AD-A248 583

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

(2)

Public reporting burden ... 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering ... collection of information. Send comments regarding this burden estimate or any other aspect of this collection ... Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway ... Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE 2/27/92	3. REPORT TYPE AND DATES COVERED Final Technical 1 Aug 88 - 30 Sep 91

4. TITLE AND SUBTITLE
Mathematical & Numerical Aspects of Quasi-Neural Networks (U)

5. FUNDING NUMBERS
61102F
2304/K8

6. AUTHOR(S)
Professor Edward K. Blum

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Mathematics Department
University of Southern California
Los Angeles, CA 90089-1113

8. PERFORMING ORGANIZATION REPORT NUMBER
AFOSR-TR- 92 0226

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NM, Building 410
Bolling Air Force Base
Wahington, DC 20332-6448

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AFOSR-88-0245

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release
Distribution unlimited

12b. DISTRIBUTION CODE
UL

13. ABSTRACT *(Maximum 200 words)*

Basic results were obtained and published on the mathematical characterization of feed-forward and recurrent neural networks. It was shown how to approximate any continuous function by a 3-layer feed-forward network. The fixed-points and oscillations of recurrent networks were analyzed.

DTIC
ELECTE
APR 16 1992
S D D

92-09766

92 4 15 114

14. SUBJECT TERMS
Neural networks, dynamical systems

15. NUMBER OF PAGES
4

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	SAR UL

GENERAL INSTRUCTIONS FOR COMPLETING SF 298

The Report Documentation Page (RDP) is used in announcing and cataloging reports. It is important that this information be consistent with the rest of the report, particularly the cover and title page. Instructions for filling in each block of the form follow. It is important to ***stay within the lines*** to meet ***optical scanning requirements***.

Block 1. Agency Use Only *(Leave blank)*.

Block 2. Report Date. Full publication date including day, month, and year, if available (e.g. 1 Jan 88). Must cite at least the year.

Block 3. Type of Report and Dates Covered. State whether report is interim, final, etc. If applicable, enter inclusive report dates (e.g. 10 Jun 87 - 30 Jun 88).

Block 4. Title and Subtitle. A title is taken from the part of the report that provides the most meaningful and complete information. When a report is prepared in more than one volume, repeat the primary title, add volume number, and include subtitle for the specific volume. On classified documents enter the title classification in parentheses.

Block 5. Funding Numbers. To include contract and grant numbers; may include program element number(s), project number(s), task number(s), and work unit number(s). Use the following labels:

C	Contract	PR	Project
G	Grant	TA	Task
PE	Program Element	WU	Work Unit Accession No.

Block 6. Author(s). Name(s) of person(s) responsible for writing the report, performing the research, or credited with the content of the report. If editor or compiler, this should follow the name(s).

Block 7. Performing Organization Name(s) and Address(es). Self-explanatory.

Block 8. Performing Organization Report Number. Enter the unique alphanumeric report number(s) assigned by the organization performing the report.

Block 9. Sponsoring/Monitoring Agency Name(s) and Address(es). Self-explanatory.

Block 10. Sponsoring/Monitoring Agency Report Number. *(If known)*

Block 11. Supplementary Notes. Enter information not included elsewhere such as: Prepared in cooperation with...; Trans. of...; To be published in.... When a report is revised, include a statement whether the new report supersedes or supplements the older report.

Block 12a. Distribution/Availability Statement. Denotes public availability or limitations. Cite any availability to the public. Enter additional limitations or special markings in all capitals (e.g. NOFORN, REL, ITAR).

- DOD - See DoDD 5230.24, "Distribution Statements on Technical Documents."
- DOE - See authorities.
- NASA - See Handbook NHB 2200.2.
- NTIS - Leave blank.

Block 12b. Distribution Code.

- DOD - Leave blank.
- DOE - Enter DOE distribution categories from the Standard Distribution for Unclassified Scientific and Technical Reports.
- NASA - Leave blank.
- NTIS - Leave blank.

Block 13. Abstract. Include a brief *(Maximum 200 words)* factual summary of the most significant information contained in the report.

Block 14. Subject Terms. Keywords or phrases identifying major subjects in the report.

Block 15. Number of Pages. Enter the total number of pages.

Block 16. Price Code. Enter appropriate price code *(NTIS only)*.

Blocks 17. - 19. Security Classifications. Self-explanatory. Enter U.S. Security Classification in accordance with U.S. Security Regulations (i.e., UNCLASSIFIED). If form contains classified information, stamp classification on the top and bottom of the page.

Block 20. Limitation of Abstract. This block must be completed to assign a limitation to the abstract. Enter either UL (unlimited) or SAR (same as report). An entry in this block is necessary if the abstract is to be limited. If blank, the abstract is assumed to be unlimited.

Final Progress Report on Research Supported by Grant AFOSR-88-0245.

Principal Investigator: Professor Edward K. Blum, Mathematics Department, University of Southern California, Los Angeles, CA 90089-1113.

Title of Project: Mathematical and Numerical Analysis Aspects of Quasi-Neural Networks.

Significant results were obtained on feed-forward and recurrent networks. Five technical papers were accepted for publication in leading journals.

1. Feed-forward networks. The P.I. and his Ph.D. student, Leong Li, obtained a basic result on feed-forward networks. This is described in the paper "Approximation theory and feed-forward networks", which has been published in the journal, *Neural Networks* (1991) 4, 511-515. The basic result is that feed-forward networks of McCulloch-Pitts neurons yield arbitrarily close piecewise-constant approximations to continuous and L_2-functions. The fundamental idea of piecewise-constant approximation was not recognized by previous researchers. By using this idea, Blum and Li were able to give a simple technique for constructing 3-layer networks to uniformly approximate arbitrary real continuous functions of n variables and also arbitrary L_2-functions in the mean square norm with arbitrary accuracy. This technique should be useful in many of the applications of feed-forward networks. Li has also published a second paper in this topic.

2. Dynamical behavior of recurrent neural networks. Recurrent neural networks, that is, networks with feedback loops, exhibit all the phenomena found in classical dynamical systems governed by systems of ordinary differential equations (ODE's). In fact, if the neuron activities are modeled by ODE's, then the network is modeled as a system of coupled ODE's. However, it is also of

Codes
/or
Special
A-1

interest to model the neurons as analog elements which have states (outputs) in the real interval $I=[0,1]$ governed by an input-output function of sigmoidal type and operating in discrete-time. If the operation is synchronous, then the network is modeled as a system of difference equations rather than differential equations. Thus, the dynamics is that of interation of a map $F: I^m \rightarrow I^m$, according to the general equation $x(t+1)=F(x(t))$, $t=0,1,2,\ldots$, where $x(t)$ is the state vector at time t. In many application (e.g., associative memories, oscillators of various kinds, etc.), F has the form $F(x)=\sigma(Wx+L)$, where W is a real "weight" matrix, L is the "threshold" vector and sigma is a sigmoidal-type nonlinear vector function. In these applications, one is interested in the existence and stability of fixed-points and periodic orbits in phase space (i.e. oscillations). There is a natural parameter in such networks, namely, the "gains" of the neurons, as represented by the maximum slopes, a_i, of the sigmoidal functions. One is also interested in the changes in the dynamics induced by variations in the gains, in particular, in the occurrence of bifurcation points at which stable equilibria become unstable or at which stable oscillations appear and disappear.

The P.I. and his Ph.D. student, Xin Wang, have studied the dynamics of such networks and obtained a number of new results which should be useful in applications. These are described in detail in the paper "Stability of Fixed Points and Periodic Orbits and Bifurcations in Analog Neural Networks", accepted for publication in the journal, *Neural Networks.*

The case of symmetric W has received attention in the recent literature. We consider the simplest instance of such a network having only 2 neurons coupled to operate like an analog multivibrator circuit. We show that the dynamics of fixed points and periodic orbits (oscillations) can be completely characterized by elementary analysis without introducing a Liapunov function, the usual tool. This analysis can be extended to special networks built from these 2-neuron modules. The same methods of analysis are then applied to non-symmetric W. For example, it is shown how to construct ring oscillators of any period. The case of coupled oscillators and forced oscillators is also subjected to the same analysis. By varying the gains of the sigmoidal neurons we obtain interesting bifurcations into oscillatory behavior.

3. Chaotic dynamics in neural networks. It is known that neural networks are capable of chaotic behavior. New results are reported in the paper, "Period-doublings to Chaos in a Simple Neural Network: A Mathematical Proof" by Xin Wang (Ph.D. student of the P.I.). Usually, chaotic dynamics is demonstrated by simulation. In this part of the project, Wang was able to prove that chaos occurs in a 2-neuron network with self-loops. Thus, this simple network differs from the 2-neuron network studied by Blum and Wang described above. However, the dynamics is again discrete-time iteration of a nonlinear sigmoidal map. By restricting the weight matrix, W to be singular, it is possible to reduce the dynamics to iteration of a family of 1-dimensional maps parametrized by the gain, μ of the neurons. Appealing to the known theory of 1-dimensional maps, Wang is able to prove that the 2-neuron networks with singular W follow the period-doubling route to chaos as μ increases. To illustrate the theoretical result, Wang computes the bifurcation diagrams for two networks with different W 's and demonstrates the occurrence of chaos as μ increases.

4. Discrete-time versus continuous-time dynamics. Blum and Wang compare the dynamical properties of continuous-time neural networks defined by systems of differential equations of the leaky integrator type with the dynamical properties of the approximating difference equations. Quantitative results in the step-size of the discrete-time numerical method to guarantee the same dynamics are obtained.

Publications:

1. Blum, E.K. (1989). Approximation of Boolean functions by sigmoidal networks, Neural Computation 1, 532-540.

2. Blum, E.K. (1990). Mathematical aspects of outer-product asynchronous content-addressable memories, Bio. Cyber. 62, 337-348.

3. Blum, E.K. and Li, L.K. (1991). Approximation theory and feed-forward networks, Neural Networks 4, 511-515.

4. Blum, E.K. and Wang, X. (1992). Stability of fixed-points and periodic orbits and bifurcations in analog neural networks, Neural Networks (In press).

5. Blum, E.K. and Wang, X. (1992). Discrete-time versus continuous-time models of neural networks, J. Computer & System Sciences, (In press).

6. Wang, X. (1992). Periodic-doublings to chaos in a simple neural network: a mathematical proof, Complex Systems.