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WAVE PROPAGATION IN LATERALLY VARYING
MEDIA: A MODEL EXPANSION METHOD

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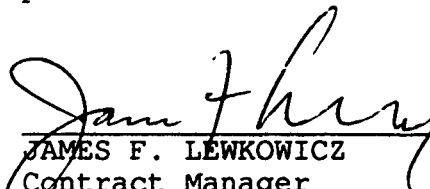


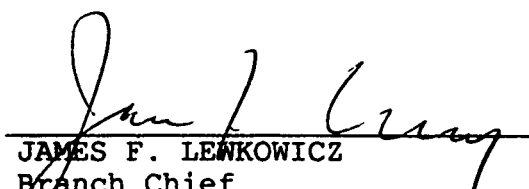
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13. ABSTRACT (Maximum 200 words) A general approach, using modes defined on subregions of the medium, has been developed to model seismic wave propagation in media with vertically and horizontally variable elastic and anelastic properties. The approach is also applicable to acoustic waves in fluid media and electromagnetic wave propagation in laterally varying media. The restriction on the medium variability is that it can be represented by step function variations in its properties in both the vertical and horizontal directions. The basic method makes use of normal mode expansion of the wave field in each partitioned sub-region of the medium within which the medium is uniform in the lateral directions. Thus the medium is partitioned into laterally uniform zones and complete normal mode solutions are obtained for each horizontally layered zone. In the analytical development the "zonal eigenvalues and eigenfunctions" are generated by treating each zone as a layered half space or radially layered sphere, as is appropriate for medium geometry. The resulting set of modes are then used as a bases for expansions of the wave fields in the layered subregions. The modes are then used as bases for expansions of the wave fields in each zone at the common boundaries				
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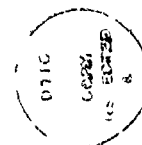
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between the zones where continuity of displacement and traction is required. This results in the definition of a "lateral propagator" of the wave field when applied to all the zones making up the entire medium and is, in application, very similar to the classical "vertical propagator" method. The method is exact when the lateral variations are actually discontinuous step changes in properties. When the actual changes can be approximated as a sequence of steps the method should be superior in computational accuracy and speed to numerical methods.

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Wave Propagation in Laterally Varying Media: A Modal Expansion Method

by

Charles B. Archambeau

Zonal Partitioning and Green's Function Representations

Consider a two dimensionally varying elastic-anelastic medium, as indicated in Figure 1. In each zone V_α , $\alpha = 1, 2, \dots, M$, the medium varies in the vertical direction (z), but is uniform in the horizontal direction (y or ρ). The supposition is that the laterally varying medium can be approximated by a series of step variations in material properties in the same way as is done in the vertical direction.

In V_α we have for the frequency domain displacement field $^{(\alpha)}u$ at any point r within V_α :

$$^{(\alpha)}u_i(r, \omega) = \frac{1}{4\pi} \int_{\Sigma_\alpha + \Sigma_{\alpha-1}} \left[t_j(r_o) ^{(\alpha)}G_j^i(r, r_o; \omega) - u_j(r_o) ^{(\alpha)}g_j^i(r, r_o; \omega) \right] d\alpha_o \quad (1)$$

where $^{(\alpha)}G_j^i$ and $^{(\alpha)}g_j^i$ are the zonal displacement and traction Greens' functions appropriate for the zone or region V_α .^{*} The vertical boundary surfaces of V_α are Σ_α and $\Sigma_{\alpha-1}$, as indicated in Figure 1. Here we assume no sources inside V_α and that the Green's functions satisfy all internal boundary conditions on all horizontal layers in V_α . (In this case there are no surface integrals over internal boundaries in (1)). Green's functions in V_α can be written in terms of the eigenvalues k_α and eigenfunctions $^{(\alpha)}\psi$ for this zone as ^{**}:

^{*} Summation over repeated coordinate indices is used throughout. Coordinate indices will appear as lower case latin subscripts and superscripts. The summation convention does *not* apply to any indices appearing in parenthesis.

^{**} Throughout this development the "sum" over the eigenvalues k_α will be written as a discrete summation but it should be understood that in an unbounded medium, such as a layered half space, part of the wave number spectrum will be continuous. In this case the "sum" over k_α must be interpreted as a generalized summation involving a regular sum over the discrete part of the spectrum plus an integration over the continuous part of the wave number spectrum.

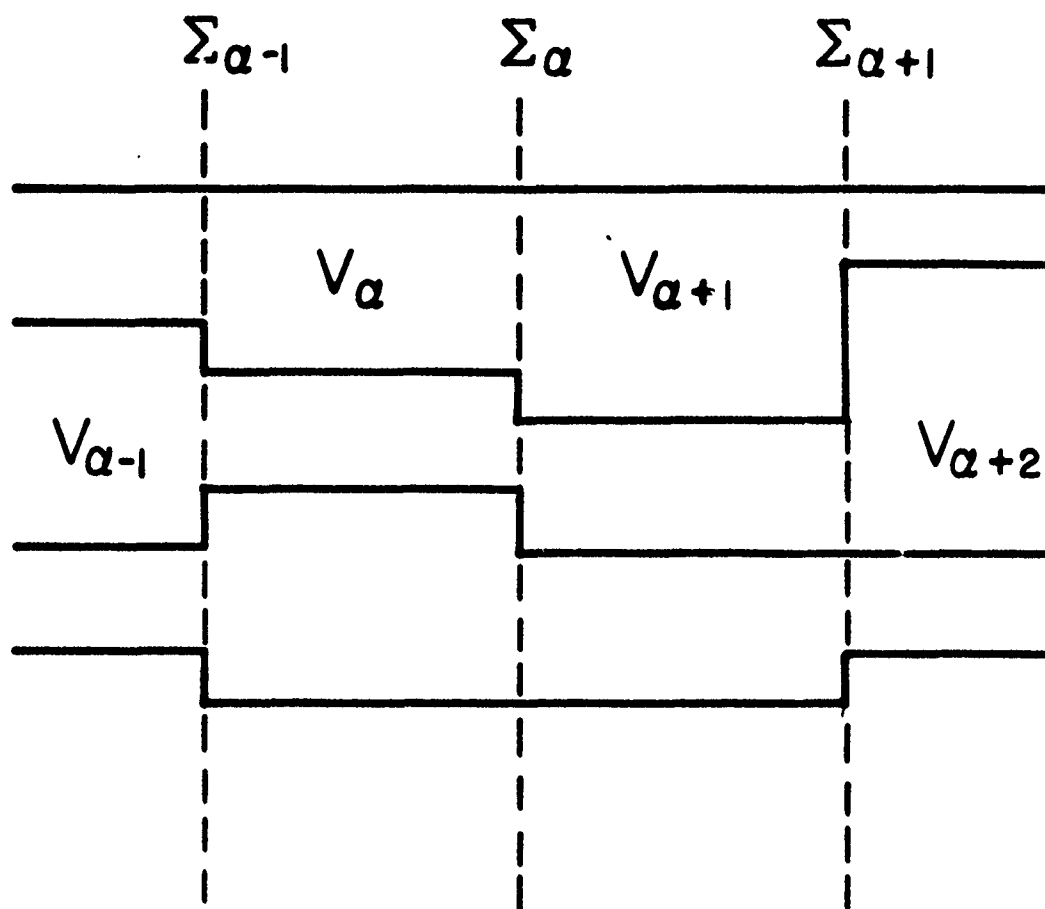


Figure 1. Zonal partitioning of a vertically and laterally varying medium into subregions of uniform horizontal layering.

$${}^{(\alpha)}G_j^i(\mathbf{r}, \mathbf{r}_0; \omega) = 4\pi \sum_{m, k_\alpha} \frac{{}^{(\alpha)}\bar{\psi}_j(\mathbf{r}_0, k_\alpha) {}^{(\alpha)}\psi_i(\mathbf{r}, k_\alpha)}{N_\alpha(k_\alpha, \omega)} \quad (2)$$

where ${}^{(\alpha)}\bar{\psi}_i$ is the complex conjugate of ${}^{(\alpha)}\psi_i$ and N_α is a normalization constant which may be a function of frequency ω and the wave number k_α . Since the ${}^{(\alpha)}\psi_j$ are eigenfunctions for the region V_α , this Green's function satisfies all boundary conditions along the horizontal boundaries in V_α . (For details see Harvey, 1983.)

Further, since:

$${}^{(\alpha)}g_j^i(\mathbf{r}; \mathbf{r}_0; \omega) = n_k^{(o)} \left[C_{kj}^{(o)} \frac{\partial {}^{(\alpha)}G_n^i(\mathbf{r}, \mathbf{r}_0; \omega)}{\partial x_l^{(o)}} \right]$$

where $n_k^{(o)}$ is the surface normal to Σ_α and $\Sigma_{\alpha-1}$ and $x_l^{(o)}$ are *source coordinate* variables, then

$${}^{(\alpha)}g_j^i(\mathbf{r}; \mathbf{r}_0; \omega) = 4\pi \sum_{m, k_\alpha} \frac{{}^{(\alpha)}\bar{\Psi}_j(\mathbf{r}_0, k_\alpha) {}^{(\alpha)}\psi_i(\mathbf{r}, k_\alpha)}{N_\alpha(k_\alpha, \omega)} \quad (3)$$

Here m is the angular index for cylindrical coordinates, k_α the horizontal wave number corresponding to the modes in V_α and where:

$${}^{(\alpha)}\bar{\Psi}_j(\mathbf{r}_0, k_\alpha) = n_k^{(o)} C_{kj}^{(o)} \frac{\partial}{\partial x_l^{(o)}} \left[{}^{(\alpha)}\psi_n(\mathbf{r}_0, k_\alpha) \right] \quad (4)$$

Because of the horizontal layering in V_α , the eigenfunctions ${}^{(\alpha)}\psi_j$ and ${}^{(\alpha)}\Psi_j$ are defined sectionally, that is:

$${}^{(\alpha)}\psi_j = \left\{ {}^{(\alpha)}\psi_j^{(s)}(z) \mid z_{s-1} \leq z \leq z_s \right\}_1^N$$

with (s) the horizontal layer index in V_α .

For the horizontally layered region V_α we have that:

$$\left. \begin{aligned} {}^{(\alpha)}G_j^i &= {}^{(\alpha)}G_R^i + {}^{(\alpha)}G_L^i \\ {}^{(\alpha)}g_j^i &= {}^{(\alpha)}g_R^i + {}^{(\alpha)}g_L^i \end{aligned} \right\} \quad (5)$$

Here $^{(\alpha)}G_j^i$ and $^{(\alpha)}\bar{G}_j^i$ are the Rayleigh and Love type Green's displacement functions (with similar names for the associated Green's tractions) and where

$$\left. \begin{aligned} ^{(\alpha)}G_j^i(\mathbf{r}, \mathbf{r}_0; \omega) &= 4\pi \sum_{m, Rk_\alpha} \frac{^{(\alpha)}\Psi_j(\mathbf{r}_0, Rk_\alpha) ^{(\alpha)}\bar{\Psi}_i(\mathbf{r}, Rk_\alpha)}{N_\alpha^{(R)}(k_\alpha, \omega)} \\ ^{(\alpha)}\bar{G}_j^i(\mathbf{r}, \mathbf{r}_0; \omega) &= 4\pi \sum_{m, Lk_\alpha} \frac{^{(\alpha)}\bar{\Psi}_j(\mathbf{r}_0, Lk_\alpha) ^{(\alpha)}\Psi_i(\mathbf{r}, Lk_\alpha)}{N_\alpha^{(L)}(k_\alpha, \omega)} \end{aligned} \right\} \quad (6)$$

with Rk_α and Lk_α representing the Rayleigh and Love type mode eigenvalues. Likewise

$$\left. \begin{aligned} ^{(\alpha)}g_j^i(\mathbf{r}, \mathbf{r}_0; \omega) &= 4\pi \sum_{m, Rk_\alpha} \frac{^{(\alpha)}\bar{\Psi}_j(\mathbf{r}_0, Rk_\alpha) ^{(\alpha)}\Psi_i(\mathbf{r}, Rk_\alpha)}{N_\alpha^{(R)}(k_\alpha, \omega)} \\ ^{(\alpha)}\bar{g}_j^i(\mathbf{r}, \mathbf{r}_0; \omega) &= 4\pi \sum_{m, Lk_\alpha} \frac{^{(\alpha)}\bar{\Psi}_j(\mathbf{r}_0, Lk_\alpha) ^{(\alpha)}\Psi_i(\mathbf{r}, Lk_\alpha)}{N_\alpha^{(L)}(k_\alpha, \omega)} \end{aligned} \right\} \quad (7)$$

In cylindrical coordinates (ρ, ϕ, z) , the eigenfunctions are (see, for example, Harvey, 1981):

$$\left. \begin{aligned} ^{(\alpha)}\Psi(\mathbf{r}, Rk_\alpha) &= ^{(\alpha)}D_m(z; Rk_\alpha) \mathbf{P}(Rk_\alpha \rho, \phi) \\ &\quad + ^{(\alpha)}E_m(z; Rk_\alpha) \mathbf{B}(Rk_\alpha \rho, \phi) \\ ^{(\alpha)}\bar{\Psi}(\mathbf{r}, Rk_\alpha) &= ^{(\alpha)}R_m(z; Rk_\alpha) \mathbf{P}_m(Rk_\alpha \rho, \phi) \\ &\quad + ^{(\alpha)}S_m(z; Rk_\alpha) \mathbf{B}_m(Rk_\alpha \rho, \phi) \end{aligned} \right\} \quad (8)$$

$$\left. \begin{aligned} ^{(\alpha)}\Psi(\mathbf{r}, Lk_\alpha) &= ^{(\alpha)}F_m(z; Lk_\alpha) \mathbf{C}_m(Lk_\alpha \rho, \phi) \\ ^{(\alpha)}\bar{\Psi}(\mathbf{r}, Lk_\alpha) &= ^{(\alpha)}T_m(z; Lk_\alpha) \mathbf{C}_m(Lk_\alpha \rho, \phi) \end{aligned} \right\} \quad (9)$$

Here $\mathbf{P}$, $\mathbf{B}$ and $\mathbf{C}$ are the vector cylindrical harmonics defined as:

$$\left. \begin{aligned} \mathbf{P}_m(k\rho, \phi) &\equiv \hat{\mathbf{e}}_z J_m(k\rho) e^{im\phi} \\ \mathbf{B}_m(k\rho, \phi) &\equiv \left[\hat{\mathbf{e}}_\rho \frac{\partial}{\partial(k\rho)} + \hat{\mathbf{e}}_\phi \left[\frac{1}{k\rho} \right] \frac{\partial}{\partial\phi} \right] J_m(k\rho) e^{im\phi} \\ \mathbf{C}_m(k\rho, \phi) &\equiv \left[\hat{\mathbf{e}}_\rho \left[\frac{1}{k\rho} \right] \frac{\partial}{\partial\phi} - \hat{\mathbf{e}}_\phi \frac{\partial}{\partial(k\rho)} \right] J_m(k\rho) e^{im\phi} \end{aligned} \right\} \quad (10)$$

where

$$J_m(k\rho) = H_m^{(1)}(k\rho) + H_m^{(2)}(k\rho)$$

with J_m the cylindrical Bessel function and $H_m^{(1)}$ and $H_m^{(2)}$ the cylindrical Hankel functions. These vector functions are clearly such that $\mathbf{P}_m \cdot \mathbf{B}_m = \mathbf{P}_m \cdot \mathbf{C}_m = \mathbf{B}_m \cdot \mathbf{C}_m = 0$ and also have the usual functional orthogonality. (e.g. Stratton 1941, Morse and Feshbach, 1953). Here $\hat{e}_z$, $\hat{e}_\rho$ and $\hat{e}_\phi$ are the unit vectors in cylindrical coordinates. The various "stress-displacement" functions $^{(\alpha)}D_m$, $^{(\alpha)}E_m$, $^{(\alpha)}R_m \dots ^{(\alpha)}T_m$ in (8) are the same as those usually appearing in the ordinary developments for a laterally *homogeneous* layered half space -- such as described in Harkrider (1964); Ben Menahem and Singh (1972), or Harvey (1981).

Similar representations for the eigenfunctions can be given in cartesian and spherical coordinates. (In the latter case the eigenfunctions ${}_R\psi$ and ${}_L\psi$ are usually termed spheroidal and torsional; and $\mathbf{P}$, $\mathbf{B}$ and $\mathbf{C}$ become vector spherical harmonics). The choice of cylindrical coordinates implies rotational symmetry, that is that the medium is partitioned into zones V_α which are cylindrical shells, with Figure 1 depicting a cross section at fixed ϕ . If cartesian coordinates are used, then Figure 1 represents a cross section at constant y , with properties constant in the $\pm y$ directions. In the development that immediately follows cylindrical coordinates will be used; however the cartesian and spherical representations are also appropriate and the development and results are analogous to those for the cylindrical choice.

"Forward" and "Backward Propagating" Mode Expansions

In addition to the eigenfunction expansions of the Green's functions in V_α , we can also expand the displacements and tractions, appearing in (1) in terms of eigenfunctions in V_α . In particular, $^{(\alpha)}u_j(\mathbf{r}_o)$ and $^{(\alpha)}t_j(\mathbf{r}_o)$ may be expanded in terms of "forward" and "backward" propagating modes as:

$$\left. \begin{aligned} {}^{(\alpha)}u_j(r_o, \omega) &= {}^{(\alpha)}u_j^{(1)}(r_o, \omega) + {}^{(\alpha)}u_j^{(2)}(r_o, \omega) \\ {}^{(\alpha)}t_j(r_o, \omega) &= {}^{(\alpha)}t_j^{(1)}(r_o, \omega) + {}^{(\alpha)}t_j^{(2)}(r_o, \omega) \end{aligned} \right\} \quad (11)$$

where the superscripts (1) and (2) denote modes propagating in the positive and negative radial (ρ) directions. Specifically,

$$\left. \begin{aligned} {}^{(\alpha)}u_j(r_o, \omega) &= \sum_{m', k'_a} \left[{}^{(\alpha)}a_{m'}^{(1)}(k'_a) {}^{(\alpha)}\psi_j^{(1)}(r_o, k'_a) + {}^{(\alpha)}a_{m'}^{(2)}(k'_a) {}^{(\alpha)}\psi_j^{(2)}(r_o, k'_a) \right] \\ {}^{(\alpha)}t_j(r_o, \omega) &= \sum_{m', k'_a} \left[{}^{(\alpha)}a_{m'}^{(1)}(k'_a) {}^{(\alpha)}\psi_j^{(1)}(r_o, k'_a) - {}^{(\alpha)}a_{m'}^{(2)}(k'_a) {}^{(\alpha)}\psi_j^{(2)}(r_o, k'_a) \right] \end{aligned} \right\} \quad (12)$$

where

$$\begin{aligned} {}^{(\alpha)}\psi^{(p)}(r_o, k'_a) &= \left[{}^{(\alpha)}D_{m'}(z_o; k'_a) P_m^{(p)} + {}^{(\alpha)}E_{m'}(z_o; k'_a) B_m^{(p)} + {}^{(\alpha)}F_{m'}(z_o; k'_a) C_m^{(p)} \right] e^{im'\phi} \\ {}^{(\alpha)}\psi^{(p)}(r_o, k'_a) &= \left[{}^{(\alpha)}R_{m'}(z_o; k'_a) P_m^{(p)} + {}^{(\alpha)}S_{m'}(z_o; k'_a) B_m^{(p)} + {}^{(\alpha)}T_{m'}(z_o; k'_a) C_m^{(p)} \right] e^{im'\phi} \end{aligned} \quad (13)$$

with

$$\left. \begin{aligned} P_m^{(p)} &= \hat{e}_z H_m^{(p)}(k'_a \rho) ; p = 1, 2 \\ B_m^{(p)} &= \left[\hat{e}_\rho \frac{\partial}{\partial(k'_a \rho)} + \hat{e}_\phi \left[\frac{im'}{k'_a \rho} \right] \right] H_m^{(p)}(k'_a \rho) \\ C_m^{(p)} &= \left[\hat{e}_\rho \left[\frac{im'}{k'_a \rho} \right] - \hat{e}_\phi \frac{\partial}{\partial(k'_a \rho)} \right] H_m^{(p)}(k'_a \rho) \end{aligned} \right\} \quad (14)$$

The coefficients ${}^{(\alpha)}a_m^{(p)}(k_a)$ are to be determined from boundary conditions at Σ_α and $\Sigma_{\alpha-1}$. These conditions bring the continuity of displacement and traction on these surfaces. On the other hand, of course, all the functions ${}^{(\alpha)}D_m^{(p)}$, ${}^{(\alpha)}E_m^{(p)}$, ${}^{(\alpha)}F_m^{(p)}$, ${}^{(\alpha)}R_m^{(p)}$, ${}^{(\alpha)}S_m^{(p)}$, and ${}^{(\alpha)}T_m^{(p)}$ are known functions of the coordinate variables and the intrinsic material properties of the internal horizontal layers, since they are provided by the usual one-dimensional propagator approach in a layered half space (e.g., Harvey, 1981). The explicit forms of the functions are included in the

Appendix 1.

Given that G_j^i and g_j^i in (1) can be split into Rayleigh and Love type Green's functions, as defined in (5)-(9), then it follows that $^{(\alpha)}u_j$ can also be split into modal sums involving only $^{(\alpha)}\psi_j$ and $^{(\alpha)}\bar{\psi}_j$. Therefore:

$$^{(\alpha)}u_j = \sum_{p=1}^2 {}^{(\alpha)}u_j^{(p)} = \sum_{p=1}^2 \left[{}^{(\alpha)}u_j^{(p)} + {}^{(\alpha)}u_j^{(p)} \right] \quad (15)$$

where

$$\begin{aligned} \sum_{p=1}^2 {}^{(\alpha)}u_j^{(p)}(r_o, \omega) &= \sum_{m', Rk_\alpha} \left[{}^{(\alpha)}a_{m'}^{(1)}(Rk_\alpha) {}^{(\alpha)}\psi_j^{(1)} + {}^{(\alpha)}a_{m'}^{(2)}(Rk_\alpha) {}^{(\alpha)}\psi_j^{(2)} \right] \\ \sum_{p=1}^2 {}^{(\alpha)}u_j^{(p)}(r_o, \omega) &= \sum_{m', Lk_\alpha} \left[{}^{(\alpha)}a_{m'}^{(1)}(Lk_\alpha) {}^{(\alpha)}\psi_j^{(1)} + {}^{(\alpha)}a_{m'}^{(2)}(Lk_\alpha) {}^{(\alpha)}\psi_j^{(2)} \right] \end{aligned} \quad (16)$$

with

$$\left. \begin{aligned} {}^{(\alpha)}\psi_j^{(p)}(r_o, Rk_\alpha) &= \left[{}^{(\alpha)}D_{m'}(z_o; Rk_\alpha) \mathbf{P}_{m'}^{(p)} + {}^{(\alpha)}E_{m'}(z_o; Rk_\alpha) \mathbf{B}_{m'}^{(p)} \right] e^{im'\phi} \\ {}^{(\alpha)}\bar{\psi}_j^{(p)}(r_o, Lk_\alpha) &= {}^{(\alpha)}F_{m'}(z_o; Lk_\alpha) \mathbf{C}_{m'}^{(p)} e^{im'\phi} \end{aligned} \right\} \quad (17)$$

A similar decomposition applies to the traction $^{(\alpha)}t$.

It is important to note that the eigenfunctions used to expand the Green's functions in equations (2) - (7) are appropriate for the horizontally layered zone in V_α and are themselves normalized such that:

$$\begin{aligned} \langle {}^{(\alpha)}\psi_j^{(p)}(k_\alpha), {}^{(\alpha)}\psi_j^{(p)}(k'_\alpha) \rangle &= \int_{V_\alpha} {}^{(\alpha)}\psi_j^{(p)}(k_\alpha \mathbf{r}) {}^{(\alpha)}\bar{\psi}_j^{(p)}(k'_\alpha \mathbf{r}) dV = \delta(k_\alpha - k'_\alpha) \delta_{m'}^{m'} \\ \langle {}^{(\alpha)}\bar{\psi}_j^{(p)}(k_\alpha), {}^{(\alpha)}\bar{\psi}_j^{(p)}(k'_\alpha) \rangle &= \int_{V_\alpha} {}^{(\alpha)}\bar{\psi}_j^{(p)}(k_\alpha \mathbf{r}) {}^{(\alpha)}\psi_j^{(p)}(k'_\alpha \mathbf{r}) dV = \delta(k_\alpha - k'_\alpha) \delta_{m'}^{m'} \end{aligned} \quad (19)$$

where $^{(\alpha)}\bar{\psi}_j$ denotes the complex conjugate of $^{(\alpha)}\psi_j$ and the right hand side involves the usual

delta functions. Therefore the normalization factors appearing in the Green's function expansions are free parameters that may be chosen so as to appropriately normalize the *zonal* Green's functions in V_α , $\alpha = 1, 2, \dots, M$.

To obtain the appropriate normalization factors for ${}^{(\alpha)}G_j^I$ and ${}^{(\alpha)}G_j^J$ and, in addition, to express these Green's functions in forms that are convenient for use with the expanded form for ${}^{(\alpha)}u_j$ in (15)-(16), it is useful to adopt an expansion form for the Green's functions that is similar to that for ${}^{(\alpha)}u_j$ in (15). That is, using both ${}^{(\alpha)}\psi_j^{(1)}$ and ${}^{(\alpha)}\psi_j^{(2)}$ in the expansion for ${}^{(\alpha)}G_j^I$, we express the Green's functions as:

$$\left. \begin{aligned} {}^{(\alpha)}G_j^I(\mathbf{r}, \mathbf{r}_0; \omega) &= {}^{(\alpha)}G_{j1}^{(1)} + {}^{(\alpha)}G_{j1}^{(2)} \\ {}^{(\alpha)}G_j^J(\mathbf{r}, \mathbf{r}_0; \omega) &= {}^{(\alpha)}G_{j2}^{(1)} + {}^{(\alpha)}G_{j2}^{(2)} \end{aligned} \right\} \quad (20)$$

where:

$$\left. \begin{aligned} {}^{(\alpha)}G_{j1}^{(p)} &= 4\pi \sum_{m, Rk_\alpha} \frac{{}^{(\alpha)}\bar{\psi}_j^{(p)}(\mathbf{r}_0, Rk_\alpha) {}^{(\alpha)}\psi_j^{(p)}(\mathbf{r}, Rk_\alpha)}{R N_p^{(\alpha)}(k_\alpha, \omega)} \\ {}^{(\alpha)}G_{j2}^{(p)} &= 4\pi \sum_{m, k_\alpha} \frac{{}^{(\alpha)}\bar{\psi}_j^{(p)}(\mathbf{r}_0, Lk_\alpha) {}^{(\alpha)}\psi_j^{(p)}(\mathbf{r}, Lk_\alpha)}{L N_p^{(\alpha)}(k_\alpha, \omega)} \end{aligned} \right\} \quad (21)$$

and similarly for ${}^{(\alpha)}g_j^I$ and ${}^{(\alpha)}g_j^J$, the Green's tractions.

Orthogonality and Normalization Relations for Zonal Eigenfunctions

We can use (15)-(16) in (1) and also substitute (20)-(21) into this representation integral. Since the representation given by (1) should be of the form of the expansion in (15), we should obtain by proper choice of the normalization factors, $R N_p^{(\alpha)}$ and $L N_p^{(\alpha)}$, exactly the expansion given in (15) in terms of forward and backward propagating modes. In particular, from (1) we have:

$${}^{(\alpha)}u_j(\mathbf{r}, \omega) = {}^{(\alpha)}u_{j1}(\mathbf{r}, \omega) + {}^{(\alpha)}u_{j2}(\mathbf{r}, \omega) \quad (22)$$

with

$$\begin{aligned} {}^{(\alpha)}u_i(\mathbf{r}, \omega) &= \int_{\Sigma_\alpha + \Sigma_{\alpha-1}} \left[{}^{(\alpha)}G_j^1 - {}^{(\alpha)}R u_j \quad {}^{(\alpha)}g_j^1 \right] d\mathbf{a}_o ; \mathbf{r} \in V_\alpha \\ {}^{(\alpha)}u_i(\mathbf{r}, \omega) &= \int_{\Sigma_\alpha + \Sigma_{\alpha-1}} \left[{}^{(\alpha)}G_j^1 - {}^{(\alpha)}L u_j \quad {}^{(\alpha)}g_j^1 \right] d\mathbf{a}_o ; \mathbf{r} \in V_\alpha \end{aligned} \quad (23)$$

Introducing the explicit eigenfunction expansions from (15)-(16) and (20)-(21), we get:

$${}^{(\alpha)}u_j(\mathbf{r}, \omega) = \sum_{p=1}^2 {}^{(\alpha)}u_j^{(p)}(\mathbf{r}, \omega) \quad (24)$$

$$\begin{aligned} {}^{(\alpha)}u_i^{(p)}(\mathbf{r}, \omega) &= \sum_{m', Rk_\alpha} {}^{(\alpha)}a_{m'}^{(p)}(Rk'_\alpha) \sum_{m, Rk_\alpha} \left\{ \frac{1}{R N_p^{(\alpha)}(k_\alpha, \omega)} \right\} \left\{ \left[\langle {}^{(\alpha)}\Psi_j^{(p)}(Rk'_\alpha), {}^{(\alpha)}\Psi_j^{(1)}(Rk_\alpha) \rangle_{\alpha, \alpha-1} \right. \right. \\ &\quad \left. \left. - \langle {}^{(\alpha)}\Psi_j^{(p)}(Rk'_\alpha), {}^{(\alpha)}\Psi_j^{(1)}(Rk_\alpha) \rangle_{\alpha, \alpha-1} \right] {}^{(\alpha)}\Psi_i^{(1)}(\mathbf{r}, Rk_\alpha) + \left[\langle {}^{(\alpha)}\Psi_j^{(p)}(Rk'_\alpha), {}^{(\alpha)}\Psi_j^{(2)}(Rk_\alpha) \rangle_{\alpha, \alpha-1} \right. \right. \\ &\quad \left. \left. - \langle {}^{(\alpha)}\Psi_j^{(p)}(Rk'_\alpha), {}^{(\alpha)}\Psi_j^{(2)}(Rk_\alpha) \rangle_{\alpha, \alpha-1} \right] {}^{(\alpha)}\Psi_i^{(2)}(\mathbf{r}, k_\alpha) \right\} \end{aligned} \quad (25)$$

Here terms of the form:

$$\langle \psi_j(k'_\alpha), \chi_j(k_\alpha) \rangle_{\alpha, \alpha-1} \equiv \langle \psi_j(k'_\alpha), \chi_j(k_\alpha) \rangle_\alpha + \langle \psi_j(k'_\alpha), \chi_j(k_\alpha) \rangle_{\alpha-1}$$

are introduced, where the inner product is defined over the surface Σ_α (or $\Sigma_{\alpha-1}$) as:

$$\langle \psi_j(k'_\alpha), \chi_j(k_\alpha) \rangle_\alpha \equiv \int_{\Sigma_\alpha} \psi_j(\mathbf{r}_o, k'_\alpha) \bar{\chi}_j(\mathbf{r}_o, k_\alpha) d\mathbf{a}_o$$

with summation over the repeated coordinate index (j) implied. An exactly analogous result holds for ${}^{(\alpha)}L u_j$; with the suffix "R" replaced by "L" in (24) and (25).

Comparing (25) with the equivalent expressions in (15) - (16), it is clear that the inner products appearing in (25) must reduce to delta functions over the angular index m and the mode eigenvalues k_α . In particular, the following orthogonality conditions apply*:

*Where it is obvious from context, the R and L identifying subscripts on the wave numbers Rk_α and Lk_α will be suppressed in order to reduce clutter in the equations.

$$\begin{aligned}
& \langle {}^{(\alpha)}\Psi_j^{(p)}(k'_\alpha), {}^{(\alpha)}\Psi_j^{(p)}(k_\alpha) \rangle_\beta - \langle {}^{(\alpha)}\Psi_j^{(p)}(k'_\alpha), {}^{(\alpha)}\Psi_j^{(p)}(k_\alpha) \rangle_\beta \\
&= \int_{\Sigma_\beta} \left[{}^{(\alpha)}\Psi_j^{(p)}(k'_\alpha \mathbf{r}_o) \cdot {}^{(\alpha)}\bar{\Psi}_j^{(p)}(k_\alpha \mathbf{r}_o) - {}^{(\alpha)}\Psi_j^{(p)}(k'_\alpha \mathbf{r}_o) \cdot {}^{(\alpha)}\bar{\Psi}_j^{(p)}(k_\alpha \mathbf{r}_o) \right] d\mathbf{a}_o \quad (26) \\
&= n_\beta \left[\mathbf{P}_m^{(p)}(k'_\alpha \rho_\beta) \cdot \bar{\mathbf{P}}_m^{(p)}(k_\alpha \rho_\beta) + \mathbf{B}_m^{(p)}(k'_\alpha \rho_\beta) \cdot \bar{\mathbf{B}}_m^{(p)}(k_\alpha \rho_\beta) \right] \delta_{k'_\alpha}^{k_\alpha} \delta_m^{m'} ;
\end{aligned}$$

with $\beta = \alpha, \alpha - 1$ and $p = 1, 2$ and where $n_\beta = 2\pi\rho_\beta$. (Here ρ_β is the constant value of the radial coordinate on the surface Σ_β .) In addition,

$$\begin{aligned}
& \langle {}^{(\alpha)}\Psi_j^{(p)}(k'_\alpha), {}^{(\alpha)}\Psi_j^{(q)}(k_\alpha) \rangle_\beta - \langle {}^{(\alpha)}\Psi_j^{(p)}(k'_\alpha), {}^{(\alpha)}\Psi_j^{(q)}(k_\alpha) \rangle_\beta = \\
& \int_{\Sigma_\beta} \left[{}^{(\alpha)}\Psi_j^{(p)}(k'_\alpha \mathbf{r}_o) \cdot {}^{(\alpha)}\bar{\Psi}_j^{(q)}(k_\alpha \mathbf{r}_o) - {}^{(\alpha)}\Psi_j^{(p)}(k'_\alpha \mathbf{r}_o) \cdot {}^{(\alpha)}\bar{\Psi}_j^{(q)}(k_\alpha \mathbf{r}_o) \right] d\mathbf{a}_o = 0 \quad (27)
\end{aligned}$$

for $\beta = \alpha, \alpha - 1$ and $p \neq q$. Formally identical relations hold for the eigenfunctions ${}^{(\alpha)}\Psi_j^{(p)}$ and are obtained by replacing the suffix "R" by "L" in (26) and (27). Here we observe that the forward and backward propagating modes are completely orthogonal sets. These conditions are equivalent to those obtained by Herrera (1964) and McGarr and Alsop (1967) and were used by Kennett (1983) in his development of a formalism for wave propagation in laterally varying media. In more explicit form, equations (26) and (27) are equivalent to:

$$\begin{aligned}
& \int_0^\infty \left[{}^{(\alpha)}R_m(z_o; k'_\alpha) {}^{(\alpha)}\bar{D}_m(z_o; k_\alpha) - {}^{(\alpha)}D_m(z_o; k'_\alpha) {}^{(\alpha)}\bar{R}_m(z_o; k_\alpha) \right] dz_o = \delta_{k'_\alpha}^{k_\alpha} \\
& \int_0^\infty \left[{}^{(\alpha)}S_m(z_o; k'_\alpha) {}^{(\alpha)}\bar{E}_m(z_o; k_\alpha) - {}^{(\alpha)}E_m(z_o; k'_\alpha) {}^{(\alpha)}\bar{S}_m(z_o; k_\alpha) \right] dz_o = \delta_{k'_\alpha}^{k_\alpha}
\end{aligned}$$

where the subscript "R" on the P-SV wave number has also been suppressed in these expressions. For the SH modes the analogous orthogonality relation is easily seen to be

$$\int_0^\infty \left[{}^{(\alpha)}T_m(z_o; k'_\alpha) {}^{(\alpha)}\bar{F}_m(z_o; k_\alpha) - {}^{(\alpha)}F_m(z_o; k'_\alpha) {}^{(\alpha)}\bar{T}_m(z_o; k_\alpha) \right] dz_o = \delta_{k'_\alpha}^{k_\alpha}$$

where the wave numbers and k_α and k'_α now refer to the SH wave number set ${}_L k_\alpha$. The "vertical eigenfunctions" in V_α are those defined in (13) and are simple exponentials in z_0 . (See Harvey, 1981.) Here also we consider the k_α to be discrete infinite sets, so that orthogonality is expressed by the Kronecker delta $\delta_{k_\alpha}^{k'_\alpha}$.

Using these orthogonality relations in (25) gives:

$$\begin{aligned} {}^{(\alpha)}_R u_j^{(1)}(\mathbf{r}, \omega) = \sum_{m', k_\alpha} {}^{(\alpha)}_R a_{m'}^{(1)}(k'_\alpha) \sum_{m, k_\alpha} \left[\frac{\xi_{k_\alpha}^{k'_\alpha} \delta_m^{m'}}{{}_R N_I^{(\alpha)}(k_\alpha, \omega)} \right] \left[n_\alpha \left\{ \mathbf{P}_m^{(1)}(k'_\alpha \rho_\alpha) \cdot \bar{\mathbf{P}}_m^{(1)}(k_\alpha \rho_\alpha) + \mathbf{B}_m^{(1)}(k'_\alpha \rho_\alpha) \cdot \bar{\mathbf{B}}_m^{(1)}(k_\alpha \rho_\alpha) \right\} \right. \\ \left. + n_{\alpha-1} \left\{ \mathbf{P}_m^{(1)}(k'_\alpha \rho_{\alpha-1}) \cdot \bar{\mathbf{P}}_m^{(1)}(k_\alpha \rho_{\alpha-1}) + \mathbf{B}_m^{(1)}(k'_\alpha \rho_{\alpha-1}) \cdot \bar{\mathbf{B}}_m^{(1)}(k_\alpha \rho_{\alpha-1}) \right\} \right] {}^{(\alpha)}_R \psi_j^{(1)}(\mathbf{r}, k_\alpha) \end{aligned}$$

So

$${}^{(\alpha)}_R u_j^{(1)}(\mathbf{r}, \omega) = \sum_{m, k_\alpha} {}^{(\alpha)}_R a_m^{(1)}(k_\alpha) {}^{(\alpha)}_R \psi_j^{(1)}(\mathbf{r}, k_\alpha)$$

provided we take:

$$\begin{aligned} {}_R N_I^{(\alpha)} = \left[n_\alpha \left\{ \mathbf{P}_m^{(1)}(k_\alpha \rho_\alpha) \cdot \bar{\mathbf{P}}_m^{(1)}(k_\alpha \rho_\alpha) + \mathbf{B}_m^{(1)}(k_\alpha \rho_\alpha) \cdot \bar{\mathbf{B}}_m^{(1)}(k_\alpha \rho_\alpha) \right\} \right. \\ \left. + n_{\alpha-1} \left\{ \mathbf{P}_m^{(1)}(k_\alpha \rho_{\alpha-1}) \cdot \bar{\mathbf{P}}_m^{(1)}(k_\alpha \rho_{\alpha-1}) + \mathbf{B}_m^{(1)}(k_\alpha \rho_{\alpha-1}) \cdot \bar{\mathbf{B}}_m^{(1)}(k_\alpha \rho_{\alpha-1}) \right\} \right] \end{aligned} \quad (28)$$

Similarly,

$${}_R u_j^{(2)}(\mathbf{r}, \omega) = \sum_{m, k_\alpha} {}^{(\alpha)}_R a_m^{(2)}(k_\alpha) {}^{(\alpha)}_R \psi_j^{(2)}(\mathbf{r}, k_\alpha)$$

provided

$${}_R N_2^{(\alpha)} = \left[n_\alpha \left\{ \mathbf{P}_m^{(2)}(k_\alpha \rho_\alpha) \cdot \bar{\mathbf{P}}_m^{(2)}(k_\alpha \rho_\alpha) + \mathbf{B}_m^{(2)}(k_\alpha \rho_\alpha) \cdot \bar{\mathbf{B}}_m^{(2)}(k_\alpha \rho_\alpha) \right\} \right] \quad (29)$$

$$+ n_{\alpha-1} \left\{ \mathbf{P}_m^{(2)}(k_\alpha \rho_{\alpha-1}) \cdot \bar{\mathbf{P}}_m^{(2)}(k_\alpha \rho_{\alpha-1}) + \mathbf{B}_m^{(2)}(k_\alpha \rho_{\alpha-1}) \cdot \bar{\mathbf{B}}_m^{(2)}(k_\alpha \rho_{\alpha-1}) \right\}$$

The results for ${}^{(\alpha)}\mathbf{u}$ are analogous and the normalization factors are:

$${}_L N_1^{(\alpha)} = \left[n_\alpha C_m^{(1)}(k_\alpha \rho_\alpha) \cdot \bar{C}_m^{(1)}(k_\alpha \rho_\alpha) + n_{\alpha-1} C_m^{(1)}(k_\alpha \rho_{\alpha-1}) \bar{C}_m^{(1)}(k_\alpha \rho_{\alpha-1}) \right] \quad (30)$$

$${}_L N_2^{(\alpha)} = \left[n_\alpha C_m^{(2)}(k_\alpha \rho_\alpha) \cdot \bar{C}_m^{(2)}(k_\alpha \rho_\alpha) + n_{\alpha-1} C_m^{(2)}(k_\alpha \rho_{\alpha-1}) \bar{C}_m^{(2)}(k_\alpha \rho_{\alpha-1}) \right] \quad (31)$$

Thus, the form of the displacement field in any one of the zones V_α is given by

$$\begin{aligned} {}^{(\alpha)}u_i(\mathbf{r}, \omega) = & \sum_{m, R k_\alpha} \left[{}^{(\alpha)}a_m^{(1)}(R k_\alpha) {}^{(\alpha)}\psi_i^{(1)}(\mathbf{r}, R k_\alpha) + {}^{(\alpha)}a_m^{(2)}(R k_\alpha) {}^{(\alpha)}\psi_i^{(2)}(\mathbf{r}, R k_\alpha) \right] \\ & + \sum_{m, L k_\alpha} \left[{}^{(\alpha)}a_m^{(1)}(L k_\alpha) {}^{(\alpha)}\psi_i^{(1)}(\mathbf{r}, L k_\alpha) + {}^{(\alpha)}a_m^{(2)}(L k_\alpha) {}^{(\alpha)}\psi_i^{(2)}(\mathbf{r}, L k_\alpha) \right] ; \mathbf{r} \in V_\alpha \end{aligned} \quad (32)$$

which is (merely) a sum of P-SV modes propagating in the forward and backward horizontal directions, plus a similar sum of SH modes. Further, the displacement field in V_α is connected to its values on the boundary surfaces Σ_α and $\Sigma_{\alpha-1}$ by the representations in (23), with the Greens functions given by the eigenfunction expansions of (20) - (21) and with the normalizations specified by (28) - (31). Use of these latter representations provide the means of determining the coefficients ${}^{(\alpha)}a_m^{(p)}$ and ${}^{(\alpha)}a_m^{(p)}$ in (32), and thereby an explicit expression of the displacement field in V_α in terms of the modes of this horizontally layered region. As will be shown, the coefficients between all the zones V_α , $\alpha = 1, 2, \dots, M$, are linked by a propagator formalism.

Zonal Boundary Conditions, Projections and Lateral Propagators

Continuity conditions expressing conservation of momentum, mass and energy apply throughout the medium, however complex the intrinsic material properties. In particular such conditions apply along the control surfaces Σ_α separating the zones of uniform lateral properties

in Figure 1. In the case of a solid medium, with welded contacts at all layer boundaries, the continuity conditions along the surface Σ_α are:

$$\begin{pmatrix} (\alpha)u_j \\ (\alpha)t_j \end{pmatrix}_\alpha = \begin{pmatrix} (\alpha+1)u_j \\ (\alpha+1)t_j \end{pmatrix}_\alpha ; j = 1, 2, 3 \quad (33)$$

where the subscript α on the matrix brackets is used to indicate evaluation on the vertical boundary Σ_α between the zones V_α and $V_{\alpha+1}$

The displacements and tractions in (33) can be expressed in terms of the eigenfunction expansion of (32). However, since the P-SV and SH modes are decoupled in V_α and $V_{\alpha+1}$, then (33) can also be expressed by the decoupled set of relations:

$$\sum_m \sum_{Rk_\alpha} \sum_{p=1}^2 \begin{pmatrix} (\alpha)A_m^{(p)}(Rk_\alpha) \begin{pmatrix} (\alpha)\psi_j^{(p)}(Rk_\alpha) \\ (\alpha)\Psi_j^{(p)}(Rk_\alpha) \end{pmatrix}_\alpha \end{pmatrix} = \sum_m \sum_{Rk_{\alpha+1}} \sum_{p=1}^2 \begin{pmatrix} (\alpha+1)A_m^{(p)}(Rk_{\alpha+1}) \begin{pmatrix} (\alpha+1)\psi_j^{(p)}(Rk_{\alpha+1}) \\ (\alpha+1)\Psi_j^{(p)}(Rk_{\alpha+1}) \end{pmatrix}_\alpha \end{pmatrix} ; j = 1, 2 \quad (34a)$$

$$\sum_m \sum_{Lk_\alpha} \sum_{p=1}^2 \begin{pmatrix} (\alpha)A_m^{(p)}(Lk_\alpha) \begin{pmatrix} (\alpha)\psi_j^{(p)}(Lk_\alpha) \\ (\alpha)\Psi_j^{(p)}(Lk_\alpha) \end{pmatrix}_\alpha \end{pmatrix} = \sum_m \sum_{Lk_{\alpha+1}} \sum_{p=1}^2 \begin{pmatrix} (\alpha+1)A_m^{(p)}(Lk_{\alpha+1}) \begin{pmatrix} (\alpha+1)\psi_j^{(p)}(Lk_{\alpha+1}) \\ (\alpha+1)\Psi_j^{(p)}(Lk_{\alpha+1}) \end{pmatrix}_\alpha \end{pmatrix} ; j = 3 \quad (34b)$$

where the expansions in P-SV and SH modes have been substituted for u_j and t_j on both sides of (33). A similar set of boundary equations apply to the other vertical boundary of V_α , on the surface $\Sigma_{\alpha-1}$, in Figure 1. (In this case the matrices are evaluated on $\Sigma_{\alpha-1}$ so the matrix indices in (34) change to $(\alpha-1)$ throughout, while on the right side of (34) all the eigenvalue and eigenfunction indices change from $(\alpha+1)$ to $(\alpha-1)$.)

We can extract expressions for individual mode coefficients $(\alpha)A_m^{(p)}$ and $(\alpha)L_m^{(p)}$, appropriate to the zone V_α , in terms of the mode coefficients in the zone $V_{\alpha+1}$ by taking integral inner products ("projections") between the displacement and traction eigenfunctions on both sides of

(34). Then we can use the (P-SV) orthogonality relations in (26) - (27), along with comparable orthogonal relations for SH modes. Specifically, using inner product bracket notation as before in equation (25) and taking the inner products between displacement and traction eigenfunctions on both sides of (34), we have:

$$\sum_{m, k_\alpha} \sum_{p=1}^2 {}^{(\alpha)}A_m^{(p)}(k_\alpha) \begin{bmatrix} \langle {}^{(\alpha)}\psi_j^{(p)}(k_\alpha), {}^{(\alpha)}\psi_j^{(s)}(k_\alpha^{(n)}) \rangle_\alpha \\ \langle {}^{(\alpha)}\psi_j^{(p)}(k_\alpha), {}^{(\alpha)}\psi_j^{(s)}(k_\alpha^{(n)}) \rangle_\alpha \end{bmatrix} = \sum_{m, k_\alpha} \sum_{p=1}^2 {}^{(\alpha+1)}A_m^{(p)}(k_{\alpha+1}) \begin{bmatrix} \langle {}^{(\alpha+1)}\psi_j^{(p)}(k_{\alpha+1}), {}^{(\alpha)}\psi_j^{(s)}(k_\alpha^{(n)}) \rangle_\alpha \\ \langle {}^{(\alpha+1)}\psi_j^{(p)}(k_{\alpha+1}), {}^{(\alpha)}\psi_j^{(s)}(k_\alpha^{(n)}) \rangle_\alpha \end{bmatrix} \quad (35)$$

where indices R or L have been suppressed but are implied, with appropriate use depending on whether $j = 1, 2$ or $j = 3$, as indicated by (34a) and (34b). (That is, this equation applies to either (34a) or (34b)). For specificity, one uses P-SV eigenfunctions and eigenvalues and a subscript "R" when considering component equations with $j = 1, 2$ and uses SH eigenfunctions and eigenvalues with subscript "L" when considering the $j = 3$ component equation.) Here $k_\alpha^{(n)}$ denotes the specific n^{th} eigenvalue of one particular mode with angular index m' .

Now we can subtract the upper matrix equation in (35) from the lower one and then make use of the orthogonality relations for P-SV modes in (26) - (27), and the obvious similar pair for the SH modes, to obtain:

$${}^{(\alpha)}A_m^{(s)}(k_\alpha^{(n)}) = \frac{1}{N_s^{(\alpha)}} \sum_{k_{\alpha+1}} \sum_{p=1}^2 {}^{(\alpha+1)}A_m^{(p)}(k_{\alpha+1}) \left\{ \begin{aligned} &\langle {}^{(\alpha+1)}\psi_j^{(p)}(k_{\alpha+1}), {}^{(\alpha)}\psi_j^{(s)}(k_\alpha^{(n)}) \rangle_\alpha - \\ &\langle {}^{(\alpha+1)}\psi_j^{(p)}(k_{\alpha+1}), {}^{(\alpha)}\psi_j^{(s)}(k_\alpha^{(n)}) \rangle_\alpha \end{aligned} \right\} ; s = 1, 2 \quad (36)$$

where we have equated the sums over m , on each side of (35), term by term. This equation again applies to either P-SV or SH modes; however, for P-SV modes $j = 1, 2$ and for SH modes, then $j = 3$. Therefore in (36) the implied summation over the coordinate index is over $j = 1$ and 2 , for the P-SV case, and for SH modes only the one term, for which $j = 3$, occurs. The free

index (s) denotes the forward and backward horizontally propagating modes, so that (36) expresses a relationship for both mode types. The factor $N_s^{(\alpha)}$ is the normalization "constant" appropriate for the different mode types. These factors are given in (28) - (29), for the forward and backward propagating P-SV modes, and in (30) - (31) for the SH modes.

It can be seen from (36) that a particular mode in V_α , at a particular eigenvalue (or wave number), will be "excited" by *all* the forward and backward propagating modes in $V_{\alpha+1}$ in the manner described by the expression on the right side in (36). Thus, all the modes in $V_{\alpha+1}$, at *all* wave numbers, will contribute to the excitation of any one mode in V_α (at a *particular* wave number) in proportion to the sum of the mode coefficients, $^{(\alpha+1)}A_m^{(p)}(k_{\alpha+1})$, weighted by the inner product factors given by the bracket term on the right side of (36). Thus the weight factors in (36) will be called *coupling coefficients*.

Considering the $k_{\alpha+1}$ eigenvalues as a discrete (infinite) set $\{k_{\alpha+1}^{(l)}\}$, as was implied for k_α by the use of $k_\alpha^{(n)}$, then we can define the discrete coupling coefficients as

$$C_{ln}^{(p,s)}(\alpha+1; \alpha) = \frac{1}{N_s^{(\alpha)}} \left[\langle ^{(\alpha+1)}\psi_j^{(p)}(k_{\alpha+1}^{(l)}) , ^{(\alpha)}\psi_j^{(s)}(k_\alpha^{(n)}) \rangle_\alpha - \langle ^{(\alpha+1)}\psi_j^{(p)}(k_{\alpha+1}^{(l)}) , ^{(\alpha)}\psi_j^{(s)}(k_\alpha^{(n)}) \rangle_\alpha \right] \quad (37)$$

and (36) becomes:

$$^{(\alpha)}a_m^{(s)}(k_\alpha^{(n)}) = \sum_l \sum_{p=1}^2 C_{ln}^{(p,s)}(\alpha+1; \alpha) ^{(\alpha+1)}a_m^{(p)}(k_{\alpha+1}^{(l)}) \quad ; \quad s = 1, 2 \quad (38)$$

The coupling coefficients can be expressed in more detail when the specific functional forms of the eigenfunctions appearing in the inner products are used in (37). In this case we can use the orthogonality of the vector cylindrical harmonics to reduce the coupling factors to simple integrals over the vertical (z) coordinate on the boundaries of each zone V_α . Specifically, from (37) for the P-SV case, using the eigenfunction expressions given earlier in (13) - (14), one

has:

$${}_R C_{\text{eff}}^{(p,s)}(\alpha+1; \alpha) = \frac{n_\alpha}{{}_R N_s^{(\alpha)}} \left[\left\{ \langle^{(\alpha+1)} D_1, ^{(\alpha)} R_n \rangle - \langle^{(\alpha+1)} R_1, ^{(\alpha)} D_n \rangle \right\} \mathbf{P}_m^{(p)}(k_{\alpha+1}^{(l)} \rho_\alpha) \cdot \bar{\mathbf{P}}_m^{(s)}(k_\alpha^{(n)} \rho_\alpha) \right. \\ \left. + \left\{ \langle^{(\alpha+1)} E_1, ^{(\alpha)} S_n \rangle - \langle^{(\alpha+1)} S_1, ^{(\alpha)} E_n \rangle \right\} \mathbf{B}_m^{(p)}(k_{\alpha+1}^{(l)} \rho_\alpha) \cdot \bar{\mathbf{B}}_m^{(s)}(k_\alpha^{(n)} \rho_\alpha) \right] \quad (39)$$

where $n_\alpha = 2\pi\rho_\alpha$, with ρ_α denoting the value of the radial coordinate on the surface Σ_α . Further the various inner products involve the "vertical eigenfunctions" defined in (13) and (17); where these inner products have explicit forms of the type:

$$\langle^{(\alpha+1)} D_1, ^{(\alpha)} R_n \rangle = \int_0^{\infty} {}^{(\alpha+1)} D_m(z_0; k_{\alpha+1}^{(l)}) {}^{(\alpha)} \bar{R}_m(z_0; k_\alpha^{(n)}) dz_0 \quad (39a)$$

with similar expressions for the other products in (39). If these products are compared to those in (26) and (27) - or more directly to the orthogonality relations involving the vertical eigenfunctions given by the equations following equation (27) - it can be seen that the inner products in (39) reduce to delta functions if the eigenfunctions in the zones V_α and $V_{\alpha+1}$ are the same; that is, if ${}^{(\alpha+1)} D_m = {}^{(\alpha)} D_m$, ${}^{(\alpha+1)} R_m = {}^{(\alpha)} R_m$, etc. This, of course, is as it must be, since only when the physical properties in the two zones are identical will the eigenfunctions be the same and it then follows that the coupling matrix must be diagonal - that is that the boundary between the two zones produces no cross mode excitation and is transparent. We see, therefore, that the analytical expression in (39) for the coupling does indeed have this required property.

The normalization factor for $C_{\text{eff}}^{(p,s)}$ is the ratio ${}_R N_s^{(\alpha)} / n_\alpha$ which can be redefined as ${}_R \tilde{N}_s^{(\alpha)}$, where from the previous expressions for ${}_R N_s^{(\alpha)}$, in (28) and (29), this constant has the form:

$${}_R N_s^{(\alpha)} = \left[\left\{ \mathbf{P}_m^{(s)}(k_\alpha^{(n)} \rho_\alpha) \cdot \bar{\mathbf{P}}_m^{(s)}(k_\alpha^{(n)} \rho_\alpha) + \mathbf{B}_m^{(s)}(k_\alpha^{(n)} \rho_\alpha) \cdot \bar{\mathbf{B}}_m^{(s)}(k_\alpha^{(n)} \rho_\alpha) \right. \right. \\ \left. \left. + \left[\frac{\rho_{\alpha-1}}{\rho_\alpha} \right] \left\{ \mathbf{P}_m^{(s)}(k_\alpha^{(n)} \rho_{\alpha-1}) \cdot \bar{\mathbf{P}}_m^{(s)}(k_\alpha^{(n)} \rho_{\alpha-1}) + \mathbf{B}_m^{(s)}(k_\alpha^{(n)} \rho_{\alpha-1}) \cdot \bar{\mathbf{B}}_m^{(s)}(k_\alpha^{(n)} \rho_{\alpha-1}) \right\} \right] \right] \quad (40)$$

In an exactly analogous fashion the coupling coefficients for the SH modes are found to be:

$${}_L C_{\text{eff}}^{(p,s)}(\alpha+1; \alpha) = \frac{n_\alpha}{{}_L N_s^{(\alpha)}} \left[\langle (\alpha+1)F_1, (\alpha)T_n \rangle - \langle (\alpha+1)T_1, (\alpha)F_n \rangle \right] C_m^{(p)}(k_{\alpha+1}^{(1)} \rho_\alpha) \cdot \bar{C}_m^{(s)}(k_\alpha^{(n)} \rho_\alpha) \quad (41)$$

where the inner products are again of the simple form:

$$\langle (\alpha+1)F_1, (\alpha)T_n \rangle = \int_0^\infty {}^{(\alpha+1)}F_m(z_0; k_{\alpha+1}^{(1)}) {}^{(\alpha)}\bar{T}_m(z_0; k_\alpha^{(n)}) dz_0 \quad (41a)$$

Further, we can again define a new normalization factor ${}_L \tilde{N}_s^{(\alpha)} = {}_L N_s^{(\alpha)} / n_\alpha$ which has the form:

$${}_L \tilde{N}_s^{(\alpha)} = \left[C_m^{(s)}(k_\alpha^{(n)} \rho_\alpha) \cdot \bar{C}_m^{(s)}(k_\alpha^{(n)} \rho_\alpha) + \left(\frac{\rho_{\alpha-1}}{\rho_\alpha} \right) C_m^{(s)}(k_\alpha^{(n)} \rho_{\alpha-1}) \cdot \bar{C}_m^{(s)}(k_\alpha^{(n)} \rho_{\alpha-1}) \right] \quad (42)$$

The computations involved in determining these coefficients are straightforward, since the cylindrical harmonics are tabulated and the integrals over the vertical coordinate z_0 involve simple integrals of exponentials that can be evaluated analytically, in closed form, for the general case.

Since (3.1) constitutes a set of two equations for $s = 1$ and $s = 2$, corresponding to forward and backward propagating modes and since the sums on the right can clearly be expressed as a product of matrices, it is natural to write the results in matrix form. Therefore we define:

$$\begin{bmatrix} {}^{(\alpha)}a_n^{(s)} \end{bmatrix} = \begin{bmatrix} {}^{(\alpha)}a_m^{(s)}(k_\alpha^{(1)}) \\ {}^{(\alpha)}a_m^{(s)}(k_\alpha^{(2)}) \\ \vdots \\ {}^{(\alpha)}a_m^{(s)}(k_\alpha^{(N)}) \end{bmatrix} ; \text{ for } s = 1 \text{ and } 2 \quad (43a)$$

and a similar column matrix of length (L) denoted $[{}^{(\alpha+1)}a_1^{(p)}]$, where the angular index m has been suppressed in writing the mode excitation matrices. Further, we can define coupling

matrices by:

$$[C_{ln}^{(p,s)}] = \begin{bmatrix} C_{11}^{(p,s)} & C_{21}^{(p,s)} & C_{L1}^{(p,s)} \\ C_{12}^{(p,s)} & C_{22}^{(p,s)} & C_{L2}^{(p,s)} \\ \vdots & \vdots & \vdots \\ C_{1N}^{(p,s)} & \dots & C_{LN}^{(p,s)} \end{bmatrix} \quad (43b)$$

for each s and p value, where $s = 1, 2$ and $p = 1, 2$. With these definitions one can write the system of equations implied by (38) in the form:

$$\begin{bmatrix} [^{(\alpha)}a_n^{(1)}] \\ [^{(\alpha)}a_n^{(2)}] \end{bmatrix} = \begin{bmatrix} [C_{ln}^{(1,1)}] & [C_{ln}^{(2,1)}] \\ [C_{ln}^{(1,2)}] & [C_{ln}^{(2,2)}] \end{bmatrix} \begin{bmatrix} [^{(\alpha+1)}a_1^{(1)}] \\ [^{(\alpha+1)}a_1^{(2)}] \end{bmatrix} \quad (44)$$

where the forward and backward propagating mode excitation coefficients are displayed explicitly. In defining the $[C_{ln}^{(p,s)}]$ matrices, and in writing the matrix result in (44), the " α indices" have been suppressed. However, when confusion can arise they should be written as $[C_{ln}^{(p,s)}(\alpha+1; \alpha)]$, etc., since the α indices change when the matrix refers to a boundary other than Σ_α . (eg. Between the zones $V_{\alpha-1}$ and V_α , on the surface $\Sigma_{\alpha-1}$, the coupling matrix is expressed as $[C_{ln}^{(p,s)}(\alpha; \alpha-1)]$).

Obviously the coupling matrices are square only if $L = N$, that is if we use as many modes in V_α as in $V_{\alpha+1}$ to represent the propagating waves. This choice will be adhered to, from this point forward, although it is not a necessary condition.

It is evident that the partitioned matrices can be written in unpartitioned form as well, where, with $L = N$, the mode coefficient matrices are of dimension $(2N \times 1)$ and the coupling matrix is square and of dimension $(2N \times 2N)$. Thus, we can also define mode coefficient matrices consisting of the (ordered) mode coefficients for the forward and backward propagating modes in the zones V_α and $V_{\alpha+1}$ as (say):

$$\begin{bmatrix} {}^{(\alpha)}m_n \end{bmatrix} = \begin{bmatrix} [{}^{(\alpha)}a_n^{(1)}] \\ [{}^{(\alpha)}a_n^{(2)}] \end{bmatrix} \quad (45a)$$

$$\begin{bmatrix} {}^{(\alpha+1)}m_l \end{bmatrix} = \begin{bmatrix} [{}^{(\alpha+1)}a_l^{(1)}] \\ [{}^{(\alpha+1)}a_l^{(2)}] \end{bmatrix}$$

and, similarly, we can define what can appropriately be called a horizontal propagator matrix:

$$H_{ln}(\alpha+1; \alpha) = \begin{bmatrix} C_{ln}^{(1,1)} & C_{ln}^{(2,1)} \\ C_{ln}^{(1,2)} & C_{ln}^{(2,2)} \end{bmatrix} \quad (45b)$$

Now the equation (44) can be written in the more compact form:

$$\begin{bmatrix} {}^{(\alpha)}m_n \end{bmatrix} = H_{ln}(\alpha+1; \alpha) \begin{bmatrix} {}^{(\alpha+1)}m_l \end{bmatrix} \quad (46)$$

and expresses the required conditions between the mode coefficients in neighboring zones.

If we take successive values of α , with α ranging from 1 to $M-1$ say, then we get

$$\begin{aligned} [{}^{(1)}m_n] &= [H_{ln}(2; 1)][{}^{(2)}m_l] \\ [{}^{(2)}m_n] &= [H_{ln}(3; 2)][{}^{(3)}m_l] \\ &\vdots \\ [{}^{(M-1)}m_n] &= [H_{ln}(M; M-1)][{}^{(M)}m_l] \end{aligned}$$

Clearly, by noting in these equations that the indices l and n are just dummy indices providing a numbering system for the eigenvalues, then

$$[{}^{(1)}m_n] = [H_{ln}(2; 1)] [H_{ln}(3; 2)] \cdots [H_{ln}(M; M-1)] [{}^{(M)}m_l]$$

by successive substitutions. Consequently, we can write, for any $\beta \geq \alpha + 1$:

$$\begin{bmatrix} {}^{(\alpha)}m_n \end{bmatrix} = \left\{ \prod_{q=\alpha+1}^{\beta} [H_{ln}(q; q-1)] \right\} \begin{bmatrix} {}^{(\beta)}m_l \end{bmatrix} \quad (47)$$

This is a propagator equation that connects the mode coefficients in any zone V_β with those in any other zone V_α . In case $\beta = \alpha + 1$ the equation (47) reduces to equation (46), which connects the coefficients in any two neighboring zones. Since the coupling coefficients composing $[H_{in}]$ can be computed from the simple eigenfunction inner products at the zone interfaces, this equation provides the means of computing mode coefficients that produce displacements and tractions satisfying all the boundary conditions along the vertical boundaries of the medium. Since the eigenfunctions used already satisfy the boundary conditions along the horizontal boundaries in each zone, then by use of the horizontal propagator relation all the boundary conditions in the laterally and vertically "layered" medium being considered can be satisfied.

Summary and Conclusions

The basic method described here makes use of normal mode expansions of the wave field in each partitioned sub-region of the medium within which the medium is uniform in the lateral directions. Thus the medium is partitioned into laterally uniform zones and complete normal mode solutions are obtained from each horizontally layered zone. In the analytical development the "zonal eigenvalues and eigenfunctions" are generated by treating each zone as a layered half space or radially layered sphere, as is appropriate for the medium geometry. The resulting set of modes are then used as bases for expansions of the wave fields in the layered subregions. The mode expansions defined on the zones are then "connected" by matching (equating) the exact Green's function representations of the wave fields in each zone at the common boundaries between the zones where continuity of displacement and traction is required. This results in the definition of a "lateral propagator" of the wave field when applied to all the zones making up the entire medium and is, in application, very similar to the classical "vertical propagator" method.

The method is exact when the lateral variations are actually discontinuous step changes in properties. When the actual changes can be approximated as a sequence of step the method should be superior in computational accuracy and speed to numerical methods.

In implementations of this method it is only necessary to compute the "zonal" normal modes once, and subsequently these zonal mode solutions can be combined in a variety of ways, using the lateral propagator equation, to produce theoretically predicted wave fields in many different laterally varying structures without the necessity of a complete recomputation of wave fields in each new structure. Further, the propagators are analytically defined so that manipulations related to inversion and perturbation calculations can be considered. For these reasons, and because of its inherent high accuracy, this method should prove useful in modeling seismic wave fields in complex media and in inversion studies. In the present study the method is developed in detail for two dimensionally variable media, using cylindrical coordinates and wave functions. However, analogous results in rectangular and spherical coordinates may be obtained using the same procedure and are appropriate for media with variability in all three spatial dimensions.

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