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# EXPONENTIAL ERROR BOUNDS ON CODES FOR NOISY CHANNELS WITH INACCURATELY KNOWN STATISTICS AND FOR GENERALIZED DECISION RULES

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Generalized decoding decision rules provide added flexibility in a decoding scheme and some advantages. In a generalized decoding decision rule, the following possibilities are considered: 1) The decoder has the option of not deciding at all, or rejecting all estimates. This is termed an erasure; 2) The decoder has the option of putting out more than one estimate. The resulting output is called a list. Only if the correct code word is not on the list do we have a list error.

Forney developed error bounds in his seminal paper of 1968 in which he used Gallager's ingenious 1965 method of bounding error probabilities.

In this paper, we consider another realistic factor, the lack of exact knowledge of the channel statistics. We assume a mismatch between the true channel transition probabilities and the nominal probabilities used in the decoding metric. We then develop error bounds under mismatch for generalized decision rules.

We also establish conditions under which the error probabilities converge to zero exponentially with the block length, in spite of the presence of mismatch.

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# 1. INTRODUCTION

One of the fundamental theorems of information theory is Shannon's coding theorem (Shannon and Weaver 1949) for noisy channels. Using random coding arguments, Shannon discovered the original version, and later, Fano (1961) stated it in a stronger version:

> For any stationary channel with finite memory, a channel capacity
> C can be defined having the following significance. For any binary
> transmission rate R smaller than C, the probability of error per digit
> can be made arbitrarily small by properly designing the channel encoder
> and decoder. Conversely, the probability of error cannot be made
> arbitrarily small when R is greater than C.

The average error probability of the best block codes on the noisy channel can be bounded as follows:

$$P_e \leq e^{-nE_r(R)} \; ,$$

where $n$ is the length of a code word and $E_r(R)$, the random coding exponent, is positive for all rates $R$ less than capacity $C$. The existence of such exponential error bounds indicates a useful communications channel. Gallager (1965) pioneered a very elegant derivation of this random coding exponent, using a novel upper bound to the error probability. In another paper, Forney (1968) generalized Gallager's exponential error bounds for generalized decoding schemes, namely decoding with erasure, list decoding, and decision feedback schemes. Much of this work uses random coding arguments in which each input message is represented by a code word constructed by selecting $n$ symbols from an alphabet of independent, identically distributed symbols. The error probability of the channel and coding scheme is averaged over the ensemble of all randomly chosen codes, and there must be at least one nonrandom code with error probability as small as the ensemble average.

In all of the previously mentioned papers and in most literature on information theory, the assumption is that the statistical model of the noisy channel, expressed by the probability transition matrix, is completely known (i.e., that the channel is statistically describable). The capacities of channels which are not so describable have been investigated by Blackwell,

Brieman, and Thomasian (1960), Stiglitz (1967), and many others. A channel for which the transition matrix can change with each use is often known as an arbitrarily-varying channel (Blackwell, Breiman, and Thomasian 1960). Of potentially practical interest is the channel, also not statistically describable, in which the probability transition matrix remains fixed over one code word. This is the so-called "fixed unknown" channel (Blackwell, Breiman, and Thomasian 1960), now called the "compound" channel (Gallager 1965).

By another approach, followed by Kazakos (1981) and elsewhere, various authors analyze the performance of transmission through noisy channels when an inaccurate version of the probability transmission matrix is used by the decoder. This is termed "mismatch." Kazakos (1981) derived upper and lower bounds for transmission through channels in the presence of mismatch. Kazakos (1981) also found the necessary and sufficient conditions for the error probability of a random code to converge to zero with increasing block length. They were expressed in terms of distances between the actual and assumed channel probability transition matrices. In the present paper, we obtain exponential error bounds for generalized decoding schemes of the type considered by Forney (1968) but for the case of mismatch.

## 2. GENERALIZED DECODING

We consider a noisy, discrete channel chosen to be memoryless for the present report. Generalizations will follow in subsequent work. Let

$$P = \{P(y_k = b \mid x_k = \alpha) = p_{b\alpha} ; \alpha = 1,\dots,A; b = 1,\dots,B\}$$

$$\sum_{b=1}^{B} P_{b\alpha} = 1$$

be the transition probability matrix of the noisy channel. Let us consider a block code of block size $n$, with the following code words:

$$\{X_1, X_2,\dots,X_M\} ; M = e^{nR} ; X_m = \left(x_{m1},\dots,x_{mn}\right) ,$$

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where $R$ = rate of the code. The probability of receiving a block $y = (y_1,...,y_n)$ when $X_m$ was transmitted is as follows:

$$P(y|X_m) = \prod_{j=1}^{n} P(y_j | X_{mj}) \ .$$ (1)

We will assume that all entries of $P$ are positive, i.e.,

$$p_o = \min_{\alpha,b} p_{b\alpha} > 0 \ .$$ (2)

Ordinarily, maximum likelihood decoding selects the message $m$ that maximizes the likelihood $P(y|X_m)$. We assume that the prior probabilities $\pi_i$ of the $M$ code words are equal: $\pi_i = M^{-1}$. Maximum likelihood decoding divides the decision space $S$ into disjoint regions $\{R_1,...,R_M\}$ by the following:

$$y \in R_m \ iff \ P(y|X_m) > P(y|X_v) \ \text{for all } v \neq m.$$ (3)

In the present report, we will assume that an inaccurate version $Q$ of the true transition probability matrix $P$ is used in decoding. Let

$$Q = \left\{ Q(y_k = b \,|\, x_k = \alpha) = q_{b\alpha} \,; \alpha = 1,...,A\,; b = 1,...,B \right\}$$ (4)

be the entries of the r...inal probability transition matrix used in decoding. Naturally, $q_{b\alpha} \neq p_{b\alpha}$ for at least one pair of entries.

For maximum likelihood decoding under mismatch, the decision space $S$ is separated into a different set of disjoint regions $\{ \tilde{R}_1 ... \tilde{R}_M \}$ such that $\bigcup_{i=1}^{M} \tilde{R}_i = S$ and $\tilde{R}_i \cap \tilde{R}_j = 0$. These regions are defined by the following:

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$$y \in R_m \ \text{iff} \ Q(y \mid X_m) \geq Q(y \mid X_v) \ \textit{for all} \ v \neq m. \tag{5}$$

(As indicated earlier, this is one form of the fixed unknown channel [Blackwell, Breiman, and Thomasian 1960].)

Two generalized decision rules were considered by Forney (1968). The first one is the inclusion of an erasure option. In this case, an additional region $R_0$ is included to represent the event that no transmitted message is to be assigned to the received symbol because the value of the latter cannot be known reliably; if $y \in R_0$, we declare an erasure. Thus, $M + 1$ outcomes are possible. The $M + 1$ regions $\{R_0, R_1, ..., R_M\}$ are disjoint and cover all the space $S$:

$$\bigcup_{j=0}^{M} R_i = S, R_i \bigcap R_j = 0, i, j = 0, 1, ..., M \ .$$

Let $E_2$ be the event of an undetected error; this is the event that $y \in R_m$ and that some code word $X_k$, $k \neq m$ was actually transmitted. That is, the decoder believes that it has correctly decoded because it has produced a code word according to the decoding algorithm. However, the code word thus produced is not the code word that was transmitted. The probability of $E_2$ can be expressed as follows:

$$P[E_2] = \sum_{m=1}^{M} \sum_{y \in R_m} \sum_{k \neq m} P(y \mid X_k) P(X_k) \ . \tag{6}$$

Let $E_1$ be the event in which the received word $y$ does not fall in the decision region $R_m$ corresponding to the transmitted code word $X_m$; the probability of $E_1$ is as follows:

$$P[E_1] = \sum_{m=1}^{M} \sum_{y \in R_m} P(y \mid X_m) P(X_m) \ . \tag{7}$$

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If $E_1$ occurs, either an undetected error or an erasure must ensue; hence, the probability of an erasure is as follows:

$$P[e] = P[E_1] - P[E_2] \geq 0 .$$

The problem in choosing the regions $\{R_1,...,R_M\}$ is now formulated. We wish to minimize $P[E_1]$ for a given $P[E_2]$ or vice versa. It is clear that increasing $R_m$ will increase $P[E_2]$ but decrease $P[E_1]$; hence, we have a variation of the Neyman-Pearson problem (van Trees 1968).

The second type of generalized decoding is list decoding. Here, the decision regions $\{R_1,...,R_M\}$ overlap; hence, for each received word $y$, a list of code words is produced. The list contains at least one code word; the size of the list varies, as will be explained. The performance of list decoding is evaluated through two event probabilities. A list error is the event in which the transmitted code word is not on the list or, equivalently, in which the received word $y$ is not in the decision region $R_m$ corresponding to the transmitted code word $X_m$. This is the event $E_1$, with probability given by Equation 7. The second probability, that some code word $X_m$ will be on the list, although some other code word $X_k$, $k \neq m$ was sent, is as follows:

$$P(X_m \text{ on list and incorrect}) = \sum_{y \in R_m} \sum_{k \neq m} P(y \mid X_k) P(X_k) .$$

The average number $\overline{L}$ of incorrect code words on the list is as follows:

$$\overline{L} = \sum_{m=1}^{M} P(X_m \text{ on list and incorrect}) = \sum_{m=1}^{M} \sum_{y \in R_m} \sum_{k \neq m} P(y \mid X_k) P(X_K) . \tag{8}$$

We observe that the expression (Equation 8) for $\overline{L}$ is identical to the expression (Equation 6) for $P(E_2)$, where $P(E_2)$ is no longer a probability but represents $\overline{L}$. In the sequel, we will use $P[E_2]$ to denote both cases.

Thus, we have a unified formulation of decoding with erasure and list decoding. Forney (1968) proved that the optimum regions $\{R_1,...,R_M\}$ found under the criterion of minimizing Equation 6 under constant of Equation 7 or vice versa are as follows:

$$R_m = \left\{ y; \left[ \sum_{k \neq m} P(y \mid X_k) P(X_k) \right]^{-1} \cdot P(y \mid X_m) P(X_m) \geq e^{nT} \right\},$$  (9)

where $n$ = block length and $T$ = an arbitrary parameter.

An equivalent way of describing the decision regions (Equation 9) is through the posterior probabilities $P(X_m \mid y)$.

$$y \in R_m \text{ iff } P(X_m \mid y) \geq u,$$  (10)

where $u = e^{nT}[1 + e^{nT}]^{-1}$.

In ordinary decoding, we decode into the code word $X_m$ for which $P(X_m \mid y)$ is greatest. With the erasure option, we guess the code word $X_m$ for which $P(X_m \mid y)$ is greatest, so long as $P(X_m \mid y) \geq u$, $u \geq 1/2$. This corresponds to $T \geq 0$. With list decoding, to minimize the average list size for a given list error probability, we put on the list all code words for which $P(X_m \mid y) \geq u$, $u \leq 1/2$. This corresponds to $T < 0$.

Thus, the regions $R_m$ defined by Equation 4 are optimal for regular decoding ($T = 0$), list decoding, ($T < 0$ and overlapping), and decoding with erasure option ($T > 0$). Note that in order to define the decision regions $R_m$, we need to know the probability transition matrix $P$. In the mismatch situation, we utilize $Q$ instead of $P$. We assume, for simplicity, from this point on, equal prior probabilities: $P(X_m) = M^{-1}$; hence, the decision regions (under mismatch) are as follows:

$$\tilde{R}_m = \left\{ y; \left[ \sum_{k \neq m} Q(y \mid X_k) \right]^{-1} Q(y \mid X_m) \geq e^{nT} \right\}, \qquad (11)$$

where $\tilde{R}_m$ denotes the decision region based on mismatch. For $Q = P$, we have $\tilde{R}_m = R_m$.

## 3. BOUNDS UNDER MISMATCH

We will now generalize Forney's upper bounds for the mismatched case. We have the following two probabilities to upperbound:

$$P[E_1] = M^{-1} \sum_{m=1}^{M} \sum_{y \in \tilde{R}_m} P(y \mid X_m), \qquad (12)$$

$$P[E_2] = M^{-1} \sum_{m=1}^{M} \sum_{y \in \tilde{R}_m} \sum_{k \neq m} P(y \mid X_k). \qquad (13)$$

Let us define the following functions:

$$S_m \triangleq S_m(y) \triangleq \sum_{k \neq m} P(y \mid X_k),$$

$$Z_m \triangleq Z_m(y) \triangleq \sum_{k \neq m} Q(y \mid X_k),$$

$$Q_m \triangleq Q(y \mid X_m),$$

and

$$P_m \triangleq P(y \mid X_m).$$

Note that

$$\sum_y S_m(y) = \sum_y Z_m(y) = M - 1. \qquad (14)$$

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Hence, if we divide $S_m(y)$ and $Z_m(y)$ by $M-1$, they become probability distributions; more specifically, they become mixtures of $M-1$ distributions, with equal mixing parameters.

If we define the indicator function as follows:

$$\Phi_m(y) = \begin{cases} 1 \text{ for } y \in \bar{R}_m, \text{i.e., for } Z_m^{-1} Q_m e^{-nT} \geq 1 , \\ 0 \qquad \text{otherwise}, \end{cases} \tag{15}$$

then, the expressions for $P[E_1]$, $P[E_2]$, can be written as follows:

$$V_1 \triangleq P[E_1] = M^{-1} \sum_{m=1}^{M} \sum_{y} [1 - \Phi_m(y)] P(y \mid X_m) , \tag{16}$$

$$V_2 \triangleq (M-1)^{-1} P[E_2] = M^{-1} \sum_{m=1}^{M} \sum_{y} \Phi_m(y) \cdot S_m(y)(M-1)^{-1} . \tag{17}$$

Note that

$$\sum_{y} S_m(y) \cdot (M-1)^{-1} = 1 \text{ and } S_m(y)(M-1)^{-1} \geq 0 .$$

Hence, the normalized $S_m$ behaves like a probability distribution function. We observe that

$$1 - V_1 = M^{-1} \sum_{m=1}^{M} \sum_{y} \Phi_m(y) P(y \mid X_m) , \tag{18}$$

$$1 - V_2 = M^{-1} \sum_{m=1}^{M} \sum_{y} [1 - \Phi_m(y)](M-1)^{-1} \cdot S_m(y) . \tag{19}$$

For $s > 0$, we have the following bounds:

$$\Phi_m(y) \leq \left( Z_m^{-1} Q_m e^{-nT} \right)^s , \tag{20}$$

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$$1 - \Phi_m(y) \leq \left( Q_m^{-1} Z_m e^{nT} \right)^s . \tag{21}$$

Using the bounds of Equations 20 and 21 in Equations 16, 17, 18, and 19, we obtain the following inequalities:

$$V_1 \leq M^{-1} \sum_{m=1}^{M} \sum_{y} \left( Q_m^{-1} Z_m e^{nT} \right)^s P_m , \tag{22}$$

$$1 - V_1 \leq M^{-1} \sum_{m=1}^{M} \sum_{y} \left( Z_m^{-1} Q_m e^{-nT} \right)^s P_m , \tag{23}$$

$$V_2 \leq M^{-1} \sum_{m=1}^{M} \sum_{y} \left( Z_m^{-1} Q_m e^{-nT} \right)^s S_m \cdot (M - 1)^{-1} , \tag{24}$$

$$1 - V_2 \leq M^{-1} \sum_{m=1}^{M} \sum_{y} \left( Z_m Q_m^{-1} e^{nT} \right)^s (M - 1)^{-1} \cdot S_m . \tag{25}$$

We define the following functions:

$$g_{1m}(s) = \sum_{y} \left[ (M - 1)^{-1} \cdot Z_m \cdot Q_m^{-1} \right]^s P_m , \tag{26}$$

$$g_{2m}(s) = \sum_{y} \left[ (M - 1) Z_m^{-1} Q_m \right]^s (M - 1)^{-1} \cdot S_m . \tag{27}$$

We then obtain the following inequalities:

$$V_1 \leq M^{-1} e^{nTs} (M - 1)^s \sum_{m=1}^{M} g_{1m}(s) , \tag{28}$$

$$1 - V_1 \leq M^{-1} e^{-nTs} (M - 1)^{-s} \sum_{m=1}^{M} g_{1m}(-s) , \tag{29}$$

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$$V_2 \leq M^{-1} e^{-nTs} (M - 1)^{-s} \sum_{m=1}^{M} g_{2m}(s) \ , \tag{30}$$

$$1 - V_2 \leq M^{-1} e^{nTs} (M - 1)^{s} \sum_{m=1}^{M} g_{2m}(-s) \ . \tag{31}$$

We will utilize a form of Jensen's inequality (Korevaar 1968), which states the following:

$$\left[ \sum_j \alpha_j^q \right]^{1/q} \leq \sum_j \alpha_j \ , \ q \geq 1, \alpha_j \geq 0 \ .$$

If we introduce a new parameter $p \geq s$, we have, by Jensen's inequality, the following:

$$g_{1m}(s) = \sum_y \left\{ \left[ (M - 1)^{-1} \sum_{k \neq m} Q_k \right]^{s/p} \right\}^p Q_m^{-s} \cdot P_m$$

$$\leq \sum_y P_m Q_m^{-s} \left\{ \sum_{k \neq m} \left[ (M - 1)^{-1} Q_k \right]^{s/p} \right\}^p$$

$$\leq \sum_y P_m^{1-s} \left[ P_m Q_m^{-1} \right]^s \left\{ \sum_{k \neq m} \left[ (M - 1)^{-1} Q_k \right]^{s/p} \right\}^p \ . \tag{32}$$

Let also $q \geq 1 - s$. By the same argument,

$$g_{2m}(s) = \sum_y \left[ (M - 1) Z_m^{-1} Q_m \right]^s (M - 1)^{-1} S_m$$

$$= \sum_y \left[ S_m Z_m^{-1} \right]^s Q_m^s \left[ (M - 1)^{-1} S_m \right]^{1-s}$$

$$= \sum_y \left[ S_m Z_m^{-1} \right]^s Q_m^s \left\{ \left[ (M - 1)^{-1} \sum_{k \neq m} P_k \right]^{(1-s)/q} \right\}^q$$

$$\leq \sum_y \left[ S_m Z_m^{-1} \right]^s Q_m^s \left\{ \sum_{k \neq m} \left[ (M - 1)^{-1} P_k \right]^{(1-s)/q} \right\}^q \ . \tag{33}$$

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We consider probability transition matrices for which a finite lower bound exists to every entry. Then, there exists a finite number $B$ that is an upper bound to $\left[ P_m Q_m^{-1} \right]^s$ , $\left[ S_m Z_m^{-1} \right]^s$ . We obtain the following upper bounds:

$$g_{1m}(s) \leq B \cdot \sum_y P_m^{1-s} \left\{ \sum_{k \neq m} \left[ (M-1)^{-1} Q_k \right]^{s/p} \right\}^p ,$$ (34)

$$g_{2m}(s) \leq B \cdot \sum_y Q_m^s \left\{ \sum_{k \neq m} \left[ (M-1)^{-1} P_k \right]^{(1-s)/q} \right\}^q .$$ (35)

Multiplying by $(M-1)^s$, we find the following:

$$(M-1)^s g_{1m}(s) \leq B \sum_y P_m^{1-s} \left[ \sum_{k \neq m} Q_k^{s/p} \right]^p ,$$ (36)

$$(M-1) \cdot (M-1)^{-s} g_{2m}(s) \leq B \cdot \sum_y Q_m^s \left[ \sum_{k \neq m} P_k^{(1-s)/q} \right]^q .$$ (37)

Note from Equations 26 and 28 that the products $(M-1)^s g_{1m}(s)$, $(M-1)^{-s} g_{2m}(s)$ appear at the upper bounds; hence, we are interested in bounding them directly.

At this point, we need to resort to random coding arguments. As is customary, and following Gallager's (1965) and Forney's (1968) approaches, we choose a code at random by choosing each input letter of each code word by a random selection in which the probability of choosing input $x_k$ is $p_k$. Denoting the average by an overbar, a modification of the approach in Forney's paper (Forney 1968) yields the following:

$$(M-1)^s \bar{g}_{1m}(s) \leq B \cdot e^{\rho n R} \cdot \left[ \sum_j \left( \sum_k p_k p_{jk}^{1-s} \right) \left( \sum_v p_v q_{jv}^{s/p} \right)^p \right]^n ,$$ (38)

where

$$P = \{ p_{jk} \} , \quad Q = \{ q_{jv} \}$$

are the true and assumed channel probability transition matrices, respectively. This is in agreement with Forney's bound if $P = Q$ (matched case) and, hence, $B = 1$.

A similar upper bound is produced for $\{\bar{g}_{2m}(s)\}$:

$$(M - 1)(M - 1)^{-s}\bar{g}_{2m}(s) \le B \cdot \exp(qnR)$$

$$\cdot \left[ \sum_j \left( \sum_k p_k p_{jk}^{(1-s)/q} \right)^q \left( \sum_v p_v q_{jv}^s \right) \right]^n , \tag{39}$$

where

$$q \ge 1 - s, \ p \ge s \ge 0, s < 1$$

and $R = $ code rate; $R = n^{-1} \log M$.

The upper bounds for $V_1$, $V_2$, averaged over the random choice of code words, are as follows:

$$\bar{V}_1 \le B \cdot e^{n(Ts-pR)} \cdot \left[ \sum_j \left( \sum_k p_k p_{jk}^{1-s} \right) \left( \sum_v p_v q_{jv}^{s/p} \right)^p \right]^n , \tag{40}$$

$$(M - 1) \cdot \bar{V}_2 \le B \cdot e^{n(qR-Ts)} \cdot \left[ \sum_j \left[ \sum_k p_k p_{jk}^{(1-s)/q} \right]^q \left[ \sum_v p_v q_{jv}^s \right] \right]^n . \tag{41}$$

Note that the previous bounds (Equations 40 and 41) will converge exponentially to zero if the functions

$$f_1(s) = \sum_j \left( \sum_k p_k p_{jk}^{1-s} \right) \left( \sum_v p_v q_{jv}^{s/p} \right)^p , \ 1 \ge p \ge s \ge 0 \tag{42}$$

and

$$f_2(s) = \sum_j \left[ \sum_k p_k p_{jk}^{(1-s)/q} \right]^q \left( \sum_v p_v q_{jv}^s \right) , \ q \ge 1 - s, s \ge 0 . \tag{43}$$

are both less than *1* for some $(s, p, q) = (s^o, p^o, q^o)$. This is a sufficient condition for both quantities $\overline{V}_1, \overline{V}_2$ to converge to zero for some random block code of size *n*, as $n \rightarrow \infty$. We observe that for any *p*, $f_1(0) = 1$ and for *q=1*, $f_2(0) = 1$.

A sufficient condition that both $V_1$ and $V_2$ converge exponentially to zero in spite of mismatch is that for a pair $(P, Q)$, we have the following:

$$min \; f_1(s) \cdot (Ts - pR) < 1 \; , \tag{44}$$

$$min \; f_2(s) \cdot (qR - Ts) < 1 \; , \tag{45}$$

where the *min* are over all three parameters $(p, q, s)$.

Due to the stated properties of $f_1(s)$, $f_2(s)$, it is always guaranteed that for *P*, *Q* sufficiently close, the bounds of Equations 40 and 41 will converge exponentially to zero as *n* becomes infinite.

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