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SENSITIVITY ANALYSIS OF THE MODERN NAVAL COMBAT MODEL

by

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Sensitivity Analysis of the Modern
Naval Combat Model

by

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ABSTRACT

This thesis describes, extends, and explores the validity of Hatzopoulos Naval Combat Model of modern surface warship missile engagements.

An extensive sensitivity analysis is conducted to determine how the model's output is affected by changes in force alertness and scouting effectiveness. The approach taken is to analyze the sensitivity of combat (missile exchange) results first through the use of ratios, and second by examining partial derivatives.

Two ratios are developed. The first is a ratio of remaining staying power after the exchange of salvoes. The second is a fractional exchange ratio, which compares the fraction of combat power remaining on the two sides after an exchange.

The robustness of the fractional exchange ratio as an indicator of success in naval salvo warfare was demonstrated.

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Finally, I am grateful to my wife who provided not only support but also assisted in producing the big tables at the end of the thesis.

I dedicate this thesis to my lovely wife Poly, my son Panos, and my daughter Joanna.

I. INTRODUCTION

A. BACKGROUND

A mathematical model is a mathematical construct which is designed to study a particular real-world system or phenomenon. According to Giordano and Weir, a mathematical model can be a formula, an equation, or a system of equations that describes how the underlying factors are interrelated [Ref. 1:p. 32]. The purpose of a model is to describe, explain, or predict. Models also are used to carry out sensitivity analyses, helping to provide a starting point for making decisions.

The goodness of a model depends on how well it succeeds in its intended purpose. The closer it approximates the real-world situation it represents, the greater is its value. As noted by Hughes, "A model is useful if a better decision can be made with the information that it adds" [Ref. 2:p. 17].

A military model is a special type of model. Such models must be able to represent complicated scenarios, must be simple to activate and use, and must produce reasonable results. A naval battle model is one type of military model. Its purpose is to help the tactical commander in thinking about how best to apply his forces to win a naval engagement (or at least minimize the losses). A naval battle model must be characterized by simple measures of the aggregate combat power and staying power of the opposite forces. In addition to mathematical models, models for naval battle planning include fleet exercises, interactive war games, and computer simulations [Ref. 2:p. 165].

B. A NAVAL COMBAT MODEL

One comprehensive mathematical model of naval combat was developed by Lt. Thomas Beall, based on Hughes's naval warfare concept [Ref. 3:p. 17]. Beall's model provides an excellent example of a useful military model designed specifically for naval combat. We now summarize his model. The material in this section is a summary of the work done by Lt. Thomas Beall [Ref. 3].

1. Definitions

The following definitions are used by Beall in defining his model:

- <u>Firepower kill</u>: A platform has suffered a <u>firepower kill</u> if its combat power diminishes to zero, so it cannot contribute combat power to its force.

- <u>1000 - pound bomb equivalent (TPBE)</u>: TPBE is equal to the explosive power of 660 pounds of TNT and is a unit of destruction (that is, the explosive power of a 1000-pound bomb in World War II). The explosive power of all weapons is expressed in multiples of TPBE.

- Staying power (SP): The staying power of a platform is the number of TPBE hits necessary to inflict a firepower kill on that platform.

- Weapon effectiveness (PC): Weapon effectiveness is the probability that a single shell fired from a group's main battery gun will strike the target.

- Theoretical combat power (FC): The theoretical combat power of a given weapon type is the number of TPBEs per minute which a platform can fire in a single salvo.

- Effective combat power (EFC): Effective combat power is the number of TPBEs per minute fired from a group's main battery guns which strike their targets.

- Indices used in the model are as follows:

  i = Weapon

  j = Platform (ship)

  k = Group

  l = Blue force

  l' = Red force.

2. Characteristic Values for a Platform

An important respect of Beall's thesis is that he used historical combat data to determine the values to use in his model.

a. Staying power. The staying power (SP) of platform j in group k of force l is computed as a function of its full load displacement. This is a characteristic of each platform (ship):

$$SP_{jkl} = 0.07 \times (\,\textit{full load displacement}\,)^{1/3} \qquad (1.1)$$

b. Theoretical combat power. The theoretical combat power (FC) is the number of TPBEs fired per minute by weapon i of platform j in group k of force l:

$$FC_{ijkl} = \frac{weight}{660\ lbs} \times wtg, \qquad (1.2)$$

where : weight = Explosive weight which the weapon fires per per minute in pounds of TNT,

wtg = 2.5, for gunnery ordnance.

The theoretical combat power of a platform j in group k of force l is given by summing the theoretical combat power of each individual weapon of the platform.

$$FC_{jkl} = \sum_{i \in j} FC_{ijkl}. \qquad (1.3)$$

The aggregate staying power (SP) and the theoretical combat power (FC) of a group k in force l, calculated as a single unit, are given by:

$$SP_{kl} = \sum_{j \in k} SP_{jkl} \qquad \forall k\ ,\ \forall l \qquad (1.4)$$

and

$$FC_{kl} = \sum_{j \in k} FC_{jkl} \qquad \forall k\ ,\ \forall l. \qquad (1.5)$$

c. Effective combat power. The effective combat power (EFC) of a group k in force l is computed as follows:

$$EFC_{kl} = FC_{kl} \times PC_{kl}. \tag{1.6}$$

3. Model Description

The terms $SP_{kl}(t)$ and $FC_{kl}(t)$ represent the aggregate staying power and theoretical combat power of a group k in force l at time step t. If the force l' is the attacking force, the aggregate staying power (TS(t)) of the group under attack and the aggregate effective combat power (AEFC(t)) of the attacking group are as follows:

$$TS(t) = \sum_{k \text{ being attacked by } l'} SP_{kl}(t-1), \tag{1.7}$$

and

$$AEFC(t) = \sum_{k \text{ firing } l'} FC_{kl'}(t-1) \times PC_{kl'} \tag{1.8}$$

where: $SP_{kl}(t\text{-}1)$ = Staying power of group k of force l at the end of time step (t-1).

$FC_{kl'}(t\text{-}1)$ = Theoretical combat power of group k of force l' at the end of time step (t-1).

The defender's continuous fire loss percentage (LC) is computed as the ratio of AEFC to TS:

$$LC = \frac{AEFC}{TS} . \tag{1.9}$$

Therefore, the staying power (SP) and the theoretical combat power (FC) can be computed for each iterative time step as follows:

$$SP_{kl}(t) = \begin{cases} SP_{kl}(t-1) \times (1-LC) & \forall k \text{ under attack} \\ SP_{kl}(t-1) & \text{otherwise ,} \end{cases} \tag{1.10}$$

and

$$FC_{kl}(t) = \begin{cases} FC_{kl}(t-1) \times (1-LC) & \forall k \text{ under attack} \\ FC_{kl}(t-1) & \text{otherwise .} \end{cases} \tag{1.11}$$

The total values of each force at all discrete time steps t can be used to represent the aggregate staying power (SP) and the theoretical combat power (FC):

$$SP_1(t) = \sum_k SP_{k1}(t) , \tag{1.12}$$

and

$$FC_1(t) = \sum_k FC_{k1}(t) . \tag{1.13}$$

C. HUMAN FACTORS IN A NAVAL COMBAT MODEL

Although Beall's model is both comprehensive and useful, it does not include either human-related combat factors or modern missiles. In 1990 Lt. Epaminondas Hatzopoulos developed a Modern Naval Combat Model based on Beall's model, but including various human-related factors such as scouting effectiveness, training, morale, and leadership [Ref. 4:p. 49]. Hatzopoulos's model extends Beall's model in four ways:

- It includes missiles, the most effective weapon of today's naval battles.
- It takes into account the defensive ability of both forces.

- It incorporates scouting effectiveness and alertness in defense for both opponents.
- It incorporated several important human factors that affect the outcome of a battle. These factors are discussed below.

1. **Scouting Effectiveness**

Scouting has played an important role in naval history from the earliest times of sailing ships to the present time of missile warships. Scouting is the gathering of useful combat information, such as the precise position of the enemy and his combat capabilities. Good scouting can result in victory for an otherwise inferior fleet. One aim of this thesis will be to determine under what battle conditions a scouting advantage will win.

Scouting provides a distinct advantage to the force which is most effective at it. On the other hand, scouting may also reduce the number of forces which can be drawn upon for firepower if they are engaged in reconnaissance activities.

2. **Training and Experience**

Well-trained troops perform better in difficult situations. If a large differential in training and experience exists between two opposing sides, battle outcome may be determined by this factor.

In his book <u>Fleet Tactics</u>, Hughes includes an observation from Aristotle's <u>Ethics</u> which emphasizes the importance of training: "We learn how to do things by doing the things we are learning how to do" [Ref. 5:p. 49].

All military personnel must be expertly trained in their domains. Training must begin in peacetime and continue until the time comes in battle when what has been learned is used. This is especially true aboard a warship, where each person works individually, yet all those in a group must know what to do and when to do it, after battle begins.

3. **Morale**

According to Watson, several factors affect morale during combat. These are summarized as follows:

- <u>The results of the first encounter</u>. If the first battle has been fought and won, this successful encounter helps morale rise.
- <u>The emotional support provided by informal leaders</u> (those who "take charge," whether or not they have formal authority).
- <u>The number of casualties incurred</u>. Reducing physical casualties helps greatly in maintaining high morale.
- <u>The cohesiveness of the group</u>. Morale is much higher if personnel are trained in small groups and kept together all the time. These "teams" have better <u>esprit de corps</u>. [Ref. 6:p. 231]

It is difficult to quantify morale. However, Dupuy has proposed a set of numerical values for five levels of morale [Ref. 7:p. 231]. These are shown in Table I.

TABLE I. DUPUY'S QUANTIFICATION OF MORALE LEVELS

Level of Morale	Assigned Value
Excellent Morale	1.0
Good Morale	0.9
Fair Morale	0.8
Poor Morale	0.7
Panic	0.2

4. Leadership

The United States Army Field Manual 22-100 states that "leadership traits are distinguishing personality qualities which, if demonstrated in daily activities, help the commander to earn the respect, confidence, willing obedience, and loyal cooperation of the men" [Ref. 8:p. 8]. Thus leadership is a phenomenon comprising many factors. The way a leader is perceived is a function of human temperament, group dynamics, and the situation. Hatzopoulos showed, through his equations, the *manner* in which leadership plays its role in combat, and in this way offers hope that the *value* of good leadership can be quantified [Ref. 4].

D. MODEL VALIDATION

Most military models have a credibility problem. This stems from difficulties in validating them. Determining what is to be the standard of reality, and how to measure it, is perhaps the most difficult problem encountered in the validation process.

Two kinds of real-world data are currently available to use for validating a battle model: training exercise results, and the results of historical naval battles. Exercise results are usually complete, specific, and can provide fairly accurate and numerous data. However, they are only as valid as the assumptions made in designing, planning, and carrying out the exercise. Actual historical results have more validity, but reliable data is difficult to acquire (especially about the enemy) and often very difficult interpret: wartime data is "dirty data." [Ref. 2:p. 293]

Hatzopoulos's model appears to be a reasonable one. However, as Hatzopoulos points out, it must be validated before its usefulness can be judged [Ref. 4:p. 84]. He proposes two ways his model might be validated:

- Perform an extensive sensitivity analysis on the model to determine how sensitive the outcomes predicted by the model are to the model parameters.
- Analyze the data from a small number of existing historical missile or pulse naval battles using the model. The validity of the model will be assessed and a better sense of appropriate values for some of the model parameters obtained.

E. THESIS GOAL AND SCOPE

The goal of this thesis is to initiate the validation of Hatzopoulos's Modern Naval Combat Model by carrying out an extensive sensitivity analysis. This validation has been accomplished in two steps.

First, the model was used to analyze the hypothetical data from several missile and pulse weapon naval battles. Model results were then examined from the standpoint of "reasonableness."

Second, sensitivity analyses were carried out to determine how sensitive the model's output is to changes in the inputs for various model parameters. The term sensitivity, as noted by Giordano and Weir, refers to the degree of change in a model's conclusions as some condition upon which they depend is varied; the greater the change, the more sensitive is the model to that condition [Ref. 1:p. 40].

Chapter II summarizes Hatzopoulos's Modern Naval Combat Model and its origins. The major equations and submodels are described to provide clear understanding of how they are to be interpreted and how they interact.

Chapter III describes a measure of combat effectiveness developed by Barr, Weir, and Hoffman, as described in their paper, "Evaluation of Combat." They refer to their measure as the "battle trace," and it is based on the Lanchester family of models.

Chapter IV reports tests done to determine how sensitive Hatzopoulos' Modern Naval Combat Model is to changes in input parameters (scouting effectiveness and troop alertness). Conclusions and results are provided in Chapter V.

The scope of this study is limited as noted above. Other possible validation techniques based on exercise results are not used. Historical data drawn from real combat would undoubtedly be the most powerful information for validating a battle model.

II. MODERN NAVAL COMBAT MODEL: DESCRIPTION AND IMPLEMENTATION

A. INTRODUCTION

This chapter summarizes and discusses the Modern Naval Combat Model developed by Hatzopoulos. The model represents missile combat between ships, using typical surface-to-surface antiship missiles [Ref. 4]. The model also includes the effects of certain human factors issues such as scouting effectiveness, training, morale, and leadership. The material in this section is a summary extraction of the work due to Lt. Epaminondas Hatzopoulos [Ref. 4].

B. MODEL DESCRIPTION

1. Definitions

- Firepower kill: A platform has suffered a firepower kill if its combat power diminishes to zero, so it cannot contribute combat power to its force.
- 1000 - pound bomb equivalent (TPBE): TPBE is equal to the explosive power of 660 pounds of TNT and is a unit of destruction (that is, the explosive power of a 1000-pound bomb in World War II). The explosive power of all weapons is expressed in multiples of TPBE.

- Staying power (SP): The staying power of a platform is the number of TPBE hits necessary to inflict a firepower kill on the platform.

- Weapon effectiveness (PC): Weapon effectiveness is the probability that a single shell fired from a group's main battery gun will strike the target.

- Theoretical combat power (P): The theoretical combat power of a given weapon type is the number of TPBEs per minute which a platform can fire in a single salvo.

- Effective combat power (E): Effective combat power is the number of TPBEs per minute fired from a group's main battery guns which strike their targets.

- Indices

  j = Platform of the Blue force,

  j' = Platform of the Red force,

  k = Group of platforms constituting the Blue force,

  k' = Group of platforms constituting the Red force,

  b = Blue force,

  r = Red force.

2. Computation of Individual Platform Values

a. Staying power. The staying power (SP) is the number of TPBE hits a platform can absorb before suffering a firepower kill. Staying power values used for Beall's model are drawn from World Wars I and II. Newer data values are not generally available, especially for missiles. Thus Hatzopoulos uses Beall's formula for his approximation and the nominal missile used for this model is assumed to have a destructive value of one TPBE. The staying power of platform j in group k for the Blue force is given by the following formula:

$$SP_{jkb} = 0.070 \times (\mathit{full\ load\ displacement})^{1/3}. \qquad (2.1)$$

b. Theoretical combat power. The theoretical combat power (P) is the number of missiles that can be fired from a unit in a single salvo. The theoretical combat power of unit j in group k for the Blue force against Red force is given by the following formula:

$$P_{jkb} = M_{jkb} \times W_m \qquad (2.2)$$

where : M_{jkb} = Theoretical number of a standard or nominal missiles that a unit j in group k in the Blue force can fire in a single salvo.

W_m = A multiplicative factor to be used for missiles all based on approximately the same technology, to account for different weights of explosive material.

If one side uses a missile with twice as much explosive material as the nominal missile, then W_m is 2.0, so that side has double the theoretical combat power. The multiplier W_m can be ignored if both sides use a missile roughly equivalent to the nominal one-TPBE missile.

c. Effective combat power. The effective combat power (E) or combat effectiveness is the number of missiles that hit their target per salvo. The effective combat power of platform j' in group k' of the Red force is given by:

$$E_{j'k'r} = M_{j'k'b} \times W_m \times PR_{j'k'r} \tag{2.3}$$

where : $PR_{j'k'r}$ = The probability that a missile fired from unit j' in group k' of the Red force hits its target.

The value of PR can be calculated as follows:

$$PR_{j'k'r} = H - \left(H \times \frac{N_{jkb}}{M_{j'k'r}}\right) \tag{2.4}$$

where : H = Firing accuracy, given for each type of missile. For the same type of missile, H is the same for all units in the force.

N_{jkb} = Number of missiles which the j platform in k group of the Blue forces can shoot down per salvo (the best that can be done).

Substituting Equation (2.4) in Equation (2.3), the effective combat power (E) of platform j' of group k' of the Red force can be written as:

$$E_{j'k'r} = (M_{j'k'r} \times W_m \times H) - (N_{jkb} \times W_m \times H). \qquad (2.5)$$

Frequently the defender can determine which missiles are threats in modern naval missile combat. Then only the ones that will strike the defender are targeted to be shot down. In this case, which closely corresponds to the use of point-defense weapons, the attacker's firing accuracy H does not apply to the second term on the righthand side of Equation (2.5). The following modified equation is therefore used in the model, as closely satisfying the combat circumstance under study:

$$E_{j'k'r} = (M_{j'k'r} \times W_m \times H) - (N_{jkb} \times W_m). \qquad (2.6)$$

Equation (2.6) represents the effective combat power (E) of a single Red platform firing against a single defended Blue platform. This power is measured in hits inflicted on the Blue platform. Hatzopoulos notes that it also would be convenient to define the effective combat power of the attacking force in terms of the destroyed staying power of the defending platform. This is done by dividing Equation (2.6) by the staying power (SP) of defending (Blue) platform. The resulting fraction of the staying power destroyed is referred to as LOSS. If Red is attacking and Blue defending, the fraction of destroyed staying power of platform j in group k of the Blue force is as follows:

$$
\begin{aligned}
LOSS_{jkb} &= \frac{(M_{j'k'r} \times W_m \times H) - (N_{jkb} \times W_m)}{SP_{jkb}} \\
&= \frac{W_m}{SP_{jkb}} \times [(M_{j'k'r} \times H) - N_{jkb}] \qquad (2.7) \\
&= \frac{E_{j'k'r}}{SP_{jkb}} .
\end{aligned}
$$

The value of $LOSS_{jkb}$ must be between 0.0 and 1.0. If the value of LOSS is a negative number (the Blue platform can shoot down more missiles than the Red platform can fire in one salvo) and we set the Blue $LOSS_{jkb}$ equal to zero. When $LOSS_{jkb}$ has a value greater than 1.0, this means that the Red platform fired more missiles than needed to destroy completely all of the Blue platform's staying power.

3. Incorporation of Human Factors

a. Scouting and Alertness

As noted before, Hatzopoulos incorporates several human factors into his model. He defines σ to be the scouting function, representing the degree to which a force is able to gather useful information about the enemy. The function σ has values between 0.0 and 1.0 and is applied to the attacking force. For the defending force , a function τ is defined as the level of alertness, with values again ranging between 0.0 and 1.0. When these functions are included in Equation (2.7), it becomes:

$$LOSS_{jkb} = \frac{W_m}{SP_{jkb}} \times (\sigma_r \times M_{j'k'r} \times H - \tau_b \times N_{jkb}) \qquad (2.8)$$

where : σ_r = Scouting function of the attacking Red force.

τ_b = Alertness modifier for the defending Blue force.

Let us examine three extreme situations.

<u>Case 1</u>

σ_r = 1.0; the attacking force is fully informed of its opponent's posture.

τ_b = 1.0; the defending force is fully alert.

In this case Equation (2.8) reduces to Equation (2.7).

<u>Case 2</u>

σ_r = 0.0; the attacking force has no information about the enemy.

Therefore, there are no hits and the $LOSS_{jkb}$ is negative, becoming 0.0 by our convention.

<u>Case 3</u>

σ_r = 1.0; the attacking Red force is fully informed of its opponent's posture and ambushes the Blue force through the use of effective scouting.

τ_b = 0.0; the defending Blue force has no information about the enemy, so the Blue force's level of alertness is zero.

In this case Equation (2.8) becomes:

$$LOSS_{jkb} = \frac{W_m}{SP_{jkb}} \times (\sigma_r \times M_{j'k'b} \times H)\,. \tag{2.9}$$

b. Training, Morale, and Leadership

Hatzopoulos uses a multiplicative degrader **m** to introduce effects of training, morale, and leadership into the model. The factor **m** has values between 0.0 and 1.0, and is applied to the ability of the attacker to fire his missiles. Similarly, a factor **n** (again with value between 0.0 and 1.0) represents the ability of the defender to shoot down missiles, as this ability is influenced by training, morale, and leadership. When these further refinements are included, Equation (2.7) becomes:

$$LOSS_{jkb} = \frac{W_m}{SP_{jkb}} \times \left[(\sigma_r \times M_{j'k'r} \times m_{j'k'r} \times H) - (\tau_b \times N_{jkb} \times n_{jkb})\right]. \tag{2.10}$$

Hatzopoulos now computes the remaining staying power (SP) and theoretical combat power (P) of platform j in group k for the Blue force at the end of time step t. This is done using $LOSS_{jkb}$ as defined in Equation 2.10, yielding the results:

$$SP_{jkb}(t) = \begin{cases} SP_{jkb}(t-1) \times (1-LOSS_{jkb}(t)) & \forall j \text{ under attack} \\ SP_{jkb}(t-1) & \text{otherwise,} \end{cases} \tag{2.11}$$

$$P_{jkb}(t) = \begin{cases} P_{jkb}(t-1) \times (1-LOSS_{jkb}(t)) & \forall j \text{ under attack} \\ P_{jkb}(t-1) & \text{otherwise.} \end{cases} \tag{2.12}$$

When a unit suffers a hit, its ability to shoot down missiles (N) at time step t is reduced. The value of N(t) is updated as follows:

$$N_{jkb}(t) = \begin{cases} N_{jkb}(t-1) \times (1-LOSS_{jkb}(t)) & \forall j \text{ under attack} \\ N_{jkb}(t-1) & \text{otherwise.} \end{cases} \tag{2.13}$$

4. Aggregation of Units into Groups

The terms "group" refers to subdivision of a force and may consist of several units. Hatzopoulos next considers a group firing as a single unit. In his model the aggregate staying power (SP) of several platforms comprising group k of the attacking Red force is given by:

$$SP_{kb} = \sum_{j \in k} SP_{jkb} . \tag{2.14}$$

The aggregate fractional loss of group k of the defending Blue force is given by:

$$LOSS_{jkb} = \frac{\sigma_r \times H \times W_m \times \sum_{j' \text{ can be used}} M_{j'k'r} \times m_{j'k'r} - \tau_b \times W_m \times \sum_j N_{jkb} \times n_{jkb}}{SP_{kb}} . \tag{2.15}$$

When $SP_{kb}(t-1)$ is the staying power of group k of the Blue force at the end of time step (t-1), the aggregate staying power (SP) of all Blue groups under attack is given by:

$$SP_{kb}(t) = \sum_{k \text{ being attacked}} SP_{kb}(t-1) . \tag{2.16}$$

The aggregate theoretical combat power (P) of the attacking Red group is given by:

$$P_{k'r}\ (t) = \sum_{k'\ is\ firing} P_{k'r}\ (t-1)\ . \qquad (2.17)$$

The aggregate remaining staying power (SP) and theoretical combat power (P) are computed from the previous time step as follows:

$$SP_{kb}(t) = \begin{cases} SP_{kb}(t-1) \times (1-LOSS_{kb}(t)) & \forall k \ under\ attack \\ SP_{kb}(t-1) & otherwise. \end{cases} \qquad (2.18)$$

$$P_{kb}(t) = \begin{cases} P_{kb}(t-1) \times (1-LOSS_{kb}(t)) & \forall k \ under\ attack \\ P_{kb}(t-1) & otherwise. \end{cases} \qquad (2.19)$$

At the end of each discrete time step t we calculate the value of the unit's ability to shoot down missiles (N) as follows:

$$N_{kb}(t) = \begin{cases} N_{kb}(t-1) \times (1-LOSS_{kb}(t)) & \forall k \ under\ attck \\ N_{kb}(t-1) & otherwise. \end{cases} \qquad (2.20)$$

Finally, Hatzopoulos's model is used to compute the remaining total values for each force at every discrete time step t. The resulting values represent the aggregate remaining staying power (SP) and theoretical combat power (P). These final results are used to determine the outcome of this naval combat, and are given by the following formulas:

$$SP_b(t) = \sum_k SP_{kb}(t), \tag{2.21}$$

$$P_b(t) = \sum_k P_{kb}(t). \tag{2.22}$$

When an exchange of salvos occurs, Red against Blue and Blue against Red, there is a set of equations for SP_r, P_r, and N_r identical to Equations (2.21) and (2.22), derived by everywhere interchanging b and r.

C. MODEL IMPLEMENTATION

The Hatzopoulos Modern Naval Combat Model is implemented as a computer program coded in Fortran 77. Input parameters are provided by the user. Several assumptions are implicit in the model, as implemented in this thesis.

- Both forces have and use the same type of missiles.
- Each force consists of only one group.

- Each force receives one pulse (missile hit) during each discrete time step t. Both forces can fire simultaneously or one force can return fire after it has received its opponent's pulse.

The program computes and provides four kinds of outputs.

- Determination of whether the group on each side is hit by one pulse or not at all during each time step.

- The fractional losses of a particular group if it is hit, which determines the remaining staying power and new offensive and defensive theoretical combat power values.

- Total losses, remaining staying power, and theoretical combat power for both forces at the end of each discrete time step t.

- Cumulative remaining staying power and theoretical combat power for each force at the end of each time step, based on calculated values from the previous time step.

III. THE BATTLE TRACE MODEL

It is difficult to determine the effects of changes in weapons and tactics during a battle. This is due to variations in the battles themselves, which result in unstable measures of effectiveness. To overcome this problem, Barr, Weir, and Hoffman propose what they refer to as the "battle trace" as a dynamic measure of combat effectiveness. That is, the battle trace measures ongoing battle results, evaluated as a function of time into the battle [Ref. 9]. This section describes the battle trace and briefly explains how it is derived.

A. LANCHESTER SQUARE-LAW MODEL

Lanchester models are analytical models of battles in which the casualty process is envisioned as a continuous erosion of force levels on each side, due to fires from the opposing side [Ref. 9:p. 5]. Lanchester examined two general cases of combat, aimed fire and area fire [Ref.10]. Aimed fire, used by Barr, Weir, and Hoffman in their model, assumes that individual targets are identified and attacked by any number of opposing systems.

Lanchester hypothesized that combat between two homogeneous forces under the conditions of modern warfare could be modelled by a system of first order differential equations. These two equations are referred to as the Lanchester square-law model.

$$\frac{dx}{dt} = -ay, \quad \text{where} \quad a > 0, \text{ and } x(0) = x_0$$

and (3.1)

$$\frac{dy}{dt} = -bx, \quad \text{where} \quad b > 0, \text{ and } y(0) = y_0.$$

In the Lanchester system, X and Y refer to the two forces, and x(t) and y(t) tc the strengths of these forces at time t. The positive constants a and b in Equations (3.1) are called the attrition rate coefficients.

B. DEVELOPMENT OF THE BATTLE TRACE MODEL

For development of the battle trace model, several assumptions are made concerning force strengths [Ref.9:p. 9].

- The strength of a force is simply the number of combatants in operation at time t, for both forces.
- The functions x(t) and y(t) can take on real values (rather than simply non-negative integer values).
- The functions x(t) and y(t) are both continuously differentiable functions of t.

Taking the ratio of equations (3.1) results in the following equation.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{bx}{ay}. \tag{3.2}$$

Separating the variables in equation (3.2) we have:

$$ay\,dy = bx\,dx. \tag{3.3}$$

Equation (3.3) can be integrated from time 0 to time t. That is,

$$a\int_0^t y\,dy = b\int_0^t x\,dx. \tag{3.4}$$

This yields the result

$$a(y_t^2 - y_0^2) = b(x_t^2 - x_0^2). \tag{3.5}$$

in which $y_0 > y_t$ and $x_0 > x_t$.

Letting $c = ay_0^2 - bx_0^2$ we have the solution,

$$ay^2 - bx^2 = c, \tag{3.6}$$

where $y = y_t$ and $x = x_t$ depend on the time $t \geq 0$.

The trajectories for $c \neq 0$ are hyperbolas. When c = 0, the trajectory is the straight line $y = \sqrt{\frac{b}{a}}x$. When c > 0, the trajectory intersects the y-axis at $y = \sqrt{\frac{c}{a}}$. At that point the Y force wins because the X force has been totally eliminated. On the other hand, if c < 0, the X force wins with a final strength level of $x = \sqrt{-\frac{c}{b}}$.

Based on the above analysis, the necessary and sufficient condition for a force to win can be stated as:

$$\text{X wins if} \quad x_0 > y_0\sqrt{\frac{a}{b}}$$

and

$$\text{Y wins if} \quad x_0 < y_0\sqrt{\frac{a}{b}}.$$

The original Lanchester square-law equations (3.1) can be differentiated and the terms rearranged to obtain the second-order differential equation

$$\frac{d^2y}{dt^2} - aby = 0 . \tag{3.7}$$

Solving this equation using standard methods in differential equation yields:

$$y(t) = y_0 \cosh\sqrt{ab}\, t - x_0\sqrt{\frac{a}{b}} \sinh\sqrt{ab}\, t.$$

Thus it can be shown that:

$$\frac{y(t)}{y_0} = \cosh\sqrt{ab}\, t - \frac{x_0}{y_0}\sqrt{\frac{a}{b}} \sinh\sqrt{ab}\, t. \qquad (3.8)$$

Equation (3.8) implies that force Y's normalized force level, $\frac{y(t)}{y_0}$, depends on an engagement parameter

$$E = \frac{x_0}{y_0}\sqrt{\frac{a}{b}}, \qquad (3.9)$$

and a time parameter

$$T = \sqrt{ab}\, t. \qquad (3.10)$$

The constant $\sqrt{ab}$ represents the intensity of the battle and controls how quickly the battle is driven to conclusion. That is, the larger the value of $\sqrt{ab}$ the shorter is the length of the battle. The ratio $\frac{a}{b}$ represents the relative effectiveness of the individual combatants on the two opposing sides [Ref. 9:p. 14].

If the battle is occurring in discrete time steps, then at each time step t the fraction $\left(\frac{y(t)}{x(t)}\right)^2$ can be compared to the ratio $\frac{b}{a}$. This results in the following equation:

$$\left(\frac{x}{y}\right)^2\left(\frac{b}{a}\right) = R, \tag{3.11}$$

This observation suggests comparing $\frac{bx^2}{ay^2}$ against 1.

- If the ratio R is less than 1, the Y force is winning.
- If the ratio R is greater than 1, the X force is winning.
- If the ratio R is equal to 1, then there is a draw.

Since we do not generally know the values of a and b, it is worthwhile to examine the ratio further. It can be shown that:

$$\frac{bx^2}{ay^2} = \frac{(-bx)}{(-ay)}\cdot\frac{x}{y} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}\cdot\frac{x}{y} \approx \frac{\Delta y}{\Delta x}\cdot\frac{x}{y} = \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}}. \tag{3.12}$$

In Equation (3.12), Δy and Δx represent Y's and X's losses, respectively, during a time interval of Δt. The result of Equation (3.12) can be combined with (3.11) to give:

$$R = \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{X}} . \qquad (3.13)$$

The value R is called the Fractional Exchange Ratio.

In summary:

- If R < 1, the Y force is winning.
- If R > 1, the X force is winning.
- If R = 1, the battle is stalemated.

C. USE OF THE BATTLE TRACE MODEL

Barr, Weir, and Hoffman refer to the plot of R versus time t over the course of a given battle as the battle trace of the battle [Ref. 9:p. 18]. That is, for a battle with duration of 1 hour, the ratio R can be computed at, say, every Δt = 5 minutes, based on force attrition rates. The resulting plot dynamically shows the time regions where X is winning and those where Y is winning without specific knowledge of coefficients **a** and **b**, or assumptions that they are constants in time. Thus, instead of a single battle outcome it is possible to trace the ebb and flow of battle progress.

IV. ANALYSIS OF MODERN NAVAL COMBAT MODEL

The primary goal of this study is to determine how sensitive the Hatzopoulos Modern Naval Combat Model is to changes in input parameters. Of particular interest is the sensitivity of the model to two human-related factors: scouting effectiveness and troop alertness. The general approach taken here is to analyze the sensitivity through the use of ratios. Two ratios are developed for this purpose.

The first is a ratio of one force's <u>remaining staying power</u> to that of the other force, following each salvo. The second is a <u>fractional exchange ratio</u>. This fractional exchange ratio represents the proportion of each force that has been lost after an exchange of missiles. These two ratios are discussed below, along with the results of their use in determining model sensitivity to the factors of interest.

A. RATIO OF REMAINING STAYING POWER

1. Development of Ratio

Following each salvo both forces have a remaining staying power (see Equation 2.18) which can be calculated using the following equation:

$$SP_r(t) = SP_r(t-1) \times (1 - LOSS_r(t)). \qquad (4.1)$$

Where : $SP_r(t)$ = Remaining staying power of the Red force after time step t (which is equivalent to one discrete salvo exchanged between the two forces).

$LOSS_r(t)$ = Percentage loss of the Red force at the end of time step t.

Solving Equation (4.1) for $LOSS_r(t)$ gives the following expression.

$$LOSS_r(t) = -\frac{SP_r(t) - SP_r(t-1)}{SP_r(t-1)} = \frac{\Delta R}{R}. \qquad (4.2)$$

This represents the relative loss of the Red force during the time step t-1 to t. Similarly, the relative loss of the Blue force is:

$$LOSS_b(t) = -\frac{SP_b(t) - SP_b(t-1)}{SP_b(t-1)} = \frac{\Delta B}{B}. \qquad (4.3)$$

The ratio RR(t) of the Red force loss to that of the Blue force can now be defined. The resulting ratio RR(t) of remaining staying power is equivalent to the ratio used in the Weir, Barr, and Hoffman Battle Trace model (Chapter III):

$$RR(t) = \frac{LOSS_r(t)}{LOSS_b(t)} = \frac{\dfrac{SP_r(t) - SP_r(t-1)}{SP_r(t-1)}}{\dfrac{SP_b(t) - SP_b(t-1)}{SP_b(t-1)}} . \qquad (4.4)$$

Therefore, the following conclusions can be reached for the Hatzopoulos salvo model:

- If RR < 1, the Red force is winning.
- If RR > 1, the Blue force is winning.
- If RR = 1, the battle is a stalemate.

As may be observed, these are the same results as for the Battle Trace model in Equation (3.13).

2. Computer Implementation

The original Hatzopoulos computer program was modified to calculate the ratio RR(t), as shown in Equation (4.4), at each discrete time step t (see Appendix A). The program generates a plot of the remaining staying power ratio for each discrete time step t. Points on the line representing the ratio value 1.0 indicate time increments when both forces have the same remaining staying power and are stalemated (temporarily). Points below 1.0 represent times when the Red force is winning; points above 1.0 indicate when the Blue force is winning.

3. Testing Model Sensitivity to Scouting Effectiveness and Alertness

This program has been used to test the sensitivity of the Hatzopoulos model to the human factors of alertness and scouting. For the scenario used for these tests, both forces consist of the same number of fast frigates (FFs). All ships have exactly the same specifications and carry the same type of missiles. Values used for both forces in the sensitivity tests are presented in Table II. As may be observed, all of the constant model parameters are exactly the same for both forces. It is assumed that both ships always fire simultaneously.

TABLE II. CHARACTERISTICS OF RED AND BLUE SHIPS

Factor	Blue Force	Red Force
Number of Ships (B or R)	5	5
Full Load Displacement	4000	4000
Missiles per Salvo (M)	3	3
Multipl. Degrader for Missiles per Salvo (m)	0.65	0.65
Missiles Shot Down per Salvo (N)	2	2
Multipl. Degrader for Missiles Shot Down per Salvo (n)	0.60	0.60
Break Point	0.30	0.30
Probability of Hit (H)	0.80	0.80

This scenario is used to test the sensitivity of the Hatzopoulos model to changes in alertness and in scouting. This is done in a two-step process. In the first, scouting effectiveness values are held constant, while alertness values are varied. In the second, alertness values remain constant while scouting effectiveness values change.

Case 1

For this case, Red force scouting effectiveness σ_R is fixed at a value of 0.90. Blue force scouting effectiveness

σ_B is given a fixed value of 0.85. Red force alertness τ_R sequentially was given values of 0.50, 0.70, 0.75, 0.80, 0.85, and 0.90. Blue force alertness τ_B then was modified in a consistent manner to determine the " critical point" when the winner changes from one side to the other.

For the first test, the alertness of the Red force was fixed at $\tau_R = 0.50$, while Blue force alertness was varied from $\tau_B = 0.45$ to $\tau_B = 0.61$ in steps of 0.01. Figure 1 provides the results. When the alertness of the Blue force changes from $\tau_B = 0.56$ to $\tau_B = 0.57$, then the winner changes from the Red force to the Blue force.

For this particular case, the break point for both forces has been set at 0.20, instead of the 0.30 shown in Table II. This was done so that the break points would not be reached after the first salvo.

With this change, when the alertness of Blue force moves from $\tau_B = 0.51$ to $\tau_B = 0.60$ both forces reach this 0.20 break point. The winner of the battle, however, changes at 0.57. Finally, at $\tau_B = 0.61$ the Blue force wins with a total loss of 0.65.

Red force alertness was next given a value of $\tau_R = 0.70$, while Blue force alertness was varied from

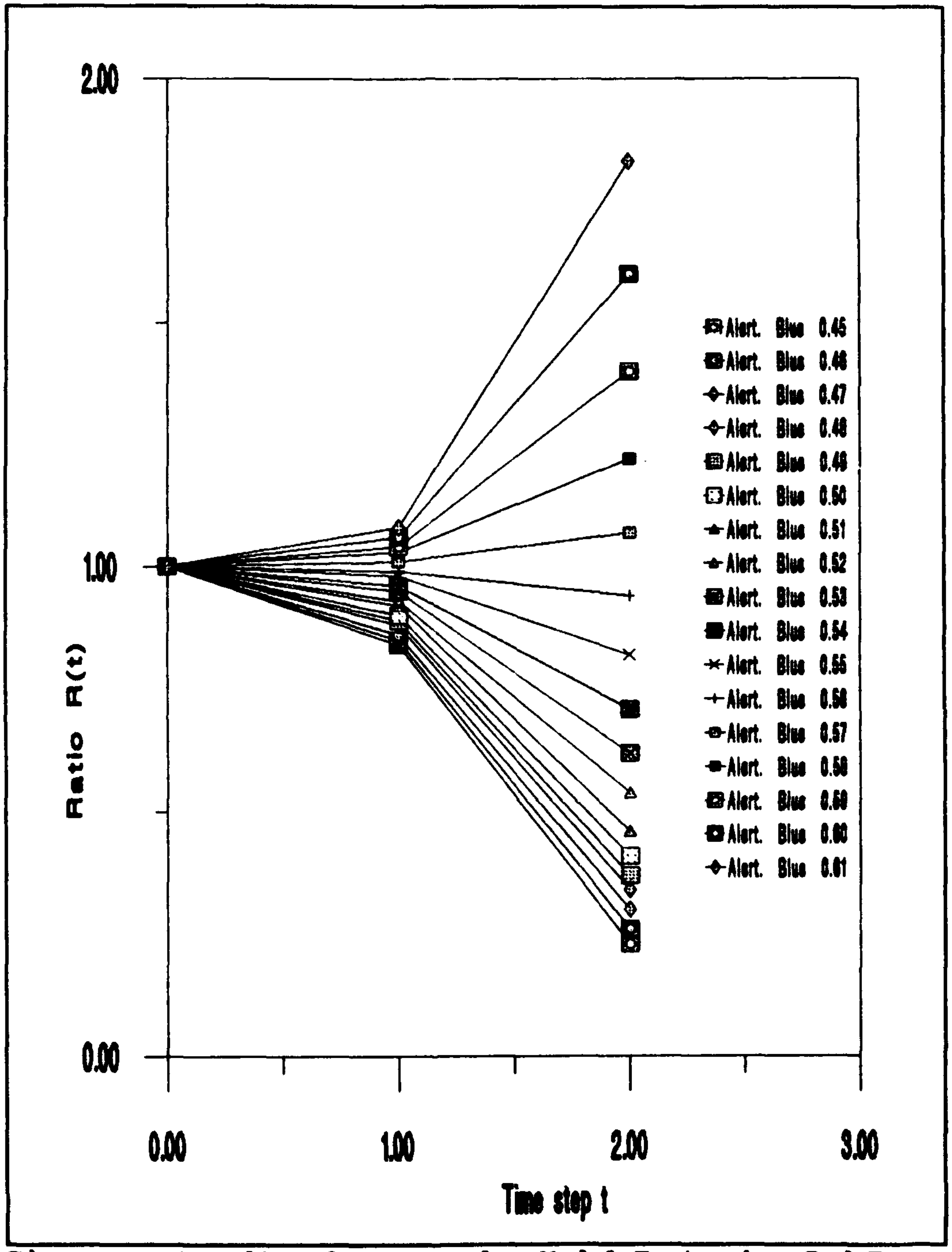

Figure 1. Results of Hatzopoulos Model Tests when Red Force Alertness is Fixed at 0.50 While Blue Force Alertness varies from 0.45 to 0.61.

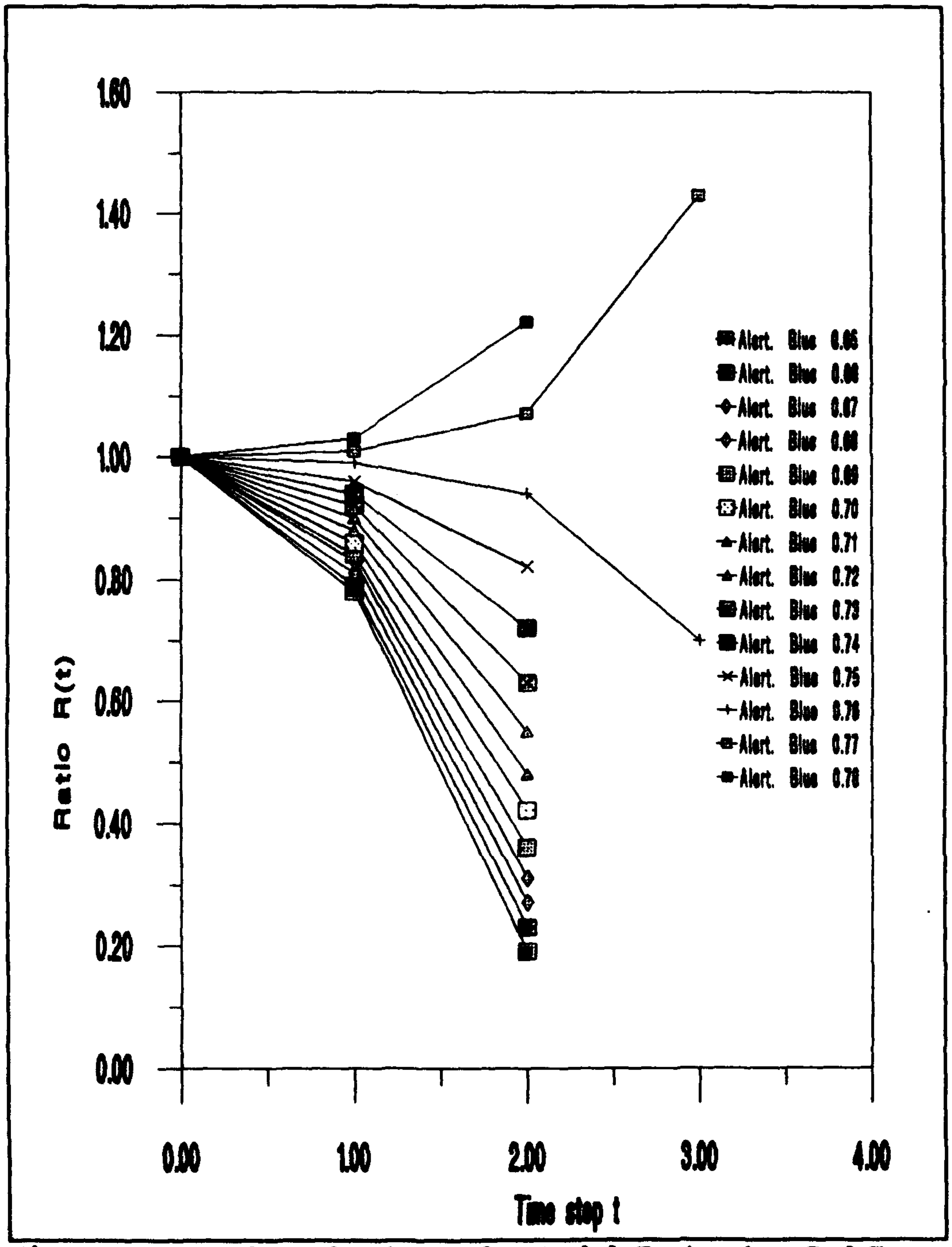

Figure 2. Results of Hatzopoulos Model Tests when Red Force Alertness is Fixed at 0.70 While Blue Force Alertness varies from 0.65 to 0.78.

$\tau_B = 0.65$ to $\tau_B = 0.80$ in 0.01 increments (Figure 2). The critical point occurs when the Blue force alertness is $\tau_B = 0.76$. At $\tau_B = 0.77$, the winner of the battle changes to the Blue force.

For the remaining tests for this case, Red force alertness was varied from $\tau_R = 0.75$ to $\tau_R = 0.90$ in steps of 0.05. Blue force alertness was initialized at $\tau_B = \tau_R - 0.05$, and increased in increments of 0.01 until Blue force wins the battle. This occurs at $\tau_B = \tau_R + 0.08$. That is, at $\tau_B = \tau_R + 0.07$, the winner changed to Blue force for these four tests. Results are shown in Figures 3, 4, 5, and 6.

Case 2

For this case the alertness of Red force τ_R was fixed at a value of 0.90 and Blue force alertness τ_B at 0.85. Red force scouting effectiveness σ_R was varied systematically from 0.70, to 0.80, to 0.90. Blue force scouting effectiveness, σ_B, was then varied in a consistent manner to determine when the battle outcome changed from one winner to the other.

Red force scouting effectiveness was set initially at $\sigma_R = 0.70$ while the scouting function of the Blue force was increased from $\sigma_B = 0.65$ in increments of 0.01 (Figure 7). At $\sigma_B = 0.74$, the winner changed to the Blue force. Until the

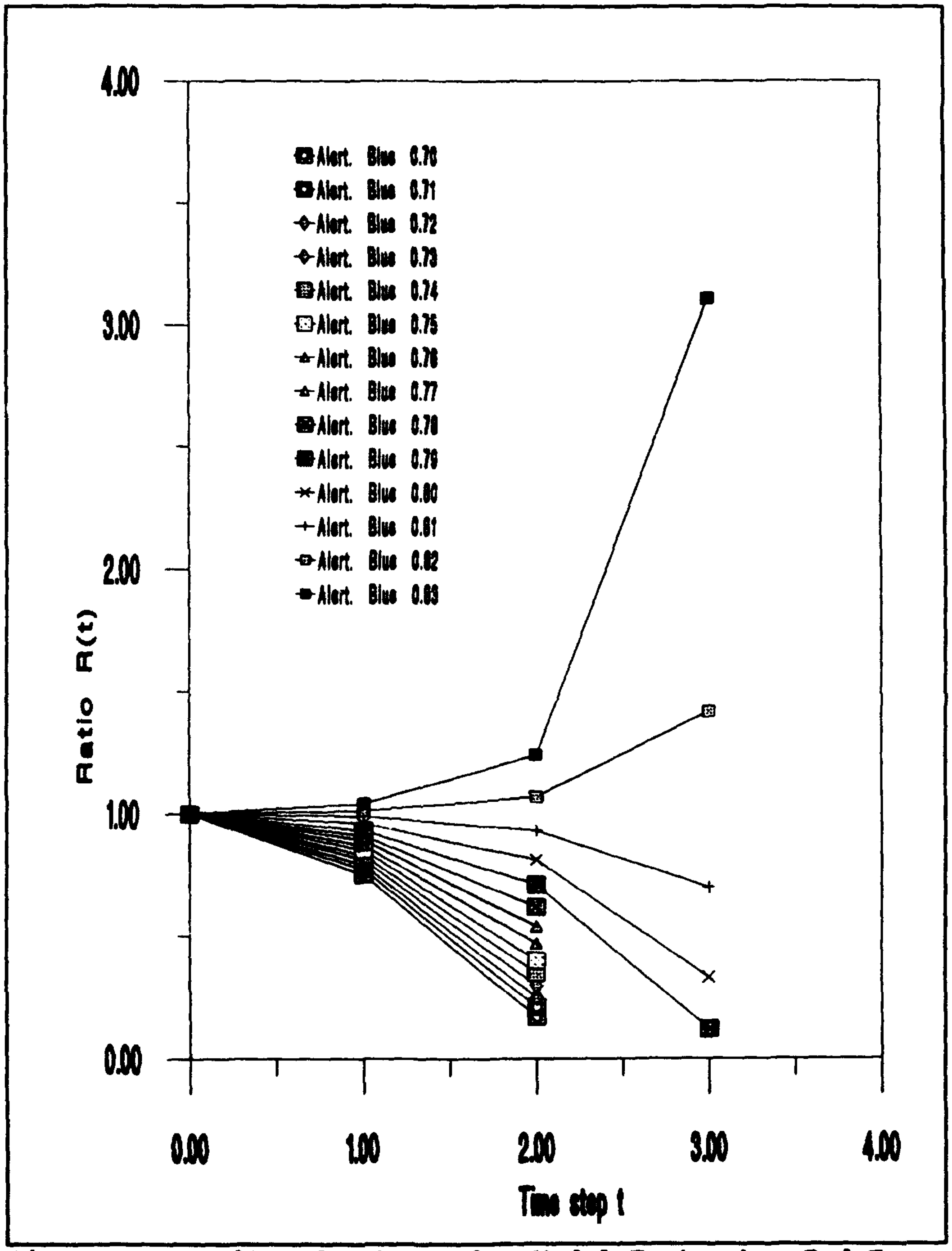

Figure 3 Results of Hatzopoulos Model Tests when Red Force Alertness is Fixed at 0.75 While Blue Force Alertness varies from 0.70 to 0.83.

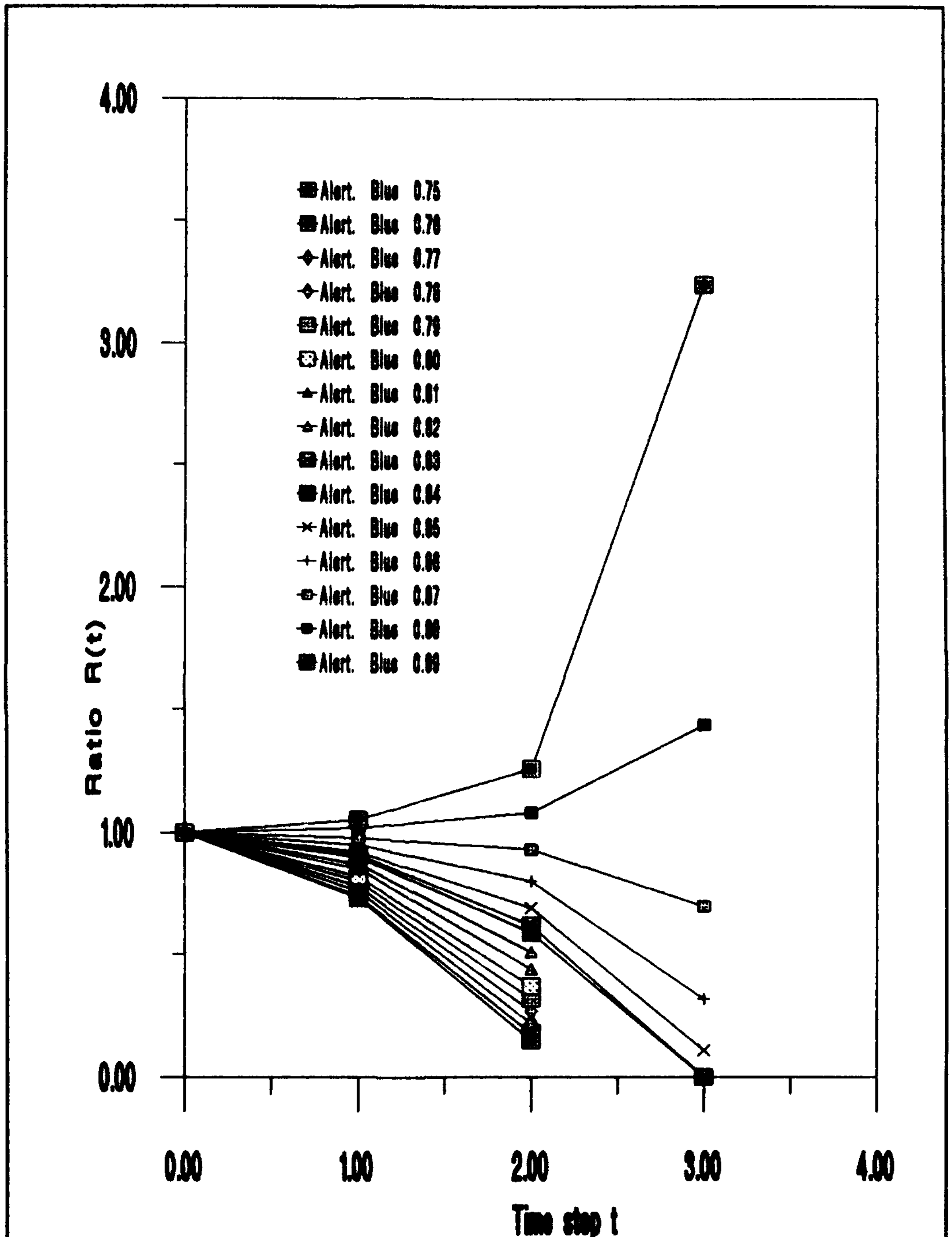

Figure 4 Results of Hatzopoulos Model Tests when Red Force Alertness is Fixed at 0.80 While Blue Force Alertness varies from 0.75 to 0.89.

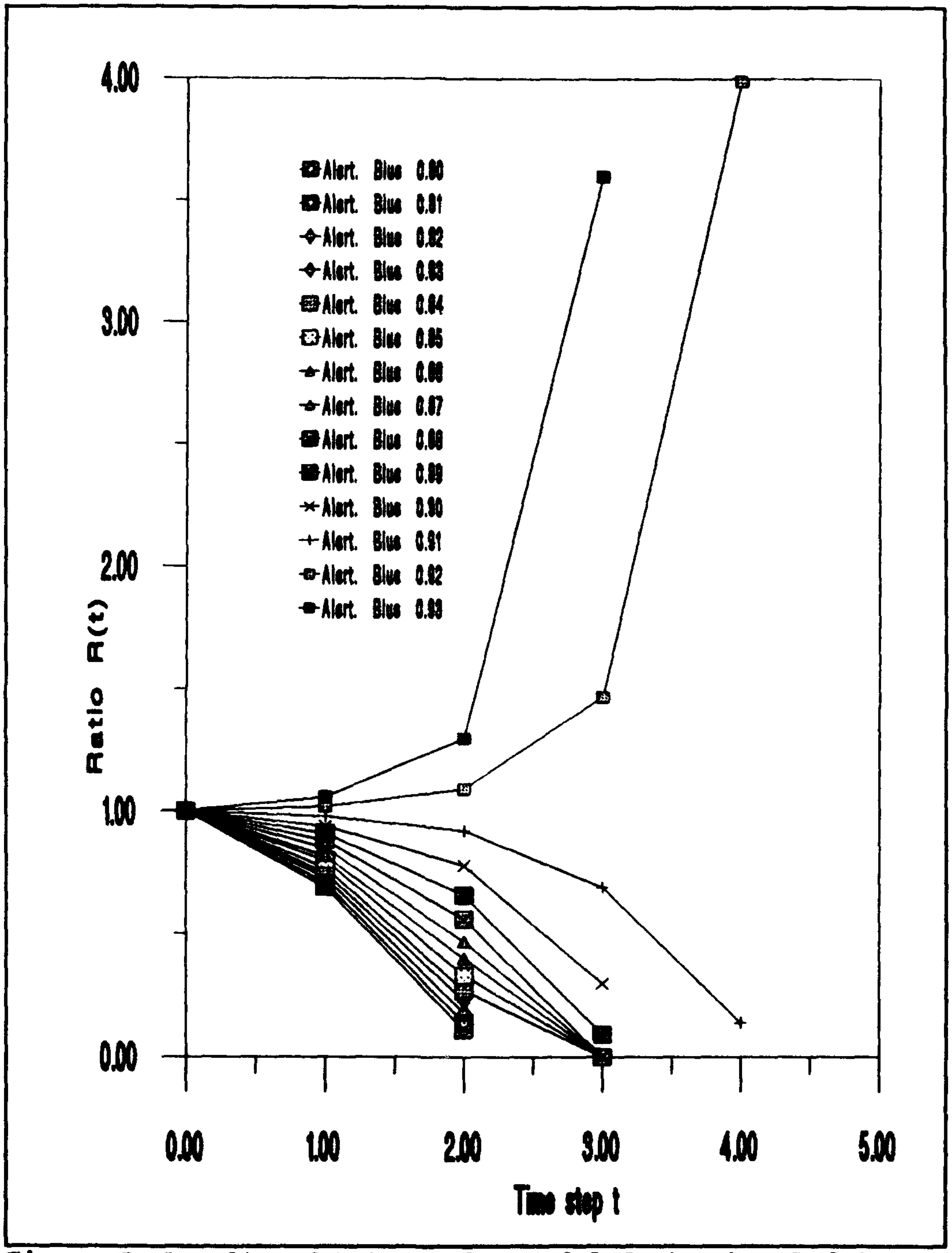

Figure 5 **Results of Hatzopoulos Model Tests when Red Force Alertness is Fixed at 0.85 While Blue Force Alertness varies from 0.80 to 0.93.**

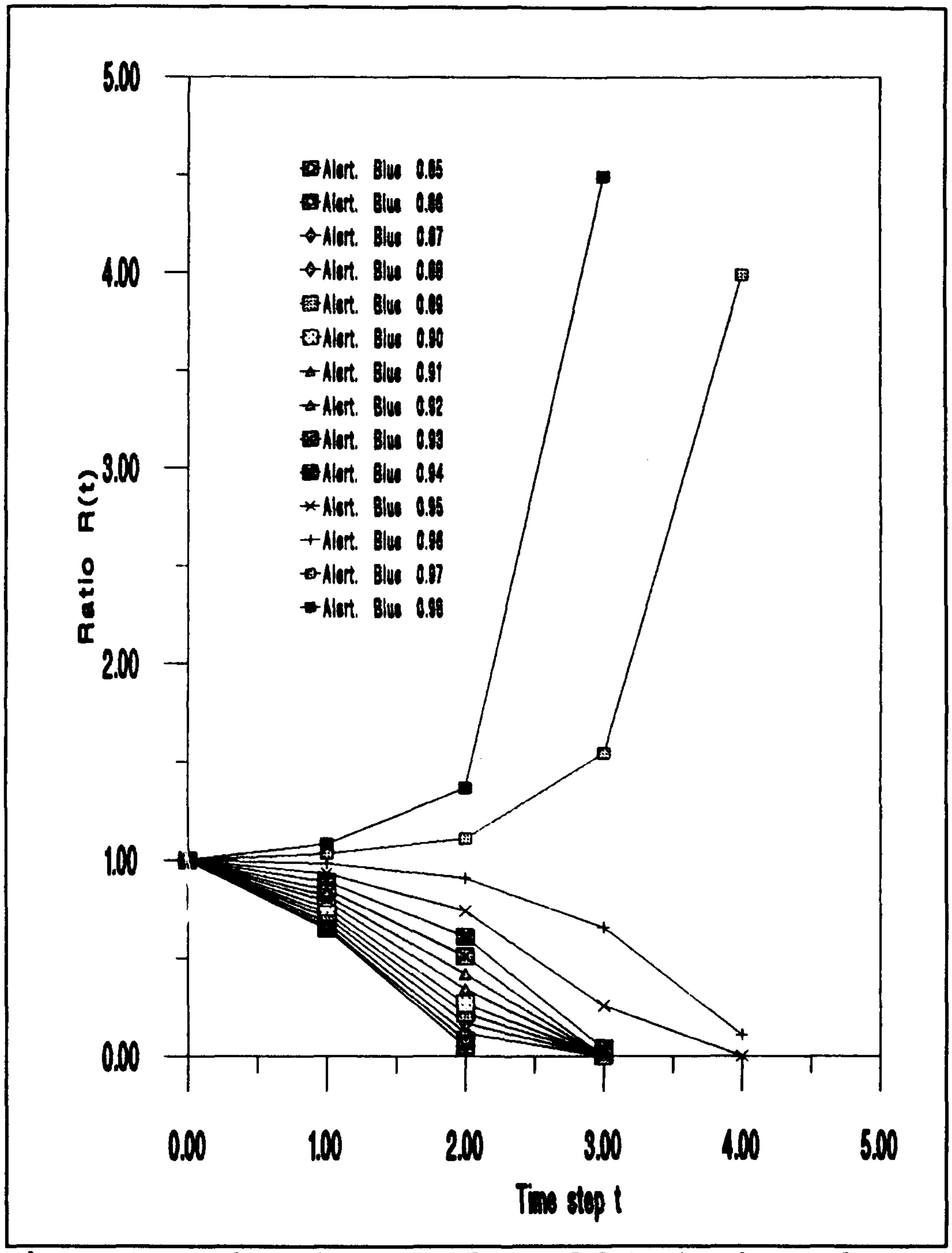

Figure 6 Results of Hatzopoulos Model Tests when Red Force Alertness is Fixed at 0.90 While Blue Force Alertness varies from 0.85 to 0.98.

scouting effectiveness of the Blue force rose to $\sigma_B = 0.70$, the total loss for the Red force was zero.

Next, the scouting effectiveness of the Red force was set at $\sigma_R = 0.80$. Blue force scouting effectiveness was increased from $\sigma_B = 0.75$ in steps of 0.01 (Figure 8). The winner changed to the Blue force at an effectiveness value of $\sigma_B = 0.84$.

Finally, Red force scouting effectiveness was given a value of $\sigma_R = 0.90$ (Figure 9). Scouting effectiveness of the Blue force was increased from $\sigma_B = 0.85$ in steps of 0.01. At $\sigma_B = 0.94$, both forces reached their break points and the winner changed to the Blue force.

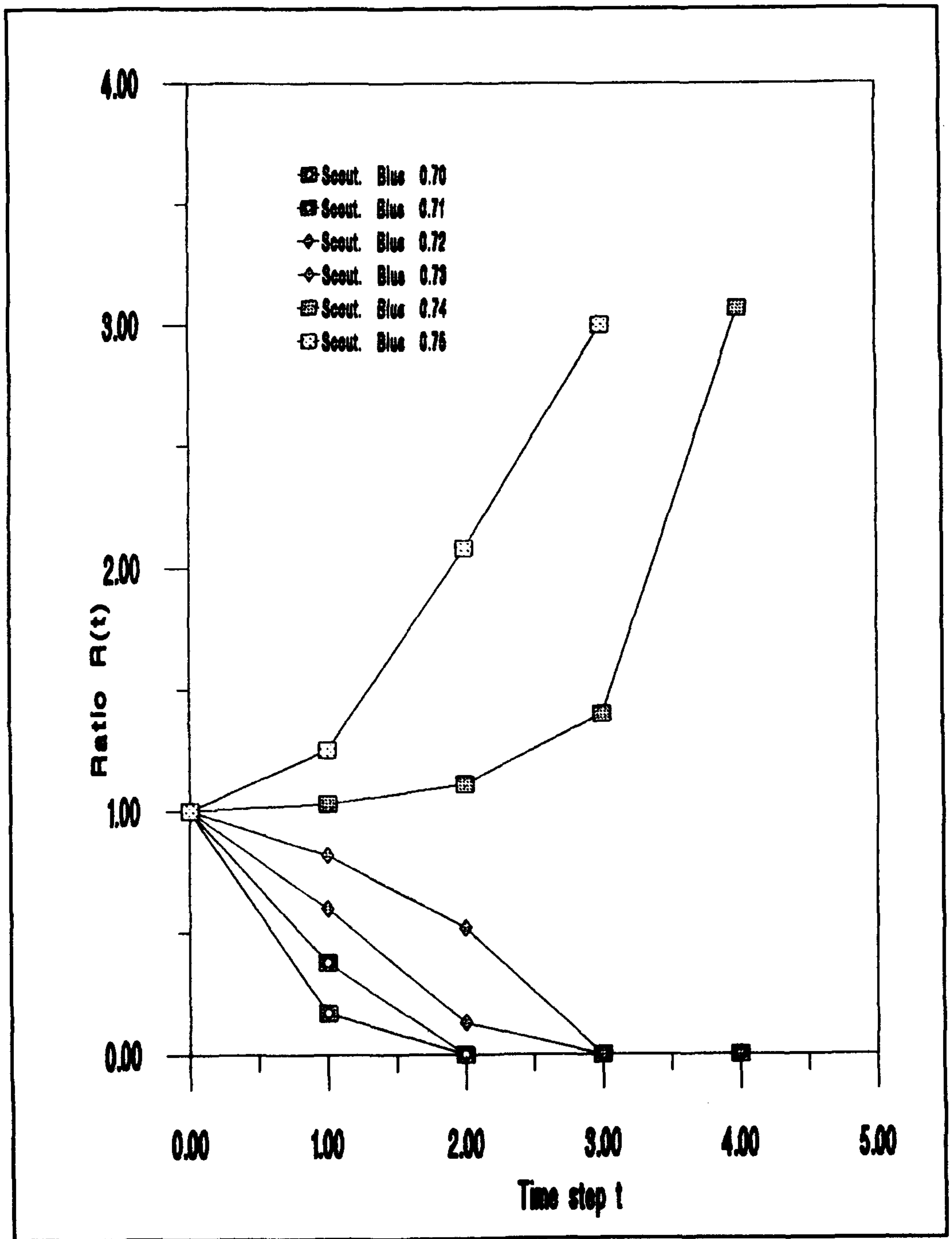

Figure 7 Results of Hatzopoulos Model Tests when Red Force Scouting is Fixed at 0.70 While Blue Force Scouting varies from 0.70 to 0.75.

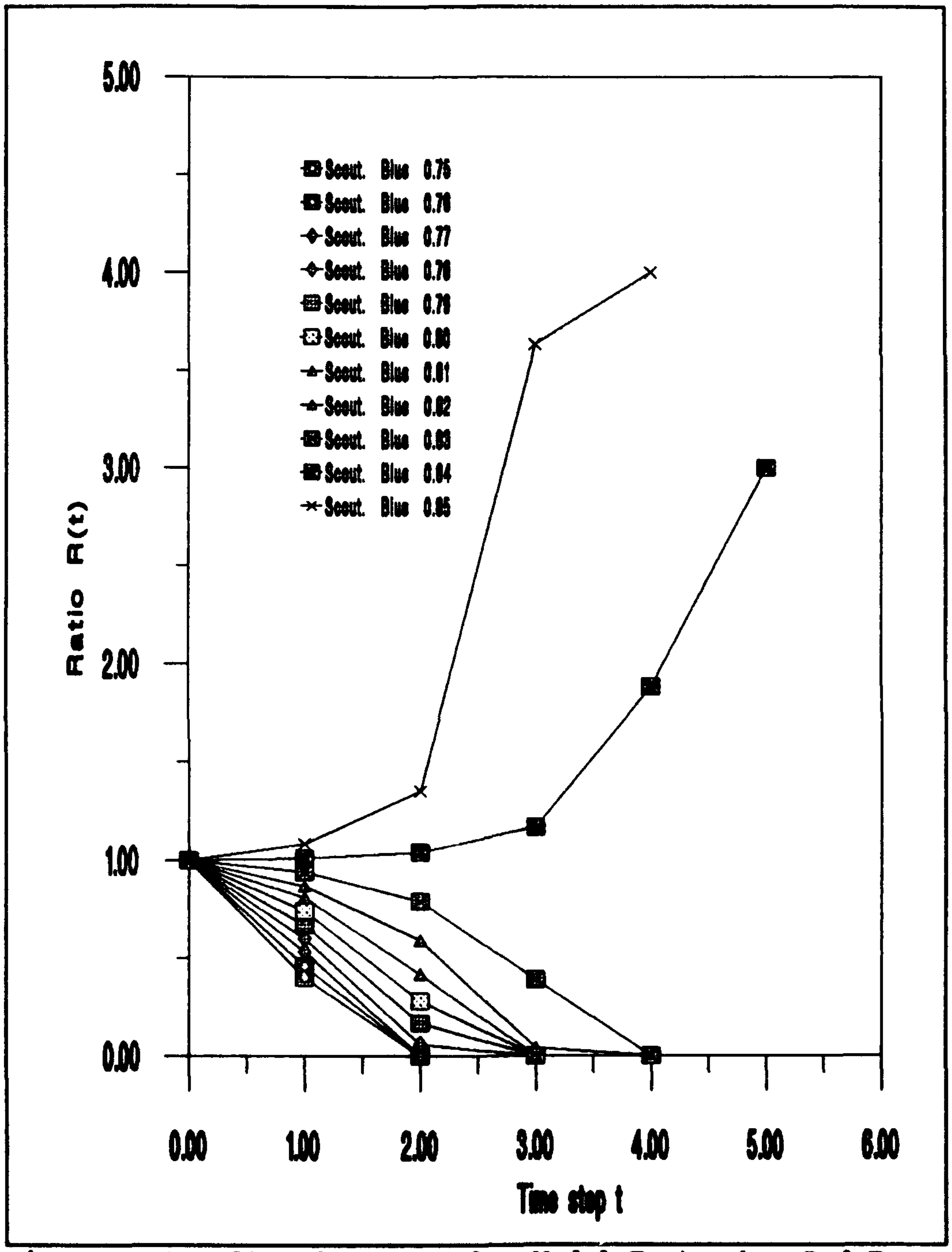

Figure 8 Results of Hatzopoulos Model Tests when Red Force Scouting is Fixed at 0.80 While Blue Force Scouting varies from 0.75 to 0.84.

B. FRACTIONAL EXCHANGE RATIO

1. Development of Ratio

Let us reconcile the terminology of Chapter II (Hatzopoulos Model) with Chapter III (Weir, Barr, and Hoffman). Let the Red and Blue forces comprise R and B identical platforms respectively. Then we have seen from Equations (4.2) and (4.3) that the fraction of identical platforms lost to R and B, respectively, is:

$$LOSS_{k'r} = \frac{\Delta R}{R},$$

and

$$LOSS_{kb} = \frac{\Delta B}{B}.$$

This holds because $SP_{k'r} = \sum_{j'} SP_{j'k'r} = \alpha_1 R$, where α_1 is assumed to be the same for each R platform. Similarly, $SP_{kb} = \sum_{j} SP_{jkb} = \beta_1 B$, when β_1 is assumed the same for each B platform.

In the Hatzopoulos model, if we further assume that both forces use missiles equivalent to a nominal one-TPBE missile, then the missile technology multiplier W_m can be ignored. Given two forces, R and B, the fractional exchange

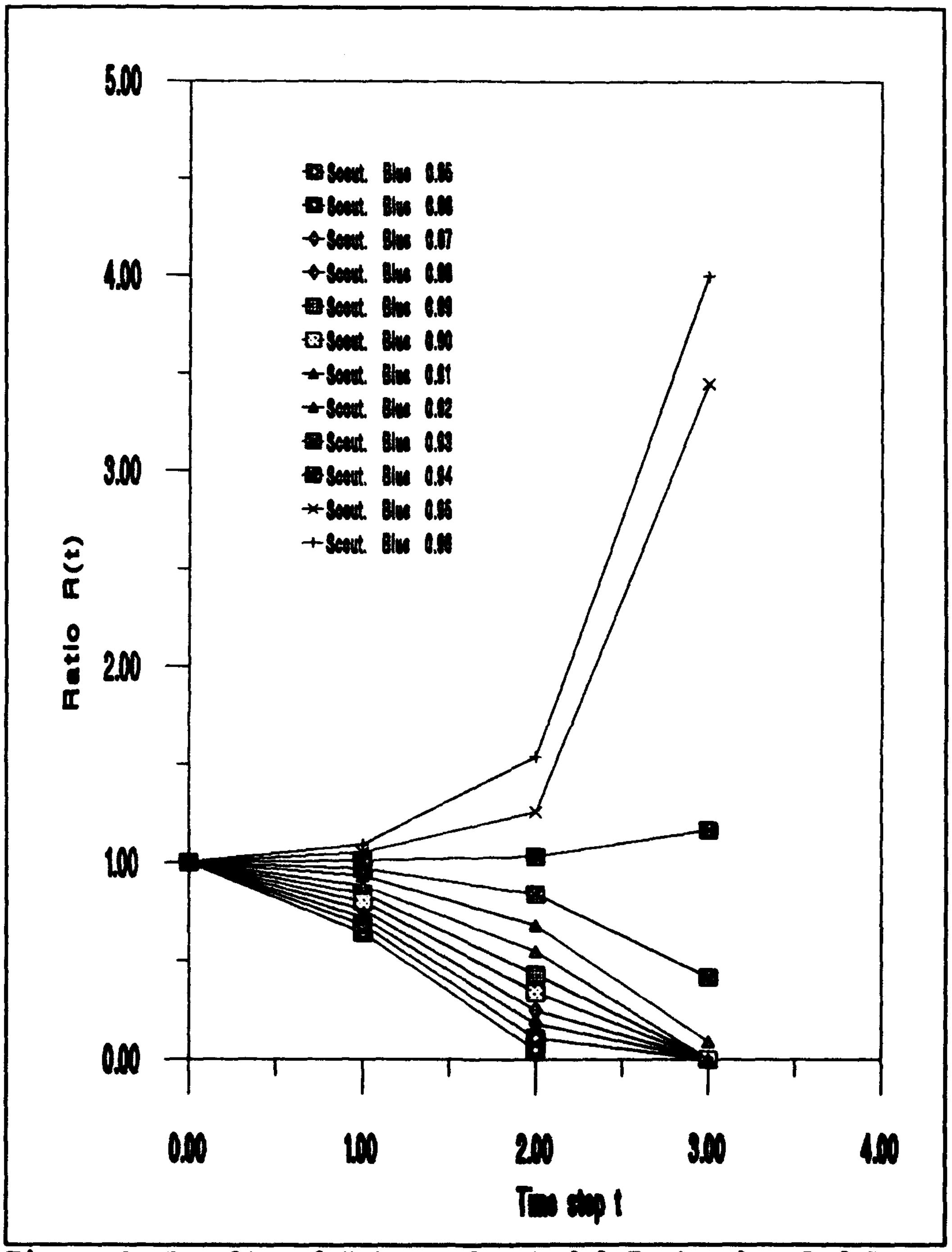

Figure 9 Results of Hatzopoulos Model Tests when Red Force Scouting is Fixed at 0.90 While Blue Force Scouting varies from 0.85 to 0.94.

ratio for each force from Equation (2.8) is:

$$\frac{\Delta R}{R} = \frac{\sigma_B \times M_B \times m_B \times H - \tau_R \times N_R \times n_R}{\alpha_1 \times R}, \tag{4.5}$$

and

$$\frac{\Delta B}{B} = \frac{\sigma_R \times M_R \times m_R \times H - \tau_B \times N_B \times n_B}{\beta_1 \times B} \tag{4.6}$$

where : α_1 , β_1 = Unit staying power (i.e., the number of TPBE hits necessary to inflict a firepower kill on a platform) of each ship of forces R and B, respectively.

$\alpha_1 \times R$, $\beta_1 \times B$ = Total staying power of forces R and B, respectively.

ΔR , ΔB = Theoretical per-salvo "delta" losses of forces R and B, respectively.

R , B = The number of units (ships) of forces R and B, respectively.

M_R , M_B = Theoretical number of missiles that each force R and B, respectively, can fire in a single salvo.

m_R , m_B = Multiplicative degrader for each force R and B, respectively, which introduces the effects of training, morale, and leadership.

N_R , N_B = Number of missiles which each force R and B, respectively, can shoot down per salvo (the best that can be done).

n_R , n_B = Multiplicative degrader for each force R and B, respectively, which represents the ability of the defender to shoot down missiles.

H = Firing accuracy, given for each type of missile. For the same type of missile, H is the same for all units in the force.

Since it is now assumed that all ships on a side are identical, additional parameters can be defined. First, we can say that:

α_2 , β_2 = Unit salvo striking power in hits (good shots) for each ship of the R and B forces, respectively.

Since all units have the same salvo striking power, the total salvo striking power for force R is $\alpha_2 \times R$. Because the M_R is the theoretical number of missiles that force R can fire in a single salvo, then $M_R \times m_R$ in the Hatzopoulos model (Equation 2.10) is the total salvo striking power in hits (good shots) for the R force which we define as $\alpha_2 \times R$. Similarly, for the B force, total salvo striking power is $\beta_2 \times B$, which would be the same as $M_B \times m_B$.

If all ships of both forces are equivalent, we can also say that:

α_3 , β_3 = Unit defensive power (power defense) for each ship of the R and B forces, respectively.

Since each unit on a side has the same defensive power, then the total defensive power for the R force is $\alpha_3 \times R$. Because the N_R is the theoretical number of missiles which the force R can shoot down per salvo (the best that can be done), the term $N_R \times n_R$ in Equation (2.10) of the Hatzopoulos Model is the total defensive power for the R force which we define as $\alpha_3 \times R$. Similarly, for the B force, total defensive power is $\beta_3 \times B$, which is the same as $N_B \times n_B$.

Using these assumptions and definitions, the fractional exchange ratio for each force can be expressed as:

$$\frac{\Delta R}{R} = \frac{\sigma_B \times \beta_2 \times B \times H - \tau_R \times \alpha_3 \times R}{\alpha_1 \times R}, \qquad (4.7)$$

and

$$\frac{\Delta B}{B} = \frac{\sigma_R \times \alpha_2 \times R \times H - \tau_B \times \beta_3 \times B}{\beta_1 \times B}. \qquad (4.8)$$

The fractional exchange ratio (FER) between the two forces is written as:

$$FER = \frac{\frac{\Delta R}{R}}{\frac{\Delta B}{B}}. \qquad (4.9)$$

Equation (4.9) combined with Equations (4.7) and (4.8) gives:

$$FER = \frac{\frac{\Delta R}{R}}{\frac{\Delta B}{B}} = \frac{\sigma_B \times \beta_2 \times B \times H - \tau_R \times \alpha_3 \times R}{\sigma_R \times \alpha_2 \times R \times H - \tau_B \times \beta_3 \times B} \times \frac{\beta_1 \times B}{\alpha_1 \times R}. \qquad (4.10)$$

Equation (4.10) is simplified first by expressing the number of ships in the R force as a function of the number of ships in the B force. That is, using a multiplier k, the number of ships in the R force is given by $R = k \times B$, where k is a real number greater than zero. Second, if it is assumed that the unit staying power for all ships in both forces is exactly the same, then $\alpha_1 = \beta_1$. Then the fractional exchange ratio (FER) can be expressed as:

$$FER = \frac{1}{k} \times \frac{\sigma_B \times \beta_2 \times H - \tau_R \times \alpha_3 \times k}{\sigma_R \times \alpha_2 \times H \times k - \tau_B \times \beta_3}. \tag{4.11}$$

Since the two factors, scouting effectiveness σ and alertness τ, are of especial interest, it is important to consider how FER changes as these factors change. Calculating the partial derivatives of FER with respect to the variables σ_R, σ_B, τ_R, and τ_B, , we have:

$$\frac{\partial(FER)}{\partial\sigma_R} = \frac{1}{k} \times \frac{-\alpha_2 \times H \times k \times (\sigma_B \times \beta_2 \times H - \tau_R \times \alpha_3\ k)}{(\sigma_R \times \alpha_2\ H \times k - \tau_B\ \beta_3)^2}, \tag{4.12}$$

$$\frac{\partial(FER)}{\partial\tau_R} = \frac{1}{k} \times \frac{-\ \alpha_3 \times k}{\sigma_R \times \alpha_2 \times H \times k - \tau_B \times \beta_3}, \tag{4.13}$$

$$\frac{\partial(FER)}{\partial\sigma_B} = \frac{1}{k} \times \frac{\beta_2 \times H}{\sigma_R \times \alpha_2 \times H \times k - \tau_B \times \beta_3}, \tag{4.14}$$

and

$$\frac{\partial(FER)}{\partial\tau_B} = \frac{1}{k} \times \frac{\beta_3 \times (\sigma_B \times \beta_2 \times H - \tau_R \times \alpha_3 \times k)}{(\sigma_R \times \alpha_2 \times H \times k - \tau_B \times \beta_3)^2}. \tag{4.15}$$

The relative sensitivity of Hatzopoulos's model to changes in scouting effectiveness, when compared to changes in alertness, can also be examined through consideration of another ratio. The parameters ρ_1 and ρ_2 are defined as:

$$\rho_1 = \frac{\dfrac{\partial(FER)}{\partial\sigma_R}}{\dfrac{\partial(FER)}{\partial\tau_R}} = \frac{\alpha_2 \times H \times (\sigma_B \times \beta_2 \times H - \tau_R \times \alpha_3 \times k)}{\alpha_3 \times (\sigma_R \times \alpha_2 \times k \times H - \tau_B \times \beta_3)}, \tag{4.16}$$

and

$$\rho_2 = \frac{\dfrac{\partial(FER)}{\partial\sigma_B}}{\dfrac{\partial(FER)}{\partial\tau_B}} = \frac{\beta_2 \times H (\sigma_R \times \alpha_2 \times H \times k - \tau_B \times \beta_3)}{\beta_3 \times (\sigma_B \times \beta_2 \times H - \tau_R \times \alpha_3 \times k)}. \tag{4.17}$$

When the ratio ρ_1 is greater than 1.0 then changes to the model's parameters for force R are more sensitive to σ_R than to τ_R. When the value of the ratio is less than 1.0, the model is more sensitive to τ_R. The same is true for ρ_2.

2. Computer Implementation

A computer program was coded in Fortran 77 to examine the ratios ρ_1 and ρ_2 for all possible values of σ_R , τ_R , σ_B, and τ_B from 0.60 to 0.95 in increments of 0.05 (Appendix B). The program gives an output table with a value of 1.0 at each point where the ratio is more sensitive to scouting than to alertness. Examples of the use of this program and its output follow.

3. Testing Model Sensitivity to Scouting Effectiveness and Alertness

The program (Appendix B) was developed to calculate the ratios ρ_1 and ρ_2 shown in Equations (4.16) and (4.17), respectively. A single scenario is used to examine where the model is more sensitive to scouting effectiveness, σ_R and σ_B, than to alertness, τ_R and τ_B, respectively.

For the scenario used for these tests both forces are assumed to have the same unit striking power, α_1 and β_1, for each ship of the two forces, R and B, respectively. The exact number of ships of each individual force does not affect the outcome. However, the multiplier k in Equation (4.11) does need to be included.

Model sensitivity is tested for two situations. In the first situation, the unit salvo striking power, α_2 and β_2, and the unit defensive power, α_3 and β_3, for each ship of both forces are taken to be exactly equal. In the second situation, the unit salvo striking power for the B force is greater than for R; that is, $\beta_2 > \alpha_2$. The unit defensive power for the B force is also greater than for the R force; that is, $\beta_3 > \alpha_3$.

Program output consists of a series of tables (Appendices C and D). Each table has exactly 64 cells. Each cell describes the scouting effectiveness and alertness of either force R or force B when the scouting effectiveness and alertness for the opposing force, B or R, is varied from 0.60 to 0.95 in increments of 0.05. The columns in each cell represent alertness and the rows represent scouting effectiveness, making up a matrix with a maximum of 64 values.

In each of the 64 cells, the program places a value of 1.0 in the matrix wherever the model is more sensitive to scouting effectiveness, σ, than to alertness τ. No value is shown in the matrix when the model is more sensitive to alertness τ than to scouting effectiveness σ. Although 64 cells are possible (due to the number of conditions considered) not all of the tables include that many cells. When the matrix has no values of 1.0 at all, these cells are simply omitted. They represent the cases where the model is sensitive only to alertness τ and not sensitive to scouting effectiveness σ at all.

Case 1

The Hatzopoulos model is tested first for the situation where both forces have the same unit salvo striking power (that is, $\alpha_2 = \beta_2 = 3$) and the same unit defensive power ($\alpha_3 = \beta_3 = 2$). Three major conditions are considered. In the first condition, the R force is only 0.75 the numerical strength, here often the numerical strength is simply called the "strength," of the B force (that is, if the B force has four ships, then the R force has only three). In the second condition, the strengths of the R and B forces are equal. In the third condition, the R force strength is 1.50 times that of the B force (that is, if the B force has four ships, the R force has six).

The program produces two tables for each of these three relative force strength conditions for each situation. Thus a total of six tables is produced during analysis of model sensitivity to the Case 1 situation (Appendix C). For each force strength condition (for example, R = 0.75B), the first table includes up to 64 cells, each with a matrix of sensitivity values (either 1.0 or none) that results from holding B force alertness τ_B and scouting effectiveness σ_B constant at given values (for example, $\sigma_B = 0.60$ and $\tau_B = 0.60$) while varying these two factors for the R force (for example, from 0.60 to 0.95). The second table produces a similar result for holding R force τ_R and σ_R values constant while varying B force alertness and scouting effectiveness.

Let us now examine each table. In the first condition the R force is 0.75 of the strength of the B force, and the table (Appendix C, Table Ia), shows that the majority of cells contain no 1.0 values. This means that, under most of the conditions considered, the model is more sensitive to Red force alertness than to its scouting effectiveness. As both the scouting effectiveness and alertness of the Red force increase, conditions are tested when the model becomes more sensitive to scouting effectiveness than to the alertness of the Red force.

In the second table (Appendix C, Table Ib), assuming the same force condition, the opposite situation is observed. Most cells contain many 1.0 values in their matrices. This indicates that, for most of the conditions considered, the model is more sensitive to scouting effectiveness than to the alertness of the Blue force. As the alertness of the Blue force increases the model demonstrates more sensitivity to Blue force alertness than to its scouting effectiveness.

In the next two tables we assume, the strengths of both forces are equal. Observe that both tables (Appendix C, Table IIa and Table IIb) are exactly the same. As the scouting effectiveness and alertness of both forces increase, the model becomes increasingly more sensitive to scouting effectiveness than to alertness of the two forces.

The third condition is where the strength of Red force is 1.50 times that of the Blue force. The first table for this condition (Appendix C, Table IIIa) contains only seven cells in which the matrix includes at least one 1.0, only occurring at very high values of scouting effectiveness and alertness for the Red force. This indicates that the model is more sensitive to Red force scouting effectiveness than to its alertness. The second table (Appendix C, Table IIIb) demonstrates that as the scouting effectiveness of Red force increases, then the model is more sensitive to the alertness than to the scouting effectiveness of Blue force.

Case 2

Next the Hatzopoulos model was tested for the situation where the unit salvo striking power of force B is greater than that of R force (that is, $\beta_2=4 > \alpha_2=3$) and B's unit defensive power is also greater (that is, $\beta_3=3 > \alpha_3=2$). Three major conditions are considered. In the first condition, the strengths of both forces are equal. In the second condition, the R force numerical strength is 1.50 times that of the B force (that is, if the B force has four ships, the R force has six). In the third condition, the R force strength is two times that of the B force (that is, if the B force has three ships, the R force has six).

The program produces two tables for each of these three relative force strength conditions. Thus a total of six tables is produced during analysis of model sensitivity to the Case 2 situation (Appendix D). For each force strength condition (for example, R = 1.50B), the first table includes up to 64 cells, each with a matrix of sensitivity values (either 1.0 or none) that results from holding B force alertness τ_B and scouting effectiveness σ_B constant at given values (for example, $\sigma_B = 0.60$ and $\tau_B=0.60$) while varying these same two factors for the R force (for example, from 0.60 to 0.95). The second table produces a similar result for holding R force τ_R and σ_R values constant while varying B force alertness and scouting effectiveness.

In the first condition (the strengths of both forces are equal), only cells representing the situation where the scouting effectiveness of the Red force is greater than 0.80 and its alertness is greater than 0.75 does the matrix contain at least one 1.0 (Appendix D, Table Ia). In all other cases the model is more sensitive to the alertness than to the scouting effectiveness of the Red force. In the second table (Appendix D, Table Ib), as scouting effectiveness increases, the model is more sensitive to Blue force alertness than to its scouting effectiveness. On the other hand, as the alertness of the Blue force increases, the model becomes more sensitive to Blue force scouting effectiveness than to its alertness in the situations where the alertness and scouting effectiveness of the Red force is very high.

The second condition represents the situation where the strength of the Red force is 1.50 times that of the Blue force. As may be seen in the first table (Appendix D, Table IIa), as the scouting effectiveness and the alertness of the Red force increases, the model becomes more sensitive to the scouting effectiveness than to the alertness of the Red force. In the second table (Appendix D, Table IIb), it becomes obvious that, as the scouting effectiveness of the Blue force increases, the model becomes more sensitive to the scouting effectiveness than to the alertness of the Blue force, when the alertness of Blue force is fixed. Otherwise, when both the scouting effectiveness and the alertness of Blue force increase, the model becomes more sensitive to Blue force alertness than to its scouting effectiveness.

In the third condition, the strength of Red force is twice that of the Blue force. The first table (Appendix D, Table IIIa) indicates that, as the alertness of the Red force increases while holding its scouting effectiveness fixed, the model becomes more sensitive to Red force alertness than to its scouting effectiveness. As the scouting effectiveness of the Red force increases, the model becomes more sensitive to the scouting effectiveness of the Red force. This is true especially for the cases where both the alertness and scouting effectiveness of the Blue force are very high. Inspection of the second table (Appendix D, Table IIIb) indicates that the model is generally more sensitive to its alertness than to the scouting effectiveness of the Blue force. The exceptions are those cells representing the situation where the alertness and scouting effectiveness of the Blue force are both very high (at least 0.75 and 0.80, respectively).

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The goal of this analysis was to determine how sensitive the Hatzopoulos Modern Naval Combat Model is to changes in scouting effectiveness and alertness. The general approach taken was to analyze model sensitivity through the use of two ratios developed for this purpose.

Several conclusions based on the analyses are provided below, to show the utility and power of the model once a real tactical situation is known. However, it should be emphasized that these conclusions cannot be generalized to all situations. The model includes six keys variables on each side. These 12 variables, all interactive, take values that depend on the characteristics and relative strengths of the two opposing forces. Although the values for the other variables are representative, only <u>scouting effectiveness</u> and <u>alertness</u> have been examined in this study.

1. Ratio of Remaining Staying Power

The first measure introduced is the ratio of one force's <u>remaining staying power</u> to that of the other force, following each salvo. For this study both forces are assumed to consist of the same number of units with the same platform characteristics. The sensitivity of this ratio is tested with the Hatzopoulos Model in two cases.

In the first case, scouting effectiveness values of both forces are fixed, while alertness values are varied. Red force scouting effectiveness σ_R is fixed at a value of 0.90. Blue force scouting effectiveness σ_B is given a lower fixed value of 0.85. Red force alertness τ_R is sequentially given values of 0.50, 0.70, 0.75, 0.80, 0.85, and 0.90. Blue force alertness τ_B then is varied in a consistent manner to determine the "critical point" when the winner changes from the Red force to the Blue force according to the criterion of remaining staying power.

In this process the Blue force alertness is initialized at $\tau_B = \tau_R - 0.05$, and increased in increments of 0.01 until the Blue force wins the battle. This consistently occurred at $\tau_B = \tau_R + 0.07$.

In the second case, alertness values of both forces are held fixed, while scouting effectiveness values are varied. For this case the alertness of Red force τ_R is fixed at a value of 0.90 and the Blue force alertness τ_B at 0.85. Red force scouting effectiveness σ_R is then varied systematically from 0.70, to 0.80, to 0.90. Blue force alertness τ_B is then modified to determine when the battle outcome changes the winner from the Red force to Blue force.

In this case the Blue force scouting effectiveness is set initially at $\sigma_B = \sigma_R - 0.05$, and increased in increments of 0.01 until the Blue force wins. This consistently occurred at $\sigma_B = \sigma_R + 0.04$.

2. Fractional Exchange Ratio

The second ratio explored is the <u>fractional exchange ratio</u>, which compares the proportion of each force that no longer can fight effectively after an exchange of missiles. By means of the application of partial derivatives, this ratio was tested to see when the model is more sensitive to scouting effectiveness σ than to alertness τ in two situations.

<u>a. Situation Where Both Forces are Equal</u>

In the first situation, both forces have the same unit salvo striking power ($\alpha_2 = \beta_2$) and the same unit defensive power ($\alpha_3 = \beta_3$). Three major variations are examined, which can be summarized as shown in Table III.

TABLE III. MODEL SENSITIVITY WHEN STRIKING AND DEFENSIVE POWER ARE EQUAL

$(\beta_2 = \alpha_2,\ \beta_3 = \alpha_3)$

Variation	Red Force Factor for which model is Most Sensitive	Blue Force Factor for which Model is Most Sensitive
Red Outnumbered [R = 0.75B]	τ_R	σ_B
Approx. Equality of Numbers [R = B]	σ_R	σ_B
Red Superior [R = 1.50B]	σ_R τ_R (*if* $\tau_R\uparrow$)	τ_B

In the first variation, the Red force size is only 0.75 that of the Blue force. The model was found to be more sensitive to the Red force alertness τ_R than to its scouting effectiveness σ_R, but more sensitive to the Blue force scouting effectiveness σ_B than to its alertness τ_B.

The model results in this variation are expected because the larger force can have higher scouting effectiveness and then increase its alertness to be ready for a battle. For example, assume the Blue force has eight ships and the Red force has six ships in an area. From the point of view of the Blue force, because the force is bigger, it makes sense that the alertness level for each ship can be less and yet the total defensive force of Blue's ships is still able to strike down the enemy missiles. Yet if the Blue force is going to win, it needs to strike out against the ships in the Red force, so Blue needs to know Red positions, and strike strongly. That is, the scouting effectiveness increase has a greater payoff for the Blue force than will an alertness increase.

On the other hand, the smaller Red force must concentrate on alertness. In our example, because Red is smaller, it must improve the alertness level for each ship first. The payoff in survivability is greater than the payoff from a better offensive strike.

In the second variation, the total strengths of the Red and Blue forces are equal. The model showed that, as the scouting effectiveness σ and alertness τ of both forces increase, the results become increasingly more sensitive to the scouting effectiveness σ rather than to the alertness τ of the two forces.

Based on this variation, we can conclude the following from the model results. When scouting effectiveness and alertness are low, we observe weak offensive and defensive power on both sides. In these particular circumstance it is more advantageous to strengthen the defense by greater alertness, because a stronger defense contributes more to deflect the enemy's (relatively weak) offense than an offensive power increase (bigger scouting effectiveness) contributes to getting through the enemy's defense. As scouting effectiveness and alertness increase, defensive firepower cannot match the offensive improvement, which will "saturate" the defense. Therefore, when scouting effectiveness and alertness both are high, a more advantageous ratio of remaining staying power results from continued improvement of scouting effectiveness, although losses to both sides will be severe.

In the third variation, the Red force strength is 1.50 times that of the Blue force. The model now is found initially to be more sensitive to the Red force alertness τ_R than to its scouting effectiveness σ_R. However, as the alertness of the Red force τ_R increases, the model becomes more sensitive to Red *force* alertness τ_R than to its scouting effectiveness σ_R. On the other hand the model begins and remains more sensitive to the alertness τ_B of the weaker Blue force than to its scouting effectiveness σ_B.

In this variation, the situation is approximately the opposite of the first variation. For example, assume the Blue force has six ships and the Red force has nine ships in an area. From the point of view of the Red force, because the force is bigger the alertness level for each ship can be less and yet total defensive force of Red's ships will still strike down most or all enemy missiles. But the Red force total alertness must first be made high enough to defeat Blue's strike. Only after that can the Red force give priority to strike out against the Blue force. Yet if the Red force is going to win Red needs to know Blue positions, and strike strongly. That is, the scouting effectiveness increase for Blue pays off more than will an alertness increase, but only after alertness is made high enough to defend the Blue force.

On the other hand, the smaller Blue force must concentrate on alertness. In our example, from the point of view of Red force, because the force is smaller, it makes sense to improve the alertness level for each ship. The payoff in survivability is greater than the payoff in a better offensive strike.

b. Situation in Which Each Blue Unit is Stronger

In the second situation, the unit salvo striking power of the Blue force is assumed to be greater than that of the Red force ($\beta_2 > \alpha_2$) and the unit defensive power of the Blue force is also assumed to be greater ($\beta_3 > \alpha_3$). Three major variations were examined which can be summarized as shown in Table IV.

TABLE IV. MODEL SENSITIVITY WHEN STRIKING AND DEFENSIVE POWER OF BLUE UNITS ARE GREATER THAN RED UNITS

$(\beta_2 > \alpha_2, \beta_3 > \alpha_3)$

Variation	Red Force Factor for which Model is Most Sensitive	Blue Force Factor for which Model is Most Sensitive
Equality of Numbers [R = B]	τ_R	τ_B except σ_B if σ_A is high
Red superior to Blue [R = 1.50B]	σ_R	σ_B if σ_B and τ_B are high; τ_B if σ_B and τ_B are low
Red twice as many as Blue [R = 2B]	τ_R except σ_R if σ_B is high and τ_R is low	τ_B

NOTE: In situations not explained, the pattern is too complex to reduce to table form.

In the first variation, the numerical strengths of both forces are equal, but the Blue force has greater unit striking and defensive power and therefore Blue combat power is greater overall. Results indicated that the model was consistently more sensitive to Red alertness τ_R than to Red scouting effectiveness σ_R. Blue also was usually better of the improve alertness τ_B. But as scouting effectiveness σ of both forces increased, the model results became more sensitive to the Blue force scouting effectiveness σ_B than alertness τ_B.

The weaker Red force should give first priority to alertness in nearly all circumstance. In our example a higher Red alertness level for each ship will pay off in a Force Exchange Ratio (FER) greater than the payoff from a better offensive strike.

In most cases Blue also improves the FER to its best advantage by improving defenses through alertness τ_B. When the enemy's scouting effectiveness σ_B is high, however, Blue cannot get enough advantage from improving alertness τ_B, so is better off by increasing its offense through better scouting effectiveness σ_B.

In the second variation, the Red force numerical strength is 1.50 times that of the Blue force. As the force-wide scouting effectiveness ($R \times \sigma_R$) and the alertness ($R \times \tau_R$) of the Red force increase, the results show that the model is nearly always more sensitive to the whole Red force's scouting effectiveness than to Red alertness. Blue's situation is complex. As the scouting effectiveness $B \times \sigma_B$ of the Blue force increases, the model becomes more sensitive to scouting effectiveness σ_B than to the alertness τ_B of the smaller but individually more capable Blue force. On the other hand, when both the baseline alertness τ_B and scouting effectivenessσ_B

of the Blue force are low, Blue force alertness τ_B is more important than scouting effectiveness σ_B.

In this variation Red is numerically greater than Blue, but Blue has greater unit striking and defensive power. For example, assume the Blue force has six ships and the Red force has nine ships in an area. From the point of view of Red, because his force is more numerous, the alertness level for each ship can be less and yet total defensive force of all Red's ships will still strike down most enemy missiles. Yet for Red to win, it needs to strike strongly by increasing its scouting effectiveness σ_R.

Blue is numerically smaller, but has greater individual unit power. Blue's situation is complex. The reader would be well advised to refer to Table IIb and draw his own conclusions for variation two.

In the third variation, Red's numerical strength is twice that of Blue. As the alertness τ_R of the Red force increases, the results indicate that the model becomes more sensitive to Red force alertness τ_R than to its scouting effectiveness σ_R. On the other hand, the model is constantly more sensitive to the alertness τ_B of the Blue force than to scouting effectiveness σ_B.

In most cases Red improves its FER to its best advantage by improving defenses through alertness τ_R. This, paradoxically, is especially so when alertness τ_R is already high. But if Blue's scouting effectiveness σ_B is high and Red's alertness τ_R is low, Red improves its FER more by an increase in its scouting effectiveness σ_R. This is because a marginal increase in its defenses cannot stop Blue's missiles sufficiently to be worthwhile.

The smaller Blue force must concentrate on alertness. The payoff in survivability will be greater than the payoff from a better offensive strike.

Finally, it is worth stating again that these conclusions are not general. They have been drawn to show the power of the Staying Power Ratio and Force Effectiveness Ratio, once the actual opposing forces and their characteristics have been estimated.

B. RECOMMENDATIONS FOR FURTHER RESEARCH

The possibilities for further parametric study and sensitivity analysis have not been exhausted in this thesis. Our recommendations for future research are as follows:

- Perform a similar sensitivity analysis with the model when the scouting effectiveness and alertness of both forces vary during the battle. One case of special interest is that in which scouting effectiveness decreases (because of confusion after the first attack), while alertness increases (because the force is more vigilant after the first attack).

- Extend the methodology developed in this thesis to perform a further sensitivity analysis on the model in different situations. An example would be to hold fixed the scouting effectiveness and alertness of both forces while varying the other parameters: staying power, striking power, and defensive power of each unit.

- Perform a similar sensitivity analysis when the two forces are different numerically.
- Perform a similar sensitivity analysis when the two forces do not fire simultaneously.
- Use historical or wargaming data to validate the model and examine its sensitivity to human factors, scouting effectiveness, and alertness.

APPENDIX A. COMPUTER PROGRAM TO CALCULATE RR(T)

The following is a program listing of the computer implementation of the naval combat model. It has been modified to calculate the ratio R(t), as described in Chapter IV, Section A.2. The program was coded in Fortran 77 and run on an IBM 3033 AP mainframe computer at the Naval Postgraduate School.

```
      PROGRAM NAVCOM1

*     ASSUMPTIONS
****************
*     1. Same type of missiles for both forces (the average
*        missile)
*     2. Each force is consisting of one group
*     3. In the duration of each discrete time step we assume
*       that both forces receive one pulse, either both forces
*        fire simultaneously or the one force returns fire,
*        after it has already received its opponent's pulse
*        (with reduced capabilities).
*
*
*
```

```
      INTEGER NB,NR,DB,DR,BUNITS,RUNITS,NPULSE,W,I,Z,K,L,Q,S,J
      REAL     LOSSB(20),LOSSR(20),SPB,SPR,SFB,SFR,R(20)
      REAL  TOTSPB,TOTSPR,REMSPB(20),REMSPR(20),BRPNTB,BRPNTR
      REAL X(20),MB,MR,TLOSSB(20),TLOSSR(20),AFB,AFR,UPDNB(20)
      REAL  UPDNR(20), NDB,NDR,H,MDB,MDR,REMPB(20),REMPR(20)
*     INITIALIZATION
      PRINT *, 'PLEASE, ENTER THE FOLLOWING DATA FOR BOTH'
      PRINT *, 'OPPONENTS BE CAREFUL, THE FIRST VALUE YOU ENTER'
      PRINT *, 'TO BE FOR THE BLUE FORCE AND THE SECOND FOR THE'
      PRINT *, 'RED FORCE NUMBER OF UNITS IN EACH FORCE'
      READ  *,  BUNITS,RUNITS
      PRINT *, 'FULL LOAD DISPLACEMENT FOR BOTH FORCES'
      READ  *,  DB,DR
      PRINT *, 'SCOUTING FUNCTION FOR BOTH FORCES'
      READ  *,  SFB,SFR
      PRINT *, 'ALERTNESS EFFECTIVENESS FOR BOTH FORCES'
      READ  *,  AFB,AFR
      PRINT *, 'NUMBER OF MISSILES A UNIT  CAN FIRE PER SALVO'
      PRINT *, 'FOR BOTH FORCES (REAL)'
      READ  *,  MB,MR
      PRINT *, 'MULTIPLICATIVE DEGRADER FOR M FOR BOTH FORCES'
      READ  *,  MDB,MDR
      PRINT *, 'NUMBER OF MISSILES A UNIT CAN SHOT DOWN IN ONE'
      PRINT *, 'SALVO FOR BOTH FORCES (INTEGER)'
      READ  *,  NB,NR
```

```
PRINT *, 'MULTIPLICATIVE DEGRADER FOR N FOR BOTH FORCES'
READ  *,  NDB,NDR
PRINT *, 'THE BREAK POINT FOR BOTH FORCES (REAL BETWEEN'
PRINT *, '0.0 AND 1.0, NOTE: THIS IS THE PERCENTAGE OF'
PRINT *, 'THE INITIAL STAYING POWER BELOW WHICH THE'
PRINT *, 'BATTLE IS CONSIDERED TERMINATED.'
PRINT *, 'IF YOU DO NOT WISH TO ASSIGN VALUES FOR'
PRINT *, 'BREAK POINT ENTER 0.0, 0.0'
READ  *,  BRPNTB,BRPNTR
PRINT *,  'PROBABILITY OF HIT VERSUS UNDEFENDED TARGET'
READ  *,  H
PRINT *, 'NUMBER OF DISCRETE TIME STEPS FOR THE PROGRAM'
PRINT *, 'TO BE EXECUTED'
READ  *,  NPULSE

SPB = 0.070*((REAL(DB)**(1.0/3.0))
SPR = 0.070*((REAL(DR)**(1.0/3.0))
TOTSPB = SPB*REAL(BUNITS)
TOTSPR = SPR*REAL(RUNITS)

I = 0
LOSSB(I) = 0.0
LOSSR(I) = 0.0
TLOSSB(I) = 1.0
TLOSSR(I) = 1.0
UPDNB(I) = NB
```

```
      UPDNR(I) = NR
      REMSPB(I) = TOTSPB
      REMSPR(I) = TOTSPR
      REMPB(I) = MB
      REMPR(I) = MR

      PRINT *, 'ARE BOTH FORCES FIRING SIMULTANEOUSLY THE '
      PRINT *, ' PULSES? (1 IF YES, OR 0 IF NO)'
      READ  *,  W
      IF(W.EQ.0) GO TO 15
*
*     PROGRAM EXECUTION
*     BOTH FORCES FIRE SIMULTANEOUSLY
*
10    I = I + 1
      LOSSB(I) = (SFR*REAL(RUNITS)*MR*MDR*H - AFB*REAL(BUNITS)*
     +UPDNB(I-1)*NDB)/REMSPB(I-1)
      IF(LOSSB(I).LT.0.0) LOSSB(I) = 0.0
      IF(LOSSB(I).GE.1.0) LOSSB(I) = 0.999
      REMSPB(I) = REMSPB(I-1)*(1.0 - LOSSB(I))
      REMPB(I) = REMPB(I-1)*(1.0 - LOSSB(I))
      UPDNB(I) = UPDNB(I-1)*(1.0 - LOSSB(I))
      TLOSSB(I) = 1.0 - (REMSPB(I)/TOTSPB)

      LOSSR(I) = (SFB*REAL(BUNITS)*MB*MDB*H - AFR*REAL(RUNITS)*
     +UPDNR(I-1)*NDR)/REMSPR(I-1)
```

```
      IF(LOSSR(I).LT.0.0) LOSSR(I) = 0.0
      IF(LOSSR(I).GE.1.0) LOSSR(I) = 0.999
      REMSPR(I) = REMSPR(I-1)*(1.0 - LOSSR(I))
      REMPR(I) = REMPR(I-1)*(1.0 - LOSSR(I))
      UPDNR(I) = UPDNR(I-1)*(1.0 - LOSSR(I))
      TLOSSR(I) = 1.0 - (REMSPR(I)/TOTSPR)

      IF(((1.0 - TLOSSB(I)).LE.BRPNTB).AND.((1.0 - TLOSSR(I)).
     +LE.BRPNTR)) GO TO 991
      IF((1.0 - TLOSSB(I)).LE.BRPNTB) GO TO 992
      IF((1.0 - TLOSSR(I)).LE.BRPNTR) GO TO 993
      MB = REMPB(I)
      MR = REMPR(I)

      IF(I.LT.NPULSE) GO TO 10

15    CONTINUE
      IF(W.EQ.0) THEN
          PRINT *, 'WHICH FORCE FIRES FIRST? NOTE:THAT MEANS '
          PRINT *, 'THAT THE OTHER FORCE RECEIVES THE PULSE '
          PRINT *, 'FIRST AND THEN RETURNS THE FIRE (0 FOR RED'
           PRINT *, ' 1 FOR BLUE)'
          READ  *,  Z
          IF(Z.EQ.1) GO TO 30
*
*     RED FORCE FIRE FIRST
```

```
*
20  I = I + 1
    LOSSB(I) = (SFR*REAL(RUNITS)*REMPR(I-1)*MDR*H - AFB*
   +REAL(BUNITS)*UPDNB(I-1)*NDB)/REMSPB(I-1)
    IF(LOSSB(I).LT.0.0) LOSSB(I) = 0.0
    IF(LOSSB(I).GE.1.0) LOSSB(I) = 0.999
    REMSPB(I) = REMSPB(I-1)*(1.0 - LOSSB(I))
    REMPB(I) = REMPB(I-1)*(1.0 - LOSSB(I))
    UPDNB(I) = UPDNB(I-1)*(1.0 - LOSSB(I))
    TLOSSB(I) = 1.0 - (REMSPB(I)/TOTSPB)
    IF((1.0 - TLOSSB(I)).LE.BRPNTB) GO TO 992]

    LOSSR(I) =(SFB*REAL(BUNITS)*REMPB(I-1)*MDB*H - AFR*
   +REAL(RUNITS)*UPDNR(I-1)*NDR)/REMSPR(I-1)
    IF(LOSSR(I).LT.0.0) LOSSR(I) = 0.0
    IF(LOSSR(I).GE.1.0) LOSSR(I) = 0.999
    REMSPR(I) = REMSPR(I-1)*(1.0 - LOSSR(I))
    REMPR(I) = REMPR(I-1)*(1.0 - LOSSR(I))
    UPDNR(I) = UPDNR(I-1)*(1.0 - LOSSR(I))
    TLOSSR(I) = 1.0 - (REMSPR(I)/TOTSPR)
    IF((1.0 - TLOSSR(I).LE.BRPNTR*TOTSPR) GO TO 993
    IF(I.LT.NPULSE) GO TO 20

30  CONTINUE
    IF(Z.EQ.1) THEN
40  I = I + 1
```

```
*
*   BLUE FORCE FIRE FIRST
*
    LOSSR(I) = (SFB*REAL(BUNITS)*REMPB(I-1)*MDB*H - AFR*
   +REAL(RUNITS)*UPDNR(I-1)*NDR)/REMSPR(I-1)
    IF(LOSSR(I).LT.0.0) LOSSR(I) = 0.0
    IF(LOSSR(I).GE.1.0) LOSSR(I) = 0.999
    REMSPR(I) = REMSPR(I-1)*(1.0 - LOSSR(I))
    REMPR(I) = REMPR(I-1)*(1.0 - LOSSR(I))
    UPDNR(I) = UPDNR(I-1)*(1.0 - LOSSR(I))
    TLOSSR(I) = 1.0 - (REMSPR(I)/TOTSPR)
    IF((1.0 - TLOSSR(I)).LE.BRPNTR) GO TO 993

    LOSSB(I) = (SFR*REAL(RUNITS)*REMPB(I-1)*MDR*H - AFB*
   +REAL(BUNITS)*UPDNB(I-1)*NDB)/REMSPB(I-1)
    IF(LOSSB(I).LT.0.0) LOSSB(I) = 0.0
    IF(LOSSB(I).GE.1.0) LOSSB(I) = 0.999
    REMSPB(I) = REMSPB(I-1)*(1.0 - LOSSB(I))
    REMPB(I) = REMPB(I-1)*(1.0 - LOSSB(I))
    UPDNB(I) = UPDNB(I-1)*(1.0 - LOSSB(I))
    TLOSSB(I) = 1.0 - (REMSPB(I)/TOTSPB)
    IF((1.0 - TLOSSB(I)).LE.BRPNTB*TOTSPB) GO TO 992
    IF(I.LT.NPULSE) GO TO 40
    ENDIF
    ENDIF
```

```
      CALL EXCMS('FILEDEF 10 DISK NAVCOM1 OUTPUT A')
      WRITE(10,50)
       WRITE(10,60)
       DO 1000 Q = 1,I
         WRITE(10,70) Q,TLOSSR(Q),REMSPR(Q)
1000   CONTINUE
       WRITE(10,55)
       DO 1500 S = 1,I
         WRITE(10,70) S,TLOSSB(S),REMSPB(S)
1500   CONTINUE
50     FORMAT(5X,'RED FORCE')
55     FORMAT(5X,'BLUE FORCE')
60     FORMAT('0','#PULSES',2X,'TOTAL LOSS',2X,'REM. STAYING
      + POWER')
70     FORMAT(4X,I2,6X,F5.2,8X,F5.2)

       DO 100 K = 1,I
         IF((REMSPR(K-1).EQ.0.0).OR.(REMSPB(K-1).EQ.0.0).OR.
      +         ((REMSPB(K) - REMSPB(K-1)).EQ.0.0)) THEN
            IF(R(K-1).GT.1.0) THEN
               R(K) = R(K-1) + 1.0
            ELSE
               R(K) = 0.0
            ENDIF
            GO TO 100
         ENDIF
```

```
          R(K) = ((REMSPR(K) - REMSPR(K-1))/REMSPR(K-1))/
     +             ((REMSPB(K) - REMSPB(K-1))/REMSPB(K-1))
          WRITE(10.75) K, R(K)
75        FORMAT('R(',I2,'):',F5.2)
100    CONTINUE

       DO 280 J =1,I
          X(J) = REAL(J)
280    CONTINUE
       CALL COMPS
       CALL PLOTD(X,R,I,.TRUE.,'LINLIN','REM. STAYING POWER',
     + '(DREMSPR/REMSPR)/(DREMSPB/REMSPB)','#SALVOS',
     + 'RATIO OF REMAINING STAYING POWER OF BOTH FORCES$')
       CALL DONEPL
       STOP
*

991    CALL EXCMS('FILEDEF 10 DISK NAVCOM1 OUTPUT A')
       WRITE(10,996) 'BOTH FORCES REACHED THEIR BREAK POINT'
       WRITE(10,500)
       WRITE(10,600)
996    FORMAT(3X,A)
       DO 2000 Q = 1,I
          WRITE(10,700) Q,TLOSSR(Q),REMSPR(Q)
2000   CONTINUE
       WRITE(10,550)
```

```
      DO 2500 S = 1,I
        WRITE(10,700) S,TLOSSB(S),REMSPB(S)
2500  CONTINUE
500   FORMAT(5X,'RED FORCE')
550   FORMAT(5X,'BLUE FORCE')
600   FORMAT('0','#PULSES',2X,'TOTAL LOSS',2X,'REM. STAYING
     + POWER')
700   FORMAT(4X,I2,6X,F5.2,8X,F5.2)

      DO 200 K = 1,I
        IF((REMSPR(K-1).EQ.0.0).OR.(REMSPB(K-1).EQ.0.0).OR.
     +          ((REMSPB(K) - REMSPB(K-1)).EQ.0.0)) THEN
           IF(R(K-1).GT.1.0) THEN
              R(K) = R(K-1) + 1.0
           ELSE
              R(K) = 0.0
           ENDIF
           GO TO 200
        ENDIF
        R(K) = ((REMSPR(K) - REMSPR(K-1))/REMSPR(K-1))/
     +            ((REMSPB(K) - REMSPB(K-1))/REMSPB(K-1))
        WRITE(10.750) K, R(K)
750     FORMAT('R(',I2,'):',F5.2)
200   CONTINUE
```

```
      DO 380 J =1,I
         X(J) = REAL(J)
380   CONTINUE
      CALL COMPS
      CALL PLOTD(X,R,I,.TRUE.,'LINLIN','REM. STAYING POWER',
     + '(DREMSPR/REMSPR)/(DREMSPB/REMSPB)','#SALVOS',
     + 'RATIO OF REMAINING STAYING POWER OF BOTH FORCES$')
      CALL DONEPL
      STOP
*

992   CALL EXCMS('FILEDEF 10 DISK NAVCOM1 OUTPUT A')
      WRITE(10,899) 'RED FORCE REACHED THEIR BREAK POINT.'
      WRITE(10,899) 'BLUE FORCE WIN'
      WRITE(10,250)
      WRITE(10,260)
899   FORMAT(3X,A)
      DO 210 Q = 1,I
         WRITE(10,270) Q,TLOSSR(Q),REMSPR(Q)
210   CONTINUE
      WRITE(10,255)
      DO 220 S = 1,I
         WRITE(10,270) S,TLOSSB(S),REMSPB(S)
220   CONTINUE
250   FORMAT(5X,'RED FORCE')
255   FORMAT(5X,'BLUE FORCE')
```

```
260  FORMAT('0','#PULSES',2X,'TOTAL LOSS',2X,'REM. STAYING
    + POWER')
270  FORMAT(4X,I2,6X,F5.2,8X,F5.2)

     DO 4000 K = 1,I
        IF((REMSPR(K-1).EQ.0.0).OR.(REMSPB(K-1).EQ.0.0).OR.
    +          ((REMSPB(K) - REMSPB(K-1)).EQ.0.0)) THEN
           IF(R(K-1).GT.1.0) THEN
                  R(K) = R(K-1) + 1.0
           ELSE
                  R(K) = 0.0
           ENDIF
           GO TO 4000
        ENDIF
        R(K) = ((REMSPR(K) - REMSPR(K-1))/REMSPR(K-1))/
    +             ((REMSPB(K) - REMSPB(K-1))/REMSPB(K-1))
        WRITE(10.7) K, R(K)
7       FORMAT('R(',I2,'):',F5.2)
4000 CONTINUE

     DO 580 J =1,I
        X(J) = REAL(J)
580  CONTINUE
     CALL COMPS
     CALL PLOTD(X,R,I,.TRUE.,'LINLIN','REM. STAYING POWER',
    + '(DREMSPR/REMSPR)/(DREMSPB/REMSPB)','#SALVOS',
```

```
     + 'RATIO OF REMAINING STAYING POWER OF BOTH FORCES$')
      CALL DONEPL
      STOP
*
      END
```

APPENDIX B. COMPUTER PROGRAM TO CALCULATE FOR VARIOUS VALUES OF SCOUTING AND ALERTNESS

The following is a listing of program codes written to examine all possible values of σ_R, τ_R, σ_B, and τ_B from 0.60 to 0.95 in increments of 0.05, as described in Chapter IV, Section B.2. The program was coded in Fortran 77 and run on an IBM 3033 AP mainframe computer at the Naval Postgraduate School.

```
      PROGRAM SENSIT
*     ASSUMPTIONS
******************
*     ALL SHIPS OF BOTH FORCES ARE EQUIVALENT
*     ALL UNITS HAVE THE SAME STRIKING POWER
*     ALL UNITS HAVE THE SAME DEFENSIVE POWER
*     ALL UNITS HAVE THE SAME STAYING POWER
*     THE RED FORCE RESPECTIVE IN THE PROGRAM BY ALFA AND THE
*     BLUE FORCE BY THE BETA

      INTEGER   ALFA2,ALFA3,BETA2,BETA3,I,J,V,W
      REAL      K,H,SCALFA,SCBETA,ALTALFA,ALTBETA,ROENA,RODUO
      REAL    ARRAY1(10,10),ARRAY2(10,10)
```

```
      PRINT *,  'PLEASE, ENTER THE FOLLOWING DATA FOR BOTH
OPPONENTS'
      PRINT *, 'BE CAREFUL, THE FIRST VALUE YOU ENTER TO BE FOR'
      PRINT *, 'THE ALFA FORCE AND THE SECOND FOR THE BETA
     +           FORCE'
      PRINT *, '-----------------------------------------------'
      PRINT *, 'ENTER THE MULTIPLIER FOR BETA FORCE'
      READ  *,  K
      PRINT *, 'ENTER THE UNIT SALVO STRIKING POWER IN HITS '
      PRINT *, 'OF BOTH FORCES (INTEGER)'
      READ  *,  ALFA2,BETA2
      PRINT *, 'ENTER THE DEFENSIVE POWER FOR BOTH FORCES'
      READ  *,  ALFA3,BETA3
      PRINT *, 'ENTER THE PROBABILITY OF HIT VS UNDEFENDED
     +      TARGET'
      READ  *,  H
*
      CALL EXCMS('FILEDEF 10 DISK SENSIT OUTPUT A')

      IF(K.EQ.1) THEN
         WRITE(10,5)'THE TWO(2) FORCES ARE EQUAL'
      ELSE
         WRITE(10,12)'THE A FORCE IS ',K,'TIMES AS MUCH AS B
     + FORCE'
      ENDIF
*
```

```
      WRITE(10,15)'THE UNIT STRIKING POWER IN HITS OF ALFA
     +FORCE IS',ALFA2
      WRITE(10,16)'  AND FOR THE BETA FORCE IS ',BETA2
      WRITE(10,17)'THE UNIT DEFENSIVE POWER OF ALFA FORCE IS'
     +          ,ALFA3
      WRITE(10,18)'  AND FOR THE B FORCE IS ',BETA3
*
      DO 800 V = 1,8
          SCBETA = 0.05*REAL(V+5) + 0.30
          DO 700 W = 1,8
              ALTBETA = 0.05*REAL(W+5) + 0.30
              I = 0
100           CONTINUE
              I = I + 1
              SCALFA = 0.05*REAL(I+5) + 0.30
              J = 0
200           CONTINUE
              J = J + 1
              ALTALFA = 0.05*REAL(J+5) + 0.30
              IF((K*REAL(ALFA2)*H*SCALFA).EQ.
      +                   (ALTBETA*REAL(BETA3))) THEN
                  ROENA = INF
                  GO TO 300
              ENDIF
              ROENA = (REAL(ALFA2)*H*(SCBETA*REAL(BETA2)*H
      +       - K*REAL(ALFA3)*ALTALFA))/(REAL(ALFA3)*
```

```
     +      (K*REAL(ALFA2)*SCALFA*H - ALTBETA*REAL(BETA3)))
 300        IF(ROENA.GT.1.0) THEN
               ARRAY1(I,J) = 1.0
            ELSE
               ARRAY1(I,J) = 0.0
            ENDIF
            IF(J.LT.8) GO TO 200
            IF(I.LT.8) GO TO 100


            WRITE(10,400)'FOR SCOUTING OF BETA FORCE',SCBETA
     +                     ' AND ALERTNESS OF BETA FORCE',ALTBETA
            WRITE(10,350)
            WRITE(10,320)'ALTALFA'
            WRITE(10,310)
            WRITE(10,250)
 250        FORMAT(1X,'SCALFA')
 310        FORMAT(10X,'-----------------------------------')
 320        FORMAT(1X,A,'  0.60 0.65 0.70 0.75 0.80 0.85 0.90
     +       0.95 ')
 350        FORMAT(10X,'------------------------------------'
 400        FORMAT(1X,A,F5.2,A,F5.2)
            DO 600 I = 1,8
               WRITE(10,500) 0.05*REAL(I+5)+0.3,
     +          (ARRAY1(I,J),J=1,8)
 500           FORMAT(1X,F5.2,3X,8F5.1)
```

```
600         CONTINUE
            WRITE(10,*)
700     CONTINUE
800   CONTINUE
*
      IF(K.EQ.1) THEN
        WRITE(10,5)'THE TWO(2) FORCES ARE EQUAL'
      ELSE
        WRITE(10,12)'THE A FORCE IS ',K,'TIMES AS MUCH AS B
   +    FORCE'
      ENDIF
*
      WRITE(10,15)'THE UNIT STRIKING POWER IN HITS OF ALFA
     +FORCE IS',ALFA2
      WRITE(10,16)'  AND FOR THE BETA FORCE IS ',BETA2
      WRITE(10,17)'THE UNIT DEFENSIVE POWER OF ALFA FORCE IS'
   +   ,ALFA3
      WRITE(10,18)'  AND FOR THE B FORCE IS ',BETA3
5     FORMAT(1X,A)
12    FORMAT(1X,A,F5.2,A)
15    FORMAT(1X,A,I2)
16    FORMAT(1X,A,I2)
17    FORMAT(1X,A,I2)
18    FORMAT(1X,A,I2)
*
      DO 80 I = 1,8
```

```
      SCALFA = 0.05*REAL(I+5) + 0.30
      DO 70 J = 1,8
         ALTALFA = 0.05*REAL(J+5) + 0.30
         V = 0
10       CONTINUE
         V = V + 1
         SCBETA = 0.05*REAL(V+5) + 0.30
         W = 0
20       CONTINUE
         W = W + 1
         ALTBETA = 0.05*REAL(W+5) + 0.30
         IF((K*REAL(BETA2)*H*SCBETA).EQ.
     +             (ALTALFA*REAL(ALFA3))) THEN
            RODUO = INF
            GO TO 30
         ENDIF
         RODUO = (REAL(BETA2)*H*(SCALFA*REAL(ALFA2)*H
     +   - K*REAL(BETA3)*ALTBETA))/(REAL(BETA3)*
     +   (K*REAL(BETA2)*SCBETA*H - ALTALFA*REAL(ALFA3)))
30       IF(RODUO.GT.1.0) THEN
            ARRAY2(V,W) = 1.0
         ELSE
            ARRAY2(V,W) = 0.0
         ENDIF
         IF(W.LT.8) GO TO 20
         IF(V.LT.8) GO TO 10
```

```
          WRITE(10,40)'FOR SCOUTING OF ALFA FORCE',SCALFA
     +                ' AND ALERTNESS OF ALFA FORCE',ALTALFA
          WRITE(10,35)
          WRITE(10,32)'ALTBETA'
          WRITE(10,31)
          WRITE(10,25)
25        FORMAT(1X,'SCBETA')
31        FORMAT(10X,'-------------------------------------')
32        FORMAT(1X,A,'  0.60 0.65 0.70 0.75 0.80 0.85 0.90
     + 0.95 ')
35        FORMAT(10X,'-------------------------------------
     + -----')
40        FORMAT(1X,A,F5.2,A,F5.2)
          DO 600 V = 1,8
             WRITE(10,50) 0.05*REAL(V+5)+0.3,
     + (ARRAY2(V,W),W=1,8)
50           FORMAT(1X,F5.2,3X,8F5.1)
60        CONTINUE
          WRITE(10,*)
70     CONTINUE
80   CONTINUE

     STOP
     END
```

APPENDIX C

Tables I, II, and III show the output of the Appendix B computer program (that is, the ratios ρ_1 and ρ_2) as described in Chapter IV, Section B.3. The values represented in this APPENDIX are: $\alpha_2 = \beta_2 = 3$ and $\alpha_3 = \beta_3 = 2$.

Each table represents a different force condition, as described below. The description appears in the upper left corner of each table.

Tables Ia and Ib

The Alpha force is 0.75 times as large as the Beta force.

The Unit Striking Power in hits of the Alpha force is 3 and for the Beta force is 3.

The Unit Defensive Power of the Alpha force is 2 and for the Beta force is 2.

Tables IIa and IIb

Both forces are equal in size.

The Unit Striking Power in hits of the Alpha force is 3 and for the Beta force is 3.

The Unit Defensive Power of the Alpha force is 2 and for the Beta force is 2.

Tables IIIa and IIIb

The Alpha force is 1.50 times as large as the Beta force.

The Unit Striking Power in hits of the Alpha force is 3 and for the Beta force is 3.

The Unit Defensive Power of the Alpha force is 2 and for the Beta force is 2.

The following variable names are used in accompanying Tables I, II, and III instead of the terms used in Chapter IV. Their location is as illustrated in the sample below.

A force	=	Red force
B force	=	Blue force
ALTALFA	=	Alertness of Red force
ALTBETA	=	Alertness of Blue force
SCALFA	=	Scouting effectiveness of Red force
SCBETA	=	Scouting effectiveness of Blue force.

TABLE Ia, IIa, IIIa

FOR SCOUTING OF ALFA FORCE.... AND ALERTNESS OF ALFA FORCE....

ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCBETA								
0.60								
.								
.								
0.95								

TABLE Ib, IIb, IIIb

FOR SCOUTING OF BETA FORCE.... AND ALERTNESS OF BETA FORCE....

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60								
.								
.								
0.95								

TABLE Ia

TABLE Ib

[illegible]

TABLE IIa

TABLE IIb

TABLE IIIa

[illegible]

TABLE IIIb

THE A FORCE IS 1.50 TIMES AS MUCH AS B FORCE

[illegible]

[illegible]

[illegible]	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

[illegible]	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

[illegible]	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

[illegible]	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

[illegible]	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

[illegible]	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

[illegible]	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

[illegible]	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0	1.0						
0.65	1.0							
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

[illegible]

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0	1.0						
0.65	1.0							
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

APPENDIX D

Tables I, II, and III show the output of the Appendix B computer program (that is, the ratios ρ_1 and ρ_2) as described in Chapter IV, Section B.3. The values represented in this APPENDIX are: $\beta_2 = 4 > \alpha_2 = 3$ and $\beta_3 = 3 > \alpha_3 = 2$.

Each table represents a different force condition, as described below. The description appears in the upper left corner of each table.

Tables Ia and Ib

Both forces are equal in size.

The Unit Striking Power in hits of the Alpha force is 3 and for the Beta force is 4.

The Unit Defensive Power of the Alpha force is 2 and for the Beta force is 3.

Tables IIa and IIb

The Alpha force is 1.50 times as large as the Beta force.

The Unit Striking Power in hits of the Alpha force is 3 and for the Beta force is 4.

The Unit Defensive Power of the Alpha force is 2 and for the Beta force is 3.

Tables IIIa and IIIb

The Alpha force is 2 times as large as the Beta force.

The Unit Striking Power in hits of the Alpha force is 3 and for the Beta force is 4.

The Unit Defensive Power of the Alpha force is 2 and for the Beta force is 3.

The following variable names are used in accompanying Tables I, II, and III instead of the terms used in Chapter IV. Their location is as illustrated in the sample below.

A force = Red force

B force = Blue force

ALTALFA = Alertness of Red force

ALTBETA = Alertness of Blue force

SCALFA = Scouting effectiveness of Red force

SCBETA = Scouting effectiveness of Blue force.

TABLE Ia, IIa, IIIa

FOR SCOUTING OF ALFA FORCE.... AND ALERTNESS OF ALFA FORCE....

ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCBETA								
0.60								
.								
.								
0.95								

TABLE Ib, IIb, IIIb

FOR SCOUTING OF BETA FORCE.... AND ALERTNESS OF BETA FORCE....

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60								
.								
.								
0.95								

TABLE Ia

[illegible]
[illegible]
[illegible]
[illegible]
[illegible]

FOR SCOUTING OF ALFA FORCE 0.80 AND ALERTNESS OF ALFA FORCE 0.60

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.80 AND ALERTNESS OF ALFA FORCE 0.65

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.80 AND ALERTNESS OF ALFA FORCE 0.70

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.80 AND ALERTNESS OF ALFA FORCE 0.75

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.80 AND ALERTNESS OF ALFA FORCE 0.80

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.80 AND ALERTNESS OF ALFA FORCE 0.85

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.80 AND ALERTNESS OF ALFA FORCE 0.90

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.80 AND ALERTNESS OF ALFA FORCE 0.95

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.85 AND ALERTNESS OF ALFA FORCE 0.60

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.85 AND ALERTNESS OF ALFA FORCE 0.65

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.85 AND ALERTNESS OF ALFA FORCE 0.70

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.85 AND ALERTNESS OF ALFA FORCE 0.75

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.85 AND ALERTNESS OF ALFA FORCE 0.80

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.85 AND ALERTNESS OF ALFA FORCE 0.85

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.85 AND ALERTNESS OF ALFA FORCE 0.90

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.85 AND ALERTNESS OF ALFA FORCE 0.95

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0	1.0						
0.65	1.0							
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.90 AND ALERTNESS OF ALFA FORCE 0.60

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.90 AND ALERTNESS OF ALFA FORCE 0.65

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.90 AND ALERTNESS OF ALFA FORCE 0.70

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.90 AND ALERTNESS OF ALFA FORCE 0.75

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.90 AND ALERTNESS OF ALFA FORCE 0.80

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.90 AND ALERTNESS OF ALFA FORCE 0.85

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0	1.0						
0.65	1.0							
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.90 AND ALERTNESS OF ALFA FORCE 0.90

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0	1.0						
0.65	1.0							
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.90 AND ALERTNESS OF ALFA FORCE 0.95

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0	1.0	1.0					
0.65	1.0	1.0						
0.70	1.0							
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.95 AND ALERTNESS OF ALFA FORCE 0.60

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.95 AND ALERTNESS OF ALFA FORCE 0.65

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.95 AND ALERTNESS OF ALFA FORCE 0.70

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.95 AND ALERTNESS OF ALFA FORCE 0.75

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.95 AND ALERTNESS OF ALFA FORCE 0.80

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0	1.0						
0.65	1.0							
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.95 AND ALERTNESS OF ALFA FORCE 0.85

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0	1.0						
0.65	1.0							
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.95 AND ALERTNESS OF ALFA FORCE 0.90

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0	1.0	1.0					
0.65	1.0	1.0						
0.70	1.0							
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF ALFA FORCE 0.95 AND ALERTNESS OF ALFA FORCE 0.95

SCBETA \ ALTBETA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.60	1.0	1.0	1.0	1.0				
0.65	1.0	1.0	1.0					
0.70	1.0	1.0						
0.75	1.0							
0.80								
0.85								
0.90								
0.95								

TABLE Ib

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
[illegible]								
0.60								
0.65								
0.70								
0.75								
0.80	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.85	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.90	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.95	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
[illegible]								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.90	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.95	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
[illegible]								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.95	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
[illegible]								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

ALPHA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
[illegible]								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

TABLE II a

[illegible]

TABLE IIb

TABLE IIIa

TABLE IIIb

[illegible]

FOR SCOUTING OF BETA FORCE [illegible] AND ALERTNESS OF BETA FORCE 0.60

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE [illegible] AND ALERTNESS OF BETA FORCE 0.65

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE [illegible] AND ALERTNESS OF BETA FORCE 0.70

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE [illegible] AND ALERTNESS OF BETA FORCE 0.75

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE [illegible] AND ALERTNESS OF BETA FORCE 0.80

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE [illegible] AND ALERTNESS OF BETA FORCE 0.85

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60	1.0	1.0						
0.65	1.0							
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE [illegible] AND ALERTNESS OF BETA FORCE 0.90

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60	1.0	1.0						
0.65	1.0							
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE [illegible] AND ALERTNESS OF BETA FORCE 0.95

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60	1.0	1.0	1.0					
0.65	1.0	1.0						
0.70	1.0							
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.90 AND ALERTNESS OF BETA FORCE 0.60

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.90 AND ALERTNESS OF BETA FORCE 0.65

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.90 AND ALERTNESS OF BETA FORCE 0.70

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.90 AND ALERTNESS OF BETA FORCE 0.75

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.90 AND ALERTNESS OF BETA FORCE 0.80

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.90 AND ALERTNESS OF BETA FORCE 0.85

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60	1.0	1.0						
0.65	1.0							
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.90 AND ALERTNESS OF BETA FORCE 0.90

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60	1.0	1.0						
0.65	1.0							
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.90 AND ALERTNESS OF BETA FORCE 0.95

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60	1.0	1.0	1.0					
0.65	1.0	1.0						
0.70	1.0							
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.95 AND ALERTNESS OF BETA FORCE 0.60

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.95 AND ALERTNESS OF BETA FORCE 0.65

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.95 AND ALERTNESS OF BETA FORCE 0.70

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60								
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.95 AND ALERTNESS OF BETA FORCE 0.75

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60	1.0							
0.65								
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.95 AND ALERTNESS OF BETA FORCE 0.80

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60	1.0	1.0						
0.65	1.0							
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.95 AND ALERTNESS OF BETA FORCE 0.85

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60	1.0	1.0						
0.65	1.0							
0.70								
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.95 AND ALERTNESS OF BETA FORCE 0.90

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60	1.0	1.0	1.0					
0.65	1.0	1.0						
0.70	1.0							
0.75								
0.80								
0.85								
0.90								
0.95								

FOR SCOUTING OF BETA FORCE 0.95 AND ALERTNESS OF BETA FORCE 0.95

ALTALFA	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
SCALFA								
0.60	1.0	1.0	1.0	1.0				
0.65	1.0	1.0	1.0					
0.70	1.0	1.0						
0.75	1.0							
0.80								
0.85								
0.90								
0.95								

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