a given definition of $\vec{M}, \vec{R}, U, \mu$, and $\vec{W}$. Using the usual convention, this conditional density may be denoted as, $f_{DC}(\vec{D}|\vec{M}, \vec{R}, U, \mu, \vec{W})$.

Thus, in order to synthesize the function desired for the highest level of reliability analysis (the joint probability of the total loads, $f_D(\vec{D})$), it is necessary to define the joint probability density of the parameters, $\vec{M}, \vec{R}, U, \mu$, and $\vec{W}$.

The list of parameters is a mixed bag in the sense that the purely structural parameters, $\vec{R}$, are probably not related in a statistical sense to any of the others. It appears reasonable to assume that the random aspects of $\vec{R}$ are statistically independent of the other operational and climatological variables; that is, for example, uncertainties in estimates of flexural rigidities exist completely independently of how it is chosen to operate the ship, or the wave climate. On this independence assumption, the joint probability density of the parameters, $\vec{M}, \vec{R}, U, \mu$, and $\vec{W}$ becomes a product of the form: $f_R(\vec{R}) f_O(\vec{M}, U, \mu, \vec{W})$, where $f_R(\vec{R})$ is the density of $\vec{R}$, and $f_O(\vec{M}, U, \mu, \vec{W})$ is the joint density of the operational and climatological variables.

Then the operation to recover $f_D(\vec{D})$ becomes:

$$f_D(\vec{D}) = \int \dots \int f_{DC}(\vec{D}|\vec{M}, \vec{R}, U, \mu, \vec{W}) f_R(\vec{R}) f_O(\vec{M}, U, \mu, \vec{W}) d\vec{M} d\vec{R} dU d\mu d\vec{W} \quad (1)$$

where the domains of integration must include the entire domains of each of the parameters.

Apart perhaps from the inclusion of load conditions and the structural properties, Eq. 1 describes the starting point for virtually all the known procedures for extrapolation to ship lives.

The basic synthesis for ship lives is very similar for the case that the hydrodynamic simulations are used in support of Level II reliability methods where means and covariances are required. In this case, the mean value of the load vector is the statistical expectation, $E[\vec{D}]$, and the covariance matrix is of the form of the expectation, $E[\vec{D}^T \vec{D} - E[\vec{D}]^T E[\vec{D}]]$, which will be denoted more compactly by $\text{Cov}(\vec{D})$. As in the case of the density of loads, the basic hydrodynamic simulation would, at most, yield

conditional estimates of the mean load vector and the covariance matrix for a given definition of $\vec{M}, \vec{R}, U, \mu$, and $\vec{W}$. The conditional means and covariance matrix will be denoted by $E[\vec{D}|\vec{M}, \vec{R}, U, \mu, \vec{W}]$ and $\text{Cov}(\vec{D}|\vec{M}, \vec{R}, U, \mu, \vec{W})$ respectively. The material in Appendix B demonstrates that the lifetime means and covariances may be derived from the conditional values by operations almost the same as those of Eq. 1:

$$E[\vec{D}] = \int \cdots \int E[\vec{D}|\vec{M}, \vec{R}, U, \mu, \vec{W}] f_R(\vec{R}) f_O(\vec{M}, U, \mu, \vec{W}) d\vec{M} d\vec{R} dU d\mu d\vec{W}$$

$$\text{Cov}(\vec{D}) = \int \cdots \int \text{Cov}(\vec{D}|\vec{M}, \vec{R}, U, \mu, \vec{W}) f_R(\vec{R}) f_O(\vec{M}, U, \mu, \vec{W}) d\vec{M} d\vec{R} dU d\mu d\vec{W} \quad (2)$$

PRACTICAL ISSUES

Though most of the synthesis methods in the literature have as their starting point relations similar to Eqs. 1 or 2, the details due to various developers vary, sometimes greatly. The reason is that simplifications and approximations must be resorted to in order to get answers, and the various developers have tended to do the simplifications differently. Only one of the most straightforward and practical approaches (essentially that of Sikora,⁸ *et al*) will be touched upon here.

The joint density, $f_O(\vec{M}, U, \mu, \vec{W})$, is the continuous version of what may be called the "operational" or "mission" profile. As a practical matter we can only estimate the joint frequency of occurrence of the speed, heading, and wave parameters for discrete ranges of the parameters. Similarly, because of the nature and high dimensionality of the load vector, $\vec{M}$, we can practically only consider a finite number of possible ship load conditions.

The straightforward approach is to break up the domains of each parameter into a finite number of sub-domains, for example;

$$\begin{aligned} \vec{M}_i - \delta\vec{M}_i/2 < \vec{M} \leq \vec{M}_i + \delta\vec{M}_i/2 & \quad \text{for } i = 1, \mathcal{I} \\ U_j - \delta U_j/2 < U \leq U_j + \delta U_j/2 & \quad \text{for } j = 1, \mathcal{J} \\ \mu_k - \delta\mu_k/2 < \mu \leq \mu_k + \delta\mu_k/2 & \quad \text{for } k = 1, \mathcal{K} \\ \vec{W}_\ell - \delta\vec{W}_\ell/2 < \vec{W} \leq \vec{W}_\ell + \delta\vec{W}_\ell/2 & \quad \text{for } \ell = 1, \mathcal{L} \end{aligned}$$

where, for example, $\vec{M}_i$ is a central or typical value of $\vec{M}$ within the sub-domain $\delta\vec{M}_i$. Then the probability that, *simultaneously*, $\vec{M}$ is in sub-domain $\delta\vec{M}_i$, U is in sub-domain δU_j , μ is in sub-domain $\delta\mu_k$, and $\vec{W}$ is in sub-domain $\delta\vec{W}_\ell$, may be written:

$$p_O(\vec{M}_i, U_j, \mu_k, \vec{W}_\ell) = \int_{\vec{M}_i - \delta\vec{M}_i/2}^{\vec{M}_i + \delta\vec{M}_i/2} \int_{U_j - \delta U_j/2}^{U_j + \delta U_j/2} \int_{\mu_k - \delta\mu_k/2}^{\mu_k + \delta\mu_k/2} \int_{\vec{W}_\ell - \delta\vec{W}_\ell/2}^{\vec{W}_\ell + \delta\vec{W}_\ell/2} f_O(\vec{M}, U, \mu, \vec{W}) d\vec{M} dU d\mu d\vec{W} \quad (3)$$

Apart from the approximations which are necessary when making generalizations from finite samples of operational and wave parameters, the quantity $p_O(\vec{M}_i, U_j, \mu_k, \vec{W}_\ell)$ represents the joint frequency of occurrence which may be estimated in practice.

Since $\vec{R}$ is a vector quantity of fairly high dimension, $f_R(\vec{R})$ itself is formally a joint density. At this point it is not clear how the variability in structural parameters will affect the whipping part of the loads, and consequently what detailed computations are implied. It may be that some analytical approximation to this density function, or possibly some simplified treatment of the effects of variability in the flexural rigidities, may in time become available. However, for the present illustrative purposes, it will be assumed that the domain of $\vec{R}$ can be broken into a finite number of sub-domains in a way similar to carried out for the operational variables above:

$$\vec{R}_m - \delta\vec{R}_m/2 < \vec{R} \leq \vec{R}_m + \delta\vec{R}_m/2 \quad \text{for } m = 1, \mathcal{M}$$

With this assumption the frequency of occurrence of each of the discrete values of $\vec{R}$ would be written:

$$p_R(\vec{R}_m) = \int_{\vec{R}_m - \delta\vec{R}_m/2}^{\vec{R}_m + \delta\vec{R}_m/2} f_R(\vec{R}) d\vec{R} \quad (4)$$

Now if the various sub-domains are chosen in such a way that the conditional density of Eq. 1 may be broken up into a series of functions, each of which are invariant with $\vec{M}, \vec{R}, U, \mu$ and $\vec{W}$ *within each sub-domain*, Eq. 1 may be put in the form of a summation.

In particular, define

$$f_{DC}(\vec{D}|\vec{M}, \vec{R}, U, \mu, \vec{W}) \approx f_{DC}(\vec{D}|\vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_\ell)$$

within each of the sub-domains $\delta\vec{M}_i, \delta\vec{R}_m, \delta U_j, \delta\mu_k$ and $\delta\vec{W}_\ell$ which are defined for each combination of i, j, k, ℓ and m .

Then, using Eqs. 3 and 4, Eq. 1 becomes approximately:

$$f_D(\vec{D}) \approx \sum_{i=1}^I \sum_{j=1}^J \sum_{k=1}^K \sum_{\ell=1}^L \sum_{m=1}^M f_{DC}(\vec{D}|\vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_\ell) p_R(\vec{R}_m) p_O(\vec{M}_i, U_j, \mu_k, \vec{W}_\ell) \quad (5)$$

Similarly, if a sequence of central values of the conditional expectation and the conditional covariance matrix are defined so that

$$\begin{aligned} E[\vec{D}|\vec{M}, \vec{R}, U, \mu, \vec{W}] &\approx E[\vec{D}|\vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_\ell], \text{ and,} \\ \text{Cov}(\vec{D}|\vec{M}, \vec{R}, U, \mu, \vec{W}) &\approx \text{Cov}(\vec{D}|\vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_\ell) \end{aligned}$$

are constant within each of the sub-domains which are defined for each combination of i, j, k, ℓ and m , the integrals of Eqs. 2 may be replaced by summations similar to those of Eq. 5.

Figure 2 on page 12 is meant to illustrate the main steps in a practical synthesis, in accordance with Eq. 5 and the prior development.

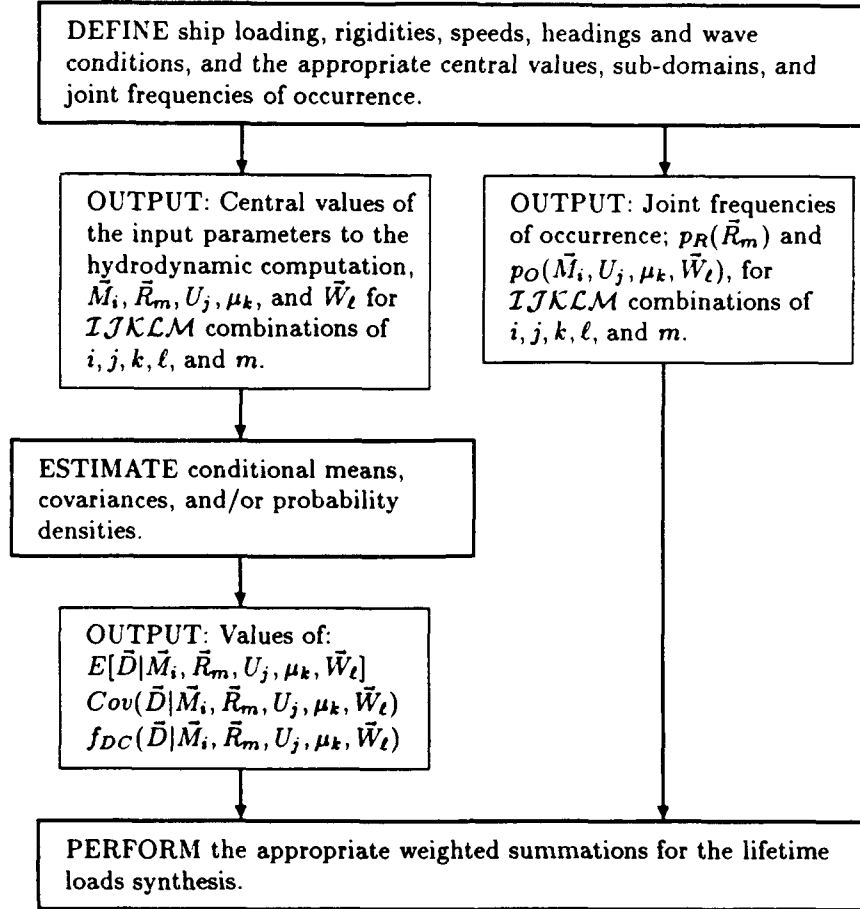


Fig. 2. Practical lifetime loads synthesis.

The results of the first step illustrated in Fig. 2 ("definition") may be logically separated into two parts; 1) the central values of the parameters which are input to the estimation step, and 2) the frequencies of occurrence which are finally used in the evaluation of the lifetime synthesis summations. The "estimation" step, Fig. 2, represents the hydrodynamic simulation, and is the most computationally demanding of the three steps illustrated. The output of the estimation step is the conditional means, covariances and/or densities, and forms the last of the required input to the weighted sums which comprise the final lifetime loads synthesis in the form of summations similar to Eq. 5.

The assumptions used in arriving at Eq. 5 illustrate some of the potential problems inherent in the definition step. The selection of the sub-domains of the operational profile, $p_O(\bar{M}_i, U_j, \mu_k, \bar{W}_\ell)$, can be quite influential on the final answer. First, it is desirable that the conditional densities, means, and covariances be approximately invariant over each sub-domain. The variables U and μ are bounded, and perhaps present minimal problems. The load conditions, $\bar{M}$, are somewhat more difficult to bound, and may

ultimately present problems.

On the other hand, the wave conditions, $\vec{W}$, are essentially unbounded at the fringes of the total domain where conditions are likely to be severe. How to choose the fringe domains for the waves is a major problem, as it is related to the classical, essentially unsolvable, problem of how high natural waves can get. Indeed, Guides Soares and Trovão⁹ find that "design" values found by means of the type of "long-term" synthesis developed in Lewis,⁴ may vary by up to 50% according to the source of the wave climate information. (Different authors have found different values for frequency of occurrence of various wave states for the same ocean area, and there is little objective information with which to determine who is most correct.) In addition to the choice of wave subdomains, the statistical validity of the wave parameterization within the domains may be a source of uncertainty in the final estimate, although a number of authors (including Guides Soares and Trovão⁹) suggest that the effect upon the final answer is relatively small.

The dimensions of the summations in Eq. 5 indicate a major practical problem in the estimation of the conditional means, covariances and densities. What Eq. 5 demands is that the conditional density of loads (or the means and covariances) be evaluated for $IJKLM$ combinations of input parameters. In existing synthesis methods for seakeeping operability, the product JKL tends to be about 2000. It is doubted that the corresponding number for structural reliability analyses will be significantly less (the number of evaluations used by Sikora⁸ is of this order). If it is assumed that the variability of $\vec{R}$ has been swept away somehow ($M = 1$), but that the number of loading conditions, I , can vary between 1 and 5, a first estimate of the range of $IJKLM$ is between 2000 and 10000. Each such "evaluation" involves the estimation of either means and covariances, or a joint probability density of response while the ship is proceeding under statistically stationary wave conditions.

SYNTHESIS STRATEGY

The most demanding of the technical problems in evaluating statistics for each statistically stationary condition is that of estimating the component of loads due to the transient response to slamming and impact. The preconditions for such response amount to a threshold problem; that is, the ship motions must attain some level of severity before wave impacts are likely to be severe enough to excite significant transient vibration of the hull. The slamming induced loads are thus nonlinear in an essential way.

It would obviously be beneficial, as a first approximation, if an approach to the statistics of loads resulting from hull vibration could be made in the frequency domain. Some attempts in this direction have been made, and integrated into procedures and codes which appear to be proprietary to the American Bureau of Shipping,⁷ (ABS). It appears that one of these procedures addresses the "springing" phenomenon, which is actually not related to wave impact. Springing is thought to be a resonant structural

response to excitation by relatively short waves, and by sum frequency interactions between longer wave components. The phenomenon is of importance to very limber ships, and thus probably not of importance in naval design.

The other procedure said to be a part of the ABS suite addresses the effects of bottom slamming, probably according to the theory presented by Ferro and Mansour¹⁰. Unfortunately, this theory is difficult to follow, and seems to have had little or no open literature verification. The assumptions of this theory are intended to approximate bottom slamming and it is doubted that it is appropriate to the influence of flare shock which may be of more importance to combatants.

Though further investigation into frequency domain methods for the estimation of the statistics of combined wave induced and slamming loads may be justified, the present consensus appears to be that the practical computation of such loads may only be carried out in the time domain, and this approach will be assumed in what is to follow.

Thus the ideal means for carrying out the required evaluations would be a non-linear time domain ship motions/loads/elastic response code. Given such a code, the mechanics of each evaluation would be to compute a long sample of loads for each statistically stationary condition required, and use each result as an "experiment" from which means, covariances, and possibly the parameters of probability density functions would be estimated. Such a procedure is essentially a Monte-Carlo simulation, and as such, is subject to sampling error, so that the length of time each evaluation represents becomes important.

On the basis of experience with model and full scale seakeeping experiments, if we want a reasonable (say plus or minus 15 percent) estimate of the population means and covariances of the hypothetical stationary process which would be modeled for each evaluation, we would need to simulate at least the equivalent of 0.5 hours real time. If in addition, we wish to estimate a probability density, we probably need to increase the sample by a factor of ten at least for a marginally accurate determination. Thus, using the range estimated in the last section, a first estimate of the total equivalent real time implied by the *IJKLM* evaluations demanded by the procedure illustrated in Fig. 2 ranges from 1000 to 50000 hours, real time.

Clearly, to achieve an affordable and practical approach to the problem with an *exclusive* use of nonlinear time domain simulations, the time domain code used must run at something of the order of 1000 times faster than real time, even assuming a state-of-art supercomputer.

The present understanding of the state-of-art in "fully" nonlinear time domain simulations suggests that this order of simulation speed is not now achievable, and in fact may not be achievable until such time as computers speed up by a factor of the order of 10000.

Accordingly, the notion of a "synthesis strategy" has to do with what can be done in the mean-time, without compromising the later incorporation of hydrodynamic or

computer technology under development, or to be developed.

INITIAL SEPARATION OF FUNCTION

The previous section and Fig. 2, define three major steps in a practical lifetime synthesis. The first consideration in the specification of a practical synthesis strategy is to keep the three steps separate; that is, with respect to computer codes, the definition step produces files of input for the hydrodynamic estimation and lifetime synthesis codes, and the hydrodynamic estimation code produces files of additional input for the lifetime synthesis.

With such a separation of function, as long as the hydrodynamic computer codes conform to agreed standards for input and output, changes in hydrodynamic technique do not require changes in other parts of the procedure. It is not the intention here to spell out the details of what exact form the input and output should be. These details can vary considerably without violating the postulated separation of function.

STRATEGY FOR CONDITIONAL LOAD COMPUTATIONS

The basic requirement in the development of the operational profile is to estimate the frequency of occurrence of *all* climatological and operational conditions—not just those which present critical (or necessarily even technically interesting) conditions. It is clear on the basis of existing data on world wide wave climatology that the usual large ship will spend a large part of its life in relatively mild wave conditions where wave impacts are unlikely, and in which purely wave induced loads are relatively low. It is also known that even when waves are sufficiently high, impacts are most likely to occur at high ship speed in head or bow seas.

Thus, for relatively large ships it seems likely that the computation of slamming loads will be unnecessary for a large portion of the population of wave conditions spelled out in the operational profile.

Turning to the purely wave induced loads, it has been known on the basis of model experiments made over the last 25 years that a linear model of wave induced loads is not normally insanely wrong. It is expected that, when and if fully nonlinear time domain computations can be made, the difference between nonlinear and linear estimates will be large only for extreme wave conditions. It may even be that the fully nonlinear computations for the majority of wave conditions defined by the operational profile will hardly be worth their extra cost. So long as this type of computation cannot be afforded for the entire operational profile, it makes some sense to reserve the use of fully nonlinear methods to the investigation of the loads at the extreme fringe of the domain of wave definition.

These considerations suggest a tentative strategy for the conditional computations in which codes of increasing complexity and cost are used in combination. For present purposes the hydrodynamic technology in hand and under development can be sorted into three levels:

1. *Linear Frequency Domain Methods.* At this level, only wave induced loads may be estimated. The estimation of process covariances (which is not now done) can be done in the frequency domain, and this level of code should be fast enough to be afforded for the entire operational profile. The existing hydrodynamic state of art at this level involves "strip" hydrodynamics. An emerging state of art involves three dimensional panel methods in which many of the deficiencies of strip hydrodynamics are overcome. The frequency domain panel methods are more costly to run than the strip methods, but probably still well within the bounds of affordability.
2. *Quasi-Linear Time Domain Methods.* Codes at this level have been available in principle for many years. Within the linear assumption, motions and rigid body loads may be estimated in the time domain, and it is further assumed that any wave impacts which may occur, as well as the resulting structural deflections, will not affect the motions. With these assumptions, the computed relative motion between ship and wave motions allows further engineering estimates of the magnitude of wave impact pressures and impulsive forces on the hull. These computed unsteady forces may in turn be used to excite a linear beam model of the hull girder, from which estimates of the additional loads due to wave impact may be made. The primary advantage of this type of engineering model is that (with a pre-computation of some basic results from linear ship motions theory) the resulting codes can probably be made to run at or, perhaps, faster than real time.
3. *Fully Nonlinear Time Domain Methods.* At this level, *in theory*, all the hydrodynamic nonlinearities, including impacts, are accounted for, and the hull girder may be modeled as in quasi-linear codes, or even (in principle) by nonlinear structural models. At the moment *no* such code appears to exist, though one or two are under serious development and well advanced. The code which is closest to being reduced to practice is actually not fully nonlinear, and runs a great deal slower than real time on a very fast machine.

Though by no means all of the required assumptions can be verified at present, a specific strategy for the conditional hydrodynamic computations is advanced as a starting point. Figure 3 on page 17 identifies the steps in this strategy as an expansion of the "Estimation" step of the lifetime loads synthesis, Fig. 2. The input to the process is composed of the central values of the loading, structural, speed, heading and wave parameters identified in the initial part of the synthesis. The final output (at the lower right corner of the diagram, Fig. 3) is composed of the final estimates of the means and covariances, and/or probability densities required in the lifetime loads synthesis.

The first step in the process is the computation of means, covariances and measures of slamming incidence for the entire operational profile by means of linear frequency

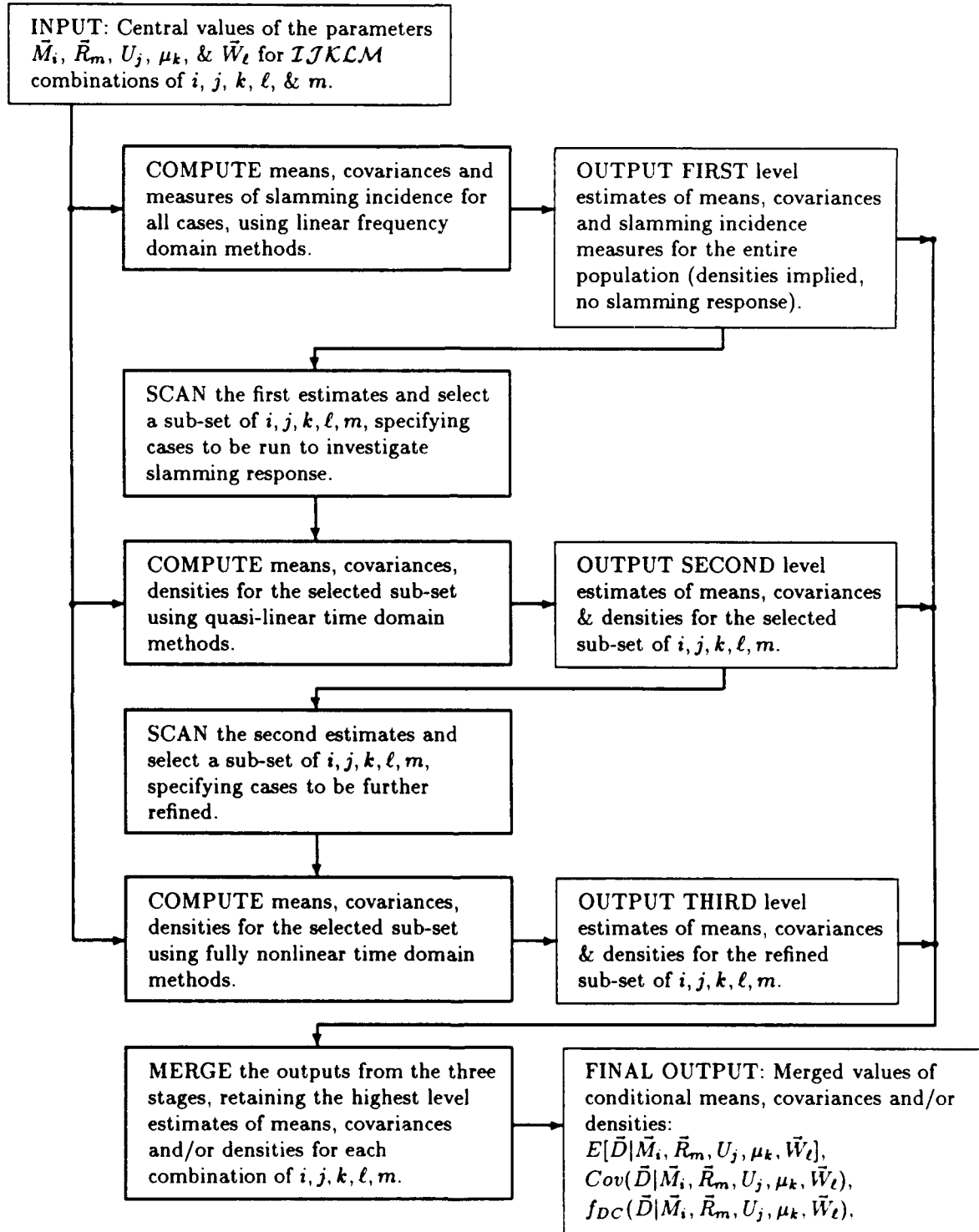


Fig. 3. Strategy for Conditional Computations.

domain methods. As indicated in the output box for this first level analysis, probability densities are implied for this step, since it is assumed that both the waves and the loads response are stationary Gaussian processes. The result is equivalent to that obtained for reliability studies in most of the literature. At this level, no estimates of whipping response are included, and it is expected that the answers for the extreme fringes of the wave definition domain may be in error for a variety of reasons (the omission of vibration, the effect of nonlinearities in the motion and wave excitation, and the effect of wave nonlinearities, to name a few possibilities).

The next step in the tentative procedure is perhaps the most practically important. It is to scan the first level results, and identify a sub-set of conditions in which whipping response is expected, or which represent cases of extreme wave induced loads. The hope here is to *literally* decimate the number of conditions to be studied further; that is, to reduce the number of combinations of input parameters from between 2000 and 10000 to a number between 200 and 1000.

The objective of the second computational step shown in Fig. 3 is to exercise the fastest possible quasi-linear time domain code on the sub-set of conditions defined in the scan of the first level results. If, for instance, a quasi-linear code which runs 10 times faster than real time can be achieved, and a half hour of real time per condition is adequate, the total computer time for this part of the procedure might be reduced to between 10 and 50 hours—not a trivial amount of effort, but one which might be affordable and sufficiently timely within a design exercise. The output of this part of the procedure would be a second estimate of means, covariances, etc. for the sub-set defined in the first scan.

The next two phases of the procedure are similar to the above in that the second level results are scanned with the intention to decimate the second level sub-set to a reduced sub-set of cases which might be practical to investigate with fully nonlinear methods. For example, a second literal decimation might result in between 20 and 100 cases to investigate. In this case, if the code runs 100 times slower than real time and a half hour is simulated for each case, the total final computer time requirement might still be between 1000 and 5000 hours with present day machines—a great deal of effort, yet quite possibly justifiable in later stages of serious design of an important ship, or in the “calibration” of the faster quasi-linear codes.

The final phase of the tentative strategy is a simple merging of the output from the various stages, retaining the results from the highest level estimates.

The strategy outlined in Fig. 3 is obviously not the only way to organize the tasks. It is hypothesized as a possible way to achieve the objectives in the near term, with presently available computers and codes which are either in-hand or thought to be developable in the very near term. It is possible to visualize a number of different feedback loops, and it is entirely possible that the levels might be expanded to accommodate additional code types which are intermediate in speed. It is also possible to visualize the elimination of stages as computers and or codes speed up. However, as noted at the

outset, this is just the sort of flexibility which is required to accommodate advances in technology.

It seems clear, to the writer at least, that the near term development of analytical means of estimating the load means and covariances required for reliability based design requires that codes of all levels of sophistication be investigated, developed and acquired.

CONDITIONAL SYNTHESIS

In an earlier section, it was noted that the operational profile which enables a lifetime synthesis must practically be defined in terms of the joint frequency of occurrence of the parameters $\vec{M}, \vec{R}, U, \mu$ and $\vec{W}$ in discrete "cells", or "joint sub-domains". It was assumed that these joint sub-domains can be chosen in such a way that it is reasonable to compute a single value of the mean vector, the covariance matrix, and/or to compute a single joint density function, which represent all actual conditions which may occur within each cell.

Then, to summarize notation indicated earlier, the "conditional synthesis" involves the estimation of ingredients for the lifetime synthesis, Fig. 2, in the form of the conditional mean, covariance and density function of the load vector, $\vec{D}$;

$$\begin{aligned} E[\vec{D}|\vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_\ell], \quad \text{and,} \\ \text{Cov}(\vec{D}|\vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_\ell), \quad \text{and,} \\ f_{DC}(\vec{D}|\vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_\ell), \end{aligned}$$

for *each* of the joint sub-domains which are defined by all combinations of the indices i, j, k, ℓ and m in the parameter sub-domains:

$$\begin{aligned} \vec{M}_i - \delta\vec{M}_i/2 < \vec{M} \leq \vec{M}_i + \delta\vec{M}_i/2 & \quad \text{for } i = 1, \mathcal{I} \\ U_j - \delta U_j/2 < U \leq U_j + \delta U_j/2 & \quad \text{for } j = 1, \mathcal{J} \\ \mu_k - \delta\mu_k/2 < \mu \leq \mu_k + \delta\mu_k/2 & \quad \text{for } k = 1, \mathcal{K} \\ \vec{W}_\ell - \delta\vec{W}_\ell/2 < \vec{W} \leq \vec{W}_\ell + \delta\vec{W}_\ell/2 & \quad \text{for } \ell = 1, \mathcal{L} \\ \vec{R}_m - \delta\vec{R}_m/2 < \vec{R} \leq \vec{R}_m + \delta\vec{R}_m/2 & \quad \text{for } m = 1, \mathcal{M} \end{aligned}$$

where, for example, $\vec{M}_i$ is a central or typical value of $\vec{M}$ within the sub-domain $\vec{M}_i \pm \delta\vec{M}_i/2$.

For example, if a joint sub-domain pertains to "a full load condition" at "design speed" in a "head Sea State 6", the estimated conditional means, covariances and joint densities should, at least ideally, take into account the full range of possible loadings, speeds, headings, and wave conditions which fit the definitions of "full load", "design speed", "head seas" and "Sea State 6" which were utilized in making up the operational profile.

Thus far, the several physical sources (and their time scales) which may contribute to the loads have been ignored—the quantities above are assumed to represent statistics of the *total* load vector. The purpose of this section is to discuss how the contributions from the various physical sources might be combined.

COMBINATION OF DEAD AND DYNAMIC LOADS

It was noted at the outset that it is customary (and natural) to divide loads into "dead" (or static) loads and "dynamic" loads, and this division is reflected in the diagram of reliability analysis elements, Fig. 1. If the vector of dead loads is denoted as $\vec{D}_{SW}$ and the vector of dynamic loads is denoted as $\vec{D}_{DY}$, the total load vector desired may be written,

$$\vec{D} = \vec{D}_{SW} + \vec{D}_{DY}.$$

One of the two major ingredients in the dead load computation is the mass distribution, $\vec{M}$. Because the mass distribution is also a major ingredient in the dynamic loads, a strict analysis must conclude that dead and dynamic load analyses are not wholly separable, and that the conventional separation implied by the diagram of Fig. 1 is not as clear as might be desired.

Dead loads result from a decision to load the ship in a particular way at the beginning of a mission, or even a series of missions. The dynamic loads result largely from whatever waves happen to be encountered during the course of the mission. In effect, the time scales of dead and dynamic loads are considerably different, and the root causes of whatever they turn out to be are relatively independent. Though not by any means rigorous, it thus appears reasonable to consider the dead loads for a fixed load condition, $\vec{M}_i$, to be statistically independent of the dynamic loads corresponding to that same condition. Under this independence assumption, the conditional joint density of the dead and dynamic load vectors is of the form:

$$f'(\vec{D}_{SW}, \vec{D}_{DY} | \vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_t) \approx f_{SW}(\vec{D}_{SW} | \vec{M}_i) \cdot f_{DY}(\vec{D}_{DY} | \vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_t)$$

where, since the dead loads depend only upon $\vec{M}_i$, $f_{SW}(\vec{D}_{SW} | \vec{M}_i)$ is the conditional density of the dead load vector, $\vec{D}_{SW}$ for mass distribution, $\vec{M}_i$, and the conditional density of the dynamic load vector, $\vec{D}_{DY}$, is $f_{DY}(\vec{D}_{DY} | \vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_t)$.

Under the independence assumption the conditional density of the total load vector, $\vec{D}$, becomes a convolution of the conditional densities of the dead and dynamic load vectors. The result in vector form may be written:

$$f_{DC}(\vec{D} | \vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_t) = \int f_{SW}(\vec{D}_{SW} | \vec{M}_i) f_{DY}(\vec{D} - \vec{D}_{SW} | \vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_t) d\vec{D}_{SW} \quad (6)$$

where the integration is over the domain of $f_{SW}(\vec{D}_{SW} | \vec{M}_i)$, and in practice, the vectors would probably be expanded into their components so that Eq. 6 involves a multiple integral in the ordinary sense.

Turning to the items needed for Level II reliability, the mean vector and covariance matrix of the dead loads may be defined as $E[\vec{D}_{SW} | \vec{M}_i]$ and $\text{Cov}(\vec{D}_{SW} | \vec{M}_i)$. Similarly, $E[\vec{D}_{DY} | \vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_t]$ will be defined as the mean vector of the dynamic loads, and

the covariance matrix defined as $\text{Cov}(\vec{D}_{DY}|\vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_\ell)$. Then, assuming that the mean vectors and covariance matrices of the dead and dynamic load vectors can be estimated separately under the independence assumption, the mean and covariance matrix of the total load vector become:

$$\begin{aligned} E[\vec{D}|\vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_\ell] &= E[\vec{D}_{SW}|\vec{M}_i] + E[\vec{D}_{DY}|\vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_\ell] \\ \text{Cov}(\vec{D}|\vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_\ell) &= \text{Cov}(\vec{D}_{SW}|\vec{M}_i) + \text{Cov}(\vec{D}_{DY}|\vec{M}_i, \vec{R}_m, U_j, \mu_k, \vec{W}_\ell) \end{aligned} \quad (7)$$

where the conventional rules of vector and matrix addition are observed.

Against the following discussion it might be noted that Eqs. 6 and 7 are consistent with intuition if the variability of the dead loads tends toward zero; that is, if the dead loads are taken to be almost constant. In this case the density of $\vec{D}_{SW}$ in Eq. 6 tends toward a multi-dimensional delta function, and the effect of the integration is just to re-define the random variable in the density of $\vec{D}_{DY}$ by an axis transformation. Similarly, in Eqs. 7 the mean vector of the total loads would be the mean of the dynamic loads shifted by a constant, and the covariance matrix of the total load would be equal to the covariance matrix of the dynamic load since negligible variability in dead load would be assumed.

Equations 6 and 7 reflect the treatment of dead and dynamic loads seen in most of the literature. As implied by the various caveats noted, it is not a rigorous treatment, but even the complications of Eq. 6 are preferable to alternatives which do not involve the independence assumption.

In an engineering sense, the adequacy of this treatment depends upon the magnitude of variation of dead loads within the mass distribution sub-domain, and the sensitivity of the wave loads to minor changes in mass distribution.

To make the approach yield a reasonable approximation, the mass distribution sub-domain should be narrow enough that the estimates of dynamic wave loads for a single central value of mass distribution apply reasonably well to the entire sub-domain. There is no apparent way to generalize how the mass distribution sub-domain choice should be made for all ships.

The dead loads are an important part of the total vertical load components for the majority of ships. In commercial forms the variability from voyage to voyage relative to the "design" dead or "still water" load can be substantial. Guides Soares and Moan¹¹, in analyzing this variability according to a Normal assumption, find standard deviations of still water loads amounting to between 20 and 30 percent of "design" values. Since the design values tend to be extreme relative to the majority of actual loads, the same analysis indicates standard deviations between 30 and 60 percent of the mean value. Of course, the exact values are dependent upon ship type and service, but the magnitude of the variability appears to be too large to ignore in reliability analyses for commercial ships.

It is not known whether estimates of dynamic wave loads for a single nominal mass distribution would really be an adequate approximation for most commercial ships.

It is known that estimates of dynamic wave loads for tankers in loaded condition are not likely to be good approximations for the ballast condition. (Some have advanced the opinion that estimates of dynamic wave loads vary significantly only with ship displacement).

Some Naval auxiliaries and sea-lift ships are likely to have the same magnitude of variability in dead loads as in the commercial case.

Though it is thought that combatants are likely to have a much smaller magnitude of dead load variability, there may be absolutely no available data on the matter.

It seems likely that the selection of ship loading cases will not necessarily be a trivial exercise, and at the very least must be done differently than in present practice if reasonably consistent reliability analyses are desired. In particular, instead of a single extreme load condition, the mean values and standard deviations of vertical moment and shear would be required, as well as some sort of estimate of their correlation coefficient. These items would define an approximating multi-dimensional normal density if an application of Eq. 6 was contemplated, and an estimate of the density of dynamic loads was available. Much of this part of the problem would involve "what-if" naval architecture which does not now seem to be practiced.

COMBINATION OF SPEED AND WAVE INDUCED LOADS

Earlier, it was noted that loads due to speed induced changes in effective buoyancy distribution can be large enough to consider for high speed ships, and the inclusion of this source of loads was implied by the diagram, Fig. 1. This source of loads is not accounted for in the literature so far seen, so that the problem of combining such loads with those due to wave action does not appear to have a prior model.

Of course, if the mode of computation of the dynamic loads involves a fully nonlinear (third level) time domain method, it would be expected that the speed induced loads will be automatically included, and there will be no "combination problem". (A consistent nonlinear motions and load code must solve the steady state wave generation problem as well as the unsteady, wave excited problem.) However, such code is not now available, and is likely not to be affordable for anything but special problems for some time to come.

Thus, as a practical matter, it will be necessary to add the speed induced component to the results of the initial stages of the synthesis in which linear and quasi-linear wave induced loads codes are employed. In this context then, the speed induced load vector will be defined as, $\vec{D}_U$, the wave induced load vector will be defined as, $\vec{D}_W$, with the understanding that the "dynamic load vector, $\vec{D}_{DY}$ is:

$$\vec{D}_{DY} = \vec{D}_U + \vec{D}_W.$$

If it is legitimate to make an independence assumption similar to that made in the case of the combination of dead and dynamic loads: that is if $\vec{D}_U$ and $\vec{D}_W$ are

independent random variables, then the conditional density of the dynamic loads would become:

$$f_{DY}(\bar{D}_{DY}|\bar{M}_i, \bar{R}_m, U_j, \mu_k, \bar{W}_t) = \int f_U(\bar{D}_U|\bar{M}_i, U_j) f_W(\bar{D}_{DY} - \bar{D}_U|\bar{M}_i, \bar{R}_m, U_j, \mu_k, \bar{W}_t) d\bar{D}_U \quad (8)$$

where the integration is over the domain of $\bar{D}_U$, $f_U(\bar{D}_U|\bar{M}_i, U_j)$ denotes the density of the speed induced load vector, and $f_W(\bar{D}_W|\bar{M}_i, \bar{R}_m, U_j, \mu_k, \bar{W}_t)$ denotes the density of the wave induced load vector.

Similarly, under the independence assumption, the mean vector and covariance matrix of the dynamic load vector would become:

$$\begin{aligned} E[\bar{D}_{DY}|\bar{M}_i, \bar{R}_m, U_j, \mu_k, \bar{W}_t] &= E[\bar{D}_U|\bar{M}_i, U_j] + E[\bar{D}_W|\bar{M}_i, \bar{R}_m, U_j, \mu_k, \bar{W}_t] \\ \text{Cov}(\bar{D}_{DY}|\bar{M}_i, \bar{R}_m, U_j, \mu_k, \bar{W}_t) &= \text{Cov}(\bar{D}_U|\bar{M}_i, U_j) + \text{Cov}(\bar{D}_W|\bar{M}_i, \bar{R}_m, U_j, \mu_k, \bar{W}_t) \end{aligned} \quad (9)$$

where $E[\bar{D}_U|\bar{M}_i, U_j]$ denotes the conditional mean and $\text{Cov}(\bar{D}_U|\bar{M}_i, U_j)$ denotes the conditional covariance matrix of the speed induced loads; and where the corresponding conditional values for the wave induced loads are $E[\bar{D}_W|\bar{M}_i, \bar{R}_m, U_j, \mu_k, \bar{W}_t]$ and $\text{Cov}(\bar{D}_W|\bar{M}_i, \bar{R}_m, U_j, \mu_k, \bar{W}_t)$.

The reference level for the vector of speed induced loads, $\bar{D}_U$, is the dead load vector; that is, the speed induced loads are defined as the difference between the loads which would be experienced when the ship is at speed in calm water and those experienced when the ship is moored in calm water. It is probably safe to assume that the speed induced load vector has only two non-zero elements—the vertical moments and shears. In addition to the geometry of the hull which is understood throughout, this component depends only upon speed and some integral properties of the mass distribution, namely the total mass or displacement and the longitudinal center of gravity. For a given speed, displacement and LCG, the speed induced loads are *not* random variables.

Thus, if speed induced loads are estimated for the central values of mass distribution, $\bar{M}_i$ and ship speed, U_j , which are defined for a joint sub-domain under the practical synthesis procedure described earlier, the result will be a *constant* vector.

In the event that the joint sub-domains of mass distribution and speed are compact enough, this constant result should be a reasonable approximation. In this case, the independence assumption is fulfilled, the density of $\bar{D}_U$ in Eq. 8 tends toward a multi-dimensional delta function, and the effect of the integration is just to re-define the random variable in the density of $\bar{D}_W$ by an axis transformation. Similarly, in Eqs. 9 the mean vector of the dynamic loads would be the mean of the wave loads shifted by a constant. Because negligible variability in speed induced load is implied in this case, the covariance matrix of the dynamic load would be equal to the covariance matrix of the wave induced loads.

On the other hand, in the event that the sub-domains of speed and mass distribution are not very compact, some different approximation problems arise. Speed induced loads are expected to vary relatively slowly with ship speed. Thus, the first possibility is that the variation of the speed induced loads is very small within each speed sub-domain, and in this case the deterministic constant approximation above would still be justified.

If not, an estimation procedure for the speed induced loads formally results in a *deterministic* load vector, $\vec{D}'_U$ of the form:

$$\vec{D}'_U = (g_1(U, \vec{M}), g_2(U, \vec{M}), 0, 0, 0) \text{ for: } U_j - \delta U/2 < U < U_j + \delta U/2 \\ \vec{M}_i - \delta \vec{M}_i/2 < \vec{M} < \vec{M}_i + \delta \vec{M}_i/2$$

where in general the functions, $g_1(\cdot)$ and $g_2(\cdot)$, assumed to be the vertical load components, would be the numerically defined results from what amounts to a wave pattern resistance computation. As a deterministic vector, $\vec{D}'_U$ has no associated density function, and thus something else would have to be assumed to produce the speed induced load vector as a random variable.

The operational profile specifies only the frequency of occurrence of ship speed within a given range of speed, not what happens inside the range. It may be reasonable to assume for a given ship speed sub-domain, U_j , that any speed within the range, $U_j \pm \delta U/2$ is equally likely, or equivalently, that the density of U is uniform within the sub-domain. Because any function of a random variable is also a random variable, this assumption allows a numerical approximation of the joint density of $\vec{D}_U$ for fixed mass distribution by what amounts to a change of the uniform distribution variable. Unless the functions, $g_1(\cdot)$ and $g_2(\cdot)$, are monotonic within the speed range, the change of variable is rather difficult to carry out directly. However, because the initial uniform density is strictly limited by the speed sub-domain limits, a reasonable numerical rendition of $f_U(\vec{D}_U | \vec{M}_i, U_j)$ for a given mass distribution, and the corresponding mean and covariance, may be quite expeditiously be carried out by Monte Carlo simulation.

It may well be that a similar assumption on the mass distribution (amounting to variations in displacement and LCG within the mass distribution sub-domain) would be reasonable, and it would be expected that the influence of random variations in mass distribution might, if necessary, be handled in the same way.

If there were negligible waves within the joint sub-domain, the treatment just mentioned would yield the total dynamic load statistics. If not, the application of Eqs. 8 and 9 provides a non-rigorous way to proceed. The basic problem with applying Eqs. 8 and 9 is identical to the problem of combining dead and dynamic loads which was discussed in the last section. The assumption used in producing Eqs. 8 and 9 was that the speed induced and wave loads are statistically independent within the joint sub-domain of interest. If the range of speed within the joint sub-domain is large, this assumption is unlikely to hold, and the approach will be inconsistent. Similarly to the case of dead and dynamic loads, the adequacy of the approximation will depend upon the sensitivity of both the speed induced and wave loads to speed and mass distribution. It seems

likely that if the speed range in the sub-domain is vary large, it will be necessary to further divide the sub-domain for *both* the speed induced and dynamic loads—at least to the point where the above becomes reasonable in an engineering sense—at which point this particular worry may become moot.

WAVE INDUCED AND SLAMMING LOADS

Of the general load sources identified, only the wave induced and slamming induced loads remain. It was noted earlier that the slamming induced loads are nonlinear in an essential way, and that the present consensus appears to be that the combination of such loads with the wave induced loads may only be carried out in the time domain. Given the effort which appears to be required to investigate what might happen during an exposure time of the order of a ship life, a synthesis strategy was hypothesized in which state-of-art linear frequency domain methods would be used to define wave induced loads for most of the ship life, and to define those conditions which would bear investigation by means of time domain simulations of various levels of complexity. Accordingly the “combination” of wave induced and slamming loads occurs only in the context of the methods of analysis of time domain results.

The context of the following discussion will be the estimation of the conditional density, $f_W(\bar{D}_W|\bar{M}_i, \bar{R}_m, U_j, \mu_k, \bar{W}_t)$ and/or the mean, $E[\bar{D}_W|\bar{M}_i, \bar{R}_m, U_j, \mu_k, \bar{W}_t]$, and covariance, $\text{Cov}(\bar{D}_W|\bar{M}_i, \bar{R}_m, U_j, \mu_k, \bar{W}_t)$, all of which were previously defined. For fixed central values of the parameters and generalized condition vectors, the wave induced load vector, $\bar{D}_W$, is a random variable because of the definition of the wave condition vector, $\bar{W}_t$, which represents the defining parameters of a stationary random wave condition. For purposes of the present discussion, the other parameters are constant.

Frequency Domain Estimates

The present state-of-art in analytic computations of loads for reliability type analyses appears to be almost wholly based in linearized frequency domain techniques. Within this framework, the wave field is assumed to be Gaussian and zero mean, and the various components of the vector of loads are assumed to be the result of linear operations on a wave field defined by a variance spectrum. Because the various components of the load vector are related, the assumptions of this framework result in a vector *process*, $\bar{D}_W^*$, say, the components of which are jointly Gaussian and zero mean. The joint density of this vector process, if sampled at random instants, is a multi-variate Gaussian density whose parameters are variances and covariances.

The linear operations are represented in the frequency domain by “transfer functions”, and the operation to compute the variances of each component of the load vector amount to an integral of the wave spectrum weighted by the squared modulus of the transfer function. None of the known linear frequency domain ship motions and loads programs routinely compute the covariances which are required to complete the covariance matrix defining the joint density of the vector process, $\bar{D}_W^*$. However, the reason

that this is so is that (perhaps up until now) there has been no compelling reason to do the computation. The computation of the process covariances is similar to the variance computation, and, for a 5 by five covariance matrix, would increase the work of the usual already very fast variance computation only by a factor of about 3; that is, would probably not present any insurmountable additional computing demands.

However, the load vector, $\vec{D}_w$, which appears to be implied in the reliability literature is *not* the vector Gaussian process, $\vec{D}_w^*$. It has been pointed out that there are some different statistical definitions of the load vector, but all so far involve the *maxima* of the process.

If a vector Gaussian process is assumed, the marginal densities of the maxima of each component of the vector are known in analytical form. It is conventional to assume that each component of the vector process is sufficiently narrow banded that the marginal density of the maxima may be described by a Rayleigh density function, whose only parameter is the *process* variance. Pierce¹² cites Middleton¹³ in indicating that the Rayleigh density is exact for the "analytic envelope" of non-narrow band processes. Because the maxima of a non-narrow band process do not necessarily coincide with the extrema of the associated analytic envelope, the Rayleigh model for maxima tends to be somewhat conservative for non-narrow band processes—but the degree of conservatism is likely be only in single digits of percent. Of course, it is doubtful that the basic assumption will hold, even approximately, for the case that significant slamming loads are superimposed upon the wave induced loads.

Leaving aside the question of the effect of slamming loads, the assumptions noted above allow a reasonable extrapolation of process variances to the marginal densities of the maxima of each load component. These densities are defined relative to the experiment of picking a maximum at random from the process.

It appears that the definition often adopted for the wave induced loads in the literature is that the marginal density of each load component reflect the largest maximum to be experienced in a set amount of exposure time. The density of the largest maximum within a given exposure time is usually derived under the (erroneous but conservative) assumption that succeeding maxima are statistically independent. With this assumption, the implied number of maxima sampled is typically set equal to the expected number of excursions in the given time, and the appropriate density is obtained from extreme value theory (with a substantial assist from the fact that the Rayleigh density has a very simple analytical form).

It is to be noted that no mention of covariances or joint densities has been made after the discussion of *process* covariances and joint density. Prior work in the field has provided models for the statistical quantities which are required—but only for *individual* components of the load vector. There appear to have been *no* treatments of the *joint* densities or *covariances* which the basic reliability framework suggests are needed for cases in which components of the load vector must be combined.

There are thus some questions about the statistical combination of the various ele-

ments in the load vector which, it seems, will have to be addressed. The questions take different forms depending upon the meaning of the elements of the final load vector.

As applied in the basic reliability model, it appears that the "joint density" function has the conventional meaning. For example, assume that the wave induced load vector, $\vec{D}_W$, is sampled, the sampled components are denoted by $(d'_1, \dots, d'_5)$, and the conditional joint density of the wave induced load vector is denoted for convenience as $f_W(d_1, \dots, d_5)$. If the *event* that the i^{th} sampled component falls within a small interval containing d_i is defined as:

$$(d_i - \delta d_i/2 < d'_i < d_i + \delta d_i/2) \quad \text{for } i = 1, 5$$

then, the joint density is defined in terms of the probability of the intersection of all such events, or:

$$\begin{aligned} P[(d_1 - \delta d_1/2 < d'_1 < d_1 + \delta d_1/2) \cap \dots \cap (d_5 - \delta d_5/2 < d'_5 < d_5 + \delta d_5/2)] \\ \approx f_W(d_1, \dots, d_5) \delta d_1 \dots \delta d_5 \end{aligned}$$

In other words, the joint density is proportional to probability that the sampled components *simultaneously* fall within small intervals containing $(d_1, \dots, d_5)$.

The common sense definition of "simultaneous" in the context of the combination of load components is "simultaneous in time". For example, a simplified approach to deck edge stresses might involve an effective tension or compression which is a linear combination of vertical and lateral bending moments with distances from neutral axes. The interest in this case is at instants of time where the effective tension or compression load reaches its maximum, not necessarily when either of the contributing components do.

The definition of the joint density raises no questions if the load vector is supposed equal to the vector process described earlier. The joint density assumed in that case has exactly the above definition for samplings of each component at identical instants of time.

Conceptual problems with the meaning of "simultaneous" arise when it is assumed that the components of the load vector involve maxima. To take the simplest example, assume that the components of the load vectors are the *maxima* of each individual component. Unless the various components are scalar functions of one another, the chances are nearly zero that a maximum of components d_2 through d_5 will exist at the same time as a maximum of d_1 ; that is, the time of a randomly picked maximum from the process underlying one of the components will in general not coincide with times of the maxima of any of the other components.

This conceptual problem is even more extreme if the components of the load vector are assumed to be defined as the largest maximum during a given exposure. The chances that all such components, or even pairs of components will reach such extremes simultaneously are almost nil.

It might be that a model for the "joint density" which would make structural sense would be a series of functions expressing the joint probability that one component has reached a maximum within a certain range while, simultaneously, the other components assume ranges of values defined by the underlying vector process. An analytical form for such densities is not known to the writer; it may exist, or may have to be derived if needed.

Yet another possibility is to define the components of the load vector as the *envelope* of the underlying process. The joint density of the envelopes of pairs of Gaussian processes would be the analytical model for this possibility. (Unfortunately, such an analytical model may well not exist.) However, the consequences of such an assumption are fairly clear intuitively. The apparent instantaneous periods of oscillation of the components of the vector load process would be expected to be nearly the same. Thus at an instant when one component reaches some sort of extreme as defined by its envelope, the envelopes of other components will be defining the approximate level of the nearest in time maxima of the other components. Since a Gaussian process is symmetric, it would not be possible to specify whether the implied level of the other components was in the same or the opposite sense to the first component. It would be expected that the envelopes of pairs of Gaussian processes would be relatively highly (positively or negatively) correlated and the probable net result might well be a more conservative than justified approach to the combined load of interest, yet this approach may be worth some further investigation.

The problem may be summarized by saying that if we stick to the definition of the load vector which has been used previously (the extreme load expected in a given exposure time) it is not clear that we really know how to estimate a meaningful joint density (or alternately, meaningful estimates of the off-diagonal terms in the covariance matrix).

Time Domain Estimates

In the time domain, the procedures required for the determination of statistical parameters of the load vector should be very much the same as those which would be used in the reduction of model or full scale data. To an extent, the "data reduction" procedures appropriate in the reliability context appear undeveloped.

If the load vector is simulated in the time domain, there would result a time series for each load element. The estimation of the analogous *process* parameters is a straight forward estimate of variances as the mean squared deviation from the mean of each time series. Though not routinely done, the estimation of the sample process covariances is no more demanding. If the duration of the sample is sufficiently long, reasonable estimates of the process covariance matrix are feasible. If the simulation is nonlinear in any way, the primary conceptual problem is whether or not the sample means and covariances are an adequate description of the corresponding process parameters. Much more than a minimal sample has to be generated in order to make an objective determination

of whether or not this is true, or to define the deviations from the usually assumed Gaussian process.

When it comes to the estimation from time domain samples of the highest excursion in a given exposure time, some means of *extrapolation* is required unless we are prepared to run the simulation for multiples of the exposure time. All the problems with the definition of the covariance functions and joint densities which were noted in conjunction with frequency domain estimates are present in the time domain case, but in varying degrees. Statistics of maxima, and some sort of correlation measure, may be derived from a simulation for almost any definition of load vector. The potential problems arise when it is required to generalize the results to the hypothetical infinite process. For this purpose, for individual components it is common to fit recognized density functions to the results of the data reduction (the Weibull distribution for example). It is no more clear how a *joint* density may be constructed within this extrapolation philosophy than it was in the case of frequency domain estimates.

SUMMARY

The objective of the present task is to support the key overall goal of establishing reliability based design procedures; in particular, to develop or acquire analytical methods for the computation of hydrodynamic loads. The primary reason for the present review was to try to insure that the products of this task would be consistent with the requirements of the structural designer and the reliability analyst, and be practically useful in the relatively near future.

For present purposes the type of loads considered has been limited to primary bending and shears in the hull girder, though, in principle, other physically induced loads are not excluded. The commercial literature tends to lump thermally induced stresses with loads, but, it is thought, without a convincing rationale for their inclusion in reliability based design. The present development has ignored such effects. If thermal effects are important at all in Naval design, the issue needs much more serious consideration.

This review has been conducted in a "top-down" order, under the basic assumption that the goal is to conduct "reliability analyses" in accordance with the basic reliability theory. The present interpretation of this theory is that, up to a point, loads and resistances may be considered separately under the (reasonable if not totally valid) assumption that loads and resistance are statistically independent.

The basic reliability theory also implies that the form of the answer to the loads part of the problem should be joint probability densities and/or means and covariances of all the loads considered, and that the sample space of these statistics for a given ship be all the possible values which may occur during the life of an infinite number of sister ships.

The need for joint densities and covariances arises when two or more load components must be combined in some way to produce an effective load on some part of the structure (the estimation of deck edge stresses is a simple example). The combination

of load components in a reliability context does not seem to have been treated in the marine reliability literature, so that, to a great extent, the estimation of covariances and joint densities is presently an unrealized goal.

In general, the primary loads are functions of a large number of parameters (some of which are vectors) which may be generalized as:

- Mass distribution along the ship
- Flexural rigidity distribution along the ship,
- Ship speed and heading to waves, and
- Wave conditions (the parameters which define the waves as a particular stationary random process).

In the context of ship lives, all of these parameters may be considered to be random. The practical approach to the synthesis of lifetime load statistics involves the prior estimation of frequency of occurrences of all possible combinations of these generalized parameters in discrete ranges or cells; that is, the determination of an operational profile.

Some comments and unresolved issues are worth noting:

1. The mass distribution (or functions of it) is important to the estimation of loads from all sources (dead loads, speed induced, and wave induced). How, exactly, to obtain the data with which to represent the possible mass distributions in a statistical way, even approximately, is not a trivial problem, and involves naval architecture which does not now seem to be practiced.
2. The inclusion of flexural rigidities in the list of load inputs is in direct conflict with the initial assumption that loads and resistances are statistically independent. Though it is possible that sensitivity studies will reveal that the influence of variability in fabrication and material properties upon the important integrated properties of the flexural rigidity distribution will be minor, this is an issue which is unresolved.
3. It has been shown that the variation, as obtained by various authors, in the frequency of occurrence of the parameters which define various wave conditions can have a large effect upon expected lifetime loads. Since the extreme "wave condition" cells in the operational profile are unbounded, it is likely that the choice of these "fringe cells" will have visible influence upon the results.

The practical approach to the synthesis of lifetime load statistics noted herein is a matter of weighting conditional statistics which are computed for each "cell" in the operational profile. There seems to be no prior guidance in the matter of uncertainty

in the basic frequency estimates in the profile. This, also, is an issue which it will be advisable to investigate further.

Some simple arithmetic involving the probable number of cells in the operational profile and the probable minimum amount of real time which must be simulated suggests that, if the latest generation of nonlinear time-domain hydrodynamic simulations were used *exclusively*, the computer time requirements for the generation of lifetime statistics are orders of magnitude more than could be considered practical in a design exercise. The basic conclusion is that, at least for the foreseeable future, we must count on using hydrodynamic technology of *all* levels of sophistication. A tentative strategy has been outlined which might accomplish the desired results this way.

The conditional synthesis of the required statistics within each cell of the operational profile has been reviewed. Some issues are worth summarizing.

1. Prior guidance in the literature suggests that dead and dynamic loads may be combined as though they are statistically independent. Because both depend, at least in part, upon the mass distribution, this approach is expedient, but not consistent. It may be that the extent of the cells chosen for mass distribution may have to be determined in practice partially on the basis of whether or not the dynamic computations for a single central value of the mass distribution cell will apply reasonably to the entire cell.
2. Some speculation upon how speed induced loads may be statistically incorporated are given on the basis of a tenuous independence assumption. The practical validity would depend upon sensitivity investigations similar to those for the combination of dead and dynamic loads.
3. The prior literature suggests that the appropriate statistic for each load component is the value of an extreme excursion in a given exposure time. The value of exposure time seems not totally agreed upon, and needs a rationalization. If this definition of the required statistic is adopted, reasonable means of estimation of the required densities, means and variances are available, but it appears that we do not know how to estimate or make an extrapolation for the *joint* densities or covariances. Some options are suggested, but the issue is open.

APPENDIX A

RELIABILITY ANALYSIS FROM A LOADS PERSPECTIVE

Reliability analyses are categorized into three "Levels" according to the degree of complication. The present review from the loads perspective will be in the order of highest to lowest levels, abstracting material from Melton¹ and Mansour *et al.*²

Level III Reliability Analysis

"Level III", or "direct integration" reliability theory is the classical "exact" approach. In all approaches to reliability based design *all* the parameters which define the "resistance" (capability, or strength) and "loads" (demand) are (at least initially) considered to be random variables. For present purposes, the loads of interest will be denoted, $d_1, d_2, \dots, d_m$, and the various parameters which define the resistance for a given mode of failure by $r_1, r_2, \dots, r_n$. Resistance and loads may be related by means of a "limit state" function,

$$G(r_1, r_2, \dots, r_n, d_1, d_2, \dots, d_m) = \mathcal{G}. \quad (10)$$

The limit state function, $G(\cdot)$ is constructed so as to relate the parameters, for the limit (failure) mode of interest, in such a way that "failure" occurs anywhere in the region

$$\mathcal{G} \leq 0.$$

The limit state function is normally an algebraic expression. To illustrate by a one dimensional example, if R denotes the ultimate strength of an element in pure tension, and L denotes the tensile load;

$$G(R, L) = \mathcal{G} = R - L,$$

and failure occurs when $R \leq L$.

If the joint probability density of the resistance and load parameters is known or can be found, the "probability of failure" (or more properly, the probability that the limit state will be reached or exceeded) becomes:

$$P_f = \int \dots \int_{\mathcal{G} \leq 0} f(r_1, \dots, r_n, d_1, \dots, d_m) dr_1 \dots dr_n d(d_1) \dots d(d_m),$$

where $f(r_1, \dots, r_n, d_1, \dots, d_m)$ is the joint probability density, and the notation on the integrals means that the integration is performed over the *region* where $\mathcal{G} \leq 0$.

It is conventional (and reasonable) to assume that the set of resistance variables and the set of load variables are statistically independent. Under this assumption, the joint probability density of all the variables may be written as the product of the joint probability density of the resistance variables, $f_R(r_1, \dots, r_n)$, and the joint probability density of the load variables, $f_D(d_1, \dots, d_m)$. Thus, under the independence assumption the probability of failure becomes:

$$P_f = \int \cdots \int_{\mathcal{G} \leq 0} f_R(r_1, \cdots r_n) \cdot f_D(d_1, \cdots d_m) dr_1 \cdots dr_n d(d_1) \cdots d(d_m). \quad (11)$$

Thus, under Level III methods, the probability of failure is the final object of the reliability analysis. The work of Level III reliability analysis may be partitioned into the following main elements:

- I. **MODELING:** The (nontrivial) definition of limit state functions, Eq. 10, by the structural analyst for all the possible failure modes.
- II. **RESISTANCE SYNTHESIS.** The synthesis of the joint probability density of the relevant resistance parameters, $f_R(r_1, \cdots r_n)$.
- III. **LOADS SYNTHESIS.** The synthesis of the joint probability density of load parameters, $f_D(d_1, \cdots d_m)$.
- IV. **RELIABILITY ANALYSIS.** The final coordination of the syntheses of Elements II and III, the invocation of the reliability analysis machinery to evaluate Eq. 11 for failure probabilities, and the interpretation of results from the various assumed limit states.

The load analyst's job is clearly confined to element III, the synthesis of joint densities of load variables.

The major problem with the Level III approach is that it is philosophically impossible to "calculate" probability densities, or even to verify approximations with anything near 100% confidence. The low probability "tails" of the densities are important, and are the regions where the least confidence will exist. The practical result is that any Level III analysis requires *extrapolations* to events of very low probability, and in practice this means fitting available data to analytical probability density functions.

Level II Reliability Analysis

Level II analysis was developed in response to the major problem with Level III analyses just mentioned. The objective was to develop a method which would be relatively free of the details of the probability density functions.

In Level II analysis limit state functions must first be defined in exactly the same form as in Level III analyses. The value of the limit state function, $\mathcal{G}$, Eq. 10, may be thought of as a safety margin. Since $\mathcal{G}$ is a random variable it is legitimate to consider the probability distribution of $\mathcal{G}$, as well as its statistical mean value, $\mu_{\mathcal{G}}$, and variance, $\sigma_{\mathcal{G}}^2$. A reliability index is defined to be $\gamma = \mu_{\mathcal{G}}/\sigma_{\mathcal{G}}$. If the probability distribution of $\mathcal{G}$ is Normal, the probability of failure may be computed with a knowledge of this reliability index. If the probability distribution of $\mathcal{G}$ is not known, it is expected that there will be a corresponding probability of failure for each value of γ , and thus the reliability index (the objective of the Level II analysis) may be taken as a valid relative measure of safety.

The first general approach to Level II reliability is called the "mean-value first-order second-moment method". In this approach the limit state function is expanded in a Taylor series about some linearization point in the space of the resistance and load variables, retaining only the first order terms. The linearization point is taken to be the vector of statistical mean values of the resistance and load values. To be specific for present purposes, $\mathbf{R}$ will denote the vector of resistance variables, $(r_1, \dots, r_n)$, and $\mathbf{D}$ will denote the vector of load variables, $(d_1, \dots, d_m)$. The linearizing point may be denoted by $\bar{\mathbf{R}}, \bar{\mathbf{D}}$, where $\bar{\mathbf{R}}$ denotes the vector of the means of the individual resistance variables, $(\bar{r}_1, \dots, \bar{r}_n)$, and $\bar{\mathbf{D}}$ denotes the vector of the means of the individual load variables, $(\bar{d}_1, \dots, \bar{d}_m)$.

Writing out the first order expansion of Eq. 10,

$$\begin{aligned} \mathcal{G} = G(r_1, \dots, r_n, d_1, \dots, d_m) &\simeq G(\bar{r}_1, \dots, \bar{r}_n, \bar{d}_1, \dots, \bar{d}_m) \\ &+ \sum_{i=1}^n (r_i - \bar{r}_i) \left[\frac{\partial G}{\partial r_i} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} \\ &+ \sum_{i=1}^m (d_i - \bar{d}_i) \left[\frac{\partial G}{\partial d_i} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} \end{aligned} \quad (12)$$

where the notation:

$$\left[\frac{\partial G}{\partial d_i} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}}$$

signifies the partial derivative of the limit state function, evaluated at the linearization point, $\bar{\mathbf{R}}, \bar{\mathbf{D}}$.

The statistical expectation of Eq. 12, yields an approximate value of $\mu_{\mathcal{G}}$,

$$\mu_{\mathcal{G}} \simeq G(\bar{r}_1, \dots, \bar{r}_n, \bar{d}_1, \dots, \bar{d}_m). \quad (13)$$

The variance of $\mathcal{G}$ is the statistical expectation of $(\mathcal{G} - \mu_{\mathcal{G}})^2$. Then from Eq. 12, the approximate variance of $\mathcal{G}$ becomes,

$$\begin{aligned} \sigma_{\mathcal{G}}^2 &\simeq \sum_{i=1}^n \sum_{j=1}^n \left[\frac{\partial G}{\partial r_i} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} \left[\frac{\partial G}{\partial r_j} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} (\rho_{r_i r_j} \sigma_{r_i} \sigma_{r_j}) \\ &+ \sum_{i=1}^n \sum_{j=1}^m \left[\frac{\partial G}{\partial r_i} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} \left[\frac{\partial G}{\partial d_j} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} (\rho_{r_i d_j} \sigma_{r_i} \sigma_{d_j}) \\ &+ \sum_{i=1}^m \sum_{j=1}^n \left[\frac{\partial G}{\partial d_i} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} \left[\frac{\partial G}{\partial r_j} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} (\rho_{d_i r_j} \sigma_{d_i} \sigma_{r_j}) \\ &+ \sum_{i=1}^m \sum_{j=1}^m \left[\frac{\partial G}{\partial d_i} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} \left[\frac{\partial G}{\partial d_j} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} (\rho_{d_i d_j} \sigma_{d_i} \sigma_{d_j}) \end{aligned}$$

where σ_{r_i} and σ_{d_i} are the standard deviations of the i^{th} load and resistance variables, and $\rho_{r_i d_j}$, for example, denotes the statistical correlation coefficient between the i^{th} resistance variable and the j^{th} load variable.

It is reasonable to assume that the sets of resistance and load variables are statistically independent, or are at least uncorrelated. Thus, under this general assumption,

$$\rho_{r_i d_j} = \rho_{d_i r_j} \equiv 0,$$

for all combinations of i and j . Then the expression for the variance of G becomes:

$$\begin{aligned} \sigma_G^2 \simeq & \sum_{i=1}^n \sum_{j=1}^n \left[\frac{\partial G}{\partial r_i} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} \left[\frac{\partial G}{\partial r_j} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} (\rho_{r_i r_j} \sigma_{r_i} \sigma_{r_j}) \\ & + \sum_{i=1}^m \sum_{j=1}^m \left[\frac{\partial G}{\partial d_i} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} \left[\frac{\partial G}{\partial d_j} \right]_{\mathbf{R}=\bar{\mathbf{R}}, \mathbf{D}=\bar{\mathbf{D}}} (\rho_{d_i d_j} \sigma_{d_i} \sigma_{d_j}) \end{aligned} \quad (14)$$

Equations 13 and 14 indicate the quantities which must be defined for the individual resistance and load variables before the mean-value first-order second-moment reliability analysis can be made.

For the load variables, the items required are the vector of mean values, $\bar{\mathbf{D}} = (\bar{d}_1, \dots, \bar{d}_m)$, and the m by m *Covariance matrix*, the elements of which are $(\rho_{d_i d_j} \sigma_{d_i} \sigma_{d_j})$ for $i \neq j$, and $\sigma_{d_i}^2$ for $i = j$.

The items required for the resistance variables are exactly similar.

In the mean-value first-order second-moment approach, significant errors are introduced by nonlinear limit state functions, the answer depends upon the exact formulation of the limit state function, and the safety index is explicitly related to the probability of failure only if all variables are Normal. An advanced version of Level II analysis, called "advanced first-order second-moment", was developed to mitigate these problems.

In the "advanced first-order second-moment" version, the random resistance and load variables are transformed to a "reduced" set of new random variables with unit variance and zero mean in accordance with the given values of dimensional means and variances. If there are correlated variables, the correlation coefficients provide the key to a mathematical rotation of coordinates, which allows a further transformation to a set of reduced variables which are also uncorrelated. Once the transformations are carried out, the limit state function is transformed to a function of the reduced, uncorrelated, variables, and the reliability analyses proceeds with the re-defined function toward a final estimate of the reliability index.

It will be assumed that the required transformations are part of the reliability machinery. Under this assumption, the resistance and load information required for the advanced method is the same as noted previously.

The work of Level II reliability analysis may be partitioned into elements similar to those of Level III:

- I. MODELING: The (nontrivial) definition of limit state functions, Eq. 10, by the structural analyst for all the possible failure modes.

- II. RESISTANCE SYNTHESIS. The synthesis for the relevant resistance parameters of the corresponding means and covariance matrix.
- III. LOADS SYNTHESIS. The synthesis for the relevant load parameters of the corresponding means and covariance matrix.
- IV. RELIABILITY ANALYSIS. The final coordination of the syntheses of Elements II and III, the invocation of the reliability analysis machinery to evaluate reliability indices, and the interpretation of results from the various assumed limit states.

It appears that the "calibration" of Level II analysis requires some approximations to a Level III analysis to be available to support the argument about the required magnitude of the reliability index.

Level I Reliability Codes

Level I design codes appear to be relatively simple relationships between strength and loads, which incorporate a considerable number of "partial safety factors". Such codes appear to be the end product of a great deal of "calibration" by means of higher level reliability analyses. A tentative code to cover everything required for ships seems not available, though some starts have been made by classification societies and the British navy.

Because the partial safety factors are partially empirical, the *method* of estimating the "loads" in a Level I code is obviously an important part of the code specification. What form this method might take in a Level I code for the US Navy very much remains to be seen. Accordingly, no generalizations are warranted—and will not be until a substantial amount of higher level reliability analysis is carried out.

APPENDIX B

SYNTHESIS OF LIFETIME MEANS AND COVARIANCES

As noted in the main text, the total load vector, is defined as $\vec{D} = [d_1, \dots, d_5]$. The fundamental object of the loads synthesis is to synthesize lifetime statistics of the load vector, $\vec{D}$, under the assumption that the population is defined by all the possible load vectors which could occur during the life of an infinite number of sister ships.

Within the present state-of-art, if the inputs to a single hydrodynamic simulation for statistically stationary wave conditions (a vector $\vec{A}$, say) themselves vary randomly over ship lives, the single hydrodynamic simulation cannot result in more than a conditional probability density of the load vector, given fixed values of $\vec{A}$.

Supposing the conditional density derived from the simulation to be $f_{DC}(\vec{D}|\vec{A})$, and the (joint) density of $\vec{A}$ to be $f_A(\vec{A})$, the theoretical expression for the density of the lifetime loads becomes:

$$f_D(\vec{D}) = \int_{\vec{A}} f_{DC}(\vec{D}|\vec{A}) f_A(\vec{A}) d\vec{A} \quad (15)$$

where the domain of integration is the domain of $\vec{A}$.

Defining the statistical expectation of a function, $g(\vec{D})$, of $\vec{D}$ by $E[g(\vec{D})]$,

$$E[g(\vec{D})] = \int_{\vec{D}} g(\vec{D}) f_D(\vec{D}) d\vec{D} \quad (16)$$

in which the domain of integration is the domain of $\vec{D}$.

Now substituting the expression for $f_D(\vec{D})$, Eq. 15, into Eq. 16, and interchanging the order of integration,

$$E[g(\vec{D})] = \int_{\vec{A}} \left(\int_{\vec{D}} g(\vec{D}) f_{DC}(\vec{D}|\vec{A}) d\vec{D} \right) f_A(\vec{A}) d\vec{A} \quad (17)$$

The integral within the large braces in Eq. 17 may be called the *conditional* expectation,

$$E[g(\vec{D})|\vec{A}] = \int_{\vec{D}} g(\vec{D}) f_{DC}(\vec{D}|\vec{A}) d\vec{D}$$

and thus,

$$E[g(\vec{D})] = \int_{\vec{A}} E[g(\vec{D})|\vec{A}] f_A(\vec{A}) d\vec{A} \quad (18)$$

Thus, the synthesis of lifetime expectations from conditional expectations, Eq. 18, is of exactly the same form as that of probability densities from conditional densities, Eq. 15. Essentially, the synthesis involves weighting conditional expectations in accordance with the density of $\vec{A}$ over the domain of $\vec{A}$.

For Level II reliability analyses it is required to estimate the means and covariances of the load vector with respect to the lifetime population. The mean value of the vector

is defined by the expectation, $E[\tilde{D}]$, so that $g(\tilde{D}) = \tilde{D}$ for this case, and the synthesis would involve the weighting of the conditional expectations, $E[\tilde{D}|\tilde{A}]$.

Similarly, the ij^{th} element of the covariance matrix is the expectation, $E[(d_i - \bar{d}_i)(d_j - \bar{d}_j)]$, so that $g(\tilde{D}) = (d_i - \bar{d}_i)(d_j - \bar{d}_j)$, and this synthesis would involve the weighting of the conditional expectations, $E[(d_i - \bar{d}_i)(d_j - \bar{d}_j)|\tilde{A}]$.

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