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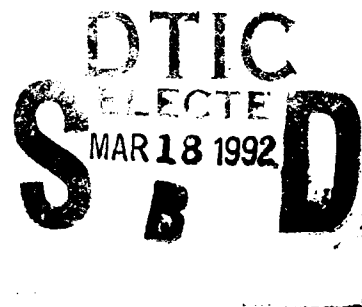
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Structural Design Methodology for Large Space Structures

by
Ralph J. Dornsife

The Department of Defense requires research and development in designing, fabricating, deploying, and maintaining large space structures (LSS) in support of Army and Strategic Defense Initiative military objectives. Because of their large size, extreme flexibility, and the unique loading conditions in the space environment, LSS will present engineers with problems unlike those encountered in designing conventional civil engineering or aerospace structures. LSS will require sophisticated passive damping and active control systems in order to meet stringent mission requirements. These structures must also be optimally designed to minimize high launch costs.

This report outlines a methodology for the structural design of LSS. It includes a definition of mission requirements, structural modeling and analysis, passive damping and active control system design, ground-based testing, payload integration, on-orbit system verification, and on-orbit assessment of structural damage. In support of this methodology, analyses of candidate LSS truss configurations are presented, and an algorithm correlating ground-based test behavior to expected microgravity behavior is developed.



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FOREWORD

This research was conducted for Headquarters, U.S. Army Corps of Engineers (HQUSACE) under Project 4A162731AT41, "Military Facilities Engineering Technology"; Technical Area E, "Echelons Above Corps Support"; Work Unit 056, "Design Issues in Large Space Structures." The HQUSACE Technical Monitor was Mr. Jerry Lundein, CERD-M.

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STRUCTURAL DESIGN METHODOLOGY FOR LARGE SPACE STRUCTURES

1 INTRODUCTION

Background

For many years, structural engineers have had a substantial body of literature available to assist them in the design of structures attached to the Earth. Analysis and design courses are a part of any structural engineering curriculum. Textbooks explain how to analyze a wide variety of structures subjected to gravity loadings as well as lateral dynamic loads produced by wind and seismic forces. Building codes have been written to ensure the safety and serviceability of structures. These codes are constantly being updated to include the latest material properties, research findings, and serviceability and failure observations. Some textbooks study structural analysis; others study design using specific materials such as concrete, steel, masonry, or wood. Standard techniques exist for designing these structures. Key criteria have been identified and can be isolated for consideration in the design process. In many cases, such designs have become so standard that engineers have automated the design procedure through the use of personal computers and hand calculators. Furthermore, computer aided design and drafting has come into popular use during the past decade.

The structural design of extremely flexible, lightweight, large space structures (LSS) opens an entirely new area of investigation for the structural engineer. LSS are initially intended for civilian and military purposes in low-Earth orbit only. Eventually LSS may be used in other planetary orbits to support interplanetary exploration and settlement. LSS will experience loadings radically different from conventional structures attached to the Earth. Because of expensive launch costs, these structures must be very lightweight. As a consequence, they will be extremely flexible, and characterized by extremely low natural frequencies. Standard structural design techniques currently are not available for LSS.

Objective

The objective of this research was to outline a methodology for structural design of LSS. The methodology should provide a model or conceptual algorithm upon which structural design can be based once specific Army applications have been developed. The methodology should allow engineers experienced in the structural design of Earth structures to conceptually understand and make the transition to LSS structural design.

Approach

In support of the objective, USACERL conducted the following activities to study structural design methodology issues and advance the state of the art in LSS:

1. Participation in the structural design and analysis of the Low-Power Atmospheric Compensation Experiment (LACE). This study was conducted at the U.S. Naval Research Laboratory in Washington, DC and was funded by the Strategic Defense Initiative Organization (SDIO). USACERL's involvement

was primarily in the structural analysis of a long deployable mast beam. The satellite was launched and deployed successfully in February 1990.¹

2. Research in ground-based testing dynamics of structural models. The behavior of a prismatic beam on various supports in the presence of gravity was compared to the expected theoretical behavior of the same beam in the microgravity of space. (This portion of the research has been reported in journal articles.²)

3. Structural analyses using finite element methods to evaluate the dynamic characteristics of various truss geometries that could be used for platform construction. The same structures were then reanalyzed to determine the effect of fixing the truss joints against rotation.

As a complement to the approach, researchers also gained valuable associative knowledge by attending conferences and workshops to evaluate the current state of the art in critical LSS issues. These issues include control-structure interaction, passive damping technology, and active control system design. These issues are discussed in the following chapters. USACERL also developed a ground-based testing theory for LSS models and has analyzed various truss geometries. A structural design methodology flow chart is provided in Appendix A.

USACERL also participated in a program to develop a viscoelastically damped insert for truss members that incorporates an automatic thermal control system³ and a program to investigate the feasibility of an electromagnetic damping concept.⁴

Scope

This report qualitatively outlines structural design methodology for LSS and provides a technique for analysis. Although the research was intended for military LSS, the methodology is also applicable to civilian LSS.

Mode of Technology Transfer

It is recommended that information in this report be included in technical and operational documents produced as part of the Army Space Master Plan. These documents will serve system developers as part of the overall technical data package for procuring the system and its verification/certification for launch and deployment on orbit.

¹ S. Fisher, "LACE Flight Dynamics Experiment," *NASA/DOD Controls-Structures Interaction Technology 1989*, NASA Conference Publication 3041 (National Aeronautics and Space Administration [NASA], 1989), pp 427-443.

² R.J. Dornisfe, "Effect of Gravity on Dynamic Behavior of Beams," *Engineering, Construction, and Operations in Space II, Proceedings of Space 90*, Volume 2, Stewart W. Johnson, John P. Wetzel, Eds. (American Society of Civil Engineers [ASCE], 1990), pp 1142-1148; R.J. Dornisfe, "Ground Based Testing Dynamics for a Prismatic Beam in Microgravity," *Journal of Aerospace Engineering* (ASCE, April 1991), pp 165-183.

³ E.M. Austin, et al., *Passive Damping Techniques for Space Structures*, final project report to the U.S. Army Construction Engineering Research Laboratory (USACERL) (CSA Engineering Inc., April 1988).

⁴ R. Stettner and P. Mlakar, *The Development of Passive Electromagnetic Vibration Damping Systems for Large Space Structures*, final project report to the Strategic Defense Initiative Organization (SDIO) (Advanced Scientific Concepts, January 15, 1990).

2 STRUCTURAL DESIGN METHODOLOGY

Initial Analysis and Design

Large space structures will serve many military purposes. They may be used to maintain relative positions of individual components of weapons and surveillance/tracking systems, to provide stiffness for antennae, and to isolate systems from the effects of dynamic vibrations. The pointing accuracies, fast response requirements, short settling times, and shape control required of these systems will define the mission requirements and will dictate a structure's shape and size. Once the mission requirements have been specified, rough estimates of overall stiffness and damping can be evaluated.

Erectable Deployable Option

LSS are broadly classified into two categories, erectable and deployable, depending on their final assembly. Erectable structures are assembled sequentially from individual components after arriving in low-Earth orbit. Assembly could be accomplished either by astronauts performing extravehicular activity (EVA) or by a telerobotic system,⁵ possibly being assisted by artificial intelligence. Deployable structures are completely preassembled and folded to fit into an unmanned rocket or into the space shuttle payload compartment. They may unfold by mechanical or automated techniques.

LSS may need to be reconfigured at some time during their lifetimes. For example, additional system components may necessitate a larger platform structure. Launch vehicle availability must be assessed. Cargo space constraints of candidate launch vehicles must be considered. Launch on demand requirements may eliminate certain vehicles from consideration. EVA risks must be carefully weighed. All of these factors must be considered in deciding whether the LSS should be erectable or deployable. For instance, a launch on demand requirement would presently preclude the use of the Space Shuttle from which an erectable structure would be assembled by astronauts performing EVA. Until telerobotic assembly systems are developed, erectable structures can be launched only using the Space Shuttle.

Geometry and Materials

After a decision has been made as to whether the LSS will be erectable or deployable, geometry and material options (together) must be examined. Truss structures will be used extensively for LSS because of their high specific stiffness and strength. Many different types of truss geometries are possible, including repeating tetrahedron units and cube shaped units. The Space Station Freedom structure is an example of an LSS having cube shaped repeating truss units with diagonals on all sides. Many different material options will be available for fabricating the individual truss elements and joints. Aluminum and graphite epoxy composites are the most likely candidate materials because of their high stiffness-to-mass ratios. Because graphite possesses a negative coefficient of thermal expansion, it can be combined compositely with aluminum to create a structural member having a net coefficient of thermal expansion of nearly zero.

⁵ "NASA Developing Telerobotic System To Automate Assembly in Space," *Aviation Week and Space Technology* (September 3, 1990), pp 197-199.

Structural Redundancy

All structures can be classified as either stable or unstable. Generally, for Earth-based construction, unstable structures are undesirable because certain types of loading would result in large deformations and/or collapse. Unstable structures are referred to as having mechanisms. In space, mechanisms may be desirable, provided the large rotations accompanying the instability can be actively controlled, allowing separation distances to be varied. An example is the Space Shuttle Remote Manipulator System. Multibody dynamics computer codes must be used to analyze such a structure. These codes will be addressed later in this report.

Stable structures can be further classified as either statically determinate or statically indeterminate. A statically determinate structure is one for which the internal and reaction forces can be evaluated for any loading condition by using only the static equations of equilibrium. Elimination of any structural member in a statically determinate structure will lead to a collapse or mechanism. The structure is then unstable. A statically indeterminate structure, also referred to as a redundant structure, is one for which the static equations of equilibrium alone are insufficient to determine the internal forces and reactions. The additional equations required for solution usually involve the compatibility of deformations at various points in the structure. Such a structure is said to be redundant because load redistribution can occur when redundant members are lost, thus averting collapse. An indeterminate structure can be classified as to its degree of indeterminacy. For example, a structure that is indeterminate to the third degree will require three additional compatibility equations to determine the internal forces and reactions. This structure could still be stable if it lost three members.

Besides redundancy, several other important differences characterize statically determinate and indeterminate structures. A statically determinate structure on Earth can undergo differential support settlements without inducing stresses in individual members. However, the individual members of an indeterminate structure subjected to differential settlements between supports will experience accompanying stresses that will be superimposed onto dead, live, and lateral load stresses. These unanticipated stresses can result in structural damage. Individual member imperfections can result in the same effect. For example, if a member is fabricated slightly too long or too short and is erected into an indeterminate structure, members throughout the entire structure must be stressed in order to accommodate the imperfection. This will not occur with a statically determinate structure, which can accommodate the imperfection by changing its geometry slightly. Thermal gradients have a similar effect. The individual members of a statically indeterminate structure subjected to thermal loading will experience additional accompanying stresses, whereas a determinate structure can accommodate such thermal loading without additional stress by changing its geometry slightly.

Serious consideration must be given to redundancy in developing and selecting LSS truss geometries. A high degree of redundancy will enable an LSS to sustain damage or loss to certain structural members without collapse of the entire structure. However, this redundancy will result in additional member stresses when the structure is subjected to thermal gradients as it passes through alternating cycles of sunlight and darkness in orbiting the Earth. Furthermore, manufacturing tolerances will be much more critical for highly redundant structures to minimize stresses resulting from member imperfections. Also, the additional forces needed to strain an indeterminate structure to accommodate an imperfect member could seriously impede astronauts in the assembly process.

Passive Damping

Passive damping must be thoughtfully considered throughout the design process. All structures have some degree of damping as a result of their constituent material properties and assembly configuration. The designers can increase the passive damping in a structure through a variety of methods including discrete dampers, constrained-layer treatments, and free-layer treatments.⁶ Viscoelastic materials (VEM) are generally used in passive damping. The method selected depends upon the structural configuration and the conditions of loading.

Accurate understanding of the behavior and properties of VEM is critical in designing passive damping systems. Almost all VEM is susceptible to creep. This is the tendency for the material to deform as a function of time when subjected to static loading. The amount of creep in a material increases with increasing static stress levels. Also, VEM stiffness is a function of the frequency of applied loading. It exhibits greater stiffness at higher load frequencies. Furthermore, VEM properties vary considerably with temperature, possibly requiring thermal control for stability.

Link dampers and damped joints perform best for truss structures. Link dampers provide passive damping by loading the VEM in pure shear, providing a highly effective loss mechanism. Damped joints perform similarly, but have several potential advantages. One damped joint could damp several attached truss members, thus minimizing weight and reducing the complexity of VEM thermal control. Past research had concentrated on using VEM in simple shear lap truss members. To prevent creep, these links were designed into structural members subjected only to dynamic loads. However, researchers have recently developed strategies to place elastic elements in a load path parallel with the VEM.⁷ This concept allows the damped member to withstand static loading without appreciable creep. Of course, the effectiveness of the damped truss member increases as the proportion of the load resisted by the VEM to the total member load increases.

The Structural Model

After selecting the truss geometry, material, and passive damping properties, a structural model can be generated. Generally, the finite element method is used. However, because of the presence of many structural members and repeating geometries, the large size of an LSS can often lead to extremely high finite element computational costs. In the preliminary phases of the design process, the structural engineer is interested only in the overall behavior of an LSS. For this reason, continuum methods have been developed and used to analyze repetitive geometries. Basically, continuum methods transform a model composed of many discrete elements into a continuous formulation. In this smearing process, a smaller number of effective material and structural parameters replace the complex discrete model, resulting in substantially reduced computational costs. A number of methods have been used to develop continuum models.⁸ Once a preliminary design has been achieved, a more detailed finite element analysis can be performed on the structure.

⁶ E.M. Austin, et al.

⁷ E.M. Austin, et al.

⁸ American Society of Civil Engineers, *Identification of Large Space Structures on Orbit*, prepared for the U.S. Air Force Rocket Propulsion Laboratory (September 1986), pp 63-77; U. Lee, "Dynamic Continuum Modeling of Beamlike Space Structures Using Finite Element Matrices," *Proceedings of the AIAA, ASME, ASCE, AHS, ASC 30th Structures, Structural Dynamics and Materials Conference*, 3-5 April 1989 (American Institute of Aeronautics and Astronautics [AIAA], 1989), pp 1955-1962.

Successful analysis of any structure requires accurate assessment of its properties and a good understanding of its behavior under loading. Material properties such as stiffness, damping, and mass density must be known. Furthermore, knowledge of the actual distribution of continuous and discrete masses is required. A thorough understanding of the types, magnitudes, and probability of occurrence of external loads, as well as the environment in which these loads will be applied is very crucial. In addition, the structural engineer must have a thorough understanding about how the structure will perform when subjected to these loads. For example, load-deformation nonlinearities and the dead band of joints (deformation without accompanying load change) must be understood and quantified to model the structure as accurately as possible.

To simplify the mathematics, the following methodology will assume linear behavior and will be based on the use of finite element techniques. Some of the basic structural dynamics finite element theory will be explained.

After generating a model of the structure, an analytical modal analysis is performed to evaluate the structure's natural frequencies and mode shapes. A freely vibrating, undamped structure is characterized by its mass and stiffness properties. A finite element model divides a continuous structure into a model having a finite number of elements and degrees of freedom. Thus an "n" degree of freedom undamped model is characterized by an "n" x "n" stiffness matrix, $\underline{K}$, and an "n" x "n" mass matrix, $\underline{M}$. The displacements are contained in an "n" x 1 displacement matrix, $\underline{v}$. Dots (·) denote derivatives with respect to time.

The resulting matrix equation of free vibration for the multiple degree of freedom undamped structure is:

$$\underline{M} \ddot{\underline{v}} + \underline{K} \underline{v} = \underline{0} \quad [\text{Eq 1}]$$

With $\underline{v} = \underline{f}(\underline{x}, t)$ and $\underline{\nabla} = \underline{f}(\underline{x})$, you can assume the displacements to be of the form:

$$\underline{v}(t) = \underline{\bar{v}} \sin(\omega t + \theta) \quad [\text{Eq 2}]$$

so that:

$$\ddot{\underline{v}}(t) = -\omega^2 \underline{\bar{v}} \sin(\omega t + \theta) = -\omega^2 \underline{v} \quad [\text{Eq 3}]$$

Substituting will yield:

$$-\omega^2 \underline{\underline{M}} \underline{\underline{v}} + \underline{\underline{K}} \underline{\underline{v}} = \underline{\underline{O}} \quad [\text{Eq 4}]$$

or

$$[\underline{\underline{K}} - \omega^2 \underline{\underline{M}}] \underline{\underline{v}} = \underline{\underline{O}} \quad [\text{Eq 5}]$$

The solution for displacements is solved by Cramer's rule.

$$\underline{\underline{v}} = \frac{\underline{\underline{O}}}{|\underline{\underline{K}} - \omega^2 \underline{\underline{M}}|} \quad [\text{Eq 6}]$$

Therefore, nonzero amplitude free vibrations are only possible when:

$$|\underline{\underline{K}} - \omega^2 \underline{\underline{M}}| = 0 \quad [\text{Eq 7}]$$

Solving this eigenvalue problem will yield "n" natural frequencies for the finite element model. The theoretical solution to the continuous model would yield an infinite number of natural frequencies. A structure's lowest natural frequency is referred to as its fundamental frequency.

The mode shape associated with each natural frequency can be evaluated by backsubstitution. The actual amplitudes of the vibrations are indeterminate. However, the shape of the vibration system can be determined by solving for all other displacements in terms of one displacement.

For a three-dimensional stable structure in space, six zero frequency rigid body modes will exist. Structures with mechanics will have more rigid body modes. No elastic body deformation is associated with these modes.

Dynamic Loads

Dynamic loads can now be evaluated for the LSS. These loads will be produced by operating machinery, retargeting procedures, firing devices, docking operations, crew activity, meteorite and space debris impact, and thermal effects. Several load cases may need to be considered. The structure must be analyzed for these imposed loads to evaluate its response. Again, finite element modeling may be used. The damping of the structure is characterized by an "n" x "n" damping matrix, $\underline{\underline{C}}$. The dynamic loading

is represented by an "n" x 1 matrix of dynamic nodal loads, $p(t)$. These are equivalent dynamic point loads applied at specific points, or nodes, of the discretized model. The matrix equation of forced vibration for the damped structure is:

$$\underline{\underline{M}} \ddot{\underline{v}} + \underline{\underline{C}} \dot{\underline{v}} + \underline{\underline{K}} \underline{v} = \underline{p}(t) \quad [\text{Eq 8}]$$

The total displacement of the structure can be expressed as the sum of its modal components

$$\underline{v} = \underline{\phi}_1 Y_1 + \underline{\phi}_2 Y_2 + \dots + \underline{\phi}_N Y_N = \sum_{n=1}^N \underline{\phi}_n Y_n \quad [\text{Eq 9}]$$

where:

$\underline{\phi}_n$ = mode shape vector for mode "n"

Y_n = modal amplitude for mode "n"

or:

$$\underline{v} = \underline{\Phi} \underline{Y} \quad [\text{Eq 10}]$$

Substitution yields:

$$\underline{\underline{M}} \underline{\Phi} \ddot{\underline{Y}} + \underline{\underline{C}} \underline{\Phi} \dot{\underline{Y}} + \underline{\underline{K}} \underline{\Phi} \underline{Y} = \underline{p}(t) \quad [\text{Eq 11}]$$

Premultiplying yields:

$$\underline{\phi}_n^T \underline{\underline{M}} \underline{\Phi} \ddot{\underline{Y}} + \underline{\phi}_n^T \underline{\underline{C}} \underline{\Phi} \dot{\underline{Y}} + \underline{\phi}_n^T \underline{\underline{K}} \underline{\Phi} \underline{Y} = \underline{\phi}_n^T \underline{p}(t) \quad [\text{Eq 12}]$$

Orthogonality conditions apply to the stiffness and mass matrices. These conditions require that:

$$\begin{aligned}\phi_m^T M \phi_n &= 0 & m \neq n \\ \phi_m^T K \phi_n &= 0 & m \neq n\end{aligned}\quad [\text{Eq 13}]$$

If it is assumed that the orthogonality condition applying to mass and stiffness also applies to damping, then:

$$\phi_m^T C \phi_n = 0 \quad m \neq n \quad [\text{Eq 14}]$$

Applying these orthogonality conditions reduces Equation 12 to:

$$\phi_n^T M \phi_n \ddot{Y} + \phi_n^T C \phi_n \dot{Y} + \phi_n^T K \phi_n Y = \phi_n^T P(t) \quad [\text{Eq 15}]$$

With the following:

$$\begin{aligned}M_n &= \phi_n^T M \phi_n \\ K_n &= \phi_n^T K \phi_n \\ C_n &= \phi_n^T C \phi_n \\ P_n(t) &= \phi_n^T P(t)\end{aligned}\quad [\text{Eq 16}]$$

Equation 15 further reduces to:

$$M_n \ddot{Y}_n + C_n \dot{Y}_n + K_n Y_n = P_n(t) \quad [\text{Eq 17}]$$

or

$$\ddot{Y}_n + 2\xi_n \omega_n \dot{Y}_n + \omega_n^2 Y_n = \frac{P_n(t)}{M_n} \quad [\text{Eq 18}]$$

where:

$$\xi_n = \frac{C_n}{2M_n\omega_n} \quad , \quad \omega_n^2 = \frac{K_n}{M_n} \quad [\text{Eq 19}]$$

This represents a single degree of freedom equation describing the forced vibration of mode "n."

The response to this forcing function can be evaluated using the Duhamel (or convolution) integral as follows:

$$Y_n(t) = \frac{1}{M_n\omega_{Dn}} \int_0^t P_n(\tau) e^{-\xi_n\omega_n(t-\tau)} \sin\omega_{Dn}(t-\tau) d\tau \quad [\text{Eq 20}]$$

where:

$$\omega_{Dn} = \omega_n \sqrt{1 - \xi_n^2} \quad [\text{Eq 21}]$$

If initial displacement and velocity are nonzero, the free vibration response must be added to the forced response. The free vibration response is:

$$Y_n(t) = e^{-\xi_n\omega_n t} \left[\frac{Y_n(0) + Y_n'(0)\xi_n\omega_n}{\omega_{Dn}} \sin\omega_{Dn}t + Y_n(0)\cos\omega_{Dn}t \right] \quad [\text{Eq 22}]$$

The individual modal components of deflection can be added together to get the total response. This is often referred to as the mode-superposition method. Mathematically, it can be written as:

$$\underline{v}(t) = \underline{\Phi} \underline{Y}(t) \quad [\text{Eq 23}]$$

The dynamic stresses can now be checked to determine if they are within the allowable stresses for the constituent materials. If stresses are exceeded, the structure must be redesigned. Structural member sizes and materials can be changed as needed to iterate to a satisfactory solution. This process is similar to basic procedures for designing conventional civil engineering structures.

If the dynamic stresses are within allowable levels, the calculated vibratory motion of the structure must be examined to determine if mission requirements are satisfied with passive damping alone.

Maximum calculated dynamic displacements and decay times must be compared to the maximum allowed for the given system. The structure must be examined for resonance effects produced by operating equipment loads acting at frequencies coincident with one of the structure's natural frequencies.

At this stage in the design process, active control has not yet been considered. If dynamic stresses are within the allowable values, but mission requirements are not satisfied, the engineer must decide whether the mission requirements can be met solely through passive damping. If so, the passive damping system must be redesigned and the previously described analyses repeated until a satisfactory design is reached.

Active Control

If passive damping alone will be incapable of satisfying mission requirements, an active control system must be designed. Active control differs from passive damping in that external energy must be supplied to the structural system. Typically, an active control system consists of sensor and actuator components. Sensors monitor the vibration of the structure. The sensor data is processed to evaluate the required output of the actuators. The actuators generate forces that counteract the sensed vibrations, thus damping the structure's motion. To date, active control appears to be a relatively feasible way to control deflections associated with the lowest natural frequencies of a structure. However, gain instabilities, spillover effects, and increased actuator masses seem to favor the use of passive damping for controlling the higher frequency vibrations. Much improved reduced order modeling of the structure and/or greater computational capacity is required to account for higher vibration modes while minimizing spillover effects. Numerous active damping devices have been proposed including proof mass actuators (PMAs), piezoelectric materials (ceramics and thin films), magnetostrictive materials, electrostrictive materials, and shape memory alloys. Many different control strategies can be applied. Two examples are: constant gain control and constant amplitude control. In reality, the mission requirements of a typical LSS will probably be met using a combination of passive damping and active control.⁹

Including active control elements will change the structural model by adding discrete and/or distributed masses to the structure. This requires modification of the original structural model, followed by further analytical modal analysis. The designer must next perform a simulation of the active control system. Several computer programs are available to help design and simulate an active control system. The two most widely used ones are EASY5 and MATRIXx. EASY5 is generally preferred for simulation while MATRIXx is preferred for design. EASY5 has nonlinear modeling capability. However, as written, neither program has the capability to handle multibody dynamics. With input of a FORTRAN module, EASY5 can handle multibody dynamics through a linearization process.

These active control design/simulation programs require the following as input: the structure's natural frequencies and mode shapes, the locations of the controllers, a description of filter components, and hysteresis information. Therefore, the analytical modal analysis must be performed first. Engineers have developed several computer programs to facilitate the transfer of data between finite element analysis and controls design/simulation programs. Three such programs are Control-Structure-Interaction (CO-ST-*IN*), Integrated Analysis Capability (IAC), and Integrated Systems Modeling (ISM).

⁹ K.E. Richards, "PACOSS Program Status and Results," *NASA/DOD Controls-Structures Interaction Technology 1989*, NASA Conference Publication 3041 (NASA, 1989), pp 31-65.

CO-ST-IN, developed for NASA Lewis Research Center, transfers NASTRAN finite element modal analysis data to the EASY5 code. After EASY5 performs a simulation of the control system, CO-ST-IN then transfers the resulting loads produced by the active control elements back to NASTRAN for dynamic stress calculations. Engineers at NASA Lewis Research Center recently used the program to analyze the solar panels of Space Station Freedom for structural integrity and pointing accuracy.¹⁰

ISM is presently under development as a replacement for IAC. Its capabilities are anticipated to be much more extensive than either IAC or CO-ST-IN. This code will interface structural analysis, thermal analysis, control simulation/design, and optics programs. The U.S. Air Force Weapons Laboratory is developing ISM through SDIO funding.

When articulated mechanisms exist in an unstable structure, multibody dynamics computer codes must be used for analysis. The ability to perform large displacement analyses sets multibody dynamics codes apart from the small displacement based finite element methods.¹¹ TREETOPS, DISCOS, and DADS are three frequently used multibody dynamics computer codes. They do not perform control system simulation or design.

Engineers use control system simulation programs to evaluate both system response and the dynamic loads applied by the active control elements to the structure. A dynamic stress analysis must then be performed to evaluate the effects of the active control devices acting on the structure in conjunction with externally applied loads. If stresses are not within the allowable range, the designer must modify the structure and again perform the modal and dynamic stress analyses. If stresses are within the allowable range, the resulting structural response must be analyzed to determine if mission requirements have been met. If the mission requirements are not met, the active control system must be redesigned, followed by another analytical modal analysis, control system simulation, and dynamic stress analysis. Once mission requirements have been met, the design can proceed to the next step.

Deployment Analysis

If the structure is deployable, a partial deployment analysis and evaluation must be performed. Deployment loads must be calculated from drive motors or other mechanisms deploying the structure. The structure's dynamic properties will change during the deployment process. Therefore, analytical modal analyses must be performed to evaluate the natural frequencies and mode shapes as a function of time throughout the deployment process. The possibility of resonance effects must be carefully examined for deployable structures driven by motors. The operating frequency of a drive motor must be selected or varied continuously so that it is not coincident with partial deployment configuration natural frequencies.

The engineer must determine that stresses are within the allowable range throughout the deployment process. If the allowable stresses are exceeded, the structure and/or the drive motor characteristics must be modified and the system reanalyzed.

¹⁰ K. Karney, et al., "Detection of Potential Space Station Control/Structure Interaction with CO-ST-IN," *NASA/DOD Controls-Structures Interaction Technology 1989*, NASA Conference Publication 3041 (NASA, 1989), pp 211-228.

¹¹ "DADS Propels Aerospace Design in a New Direction," *Aerospace Engineering* (November 1990), pp 8-9.

Experimental Verification of the Model

Because any structural model is only an idealization of an actual structure, it is important to experimentally verify the theoretically predicted behavior. Even the best mathematical models are based on assumptions and must be verified against test data. For example, the effect of joint fixity, nonlinearities, and dead band may not have been modeled sufficiently well in the finite element analysis. Validation of theoretical models will be particularly critical for LSS having active control systems. The stability and effectiveness of such systems will be extremely sensitive to accurate identification of structural parameters.

Experimental modal analysis methods¹² have most commonly been used to verify structural behavior and to improve the fidelity of finite element models. These methods characterize a structure by relating its acceleration or displacement response to a set of input forces. Basically, the structure is mounted in a configuration having specific boundary conditions, one or more dynamic forces are applied, and the structure's displacement or acceleration response is measured. The resulting transfer functions relating response to force input can be used to modify the original theoretical model or to generate a new mathematical model. These procedures have worked quite well in the past on conventional aerospace structures.

The engineer must select hardware and testing configuration prior to testing. A suspension system must be devised for the structure to circumvent the problems peculiar to LSS that were not encountered in testing conventional aerospace structures in the past. For example, LSS may be too flexible to support even a small portion of their own weight on Earth. Also, gravity imposed dead loads will load joints to much higher levels than they will ever encounter in space. Because of dead band and slippage, joints behave nonlinearly under large load changes.¹³ Therefore, they will demonstrate much stiffer behavior in the presence of gravity than they would in space. Furthermore, an LSS may simply be too large to be supported in any existing test facility. A solution to this problem may be to perform tests on scale models.

Scale Models

Basically, two types of scale modeling exist: replica scaling and distorted scaling. All components are scaled equally in replica scaling. It may become prohibitively expensive or even physically impossible to fabricate such a model when the ratio of the size of individual components to overall structure size is very small. By scaling individual components differently, distorted scaling may provide a solution. However, much care must be exercised to maintain dynamic similitude. The key to effective distorted scale modeling is to understand the behavior of the overall structure well enough to know the scaling relationships between different structural parameters and what approximations can be made. NASA Langley Research Center has conducted research to develop a dynamically scaled model of Space Station Freedom in order to verify the capability for theoretically predicting the full-scale dynamic behavior of multibody joint dominated LSS.¹⁴

¹² D.J. Ewins, *Modal Testing: Theory and Practice* (Research Studies Press Ltd., Letchworth, Hertfordshire, England; and John Wiley & Sons, New York, 1984).

¹³ W.K. Belvin, *Modeling of Joints for the Dynamic Analysis of Truss Structures*, NASA Technical Paper 2661 (NASA, May 1987).

¹⁴ R. Letchworth, et al., "Conceptual Design of a Space Station Dynamic Scale Model," *NASA/DOD Controls-Structures Interaction Technology 1987*, AFWAL-TR-88-3052 (Wright Patterson Air Force Base [AFB], Ohio, June 1988), pp 87-119.

Testing Configuration

A common procedure used to test structures in the past was to suspend them on extremely flexible suspension systems such as long thin cables, soft mechanical springs, or air springs. A general rule has been that the lowest natural frequency of the suspension system be a factor of four to five less than the frequency of the lowest mode of the structure being tested.¹⁵ This has worked well for conventional aerospace structures. However, typical LSS will be much larger and more flexible than any structures previously tested. Satisfying the general rule stated above would require very long cables or extremely soft springs. Conventional test facilities probably would not have sufficient overhead clearance for such suspension systems.

Two alternatives exist to circumvent this problem. The first is to develop very low frequency test devices. Several zero rate spring mechanisms have been developed to provide very low, nearly linear stiffness over a limited range of axial deformation. One such device consists of a vertical spring and two precompressed horizontal springs. As the test article moves vertically from its static equilibrium position, the two horizontal springs deviate slightly from their horizontal orientation and apply a vertical force component to the test article that opposes the incremental force applied by the vertical spring. Over a wide range of deformation, the device behaves nonlinearly. A potential disadvantage of this device is its inherent damping. A hybrid device using a combination passive pneumatic and active electromagnetic system has also been developed which provides frequency-dependent stiffness.¹⁶ It has an extremely high static stiffness accompanied by very low dynamic stiffness.

The second alternative to circumventing the problem is to mathematically eliminate the effect of the suspension system. Methods must be developed to correlate test results obtained using suspension systems in a laboratory to behavior that would be expected in microgravity. The research performed by USACERL in this area will be discussed later.

Excitation devices and response sensors must be selected for the system. Several possibilities exist for exciting the structure including application of an impact, a step relaxation, a shaker input, or application of actual operating loads. Response can be measured in terms of displacement, velocity, acceleration, or strain. Accelerometers are the most widely used response sensors. A computer software system must also be selected for acquisition and processing of input and response data.

Once these decisions are made, the experimental modal analysis can proceed. The forced inputs and response outputs are measured experimentally in the time domain and transformed to the frequency domain to yield modal frequencies and complex transfer functions. From this data, the structural properties of the structure can be computed. These properties include stiffness, mass, and damping characteristics. This problem is often referred to as system identification. The structural characteristics are evaluated by measuring the response of the system to known excitations.

¹⁵ B.R. Hanks, "Dynamic Verification of Very Large Space Structures," *Second International Symposium on Aeroelasticity and Structural Dynamics*, 1-3 April 1985 (AIAA, 1986), pp 648-655.

¹⁶ D.A. Kienholz, E.F. Crawley, and T.J. Harvey, "Very Low Frequency Suspension Systems for Dynamic Testing," *Proceedings of the AIAA, ASME, ASCE, AHS, ASC 30th Structures, Structural Dynamics and Materials Conference* April 1989 (AIAA, 1989), pp 327-336.

The problem of system identification can be very difficult for complex structures. An unlimited number of physical models could behave the same way for a given excitation. The initial apparent nonexistence of a unique model for a structure does not necessarily mean that the problem has no solution; it simply indicates that more extensive testing must be performed to arrive at an optimum model that can fairly accurately predict the structure's response to any excitation. The calculation of the optimum associated mass, stiffness, and damping matrices requires high precision in the measurement of higher frequency modes. Hence, dynamic response calculated from a model in which the structural parameters have been evaluated from system identification procedures can never be expected to yield better results than those predicted by a model in which the parameters were measured directly. Furthermore, it has been argued that the stiffness, mass, and damping matrices have no real physical meanings for complex structural systems when they are determined through system identification.

Final Analysis and Design Considerations

As mentioned earlier, experimental modal testing can be used to improve the fidelity of the structural engineer's theoretical model. Various optimization schemes have been devised to do this by revising the stiffness, mass, and damping matrices of the finite element model. The structural engineer is interested in knowing how the structure will behave in microgravity. Therefore, it is necessary to have techniques to correlate ground based behavior to microgravity behavior. In microgravity, the natural frequencies of a structural system will be lower than the system's natural frequencies when supported by finite stiffness suspension devices on Earth.

A complete stress analysis and controls simulation must be performed on the revised structural model. If the resulting stresses are greater than allowable, the engineer must redesign the structure, then perform further experimental modal analysis to verify the new theoretical model. If, however, the stresses are within the allowable range, the updated model response must be analyzed to determine if all mission requirements are met. If the requirements are not met, the passive damping and/or active control system must be redesigned. The redesigned structure must then undergo experimental modal analysis to verify its theoretical model. For deployable structures, partial deployment configuration stresses must be reevaluated using the experimentally revised model. Some structural modifications may be necessary.

At this point, launch stresses must be evaluated for the structural components. Individual structural members will probably experience their highest stresses during launch when inertial effects produced by high accelerations are greatest. Launch vehicle accelerations, as well as cargo bay shape and size must be known. From this information, a packing design must be developed to minimize launch stresses. Stresses incurred by structural members during transport to the launch site must also be evaluated carefully. This is very important because the structure has not been specifically designed to support its own weight in a gravity environment. Again, a packing design must be developed for transportation to the launch site.

On-Orbit Testing and Damage Assessment

The structure is ready for launch following fabrication, transportation to the launch site, and integration of the structure into the launch vehicle. Once in low-Earth orbit, it will be either deployed or erected by astronauts or robotic systems. Only after reaching space can the structure finally respond dynamically in the microgravity environment for which it was designed. The structure can be

instrumented and dynamically tested in low-Earth orbit.¹⁷ Only then can the validity of the ground-based testing be assessed. Some researchers feel that ground-based testing will never be a feasible way to validate a theoretical model.¹⁸ They feel that gravity imposed dead loads will stress joints to much higher levels than they will ever encounter in space, resulting in highly nonlinear test behavior on Earth. Joint dead band further complicates this situation. Because of nonlinearities and dead band, structures may demonstrate much stiffer behavior in the presence of gravity than they would in space. These researchers advocate on-orbit identification of the structural system. However, while on-orbit testing is the theoretically superior method of system identification and verification for LSS, individual structural components must be fabricated on Earth before launch and subsequent installation in low-Earth orbit. Some prior degree of accuracy in estimating structural behavior in orbit is necessary to at least perform a rough optimal design of the structure to minimize its launch costs.

If the on-orbit dynamic behavior of LSS matches ground-based test predictions, engineers can continue to use ground-based testing to develop new LSS designs. If the behavior does not match well, further research must be conducted in ground-based testing equipment and procedures as well as correlation techniques. The new methods can be used to test future LSS designs before deployment in orbit. If further research provides reasonably accurate correlation of ground-based test results with actual microgravity behavior, then the improved ground-based testing procedures can be used to develop subsequent LSS designs.

However, improved ground-based testing equipment, procedures, and correlation techniques may not result in reasonably accurate estimates of microgravity behavior. The only alternatives to stiffer, heavier structures would be to develop (1) adaptive structures or (2) artificial intelligence (AI) systems for designing and assembling active control systems after reaching Earth orbit.

The first alternative, adaptive structures, is based on replacing a physical element in the load path of a structure with an adaptive element that can be adjusted on-orbit to achieve some desired structural performance.¹⁹ The adaptive elements accomplish this goal by changing the structural parameters through modification of stiffness or damping. The structure can be adjusted to characteristics selected before launch or to optimal values based on flight data. Furthermore, the structure can be changed to improve its performance for each of several events in an overall mission. One example of an adaptive element is a truss member with an integrated piezoelectric element that can vary its own length. Another example is a damping element whose damping characteristics can be varied in space by adjusting the temperature of the damping material.

The second alternative is to design a control system after reaching orbit. Because the design of an active control system requires accurate knowledge of LSS structural parameters, on-orbit identification would have to be performed first. NASA Langley Research Center is presently working on a program to conduct on-orbit identification of the baseline configuration of Space Station Freedom.²⁰ This is not

¹⁷ T.K. Hasselman, "Identification of Large Space Structures On-Orbit, A Survey by Members of the Task Committee on Methods for Identification of Large Structures in Space of the Aerospace Division of the American Society of Civil Engineers," *Model Determination of Large Space Systems Workshop*, JPL D-5574 (Jet Propulsion Laboratory [JPL], March 1988), pp 36-53.

¹⁸ B.K. Wada et al., "Adaptive Structures," *Proceedings of the AIAA, ASME, ASCE, AHS, ASC 30th Structures, Structural Dynamics and Materials Conference*, April 1989 (AIAA, 1989), pp 1-11.

¹⁹ B.K. Wada et al., "Adaptive Structures," *NASA/DOD Controls-Structures Interaction Technology 1987* (Wright Patterson AFB, June 1988), pp 163-175.

²⁰ J.W. Johnson, "The Space Station Structural Characterization Experiment," *Proceedings of the Model Determination for Large Space Systems Workshop* JPL D-5574 (JPL, March 1988), pp 401-436.

a part of the space station development process. The objective of the program is to collect research data to support future development of other LSS.

Once in orbit, LSS maintenance and repair procedures must be implemented to assure that the structure will continue to satisfy mission requirements. Debris and meteorite impact, atomic oxygen, outgassing, solar radiation, cosmic radiation, temperature fluctuation, docking maneuvers, and military hostility have the potential to structurally impair LSS. The resulting damage may reduce the structure's ability to satisfy the mission requirements. As the number of objects in low-Earth orbit and the size of LSS increases, the probability of impacts will increase significantly. For this reason, debris management in low-Earth orbit will become an increasingly important issue.

System identification techniques can be used in low-Earth orbit to assess damage to an LSS so necessary repairs can be made.²¹ Because the dynamic properties of a structure are functions of its stiffness and mass properties, structural damage will result in changes in its dynamic characteristics. The changes in dynamic properties of a structure can be identified by continuously monitoring its dynamic response. Mathematical damage functions can be formulated to assess the occurrence, location, and extent of damage incurred. These functions are particularly useful when visual inspection is difficult or impossible. In many cases, certain damage may not even be visible. For example, the hostile space environment will, over time, induce natural changes in the material properties of a structure. Continuous identification of the structure will be necessary to appropriately adapt the active control system to these changes.²² In summary, on-orbit system identification will probably play an extremely important role in assuring that the structure will be continuously capable of satisfying mission requirements.

²¹ G.D. Jeong, N. Stubbs, and J.T.P. Yao, "Assessment and Control of Structural Damage," *Proceedings of the Model Determination for Large Space Systems Workshop*, JPL D-5574 (JPL, March 1988), pp 1056-1098; J.C. Chen, "Damage Assessment Technology for Large Space Systems," *Proceedings of the Model Determination for Large Space Systems Workshop*, JPL D-5574 (JPL, March 1988), pp 1112-1128; P. Hajela and F.J. Soeiro, "Structural Damage Detection Based on Static and Modal Analysis," *Proceedings of the AIAA, ASME, ASCE, AHS, ASC 30th Structures, Structural Dynamics and Materials Conference*, April 1989 (AIAA, 1989), pp 1172-1182.

²² A. Berman, "Overview of Structural Parameter Identification for Large Space Structures Needs, Concepts, Limits, Potential," *Proceedings of the Model Determination for Large Space Systems Workshop* JPL D-5574 (JPL, March 1988), pp 54-71.

3 MODELING AND ANALYSIS OF SELECTED TRUSSES

Because of high launch costs, an LSS must be designed with a very high stiffness-to-mass ratio. Structural members can resist loads through pure tension or pure compression (axial stress), bending, torsion, and/or shear. Structural members that resist load by pure axial stress are called truss elements. The applied load-to-displacement ratio of a truss element is referred to as its stiffness. Because of the very high stiffnesses of the individual truss elements, truss structures have very high stiffness-to-mass ratios and are among the most efficient structures known to man. LSS will be constructed of trusses to exploit this efficiency.

The stiffness of an individual truss member depends on its cross sectional properties and on the properties of the material of which it is composed. The stiffness of the overall truss structure depends on the stiffness of the individual members as well as the geometric configuration of the structure. Materials researchers are developing improved composite materials with higher stiffness-to-mass ratios. USACERL has studied the effect of truss geometric configuration on overall structure stiffness using finite element methods. A number of geometries were studied in conjunction with the work reported here. For comparison, all structures were assumed to behave linearly; nonlinear responses, if any, must be addressed in a much more complex manner and are beyond the scope of this study. The effects of varying the directions of diagonal members and of joint fixity were examined as part of this study.

Five variations of a square truss with pinned joints were examined. Each variation had a different orientation of its diagonal members. These five variations are shown in Figures 1 through 5. Each truss

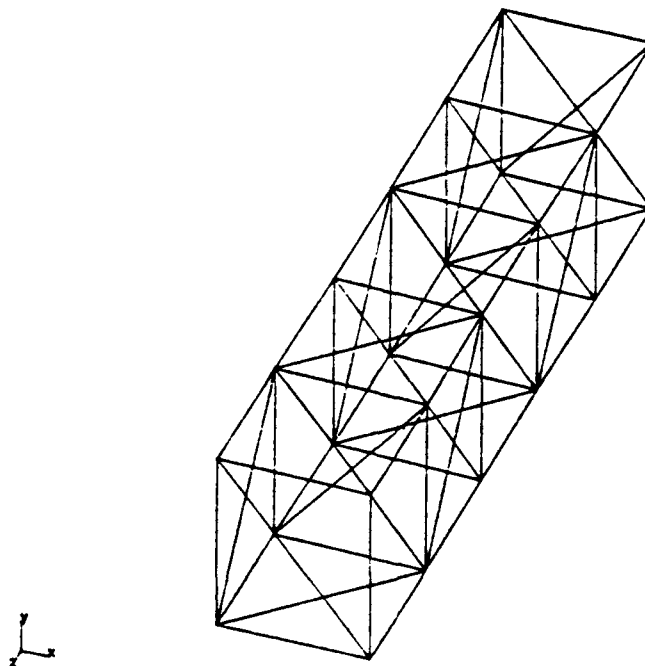


Figure 1. Square Truss Variation 1.

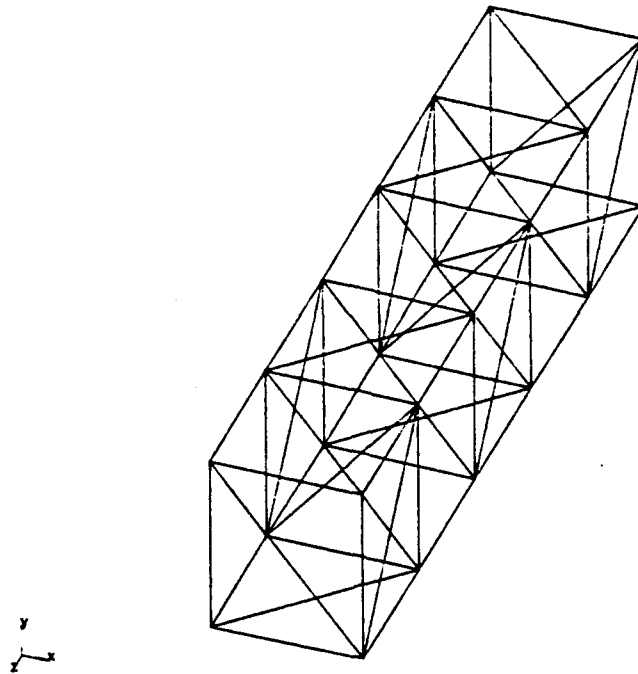


Figure 2. Square Truss Variation 2.

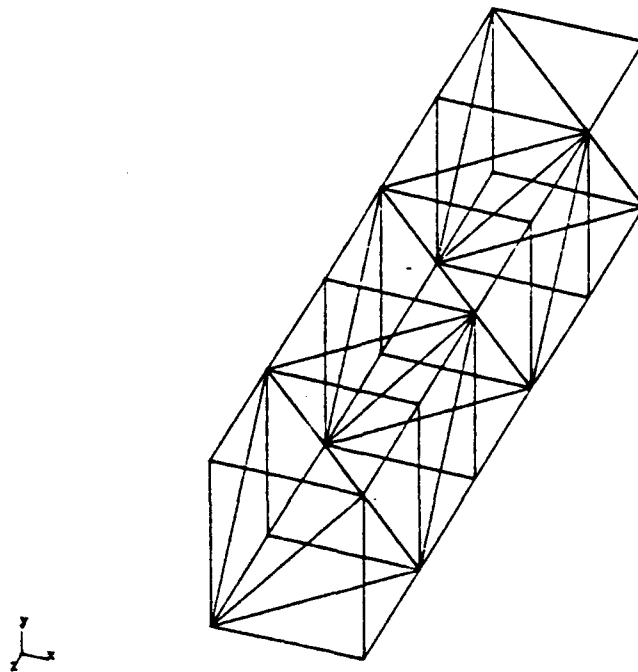


Figure 3. Square Truss Variation 3.

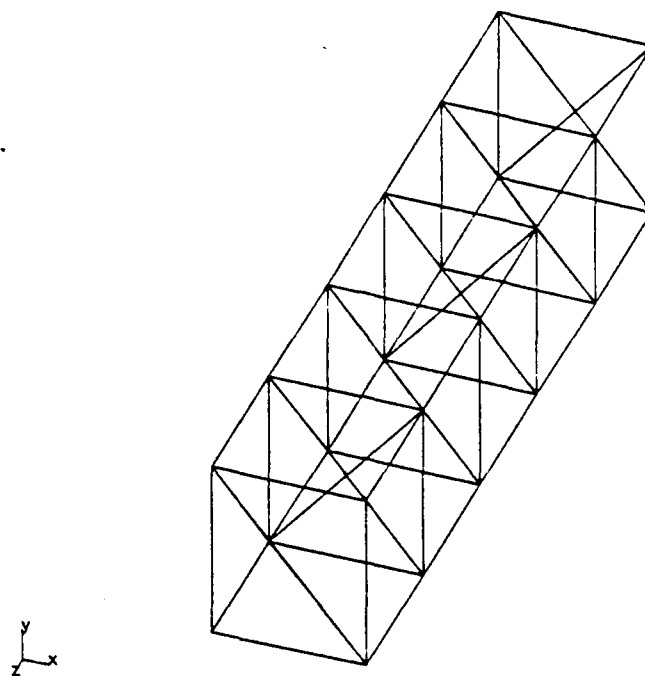


Figure 4. Square Truss Variation 4.

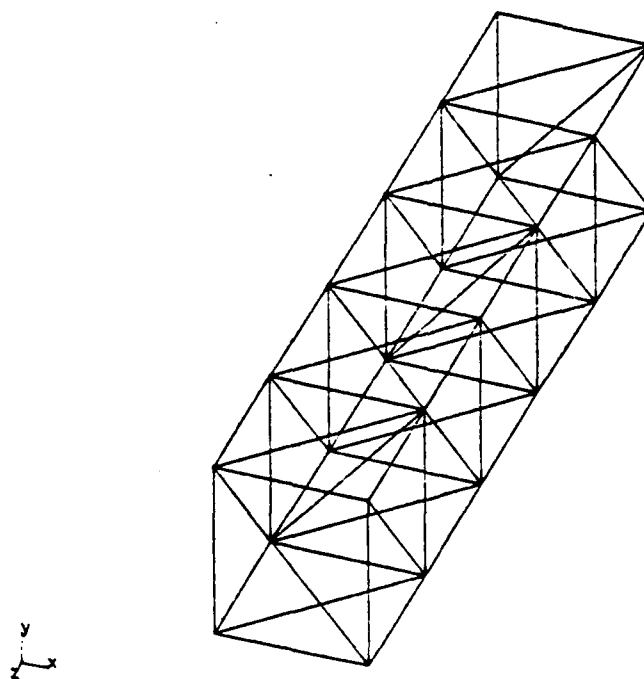


Figure 5. Square Truss Variation 5.

member is 5 meters long and consists of a hollow aluminum tube with an outside diameter of 3.81 cm and an inner diameter of 3.53 cm. Analytical modal analysis of each structure was performed using COSMOS, a finite element analysis program. Each structure was analyzed unsupported, resulting in six rigid body modes. Lumped mass matrices were used in these analyses. Because these were initial studies, the joint masses were not explicitly included. Inclusion of these masses would augment the diagonal elements of the mass matrix. This would result in slightly lower natural frequencies. However, until more refined analyses are required, the inclusion of these supplemental masses is not critical.

The primary effect of varying the diagonal orientations was to change the coupling between orthogonal bending directions. The magnitudes of the natural frequencies changed slightly. Truss variations 1, 3, 4, and 5 showed very strong coupling in the two bending directions. Variation 2 exhibited very weak coupling in bending. The fundamental elastic body mode of vibration was characterized by torsion for all five variations. Some of the higher modes were characterized by strong coupling of axial and torsional deformations. The lowest eight elastic body natural frequencies for the five truss variations are shown in Table 1. The lowest four elastic body mode shapes of variation 5 are shown in Figure 6.

The same five structures were then analyzed to study the effect of joint fixity. All truss elements in the original finite element model were replaced with beam elements capable of resisting loading through axial, bending, and torsional deformations. Such capability could be created by welding ends of truss elements to the node joint. In typical civil engineering structures, joints are usually modeled as either ideally pinned (shear connection) or ideally fixed (moment connection). In reality, the actual degree of fixity is somewhere between being ideally pinned and ideally fixed. A structure with ideally fixed joints was modeled to place a bound on the maximum additional stiffness that could be added to the structure. The ensuing finite element analysis showed that the effect on overall structural stiffness of fixing the joints was negligible. The lowest eight elastic body natural frequencies for the five variations of the square truss with fixed joints are shown in Table 2. Elastic body natural frequencies were changed only to the third decimal place, which attests to the high efficiency of truss action alone to resist loading. Particularly for lower modes, the elastic strain energy produced by axial deformation far outweighs elastic strain energy produced by bending in the structural member. For higher modes, it is expected that elastic strain energy from bending will be more significant than elastic strain energy from axial deformation. However, the lower frequency modes associated with very large deflections are usually of greater concern in LSS design.

Several other geometric configurations were examined, including the tetrahedral truss (Figure 7) and the hexagonal platform made from repeating tetrahedral units (Figure 8). For both structures, member properties were taken to be the same as those for the square truss. The lowest eight elastic body natural frequencies for these structures are shown in Tables 3 and 4.

The preceding analyses is very elementary. Accurate modeling of realistic trusses would require consideration of nonlinear joint behavior and dead band. Research on these effects is presently being expanded.²³ Including joint nonlinearities and dead band in the finite element model would require nonlinear solution procedures and specialized finite elements to model the joints. As an additional step, continuum models could be developed for the discrete models analyzed above. These continuum solutions could be compared to the finite element solutions. As mentioned earlier, continuum modeling can significantly reduce computational expense during the preliminary design phase. Also, subsequent finite element analyses could be performed to evaluate the increase in stiffness that can be obtained by constructing the hexagonal platform two or more tiers thick.

²³W.K. Belvin.

Table 1
Square Truss Natural Frequencies

VARIATION 1	
Mode	Frequency
1	20.326
2	21.698
3	21.698
4	36.816
5	36.816
6	37.916
7	49.222
8	50.914

VARIATION 2	
Mode	Frequency
1	17.413
2	21.684
3	21.755
4	32.150
5	36.952
6	37.426
7	41.206
8	43.637

VARIATION 3	
Mode	Frequency
1	20.430
2	21.726
3	21.726
4	36.897
5	36.897
6	38.457
7	51.063
8	52.354

VARIATION 4	
Mode	Frequency
1	16.362
2	20.929
3	23.153
4	31.790
5	34.643
6	39.874
7	45.777
8	46.327

VARIATION 5	
Mode	Frequency
1	16.362
2	20.929
3	23.153
4	31.790
5	34.643
6	39.874
7	45.777
8	46.327

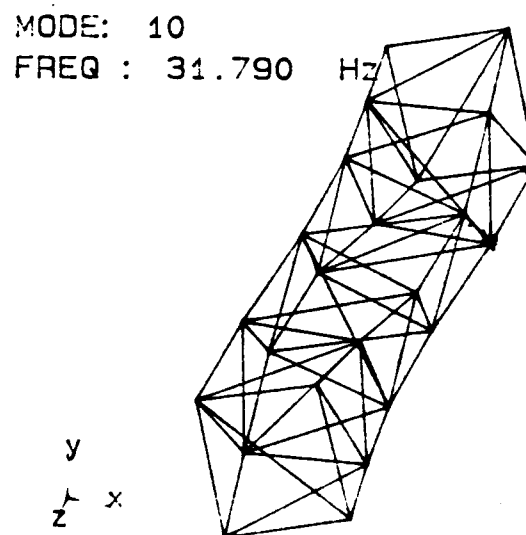
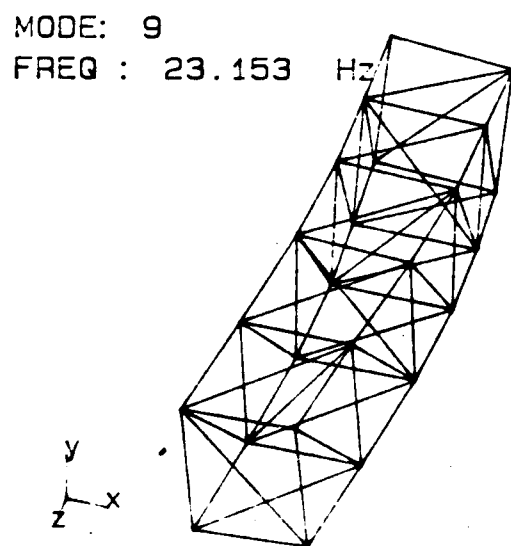
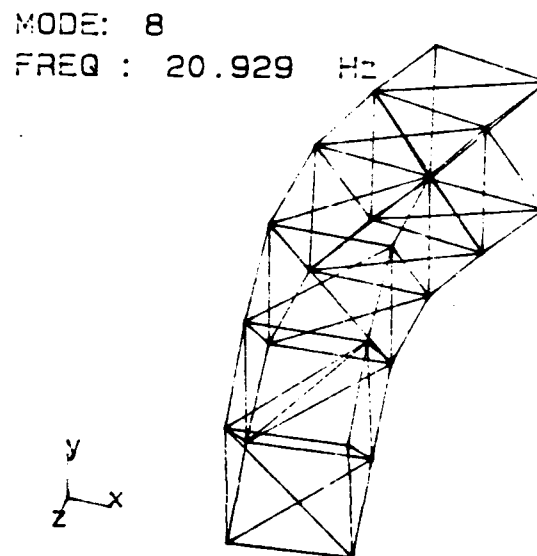
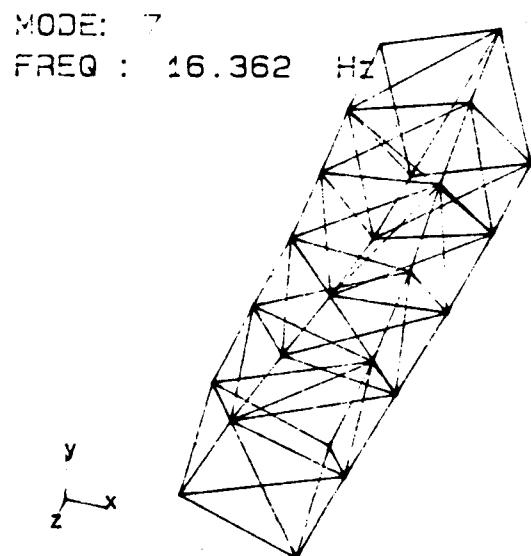


Figure 6. Square Truss Elastic Body Mode Shapes (Variation 5).

Table 2
Square Truss Natural Frequencies for Fixed Joints

VARIATION 1	
Mode	Frequency
1	20.331
2	21.703
3	21.703
4	36.830
5	36.830
6	37.928
7	49.229
8	50.934

VARIATION 2	
Mode	Frequency
1	17.419
2	21.689
3	21.760
4	32.163
5	36.966
6	37.439
7	41.230
8	43.660

VARIATION 3	
Mode	Frequency
1	20.408
2	21.732
3	21.732
4	36.911
5	36.911
6	38.469
7	51.071
8	52.374

VARIATION 4	
Mode	Frequency
1	16.368
2	20.934
3	23.157
4	31.802
5	34.657
6	39.886
7	45.796
8	46.337

VARIATION 5	
Mode	Frequency
1	16.368
2	20.934
3	23.157
4	31.802
5	34.657
6	39.886
7	45.796
8	46.337

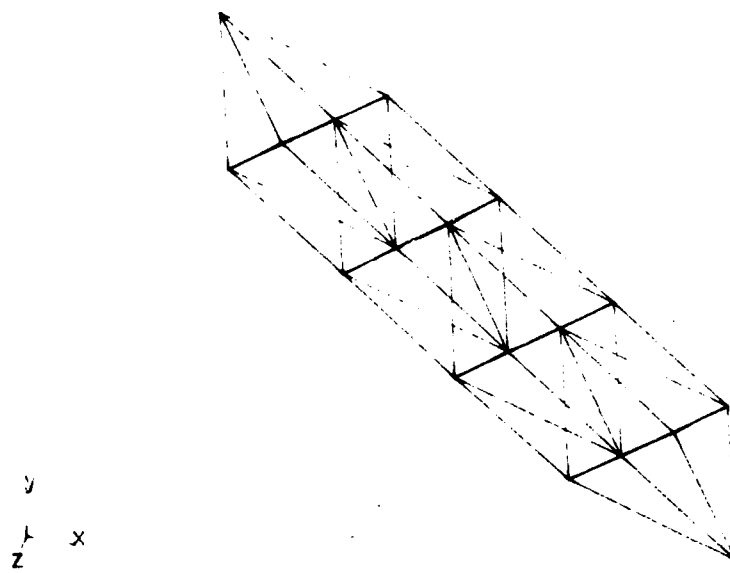


Figure 7. Tetrahedral Truss.

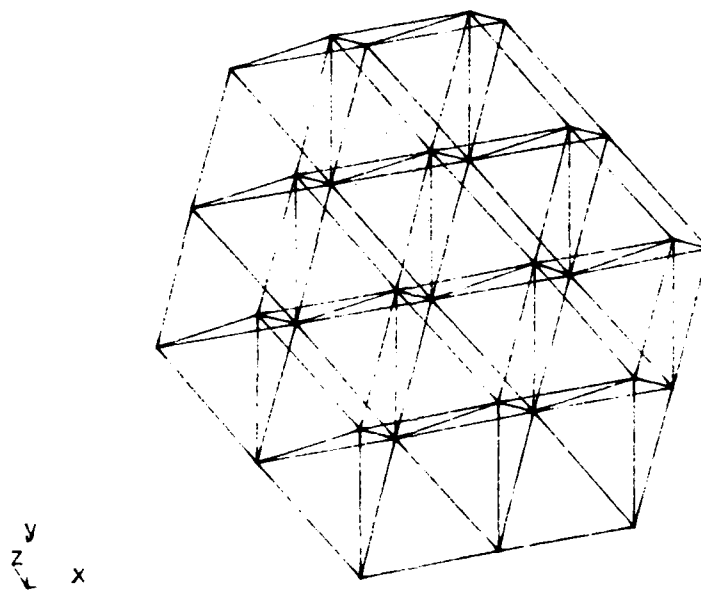


Figure 8. Hexagonal Platform.

Table 3**Tetrahedral Truss Natural Frequencies**

MODE	FREQUENCY
1	19.072
2	19.244
3	23.382
4	38.933
5	39.043
6	45.089
7	55.147
8	58.695

Table 4**Hexagonal Platform Natural Frequencies**

MODE	FREQUENCY
1	28.201
2	28.201
3	33.460
4	40.778
5	44.348
6	44.348
7	49.731
8	57.730

4 GROUND-BASED TESTING CORRELATION RESEARCH

As mentioned earlier, techniques must be developed to predict the behavior of an LSS in microgravity from ground-based dynamic tests of representative models. The models are first tested in a laboratory using suspension systems made of very soft springs or cables, zero rate spring mechanisms, or pneumatic devices having very low dynamic stiffnesses. Laboratory testing generally has examined dynamic behavior only in directions perpendicular to the gravity vector. USACERL has examined the correlation between earth-based natural frequencies in vibration directions parallel to the gravity vector and the theoretical natural frequencies expected in microgravity.

The natural frequencies of a structure depend on the physical properties of its components and the way it is supported. For example, a cantilevered beam will exhibit different natural frequencies than the same beam simply supported at its ends. Likewise, an unsupported beam in space microgravity will have different natural frequencies than the same beam resting on an elastic foundation within a gravity field. However, the unsupported beam cannot be tested on earth because finite stiffness supports are required to counteract the gravity imposed dead load.

Beam on an Elastic Foundation

For simplicity, consider a prismatic beam of length L , resting on a uniformly distributed elastic foundation.²⁴ The governing differential equation of free vibration is:

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + \kappa w(x,t) + m \frac{\partial^2 w(x,t)}{\partial t^2} = 0 \quad [\text{Eq 24}]$$

where:

- E = modulus of elasticity of beam
- I = moment of inertia of beam about bending axis
- $w(x,t)$ = deflection of beam with respect to static position
- κ = foundation stiffness per unit length
- m = beam mass per unit length
- t = variable time
- x = distance measured along beam from far left end toward right end

²⁴ L.E. Meirovitch, *Elements of Vibration Analysis* (McGraw-Hill, New York, 1986), pp 220-227.

Assuming a solution of the form:

$$w(x,t) = w(x) \sin \omega t \quad [\text{Eq 25}]$$

where: ω = circular frequency of vibration

Applying boundary conditions at the ends (bending moment and shear equal to zero) yields an eigenvalue problem. The roots are the circular natural frequencies. These are:

$$\omega_i = \left(\frac{16EI\alpha_i^4}{L^4m} + \frac{\kappa}{m} \right)^{1/2} \quad [\text{Eq 26}]$$

where:

α_1	=	0
α_2	=	0
α_3	=	2.36502035
α_4	=	3.9266023
α_5	=	5.49778715
α_6	=	7.0685828

$$\alpha_i = \frac{(2i-3)\pi}{4} \quad i = 7, 8, 9, \dots \quad [\text{Eq 27}]$$

In summary, the natural frequencies of a beam resting on an elastic foundation depend on the stiffness of the foundation relative to the stiffness of the beam, as well as the mass of that beam per unit length. The beam can experience free vibrations only at circular frequencies greater than $[\kappa/m]^{1/2}$.

Unsupported Beam in Microgravity

The theoretical solution for the unsupported beam in a microgravity environment is actually a special case of the beam on an elastic foundation, where the foundation stiffness, κ , is zero. Now,

$$\omega_i = \alpha_i^2 \left(\frac{2}{L} \right)^2 \left(\frac{EI}{m} \right)^{1/2} \quad [\text{Eq 28}]$$

where α_i is as previously defined. The lowest two circular natural frequencies are obviously zero. Thus, the square of each circular natural frequency of the beam on an elastic foundation on earth will be κ/m greater than the square of each circular natural frequency of the same beam in a space microgravity environment. Or,

$$\omega_{sp-i} = \left[\omega_{ef-i}^2 - \frac{\kappa}{m} \right]^{1/2} \quad [\text{Eq 29}]$$

where: ω_{sp-i} = i^{th} circular natural frequency of the beam in microgravity

ω_{ef-i} = i^{th} circular natural frequency of the beam on an elastic foundation in gravity

Beam Supported by Discrete Springs

Experimentally, it would be rather difficult to support a prismatic beam on a continuous elastic foundation. A much more practical situation occurs when the beam is supported by a number of discrete springs equally spaced. This problem is considerably more complicated because the effect of each spring must be considered. Instead of one governing differential equation, separate (discrete) equations will describe the dynamic behavior between each pair of springs.

The piecewise governing differential equation of free vibration for a prismatic beam, including the effect of the spring masses, can be expressed as:

$$\begin{aligned} & EI \frac{\partial^4 w(x,t)}{\partial x^4} + m \frac{\partial^2 w(x,t)}{\partial t^2} + \frac{1}{3} \mu^* \frac{\partial^2 w(0,t)}{\partial t^2} \delta(x) \\ & + \frac{1}{3} \mu^* \frac{\partial^2 w(L,t)}{\partial t^2} \delta(x-L) + \frac{1}{3} \mu \sum_{j=1}^{n-1} \frac{\partial^2 w(x,t)}{\partial t^2} \delta(x-x_j) \\ & = -k^* w(0,t) \delta(x) - k^* w(L,t) \delta(x-L) - k \sum_{j=1}^{n-1} w(x,t) \delta(x-x_j) \end{aligned} \quad [\text{Eq 30}]$$

where: $w(x,t)$ = deflection of beam with respect to static position
 μ = mass of each interior spring
 μ^* = mass of each exterior spring
 k = stiffness of each interior spring
 k^* = stiffness of each exterior spring
 n = number of equal length segments into which beam is divided
 j = 0 corresponds to left end of beam
 j = n corresponds to right end of beam
 $\delta(x-x_j) = 0.0$ for $x-x_j \neq 0.0$
 $\delta(x-x_j) = 1.0$ for $x-x_j = 0.0$

The spring mass terms are a result of the inertial effect of the springs due to their axial accelerations. The 1/3 factor is a well known approximation based on the solution of the wave equation for axial vibration of a continuous bar fixed at one end and having a tip mass attached to the free end.²⁵

Again, assuming a solution of the form:

$$w(x,t) = w(x)\sin\omega t \quad [\text{Eq 31}]$$

and applying boundary conditions at each discrete spring and at the ends yields a rather complicated eigenvalue problem. Because of the complexity of the problem and because the mathematics is very well suited to automatic computation, a FORTRAN 77 computer program was developed to solve the problem. This program searches for intervals of ω where roots exist. Interval halving is then used to close in on a root. The beam must be divided into an odd number of segments, "n." The user has the option of including or neglecting the inertial effect of the spring masses. After evaluating each circular natural frequency, the corresponding mode shape is calculated for the left half span by evaluating the relative deflection at each spring. The listing of this computer program is shown in Appendix B.

Example

Consider a 6-meter long aluminum beam having a 2.5×2.5 cm cross section. The bending stiffness, mass per unit length, and foundation stiffness required for an average dead load deflection of 15 cm are calculated as follows:

$$EI = \left(\frac{6.9 \times 10^{10} \text{ N}}{\text{m}^2} \right) \left(\frac{1}{12} \right) (.025 \text{ m})^4 = 2250 \text{ N} \cdot \text{m}^2 \quad [\text{Eq 32}]$$

$$m = \left(\frac{2690 \text{ kg}}{\text{m}^3} \right) (.025 \text{ m})^2 = 1.681 \text{ kg/m} \quad [\text{Eq 33}]$$

²⁵ W.C. Young, *Roark's Formulas for Stress and Strain* (McGraw-Hill, New York, 1989), p 715.

$$\kappa = \frac{(1.681 \text{ kg/m})(9.80665 \text{ m/s}^2)}{(0.15 \text{ m})} = 109.9 \text{ N/m}^2 \quad [\text{Eq 34}]$$

The unsupported beam in a space microgravity environment has circular natural frequencies calculated as follows:

$$\omega_{sp-i} = \alpha_i^2 \left(\frac{2}{L} \right)^2 \left(\frac{EI}{m} \right)^{1/2} \quad [\text{Eq 35}]$$

yielding: $\omega_{sp-1} = 0.0000 \text{ sec}^{-1}$ $\omega_{sp-2} = 0.0000 \text{ sec}^{-1}$
 $\omega_{sp-3} = 22.7371 \text{ sec}^{-1}$ $\omega_{sp-4} = 62.6756 \text{ sec}^{-1}$, etc.

The same beam resting on a continuous elastic foundation on earth has circular natural frequencies calculated as follows:

$$\omega_{ef-i} = \left[\omega_{sp-i}^2 + \frac{\kappa}{m} \right]^{1/2} = \left[\alpha_i^4 \left(\frac{2}{L} \right)^4 \frac{EI}{m} + \frac{\kappa}{m} \right]^{1/2} \quad [\text{Eq 36}]$$

yielding: $\omega_{ef-1} = 8.0857 \text{ sec}^{-1}$ $\omega_{ef-2} = 8.0857 \text{ sec}^{-1}$
 $\omega_{ef-3} = 24.1320 \text{ sec}^{-1}$ $\omega_{ef-4} = 63.1950 \text{ sec}^{-1}$, etc.

Consider the same beam on six discrete springs ($n=5$). To maintain the same average dead load deflection of 15 cm, each interior spring must have a stiffness of $(109.9 \text{ N/m}^2) \times (6 \text{ m}/5) = 131.88 \text{ N/m}$. Each exterior spring has half this stiffness. The resulting circular natural frequencies evaluated by the computer program are listed in Table 5. The unsupported microgravity case, elastic foundation case, and twelve discrete spring case ($n=11$) are included in the table for comparison. The program was compiled and executed on a Harris 500 minicomputer. The computer output is shown in Appendix C. Finite element modal analyses of several of these beam spring systems using NASTRAN very closely matched the solutions obtained by using this algorithm. In addition, several limiting case solutions were compared to theoretical textbook solutions. These cases included a beam with pinned ends, a beam on an elastic foundation, a beam supported on two finite stiffness discrete springs at its ends, and a free-free beam in microgravity.

Table 5
Lowest Eight Circular Natural Frequencies for
Beam with Various Support Conditions

1	Unsupported in Microgravity	Elastic Foundation	6 Discrete Springs (n=5)	12 Discrete Springs (n=11)
1	0.0000	8.0857	8.0823	8.0855
2	0.0000	8.0857	8.3968	8.1519
3	22.7371	24.1320	24.4646	24.2012
4	62.6756	63.1950	63.4146	63.2398
5	122.8686	123.1343	123.3029	123.1673
6	203.1092	203.2701	203.4568	203.2953
7	303.4101	303.5178	303.5146	303.5386
8	423.7711	423.8483	423.8836	423.8662

Observations

Several observations can be made from the tabulated results. First, natural frequencies for the discrete spring cases are very close to those for the continuous elastic foundation case. The two zero frequency rigid body modes for the beam in microgravity correspond to two very closely spaced nonzero frequency rigid body modes for the supported beam. The more discrete springs, the closer together these two frequencies are, until they become equal for the beam on an elastic foundation. Beyond the second mode, natural frequencies for the supported beam become increasingly closer to the natural frequencies of the beam in microgravity.

5 SUMMARY AND CONCLUSIONS

This report outlined a basic structural design methodology for LSS. It begins with definition of the mission requirements. From these requirements, rough estimates of overall stiffness and damping can be evaluated. An early decision must be made as to whether the structure will be erectable or deployable, based on reconfiguration requirements, launch vehicle availability, and other factors. Geometric configurations, structural material options, passive damping methods, and structural redundancy must all be considered in selecting an LSS truss configuration.

Using conventional finite element procedures, LSS can be designed iteratively, in a manner similar to the way in which conventional civil engineering structures on Earth are designed. Accurate assessment of dynamic loads, mass distribution, and structural behavior is essential for successful design of the structure and its active control systems. The basic theory of finite element modal analysis and calculation of dynamic responses was outlined. If mission requirements cannot be met with passive damping alone, an active control system must be designed. Two computer programs for designing and simulating active control systems were discussed. Several other programs for facilitating data transfer between finite element codes and active control design/simulation codes also were discussed. The necessity of using multibody dynamics programs for analyzing structures with articulated mechanisms was mentioned. For deployable structures, partial deployment configurations must be investigated for resonance effects resulting from the action of drive motors. Considering all of these factors, the extremely iterative nature of the design process is obvious.

Because any structural model is only an idealization of an actual structure, it is important to experimentally verify the theoretical model. It is envisioned that this will be done using conventional experimental modal analysis techniques while the structure is supported on some specialized suspension system. After instrumenting the structure, forced inputs and response outputs of the structure are measured, yielding modal frequencies and complex transfer functions. This procedure is referred to as system identification. The transfer functions obtained from the measured responses can be used to improve the mathematical parameters in the theoretical model. Once the theoretical model is modified, the performance of the structure acting in conjunction with its active control system can be reevaluated. Packing designs must be developed to minimize inertial stress effects on individual structural members during transportation to the launch site and during launch into space.

Controversy exists in the scientific community about the effectiveness of ground-based testing in predicting microgravity behavior of an LSS. A final conclusion on its viability cannot be made until an actual LSS is launched into low-Earth orbit, instrumented, and its dynamic response measured and compared to ground-test predictions. Until then, the only alternative to designing overly conservative structures is to rely on the ground-based testing available. If ground-based testing is shown to be ineffective in predicting microgravity behavior, further research will be required to develop improved suspension systems and correlation procedures. If these developments fail, the use of adaptive structures and/or on-orbit system identification and AI-based active control system design may be the only options to overly conservative structures.

Maintenance and repair issues must also be addressed. Space debris is becoming an increasingly critical problem for spacecraft in low-Earth orbit as the quantity of orbiting hardware increases. On-orbit system identification can be used to assess structural damage caused by this debris and other environmental effects so that repairs can be conducted.

The effect of truss geometry on overall structural stiffness was examined for a number of candidate LSS trusses. Finite element analyses concluded that joint fixity added negligibly to overall stiffness of these structures. This demonstrates the efficiency of trusses in resisting loading. Further analyses showed that changing the orientation of truss diagonals varied the degree of coupling in orthogonal bending directions.

Results of a study to examine the relationship between ground-based behavior of a structure and its microgravity behavior were presented. The structure's dynamic characteristics were shown to be dependent on its support conditions and physical properties. The structure supported on earth will always have higher natural frequencies than it would in microgravity. The theory was developed for calculating the natural frequencies and mode shapes of a prismatic beam suspended on discrete springs. This suspension configuration is relatively feasible for laboratory experimentation. Because the solution to the problem is rather difficult computationally, the algorithm was automated in a FORTRAN 77 computer program. An example problem was demonstrated.

It is recommended that this research be continued by the Air Force activity to which all Department of Defense LSS work has been assigned.

CITED REFERENCES

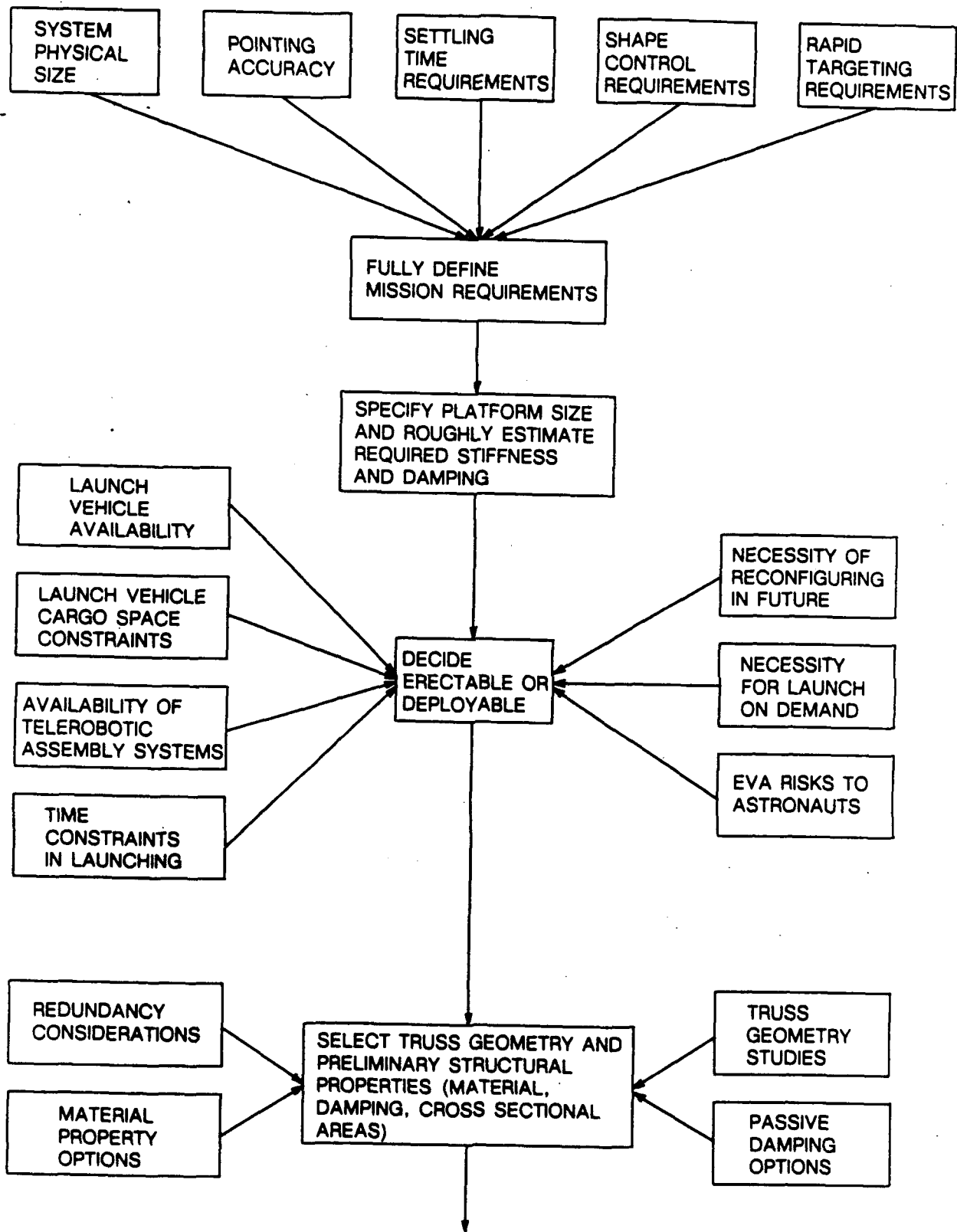
- American Society of Civil Engineers, *Identification of Large Space Structures on Orbit* (U.S. Air Force Rocket Propulsion Laboratory, September 1986), pp 63-77.
- Austin, E.M., et al., *Passive Damping Techniques For Space Structures*, final report to U.S. Army Construction Engineering Research Laboratory (USACERL) (CSA Engineering Inc., April 1988).
- Belvin, W.K., *Modeling of Joints for the Dynamic Analysis of Truss Structures*, NASA Technical Paper 2661 (National Aeronautics and Space Administration [NASA], May 1987).
- Berman, A., "Overview of Structural Parameter Identification for Large Space Structures Needs, Concepts, Limits, Potential," *Proceedings of the Model Determination for Large Space Systems Workshop*, JPL D-5574 (1988), pp 54-71.
- Chen, J.C., "Damage Assessment Technology for Large Space Systems," *Proceedings of the Model Determination for Large Space Systems Workshop*, JPL D-5574 (1988), pp 1112-1128.
- "DADS Propels Aerospace Design in a New Direction," *Aerospace Engineering* (November 1990), pp 8-9.
- Dornsife, R.J., "Effect of Gravity on Dynamic Behavior of Beams," *Engineering, Construction, and Operations in Space II, Proceedings of Space 90*, Volume 2, Stewart W. Johnson and John P. Wetzell, Eds. (American Society of Civil Engineers [ASCE], 1990), pp 1142-1148.
- Dornsife, R.J., "Ground Based Testing Dynamics for a Prismatic Beam in Microgravity," *Journal of Aerospace Engineering*, (ASCE, April 1991), pp 165-183.
- Ewins, D.J., *Modal Testing: Theory and Practice* (Research Studies Press Ltd., Letchworth, Hertfordshire, England; and John Wiley & Sons, New York, 1984).
- Fisher, S., "LACE Flight Dynamics Experiment," *NASA/DOD Controls-Structures Interaction Technology 1989*, NASA Conference Publication 3041 (NASA, 1989), pp 427-443.
- Hanks, B.R., "Dynamic Verification of Very Large Space Structures," *Proceedings of the Second International Symposium on Aeroelasticity and Structural Dynamics*, 1-3 April 1985, pp 648-655.
- Hajela, P. and F.J. Soeiro, "Structural Damage Detection Based on Static and Modal Analysis," *Proceedings of the AIAA, ASME, ASCE, AHS, ASC 30th Structures, Structural Dynamics and Materials Conference*, 3-5 April 1989 (American Institute of Aeronautics and Astronautics [AIAA], 1989), pp 1172-1182.
- Hasselman, T.K., "Identification of Large Space Structures On-Orbit: A Survey by Members of the Task Committee on Methods for Identification of Large Structures in Space of the Aerospace Division of the American Society of Civil Engineers," *Proceedings of the Model Determination of Large Space Systems Workshop*, JPL D-5574 (Jet Propulsion Laboratory [JPL], March 1988), pp 36-53.
- Jeong, G.D., N. Stubbs, and J.T.P. Yao, "Assessment and Control of Structural Damage," *Proceedings of the Model Determination for Large Space Systems Workshop*, JPL D-5574 (JPL, March 1988), pp 1056-1098.
- Johnson, J.W., "The Space Station Structural Characterization Experiment," *Proceedings of the Model Determination for Large Space Systems Workshop*, JPL D-5574 (JPL, March 1988), pp 401-436.
- Karney, K., et al., "Detection of Potential Space Station Control/Structure Interaction with CO-ST-IN," *NASA/DOD Controls-Structures Interaction Technology 1989*, NASA Conference Publication 3041 (NASA, 1989), pp 211-228.

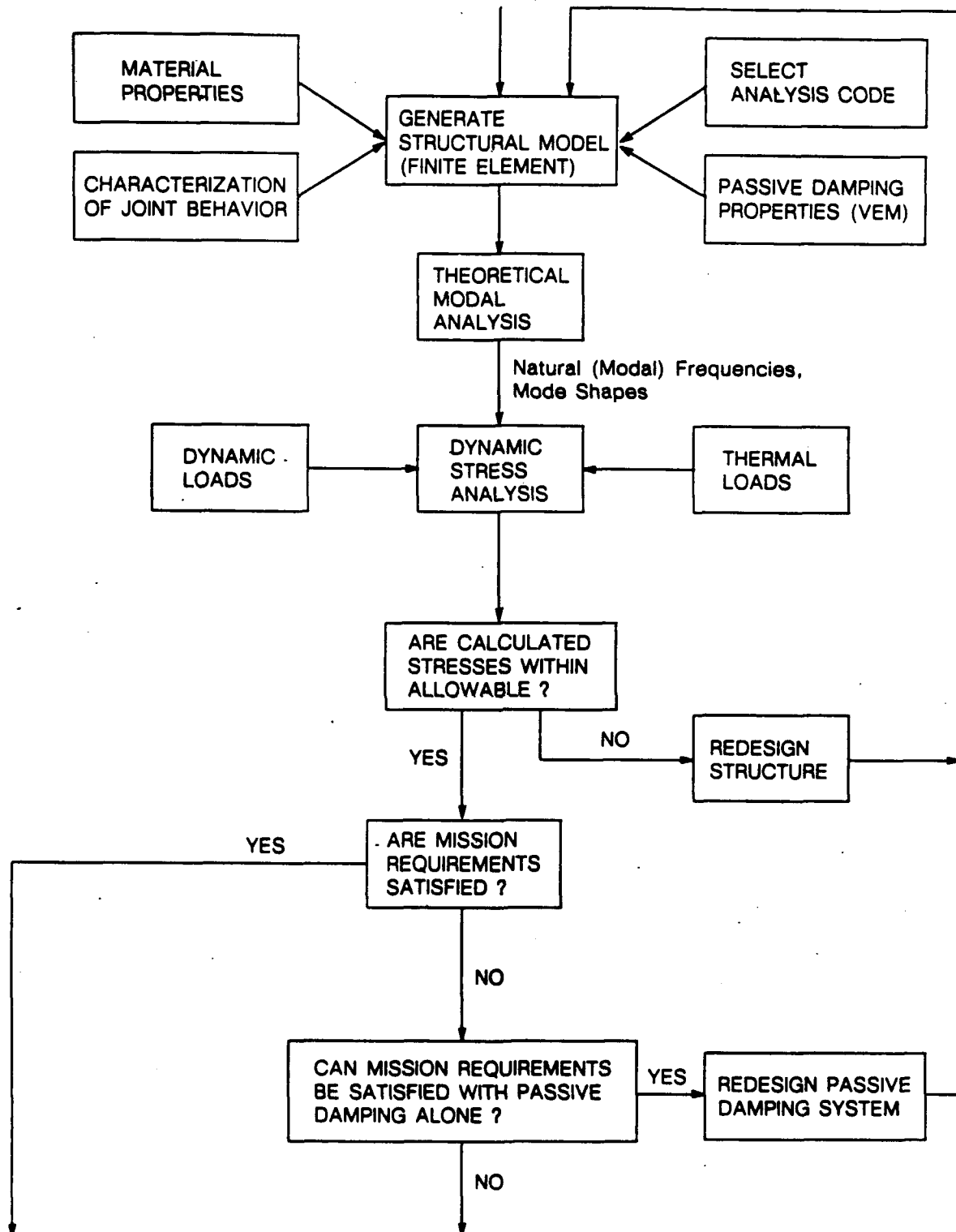
- Kienholz, D.A., E.F. Crawley, and T.J. Harvey, "Very Low Frequency Suspension Systems For Dynamic Testing," *Proceedings of the AIAA, ASME, ASCE, AHS, ASC 30th Structures, Structural Dynamics and Materials Conference*, 3-5 April 1989 (AIAA, 1989), pp 327-336.
- Lee, U., "Dynamic Continuum Modeling of Beamlike Space Structures Using Finite Element Matrices," *Proceedings of the AIAA, ASME, ASCE, AHS, ASC 30th Structures, Structural Dynamics and Materials Conference*, 3-5 April 1989 (AIAA, 1989), pp 1955-1962.
- Letchworth, R., et al., "Conceptual Design of a Space Station Dynamic Scale Model," *Proceedings of the NASA/DOD Controls/Structures Interaction Technology 1987*, AFWAL-TR-88-3052 (Wright Patterson Air Force Base [AFB], Ohio, June 1988), pp 87-119.
- Meirovitch, L.E., *Elements of Vibration Analysis* (McGraw-Hill, New York, 1986), pp 220-227.
- "NASA Developing Telerobotic System To Automate Assembly in Space," *Aviation Week and Space Technology* (September 3, 1990), pp 197-199.
- Richards, K.E., "PACOSS Program Status and Results," *NASA/DOD Controls-Structures Interaction Technology 1989*, NASA Conference Publication 3041 (NASA, 1989), pp 31-65.
- Stettner, R. and P. Mlakar, *The Development of Passive Electrodynamic Vibration Damping System For Large Space Structures*, final project report to the Strategic Defense Initiative Office, January 15, 1990 (Advanced Scientific Concepts, January 15, 1990).
- Wada, B.K., "Adaptive Structures," *Proceedings of the AIAA, ASME, ASCE, AHS, ASC 30th Structures, Structural Dynamics and Materials Conference*, 3-5 April 1989 (AIAA, 1989), pp 1-11.
- Wada, B.K., et al., "Adaptive Structures," *Proceedings of the NASA/DOD Controls/Structures Interaction Technology 1987*, AFWAL-TR-88-3052 (Wright Patterson AFB, June 1988), pp 163-175.
- Young, W.C., *Roark's Formulas for Stress and Strain* (McGraw-Hill, New York, 1989), p 715.

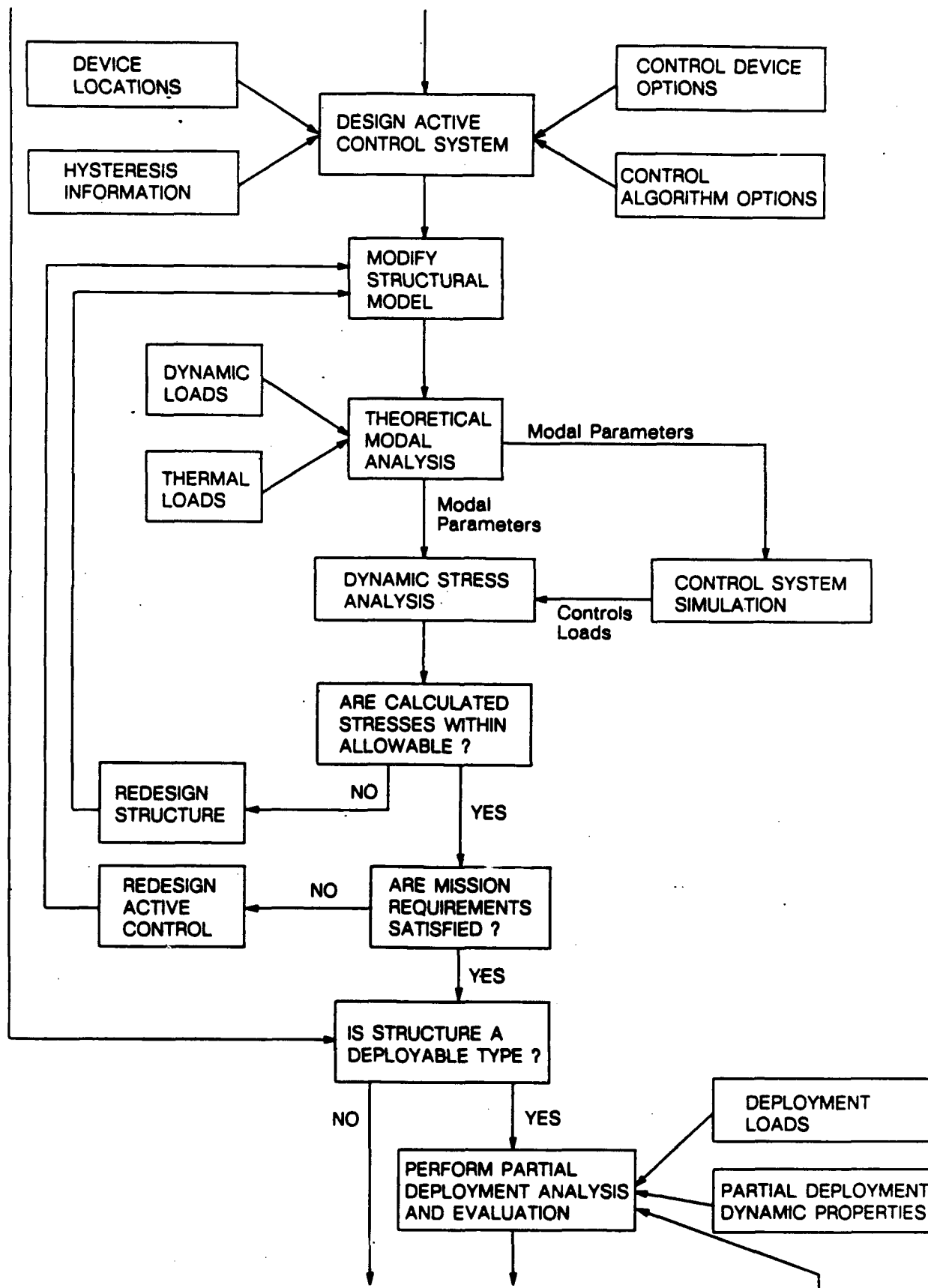
UNCITED REFERENCES

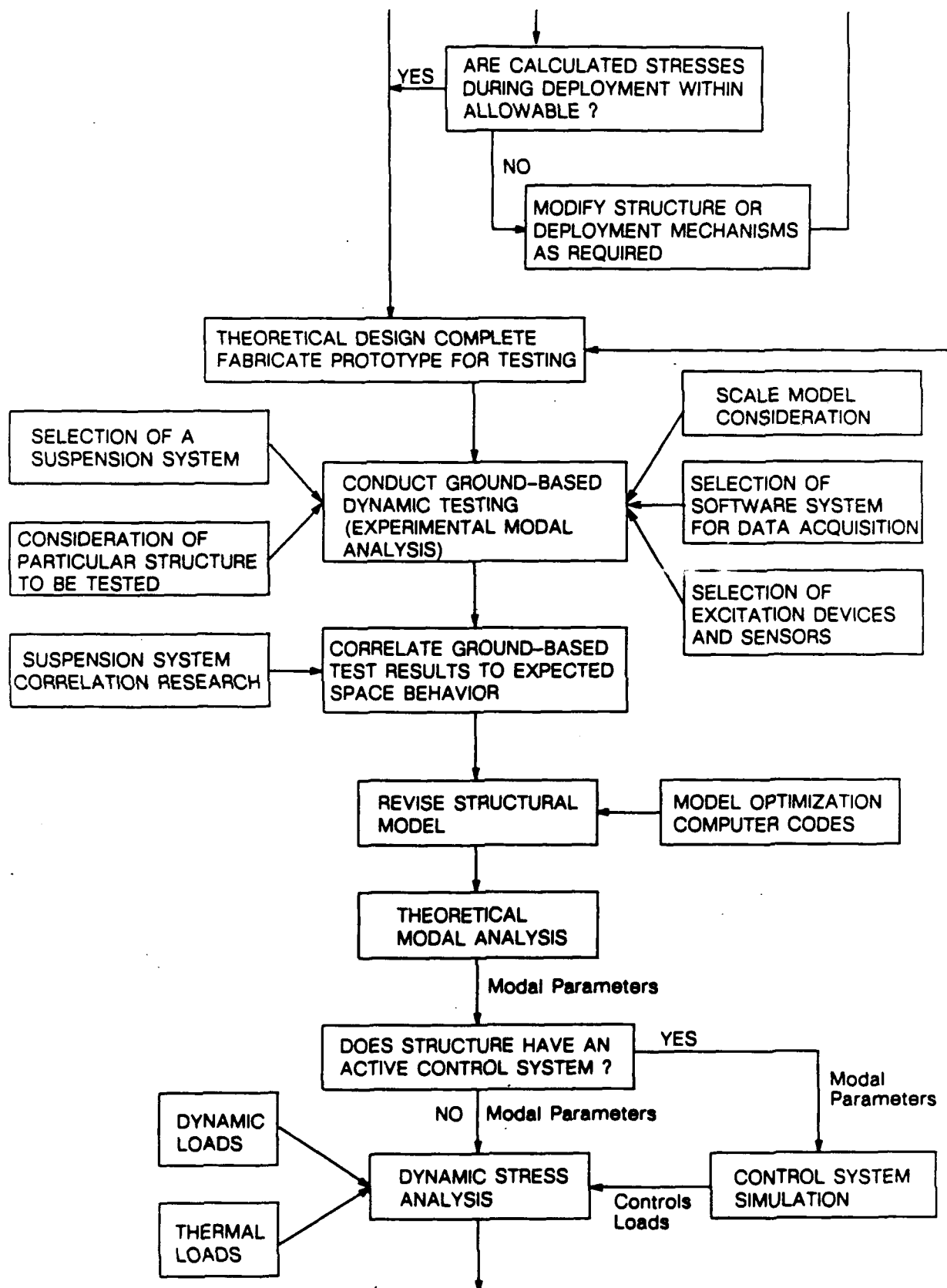
- Andreshak, D.S. and L.A. Bergman, *An Evaluation of Smart Materials for the Control of Flexible Structures*, University of Illinois Technical Report AAE 90-6, UILU ENG 90-0506 (University of Illinois, September 1990).
- Clough, R.W. and J. Penzien, *Dynamics of Structures* (McGraw-Hill, New York, 1975) pp 176-206.
- Lozar, C.C. and L.D. Stephenson, *State-of-the-Art Technologies for Construction in Space: a Review*, USACERL Technical Report M-87/17/ADA188412 (USACERL, September 1987).

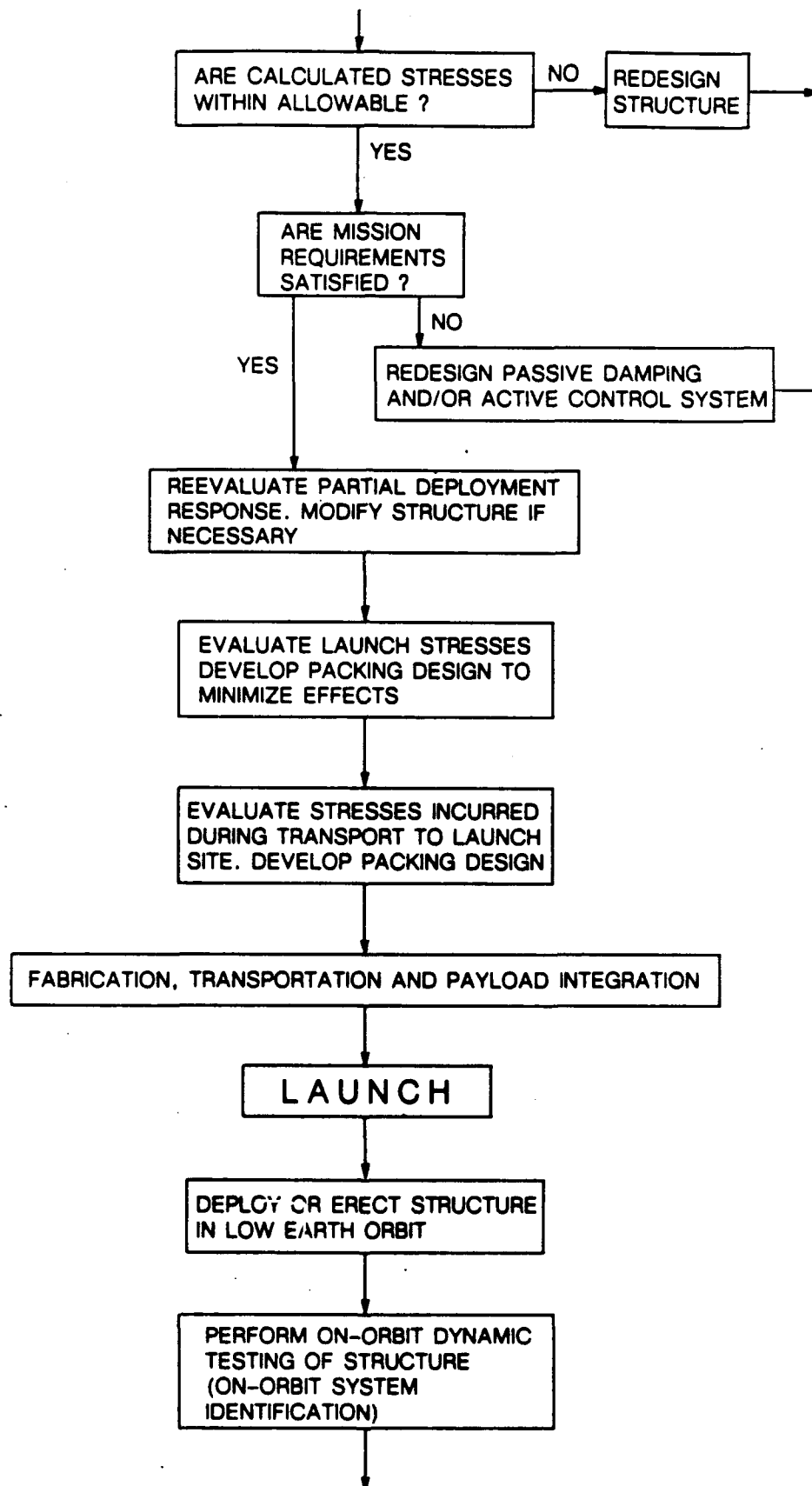
APPENDIX A: Structural Design Methodology Flow Chart

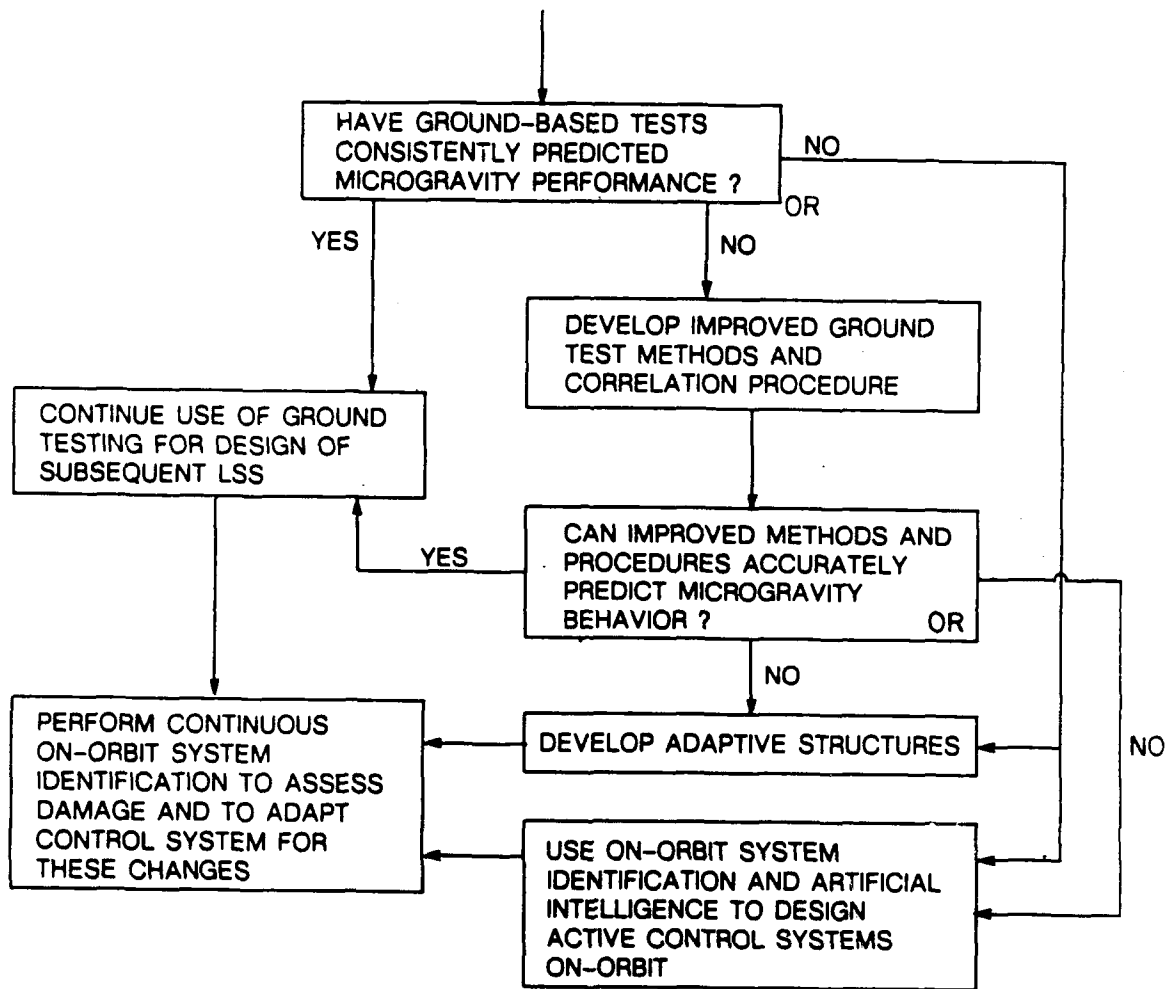












APPENDIX B: Computer Code

This computer code implements algorithms for calculating natural frequencies and mode shapes of a prismatic beam suspended on discrete springs.

USER INPUT

BEAM BENDING STIFFNESS, EI = 2250.0000
INTERIOR SPRING STIFFNESS, K = 59.9455
EXTERIOR SPRING STIFFNESS, KE = 29.9727
BEAM OVERALL LENGTH = 6.0000
BEAM MASS PER UNIT LENGTH, M = 1.6810
INTERIOR SPRING MASS, MU = 0.0000
EXTERIOR SPRING MASS, MUE = 0.0000
NUMBER OF BEAM SEGMENTS, N = 11

THIS PROGRAM WILL CALCULATE THE NATURAL FREQUENCIES AND MODE SHAPES FOR A PRISMATIC BEAM SUSPENDED ON A SERIES OF SPRINGS

*** USER IS RESPONSIBLE FOR USING CONSISTENT UNITS ***

NATURAL FREQUENCIES AND CORRESPONDING MODE SHAPES FOLLOW:

MODE NUMBER 1 NATURAL FREQUENCY (OMEGA) = 0.80855086E+01

LOCATION - X	NORMALIZED DEFLECTION
0.0000	0.99465
0.5455	0.99643
1.0909	0.99786
1.6364	0.99893
2.1818	0.99964
2.7273	1.00000

MODE NUMBER 2 NATURAL FREQUENCY (OMEGA) = 0.81519344E+01

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	0.81892
1.0909	0.63745
1.6364	0.45562
2.1818	0.27350
2.7273	0.09119

MODE NUMBER 3 NATURAL FREQUENCY (OMEGA) = 0.24201191E+02

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	0.57953
1.0909	0.17500
1.6364	-0.18013
2.1818	-0.44608
2.7273	-0.58878

MODE NUMBER 4 NATURAL FREQUENCY (OMEGA) = 0.63239836E+02

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	0.29534
1.0909	-0.30424
1.6364	-0.63365
2.1818	-0.59516
2.7273	-0.23991

MODE NUMBER 5 NATURAL FREQUENCY (OMEGA) = 0.12316729E+03

LOCATION - X	NORMALIZED DEFLECTION
0.0000	-1.00000
0.5455	0.03411
1.0909	-0.59442
1.6364	-0.54092
2.1818	0.05990
2.7273	0.62497

MODE NUMBER 6 NATURAL FREQUENCY (OMEGA) = 0.20329529E+03

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	-0.20033
1.0909	-0.65260
1.6364	-0.04002
2.1818	0.66521
2.7273	0.42427

MODE NUMBER 7 NATURAL FREQUENCY (OMEGA) = 0.30353864E+03

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	-0.39588
1.0909	-0.47842
1.6364	0.50435
2.1818	0.50091
2.7273	-0.49968

MODE NUMBER 8 NATURAL FREQUENCY (OMEGA) = 0.42386620E+03

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	-0.54238
1.0909	-0.13818
1.6364	0.70714
2.1818	-0.24674
2.7273	-0.56597

MODE NUMBER 9 NATURAL FREQUENCY (OMEGA) = 0.56426645E+03

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	-0.63216
1.0909	0.25391
1.6364	0.42457
2.1818	-0.70514
2.7273	0.33885

MODE NUMBER 10 NATURAL FREQUENCY (OMEGA) = 0.72473413E+03

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	-0.66113
1.0909	0.56988
1.6364	-0.14991
2.1818	-0.33885
2.7273	0.66247

MODE NUMBER 11 NATURAL FREQUENCY (OMEGA) = 0.90526744E+03

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	-0.62933
1.0909	0.70744
1.6364	-0.62039
2.1818	0.42371
2.7273	-0.15029

MODE NUMBER 12 NATURAL FREQUENCY (OMEGA) = 0.11058894E+04

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	-0.54111
1.0909	0.62178
1.6364	-0.66236
2.1818	0.69083
2.7273	-0.70520

MODE NUMBER 13 NATURAL FREQUENCY (OMEGA) = 0.13265046E+04

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	-0.40502
1.0909	0.33957
1.6364	-0.24706
2.1818	0.15028
2.7273	-0.05036

MODE NUMBER 14 NATURAL FREQUENCY (OMEGA) = 0.15672226E+04

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	-0.23304
1.0909	-0.05004
1.6364	0.33889
2.1818	-0.56607
2.7273	0.69122

MODE NUMBER 15 NATURAL FREQUENCY (OMEGA) = 0.18281550E+04

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	-0.03976
1.0909	-0.42370
1.6364	0.69102
2.1818	-0.62039
2.7273	0.24704

MODE NUMBER 16 NATURAL FREQUENCY (OMEGA) = 0.21080365E+04

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	0.15771
1.0909	-0.66200
1.6364	0.56708
2.1818	0.04816
2.7273	-0.61935

MODE NUMBER 17 NATURAL FREQUENCY (OMEGA) = 0.24231541E+04

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	0.35238
1.0909	-0.68697
1.6364	0.02440
2.1818	0.67401
2.7273	-0.38433

MODE NUMBER 18 NATURAL FREQUENCY (OMEGA) = 0.26231550E+04

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	0.45593
1.0909	-0.58438
1.6364	-0.34160
2.1818	0.64826
2.7273	0.20768

MODE NUMBER 19 NATURAL FREQUENCY (OMEGA) = 0.27881159E+04

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	0.52829
1.0909	-0.44834
1.6364	-0.56819
2.1818	0.39307
2.7273	0.62464

MODE NUMBER 20 NATURAL FREQUENCY (OMEGA) = 0.28081159E+04

LOCATION - X	NORMALIZED DEFLECTION
0.0000	1.00000
0.5455	0.53623
1.0909	-0.42944
1.6364	-0.58896
2.1818	0.35169
2.7273	0.68520

APPENDIX C: Computer Output

This appendix shows the computer output for a prismatic beam supported by six discrete springs.

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C      * * * * *
C      * THIS PROGRAM WILL CALCULATE THE NATURAL FREQUENCIES OF *
C      * A PRISMATIC BEAM SUSPENDED ON A SERIES OF DISCRETE *
C      * SPRINGS. BY R.J. DORNSIFE, JUNE 1988. *
C      * * * * *
C      * 27 JUNE 1988 - ANTISYMMETRIC AND SYMMETRIC CASES *
C      * * * * *
C      *
C      * EI = BENDING STIFFNESS OF BEAM = PRODUCT OF ELASTIC *
C      * MODULUS AND MOMENT OF INERTIA ABOUT BENDING AXIS *
C      *
C      * K = STIFFNESS OF EACH INTERIOR SPRING *
C      *
C      * KE = STIFFNESS OF EACH EXTERIOR SPRING *
C      *
C      * L = LENGTH OF BEAM *
C      *
C      * M = BEAM MASS PER UNIT LENGTH *
C      *
C      * N = NUMBER OF SEGMENTS INTO WHICH BEAM WILL BE DIVIDED. *
C      * THIS WILL RESULT IN (N-1) INTERIOR SPRINGS AND 2 *
C      * EXTERIOR SPRINGS. N MUST BE AN ODD NUMBER FOR *
C      * THIS FORMULATION. *
C      *
C      * NSM = USER FLAG FOR READING IN SPRING MASSES. IF GREATER *
C      * THAN ZERO, THE SPRING MASSES WILL BE INCLUDED WITH *
C      * THE BEAM MASS IN THE ANALYSIS. *
C      *
C      * MU = MASS OF EACH INTERIOR SPRING *
C      *
C      * MUE = MASS OF EACH EXTERIOR SPRING *
C      *
C      * *** USER IS RESPONSIBLE FOR USING CONSISTENT UNITS *** *
C      * * * * *
C
C      PROGRAM BEAM4
C      REAL *6 EI,K,KE,L,M,MU,MUE,OMEGA,OMEGA1,OMEGA2,
C      1 DETERM,DETERM1,DETERM2
C      READ(5,*) EI,K,KE,L,M,N,NSM
C      MU=0.0
C      MUE=0.0
C      IF(NSM .GT. 0) THEN
C        READ(5,*) MU,MUE
C      ENDIF
C      WRITE(6,701)
C      WRITE (6,702)EI,K,KE,L,M,MU,MUE,N
C      WRITE(6,703)
C      WRITE(6,704)
C      ODDCHK=(-1)**N
C      IF (ODDCHK .GT. 0) THEN
C        WRITE(6,705)
C        STOP
C      ENDIF
C      WRITE(6,706)
C      NROOTS=1
C      NFLAG=1
C      OMEGA=0.10
C
C      103 CALL EVAL(OMEGA,DETERM,EI,K,KE,L,M,MU,MUE,N,NFLAG)
C      OMEGA1=OMEGA
C      DETERM1=DETERM
C
C

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106 IF(NROOTS .GT. 2) THEN
    OMEGA=OMEGA+.5
ELSE
    OMEGA=OMEGA+.005
ENDIF
CALL EVAL(OMEGA,DETERM,EI,K,KE,L,M,MU,MUE,N,NFLAG)
OMEGA2=OMEGA
DETERM2=DETERM
C
IF (DETERM1 .LE. 0. .AND. DETERM2 .GE. 0.) GO TO 109
IF (DETERM1 .GE. 0. .AND. DETERM2 .LE. 0.) GO TO 109
GO TO 112
109 CONTINUE
CALL ZERO(OMEGA1,OMEGA2,EI,K,KE,L,M,MU,MUE,N,NROOTS,NFLAG)
NROOTS=NROOTS+1
NFLAG=-NFLAG
IF (NROOTS .GT. 30) STOP
GO TO 103
C
112 CONTINUE
OMEGA1=OMEGA
DETERM1=DETERM2
GO TO 106
C
701 FORMAT(///,3X,'USER INPUT')
702 FORMAT(///,25X,'BEAM BENDING STIFFNESS, EI = ',3X,F10.4,/,/,
1 25X,'INTERIOR SPRING STIFFNESS, K = ',1X,F10.4,/,/,25X,
2 'EXTERIOR SPRING STIFFNESS, KE = ',F10.4,/,/,25X,
3 'BEAM OVERALL LENGTH = ',10X,F10.4,/,/,25X,
4 'BEAM MASS PER UNIT LENGTH, M = ',1X,F10.4,/,/,25X,
5 'INTERIOR SPRING MASS, MU = ',5X,F10.4,/,/,25X,
6 'EXTERIOR SPRING MASS, MUE = ',4X,F10.4,/,/,25X,
7 'NUMBER OF BEAM SEGMENTS, N = ',9X,14)
703 FORMAT(////////,15X,'THIS PROGRAM WILL CALCULATE THE ',
1 'NATURAL FREQUENCIES AND MODE',/,15X,'SHAPES FOR A ',
2 'PRISMATIC BEAM SUSPENDED ON A SERIES OF SPRINGS')
704 FORMAT(///,18X,'*** USER IS RESPONSIBLE FOR USING ',
1 'CONSISTENT UNITS ***')
705 FORMAT (////////,20X, 'N MUST BE AN ODD NUMBER - EXECUTION ',
1 'TERMINATED')
706 FORMAT(////////,16X,'NATURAL FREQUENCIES AND CORRESPONDING ',
1 'MODE SHAPES FOLLOW:',/,',1')
C
END
C
C
SUBROUTINE EVAL(OMEGA,DETERM,EI,K,KE,L,M,MU,MUE,N,NFLAG)
COMMON /EIGEN/P,PHI1,PHI2,PHI3,PHI4,AA,BB,BETA,B3,KNEWSTR
REAL *6 P(100,2),C(100,3),PHI1(0:100),PHI2(0:100),BETA,
1 PHI3(0:100),PHI4(0:100),EI,K,KE,L,M,MU,MUE,OMEGA,DETERM,
2 AA,BB,B3,ALPHA,ARG,CC,DD,EE,FF,APRODUCT1,APRODUCT2,KNEW,
3 KNEWSTR
C
THIS SUBROUTINE CALCULATES THE DETERMINANT ASSOCIATED
WITH A SPECIFIED NATURAL FREQUENCY, OMEGA.
C
BETA=(OMEGA**2.*M/EI)**.25
B3=BETA**3.
N=L/REAL(N)
ALPHA=BETA*M
KNEW=K-MU/3.*OMEGA**2.
KNEWSTR=KE-MUE/3.*OMEGA**2.
DO 203 I=0,(N-1)/2
    ARG=REAL(I)*ALPHA
    PHI1(I)=.5*( COS(ARG)+COSH(ARG))
    PHI2(I)=.5*( SIN(ARG)+SINH(ARG))
    PHI3(I)=.5*(-COS(ARG)+COSH(ARG))
    PHI4(I)=.5*(-SIN(ARG)+SINH(ARG))
203 CONTINUE
C
DO 206 I=51,51+(N-1)/2
    ARG=(REAL(I-51)+.5)*ALPHA
    PHI1(I)=.5*( COS(ARG)+COSH(ARG))

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      PHI2(1)=-.5*( SIN(ARG)+SINH(ARG))
      PHI3(1)=-.5*(-COS(ARG)+COSH(ARG))
      PHI4(1)=-.5*(-SIN(ARG)+SINH(ARG))
206 CONTINUE
C
C
      C(1,1)=-KNEW*(PHI1(1)-KNEWSTR/B3/EI*PHI4(1))
      C(1,2)=-KNEW*PHI2(1)/BETA
      C(1,3)=0.
      P(1,1)=C(1,1)
      P(1,2)=C(1,2)
C
      IF(N .LT. 5) GO TO 218
C
      DO 209 I=2,(N-1)/2
        C(1,1)=-KNEW*(PHI1(I)-KNEWSTR/B3/EI*PHI4(I))
        C(1,2)=-KNEW*PHI2(I)/BETA
        C(1,3)=-KNEW/EI/B3*PHI4(I-1)
209 CONTINUE
C
      DO 212 I1=2,(N-1)/2
        P(1,1)=C(1,1)
        P(1,2)=C(1,2)
        DO 215 I2=1,I1-1
          P(1,1)=P(1,1)+P(1,2)*C(1,2+1,3)
          P(1,2)=P(1,2)+P(1,2)*C(1,2+1,3)
215 CONTINUE
212 CONTINUE
C
218 IF(NFLAG .GT. 0) GO TO 224
C
C      ANTISYMMETRIC SOLUTION
      AA=PHI1(51+(N-1)/2)-KNEWSTR/B3/EI*PHI4(51+(N-1)/2)
      BB=1./BETA*PHI2(51+(N-1)/2)
      CC=PHI3(51+(N-1)/2)-KNEWSTR/B3/EI*PHI2(51+(N-1)/2)
      DD=1./BETA*PHI4(51+(N-1)/2)
C
      DO 221 I=1,(N-1)/2
        EE=1./B3/EI*PHI4(51+(N-1)/2-I)
        AA=AA+P(I,1)*EE
        BB=BB+P(I,2)*EE
        FF=1./B3/EI*PHI2(51+(N-1)/2-I)
        CC=CC+P(I,1)*FF
        DD=DD+P(I,2)*FF
221 CONTINUE
      GO TO 230
C
C      SYMMETRIC SOLUTION
224 AA=PHI4(51+(N-1)/2)-KNEWSTR/B3/EI*PHI3(51+(N-1)/2)
      BB=1./BETA*PHI1(51+(N-1)/2)
      CC=PHI2(51+(N-1)/2)-KNEWSTR/B3/EI*PHI1(51+(N-1)/2)
      DD=1./BETA*PHI3(51+(N-1)/2)
C
      DO 227 I=1,(N-1)/2
        EE=1./B3/EI*PHI3(51+(N-1)/2-I)
        AA=AA+P(I,1)*EE
        BB=BB+P(I,2)*EE
        FF=1./B3/EI*PHI1(51+(N-1)/2-I)
        CC=CC+P(I,1)*FF
        DD=DD+P(I,2)*FF
227 CONTINUE
C
C      CALCULATE DETERMINANT
230 APRODUCT1=AA*DD
      APRODUCT2=BB*CC
      DETERM=APRODUCT1-APRODUCT2
      RETURN
      END
      SUBROUTINE ZERO(X0,X1,EI,K,KE,L,M,MU,MUE,N,NROOTS,NFLAG)
      REAL *6 X0,X1,X2,F0,F1,F2,EPS1,EPS2,EI,K,KE,L,M,MU,MUE
C
C      THIS SUBROUTINE USES THE INTERVAL HALVING METHOD TO
C      SOLVE FOR THE ZEROS OF A FUNCTION.
C

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EPS1=.000000001
NM = 0
CALL EVAL(X0,F0,E1,K,KE,L,M,MU,MUE,N,NFLAG)
CALL EVAL(X1,F1,E1,K,KE,L,M,MU,MUE,N,NFLAG)
303 CONTINUE
IF(ABS(X0-X1) .LT. ABS(EPS1*X1)) GO TO 315
X2=.5*(X0+X1)
CALL EVAL(X2,F2,E1,K,KE,L,M,MU,MUE,N,NFLAG)
IF(F2 .LT. 0. .AND. F0 .GT. 0.) GO TO 306
IF(F2 .GT. 0. .AND. F0 .LT. 0.) GO TO 306
GO TO 309
306 X1=X2
F1=F2
GO TO 312
309 X0=X2
F0=F2
312 CONTINUE
NM=NM+1
IF (NM .GT. 500) THEN
  WRITE(6,901)
  STOP
ENDIF
GO TO 303
C
315 CONTINUE
WRITE (6,902) NROOTS,X0
CALL MSHAPE(E1,L,N)
RETURN
C
901 FORMAT(1X,'STOP - TOO MANY ITERATIONS')
902 FORMAT(/////4X,'MODE NUMBER',14,8X,
1 'NATURAL FREQUENCY (OMEGA) = ',E20.8)
C
END
C
C
SUBROUTINE MSHAPE(E1,L,N)
COMMON /EIGEN/P,PH11,PH12,PH13,PH14,AA,BB,BETA,B3,KNEWSTR
REAL *6 P(100,2),PH11(0:100),PH12(0:100),PH13(0:100),
1 PH14(0:100),W(0:100),E1,L,AA,BB,WMAX,FACT,X,KNEWSTR,
2 BETA,B3
C
C THIS SUBROUTINE DETERMINES THE MODE SHAPE ASSOCIATED
C WITH A PARTICULAR NATURAL FREQUENCY BY CALCULATING
C THE RELATIVE DEFLECTION MAGNITUDE AT EACH SPRING.
C
WMAX=0.
DO 403 I=0,(N-1)/2
  W(I)=PH11(I)*KNEWSTR/B3/E1*PH14(I)-AA/B3*PH12(I)/BETA
  IF(I .LT. 2) GO TO 409
  DO 406 II=1,I-1
    FACT=1./B3/E1*PH14(I-II)
    W(I)=W(I)+FACT*(P(II,1)-P(II,2)*AA/B3)
  406 CONTINUE
  409 IF(ABS(W(I)) .GT. WMAX) WMAX=ABS(W(I))
  403 CONTINUE
C
C NORMALIZE ALL DEFLECTIONS SUCH THAT MAXIMUM SPRING
C DEFLECTION = 1.0.
C
WRITE (6,905)
DO 412 I=0,(N-1)/2
  X=REAL(I)*L/REAL(N)
  W(I)=W(I)/WMAX
  WRITE (6,906) X,W(I)
412 CONTINUE
RETURN
C
905 FORMAT(//,10X,'LOCATION - X',10X,'NORMALIZED DEFLECTION',/)
906 FORMAT(12X,F8.4,21X,F8.5)
C
END
C

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