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Universal Properties of The Resonance Curve of Complex Systems *

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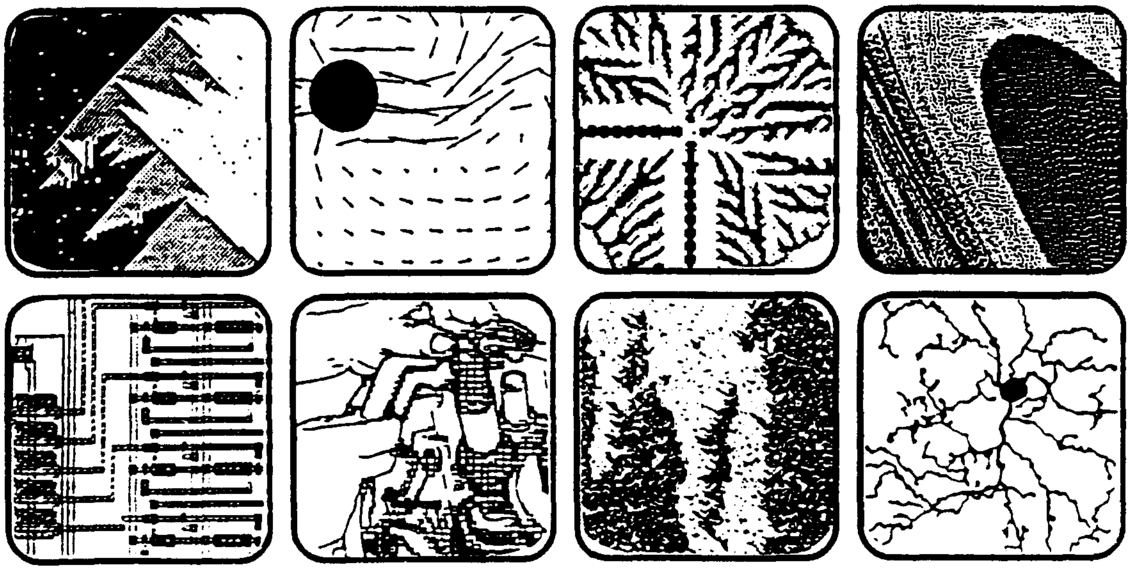

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Abstract. The dynamics of a large variety of complex systems are confined to a low-dimensional manifold. We show that the resonance curve of those systems has a universal shape. The parameters of the resonance curve can be used to characterize a complex system.

INTRODUCTION

Resonance spectroscopy has proved very successful in many fields of physics. However nonlinear oscillators usually respond to a purely periodic perturbation in a complex and often chaotic manner [1] due to the amplitude-frequency coupling. This chaotic response is small and difficult to characterize. [2] It is also not resonant, because the driving force and the oscillator are out of phase [3]. Recently, for oscillators with well-defined energies, a method has been proposed to calculate a driving force which is in phase with the oscillator velocity at all amplitudes. [4] Usually these driving forces are aperiodic and are called resonant driving forces because the reflected energy and the reaction power are zero [5]. Based on these ideas, we introduce a more general defintion of resonance that is also valid for systems without a well-defined energy function. We apply this concept in calculating resonant driving forces for the chaotic dynamics of an logistic map.

GENERAL RESONANCE SPECTROSCOPY

Let us consider a nonlinear oscillator of type

$$\ddot{y} + \eta_1 \dot{y} + \frac{dV\left(y, \vec{p}_1\right)}{dy} = F\left(t\right) \tag{0.1}$$

where η is a friction constant, $\vec{p}_1$ is a set of parameters of the nonliner potential V and F is a driving force which is independent from y. Let us

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assume that the amplitude y at time $t = 0$ is in the vicinity of a minimum of the potential. To calculate resonant driving force one has to have a model of the dynamics of the unperturbed experimental system

$$\ddot{x} + \eta_2 \dot{x} + \frac{dV(x, \vec{p}_2)}{dx} = 0 \tag{0.2}$$

In order to calculate a resonant driving force the goal equations has to be integrated [5]

$$\ddot{z} + \eta_2 \dot{z} + \frac{dV(z, \vec{p}_2)}{dz} = \eta_3 \dot{z} \tag{0.3}$$

which differs from the dynamics of the model just by an additional friction term. Usually η_3 is larger than η_2. The driving force results from $F(t) = \eta_3 \dot{z}$

A driving force is called resonant if the reaction power is zero, i.e. the transfered energy $P = F\dot{y}$ is either always positive or always negative. If the parameters of the model coincide with the parameters of the system $y(t) = z(t)$ is a solution of Eq. (0.1). In many cases this solution is a stable solution with a large basin of attraction [6]. If the initial conditions lie within the basin of attraction, the driving force given by Eq. (0.1) is a resonant driving force. The basic idea of this procedure is to find a model which can predict the experimental system's response for any perturbation of physical interest and to use this model to force the system into a certain dynamics. Usually this goal dynamics is calculated by a variation principle, e.g. to get a large energy transfer. Of course, from an experimental point of view it is interesting to search for a model with perturbations that produce a large energy transfer in order to get a good signal to noise ratio, but any other type of perturbation which produces a large response, such as a large frequency shift, might be as useful. A common feature of all these methods is the search for a model where the difference between all the goal dynamics of physical interest and the response of the system becomes as small as possible, i.e. the quantity $R = \left(\overline{y(t) - z(t)}\right)^{-1}$ should be as large as possible. The average is taken over time and over all types of goal dynamics which are of physical interest and which are in the basin of attraction of the solution $y(t) = z(t)$ if the model would be correct. A systematic search for the maximum value of R is called general resonance spectroscopy since it can be applied to systems where no well-defined energy function exists. In the next section, we apply this concept to the dynamics of a logistic map and several other maps and show that the corresponding resonance curves are sharp and have a similar, simple shape.

RESONANCE CURVES OF SOME MAPS

We investigated discrete maps of the form

$$y_{n+1} = f(y_n, c_e) + F_n \tag{0.4}$$

where c_e is the parameter of the map, where F_n is a time dependent driving force and where $n = 0, 1, 2, \ldots$. We use a map with a single unknown parameter c_m in order to model the dynamics of the unperturbed map

$$x_{n+1} = f\left(x_n, c_m\right) \tag{0.5}$$

As the goal dynamics, we use a map which provides chaotic and periodic dynamics

$$z_{n+1} = g\left(z_n, c_g\right) \tag{0.6}$$

and which satisfies the condition $\left|\frac{df(z_n, c_m)}{dz_n}\right| < 1$ in order to ensure the stability of $y_n = z_n$. The deviation between y_0 and x_0 is about 1%. The driving force is given by $F_n = -f\left(z_n, c_m\right) + g\left(z_n, c_g\right)$. Fig. 1 shows the resonance curves of several systems. The resonance curves have a sharp peak at the exact value of the parameter.

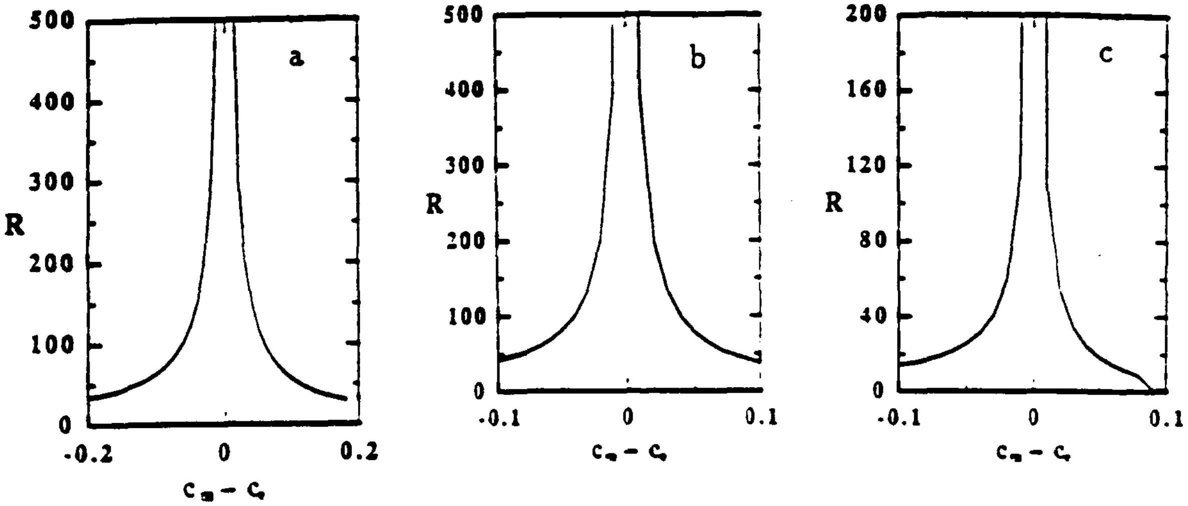

Fig.1 R versus $c_m - c_e$ for $f = c_e y_n\left(1 - y_n\right)$ where $c_e = 3.8$, $g\left(z_n, c_g\right) = f\left(4\left(z_n - \frac{3}{8}\right), c_g\right)$, $c_g = 3.7$ (a), $f = c_e\left(exp\left(-2\left(y_n - .5\right)^2\right) - exp(-.5)\right)$ where $c_e = 1.9$, $g\left(z_n, c_g\right) = f\left(2z_n - .5, c_g\right)$, $c_g = 1.8$ (b), and $f = c_e \sin\left(\pi x\right)$ where $c_e = .8$, $g\left(z_n, c_g\right) = f\left(5z_n - 2, c_g\right)$, $c_g = .6$ (c).

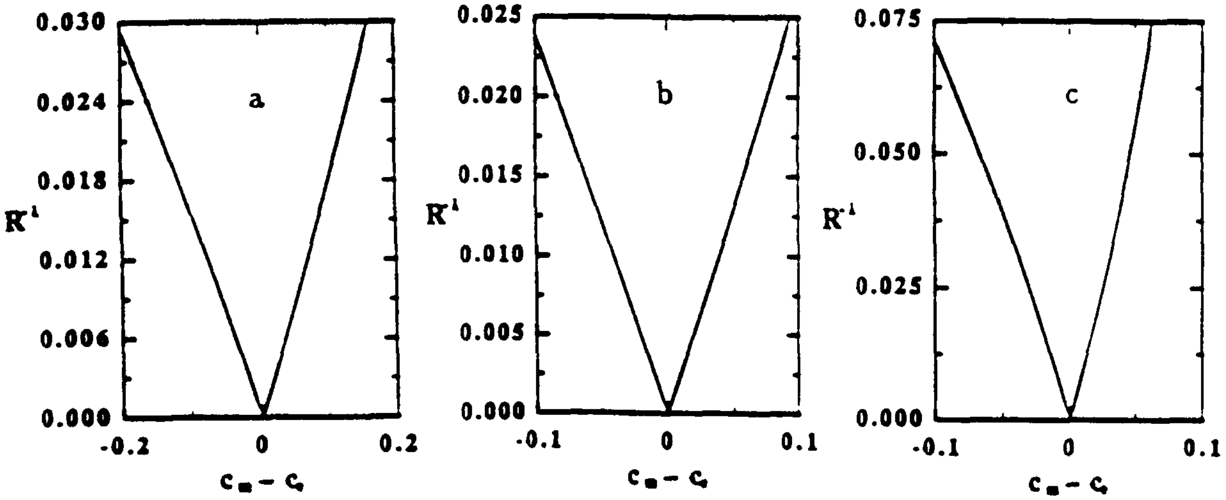

Fig. 2 R^{-1} versus $c_m - c_e$ for the same systems as in Fig. 1

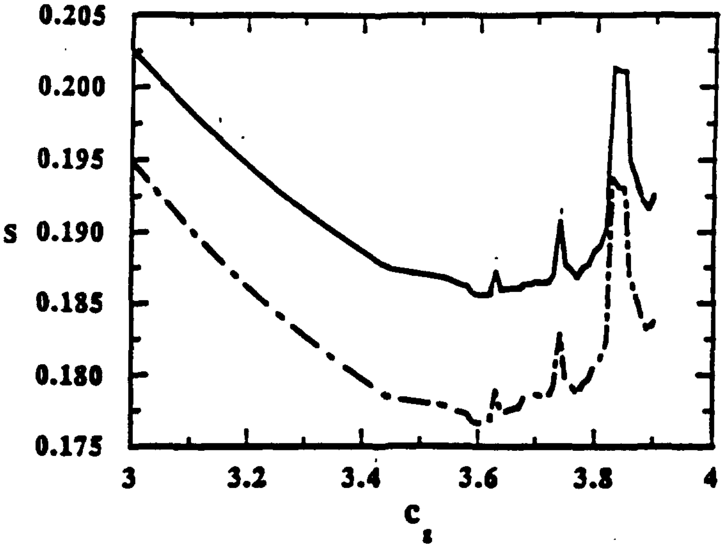

Fig.3 The slope of a linear approximation of R^{-1} (see Fig.2) versus c_g for the logistic map $f = 3y_n(1 - y_n)$. The solid line represents the magnitude of the slope above c_e, the broken line represents it below c_e.

Close to the peak of the resonance curve, R^{-1} versus the parameter of the model can be estimated by a straight line (Fig.2) This shows that the resonance curves can be estimated by a hyperbola in this parameter region. The slope of the straight line is a measure of the width of the resonance curve. Fig. 3 shows that the slope depends on c_g. When the goal dynamics is in an periodic window, the resonance curve is sharper than for neighbouring value for c_g which generate a chaotic dynamics.

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