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13. ABSTRACT (Maximum 200 words)

The problem of "locking", which arises in the approximation of parameters dependent problems has been extensively investigated. A general theoretical framework to analyze this phenomenon has been developed, and the "locking" and robustness of different finite element schemes for various problems has been characterized. Work on the p and h-p versions of the finite element method has continued. Progress here includes optimal approximation results for the p version of the boundary element method in three dimensions, an analysis of a p version mixed method for quasilinear problems, and investigation of quadrature schemes and related errors. Additional work has been conducted on singularities of solutions for the three dimensional elasticity and hydro dynamics equations in domains with edges and vertices, on the numerical evaluation of singular surface integrals in the boundary element method, and the calculation of optimal shear correction factors for plate models.

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NUMERICAL TREATMENT OF DIFFERENTIAL AND INTEGRAL EQUATIONS BY THE p AND $h-p$ VERSIONS OF THE FINITE ELEMENT METHOD

(Continuation of 'Analysis of the Performance of Mixed Finite Element Methods')

Principal Investigator: Manil Suri
Co-investigator: Christoph Schwab
Grant AFOSR 89-0252

Final Scientific Report–January, 1992

Abstract

Our research can be divided into three main groups.

1. An investigation into the problem of "locking" which arises in the approximation of parameter-dependent problems. We have developed a general theoretical framework to analyze this phenomenon and have characterized the locking and robustness of different finite element schemes for various problems.
2. Results on the p and $h-p$ versions of the finite element method.
   (a) Optimal approximation results for the p version BEM in three dimensions.
   (b) Analysis of a p version mixed method for quasilinear problems.
   (c) Analysis of the error in L_2 and lower norms.
   (d) Investigation of quadrature schemes and related errors.
3. Results obtained by C. Schwab.
   (a) Singularities of solutions for the equations of three dimensional elasticity and hydrodynamics in domains with edges and vertices.
   (b) Numerical evaluation of singular surface integrals in the boundary element method.
   (c) Calculation of optimal shear correction factors for plate models.

1. Introduction

The goals of this project, as described in the original proposal which was funded, were to investigate the p and $h-p$ versions of the finite element method in various areas, such as the numerical approximation of three-dimensional PDEs and integral equations, the investigation of mixed methods for these versions and, most importantly, the uniform approximation of parameter-dependent problems by these versions. By the p version, we mean a finite element method where the mesh is kept fixed and the polynomial degrees are raised to increase accuracy. In the $h-p$ version, both the mesh and the polynomial degrees are changed. Research on these methods (which has received strong support from AFOSR, among others) has led to various important results which show that the p and $h-p$ versions are particularly suitable for linear elliptic PDEs (like those arising in structural analysis), with extremely high rates of convergence being obtained. Various commercial codes (used in the design of aircraft, for example) have been developed on the basis of such research - we mention PROBE (developed originally by Noetic Technologies, MO and now part of MSC software) and STRIPE (developed by the Aeronautical Research Institute of Sweden).

The first, and most important part of our research has been on the approximation of parameter-dependent problems. This can often suffer from a phenomenon called "locking", which causes the deterioration of certain finite element approximations for some ranges of the parameter. Locking can cause computational results in real-life situations to turn out to be completely inaccurate and has been described by Richard H. MacNeal (chairman, MacNeal-Schwendler Corporation), as "the most virulent problem that afflicts lower-order schemes" (ICIAM 91 lecture). Our goal has been to set up a mathematical framework to define and analyze locking, our suggested strategy to overcome its effects has been, in general, to use higher-order schemes. In this context, we have shown that the p and $h-p$ versions can be particularly effective, as can be higher-order h version schemes. This research is a starting point for many of the problems we will be doing as part of our new grant.

Our second area of research deals with results obtained for the p and $h-p$ versions of the finite element method on the other topics mentioned above and some additional related topics. We have analyzed a mixed p version finite element method for a quasi-linear elliptic problem, obtaining optimal results. We have investigated the optimality of the p version in lower order norms, when applied to domains with corners. We have also obtained several results on the accuracy of the p version in the presence of numerical quadrature, both for source and eigenvalue problems. Since January, 1991, Dr. Christoph Schwab has also been funded by this grant. In joint research, we have investigated the optimal rate of convergence of the p-version boundary element method (BEM) over polyhedral surfaces in R^3 and obtained sharp rates of convergence, the first results of this kind in three dimensions.

In addition, Dr. Schwab has investigated jointly with W.L. Wendland (Germany) the numerical quadrature of singular and hypersingular integrals over curved surfaces, which arise for example in boundary element methods for three dimensional crack problems. In other work, using the Fourier analysis of hierarchical plate models, a mathematically rigorous framework for the optimal selection of the so-called shear correction factors in the Reissner Mindlin and related plate models has been given. Finally, in joint work with V.A. Kozlov and V.G. Maz'ya (Sweden), the vertex singular exponents for the Lamé- and Stokes- equations in three- dimensional domains with vertices and edges have been analyzed and sharp estimates in dependence on the local vertex geometry obtained.

2. Results achieved

In this section, we describe the results achieved under the grant over the past three years.

2.1. The problem of locking in the FEM

The numerical approximation of various parameter-dependent problems may suffer when the parameter t lies close to a limiting value, a phenomenon called *locking*. For example, consider the case of an elastic plate. As the thickness tends to zero, the solution of the 3-d elasticity problem tends to that of the Kirchoff plate and Kirchoff's hypothesis gets imposed as a constraint on both the exact and on the approximate solution. Locking occurs when there are not enough functions in the approximate subspace satisfying this constraint to maintain the level of approximability expected. A similar effect occurs when the Poisson ratio ν is close to 0.5, in which case, the constraint of *incompressibility* can cause problems.

Locking effects can have extreme consequences in terms of "real-life" computations. There have been many engineering and mathematical papers which describe various locking effects and analyze methods (mainly mixed methods) to overcome it. However, the treatment in these papers has been ad hoc, with the term "locking" being used to describe very different phenomena (compare [4] with [5], for example).

In [P5], we have developed a systematic theory of locking which includes mathematically precise definitions and also both asymptotic and practical methods of quantifying the locking and robustness of various schemes. Our theory provides a general framework for the analysis of the examples listed in the above references. An important subclass of problems we consider is one where the parameter t enters the variational problem in the following way: find $u_t \in V$ satisfying

$$a(u_t, v) + \frac{1}{t}(Cu_t, Cv) = F_t(v) \quad \forall v \in V. \tag{2.1}$$

Here, $a(\cdot,\cdot)$ is coercive on V and C is a bounded operator acting on V. As $t \to 0$, the solution u_t satisfies the constraint

$$Cu_0 = 0. \tag{2.2}$$

For example, if (2.1) represents the 2-d elasticity equations and $t \approx 1 - 2\nu$, where ν is the Poisson ratio, then $Cu \equiv \operatorname{div} u$ and the limiting case of (2.1) when $\nu = 1/2$ is Stokes' problem, whose solution is divergence free. Similarly, in the case of the Reissner-Mindlin plate,

$$Cu = C(\vec{\phi}, \omega) = \vec{\phi} - \operatorname{grad} \omega$$

(where $\vec{\phi}$ measures the shift in the normal and ω the transverse displacement), so that (2.2), as mentioned above, is simply Kirchhoff's constraint.

For problems of the form (2.1), we may often decompose the solution as

$$u_t = u_0 + (u_t - u_0) = u_0 + z_t \tag{2.3}$$

where u_0 is the solution of the limit problem (t=0). A *necessary* condition for the absence of locking is that u_0 be optimally approximated under the constraint (2.2), i.e.

$$\inf_{v \in S^N} \|u_0 - v\|_V \approx \inf_{\substack{v \in S^N \\ Cv=0}} \|u_0 - v\|_V \tag{2.4}$$

where $S^N \subset V$ is the approximate subspace used. As further shown in [P5], (2.4) is *sufficient* as well, provided the remainder z_t satisfies "Condition(α)", i.e.

$$\|z_t\|_H \leq Ct^{1/2} \tag{2.5}$$

in an appropriate norm H. We have derived several related general theorems relating locking and robustness in different norms in [P5]. In particular, we have shown, by means of several examples, that locking depends upon several factors, like the regularity of the solution, the norm in which the error is calculated, etc.

The general theory summarized above has been used by us to analyze several examples, including:

(1) In [P5], we have investigated the case of heat flow through highly anisotropic materials, where t represents the ratio of conductivities in two mutually perpendicular directions. "Condition (α)" does not hold for this problem. One of the results obtained is that locking cannot be avoided for the h version unless the mesh is properly aligned, a result similar to the one for shells in [5]. Also, it is shown that the p version is robust for this problem.

(2) The problem of locking in the Timoshenko beam, which was analyzed in [1], [3] is treated in [P5] using our theory. In particular, it is shown that Condition (α) holds, so that (2.4) is necessary *and* sufficient. For the h version, we get locking for *any* p while for the p version, no locking occurs.

(3) In [P6], we derive various results for Poisson ratio locking ($\nu \to 1/2$) when straight-sided elements in 2-d are used. In particular, we establish Condition (α) for this problem. The fact that locking is avoided using the p version or using the h version with triangular elements of degree ≥ 4 (obtained originally in [10], [7] respectively) are shown to follow immediately from our theory. Moreover, it is rigorously established that for $p \leq 3$, locking leads to exactly an $O(h)$ loss of convergence with triangular elements. We also establish various new results for two types of rectangular elements. Extensions to the 3-d case are also considered.

As noted experimentally in [9], if the elements are curved, then the effect of locking is much more severe. We have analyzed this case both when the elements can be represented by polynomial mappings and analytic mappings. Our results for the p version show that the there is no locking using polynomial mappings but the rate of convergence is "shifted", i.e. it is $C(p-s)^{-\alpha}$ instead of $Cp^{-\alpha}$. We can explicitly characterize s in terms of the degree of the mapping. We will present these results in a forthcoming paper. A sample result was presented in [P4], where locking effects for the h and p version were compared.

(4) We have also considered the Reissner-Mindlin plate problem. It is known that the solution has a boundary layer for a finite plate [2], so that Condition (α) may not hold for H regular enough. We therefore consider the cases of a plate with periodic boundary conditions and establish Condition (α) in this case. Once again, we show that the p version is free from locking, while the h version shows locking for any p ($O(h)$ loss of convergence for triangles). These and other results are presented in [P14].

(5) Various models for arches has been discussed in [12]. We have established that Condition (α) holds in all these cases. This enables us to investigate the locking of various standard finite element schemes, which will be presented in a forthcoming paper.

In the above examples, we have mainly dealt with *standard*, as opposed to *mixed* finite element schemes, since typically the former are the only ones available in terms of most commercial codes and are consequently of great practical interest. Although several kinds of locking problems have been investigated by us above, there remain important cases that do not fit into the general structure studied so far. These will be the subject of future research.

Some of the above results on locking were presented at an invited lecture at the Numerical Methods for Elliptic Systems Workshop, Helsinki Institute of Technology, Espoo, Finland (October 25-28, 1989). Also, a minisymposium entitled "The problem of locking in finite element analysis" was organized by the prinicipal investigator as part of ICIAM 91 (Washington, D.C., July 8-12, 1991), at which various aspects of locking were discussed.

2.2. Results on the p and $h-p$ versions of the FEM

The main emphasis of the research done under a previous grant (AFOSR 85-0322) was to establish various basic theoretical results for the p and $h-p$ versions of the FEM. These included optimal approximation results for various types of p and h-p subspaces, the numerical approximation of singularities by these methods, the treatment of inhomogeneous Dirichlet boundary conditions by the p version and applications to the boundary element method (BEM) for integral equations. These results were presented at an invited lecture at ICOSAHOM '89 at Villa Olmo, Italy (June 26-29, 1989) and have been summarized in the survey [P2].

During the current grant, some additional questions have been investigated. One of these is the characterization of the L_2 error for the p version, over non-convex domains. It was shown in [11] that for the h version, one does not get an $O(h)$ improvement over the H^1 error in this case. This is due to the fact that the usual duality argument breaks down when the domain is non-convex. In contrast, we have shown in [P8] that the p version gives the full $O(p^{-1})$ improvement, by using a modified duality argument. Negative norm estimates and computational results (dealing with the sharpness of our estimates) are also presented.

In [8], we investigated the stability and convergence properties of the Raviart-Thomas and Brezzi-Douglas-Marini spaces in the context of the mixed p version (and h-p version) FEM. We showed stability and optimal convergence (up to an $\epsilon > 0$) in terms of p. In [P3], we have extended our analysis of the Raviart-Thomas spaces to the case of quasilinear problems and proven optimal convergence once more for the mixed p version.

The p version of the BEM for 3-d problems has been investigated in joint work by the principal investigators. Various optimal approximation results for the different types of singularities that may occur in this situation have been established. In particular, we present convergence results for vertex, edge and combined edge-vertex singularities in the report [P9] (a more detailed paper, containing improved versions of these results together with their proofs, is in preparation).

Finally, we have been investigating the problem of numerical quadrature for the p version. This is one of the last remaining basic problems for the p version, and one of great practical importance, since for 3-d codes (like STRIPE), approximately 50% of the cost of computation is from calculating the integrals in the various matrices. In [P7], we have established that the asymptotic order of convergence for the source problem remains unchanged when commonly used types of quadrature are employed instead of exact integration. We also discuss situations where overintegration may be necessary to recover this rate of convergence, when distorted elements are used. The paper contains various computational results related to the use of mapped elements. In [P12], the corresponding results (including an optimal convergence rate) are established for the eigenvalue problem. In addition to the results reported in these references, various additional results have been obtained in collaboration with a Ph.D. student, Chang Kim, on the effect of numerical integration on lower order norms.

To summarize, at this stage, most of the basic theoretical work for the p and $h-p$ versions has been completed. This body of work will be reported in a contribution being prepared (jointly with I. Babuška) for the "Handbook of Numerical Analysis" (P. G. Ciarlet and J. L. Lions, editors). A second survey [P13], which explains the main differences between the h and the p (and $h-p$) versions will be submitted to SIAM Review shortly.

2.3. Results obtained by Dr. C. Schwab

The co-principal investigator has also completed research on several other topics, portions of which have been supported by AFOSR grant 89-0252.

The regularity of solutions to the equations of 3-d elasticity and hydrodynamics in domains with edges and vertices is governed by the singular exponents associated with them. While the edge singularities are easily obtained from 2-d theory, the vertex exponents are difficult to estimate. In [C1],[C2] we proved sharp estimates for these exponents in dependence on the geometry of the domain and established a (nonlinear) variational principle for them. This implies in particular a H^2 regularity result for convex polyhedra in three dimensions for the Lamé- and Stokes equations. Singular exponents for rotational cones have been characterized by a separation of variables technique, generalizing work by Bažant and Keer, and we computed these exponents numerically in [C3].

Furthermore, we analyzed and solved the problem of numerical integration for the singular surface integrals arising in boundary element methods. In [C4], we analyzed and classified the general form of the integrands arising in BEM (our analysis covers all operators that can possibly arise in BEM). Based on this, in [C5] we developed numerical quadrature schemes and proved error estimates in dependence on the size of the integration domain and the number of quadrature points. In forthcoming work [C7], we will present numerical examples and demonstrate the use of symbolic manipulation in the regularization of the hyper singular, finite part surface integrals. Moreover, we also present estimates of the quadrature error for the p-BEM there.

We also investgated in [C6] hierarchical plate models for homogeneous, isotropic plates. There, using the Fourier analysis of plate models developed in [6], we obtained optimal shear correction factors for the Reissner Mindlin and the next higher model (the (1,1,2)-model) in the hierarchy. We also showed that in all models of higher order, the introduction of a shear correction factor downgrades the asymptotic accuracy.

Furthermore, work is in progress regarding a coercive formulation for exterior three dimensional solid - fluid interaction problems that can then be combined with dimensional reduction techniques to obtain well posed boundary value problems for shells immersed in a fluid.

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3. Chronological list of publications by P.I.

P1 M. Suri, "Two methods for the approximation of inhomogeneous essential boundary conditions by the *p*-version of the finite element method", ***Transactions of the IMACS: Numerical and Applied Mathematics***, Editor: C. Brezinski, 531-535 (1989).

P2 I. Babuška and M. Suri, "The *p* and *h-p* versions of the finite element method. An overview", ***Computer Methods in Applied Mechanics and Engineering***, 80 (1990).

P3 F. Milner and M. Suri, "Mixed finite element methods for quasilinear second order elliptic problems: the p-version", to appear in *RAIRO Mathematical Modelling and Numerical Analysis*, 1992.

P4 M. Suri, "On the robustness of the *h*- and *p*-versions of the finite element method", *Journal of Computational and Applied Mathematics*, 35, 303-310 (1991).

P5 I. Babuška and M. Suri, "On locking and robustness in the finite element method", Technical Note BN-1112, IPST, University of Maryland, College Park (1990),to appear in *SIAM Journal of Numerical Analysis*, 1992.

P6 I. Babuška and M. Suri, "Locking effects in the finite element approximation of elasticity problems", Technical Note BN-1119, IPST, University of Maryland, College Park (1990), to appear in *Numerische Mathematik*, 1992.

P7 U. Banerjee and M. Suri, "The effect of numerical quadrature in the *p*-version of the finite element method", Research Report 91-09 (1991), University of Maryland Baltimore County, to appear in *Mathematics of Computation*, 1992.

P8 S. Jensen and M. Suri, "On the L_2 error for the *p* version of the finite element method over polygonal domains", Research Report 91-11 (1991), University of Maryland Baltimore County, to appear in *Computer Methods in Applied Mechanics and Engineering*, 1992.

P9 C. Schwab and M. Suri, "On the rate of convergence of the p-version BEM over polyhedral surfaces", *Proceedings of the 13th IMACS World Congress*, Dublin, Ireland, July 22-26, 1991.

P10 M. Suri, "On locking effects in the finite element method", *Proceedings of the 13th IMACS World Congress*, Dublin, Ireland, July 22-26, 1991.

P11 M. Suri, "The problem of locking in parameter-dependent problems", *Proceedings of Equadiff '91*, Barcelona, Spain, August 26-31, 1991.

P12 U. Banerjee and M. Suri, "The analysis of numerical integration in p-version finite element eigenvalue approximation", Technical Note 91-12, Department of Mathematics and Statistics, University of Maryland Baltimore County, Baltimore (1991), accepted by *Numerical Methods for Partial Differential Equations.*

P13 I. Babuška and M. Suri, "The p and h-p versions of the finite element method", (to be submitted to *SIAM Review*).

P14 M. Suri, I. Babuška and C. Schwab, "Locking effects for the Reissner-Mindlin plate", in preparation.

4. Publications by Co-P.I.

C1 V.A. Kozlov, V.G. Maz'ya and C. Schwab: "On singularities of solutions of the Dirichlet problem for the Lamé system near the vertex of a cone", Arch. Rat. Mech. Anal. 1992 (in press).

C2 V.A. Kozlov, V.G. Maz'ya and C. Schwab: "On singularities of solutions to the Dirichlet problem of hydrodynamics near the vertex of a 3-d cone", Preprint 90-21, Department of Mathematics and Statistics, UMBC, Dec. 1990.

C3 V.A. Kozlov, V.G. Maz'ya and C. Schwab: "On singularities of solutions to the boundary value problems of elasticity and hydrodynamics near the vertex of a rotational cone", Preprint 91-07,Department of Mathematics and Statistics, UMBC, Dec. 1991.

C4 C. Schwab and W.L. Wendland: "Kernel properties and representations of boundary integral operators", Mathematische Nachrichten, 1992, in press.

C5 C. Schwab and W.L. Wendland: "On numerical cubatures of singular surface integrals in boundary element methods", Numerische Mathematik 1992, in press.

C6 I. Babuška, J.M. d'Arcourt and C. Schwab: "Optimal Shear Correction Factors in Hierarchical Plate Modelling", Technical Note BN-1129, Oct. 1991, IPST Univ. of Maryland, College Park.

C7 R. Kieser, C. Schwab and W. Wendland: "Numerical Evaluation of Singular and Finite Part Integrals over Curved Surfaces Using Symbolic Manipulation", in preparation (will appear as Preprint in the Mathematisches Institut A, Univ. Stuttgart and in Computing 1992, issue in mem. Prof. Wacker).

5. Professional personnel associated with research

In addition to the P.I., the following received funds from the grant:
C. Schwab (co-investigator for the year 1991)
U. Banerjee (consultant and collaborator during visit)
C. Kim (graduate student - did not receive salary support from grant)

6. Talks given to disseminate information on funded research

(Talks are by P.I. unless indicated)
Invited Talks at Universities

Jan '90 University of California at San Diego, San Diego, California, "On locking and robustness in the finite element method"

Feb '90 University of Maryland, College Park, Maryland, "The problem of Poisson locking in the finite element method"

Nov '90 Indiana University, Bloomington, Indiana, "The p and h-p versions of the finite element method"

Feb '91 University of Maryland, College Park, Maryland, "Numerical Integration of Singular Integrals in the Boundary Element Method" (C. Schwab)

Mar '91 University of Texas, Arlington, Texas, "On locking and robustness in the finite element method"

Invited Talks at Conferences

June '89 ICOSAHOM '89 (International Conference on Spectral and Higher Order Methods), Villa Olmo, Italy (June 26-29, 1989), "The theoretical foundations of the p and h-p versions of the finite element method"

Oct '89	Numerical Methods for Elliptic Systems Workshop, Helsinki Institute of Technology, Espoo, Finland (October 25-28, 1989), "The h and p versions of the finite element method for problems involving numerical locking"
June '91	NATO Advanced Research Workshop on Numerical Integration, Bergen, Norway (June 17-21,1991)(C. Schwab)
July '91	*ICIAM 91, Washington, DC (July 8-12, 1991)* "A mathematical approach to locking and robustness" (co-organizer of minisymposium, "The problem of locking in finite element analysis")
July '91	13th IMACS World Congress on Computation and Applied Mathematics, Dublin, Ireland (July 22-26, 1991), "On locking effects in the finite element method"
July '91	13th IMACS World Congress on Computation and Applied Mathematics, Dublin, Ireland (July 22-26, 1991), "On the rate of convergence of the p version BEM over polyhedral surfaces"
November '91	Oberwolfach Conference on Singularities in Continuum Mechanics and Constructive Methods for their treatment, Oberwolfach, Germany, (17-22 November 1991), "The rate of convergence of the p-version BEM on polyhedral surfaces" (C. Schwab).

<u>Contributed Talks at Conferences</u>

Spring '89	Semi-annual FEM conference, Purdue University, West Lafayette, IN, "Unlocking Poisson locking"
July '89	Annual SIAM meeting, San Diego, CA, "Some results for a locking problem"
Fall '89	Semi-annual FEM conference, University of Maryland, Baltimore County campus, MD, "A mathematical formulation for numerical locking and robustness"
July '90	International Congress on Computational and Applied Mathematics, Leuven, Belgium, July 23-28, 1990, "On the robustness of the p and $h-p$ versions of the finite element method"
Fall, '90	Semi-annual FEM conference, Rutgers University, New Jersey, "On the L_2 error in the p-version of the finite element method"
Spring, '91	Semi-annual FEM conference, University of Houston, Houston, Texas, "Locking effects for nearly incompressible materials using rectangular elements"
August, '91	Equadiff '91, Barcelona, Spain, August 26-31, 1991, "The problem of locking in parameter-dependent problems"