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NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

NETWORK INTERDICTION MODELS

by

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September, 1991

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UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION Unclassified	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.
2b DECLASSIFICATION/DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION Naval Postgraduate School 6b OFFICE SYMBOL *(If applicable)* OR	7a NAME OF MONITORING ORGANIZATION Naval Postgraduate School
6c ADDRESS *(City, State, and ZIP Code)* Monterey, CA 93943-5000	7b ADDRESS *(City, State, and ZIP Code)* Monterey, CA 93943-5000
8a NAME OF FUNDING/SPONSORING ORGANIZATION 8b OFFICE SYMBOL *(If applicable)*	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c ADDRESS *(City, State, and ZIP Code)*	10 SOURCE OF FUNDING NUMBERS Program Element No / Project No / Task No / Work Unit Accession Number

11 TITLE *(Include Security Classification)*
NETWORK INTERDICTION MODELS

12 PERSONAL AUTHOR(S) STEINRAUF, Robert Louis

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT *(year, month, day)*	15 PAGE COUNT
Master's Thesis	From To	1991 September	67

16 SUPPLEMENTARY NOTATION
The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

17 COSATI CODES			18 SUBJECT TERMS *(continue on reverse if necessary and identify by block number)*
FIELD	GROUP	SUBGROUP	Network, Interdiction, Counter-narcotics, Cocaine, Drugs, Riverine, Mathematical Programming Models

19 ABSTRACT *(continue on reverse if necessary and identify by block number)*

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20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS REPORT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION Unclassified
22a NAME OF RESPONSIBLE INDIVIDUAL R. Kevin Wood	22b TELEPHONE *(Include Area code)* 408-646-2532 22c OFFICE SYMBOL OR/Wd

DD FORM 1473, 84 MAR 83 APR edition may be used until exhausted. All other editions are obsolete

SECURITY CLASSIFICATION OF THIS PAGE
Unclassified

Approved for public release; distribution is unlimited.

Network Interdiction Models

by

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Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
Septermber 1991

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ABSTRACT

Two mathematical programs are developed which determine strategies to interdict a network using limited resources. The first model identifies a set of arcs whose interdiction minimizes the maximum flow through the network, constrained by the available resources. The solution is a set of segments to interdict and a set of segments which are not interdicted, but determine the maximum flow in the interdicted network. The second model identifies a set of arcs whose interdiction isolates a targeted demand node and a maximum number of contiguous nodes about this specified node. This model is developed to take into account that the exact location of the demand node may not be known with certainty. The models are applied to a sample network that is similar to a river and road network in Bolivia where counter-narcotic interdiction operations are being conducted to stop the flow of precursor chemicals needed to manufacture cocaine. Insights drawn from the models' solutions are discussed.

THESIS DISCLAIMER

The reader is cautioned that the computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the program is free of computational and logic errors, it cannot be considered validated. any application of this program without additional verification is at the risk of the user.

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ACKNOWLEDGEMENT

I would like to express my gratitude to Professor Kevin Wood for his direction, guidance and patience while working with me on this thesis. He has made the process a valuable learning experience for me. I am also indebted to Professor Jerry Brown for his insight and support for my work. My deepest gratitude and love to go my wife, Heidi, without whose love, support and friendship, none of this would have been possible.

I. INTRODUCTION

A. BACKGROUND

The purpose of this thesis is to develop mathematical programming tools to assist counter-narcotics agents in South America who plan riverine and ground operations by helping them position their limited assets, i.e., interdiction teams on the waterways and roads, to most effectively stem the flow of chemicals used in the production of cocaine.

The United States has been engaged in a war since 1980, a war on drugs. A great deal of effort and money has been expended trying to educate the public, enforce current drug laws and assist South American countries in stemming the flow of illicit drugs out of their countries. Despite all that has been done, the situation has changed little since the war on drugs started. The producers of illicit drugs are motivated by profits and continue to find ways to increase production efficiency, decrease the chance of being detected and establish low-risk, high-volume transportation routes to move drugs. Many national and international agencies are working to disrupt the production and distribution process. The basic problem this thesis addresses is how best to interdict the flow of chemicals, which are precursors of cocaine, into a drug-producing region given the limited interdiction assets available.

1. Cocaine Production Process

One of the major drugs targeted for interdiction is cocaine. There are numerous ways to produce cocaine depending on the situation and availability of the ingredients. The production of cocaine is not tightly controlled by large organizations.

The process encompasses a large number of people working independently, who are involved in only a few of the production steps, or who provide the supplies to meet the market demand for precursor chemicals, coca leaves, or some stage of the processed cocaine. This decentralized structure makes it difficult to hinder the production of illicit drugs, because when one person in the structure is removed, others are able to move in and replace the person.

The following is a generic "recipe" for producing one kilogram of cocaine hydrochloride (HCl). First, 247 Kilograms of coca leaves are macerated in a solution of kerosene, sodium bicarbonate and sulfuric acid in simple, plastic-lined pits. The residue, 3.3 Kilograms of coca paste, is collected and transported to the next processing site. The paste is mixed with sulfuric acid, potassium permanganate and ammonium hydroxide to remove a majority of the impurities. The mixture is dried and 1.1 Kilograms of cocaine base remains. The cocaine base is then taken to a cocaine laboratory where chemists mix precise amounts of ethyl ether, acetone and hydrochloric acid to remove the final impurities and produce 1 Kilogram of cocaine HCl. Finally, the cocaine is packaged and distributed to markets in the United States and elsewhere for consumption.

2. The Role of Precursor Chemicals

The entire process of producing cocaine is based on the availability of certain chemicals, or substitutes, at each stage. These chemicals are known as "precursor chemicals". All precursor chemicals have legitimate uses in industry and large quantities are imported into the Andean countries each year. The quantities needed for illicit drug production are thought to amount to less than 1% of the total imported each year. The precursor chemicals are legally brought into the country, and then diverted for illicit use. The Drug Enforcement Agency is making major efforts to minimize the amount of chemical diverted for illicit purposes, but precursor chemicals are still diverted despite these efforts.

3. Why Focus on Precursor Chemicals?

The focus of this thesis is on precursor chemicals and not on processed cocaine or coca leaves. Without precursor chemicals to process the plentiful coca leaves, drug production stops. The chemicals are not found in the region and are required in large quantities. This necessitates using the road and river network for moving the chemicals. Coca is grown throughout much of the region, providing a ready supply. Coca leaves can also be economically transported by men and animals off the normal transportation routes, where interdiction is much more difficult. Since precursor chemicals usually are shipped in 55-gallon drums, it would be hard and less economical to move them off normal transportation routes. Once processed, however, cocaine can be moved in various size packages, limited only by the imagination of the *narcos*, i.e. persons involved in the illegal production of narcotics. Because of this flexibility, interdiction of cocaine in paste, base and HCl form is very difficult. The dependence on precursor chemicals can be considered a weak link in the illicit drug production chain.

4. Scope of Project

a. General

Bolivia is the second largest supplier of coca leaf for the international illicit cocaine market; Peru is the largest. The estimated area under coca cultivation in Bolivia is between 33,000 and 48,000 hectares which could yield 46,000 to 67,800 metric tons of coca leaves. This amount could produce approximately 92 to 135 metric tons of cocaine HCl per harvest. The coca leaf is harvested four times a year, resulting in enough coca leaves to produce up to 540 metric tons of cocaine HCl per year. (U.S. Department of Justice, 1988, pp. 3-4)

b. Area of interest

The areas of interest in this project are the Chaparé and Beni regions and the Mamoré river basin of Bolivia. (See Figure 1.) The Chaparé region in Bolivia produces an estimated 60 to 75 percent of the coca cultivated in Bolivia. The coca leaves are harvested and the paste is made in the Chaparé. The paste is shipped out of the Chaparé by the *Rios Ichilo* and *Chaparé* to the Mamoré river basin and lower Beni region, where the base is produced. The base is then converted into cocaine HCl in the upper Beni region or flown out of the country to laboratories in other South American countries, principally Columbia.

c. Interdiction opportunities

Bolivia and the United States are expanding counter-narcotic efforts to include the interdiction of precursor chemicals in Bolivia. These precursor chemicals must be transported primarily along waterways, from points of entry to the laboratories located in remote regions, providing an opportunity for interdiction. Bolivia, with assistance from the United States, maintains riverine and ground forces, the "Blue Devils", in this region for the purpose of interdicting drug and precursor chemical traffic.

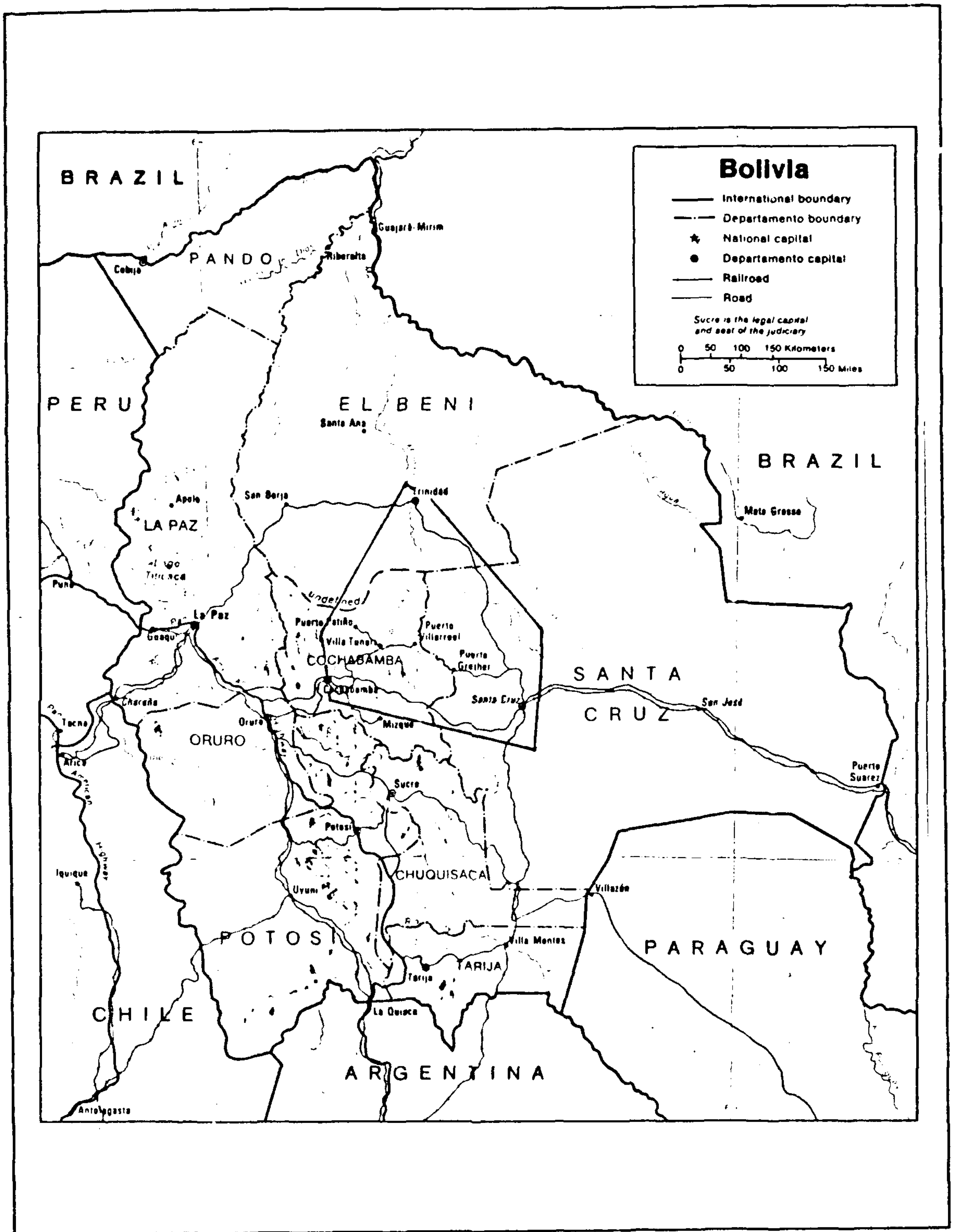

Figure 1 Map of Bolivia with the Chaparé region and the Mamoré river basin outlined.

B. THE BASIC PROBLEM

1. Geography of the Region

The broad eastern plains of Bolivia provide an excellent region for *narcos* to produce cocaine. The region is primarily tropical three-canopy rain forest. There are very few all-weather roads, but there is an elaborate system of navigable rivers, which reaches to the base of the Andes. The absence of roads makes the waterways the primary means of transporting goods throughout the region from the major cities in the area.

There is a wide variation in the physical nature of the waterways. Rivers vary in width from 12 feet to 12 miles, while tributaries can be as narrow as 1-2 feet. During the rainy season, the rivers swell and new tributaries become navigable to small boats and canoes. During the dry season, many small tributaries dry up and become dirt roads, but the major rivers are still navigable. There are several deep draft ports near the base of the Andes which are accessible all year long.

2. The Transportation Network

Given the river system, the lack of roads and the lack of mobility off the roads, the transportation routes in the region can be depicted as a network. The network used to evaluate the interdiction models developed in this thesis is based on the river and road network in the Beni region of Bolivia. This region has three major river systems which flow into the Mamoré south of Trinidad. One all-weather highway crosses the upper parts of the rivers near the foot of the Andes. However, there are several illicit airfields in the area, which could be assumed to connect with the river system by road or trail and be used for transportation of precursors. For simplicity, but without loss of generality, these airfields are not considered.

C. LITERATURE SEARCH AND ANECDOTAL BACKGROUND

1. Counter-narcotic Efforts

The following information on counter-narcotic efforts was gathered during interviews in November and December 1990 with representatives of organizations named below, during a trip to Panama and Washington, D.C. Many different government agencies are working to stem the flow of illicit drugs, and each is approaching the problem based on the strengths, characteristics and capabilities of their particular organization.

a. Key people and organization matrix

The Joint Tactical Intelligence Center at the Pentagon is working on a three-dimensional matrix which identifies key people and organizations by specific steps in cocaine production, geographical area and time. Once the linkages are known, it may be possible to disrupt the production process by targeting a few key people who are principally responsible for certain steps of the production process.

b. Transportation of precursor chemical on waterways

The DB-5 section of the Defense Intelligence Agency Center, Bolling Air Force Base, is studying on the transportation of precursor chemicals on South American waterways. They provided information on the scope of the waterway network, problems associated with the region e.g., the seasonal changes in the river network and the lack of river navigation information for the a majority of the region, and a detailed map of a portion of the Bolivian waterways in the drug production region.

c. *Global movement of precursor chemicals*

The Diversion Operations section of the Drug Enforcement Agency, tracks large-scale, global movement of precursor chemicals. They maintain a database on all chemical shipments from the United States. The agency attempts to certify that the recipients of the shipments are not connected with drug trafficking and will use the chemicals for legitimate purposes. They publish a pamphlet defining exactly what chemicals are considered precursors and outlining the requirements for shipment of precursor chemicals.

d. *U.S. Southern Command (SOUTHCOM) initiatives*

Several SOUTHCOM organizations are working on the problem of drug interdiction in South America. The Deputy Director, Drugs section is working on creating a "Think Tank" consisting of all the elements working in the area, linked together by the Command Management System (CMS). CMS will, among other things, provide data and communications links that allow time-sensitive information to be passed to all agencies involved in counter-narcotic operations in a matter of seconds. The "Think Tank" would coordinate all counter-narcotics interdiction efforts to increase effectiveness and bring together experts from different fields and provide a forum for the exchange and integration of information.

SOUTHCOM J-2, Intelligence, follows the current trends in drug production and maintains information on drug production activities in various countries. SOUTHCOM J5-RW, Wargaming, is developing drug lab interdiction scenarios for use with the JANUS-A (U.S. Army TRADOC Analysis Center, 1986) high resolution combat model, to provide training for law enforcement agencies in countries where drug production takes place.

2. Network Interdiction Models

During the 1960s and 1970s, network interdiction models were studied extensively, due in part to the problem of interdicting supply routes in the Vietnam War. Much of the work was based on related material from the 1950s. This section describes some of the models developed and techniques employed to solve the models.

Ford and Fulkerson (1962) develop the well known max flow - min cut theorem which provides the basis on which many other works build. Their theorem states that the maximum flow in a network is equal to the capacity of the minimal cut set in the network. By using a maximum flow algorithm, it is possible to determine which arcs are in the minimal cut-set. These arcs should be targeted for interdiction if the resources expended to interdict an arc do not vary significantly from one arc to another. Their theorem is also useful to determine how all paths between two specified nodes can be broken with the minimal expenditure of resources. If the network data is redefined so that the effort to interdict an arc is used as the capacity on the arc in a max flow problem, the minimal capacity cut will identify the arcs to interdict at lowest effort.

Wollmer (1970) develops two heuristic algorithms for targeting strikes against a Lines of Communication network. The problem he addresses is determining the most important arc in the network that when interdicted, increases the cost of using the network the most. The cost to the user is based on the unit-flow cost, repair time and repair cost. The first algorithm assumes the cost function is linear and the second, piecewise linear with one breakpoint. Wollmer assumes the network user is attempting to achieve a certain level of flow circulation at a minimum cost. The algorithms determine a value for each arc, which is a function of the repair cost and resulting increase in transportation cost when an arc is interdicted. Wollmer uses a combined formulation of the required flow and maximum flow problems to determine the actual cost due to interdiction. The formulation, called "minimum-cost circulation", finds a minimum cost flow to meet the required flow if the network will support the flow, and if not, the maximum flow is found. The arc with the largest value is then interdicted. If the problem allows multiple arcs to be interdicted, the algorithm is repeated with previously interdicted and unrepaired arcs at their interdicted capacities.

In another paper (1964), Wollmer presents an labeling algorithm to determine the set of n arcs to remove in a planar network which minimizes flow. The topological dual of the primal network (see Lawler (1976) for a discussion of the topological dual) is used by the algorithm. In this dual network, a node is created for each face in the primal network and dual nodes in adjacent faces are connected by dual arcs. Also, a dual source, s', and dual sink, t', are created on the exterior face of the primal network and connected to dual nodes in adjacent faces. Each dual arc is then assigned a length equal to the capacity of the primal arc it crosses. The shortest path from s' to t' in the dual network then corresponds to the minimal capacity cut in the primal network. Wollmer's methodology can be thought of as working on a modified dual network in which each dual arc is replaced by two arcs in parallel, one with length 0 and one with length equal to the capacity of the primal arc being crossed. The problem then becomes one of finding the shortest path from s' to t' in the dual network, using at most n zero-length arcs. The set of zero-length arcs in the shortest path correspond to the set of arcs to remove in the primal network to minimize the maximum flow. Wollmer's methodology is straightforward and attractive because it solves the problem in polynomial time. However, the methodology is limited because it requires a planar network and assumes that the amount of resource necessary to interdict an arc is the same for all arcs.

Helmbold (1971) uses dynamic programming to solve a generalization of Wollmer's model in which the resource necessary to interdict an arc may vary among arcs. The algorithm can be thought of as using the same modified dual network used by Wollmer, described above, but where the zero-length dual arcs now require some positive integer amount of resource to traverse. Helmbold uses a backward recursion to find the shortest path from s' to t' constrained by the resources available. The recursion applied to the modified dual network is effectively

$$F_j(x) = \min_{(j,k) \in FS(j)} \{ F_k(x - r_{jk}) \ , \ F_k(x) + L_{jk} \}$$

where $F_j(x)$ denotes the shortest path distance from node j to node t' which uses x units of resource, $FS(j)$ denotes the forward star of j, i.e., the set of arcs directed out of j, L_{jk} is the length of arc (j,k) which uses no resources, and r_{jk} is the amount of resource consumed by traversing the zero-length arc (j,k). Because the number of steps taken by the algorithm depends on the total available resources, it is a pseudo-polynomial time algorithm.

McMasters and Mustin (1970) develop another algorithm for interdicting a planar network with limited resources. The problem they address is essentially the same as Helmbold's, but their approach is different. Their algorithm determines which targets, or arcs, to interdict and how much effort to expend. They develop the topological dual of the capacitated flow network, and determine the shortest path through the fully interdicted dual. The interdiction cost is then determined and compared to the resources. If it is less than the resources, the problem is solved. If the resource constraint is exceeded, the algorithm attempts to "unspend" the resources while increasing the flow as little as possible. The resources are unspent along the shortest path until the resource constraint is met. The algorithm looks next for the second shortest path and repeats the resource check and the unspending process. The distance of the second shortest path is compared to the distance to the first distance and the smaller is saved as the best solution. The process continues until the distance of the ith shortest path is longer than the current best solution. The algorithm is designed for problems where at least one arc has a lower bound greater than zero. If all arcs can be interdicted to zero flow, the algorithm must evaluate all feasible length, loopless routes through the dual.

Preston (1972) uses dynamic programming to identify the optimal allocation of aircraft for an airstrike against a planar transportation network. He uses an exponential damage function to determine the relative cost of allocating another aircraft to interdict an arc versus the benefit of the interdiction. The topological dual is constructed from the planar network, and the shortest path through the dual network is found using the capacities of the arcs in the original network as the lengths of the corresponding dual arcs. Next, the arcs of the dual are assigned lengths equal to the fully interdicted capacities of the corresponding arcs in the original network. The set of all shortest paths, S, is found such that their lengths do not exceed the length of the shortest path found for the uninterdicted network. For each path in S, a recursive equation is used to find the optimal allocation of aircraft to the arcs, for 1 to K aircraft. The flow is determined at each level of interdiction for each path. The optimal allocation for 1 to K aircraft is then determined by comparing the level of flow in each path in S for the allocated aircraft. The final decision on aircraft allocation is based on a cost/benefit function for the addition of one more aircraft. Starting with 0 aircraft, the algorithm determines if one more aircraft is cost-effective. If the benefit is greater than the cost, another aircraft is used. It is extremely difficult to define an acceptable cost/benefit function, and once defined, the answer is very sensitive to the function used. The example used in the paper equates the dollar cost of operating an aircraft with the dollar benefit of interdicting a ton of enemy supplies. Information on the dollar cost of operating the aircraft is available, but it is difficult to place a dollar cost on a ton of interdicted supplies. Preston shows that the optimal solution could be to send no aircraft on interdiction missions because the return is not high enough. If the problem to be solved were purely economical, with outcomes based on dollar amounts, this model might useful. However, given the nature of drug interdiction, the model does not seem very useful. Furthermore, the enumeration of the set S is likely to require an exponential amount of work in general.

Durbin (1966) develops an interdiction model which evaluates flow through a network as arcs are successively destroyed and repaired. The model uses Fulkerson's Out-of-Kilter Algorithm to profile the maximum cargo flow as a function of available vehicles traveling on a given highway system. The profile solution is found by considering the inverse problem of the number of trucks required to support a certain flow. This is done by increasing the flow from 0 to the maximum throughput, evaluating the number of trucks required at pre-determined profile points. Then, based on the estimated number of trucks available to the network user, Wollmer's algorithm (Wollmer, 1974) is used to destroy the arc that reduces flow most. These steps are repeated until there are no more interdiction assets remaining or flow is stopped. The step-wise removal of arcs in this model does not ensure the solution found is optimal.

Lubore, Ratliff and Sicilia (1971) develop an algorithm that, given capacities on the arcs of a network, determines the single most vital link in the network with respect to maximum flow. The algorithm assumes that the interdiction cost for each arc is the same. Their work is an improvement over an earlier algorithm by Wollmer (1963). Wollmer's algorithm requires complete enumeration of all arcs, while this improved algorithm reduces the number of arcs considered as candidates by establishing a necessary condition for an arc to be most vital. The value of arc (x,y) is defined as the difference in maximum flow in [N;A] and [N;{A - (x,y)}] between the given source and sink. Their algorithm requires the flow in arc (i,j) to be at least as great as the flow over every arc in a minimal cut-set for some maximum flow pattern in [N;A] before it is considered. Their work is applicable in determining the single most valuable arc in the network, but it is not capable of finding the most vital set of arcs for sets with cardinality greater than 1.

In a later paper, Ratliff, Sicilia and Lubore (1975) present an algorithm which finds a set of n arcs, whose removal from the network results in the greatest decrease in throughput of the remaining system between the source and sink. This is the same problem that Wollmer (1964) addresses, but the methodology of these authors is applicable to both planar and nonplanar networks. Their algorithm uses a modified network in which all arc capacities are set at the smaller of a specified value, u, or the arc's original capacity. The value of u starts at 1 and is increased by 1 until the algorithm terminates. The minimum cut-sets are then determined for the modified network using a maximum flow algorithm. If the cardinality of the minimum cut-set is equal to n, the algorithm terminates. Otherwise u is incremented by 1, the network is again modified and the minimum cut sets are determined. When this is successful, it finds a most vital link set of cardinality 1, then 2, etc., up to n. If the procedure is not successful, the authors use a partitioning branch and bound procedure to limit the enumeration required to determine the optimal arc set. This algorithm does not take interdiction cost into account. However, if a resource constraint and a cost function were included in the model, it could possibly provide a useful solution to the problem at hand.

Golden (1978) proposes a model that uses a linear cost function to lengthen the arcs of a network based on increasing the shortest path via a least-cost investment strategy. The problem reduces to a minimum cost flow problem which can be easily solved. He complicates the model by requiring the shortest path to be increased by at least some value, τ, which ensures a predetermined level of difficulty for the network user. The value of τ can be increased or decreased to accommodate a budget constraint. The model identifies the arcs that are most cost effective in increasing the shortest path, but does not allow for the removal of arcs from the network.

There are several game-theoretic works that deal with interdiction of networks. The works by Danskin (1962) and Wollmer (1970) approach network interdiction as two-player zero sum games. While these works are of interest and might usefully be expanded upon for drug interdiction, this thesis will not consider game-theoretic models.

D. OUTLINE OF PROPOSED MODELS AND SOLUTION TECHNIQUES

1. General

The mathematical programming models proposed here differ from other techniques used to solve interdiction problems. The formulations used are better than the other methods because they more easily generalize to different problems. The *narcos'* goal is to move supplies of precursor chemicals through the network to specific locations where the chemicals are needed. The models provide solutions to the interdictor which dictate where to place the limited interdiction assets to most effectively disrupt the flow of chemicals.

The actual Mamoré river basin network is not used since not all of the information needed is available and some of the information is classified. The transportation network used to evaluate the models is semi-randomly generated by a FORTRAN program. River and road segments are created and assigned values for the capacity and the required number of interdiction assets to disrupt flow on the arc. The supply locations for precursor chemicals are set. The models, formulated in GAMS (Brooke, Kendrick and Meeraus, 1988), use data from the network to set up the equations. GAMS/ZOOM (Brooke, Kendrick and Meeraus, 1988) or XA (Sunset Software, 1987), mixed integer program solvers, are then used to solve the problem. The resulting solution is a set of river and road segments to be considered for interdiction. This thesis considers two different network interdiction models:

2. Model 1 - Minimize the Maximum Flow

This model identifies a set of river and road segments whose interdiction minimizes the maximum flow through the network, constrained by the available assets. The solution is a set of segments to interdict and a set of segments that are not interdicted. The set of segments not interdicted determine the maximum flow in the network.

3. Model 2 - Maximize the Number of Nodes Isolated

The second model identifies a set of river and road segments whose interdiction isolates a specified node and a maximum number of other nodes surrounding the specified node. This model attempts to take into account the uncertainty involved in pinpointing the exact location of a suspected drug lab. If a certain area is thought to contain a drug lab, cutting off the entire area should stop the flow of precursor chemicals to the site. The objective of this model is to maximize the number of nodes, including the specified node, isolated from the supply nodes by interdicting segments without exceeding the available assets.

II. NETWORK INTERDICTION MODELS

A. DESCRIPTION OF A NETWORK

A network is a directed graph, G = (N,A), where N and A are specially defined sets whose elements may have parameters associated with them. N is the finite set of nodes or vertices, $N = \{1,2,...,m\}$, and A is a set of ordered pairs from N called arcs or edges; $A = \{(i,j),(k,l),...,(s,t)\}$, where $i,j,k,l,...,s,t \in N$. For an arc (i,j), i is the tail node, or where the arc originates, and j is the head node, where the arc terminates. In a transportation network, an arc (i,j) can be thought of as a pipeline or river segment which carries a flow of liquid from i to j. Let $m = |N|$, and $n = |A|$.

Associated with each arc (i,j) is its capacity, u_{ij} and the cost to interdict the arc, c_{ij}. In this problem, the capacity represents the maximum amount of precursor chemicals the *narcos* can transport on a given arc without raising suspicion. The cost represents the number of interdiction assets required to effectively stop the transportation of precursor chemicals along the arc and is measured in interdiction teams.

In the first problem addressed, it is assumed that the *narcos* are trying to maximize the amount of precursor chemicals transported to drug labs or maceration pits in order to maximize the production of cocaine paste and base. In terms of the network, this is equivalent to maximizing the flow from some source node s to a demand node t, subject to arc capacities. (This can be further generalized to include multiple source and demand nodes.) This is the maximum flow problem which can be stated and solved as a linear program. The formulation of the problem is

$$
\begin{aligned}
&\textit{Maximize} \quad f \\
&s.t. \\
&\sum_{j \in N} x_{ij} - \sum_{k \in N} x_{ki} = \begin{cases} f & \text{if } i = s \\ 0 & \text{if } i \neq s, t \\ -f & \text{if } i = t \end{cases} \\
&\qquad 0 \le x_{ij} \le u_{ij} \qquad (i, j) \in A
\end{aligned}
$$

where the sums and inequalities are taken over the existing arcs in the network. The formulation constraints require conservation of flow for each node. The bounds on x_{ij} imply that the flow cannot exceed the arc's capacity, nor can it be less than zero. The variable f can be thought of as flow on an artificial arc (t,s), or *return arc*.

The goal of a network interdictor is to disrupt flow in the network. This may be done by either isolating the supply node from the demand node, or by minimizing the maximum flow between the two nodes. In order to discuss these ideas, it is useful to define a cut-set and the capacity of the cut-set. Let X be any set of nodes in the network such that X contains node s but not node t. Let $X' = N - X$. Then $(X,X') \equiv \{(i,j) : i \in X, j \in X'\}$ is called a *cut-set* separating node t from node s (Bazaraa, Jarvis and Sherali, 1990, p. 565). The max-flow min-cut theorem states that the maximum flow in the network is equal to the capacity of a minimum capacity cut-set, where the capacity of the cut-set is defined as $\sum_{(i,j):\ i \in X,\ j \in X'} u_{ij}$.

B. THE DUAL OF A MAX FLOW FORMULATION

In the classical maximum flow problem, there is an associated problem that can be formed called the dual. The dual formulation is

$$
\begin{aligned}
&\textit{Minimize} \quad \sum_{i \in N} \sum_{j \in N} u_{ij} h_{ij} \\
&\text{s.t.} \\
&\qquad w_t - w_s = 1 \\
&w_i - w_j + h_{ij} \geq 0 \quad (i, j) \in A \\
&\qquad\qquad h_{ij} \geq 0 \quad (i, j) \in A
\end{aligned}
$$

where w_i corresponds to the flow conservation equations and h_{ij} corresponds to the bounds $x_{ij} \leq u_{ij}$. The dual has a solution in which all variables are 0 or 1. If variable $w_i = 1$, node i is a member of the of the set X'. If $w_i = 0$, the node is a member of set X. $h_{ij} = 1$ indicates membership in the set of arcs directed from set X to set X'. Therefore the dual of the maximum flow linear program is a minimum capacity cut-set identification problem.

C. FORMULATION OF THE MODELS

1. Model 1 - Minimize the Maximum Flow

In the first model considered, the interdictor tries to minimize the maximum flow of chemicals the *narcos* can move from s to t, subject to the number of interdiction teams available. This can be formulated as

$$
\begin{aligned}
&\underset{\delta}{\textit{Minimize}} \quad \underset{x}{\textit{Maximize}} \quad f \\
&\text{s.t.} \\
&\sum_{j \in N} x_{ij} - \sum_{k \in N} x_{ki} = \begin{cases} f & \text{if } i = s \\ 0 & \text{if } i \neq s, t \\ -f & \text{if } i = t \end{cases} \\
&x_{ij} - u_{ij}(1 - \delta_{ij}) \leq 0 \quad (i, j) \in A \\
&\sum_{(i,j) \in A} c_{ij}\delta_{ij} \leq \Delta \\
&\qquad x_{ij} \geq 0 \quad (i, j) \in A \\
&\qquad \delta_{ij} \in \{0,1\} \ \forall \ (i, j) \in A
\end{aligned}
$$

By taking the dual of the inner maximization, and making several substitutions, Wood (1991) shows that this model can be converted into the integer programming model

$$
\begin{aligned}
& \textit{minimize} \quad \sum_{(i,j)\in A} u_{ij}\beta_{ij} \\
& \text{s.t.} \\
& \alpha_i - \alpha_j + \beta_{ij} + \delta_{ij} \ge 0 \quad \forall\,(i,j) \in A \\
& \alpha_s = 0 \\
& \alpha_t = 1 \\
& \sum_{(i,j)\in A} c_{ij}\delta_{ij} \le \Delta \\
& \alpha_i \in \{0,1\} \ \forall\, i \in N \\
& \beta_{ij},\ \delta_{ij} \in \{0,1\} \ \forall\,(i,j) \in A
\end{aligned}
$$

$\alpha_i = 1$ indicates the node is on the demand side of some cut-set (X,X') and $\alpha_i = 0$ indicates the node is on the supply side of that cut. δ_{ij} or β_{ij} take on the value of 1 if arc (i,j) crosses from X to X'. Further, arc (i,j) is interdicted if $\delta_{ij} = 1$. The flow on the uninterdicted arcs in the cut-set, $\beta_{ij} = 1$, determines the flow to the demand node. All other δ_{ij} and β_{ij} are 0. The model can be extended to handle multiple supply and demand nodes by fixing $\alpha_i = 0$ for all supply nodes i and fixing $\alpha_j = 1$ for all demand nodes j.

The model can be easily generalized to handle multiple assets necessary to interdict an arc (for instance ground forces plus the boats needed to conduct riverine operations), or multiple independent resources which can be used for interdiction (for instance local police forces or the "Blue Devils"). In the former case, with asset set K, let c_{ijk} be the amount of asset k used by interdicting arc (i,j) and let Δ_k be the amount of asset k available. Then, the constraint

$$\sum_{(i,j) \in A} c_{ij}\delta_{ij} \le \Delta$$

is replaced by

$$\sum_{(i,j) \in A} c_{ijk}\delta_{ij} \le \Delta_k \ , \qquad \forall\ k \in K$$

In the latter case, let δ_{ijk} be 1 if asset k is used to interdict arc (i,j) and let c_{ijk} and Δ_k be defined as before. Then, Model 1 is modified to

$$\begin{aligned}
& minimize \quad \sum_{(i,j) \in A} u_{ij}\beta_{ij} \\
\alpha_i - \alpha_j + \beta_{ij} + \sum_k \delta_{ijk} &\ge 0 \\
\alpha_s &= 0 \\
\alpha_t &= 1 \\
\sum c_{ijk}\delta_{ijk} &\le \Delta_k \quad \forall\ k \in K \\
\alpha_i &\in \{0,\ 1\}\ \forall\ i \in N \\
\beta_{ij} &\in \{0,\ 1\}\ \forall\ (i,j) \in A \\
\delta_{ij.} &\in \{0,\ 1\}\ \forall\ (i,j,k,) \in A
\end{aligned}$$

Clearly, Model 1 assumes that locations of the supply and demand nodes are known to the interdictor. However, if the exact locations of supply and demand nodes were known, removal of the nodes by law enforcement agencies might be preferable to interdiction of transportation routes. If the exact locations are not known, the model can still be used directly by varying the demand node locations around the suspected node and comparing the results on the placement of teams and the maximum flow. The second model deals more directly with the uncertainty of demand node locations.

2. Model 2 - Maximum Number of Nodes Isolated

Much of the information available to the network interdictors is of a general nature and does not specifically identify a laboratory location. Information may be based on aircraft flights in and out of covert airfields or information from paid informants concerning drug activity in certain regions. This information may not be sufficient to mount a raid on a suspected site, but can be sufficient to conduct interdiction operations. The model isolates as large a set of contiguous nodes as possible around a suspected demand node in an attempt to heuristically maximize the chance of isolating the true demand node, given its exact location is unknown. This is done by maximizing the number of nodes, including the suspected demand node, isolated from the source by use of the interdiction assets. The formulation of the model is

$$
\begin{aligned}
&\textit{maximize} \quad \sum_{i \in N} \alpha_i \\
&\textit{s.t.} \\
&\alpha_i - \alpha_j + \delta_{ij} \geq 0 \quad \forall\ (i,j) \in A \\
&\alpha_s = 0 \quad \forall\ s \in N_s \\
&\alpha_t = 1 \quad \forall\ t \in N_t \\
&\sum_{(i,j) \in A} c_{ij}\delta_{ij} \leq \Delta \\
&\alpha_i \in \{0,1\}\ \forall\ i \in N \\
&\delta_{ij} \in \{0,1\}\ \forall\ (i,j) \in A
\end{aligned}
$$

The first constraint forces some cut-set to be formed between the supply and demand nodes. The value of α_i is fixed for the supply and demand nodes. Multiple demand nodes can be handled in the same manner as the first model, but the model is intended for a single demand node. The last constraint is a resource constraint, where the total number of interdiction teams used cannot exceed the number available, Δ. If there are sufficient interdiction assets, this formulation will identify a cut-set that isolates the largest possible number of nodes. If, however, there are insufficient assets available to form a cut-set, the problem is infeasible since it is impossible to isolate the demand node.

As formulated, the model does not enforce the requirement that the nodes isolated be contiguous. For example, consider the network in Figure 2. There is a large segment of the graph which is a tree that does not contain the demand node but can be isolated by breaking arc *a* which consumes some of the available assets. Assume that the demand node can be isolated using a portion of the remaining interdiction assets. If the number of additional nodes that the remaining assets could isolate around the demand node is less than the number of nodes that could be isolated in the tree, the tree will be isolated.

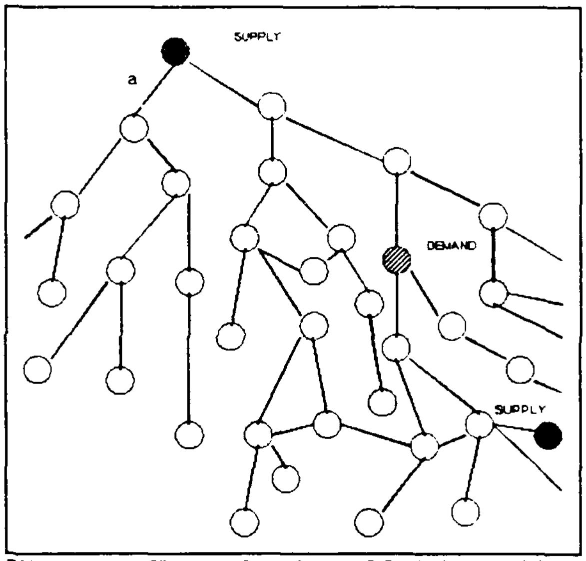

Figure 2 Example where Model 2 yields a set of non-contiguous nodes.

There are several ways to avoid this problem. If the network were a tree, it could be constructed so that all arcs were directed toward the demand node t. The following constraints would then enforce contiguity:

$$\alpha_i - \alpha_j \geq 0 \quad \forall\, (i, j) \in A$$

Another approach to encourage contiguity is to assign weights to each node, which decrease as the distance from the demand node t increases. The objective function then becomes $\Sigma\, w_i \alpha_i$, where w_i is the weight of node i. Weighting the nodes will also tend to encircle the demand node by making the nodes at distance k more attractive than the nodes at distance $k+1$. The current model counts a node at distance $k > 1$ equal to a node adjacent to the posited demand node. (We conjecture that if the weights decrease sufficiently quickly with distance, this model will, in fact, ensure contiguity of the isolated nodes.)

It should be noted that Model 2 can be generalized to handle multiple assets necessary to interdict an arc and multiple independent assets which can be used to interdict an arc in the same fashion that these generalizations can be made to Model 1.

III. APPLICATION OF MODELS

A. NETWORK USED TO EVALUATE MODELS

An actual transportation network was not available to evaluate the models. A network similar to the real-world network was constructed, using a FORTRAN program. The program randomly generates a network, based on certain user-supplied parameters. The network is built from a root node, node 1, and a random number draw determines the number of arcs, 0, 1, or 2, incident to the node. At the end of each arc, a numbered node is created. In this manner, a tree representing the river network is constructed until the network reaches a pre-specified depth. Depth refers to the distance, in terms of nodes, from node i to the root node. The interdiction costs and capacities are assigned to each arc as functions of depth. This is done to incorporate the changing river characteristics for segments that are further upstream from the root node. At the specified depth, all adjacent nodes are joined by arcs representing road segments. Arc capacities and interdiction costs for the road segments are assigned. All arcs in the network are considered undirected and are replaced with two anti-parallel, directed arcs with the same capacity and interdiction cost. The root node and the two nodes at the extremes of the road were made supply nodes by setting $\alpha_i = 0$. The network used to evaluate the model is shown in Figure 3. A listing of the network data, arc capacities and interdiction costs is contained in Annex A. Annex B contains the formulation of Model 1 in GAMS which was run on a 486 MS-DOS based computer and solved using XA. A listing of the GAMS

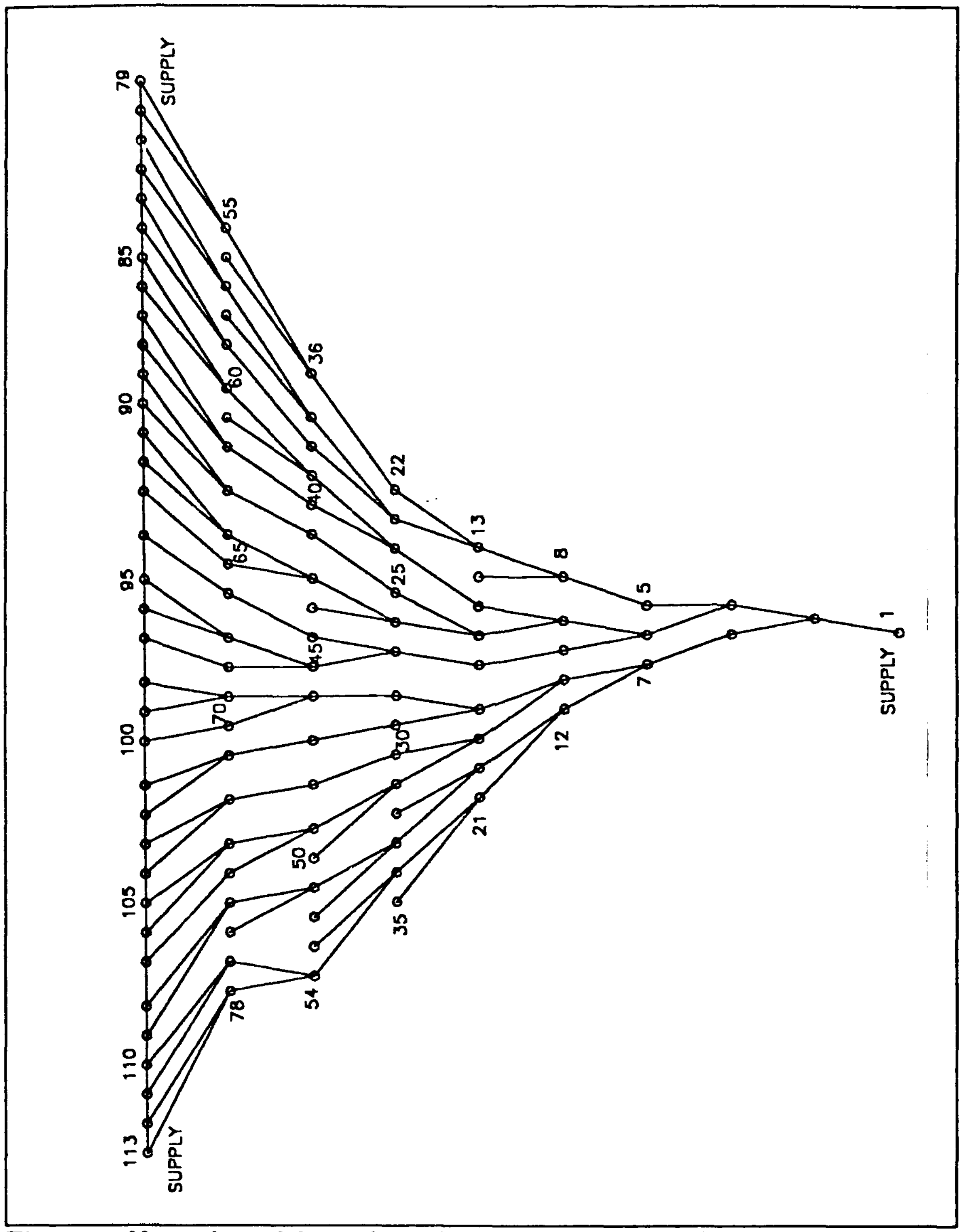

Figure 3 Network used for analysis. The road is represented by the arcs at the top of the network between nodes 79 and 113. Node 1 represents the town of Trinidad and the mouth of the river system.

formulation of Model 2 is contained in Annex C. Model 2 was run on an AMDAHL 5990 mainframe and solved using GAMS/ZOOM.

B. ANALYSIS OF MODEL 1

1. Scenario

Based on aircraft flights to covert airfields in a certain area, the counter-narcotic intelligence section suspects that there is a drug laboratory in the vicinity of node 42. The operational planner is interested in mounting an interdiction effort against the site. He also needs an idea of the possible scope of the activity at the node. Currently, this is the only suspected drug laboratory.

2. Interdiction Strategy for Node 42

By setting α_{42} = 1 and the number of teams to zero, the planner is able to use the model to determine the maximum, uninterdicted flow to the demand node. Using this figure as a baseline, he can then determine the marginal effectiveness of additional interdiction teams and the best strategy for the placement of the teams. Table 1 outlines the recommended strategy for different numbers of teams.

TABLE 1
Strategies for Interdicting Node 42, using Model 1

TEAMS USED	ARCS WHICH DETERMINE FLOW (β_{ij} = 1)	INTERDICTED ARCS (δ_{ij} = 1)	FLOW
0	(9,16) (88,89) (94,93)	None	16
1	(9,16) (88,89)	(94,93)	13
2	(9,16) (89,90)	(89,63) (94,93)	12
3	(26,42)	(90,91) (94,93)	12
4	(90,91) (94,93)	(26,42)	5
5	(90,91)	(16,26) (94,93)	2

6	(90,91)	(16,26) (94,93)	2
7	None	(26,43) (91,64) (92,64)	0

The results of the model show that the maximum flow to node 42 is 16 units. The largest marginal decrease in flow comes from employing four teams, while using less than four teams does not reduce flow by more than 25%. There is also no marginal benefit in using three teams instead of two teams or six teams instead of five since there is no change in flow. The planner notices that when the all flow is interdicted, the arcs cut are relatively close to the demand node. Using the table, the planner now has several options to choose from to reduce the flow of chemicals to the demand node.

3. Scenario with Multiple Demand Nodes

The mission planner has received information on two additional suspected drug production sites. Suspected sites are now located in the vicinity of nodes 42, 60 and 69. If possible, the planner would like to combine interdiction operations against all three sites. If this is not possible, then the planner seeks the best allocation of the teams which will decrease the throughput of precursor chemical to the drug laboratories. Of interest is the maximum uninterdicted flow to the demand nodes, both collectively and individually, and the change in flow caused by the addition of interdiction teams to the operation. The planner can evaluate the solutions provided by the model to assist in determining the optimal strategy which minimizes the flow to the demand nodes.

4. Interdiction Strategy for Nodes 42, 60 and 69

The baseline flows to the individual demand nodes are found by setting the corresponding $\alpha_i = 1$ and solving the model. The maximum flow to each node is given in the Table 2.

TABLE 2
Maximum Flow to Nodes 42, 60 and 69

NODE	ARCS WHICH DETERMINE FLOW ($\beta_{ij} = 1$)	FLOW
42	(9,16) (88,89) (94,93)	16
60	(15,24) (84,85) (89,88)	18
69	(46,69) (97,98) (100,99)	21

If each of the nodes is treated as a separate demand node with an operation directed against it, 17 teams are required to stop all the flow to the nodes. Table 3 shows the arcs to interdict and the number of teams required to achieve this.

TABLE 3
Strategies to Stop Flow to Nodes 42, 60 and 69 Individually

NODE	ARCS INTERDICTED ($\delta_{ij} = 1$)	TEAMS REQUIRED
42	(26,42) (93,65) (91,64) (92,64)	7
60	(39,60) (85,60) (86,60)	5
69	(46,69) (98,69) (99,69)	5

The zero-flow cut-sets which isolate the nodes are grouped tightly around the suspected node and do not interfere with flow to the other demand nodes. However, it may be beneficial to consider combining the operations, since it appears that interdiction efforts directed against one node could have an effect on flow to another node. To evaluate this, the planner runs the model with multiple demand nodes. Setting $\alpha_{42} = \alpha_{60} = \alpha_{69} = 1$ and solving the model for different numbers of teams yields the solutions in Table 4.

TABLE 4
Solutions with Multiple Demand Nodes using Model 1

TEAMS USED	ARCS WHICH DETERMINE FLOW ($\beta_{ij} = 1$)	INTERDICTED ARCS ($\delta_{ij} = 1$)	FLOW
0	(3,6) (46,69) (84,85) (99,100)	None	35
1	(3,6) (18,28) (84,85)	(101,100)	30
2	(1,2) (79,80) (113,112)	(79,55) (113,78)	27
3	(3,6) (18,28)	(85,60) (85,86) (101,100)	24
4	(1,2) (79,80)	(79,55) (113,78) (113,112)	20
5	(3,6) (84,85)	(46,69) (100,99)	18
6	(1,2)	(79,55) (80,55) (80,81) (113,78) (113,112)	14
7	(3,6)	(46,69) (85,60) (85,86) (100,99)	12
8	(6,9)	(46,69) (85,60) (85,86) (94,93) (98,69) (99,69)	12
9	(6,9)	(46,69) (85,60) (85,86) (94,93) (98,69) (99,69)	12

The planner notices that when eight or more teams are committed to the interdiction effort, node 69 is isolated, (Figure 4). The cut-set which isolates node 69 is the same as zero-flow cut-set found earlier. Five teams are required for this and the remaining teams interdict flow to nodes 42 and 60. When five teams are used to stop flow to node 69, the strategies dictated are the same as using three or more teams to

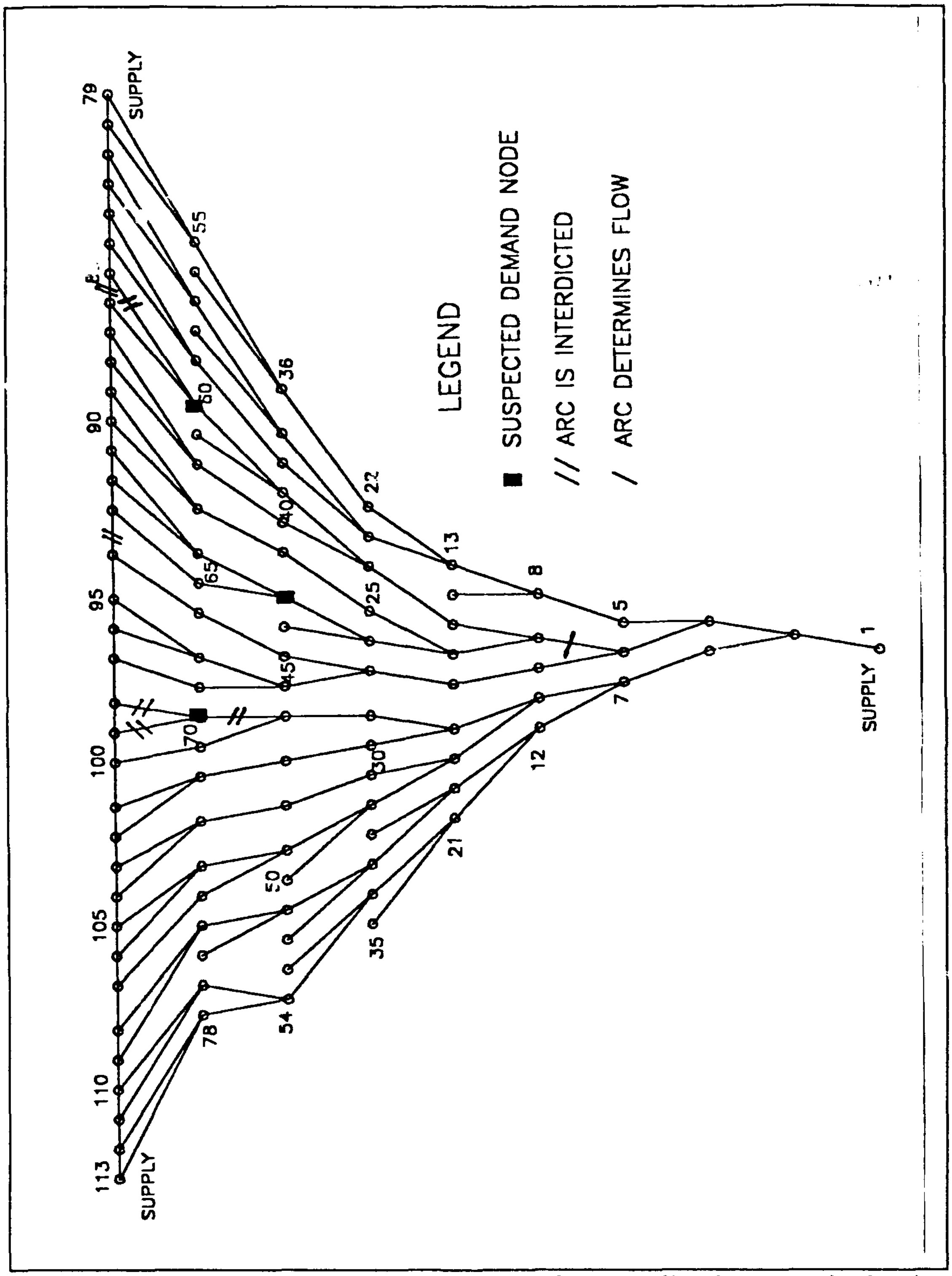

Figure 4 Solution to Model 1 with 8 teams used to interdict the network, showing node 69 isolated.

minimize the maximum flow to nodes 42 and 60. Evaluating the model for nodes 42 and 60 results in the following strategies shown in Table 5.

TABLE 5
Strategies for Interdicting Nodes 42 and 60 using Model 1

TEAMS USED	ARCS WHICH DETERMINE FLOW ($\beta_{ij} = 1$)	INTERDICTED ARCS ($\delta_{ij} = 1$)	FLOW
3	(6,9)	(85,60) (85,86) (93,94)	12
4	(6,9)	(85,60) (85,86) (93,94)	12
5	(6,9)	(85,60) (85,86) (93,94)	12
6	(6,9)	(85,60) (85,86) (93,94)	12
7	(84,85) (98,97)	(3,6)	10
8	(85,86) (98,97)	(3,6) (85,60)	8
9	(98,97)	(3,6) (85,60) (85,86)	4
10	(90,91)	(26,42) (39,60) (85,60) (86,60) (94,93)	2

Unless more than 6 teams are used to interdict flow to nodes 42 and 60, the flow will not be decreased. When more than ten teams are used, node 60 is isolated by the same zero-flow cut-set determined earlier. When 12 teams are used, the flow to both nodes 42 and 60 is reduced to zero, and each node is isolated by interdicting the zero-flow cut-sets found earlier.

5. Model 1 Insights

The model can be useful in determining the potential for movement of precursor chemicals on the existing network and the marginal change in flow based on the number of teams used. The marginal decrease in the maximum flow is large when relatively few teams are committed, and operations are less localized. But when there are sufficient assets available, the model isolates nodes independently. If the information on the demand node location is accurate, the localized solutions may yield the best solutions for minimizing flow to the demand node with the fewest teams. If the demand node location is uncertain, the localized solution may not interdict flow to the true demand node.

When multiple demand nodes are considered, the maximum flow to the combined demand nodes will be less than or equal to the sum of the maximum flows for each individual node. This is due to the nature of the network, where an arc that provides flow to one node may also provide flow to other nodes. In this case, multiple demand nodes could compete for flow. Interdiction of an arc used jointly will tend to decrease flow to both nodes. This will tend to increase the effectiveness of interdiction efforts by further restricting the flow of already limited supplies.

C. ANALYSIS OF MODEL 2

1. Scenario

The mission planner has 11 interdiction teams available for use in operations against suspected drug production sites. Intelligence indicates that there is a drug production site, or demand node, near node 42. The planner is now faced with the task of determining the optimal strategy in placing his teams to isolate node 42 and as many additional nodes around it as possible. Since the number of teams is limited, the planner is interested in the marginal increase or decrease in the number of nodes isolated if more or less teams are used. By keeping some teams back from the operation, the planner has more flexibility to respond to information about any other demand locations. The planner is also interested if the strategy developed for node 42 remains optimal if the demand node was really located at another node in the vicinity of node 42.

2. Optimal Strategy

Setting $\alpha_{42} = 1$ in the model to indicate the demand node, the basic, unweighted model can be solved. The solution from the model using all 11 teams is shown in Figure 5. The solution appears reasonable and the nodes are contiguous. To look at different solutions based on the number of teams, the planner runs the model for different allocation of teams. The Table 7 shows the nodes isolated by the optimal strategy for the indicated number of teams.

TABLE 7
Strategies to Isolate Node 42 using Model 2

TEAMS USED	INTERDICTED ARCS ($\delta_{ij} = 1$)	NODES ISOLATED ($\alpha_i = 1$)	TOTAL NODES
12	(3,6) (85,60) (85,86) (98,97)	6,9,10,15,16,17,24,25,26,27,39,40,41,42, 43,44,45,60,61,62,63,64,65,66,67,68,69, 86,87,88,89,90,91,92,93,94,95,96,97	38
11	(9,16) (40,62) (60,86) (85,86) (94,93)	16,25,26,41,42,43,62,63,64,86,87,88,89, 90,91,92,93	18
10	(9,16) (88,89) (94,93)	16,25,26,41,42,43,63,64,65,89,90,91,92, 93	14
9	(16,26) (90,91) (94,93)	26,42,43,64,65,91,92,93	8
8	(16,26) (90,91) (94,93)	26,42,43,64,65,91,92,93	8
7	INFEASIBLE	INFEASIBLE	NA

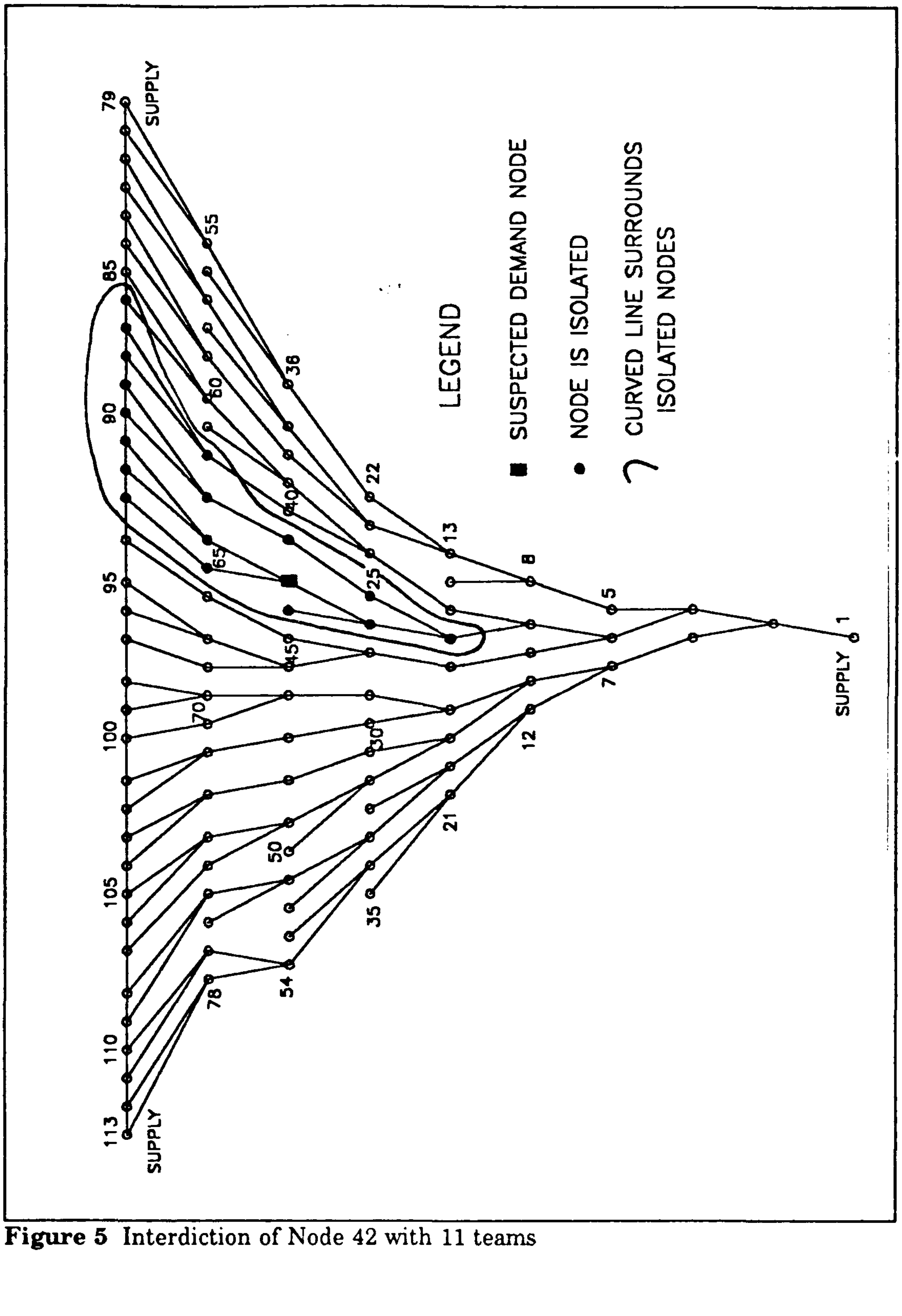

Figure 5 Interdiction of Node 42 with 11 teams

The results indicate that the planner could divert several teams from the current mission and still isolate node 42 and the nodes in the immediate vicinity. There is also no benefit in placing 9 teams instead of 8 teams, because the additional team is does not change the optimal strategy.

Based on the reliability of the information that identifies node 42 as a demand node, the planner may also want to consider that the true demand node is in the vicinity of node 42, either node 26, 41, 43, 64, or 65. By setting the number of teams to 11 and changing the demand node location, it is possible to determine the optimal strategy for each different node. In this case, the resulting optimal strategy is the same for each node listed above. With the information from the model, the planner can continue planning the mission, choosing from several alternate strategies.

3. Model 2 Insights

It is possible to draw insights from this model that can guide operational planning. In almost all cases, the optimal solution includes placing interdiction teams along the road rather than on the river segments which are incident to the road. By interdicting the road in two places, it is possible to stop flow along a significant number of river segments. Additionally, the cumulative cost of cutting each individual river segment incident to the road between two road nodes is greater than cutting the road twice. While this may be a function of the interdiction costs assigned to the arcs involved, it follows that it would be more efficient to interdict one large shipment than an equal quantity of chemicals broken down into smaller shipments. Another insight is that the placement of interdiction teams downstream from the demand node is limited to one major arc, which in conjunction with cutting the road, isolates a branch of the river. As fewer teams are available, the size of the branch decreases. Taken to an extreme with unlimited teams available, the optimal solution would be to cut all the arcs coming out of supply nodes.

IV. CONCLUSIONS AND RECOMMENDATIONS

A. BENEFITS OF THE MODELS

This thesis has developed mathematical programming models to solve the difficult problem of allocating interdiction assets to arcs in a network, in one case to minimize the maximum flow and in the other case to surround and isolate a target demand node. The formulations, as integer programs, appear easy to solve in practice and any number of solvers could be used, as opposed to the specialized algorithms described in earlier papers. Furthermore, the formulations are generalizable and allow easy comparison of various scenarios. (Generalizations and large scenarios, might, however, yield problems which are not so easy to solve.)

By changing the number of teams or the location of demand nodes, it is possible to determine the impact of change. In the first model, the addition of one more interdiction team to the effort can be weighed against the marginal change in the flow. The decision-maker can then determine if the resulting decrease in flow is worth the cost of committing that asset. Comparison of scenarios in Model 2 provides the planner with the capability to determine the minimum number of interdiction assets required to isolate the demand node. If sufficient assets are not available, the planner may then need to modify the mission or request more assets. However, if sufficient assets are available, the planner may be able to isolate the demand node with fewer assets than originally allocated to the mission. If the planner is familiar with the area of operations, he will be able to determine which allocation of assets is sufficient to cover a reasonable area around the suspected demand node.

Model 2 also allows the planner the flexibility of using a weighting scheme on the nodes surrounding the suspected demand node. This could be used if the unweighted model solution isolates a set of nodes which are not contiguous with the demand node.

B. ENHANCEMENTS TO THE MODEL

One might explicitly consider the probability of successfully interdicting a shipment of precursor chemicals that passes through an interdicted road or river segment. For instance, consider river traffic which is the primary means of transportation in the region. The river traffic includes large barges loaded with large, bulky items such as logs or 55-gallon drums. Interdiction teams could have a difficult time locating precursor chemicals if the chemicals are hidden in or around the cargo, or in false compartments on the vessel. Additionally, if traffic along a road or river is heavy, the interdiction teams may not be able to check each vehicle or vessel, but only a certain percentage.

Neither model incorporates time, which plays a major role in the success of interdiction operations. If interdiction teams stay stationary too long, they can become ineffective since the network user will find bypasses around the teams. An improved model would yield interdiction plans which are randomized over time.

An ideal model would combine attributes from both the models presented here, maximizing the nodes interdicted around a suspected demand node and minimizing the flow to that node. However, because the two models have different purposes, combining them would be difficult.

C. OTHER CONSIDERATIONS

The most critical aspect of using the models is the quality of the intelligence gathered. Multiple sources may provide contradictory intelligence. Who and what to believe is difficult to answer, and over time could change. Focusing collection assets to confirm or deny intelligence reports may mean the difference between directing interdiction efforts against an arc that is not used or not interdicting an arc which is used.

In order to use the models presented in this thesis, the network must be transformed into a set of arcs and nodes, with assigned arc capacities and interdiction costs. For the precursor chemical interdiction problem, the transportation network is quite large and changes occur seasonally. Because of this, the true network may not be known. However, a totally accurate depiction of the network might not be required and river segments could be aggregated to form a less extensive network. It would, however, be difficult to accurately assign values for the attributes to each segment and to interdict such. These values could be based on a best guess by a subject-matter expert and as interdiction operations progress and data is collected on effectiveness, the values could be updated.

A noticeable shortcoming of interdiction operations is the lack of well-defined, measurable Measure of Effectiveness (MOE). Effectiveness can be measured in terms of quantity seized, but it is difficult to estimate the quantity of goods that were not transported or diverted because of interdiction efforts. Other MOEs can be the percentage of time interdiction teams are operating or the number of contacts that result in searches. Regardless of the MOE chosen, the true effectiveness of interdiction operations will not be known unless the interdicted party provides the information.

Two other considerations involve the interruption of legitimate transportation of goods. The interdiction teams need a method of rapidly identifying liquids that are found during searches. Liquids may be found in unmarked or mislabeled containers. In either case, the liquid may have a legitimate purpose in the area to which it is being transported. If legitimate shipments are seized until positive identification is made, legal trade is disrupted. Similarly, if the local populace who rely on the waterways and roads for daily transportation are stopped and searched every time they use the transportation system, their support for the counter-narcotic efforts may decrease.

D. CONCLUSIONS

The models developed in this thesis could provide analytical tools to the Drug Enforcement Agency agents who plan riverine and ground interdiction operations in the Mamoré river basin of Bolivia which could help them position their limited interdiction assets on the waterways and roads to most effectively stem the flow of precursor chemicals. The models are easily transportable and can be solved on a 486 MS-DOS personal computer with commercial solvers. GAMS was used to formulate the models because of availability and ease of programming. GAMS/ZOOM was unable to consistently provide solutions for Model 1, but was able to solve Model 2. XA was able to solve both models more rapidly and always provided a solution. Therefore, XA is recommended over GAMS/ZOOM as the solver.

The examples in Chapter III show some of the possible uses of the models. While this thesis is just a small part of the overall counter-narcotic, drug interdiction effort, implementation of the models might improve the effectiveness of interdiction operations, resulting in moving us one step closer to a solution to our drug problem.

APPENDIX A NETWORK GENERATION

IJEDGES DATA - LISTING OF ARCS IN THE NETWORK

/

(1. 2) = 1 , (2. 1) = 1
(2. 3) = 1 , (3. 2) = 1
(2. 4) = 1 , (4. 2) = 1
(3. 5) = 1 , (5. 3) = 1
(3. 6) = 1 , (6. 3) = 1
(4. 7) = 1 , (7. 4) = 1
(5. 8) = 1 , (8. 5) = 1
(6. 9) = 1 , (9. 6) = 1
(6. 10) = 1 , (10. 6) = 1
(7. 11) = 1 , (11. 7) = 1
(7. 12) = 1 , (12. 7) = 1
(8. 13) = 1 , (13. 8) = 1
(8. 14) = 1 , (14. 8) = 1
(9. 15) = 1 , (15. 9) = 1
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(11. 18) = 11 , (18. 11) = 11	(41. 63) = 14 , (63. 41) = 14
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(18. 28) = 12 , (28. 18) = 12	(49. 73) = 10 , (73. 49) = 10
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(21. 34) = 10 , (34. 21) = 10	(55. 79) = 9 , (79. 55) = 9
(21. 35) = 9 , (35. 21) = 9	(55. 80) = 11 , (80. 55) = 11
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(23. 37) = 10 , (37. 23) = 10	(57. 82) = 8 , (82. 57) = 8
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/;

NODES DATA - TOTAL NUMBER OF NODES IN NETWORK

```
I NETWORK NODES  / 1 *  113 /;
```

SUPPLY NODES - INDICATES WHICH NODES ARE SUPPLY NODES

```
ALPHA.FX('1' ) = 0;
ALPHA.FX('79' ) = 0;
ALPHA.FX('113' ) = 0;
```

APPENDIX B MODEL 1 FORMULATION

MODEL 1 - MINIMIZATION OF MAXIMUM FLOW

```
$TITLE    The Network Interdiction Model
$STITLE        Minimize the flow
*
*    WRITTEN BY :  Robert L. Steinrauf
*                  SMC 2862, NPS
*                  Monterey, CA  93943
*                  (408) 649-1063
*
*--------------GAMS OPTIONS and DOLLAR CONTROL OPTIONS----------------

$OFFUPPER OFFSYMXREF OFFSYMLIST

 OPTIONS LIMCOL = 0, LIMROW = 0, SOLPRINT = OFF;
 OPTIONS RESLIM = 100, ITERLIM = 10000 ;
 OPTIONS OPTCR = 0.001;
*-----Definitions and Data----------------------------------------

  SET
$  INCLUDE NODES.DAT

   ALIAS (I,J);

  PARAMETERS
    ARC(I,J)

$   INCLUDE IJEDGES.DAT

  PARAMETER
    COST(I,J)
$   INCLUDE INTCOST.DAT

  PARAMETER
    UFLOW(I,J)
$   INCLUDE FLOW.DAT

  SCALAR
   TEAMS   number of interdiction teams avaible / 2/;

*-----------------Model------------------------------------------
```

```
VARIABLES
   TFLOW            'total flow to demand node';

BINARY VARIABLE
   ALPHA(I)         '1 indicates node is in T'
   DELTA(I,J)       'arc (I,J) is interdicted ';

POSITIVE VARIABLE
   BETA(I,J)        'arc (I,J) is in cutset, but not isolated' ;

   BETA.UP(I,J) = 1 ;

EQUATIONS
  OBJFLOW           'total flow through network'
  CUTSET(I,J)       'determines cutset'
  TEAMTOT           'constraint on total number of teams' ;

* minimize
  OBJFLOW..    TFLOW =E= SUM((I,J), UFLOW(I,J)*BETA(I,J)) ;

* subject to
  CUTSET(I,J)$(ARC(I,J))..
         ALPHA(I) - ALPHA(J) + BETA(I,J) + DELTA(I,J) =G= 0;

  TEAMTOT..
         SUM((I,J), DELTA(I,J) * COST(I,J)) =L= TEAMS;

 MODEL MINFLOW /OBJFLOW, CUTSET, TEAMTOT/;

$  INCLUDE ALPHA.SUP

  ALPHA.FX('42') = 1;
  ALPHA.FX('60') = 1;
  ALPHA.FX('69') = 1;

 SOLVE MINFLOW USING  MIP MINIMIZING TFLOW;

*------------Reports--------------------------------------------
 PARAMETER REPORT(*,*) Number of teams employed;
 REPORT(I,J) = DELTA.L(I,J) * COST(I,J);
 DISPLAY TFLOW.L;
 DISPLAY DELTA.L;
 DISPLAY ALPHA.L;
 DISPLAY BETA.L;
 DISPLAY REPORT;
```

APPENDIX C MODEL 2 FORMULATION

MODEL 2 - MAXIMUM SET OF ISOLATED NODES

```
$TITLE    The Network Interdiction Model
$STITLE    Maximize the area isolated
*
*    WRITTEN BY : Robert L. Steinrauf
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*                 Monterey, CA  93943
*                 (408) 649-1063
*
*-------------GAMS OPTIONS and DOLLAR CONTROL OPTIONS---------------

$OFFUPPER OFFSYMXREF OFFSYMLIST

 OPTIONS LIMCOL = 0, LIMROW = 0, SOLPRINT = OFF ;
 OPTIONS RESLIM = 500, ITERLIM = 10000 ;
 OPTIONS OPTCR = 0.01, INTEGER1 = 122 ;
 OPTIONS mip =xa   ;

*-----Definitions and Data--------------------------------------

  SET
$  INCLUDE NODES.dat

   ALIAS (I,J);

  PARAMETER
   ARC(I,J)

$  INCLUDE IJEDGES.dat

  PARAMETER
   COST(I,J)
$  INCLUDE INTCOST.dat

  SCALAR
   TEAMS   number of interdiction teams avaible / 11/;

*-----------------Model------------------------------------------
```

```
VARIABLES
   TNODE              'total nodes isolated' ;

BINARY VARIABLE
   ALPHA(I)           '1 node is isolated' ;

POSITIVE VARIABLE
   DELTA(I,J)         'arc (I,J) is interdicted ';

   DELTA.UP(I,J) $ ARC(I,J)  =  1;

EQUATIONS
  OBJNODE             'total nodes isolated in network'
  CUTSET(I,J)         'determines cutset'
  TEAMTOT             'constraint on total number of teams'  ;

* maximize
  OBJNODE..   TNODE =E= SUM(J,ALPHA(J));

* subject to
  CUTSET(I,J)$(ARC(I,J))..
            ALPHA(I) - ALPHA(J) + DELTA(I,J) =G= 0;

  TEAMTOT..
           SUM((I,J), DELTA(I,J) * COST (I,J)) =L= TEAMS;

 MODEL MAXNODE /OBJNODE, CUTSET, TEAMTOT /;

$  INCLUDE alpha.sup
  ALPHA.FX('42') = 1;

 SOLVE MAXNODE USING MIP MAXIMIZING TNODE;

*------------Reports-------------------------------------------

 PARAMETER REPORT(*,*) Number of teams employed;
 REPORT(I,J) = DELTA.L(I,J) * COST(I,J);
 option alpha:0:0:1 ;
 option delta:3:0:1 ;

 DISPLAY TNODE.L;
 DISPLAY ALPHA.L;
 DISPLAY DELTA.L;
 DISPLAY REPORT;
```

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