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13. ABSTRACT (Maximum 200 words)

Research was concerned with designing and analyzing efficient and novel iterative algorithms for solving large sparse linear systems, typically arising from the discretizations of partial differential equations, which are highly parallelizable and converge fast. These include domain decomposition algorithms and multilevel preconditioners. Some basic dense linear algebra problems, including rank-revealing QR factorizations and stable Toeplitz solvers, which have applications to signal processing were considered.

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Numerical Methods for Solving Large Sparse Eigenvalue Problems

and For the Analysis of Bifurcation Phenomena

Final Report

Principal Investigator: Tony F. Chan

October 1, 1991

U.S. Army Research Office

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Univ. of California at Los Angeles

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Description of Research: We design and analyse efficient and novel iterative algorithms for solving large sparse linear systems, typically arising from the discretizations of partial differential equations, which are highly parallelizable and converge fast. These include domain decomposition algorithms and multilevel preconditioners. We are also interested in some basic dense linear algebra problems, including rank-revealing QR factorizations and stable Toeplitz solvers, which have applications to signal processing.

Summary of Most Important Results:

1. Development of "boundary probing" interfacial preconditioners in domain decomposition, which adapts to the coefficients of the differential operator and the geometry of the subdomains. Recently, we extended this to derive a more efficient version of the vertex space method of Smith and Widlund.

2. Using a result on the relationship between the overlapping and nonoverlapping methods in domain decomposition, we prove that many domain decomposition algorithms have convergence rates that are independent of the aspect ratios of the subdomains or the amount of overlap.

3. Developed a new class of multilevel preconditioners for elliptic problems, based digital filtering ideas. They are applicable to a wide class of elliptic operators, including anisotropic operators, the biharmonic operator, convection-diffusion operators and to problems with mesh refinement. Preliminary timing results on the CM-2 are also very encouraging.

4. Analyzed a parallel multigrid method proposed by Frederickson and McBryan and show how to extend the method to anisotropic problems.

5. Developed and analyzed the performance of a Euler solver (FLO52) on hypercubes.

6. Developed look-ahead Levinson algorithms which are stable for a much more general class of Toeplitz systems, including indefinite ones, while retaining the efficiency of the classical algorithm. Fortran and MATLAB codes have been developed also.

7. Developed rank-revealing QR factorization algorithms, which can be viewed as inexpensive alternatives to the singular value decomposition. We developed applications to total least squares, subset selection, rank-deficient least squares.

What follows is a list of papers/reports supported by this contract.

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