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Brief Outline of Research Findings

Work continued on the thermodynamics of two-phase continua. The general two-phase Stefan problem with supercooling, superheating, and capillarity, was studied [T19] in collaboration with M. Soner (Carnegie Mellon). Simple solutions – illustrating the chief differences between this problem and the classical Stefan problem – were obtained for the spherically symmetric problem, under the assumption of fast diffusion, with the liquid supercooled at infinity. It is shown that: (i) for $\Omega = \mathbb{R}^3$, a ball of the solid phase of sufficiently small size disappears in finite time, but a sufficiently large ball grows without bound; (ii) for $\Omega = \mathbb{R}^3$ and the solid phase initially situated in a spherical shell of thickness ϵ , the thickness of the solid shell initially increases, but the inner radius of this region decreases to zero in finite time T ; the solid ball remaining at time T disappears at a later time or grows without bound according as ϵ is less than or greater than a critical value; in the limit $\epsilon \rightarrow 0$ the region occupied by the solid disappears infinitely fast; the problem has no solution for $\epsilon = 0$; (iii) when Ω is the region exterior to a sphere of radius R , with the boundary $r = R$ insulated and with the solid phase initially in a spherical shell of zero thickness at $r = R$, the solid phase grows without bound provided R is sufficiently large. While (ii) and (iii) are of little practical interest, they demonstrate the possibility of growth from a seed of zero volume.

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Papers and reports which appeared under this contract

- [T1]^{*} Gurtin, M.E., Multiphase thermomechanics with interfacial structure. 1. Heat conduction and the capillary balance law. *Archive for Rational Mechanics and Analysis*, 104, 195–221 (1988).
- [T2]^{*} Gurtin, M.E., On a nonequilibrium thermodynamics of capillarity and phase, *Quarterly of Applied Mathematics*, 97, 129–145 (1989).
- [T3]^{*} Gurtin, M.E., On diffusion in two-phase systems: the sharp interface versus the smooth transition layer, *Proceedings CNRS–NSF Symposium, Nice* (1988), Springer Lecture Notes in Physics 344, (1989).
- [T4]^{*} Browning, R.V., M.E. Gurtin, and W.O. Williams, A model for viscoplastic materials with temperature dependence, *International Journal of Solids and Structures*, 25, 441–457 (1989).
- [T5]^{*} Almgren, F. and M.E. Gurtin, A mathematical contribution to Gibbs's analysis of fluid phases in equilibrium, *Partial Differential Equations and the Calculus of Variations, Volume I, Essays in Honor of Ennio De Giorgi*, (eds. F. Colombini, A. Marino, L. Modica, S. Spagnolo) Birkhauser, Boston, 1989.
- [T6]^{*} Milic, N., On non-equilibrium phase transitions in mixtures with interfacial structure, Ph.D. Thesis, Department of Mathematics, Carnegie Mellon University (December 1988).
- [T7]^{*} Gurtin, M.E., On the isothermal motion of a phase interface, *Proceedings of the Conference* (1988) "Problems Involving Change of Type", *Lecture Notes in Physics*, 359 (ed. K. Kirchgassner) Springer–Verlag, Berlin, 1990. .
- [T8]^{*} Gurtin, M.E., A. Struthers, and W.O. Williams, A transport theorem for moving interfaces, *Quarterly of Applied Mathematics*, 47, 773–777 (1989).
- [T9]^{*} Angenent, S. and M.E. Gurtin, Multiphase thermomechanics with interfacial structure. 2. Evolution of an isothermal interface, *Archive for Rational Mechanics and Analysis*, 108, 323–391 (1989).
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- [T12]^{*} Gurtin, M.E. and W.J. Hrusa, On the thermodynamics of viscoelastic materials of single–integral type, *Quarterly of Applied Mathematics*. Forthcoming.

- [T13]* Gurtin, M.E., On the continuum mechanics of the motion of a phase interface. *Proceedings Seventh Army Conference on Applied Mathematics and Computing*, West Point (1989).
- [T14]* Gurtin, M.E. and A. Struthers, Multiphase thermodynamics with interfacial structure. 3. Evolving phase boundaries in the presence of bulk deformation, *Archive for Rational Mechanics and Analysis*, 112, 97–160 (1990).
- [T15]* M.E. Gurtin and W.J. Hrusa, Global existence in one-dimensional nonlinear viscoelasticity with heat conduction, *Journal of Integral Equations*, 2, 431–460 (1990).
- [T16]* F. Davi and M.E. Gurtin, On the motion of a phase interface by surface diffusion, *Zeitschrift für angewandte Mathematik und Physik*, 41, 782–811 (1990).
- [T17]* M.E. Gurtin, On thermomechanical laws for the motion of a phase interface, *Zeitschrift für angewandte Mathematik und Physik*. Forthcoming.
- [T18]* Gurtin, M.E., Evolving phase boundaries in the presence of deformation and surface stress, *Proceedings IMA Workshop on Evolving Phase Boundaries*, U. Minnesota (1990).
- [T19] Gurtin, M.E. and H.M. Soner, Some remarks on the Stefan problem with surface structure, *Quarterly of Applied Mathematics*. Forthcoming.

* Discussed in previous progress reports.

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