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A SUMMER PROGRAM IN MATHEMATICS
AND
COMPUTER SCIENCE
FOR
ACADEMICALLY ORIENTED STUDENT

JUNE 24 - JULY 26, 1991

FUNDED BY
THE OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY

AT
UNIVERSITY OF THE DISTRICT OF COLUMBIA
WASHINGTON, D. C.

Final Report

of

A Summer Program in Mathematics and Computer Science

for

Academically Oriented Students

June 24 - July 26, 1991

Funded by

The Office of Naval Research

Department of the Navy

Report by

Bernis Barnes

University of the District of Columbia

Washington, D. C

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INTRODUCTION

The 1991 summer intervention program in mathematics and computer science for academic oriented students was one of our most successful. The five-week program provided intensive and rigorous study for thirty-six mainly ninth and tenth grade students from the D. C. area. As in the previous nine years of the program, addressing the problems of the under-representation of minorities, especially Blacks and Hispanics in engineering, natural science, and other mathematics-based fields, was given a high priority. During the summer, the students most of whom are from the under represented groups were encouraged to strengthen their background in mathematics and to pursue careers in mathematics-based fields. They were also exposed to career opportunities in mathematics-based fields, and to how they should prepare themselves in high school to increase their career options by the time they reach college. The program faculty which has years of experience in teaching local minority students provided the encouragement and motivation, as well as, a carrying and supportive environment.

PURPOSE AND GOALS

The purpose of this project is to implement an intensive pre-college intervention program for academically talented students, mainly ninth and tenth grade students from the District of Columbia area, that is designed to increase their representation in mathematics-based careers. By offering the program at this grade level, the students are able to take more rigorous math courses while in high school.

The goals of the project are (1) to strengthen the students' backgrounds in mathematics, computer science, and statistics and operations research, (2) to improve their academic skills with an emphasis on reasoning competencies, (3) to increase their awareness of careers in mathematics-based fields and of the preparation needed for these fields, and (4) to encourage and motivate these students to enroll in calculus-track courses while in high school.

The five basic competencies in the area of reasoning are (1) the ability to identify and formulate problems, as well as, the ability to propose and evaluate ways to solve them; (2) the ability to recognize and use inductive and deductive reasoning, and to recognize fallacies in arguments; (3) the ability to draw reasonable conclusions from information found in various sources whether written, spoken, tabular, or graphic, and to defend one's conclusions rationally; (4) the ability to comprehend, develop and use concepts and generalizations; (5) the ability to distinguish between fact and opinion.

PROGRAM PERSONNEL

The program staff consisted of long-time members of the faculty of the University. They therefore have had considerable experience in working with local students---encouraging and motivating them, as well as, providing the needed carrying and supportive environment. Members of the staff were Professors Bernis Barnes and Reuben Drake of the Mathematics Department who taught the General Mathematics classes and coordinated the career education component, Professor William Rice of the Mathematics Department who taught the Statistics and Operations Research classes, and Professor Gail Finley of the Computer science Department who taught the Computer Science classes. Professor Barnes also served as director of the program.

PARTICIPANTS

The participants were selected from a pool of applicants most of whom were recommended by their current mathematics teacher. Three students were recommended by the director of the pre-engineering program of the D. C. public schools. The students were rated on sixteen items of a questionnaire which addressed attitude, achievement, interest, abstract reasoning, study skills, etc. (The complete set of items appear in Appendix B.)

Using the information provided, forty students were selected. Of the six students who did not show up, three were replaced and three were dropped after they fail to show up the first three days of the program. Another student dropped out later in the summer. Of the thirty-six remaining students seventeen were female and nineteen were males. Eleven of the students were from private or parochial schools and twenty-five were from public schools; thirteen had recently completed the eighth grade, twenty-one had recently completed the ninth grade, and two had recently completed the tenth grade; Thirty-one participants were Black, two were white, and two were Hispanic, and one Asian. The ages of all but three of the students were fourteen and fifteen. The remaining three students were sixteen.

PROGRAM COMPONENTS

There were two major program components: (1) the academic component which consisted of three courses---General Mathematics, Computer Science, and Statistics and Operations Research---which were designed to improve the students' backgrounds in these fields while enhancing their reasoning skills; and (2) the career education component which was designed to provide participants an opportunity to see "mathematics at work" in every day life through films, videos, etc. and to visit the workplace of and to interact directly with professional in mathematics-based fields through scheduled field trips and a forum.

1. Academic Component: The academic component consisted of the three courses. Both the computer science and statistics courses met for one hour and fifteen minutes each morning, and the mathematics course met for one and one-half hours each afternoon, except on Fridays. The computer classes were held in a computer laboratory (a classroom with eighteen IBM PC's with graphic capability and several terminals) three days a week, and the statistics classes were held in the laboratory two days a week.

The goals,topics and some exercises for the courses follow. (Sample curriculum materials are in Appendix C.)

General Mathematics

This course was basically the same as it has been the pass nine years. Finite mathematical systems were used to introduce advanced topics in mathematics. For example, in abstract algebra, the students constructed operation tables for the transformations ("rigid motions") of 3 x 3 magic squares and of equilateral triangles. They also determined the effect of each group property on the rows and columns of finite operation tables. In topology, the students counted the number of topologies on a given finite set. They also identified cluster and interior points, as well as, the closure and boundary of a set for a given finite set.

Also, each student was required to complete a calendar of twenty-five problems, one problem for each day in the program. The solutions to these problems required knowledge of concepts and procedures used in general mathematics, algebra and geometry.

In addition to increasing the students' knowledge, understanding and skills of the topics offered, the goals of the course were to improve their

- skills in recognizing patterns and drawing conclusions,
- facility with the technical language of mathematics,
- knowledge of the structural nature of mathematics, and
- skills in techniques of formulating and solving problems.

Most of the topics taught in the course were not topics that are usually taught to students at this level. But, in general, the students performed well and showed significant interest in the subject matter.

Course Content

- o Recognizing Patterns
- o Deductive and Inductive Reasoning
- o Functions Defined by Sets of Ordered Pairs and by Rules
- o Properties of Groupoids and Groups on Finite Sets
- o Cyclic Groups and Generators, Cosets, Normal Groups, and Factor Groups
- o Endomorphisms on Finite Groups, and the Operation Tables of Endomorphisms
- o Topologies on Finite Sets, the Cluster Points, Interior Points, Boundary Points, and Closure
- o Continuous Functions on Finite Sets

Some of the classroom and homework exercises used in the mathematics course are as follows:

o Given worksheets with a variety of patterns and logic exercises, the students identified the patterns and derived logical conclusions. Also, the students were required to recognize patterns and derive logical conclusions throughout the course.

o Given two finite sets A and B, the students identified subsets of the Cartesian product which were
(a) functions of A into B,
(b) functions of A onto B,
(c) functions of A into B which were 1 to 1,
(d) relations that were not functions, and
(e) constant functions.

o Given collections of ordered pairs that defined functions, the students
(a) identified the domain of the function,
(b) identified the image of the function,
(c) gave the inverse image of the function, and
(d) stated whether or not the inverse of the function was itself a function and justified their answers.

o Given a table of three integral values of a linear function, the students
(a) wrote corresponding equation of the function,
(b) gave the slope of the line,
(c) sketched the graph of the function, and
(d) gave the x and y intercepts of the line.

o Given several graphs of relations, the students identified those graphs which represented functions.

o Given information regarding the slope of a line, such as positive slope, negative slope, slope of '0', slope does not exist, the students gave the direction of the corresponding line.

o Given a function $f:X \rightarrow Y$ where X and Y are finite sets, the students listed the elements of $f[X]$ and of its inverse image $f^{-1}[Y]$, and showed that $f[A \cup B] = f[A] \cup f[B]$ and that $f^{-1}[A \cup B] = f^{-1}[A] \cup f^{-1}[B]$.

o Given a 3 by 3 array of squares, the students listed all the 3 by 3 magic squares, established relationships between the magic squares, and constructed an operation table for the composition of the transformations of the magic squares.

- o Given the definition of Roup (a group without the associative property), the students identified examples and non-examples in a set of exercises, and justified their answers.

- o Given a set with three elements, the students determined the number of groupoids which can be defined on the set, and constructed tables of operations which satisfied a given property or given properties.

- o Given the operation table for the composition of the symmetries of the square, the students determined subgroups, cyclic subgroups and their generators, left and right cosets, normal subgroups, and factor groups.

- o Given a finite group with three or four elements, the students determined the number of group endomorphisms. Given two finite groups, the students determined which were homomorphic; and when they were, they determined the homomorphism.

- o Given the subsets of a three element set, the students identified each collection of subsets of the set which defined a topology on the set.
- o Given a topology on a three element set, the students determined the limit points, interior points, boundary points and the closure of each subset of the set.

- o Given a function $f:X \to X$ on a finite set X and a topology G on X, the students determined the points at which the function was continuous.

Method of Teaching

A variety of teaching strategies were used in the mathematics course. Although most of the sessions were student centered and discovery oriented, lecture demonstrations were given when appropriate. Worksheets were used to focus and guide the classroom sessions, and to provide homework assignments.

Computer Science

The primary computer language of this course has shifted from BASIC to Pascal. However, the course continues to use the fundamentals of programming to introduce the computer as a tool to aid in problem solving. Throughout the summer, each student had access to personal computers and computer terminals in a laboratory type hands-on experience. The equipment was used to test and run the programs the students wrote, and to run some computer packages.

The goals of the computer course were to prepare students

- o to be literate in the language and hardware of computer science,

- o to develop algorithms using flowcharts and pseudo code,

- o to understand more complex algorithm development using top-down structured design, and o to construct and debug programs in the BASIC and Pascal languages that employ control statements, string variables and arrays and that use data files, graphic techniques and subroutines.

Course Content

Unit I: Introduction

- o Orientation and tour of campus computing facilities
- o Terminology
- o Introduction to algorithm development
- o Lab: Personal computer (PC) operation; Graphic modules to introduce algorithm development

Unit II: The Pascal language and simple algorithms

- o Top-down design
- o Simple Pascal programs with input, output and assignment
- o Numeric and character string variables
- o Simple procedures without parameters
- o Lab: Implementation of simple Pascal programs

Unit III: Pascal decision and control structures

- o Boolean variables
- o Decisions

  IF...THEN

  IF...THEN...ELSE
- o Looping and iteration

  FOR...loops

  WHILE...loops
- o Accumulation
- o Lab: Programs requiring decisions and loops;

  Programs using simple procedures and graphics within decisions and loops

Unit IV: Top-down structured design and step wise refinement

- o Problem solving

o Procedures with parameters

o External files

o Lab: More complex problems with skills to date; Programs for games and graphics

Unit V: Arrays, tables and data structures

o Use of subscripted variables in single arrays

o Use of external data files

o Introduction to data structures

o Searching and sorting

o Lab: Assigned problems with array implementation; Modules demonstrating data structures; Modules demonstrating computer science concepts

Daily Activities: Lecture/discussion sessions T, Th; Lab sessions M,W,F

Programming Language Tool: The Pascal language is the one used in high schools for the advanced placement course in computer science and is also the most widely used language in college first-year courses. In addition, it is a language which supports the proper learning of structured program development.

Algorithm Development: Top-down structured design for algorithm development is widely accepted in educational circles as a tool to produce more readable and more error-free programs. The students will spend a considerable amount of time implementing design strategies with top-down structure and step-wise refinement. Procedure implementation and graphics will be the focus of their program development.

Method of Teachi ɡ

Lecture and lecture discussion methods reinforced by problem worksheets were used. Lab activities provided practical implementation of the concepts introduced in the lectures.

It is expected that course participants may have various levels of prior experience. To accommodate this situation, the more advanced student may move at his or her own pace through the earlier assignments to more complex problems and projects. It is not reasonable to expect a beginning student to do a great deal with arrays or to independently do many problems beyond that level while a more advanced student may begin with emphasis in that area and continue further. Some lectures and discussions continue during the lab period for advanced projects. The notion of top-down design and some control structures are expected to be relatively new for most participants therefore lecture sessions are the same for most participants.

Statistics and Operations Research

The S'atistics and Operations Research course continues to used quantitative techniques as tools in decision making. The course focused on analyzing, interpreting and utilizing data. Specifically, the students used statistics techniques to summarize data and to make inference based on that data. They also learned to construct models for research.

The goals of the Statistics and Operations Research course were

- o exploring techniques of assigning numbers that represent chances or probabilities of various events of interest,
- o exploring techniques of organizing, summarizing and displaying data to reveal its patterns and relationships, and
- o integrating the computer throughout the course, i.e. the students learned to transfer the skills developed in their computer course into problem solving tools in statistics.

Course Content:

Unit I: Exploring Probability

- o Experimenting with Chance
- o Knowing our Chance in Advance---Theoretical Probability
- o Complementary Events and Odds
- o Compound Events
  - Multiplying Probabilities
  - Adding Probabilities

Unit II: Exploring Data: An Introduction to Statistics

- o Line Plots
- o Stem-and-Leaf Plots
- o Median, Mean, Quartiles, and Outliers
- o Box Plots
- o Scatter Plots

o Lines on Scatter Plots

o Smoothing Plots Over Time

Some of the classroom and homework exercises used in the course are as follows:

o Line plots are quick, simple ways to organize data. From a line plot it is easy to spot the largest and smallest values, outliers, clusters, and gaps in the data. It is also possible to find the relative position of particular points of interest.

o Steam-and-leaf plots are used as a substitute for the less informaiive histograms and bar graphs. From a stem-and-leaf plot it is easy to identify the largest and smallest values, outliers, clusters, gaps, the relative position of any important value, and the shape of the distribution.

o The median and the mean are single numbers that summarize the location of the data. Neither alone can tell the whole story about the data, but sometimes we do want a single, concise, summary value. The lower quartile, median, and upper quartile divide the data into four parts with approximately the same number of observations in each part. The interquartile range, the third quartile minus the first quartile, is a ,measure of how spread out the data are.

o The box plots is a useful technique for focusing on the relative positions of different sets of data and thereby compare them more easily.

o Scatter plots are the best way to display data in which two numbers are given for each person or item. When analyzing a scatter plot, look for clusters of points, points that do not follow the general pattern, and positive, negative, or no association.

o The scatter plot is also the basic method for learning about relationships between two variables. This topic treats the cases where the interpretation becomes clearer by adding a straight line to the plot.

o Smoothing is a technique that can be used with time series data where the horizontal axis is marked off in years, days, hours, ages, and so forth. We can use medians to obtain smoothed values, and these smoothed values can remove much of the sawtooth effect often seen in time series data. As a result, a clearer picture of where values are increasing and decreasing emerges.

Method of Teaching

The teaching strategies were student centered. Exercise sheets from the texts were used to focus and guide the classwork and class discussions, and to provide homework assignments.

TEXTS: Newman, Claire M., Obremski, Thomas E., and Scheaffer, Richard L., Exploring Probability, Dale Seymour Publications, 1987.

Landwehr, James M., and Watkins, Ann E., Exploring Data, Dale Seymour Publications, 1986.

2. Career Education Component: This component provided the students an opportunity to increase their awareness of the many career opportunities that are available in mathematics-based fields, and of the preparation that is necessary to pursue these careers. One type of activity was the field trips to the Naval Research Laboratory in

Washington, D.C. and the David Taylor Naval Ship Research and Development center in Carderock, Maryland where the students visited the workplace of scientists, and interacted with some of them. At David Taylor, the students visited test facilities such as the mile long tank and the wave generating circular tanks; while at the Naval Research Laboratory, Dr. George Carruthers gave a lecture on exploring outer space.

In addition to the tours, students viewed video tapes and a film, and discussed career opportunities in mathematics-based fields. The students viewed the video tapes: "The Challenge of the Unknown," developed through a grant to the American Association of the Advancement of Science, a motivational video by the National Urban Coalition which features successful blacks such as Frederick Gregory and Eleanor Holmes Norton, and the video by George Polya, "How to Solve It." The students also viewed the film "Donald in Mathemagic Land" which shows how mathematics grows from simple ideas.

This component also includes a career awareness forum which is a highlight of the program. At the forum, each of the panelists discussed the mathematics preparation necessary for his/her present position. The panelists were Dr. Elvira Doman in biochemistry, Miss Judith Richardson in computer science, and Dr. John Alexander in mathematics.

PROGRAM EVALUATION

As with any academic program, some students' assessment were favorable while other were not as favorable. The majority of the students, however, evaluated the overall program, the instructional component, the teaching materials and the extra-curricular activities positively. Specifically, 94% of the students felt that the program increased their understanding and appreciation of mathematics and computer science; 82% felt that it has prepared them to perform better in their high school courses; 82% felt that it has inspired them to pursue the more challenging mathematics courses while in high school; and 65% felt that it has prepared them to reason more clearly. 77% of the students felt that the program made them aware of their career options. Also, 94% of the students felt that the subject matter and class size was about right. But most of the students felt that the program and class periods were too long.

Both Professors Rice and Finley, who have taught several summers in the program, stated on several occasions that this was one of the best group of students we have had. The faculty generally agreed that most of the anticipated outcomes of the program were reached: (a) the students became more proficient in reasoning skills; (b) the students became more aware of the inter-relation of mathematics and other sciences; (c) the students became more aware of how the computer is used as an aid in solving problems in statistics and other disciplines, and (d) the students became more aware of the capabilities of UDC and the Navy and the opportunities available to them.

Throughout the program, the students were encouraged to remain in the calculus-track while in high school, as they would then have more career options available to them when they reach college.

APPENDIX A

A Summer Program in Mathematics and Computer Science

1991

Sigma

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 to 10:15	Prof. Finley Computer Science Lab Room 101/Bldg. 32	Prof. Finley Computer Science Room 209/Bldg. 42	Prof. Finley Computer Science Lab Room 101/Bldg. 32	Prof. Finley Computer Science Room 209/Bldg. 42	Prof. Finley Computer Science Lab Room 101/Bldg. 32
10:15 to 11:30	Prof. Rice Statistics Room 208/Bldg. 32	Prof. Rice Statistics Lab Room 101/Bldg. 32	Prof. Rice Statistics Room 208/Bldg. 32	Prof. Rice Statistics Lab Room 101/Bldg. 32	Prof. Rice Statistics Room 208/Bldg. 32
11:30 to 12:30	Lunch	Lunch	Lunch	Lunch	Lunch
12:30 to 2:00	Profs. Drake/Barnes General Mathematics Room 209/Bldg. 42	Profs. Drake/Barnes General Mathematics Room 209/Bldg. 42	Profs. Drake/Barnes General Mathematics Room 209/Bldg. 42	Profs. Drake/Barnes General Mathematics Room 209/Bldg. 42	Career Education Field Trips: July __, 1991 David Taylor Research Center July __, 1991 Navy Research Laboratory

A Summer Program in Mathematics and Computer Science

1991

Theta

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 to 10:15	Prof. Rice Statistics Room 208/Bldg. 32	Prof. Rice Statistics Lab Room 101/ Bldg. 32	Prof. Rice Statistics Room 208/Bldg. 32	Prof. Rice Statistics Lab Room 101/Bldg. 32	Prof. Rice Statistics Room 208/Bldg. 32
10:15 to 11:30	Prof. Finley Computer Science Lab Room 101/Bldg. 32	Prof. Finley Computer Science Room 209/Bldg. 42	Prof. Finley Computer Science Lab Room 101/Bldg. 32	Prof. Finley Computer Science Room 209/Bldg. 42	Prof. Finley Computer Science Lab Room 101/Bldg. 32
11:30 to 12:30	Lunch	Lunch	Lunch	Lunch	Lunch
12:30 to 2:00	Profs. Drake/Barnes General Mathematics Room 208/Bldg. 32	Profs. Drake/Barnes General Mathematics Room 208/Bldg. 32	Profs. Drake/Barnes General Mathematics Room 208/Bldg. 32	Profs. Drake/Barnes General Mathematics Room 208/Bldg. 32	Career Education Field Trips: July __, 1991 David Taylor Research Center July __, 1991 Navy Research Laboratory

APPENDIX B

University of the District of Columbia
College of Physical Science Engineering and Technology

Department of Mathematics
4200 Connecticut Avenue, N.W.
Washington, D.C. 20008

Telephone (202) 282-3171

April 12, 1991

Dear Principal:

The Department of Mathematics and the Department of Computer Science at the University of the District of Columbia are pleased to announce a program, entitled, **A Five-Week Summer Program in Mathematics and Computer Science for Academically Oriented Students**, scheduled June 24 through July 26, 1991. This program will provide a five-week, intensive, exciting and rigorous academic program in mathematics, computer science and operations research for forty (40) ninth and tenth grade students and is funded by the Office of Naval Research, Department of the Navy.

Please encourage the teachers of mathematics at your school to recommend no more than two students who are capable of success in this postmarked by May 10, 1991. Application forms are included and must be completed by the appropriate mathematics teachers.

Thank you in advance for your prompt consideration. We look forward to hearing from the teachers at your school.

Sincerely,

Bernis Barnes

Bernis Barnes
Project Director

Selection of Participants for the Summer Program

To be considered for this program the student must be:

1. recommended by his/her mathematics teacher.
2. passing to the ninth or tenth grade.
3. motivated to work hard.
4. a serious student.

Stipends

Each student will receive a stipend of $250.00 for participating in the five week program.

Applications

Address applications to:

A Summer Program
Department of Mathematics
University of the District of Columbia
4200 Connecticut Avenue, N.W.
Washington, D.C. 20008

Deadline

Applications must be postmarked no later than May 10, 1991. Students will be notified by May 24, 1991

Further Information

Professor Bernis Barnes 282-3171
Dr. Alvin Darby 282-7427

The Department of Mathematics and The Department of Computer Science

present

A Summer Program in Mathematics and Computer Science for Academically Oriented Students

June 24 – July 26, 1991

Funded by the Office of Naval Research
Department of Navy

A program in mathematics and computer science for academically talented students will be offered at the University of the District of Columbia (Van Ness Campus) this summer. This program will focus on reasoning competencies while enriching the educational experience of the students. Specifically, the program will provide a five-week, intensive, exciting and rigorous academic program in mathematics, computer science, statistics and operations research for forty (40) ninth and tenth grade students.

The Department of Mathematics and the Department of Computer Science realize that many students are capable of success in mathematics-based fields, but they have not been motivated to seek careers in those areas. We recognize the need to provide exciting programs in mathematics and computer science to intrigue these students and stimulate their interest in mathematics and mathematics-based fields. This need is most acute where the students may suffer from educational, financial, or cultural disadvantage.

This program is open to all students, without discrimination.

CURRICULUM SPECIFICS

General Mathematics

Finite mathematical systems will be used to introduce structure in algebra, topology and geometry. The student-centered classes will be designed to encourage students to investigate these topics. The focus will be on the language, patterns and logical nature of mathematics

Computer Science

The fundamentals of programming, flow charting and the BASIC language will be used to introduce the computer as a tool to aid in solving problems in many disciplines. Each student will have access to a computer terminal in a laboratory type hands-on experience.

Statistics and Operations Research

Students will use quantitative techniques as a tool in decision making. They will study statistical and OR techniques and apply them to management type problem solving. This component will be computer based and will focus on interpretation and utilization of data.

Field Trips, Films and Forums

Field trips, films or forums will be scheduled on Fridays. These opportunities will be provided for students to experience the use of mathematics in the working world of the scientist and for students to interact with professionals in the field. The Office of Naval Research will work cooperatively with UDC in implementing this component.

Application Form

SUMMER PROGRAM IN MATHEMATICS AND COMPTUER SCIENCE

(To be completed by Mathematics Teacher)

Please Print or Type

______________________________ ( )Male ( )Female ____________

Student's Last Name First Middle Social Security No.

Present Grade 8th ( ) 9th ( ) Date of Birth__________

______________________________ ____________

Parent/Guardian's Last Name First Middle Home Telephone

Home Address______________________________

Number & Street City State Zip Code

____

Title of Mathematics Course in which student is currently enrolled

1) What is the best estimate you can give to the applicant's present rank in your course?

Top 10% 2nd 10% 3rd 10% 4th 10% 5th 10%

2) What is the applicant's attitude toward and interest in the course work?

Outstanding Excellent Good Average Below Average

3) What are the levels of promptness and attention to detail with which class assignments are completed by the applicant?

Outstanding Excellent Good Average Below Average

4) What is the applicant's level of abstract reasoning?

Outstanding Excellent Good Average Below Average

5) What is the applicant's level of computational skills?

Outstanding Excellent Good Average Below Average

Please check the appropriate section.

	Outstanding	Excellent	Good	Average	Below Average
Study Habits					
Self Motivation					
Organization of Time & Work					
Intellectual Curiosity					
Attention Span					
Ability to express ideas orally					
Ability to follow directions					
Ability to work independently					
Perseverance					
Attendance					
Parent Cooperation					

(Over)

Comments:

Teacher's Name (Print or Type)

Teacher's Signature

School

School Phone Number

Please Return This Form
by May 10, 1991

A Summer Program
Department of Mathematics
University of the District of Columbia
4200 Connecticut Avenue, N.W.
Washington, D. C. 20008

NAVY PROGRAM
1991
ROSTER

SIGMA

1. Omolara Adedoyin
2. Aderonke Awe
3. Dannielle Blake
4. Michael Bowman
5. Ilich Briones
6. Michael Burrell
7. Zaneta Carelock
8. Nyesha Clarke
9. Charles Gartland
10. Diallo Glover
11. James Herndon
12. Sharon Johnson
13. Joi Lucas
14. Angel Maldonado
15. Tisha McCray
16. Kia McLean
17. Yerodin Sanders
18. Jamal Wells

THETA

1. Consuella Belsches
2. Tevye Holloman
3. Kamarica Humphrey
4. Jeffrey Jackson
5. Katrice Jackson
6. Ben Jordan-Downs
7. Lonette Merriman
8. Ashley Moses
9. Gary Murray
10. Yvette Murray
11. Okechukwu Oji
12. Elizabeth Oyebode
13. [redacted]
14. Tiffani Sloan
15. John Sopko
16. Devon Stancell
17. Yusef Trowell
18. Robert Waters
19. Keisha Watson

A Summer Program in Mathematics and Computer Science
Orientation Session
June 24, 1991

Welcome .... Bernis Barnes
Program Director

Greetings .... Dr. Philip L. Brach
Dean - College of
Physical Science
Engineering & Technology

Introduction of Faculty, Staff and Students

Overview of the Program.. Professors: Bernis Barnes
Gail Finley
William Rice
Reuben Drake

Expectations of the Program...Professor Bernis Barnes

Tour of Facilities at UDC...Professor William Rice
Professor Gail Finley

Photo Identification Session.. Mr. James Stephens
Building 38, Room 206

Follow your afternoon schedule - 12:30 until 2:00

Note: The Closing Session will be held on Friday, July 26 from 12:30 until 2:30

A SUMMER PROGRAM IN MATHEMATICS AND COMPUTER SCIENCE
June 24 - July 26, 1991

Program Description

This intervention program is designed to provide a five-week intensive and rigorous academic program in mathematics (including probability and statistics), and computer science for forty (40) academically-talented ninth and tenth grade students from the District of Columbia area. It's primary purpose is to prepare these students to pursue mathematics and mathematics-based fields: by preparing them to take the more rigorous, calculus-track, mathematics courses while in high school, and by making them aware of the many career opportunities in mathematics-based fields. The goals, therefore, are (1) to strengthen the students' background in mathematics, computer science, and statistics and operation research, (2) to improve their reasoning skills, (3) to motivate them to take calculus-track courses and calculus in high school, and (4) to expose them to career opportunities in mathematics and mathematics-based fields.

There are two major components of this program: academic and career education. The academic component offers three courses--General Mathematics, Computer science, and Statistics and Operations Research. Both the computer science and statistics courses meet for one hour and fifteen minutes each morning, and the mathematics course meets for one and one-half hours each afternoon, except on Fridays. The computer classes are held in a computer laboratory (a classroom with seventeen IBM PC's and several terminals) three days a week, and the statistics classes are held in that laboratory two days a week.

General Mathematics: Finite mathematical systems are used to introduce structures in algebra, topology and geometry. The student-centered classes emphasize identifying patterns, applying reasoning skills, and using the technical language of mathematics while focusing on the logical and structural nature of mathematics. The objectives of this course are (1) to improve the students' skills in recognizing patterns, analyzing data and drawing conclusions. (2) to improve the students' facility with the technical language of mathematics, (3) to improve the students' reasoning skills, (4) to improve the students' understanding of the logical and structural nature of mathematics, (5) to improve the students' techniques of formulating and solving problems, and (6) to increase the students' knowledge of and strengthen their skills in basic mathematics.

Computer Science: The fundamentals of programming, flow charting, and the BASIC and Pascal languages are used to introduce the computer as a tool to aid in solving problems. Throughout the course, each student has access to personal computers and computer terminals in a laboratory type hands-on experience. The objectives of this course are (1) to prepare the students to be literate in computers and knowledgeable of the hardware, (2) to prepare the students to construct flowcharts for algorithmic development, and (3) to prepare the students to write and debug programs in both the BASIC and Pascal languages that make use of control statements, string variables, arrays, data files and graphic techniques.

Statistics and Operations Research: Quantitative techniques are used as tools in decision making. The students study statistical and operations research techniques and apply them in problem solving. This course is computer based and focuses on the interpretation and utilization of data. The objectives are (1) to expose students to probability theory, (2) to enable the students to transfer the skills developed in their computer course into problem solving tools in statistics, and (3) to expose students to some of the significant mathematics models underlying statistics.

Career Education includes field trips, films, videos, and a forum which are usually held on Fridays afternoon. These activities expose the students to career opportunities in mathematics-based fields: and provides them with opportunities to visit the work-place of scientists, to experience the use of mathematics in the working world of the scientist, and to interact with professional in the field.

Personnel

The program staff consists of four (4) members of the faculty of the University of the District of Columbia ---three from the Department of Mathematics and one from the Department of Computer Science---and two student assistants.

Professors: Bernis Barnes
282-3171

Reuben Drake
282-3171

Gail Finley
282-7345

William Rice
282-3171

Education Technicians: Marisha Pennington
282-3171

Anwar Mian

Assignments

Students are expected to complete all assignments. Homework assignments are carefully selected to reinforce concepts presented in class.

Attendance

Students are expected to meet their schedules daily and on time. Tardiness and absenteeism will surely interrupt the continuity of the subjected matter so carefully prepared for the students.

Excused absences may be obtained from the Department of Mathematics at 282-3171. Parents or guardians should call the office between 8:30 and 9:00 on the day of the absence.

Field Trips

The instructional phase of the program will be supplemented by field trips, films and a forum scheduled on Friday afternoons. The field trips and films will allow students an opportunity to see the use of mathematics in the working world of the scientist, and the forum will give students an opportunity to interact with professionals from the Navy and the University of the District of Columbia. Parental consent is required for students to attend the field trips.

Attire

Students are expected to wear modest attire.

No Nos

1. No radios
2. No gum
3. No tardiness
4. No food or drink in the classrooms or computer laboratories
5. No smoking
6. No cameras on field trips

Student Identification Cards

All students will be issued a student identification card. The ID card is required for use of all university services and must be available for presentation to security personnel in university buildings. Please wear your student ID card throughout the course of the program.

Health Appraisal Form

The University Health Center is authorized to provide services to minors with parental consent. Parents desiring the service should so indicate on the Health Appraisal Form. The University Health Service is located in Building 44, Room A33 and directed by Dr. Franklin.

Services for Students

Bookstore: The university bookstore is located in Building 38, Level A. The bookstore is open from 9:00 to 5:00 and provides books and supplies that might be needed by the students. Snacks are also available in the bookstore.

Library: The university maintains four libraries. The main collection is located on the Van Ness campus in Building 41, Level A. The collection includes more than 400,000 books and more than 1,000,000 items including microfilms, media materials and government documents. There are reading rooms, open stacks, microfilms and individual study carrels.

The hours of the Van Ness Library are between 8:00 a. m. and 7:00 p. m. A valid university ID is required by students using the services of the library.

Eating Facilities: Students may bring bag lunches daily. Bag lunches should be brought on days of field trips, as the trips are scheduled shortly after the morning classes.

Eating facilities in the area include the university cafeteria, located in Building 38, Level B and several fast food establishments within two blocks of the university on Connecticut Avenue.

Computer Facilities: Computer facilities are available to students in Building 32, Room 101 and in Building 41, Room 302. Students will have access to the university's computer systems via CRTS and printer terminals.

University of the District of Columbia
College of Physical Science Engineering and Technology

Department of Mathematics
4200 Connecticut Avenue, N.W.
Washington, D.C. 20008

Telephone (202) 282-3171

July 11, 1991

Parent/Guardian:

Two field trips have been planned for the students participating in the UDC-Navy sponsored program, entitled, A Summer Program in Mathematics and Computer Science . The bus transportation will be provided by the University of the District of Columbia at no cost to the students.

Our first trip is scheduled for Wednesday, July 17, 1991 from 12:15 - 3:00 P.M. for the David Taylor Research Center at Carderock, Maryland. We request your permission for your child

____________________________________ to go on this trip.
Name of Child

____
Signature of Parent or Guardian   Date

University of the District of Columbia
College of Physical Science Engineering and Technology

Department of Mathematics
4200 Connecticut Avenue, N.W.
Washington, D.C. 20008

Telephone (202) 282-3171

July 18, 1991

Parent/Guardian:

The second of the two field trips planned for the students participating in the UDC-Navy sponsored program in mathematics and computer science is scheduled for Tuesday, July 23, 1991 from 12:15 - 4:00 P.M. for the Naval Research Laboratory in Washington, D. C. Bus transportation will be provided by the University of the District of Columbia at no cost to the students.

We request your permission for your child ________________________________
Name of Child

to go on this trip.

____

Signature of Parent or Guardian Date

APPENDIX C

MATHEMATICS

UNIT:TOPOLOGICAL STRUCTURES ON FINITE SETS

This unit focuses on the topological structures on finite sets and on the technical language that is used in these systems. The topics are functions, point set topologies, basic topological concepts, and continuous functions.

The objectives for the students are

> To identify and establish relations that are functions and functions that are onto and/or one-to-one, and to determine for given functions the image and inverse image of given sets.
>
> To determine if a given collection of subsets of a set is a topology on that set, and to form collections of subsets of a given set that are topologies on that set.
>
> To identify for a given topology the interior, exterior, boundary and cluster points of a given set, and the complement and closure of that set.
>
> To use the definition of a continuous function to show that for a given topology a given function is or is not continous at a given point.

This unit will be taught in four lessons. In teaching each lesson, the emphasis will be on recognizing patterns, using reasoning skills, and interpreting and applying given definitions. The attached worksheets will be used to structure, focus and guide the classwork, and to provide homework assignments.

<u>Technical Terms</u>

set	relation
element	function
subset	into
equal sets	domain
union	range
intersection	codomain
complement	onto
cluster point	one-to-one
interior point	image
exterior point	inverse image
boundary point	topology
closure of a set	open set
one and only one	continuity
if and only if	basis

RELATIONS AND FUNCTIONS ON FINITE SETS

1. The Cartesian product of sets A and B (written A x B) is the set of all ordered pairs such that the first element of each pair is an element of A and the second element of each pair is an element of B.

Let A = {a,b,c} and let B = {1,2}, list the elements of A x B and the elements of B x A.

Also, list the elements of A x A.

2. A relation from set A into set B is a subset of A x B.

Let A = {a,b,c} and let B = {1,2}, define five relations from set A into set B.

How many relations are there from set A into set B? __________

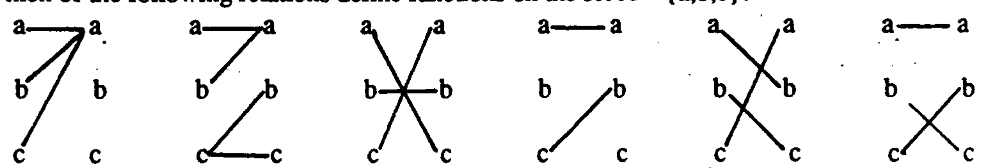
How many relations are there from set A into set A? __________

3. A function from set A into set B is a relation from A into B in which each element of A is paired with one and one element of B.

i. The set of all first components of the pairs, all the elements of A, is called the domain of the function.

ii. The set of all second components of the pairs, a subset of B, is called the range of the function.

a. Which of the following relations define functions on the set A = {a,b,c}?



What is the range of each of the functions?

How many functions are there from set A into set A? __________

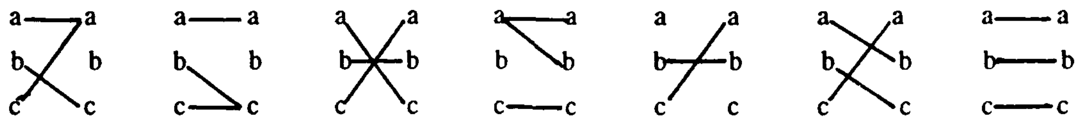
b. Let A = {a,b,c} and let B = {1,2}, how many functions are there from set A into set B? __________

c. Suppose set A has n elements and set B has m elements. determine the following:

(1). How many relations are there from set A into set B? ___________

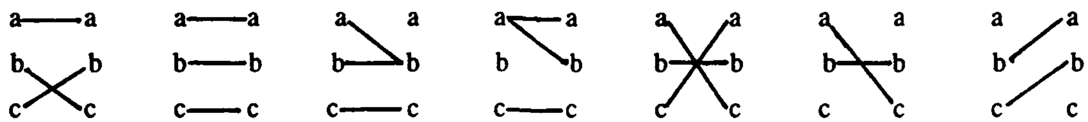
(2). How many functions are there from set A into set B? __________

4. The function $f:A\rightarrow B$ is <u>onto</u> iff for each element $y\varepsilon B$, there is an element $x\varepsilon A$ such that $f(x) = y$.
Which of the following functions are onto functions?



List the remaining onto functions $f:V\rightarrow V$ where $V = \{a,b,c\}$.

5. The function $f:A\rightarrow B$ is <u>one-to-one</u> iff $f(x) = f(y)$ implies that $x = y$.
Which of the following functions are one-to-one?



List the remaining one-to-one functions of $f:V\rightarrow V$ where $V = \{a,b,c\}$.

6. Let $f:X\rightarrow Y$, then

i. $f[X] = \{f(x)\ \varepsilon Y : x\ \varepsilon X\}$ is the image of X in Y, and

ii. $f^{-1}[Y] = \{x\ \varepsilon X : f(x)\ \varepsilon Y\}$ is the inverse image of X in Y.

Let $X = \{1,2,3,4,5\}$ and $Y = \{1,2,4\}$, and define $f:X\rightarrow Y$ by $f(2) = 1$, $f(4) = 2$ and $f(1) = f(3) = f(5) = 4$.

a. Let $A = \{1,2,3,4\}$ and $B = \{1,2,5\}$, find:

$f[A] = f[A \cap X] =$ $\qquad\qquad$ $f[B] = f[B \cap X] =$

$f[A \cup B] =$ $\qquad\qquad$ $f[A \cap B] =$

Let $A = \{1,2,3,4\}$ and $B = \{1,2,5\}$, show that:

$f[A \cup B] = f[A] \cup f[B]$ $\qquad\qquad$ $f[A \cap B] \subseteq f[A] \cap f[B]$

b. Let $A = \{1,2,3,4\}$ and $B = \{1,2,5\}$, find:

$f^{-1}[A] = f^{-1}[A \cap Y] =$ $\qquad\qquad$ $f^{-1}[B] = f^{-1}[B \cap Y] =$

$f^{-1}[A \cup B] =$ $\qquad\qquad$ $f^{-1}[A \cap B] =$

Let $A = \{1,2,3,4\}$ and $B = \{1,2,5\}$, show that:

$f^{-1}[A \cup B] = f^{-1}[A] \cup f^{-1}[B]$ $\qquad\qquad$ $f^{-1}[A \cap B] = f^{-1}[A] \cap f^{-1}[B]$

TOPOLOGIES ON FINITE SETS

Def. A set X is a subset of a set V iff each element of X is an element of V.

Which of the following sets are subsets of the set {a,b,c}?

{a,b}　　{a}　　{a,d}　　{ }　　{a,b,c}　　{d}

List the subsets of {a,b,c} that are not given above.

Def. Any collection of subsets of a finite set V is a topology on V iff

i). the collection contains the empty set and the set itself, and

ii). the collection is closed under the operations of union and intersection.

1. The collection { ø , {a,b}, {a,b,c} } of subsets of {a,b,c} is a topology on {a,b,c}.

i). The collection contains ø and {a,b,c}.

ii).

$\cup$	ø	{a,b}	{a,b,c}
ø	ø	{a,b}	{a,b,c}
{a,b}	{a,b}	{a,b}	{a,b,c}
{a,b,c}	{a,b,c}	{a,b,c}	{a,b,c}

$\cap$	ø	{a,b}	{a,b,c}
ø	ø	ø	ø
{a,b}	ø	{a,b}	{a,b}
{a,b,c}	ø	{a,b}	{a,b,c}

2. Show that the collection { ø, {a}, {b,c}, {a,b,c} } of subsets of {a,b,c} is a topology on the set {a,b,c}.

3. State the reason why each of the following collections of subsets of the set {a,b,c} is not a topology on {a,b,c}.

a. { ø, {a}, {b}, {a,b} }　　b. { {a}, {a,b}, {a,b,c} }　　c. { ø, {a}, {b}, {a,b,c} }　　d. { ø, {a,b}, {b,c], {a,b,c} }

4. Which of the following collections of subsets of {a,b,c} are topologies on {a,b,c}?

1. { ø, {a,b,c} }
2. { ø, {a}, {b}, {a,b,c} }
3. { ø, {a,b}, {a,b,c} }
4. { ø, {a}, {a,b,c} }
5. { ø, {a}, {b}, {a,b}, {a,b,c} }
6. { ø, {a}, {a,b}, {a,b,c} }
7. { ø, {a,b}, {a,c}, {a,b,c} }
8. { ø, {b}, {a,c}, {a,b,c} }
9. { ø, {a}, {b}, {c}, {a,b,c} }
10. { ø, {a}, {a,b}, {a,c}, {a,b,c} }
11. { ø, {a,b}, {a,c}, {b,c}, {a,b,c} }
12. { ø, {a}, {b}, {a,b}, {a,b,c} }
13. { ø, {b}, {a,b}, {a,c}, {a,b,c} }
14. { ø, {a}, {b}, {a,b}, {a,c}, {a,b,c} }
15. { ø, {a}, {b}, {a,c}, {a,b,c} }
16. { ø, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c} }

5. How many topologies can be defined on a set with three elements?

1. { ø, {a,b,c} }
2. { ø, {a}, {a,b,c} }
3. { ø, {a,b}, {a,b,c} }
4. { ø, {a}, {a,b}, {a,b,c} }
5. { ø, {b}, {a,c}, {a,b,c} }
6. { ø, {a}, {a,b}, {a,c}, {a,b,c} }
7. { ø, {a}, {b}, {a,b}, {a,b,c} }
8. { ø, {a}, {b}, {a,b}, {a,c}, {a,b,c} }
9. { ø, {a}, {b}, {c}, {a,b}, {a,c}, {b,c}, {a,b,c} }

Def. Let T be a topology on set V, then a collection of subsets B of V is a basis for T iff
i). B T, and
ii). every X T is the union of members of B.

6. To the right of each topology in number 5 above, give a basis for the topology.

TOPOLOGICAL SETS

For each set X ($X \subseteq V$) in the tables below, list the elements of the following sets where $V = \{1, 2, 3\}$.

1. **Complement** of set X: $X^C = \{p \in V: p \notin X\}$
2. **Interior** of set X: $Int_G(X) = \{p \in X$: there exists a set $A \in G$ such that $p \in A$ and $A \subseteq X\}$
3. **Exterior** of set X: $Ext_G(X) = \{p \in X^C$: p is an interior point of $X^C\}$
4. **Boundary** of set X: $Bd_G(X) = \{p \in V: p \notin Int_G(X)$ and $p \notin Ext_G(X)\}$
5. **Cluster points** of X: $Cp_G(X) = \{p \in V$: every set $A \in G$ which contains p contains a point of X other than p$\}$
6. **Closure** of set X: $Cl_G(X) = \{p \in V: p \in X$ or $p \in Bd_G(X)\}$
7. A set X is **open** relative to G iff every point in S is interior to X. Which of the sets X are open sets?

a. Let the topology G be the set $\{\emptyset, \{1\}, V\}$.

X	X^C	$Int_G(X)$	$Ext_G(X)$	$Bd_G(X)$	$Cp_G(X)$	$Cl_G(X)$
$\emptyset$	V	$\emptyset$	V	$\emptyset$	$\emptyset$	$\emptyset$
{1}	{2,3}	{1}	$\emptyset$	{2,3}	{2,3}	V
{2}	{1,3}	$\emptyset$	{1}	{2,3}	{3}	{2,3}
{3}	{1,2}	$\emptyset$	{1}	{2,3}	{2}	{2,3}
{1,2}	{3}	{1}	$\emptyset$	{2,3}	{2,3}	V
{1,3}	{2}	{1}	$\emptyset$	{2,3}	{2,3}	V
{2,3}	{1}	$\emptyset$	{1}	{2,3}	{2,3}	{2,3}
V	$\emptyset$	V	$\emptyset$	$\emptyset$	{2,3}	V

b. Let the topology G be the set $\{\emptyset, \{1\}, \{1,2\}, V\}$.

X	X^C	$Int_G(X)$	$Ext_G(X)$	$Bd_G(X)$	$Cp_G(X)$	$Cl_G(X)$
$\emptyset$						
{1}						
{2}						
{3}						
{1,2}						
{1,3}						
{2,3}						
V						

For each set X ($X \subseteq V$) in the tables below, list the elements of the following sets where $V = \{1, 2, 3\}$.

1. **Complement** of set X: $X^C = \{p \in V: p \notin X\}$
2. **Interior** of set X: $Int_G(X) = \{p \in X$: there exists a set $A \in G$ such that $p \in A$ and $A \subseteq X\}$
3. **Exterior** of set X: $Ext_G(X) = \{p \in X^C$: p is an interior point of $X^C\}$
4. **Boundary** of set X: $Bd_G(X) = \{p \in V: p \notin Int_G(X)$ and $p \notin Ext_G(X)\}$
5. **Cluster points** of X: $Cp_G(X) = \{p \in V$: every set $A \in G$ which contains p contains a point of X other than p$\}$
6. **Closure** of set X: $Cl_G(X) = \{p \in V: p \in X$ or $p \in Bd_G(X)\}$
7. A set X is **open** relative to G iff every point in S is interior to X. Which of the sets X are open sets?

c. Let the topology G be the set $\{ \emptyset, \{1\}, \{2,3\}, V\}$.

X	X^C	$Int_G(X)$	$Ext_G(X)$	$Bd_G(X)$	$Cp_G(X)$	$Cl_G(X)$
$\emptyset$						
{1}						
{2}						
{3}						
{1,2}						
{1,3}						
{2,3}						
V						

d. Let the topology G be the set $\{ \emptyset, \{1\}, \{1,2\}, \{1,3\}, V\}$.

X	X^C	$Int_G(X)$	$Ext_G(X)$	$Bd_G(X)$	$Cp_G(X)$	$Cl_G(X)$
$\emptyset$						
{1}						
{2}						
{3}						
{1,2}						
{1,3}						
{2,3}						
V						

CONTINUOUS FUNCTIONS

Def. Let G and H be topologies on sets V and U respectively, then the function $f{:}V \to U$ is continuous at the point $p_o \in V$ relative to the given topologies G and H iff for every set $Y \in H$ which contains $f(p_o)$, there is a set $X \in G$ which contains p_o such that $f[X \cap V] \subseteq Y$, i.e. if $p \in X \cap V$, then $f(p) \in Y$.

1. Let $G = \{ \emptyset, \{1,2\}, V\}$ and $H = \{ \emptyset, \{a\}, \{a,b\}, U\}$ be topologies on $V = \{1,2,3\}$ and $U = \{a,b,c\}$ respectively.

a. If $f{:}V \to U$ is defined by $f(1) = a$, $f(2) = a$ and $f(3) = c$, show that f is continuous at each point of V.

(1). Is f continuous at 1?

$f(1) = a$ is an element of the following sets in H: $\{a\}$, $\{a,b\}$, $\{a,b,c\}$. Thus, since there exists a set $\{1,2\} \in G$ such that $1 \in \{1,2\}$ and $f[\{1,2\}] = \{a\}$, and since $\{a\}$ is a subset of sets $\{a\}$, $\{a,b\}$ and $\{a,b,c\}$, then f is continuous at 1.

(2). Is f continuous at 2?

(3). Is f continuous at 3?

b. Let $f{:}V \to U$ be defined by $f(1) = c$, $f(2) = a$ and $f(3) = b$, show that f is not continuous at the points $2,3 \in V$.

iff $\exists$ $Y \in H$ which contains $f(p_o)$ s.t. $\forall$ $X \in G$ which contains p_o, $\exists$ $p \in V \cap X$, but $f(p) \notin Y$.

$\{a\}$	a	$\{1,2\}$	2	1	$f(1) = c \notin \{a\}$
		$\{1,2,3\}$	2	1	$f(1) = c \not\subset \{a\}$

2. Given the topologies $G = \{ \emptyset, \{1,2\}, V\}$ and $H = \{ \emptyset, \{1\}, \{1,2\}, V\}$ on the set $V = \{1,2,3\}$, determine if the function $f{:}V \to V$ defined by $f(1) = f(2) = 2$ and $f(3) = 1$ is continuous at the three points of V.

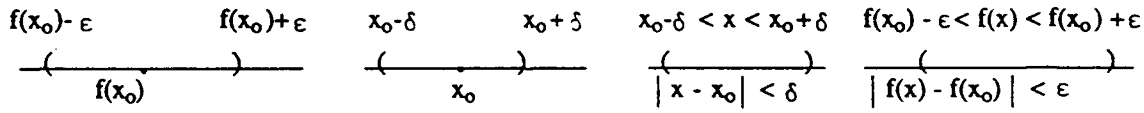
3. Let $V = \{1,2,3\}$ and let $G = \{\varnothing, \{1\}, \{1,2\}, V\}$ be a topology on V. If $f:V \to V$ is defined by $f(1) = 2$, $f(2) = 1$ and $f(3) = 3$, determine if f is continuous at each of the points of V.

4. Let $V = \{1,2,3\}$ and let $G = \{\varnothing, \{2\}, \{1,2\}, V\}$, define two functions $f:V \to V$ such that f is continuous on V.

Def. The function $f:D \to R$ is continuous at point $a \in D$ iff for any real number $\varepsilon > 0$, there exists a real number $\delta > 0$ such that if $x \in D$ and $|x - a| < \delta$, then $|f(x) - f(a)| < \varepsilon$.

iff $\forall\ Y \in H$ which contains $f(p_o)$, $\exists\ X \in G$ which contains p_o s. t. $\forall\ p \in X \cap V$, then $f(p) \in Y$.

$\forall$ ε-interval of $f(x_o)$, $\exists$ δ-interval of x_o s. t. $\forall$ x in δ-interval, then f(x) in ε-interval



5. Show algebraically that $f(x) = 2x$ is continuous at $x = 3$.
   a. For any given epsilon (ε), find a delta (δ) s. t. δ is a function of ε.

   b. Show that if $|x - 3| < \delta$, then $|f(x) - f(3)| < \varepsilon$.

6. Show algebraically that $f(x) = x^2$ is continuous at $x = 2$.

UNIT:ALGEBRAIC STRUCTURES ON FINITE SETS

This unit focuses on the algebraic structures on finite sets and on the technical language that is used in these systems. The topics are binary operations, groups and subgroups, normal and factor groups, and group endomorphisms.

The objectives for the students are

> To construct operation tables that define an operation on a finite set, that define an operation which satisfies a given property, and that define an operation which satisfies the group properties.
>
> To form magic squares, to construct an operation table of the composition of rotations and/or flips of magic squares, and to identify the subgroups of this system.
>
> To find left and right cosets of a given subgroup, find the factor groups of the normal subgroups, and construct the induced operation tables of the normal subgroups.
>
> To identify functions that are group endomnorphisms for a given group, and construct operation tables for addition and composition of the endomorphisms on groups with three elements and groups with four elements.

This unit will be taught in four lessons. In teaching each lesson, the emphasis will be on recognizing patterns, using reasoning skills, and interpreting and applying definitions. The attached worksheets will be used to structure, focus and guide the classwork, and to provide homework assignments.

<u>Technical Terms</u>

operation	row
closed	column
group	transformation
subgroup	rotation
coset	flip
associative	composition
identity	endomorphism
inverse	homomorphism
commutative	normal
left-identity	factor group
left-cancellation	function
weakly left-linear	unique
weakly left-solvable	array
left-faithful	main diagonal
magic square	operation table

UNIT: ALGEBRAIC STRUCTURES ON SETS
Binary Operations

Name ______________________________

Def. The function $+ : V \times V \to V$ is a binary operation on the set V iff to each element $(x,y) \in V \times V$, $+$ assigns a unique element of V.

1. Which of the following tables define binary operations on the set $V = \{a,b,c\}$?

$+_1$	a	b	c
a	a	a	a
b	a	a	a
c	a	a	a

$+_2$	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

$+_3$	a	b	c
a	a	b	c
b	e	e	e
c	c	a	b

$+_4$	a	b	c
a	a	a	a
b	b	b	b
c	c	c	c

$+_5$	a	b	c
a	a	b	c
b	b		a
c		b	

$+_6$	a	b
a	a	b
b	b	a

2. Define 12 additional binary operations on the set $V = \{a,b,c\}$.

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

	a	b	c
a			
b			
c			

3. How many binary operations can be defined on a set with three elements? ________

4. Complete each of the following tables so that it defines an operation on the set $V = \{a,b,c\}$ which satisfies the given property. How are the entries in the table affected by the property?

(a). If $x,y \in V$, then $x + y = y + x$.

+	a	b	c
a	a	c	a
b	c	b	b
c	a	b	c

+	a	b	c
a	c	___	___
b	c	a	b
c	b	___	b

+	a	b	c
a	a	c	___
b	___	a	___
c	b	___	a

÷	a	b	c
a			
b			
c			

Commutative- ______________________________

(b). There is an element $e \in V$ such that $e + x = x$ for every $x \in V$.

+	a	b	c
a	b	a	b
b	a	b	c
c	c	a	b

+	a	b	c
a	a	b	b
b	b	b	c
c	___	___	___

+	a	b	c
a	a	___	___
b	___	c	b
c	___	c	___

+	a	b	c
a			
b			
c			

Left-identity- ______________________________

(c). If x is in V, then there is an element $e \in V$ such that $e + x = x$.

+	a	b	c
a	c	b	a
b	a	c	c
c	b	a	a

+	a	b	c
a	b	c	a
b	b	___	___
c	___	c	b

+	a	b	c
a	___	c	b
b	b	a	___
c	b	___	a

+	a	b	c
a			
b			
c			

Local-left-identity- ______________________________

(d). If $x,y,z \in V$ and $x + y = x + z$, then $y = z$.

+	a	b	c
a	a	b	c
b	a	c	b
c	a	b	c

+	a	b	c
a	a	c	___
b	___	c	b
c	b	___	c

+	a	b	c
a	b	___	a
b	c	___	b
c	___	c	___

+	a	b	c
a			
b			
c			

Left-cancellation- ___

(e). If $x,y \in V$, then there is an element $z \in V$ such that $z + x = y$.

+	a	b	c
a	a	c	c
b	c	b	a
c	b	a	b

+	a	b	c
a	___	b	b
b	a	c	___
c	b	___	c

+	a	b	c
a	c	a	___
b	___	c	b
c	a	___	___

+	a	b	c
a			
b			
c			

Weakly-left-solvable- ___

(f). If $x,y,z \in V$, then there is an element $u \in V$ such that $x + y = u + z$.

+	a	b	c
a	c	b	a
b	a	a	b
c	b	c	c

+	a	b	c
a	c	___	a
b	___	a	b
c	b	c	___

+	a	b	c
a	b	c	b
b	___	___	b
c	___	c	c

+	a	b	c
a			
b			
c			

Weakly-left-linear- ___

(g). For an $e \in V$, there is for each $y \in V$ an element $x \in V$ such that $x + y = e$.

+	a	b	c
a	b	c	a
b	a	b	c
c	c	a	b

+	a	b	c
a	___	b	c
b	c	c	___
c	b	___	b

+	a	b	c
a	c	b	___
b	___	___	___
c	___	c	a

+	a	b	c
a			
b			
c			

Left-inverse- ___

(h). If $y,z \in V$ and $x + y = x + z$ for every $x \in V$, then $y = z$.

+	a	b	c
a	a	b	c
b	c	a	b
c	b	c	a

+	a	b	c
a	a	___	a
b	c	___	b
c	b	___	b

+	a	b	c
a	c	___	___
b	___	c	b
c	a	___	a

+	a	b	c
a			
b			
c			

Left-faithful- ___

How many commutative operations can be defined on a set with three elements? ___

How many operations with an identity can be defined on a set with three elements? ___

How many operations with inverses can be defined on a set with three elements? ___

Def. The system $(V,+)$ is a group iff the following properties hold:

i. If $x,y \in V$, then $x + y \in V$.
ii. If $x,y,z \in V$, then $(x + y) + z = x + (y + z)$.
iii. There is an element $e \in V$ such that for every $x \in V$, then $e + x = x + e = x$.
iv. For each $y \in V$, there is an $x \in V$ such that $x + y = y + x = e$, for an $e \in V$.

Complete each of the following tables so that it defines a group.

+	a	b	c
a	___	___	___
b	___	c	___
c	___	___	b

+	a	b	c
a	c	___	___
b	___	___	___
c	___	___	a

+	a	b	c
a	b	___	___
b	___	a	___
c	___	___	___

UNIT: ALGEBRAIC STRUCTURES ON SETS

Groups and Subgroups

Name________________________________

Observe the 3-by-3 squares in Figures 1 and 2 below. You may notice that the sum of numbers in each row, each column and each main diagonal is equal to 15. How are the two squares related?

4	9	2
3	5	7
8	1	6

Fig. 1

8	1	6
3	5	7
4	9	2

Fig. 2

		8
2		6

Fig. 3

6		2
8		

Fig. 4

Def. An array of squares as above is a magic square iff the sums of the numbers in each row, each column, and each main diagonal are equal.

Try your luck at completing the squares in Figures 3 and 4 so that the sum of the numbers in each row, column and main diagonal is 15. How are the squares in Figures 3 and 4 related to the square in Figure 1?

It appears that if we start with a magic square and then perform a series of flips and rotations (transformations) on it, we obtain still another square that is magic.

How many different magic squares can we get from transformations on Figure 1? Identify the kind of transformation which gives each square from Figure 1.

2		
6		8

8		
6		2

6		8
2		

2		6
		8

Further observation tells us that if we rotate the square in Figure 1 90^{o} clockwise (R_{90^o}), then flip the square that results along its verticle axis (F_v), we obtain the square in Figure 3, which is also magic.

Now, let's flip the square in Figure 1 along its verticle axis and then follow the flip with a rotation of 90^{o} clockwise. We obtain the square in Figure 4 which is also magic. But the squares in Figures 3 and 4 are different. Explain.

Show that the operation of composition on rotations and/or flips of magic squares as described above is closed. Complete the operation table below to support your answer.

I - no rotation or flip of Fig. 1
R - 90° clockwise rotation of Fig. 1
R' - 180° clockwise rotation of Fig. 1
R" - 270° clockwise rotation of Fig. 1
H - flip along the horizontal axis of Fig. 1
V - flip along the vertical axis of Fig. 1
D - flip along the top to bottom main diagonal of Fig. 1
D' - flip along the bottom to top main diagonal of Fig. 1

*	I	R	R'	R"	H	V	D	D'
I								
R								
R'								
R"								
H								
V								
D								
D'								

Def. The system (V,+) is a subgroup of the group (U,+) iff $V \subseteq U$ and the + is the binary operation on V.

1. The following tables define subgroups of ({I,R,R',R",H,V,D,D'}, *).

*	I	R'
I	I	R'
R'	R'	I

*	I	H
I	I	H
H	H	I

*	I	R	R'	R"
I	I	R	R'	R"
R	R	R'	R"	I
R'	R'	R"	I	R
R"	R"	I	R	R'

2. Construct the operation tables for the remaining subgroups of ({I,R,R',R",H,V,D,D'},

Normal and Factor Groups

Name________________________________

Given that the table to the right defines an operation on the set G = {I,R,R',R",H,V,D,D'} such that the system (G,*) is a group.

*	I	R	R'	R"	H	V	D	D'
I	I	R	R'	R"	H	V	D	D'
R	R	R'	R"	I	D	D'	V	H
R'	R'	R"	I	R	V	H	D'	D
R"	R"	I	R	R'	D'	D	H	V
H	H	D'	V	D	I	R'	R"	R
V	V	D	H	D'	R'	I	R	R"
D	D	H	D'	V	R	R"	I	R'
D'	D'	V	D	H	R"	R	R'	I

1. Def. The system (H,+) is a subgroup of the group (G,+) iff $H \subseteq G$ and + is a binary operation on H.

   a. The following tables define subgroups of (G,*).

*	I	R'
I	I	R'
R'	R'	I

*	I	H
I	I	H
H	H	I

*	I	R	R'	R"
I	I	R	R'	R"
R	R	R'	R"	I
R'	R'	R"	I	R
R"	R"	I	R	R'

   b. Construct the operation tables for the remaining subgroups of (G,*).

2. Def. Let H be a subgroup of group G. The set Ha is a right coset of H iff Ha = {ha:h ε H and "a" is a fixed element of G}.

   a. Since I*V = V and R'*V = H, then the right coset of {I,R'} is {I,R'}V = {V,H}.

   Since I*R = R and R'*R = R", then another right coset of {I,R'} is {I,R'}R = {R,R"}.

   b. List all the left and right cosets of {I,R'}.

Left Cosets	Right Cosets
I {I,R'} =	{I,R'} I =
R {I,R'} =	{I,R'} R =
R'{I,R'} =	{I,R'} R' =
R"{I,R'} =	{I,R'} R" =
H {I,R'} =	{I,R'} H =
V {I,R'} =	{I,R'} V =
D {I,R'} =	{I,R'} D =
D'{I,R'} =	{I,R'} D' =

3. Def. H is a normal subgroup of G iff for every a ε G, aH = Ha.

   a. Is {I,R'} a normal subgroup of (G,*)?

   b. Complete the operation table below of the cosets of {I,R'}.

*	{I,R'}	{R,R"}	{H,V}	{D,D'}
{I,R'}	{I,R'}	{R,R"}	{H,V}	{D,D'}
{R,R"}	{R,R"}	{I,R'}		
{H,V}	{H,V}			
{D,D'}	{D,D'}			

4. Def. (G/H,*) is a factor group of G modulo H iff H is a normal subgroup of G and the elements of G/H are the cosets of H in G.

2. List all the left and right cosets of {I,H}.

*	I	R	R'	R"	H	V	D	D'
I	I	R	R'	R"	H	V	D	D'
R	R	R'	R"	I	D	D'	V	H
R'	R'	R"	I	R	V	H	D'	D
R"	R"	I	R	R'	D'	D	H	V
H	H	D'	V	D	I	R'	R"	R
V	V	D	H	D'	R'	I	R	R"
D	D	H	D'	V	R	R"	I	R'
D'	D'	V	D	H	R"	R	R'	I

Left Cosets	Right Cosets
I {I,H} =	{I,H} I =
R {I,H} =	{I,H} R =
R'{I,H} =	{I,H} R' =
R"{I,H} =	{I,H} R" =
H {I,H} =	{I,H} H =
V {I,H} =	{I,H} V =
D {I,H} =	{I,H} D =
D'{I,H} =	{I,H} D' =

Is {I,H} a normal subgroup of ({I,R,R',R",H,V,D,D'}, *)?

Complete the operation table of the right cosets of {I,H}.

*	{I,H}	{R,D}	{R',V}	{R",D'}
{I,H}				
{R,D}				
{V,R'}				
{R",D'}				

Compare the operation tables of cosets of subgroups that are normal and subgroups that are not normal. What do you find?

3. Which of the remaining subgroups of ({I,R,R',R",H,V,D,D'}, *) are normal? List the cosets of each (factor groups).

UNIT: ALGEBRAIC STRUCTURES ON SETS

Group Endomorphisms

Name________________________________

Def. The function $f:V \to V$ is a group endomorphism on $(V,+)$ iff $f(x + y) = f(x) + f(y)$ for all $x,y \in V$.

A. Let $V = \{a,b,c\}$.

1. Show that $f_2:V \to V$ defined by $f_2(a) = a$, $f_2(b) = c$ and $f_2(c) = b$ is a group endomorphism on $(V,+)$ where the operation + is defined by the table $\longrightarrow$

+	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

$f_2(a + a) = f_2(a) + f_2(a)$
$f_2(a) = a + a$
$a = a$

$f_2(b + a) = f_2(b) + f_2(a)$
$f_2(b) = c + a$
$c = c$

$f_2(c + a) = f_2(c) + f_2(a)$
$f_2(c) = b + a$
$b = b$

$f_2(a + b) = f_2(a) + f_2(b)$
$f_2(b) = a + c$
______

$f_2(b + b) = f_2(b) + f_2(b)$
$f_2(c) = c + c$
______

$f_2(c + b) = f_2(c) + f_2(b)$
$f_2(a) = b + c$
______

$f_2(a + c) = f_2(a) + f_2(c)$

$f_2(b + c) = f_2(b) + f_2(c)$

$f_2(c + c) = f_2(c) + f_2(c)$

2. Show that $f_1:V \to V$ defined by $f_1(a) = a$, $f_1(b) = b$ and $f_1(c) = c$ is a group endomorphism on $(V,+)$ where + is defined as above.

3. Show that $f_0:V \to V$ defined by $f_0(a) = f_0(b) = f_0(c) = a$ is a group endomorphism on $(V,+)$ where + is defined as above.

4. Construct tables for the following operations on the above group endomorphisms for $x \in V$.

a. $(f_i \oplus f_j)(x) = f_i(x) + f_j(x)$

$\oplus$	f_0	f_1	f_2
f_0			
f_1			
f_2			

b. $(f_i \odot f_j)(x) = f_i(f_j(x))$

$\odot$	f_0	f_1	f_2
f_0			
f_1			
f_2			

B. Let $V = \{a,b,c,d\}$.

1. What are the group endomorphisms on $(V,+)$ where $+$ is defined by

+	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

2. Construct tables for the following operations on the above group endomorphisms.

   a. $(f_i \oplus f_j)(x) = f_i(x) + f_j(x)$ b. $(f_i \odot f_j)(x) = f_i(f_j(x))$

C. The following are the group endomorphisms on $(\{a,b,c,d\}, +)$ where $+$ is defined by

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

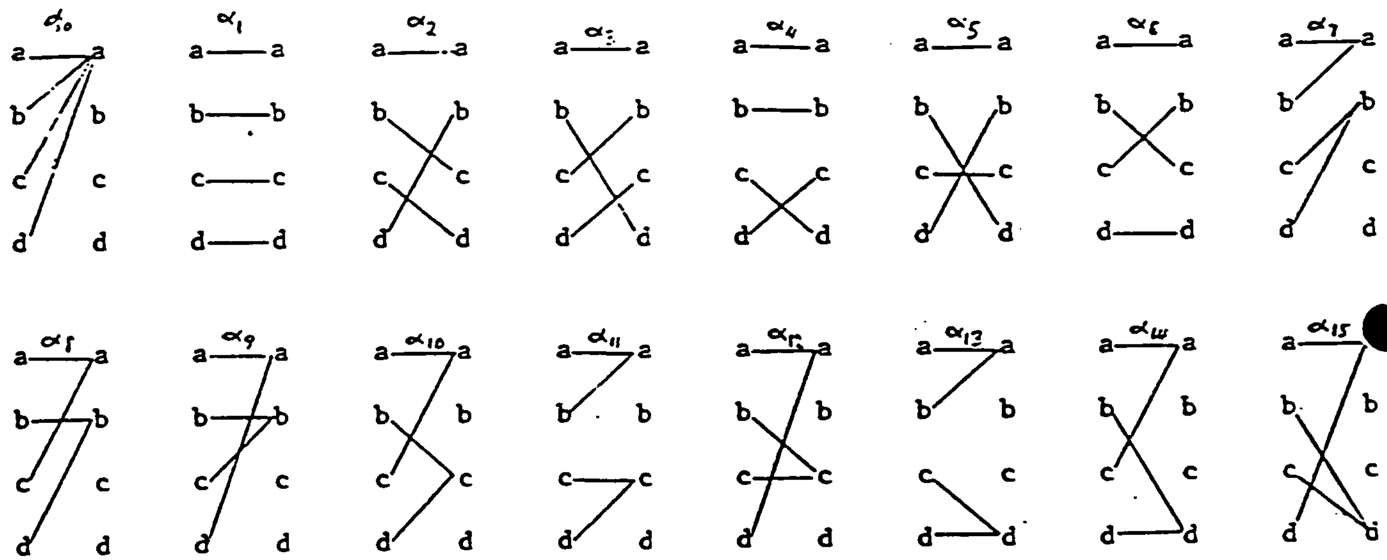
$\{a\}$

$\{a,b\}$

$\{a,c\}$

$\{a,d\}$

$\{a,b,c,d\}$



Construct tables for the following operations on the above group endomorphisms.

a. $(\alpha_i \oplus \alpha_j)(x) = \alpha_i(x) + \alpha_j(x)$ b. $(\alpha_i \odot \alpha_j)(x) = \alpha_i(\alpha_j(x))$

Unit on Groups

Vocabulary

binary operation (closure)
associative law
existence of an identity
existence of inverses
commutative law
group
abelian group
cardinality

Symbols

ϵ (identity); a^{-1} (inverse of a)

Behaviorial Objective

1. Given a Set A, an operation * and an operation table, the student will be able to determine
   a) if Set A is closed under operation *
   b) if Set A is associative under operation *
   c) If there is an identity element of Set A under operation *
   d) the inverses of each element of A under *, if such inverses exist

2. The student will be able to <u>identify</u> commutative and noncommutative groups.

3. The student will cite an example and nonexample of each of the following:
   a) a group
   b) a commutative group
   c) a infinite group

Instructional Strategy

The instructor will use the expository method. Students will be encouraged to focus on relationship and abstract commonalities from examples.

Inverses: The table shows that the set whose members are a, b and c contains an inverse for every one of its members. The inverse of a is a, since a combined with a is the identity element, i.e. a * a = a. The inverse of b is c, since b combined with c is the identity element, i.e. b * c = c * b = a. The inverse of c is b, since c combined with b is the identity element, i.e. c * b = b * c = a.

The associative property is satisfied though tedious to show.

Thus we see the the set whose members are a, b, c, taken with the operation *, has all properties that define a group. Therefore, it is a group with respect to operation *. Namely, it is closed, has an identity, each element has an inverse and the associative property is satisfied.

Item 3: The set {-1, 0, 1} with the operation addition

+	-1	0	1
-1	-2	-1	0
0	-1	0	1
1	0	1	2

Closure: A glance at the table shows that -1 + -1 = -2. Since -2 is not an element of the set {-1, 0, 1} , this set is not closed with respect to addition.

Since a defining characteristic is not satisfied the set whose members are -1, 0, 1, taken with operation addition is not a group.

Item 4: The set of whole numbers whose members are 0, 1, 2, 3, 4, ... with operation addition

A partial table is provided:

+	0	1	2	3	.	.	.
0	0	1	2	3	.	.	.
1	1	2	3	4	.	.	.
2	2	3	4	5	.	.	.
3	3	4	5	6	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

Closure: A glance at the table shows that the sum of any two elements of the set results in an element of the set. So the set of whole numbers with operation addition is closed.

Identity: 0 is clearly the identity element because when 0 is added to any element of the set of whole numbers, the element is unchanged, i.e. 0 + 3 = 3 + 0 = 3; 0 + 12 = 12 + 0 = 12.

Inverses: The only element of the set of whole numbers with an inverse under operation addition is 0. The inverse of 0 is 0 because 0 + 0 = 0, the identity. There is no whole number which when

Another way to express all of the products is to use a multiplication table for this set:

x	1	-1
1	1	-1
-1	-1	1

Since each possible product is 1 or -1 and each of these numerals belong to the set {1, -1} we may conclude that the set having members 1 and -1 is closed with respect to multiplication.

Identity - Multiplication of each element of the set by 1 leaves the element unchanged. Therefore, 1 is the identity element in the set whose members are 1 and -1.

Inverses - Examining all of the possible products using elements 1 and -1 we see that every one of these members has an inverse. The inverse of 1 is 1, since 1 x 1 = 1, the identity element, and the inverse of -1, is -1, since -1 x -1 = 1, the identity element.

Associative - Thus we see that the set whose members are 1 and -1, taken with the operation multiplication satisfies the properies of a group. Hence the set {1, -1} is closed, has an identity each element has an inverse and the associative property is satisfied.

Item 1: The set {a,b,c} with the operation * and table:

*	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

Note: Each element in the table is determined by combining the element in the row and the element in the column. For example, **b** combined with **c** under operation * results in a, i.e. **b * c = a**. Remember (element in row first) * (element in column):

columns

rows

*	a	b	c
a			
b			a
c			

Closure: A glance at table (1) shows that any two elements of the set combined with the operation * results in either **a** or **b** or **c**. That is, the result of the operation * on two members of the set {a,b,c} is also a member of the set {a,b,c}. So the set is closed with respect to operation *.

Identity: Since the result of the operation of either of the elements a or b or c by a leaves it unchanged, a is the identity element, i.e.

a * b = b * a = b
a * a = a * a = a
a * c = c * a = c

Inverses: The table shows that the set whose members are a, b and c contains an inverse for every one of its members. The inverse of a is a, since a combined with a is the identity element, i.e. a * a = a. The inverse of b is c, since b combined with c is the identity element, i.e. b * c = c * b = a. The inverse of c is b, since c combined with b is the identity element, i.e. c * b = b * c = a.

The associative property is satisfied though tedious to show.

Thus we see the the set whose members are a, b, c, taken with the operation *, has all properties that define a group. Therefore, it is a group with respect to operation *. Namely, it is closed, has an identity, each element has an inverse and the associative property is satisfied.

Item 3: The set {-1, 0, 1} with the operation addition

+	-1	0	1
-1	-2	-1	0
0	-1	0	1
1	0	1	2

Closure: A glance at the table shows that -1 + -1 = -2. Since -2 is not an element of the set {-1, 0, 1} , this set is not closed with respect to addition.

Since a defining characteristic is not satisfied the set whose members are -1, 0, 1, taken with operation addition is not a group.

Item 4: The set of whole numbers whose members are 0, 1, 2, 3, 4, ... with operation addition

A partial table is provided:

+	0	1	2	3	.	.	.
0	0	1	2	3	.	.	.
1	1	2	3	4	.	.	.
2	2	3	4	5	.	.	.
3	3	4	5	6	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

Closure: A glance at the table shows that the sum of any two elements of the set results in an element of the set. So the set of whole numbers with operation addition is closed.

Identity: 0 is clearly the identity element because when 0 is added to any element of the set of whole numbers, the element is unchanged, i.e. 0 + 3 = 3 + 0 = 3; 0 + 12 = 12 + 0 = 12.

Inverses: The only element of the set of whole numbers with an inverse under operation addition is 0. The inverse of 0 is 0 because 0 + 0 = 0, the identity. There is no whole number which when added to the other whole numbers, 1, 2, 3, 4, ..., will result in the sum of 0, the identity element with respect to addition in this set. So, not every element in the set of whole numbers has a inverse under the operation addition.

Hence, the set of whole numbers whose members are 0, 1, 2, 3, ... taken with the operation addition is not a group because the inverse property is not satisfied.

WORKSHEET I

NAME________________________________ DATE______________

Directions: Identify each of the following items as an example or nonexample of a group by checking the appropriate box. If the set with the operation is not a group (nonexample), give at least one defining characteristic that is lacking, i.e. not closed, no identity, not every element has an inverse, not associative.

1. The set {-1, 0, 1} with the operation multiplication

x	-1	0	1
-1	1	0	-1
0	0	0	0
1	-0	0	1

example /__/
nonexample /__/ ______________________________

2. The set { 0, 1 } with operation multiplication

x	0	1
0	0	0
1	0	1

example /__/
nonexample /__/ ______________________________

3. The set {-1, 2 } with the operation addition

+	-1	2
-1	-2	1
2	1	4

example /__/
nonexample /__/ ______________________________

4. The set { 1 } with the operation multiplication

x	1
1	1

example /__/
nonexample /__/ ______________________________

5. The set { -1 } with the operation multiplication

x	-1
-1	1

example /__/
nonexample /__/ ______________________________

6. The set {x, y} with the operation *

*	x	y
x	x	y
y	y	z

example /__/
nonexample /__/ ________________________________

7. The set {0,1,2} with the operation *

*	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

example /__/
nonexample /__/ ________________________________

8. The set {-2,0,2} with the operation addition

+	-2	0	2
-2	-4	-2	0
0	-2	0	2
2	0	2	4

example /__/
nonexample /__/ ________________________________

9. The set of whole numbers whose members are 0,1,2,3,... with the operation multiplication

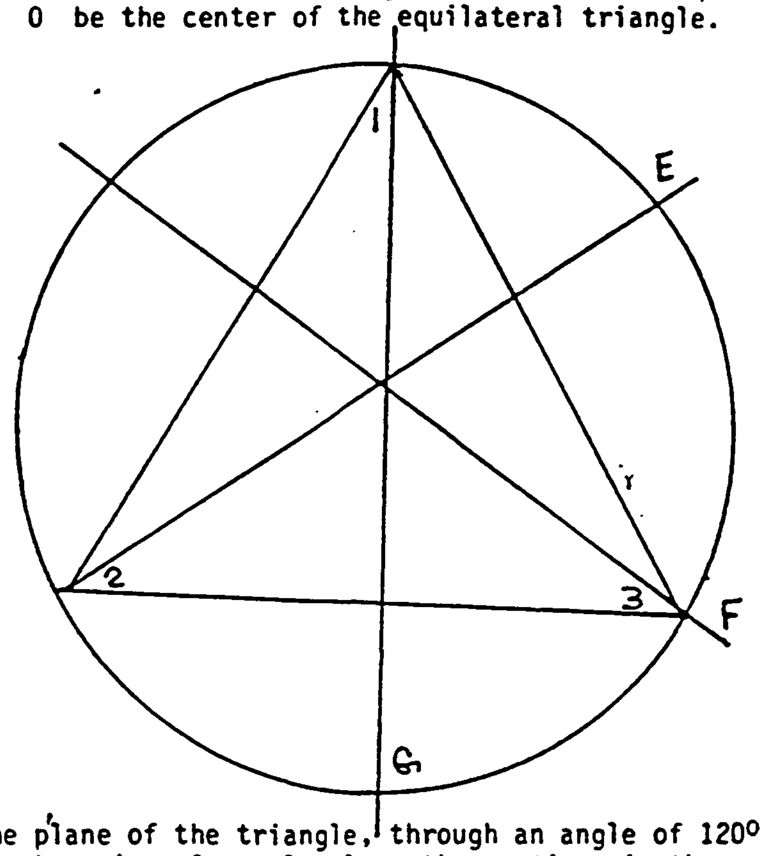
example /__/
nonexample /__/ ________________________________

10. The set of even whole numbers whose members are 0, 2, 4, 6, . . ., with the operation multiplication

example /__/
nonexample /__/ ________________________________

Rigid Motions of a Triangle

Today we will study all rigid motions of a triangle into itself. That is, we will consider motions such that the figure will look the same after the motion as before. Let us designate the vertices of the triangle as 1, 2 and 3. Let E, F and G be axis bisecting the sides of the equilateral triangle and let 0 be the center of the equilateral triangle.



A rotation, in the plane of the triangle, through an angle of 120° counterclockwise about the point 0 would place the vertices in the position 3 1 2. We may interpret the results of this rotation as mapping 1 into 3, 2 into 1 and 3 into 2.

A rigid motion of 120° counterclockwise results in the permutation

$$\alpha_1 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

Similary a rigid motion of 240° counterclockwise results in the permutation

$$\alpha_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

A rigid motion of 360°

$$\alpha_3 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

Let α_4 be the permutation which arises from a rotation through an angle of 180^0 about the line E (flip over E); α_5 the permuation arising from a flip over the line F and α_6 the permutaiton arising from a flip over the line G

$$\alpha_4 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \qquad \text{flip over E}$$

$$\alpha_5 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} \qquad \text{flip over F}$$

$$\alpha_6 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \qquad \text{flip over G}$$

Let us consider the set $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$ of permutations obtained by rigid motions of the triangle and the operation "multiplication." Two permutations $\alpha_i\ \alpha_j$ may be multiplied by performing rigid motion α_i followed by rigid motion α_j on the result.

*	α_1	α_2	α_3	α_4	α_5	α_6
α_1			α_1			
α_2			α_2			
α_3			α_3			
α_4			α_4			
α_5			α_5			
α_6	α_5	α_4	α_6	α_2	α_1	α_3

1. Complete the above table.

2. Is the set of permutations closed under operation * ?

3. Is there an identity element? If so name it______________ .

4. Give the following inverses if they exist.

$\alpha_1{}^{-1}$ = ______________

$\alpha_2{}^{-1}$ = ______________

$\alpha_3{}^{-1}$ = ______________

$\alpha_4{}^{-1}$ = ______________

$\alpha_5{}^{-1}$ = ______________

$\alpha_6{}^{-1}$ = ______________

5. Is the associative property satisfied? ________________
Show two cases to support your answer.

6. Is the set of permutations obtained by rigid motions of the triangle with the operation "multiplication" a group?

7. Is it abelian? __________. Show two cases to support your answer.

WORKSHEET III

NAME______________________________ DATE________________

Worksheet IV

Name ______________________________ Date ____________________

Suppose that for any integer we consider only the remainder resulting from division by 5, and we define two integers to be "equivalent" if they have the same remainder. We express that 12 and 17 both have the same remainder when divided by 5 by writing

$$12 \equiv 17 \pmod{5}$$

where $\equiv$ denotes "equivalent" and "mod" is an abbreviation for "modulo". Similarly, we write

$$3 \equiv 8 \pmod{5}$$

1. Consider the set $A = \{0,1,2,3,4\}$ and the binary operation "addition modulo 5", denoted by $\oplus$. Please note that if a and b are two elements of set A, then

$$a \oplus b = 2 \quad \text{if} \quad a + b \equiv 2 \pmod{5}$$

Show that the set A with binary operation "addition modulo 5" constitutes a group. Fill in the chart first.

$\oplus$	0	1	2	3	4
0					
1					
2					
3					
4					

2. Show that the set of integers $\{1,2,3,4\}$ with the binary operation "multiplication modulo 5" is a group.

$\otimes$	1	2	3	4
1				
2				
3				
4				

3. Let $p > 1$ be a prime number, i.e., a number with precisely two positive integral divisors, 1 and p, and consider the set

$$\{1,2,3,\ldots, p-1\}$$

We claim that "multiplication modulo p" is a binary operation on this set. Show that the group properties are satisfied.

UNIT ON FUNCTIONS

Reasoning skills 2-4 will be stressed

Vocabulary

set
ordered pair
Cartesian product
relations
functions
inverse of a function
domain of a function
range of a function
"into"
"onto"
1-1
constant function
real-valued function
linear function
independent variable
dependent variable
slope of a line
intercept of a line
identity function
graph of a function

Symbols

ε , f , $f{:}A \to B$, $A \xrightarrow{f} B$, $f(a)$, $a \varepsilon A$

Behavioral Objectives

1. Given two finite sets A and B the student will identify subsets of the Cartesian product which are
   (a) functions of A into B
   (b) functions of A onto B
   (c) functions of A into B which are 1-1
   (d) relations
   (e) constant functions

2. Given a collection of ordered pairs that represent a function, the student will be able to
   (a) identify the domain of the function
   (b) identify the range of the function
   (c) give the inverse of the function
   (d) state whether or not the inverse of the function is itself a function and justify the answer.

3. Given a table of three intergral values of a linear function the student will be able to
   (a) write a corresponding equation of the function
   (b) give the slope of the line determined by the function
   (c) sketch the graph of the function
   (d) give the x- and y- intercept of the line

4. Given several graphs of relations,the student will be able to identify those graphs which represent functions.

5. Given information regarding the slope of a line, i.e., positive slope, negative slope, slope of '0', slope does not exist, the student will be able to discuss the direction of the corresponding line.

6. Given a table of three integral values of a quadratic function, the student will be able to
   a) write a corresponding equation of the function

7. Given at least five terms of a sequence, the student will find the next number and the n^{th} term.

Scanigs

NAME ____________________________

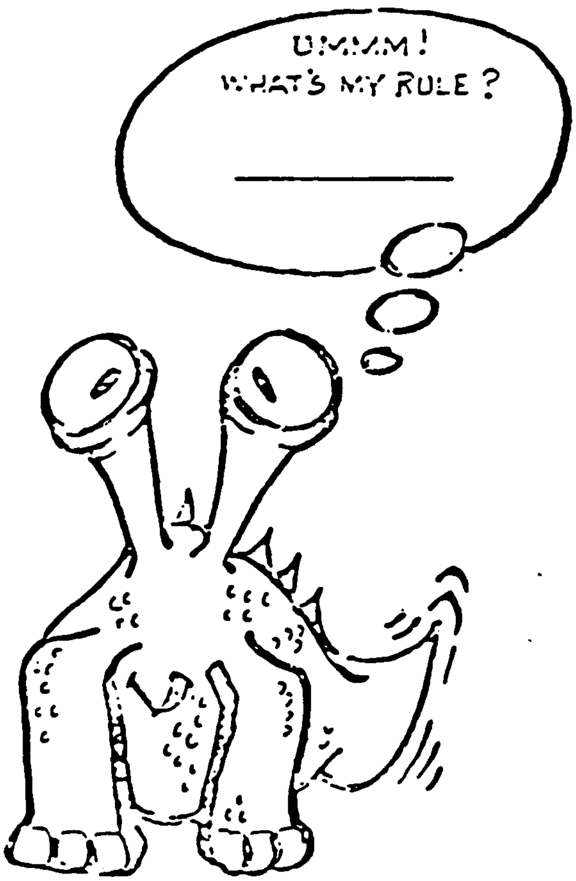
All of these are scanigs.

21 → 35 71 → 85

41 → 55 22 → 36

Make scanigs out of these:

1. 3 → ☐
2. 19 → ☐
3. 47 → ☐
4. 55 → ☐
5. 0 → ☐
6. 108 → ☐



Make your own scanigs!

○ → ☐

○ → ☐

RELATIONS AND FUNCTIONS ON SETS

1. The Cartesian product of sets A and B is the set of all ordered pairs such that the first element of each pair is an element of A and the second element of each pair is an element of B (written $A \times B$).

   If $A = \{a,b,c\}$ and $B = \{1,2\}$, list the elements of $A \times B$ and the elements of $B \times A$.

   Also, list the elements of $A \times A$.

2. A relation from set A into set B is a subset of $A \times B$.

   If $A = \{a,b,c\}$ and $B = \{1,2\}$, define five relations from set A into set B.

   How many relations are there from set A into set B?

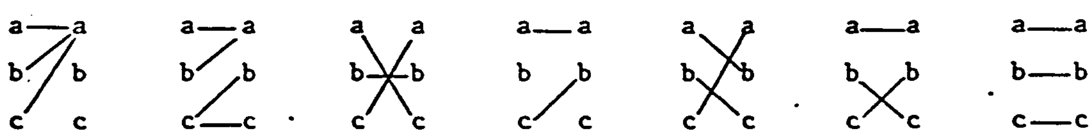
   How many relations are there from set A into set A?

3. A function from set A into set B is a relation from A into B in which each element of A is paired with one and only one element of B.

   i. The set of all first components of a function, all the elements of A, is called the domain of the function.

   ii. The set of all second components of a function, a subset of B, is called the range of the function.

   Which of the following relations define functions on the set $A = \{a,b,c\}$?



   What is the range of each of the functions?

   How many functions are there from set A into set A?

   If $A = \{a,b,c\}$ and $B = \{1,2\}$, how many functions are there from set A into set B?

   If set A has n elements and set B has m elements, determine the following:

   a. How many relations are there from set A into set B?

   b. How many functions are there from set A into set B?

WORKSHEET I

NAME____ DATE______________

1. If $A = \{1,2,3\}$ and $B = \{a,b,c\}$ the Cartesian product of A and B, denoted AXB, is

$\{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c), (3,a), (3,b), (3,c)\}$

a. Give a subset of AXB which represents a function from A into B.

____

b. Give a different subset of AXB which represents a function from A onto B.

____

c. Give a different subset a AXB which represents a 1-1 function from A into B.

____

d. Give a different subset of AXB which represents a relation which is not a function.

____

e. Give a different subset of AXB which represents a function from A into B which is a constant function.

____

f. Give a different subset of AXB which represents a function from A into B which is not onto.

____

2. Let $A = \{1,2,3,...20\}$ and $B = \{1,2,3,...100\}$ and $f:A \rightarrow B$ such that $f(a) = 2a$ for every $a \in A$.

a. Give the other elements of $f\{(1,2)$ ______________$\}$

b. Give the domain of f.______________

c. Is f onto? ________________.

d. Is f 1-1? ________________

e. Give the inverse of f, denoted f^{-1}

____

f. Give the domain of f^{-1}

____

g. Give the range of f^{-1}

____

h. Is f^{-1} a function?

____

Justify your answer.

____

____

WORKSHEET II
"GUESSING FUNCTIONS"

NAME______________________________________ DATE____________

Find the rule associated with each integer chart and complete the accompanying statement:

(1)

x	f(x)
1	6
2	11
3	16

Rule: f(x) = 5x + 1
For every unit increase in x, there is a 5 unit increase in f(x)

(2)

Δ	□
2	1
3	0
4	-1

Rule:____________
For every unit increase in Δ, there is a ____ unit __________ in □.

(3)

x	y
3	13
4	15
5	17

Rule:____________
For every unit increase in x, there is a ______ unit ________ in y.

(4)

z	f(z)
2	0
4	1
5	3/2

Rule:____________
For every unit increase in z, there is a ______ unit __________ in f(z).

(5)

Δ	□
2	-3
3	-7
4	-11

Rule:____________
For every unit increase in Δ, there is a ______ unit __________ in □.

(6)

x	f(x)
3	-5
5	-7
7	3/2

Rule:____________
For every unit increase in x, there is a _____ unit__________ in f(x).

(7)

x	f(x)
1	-4/7
3	-2
4	-2 5/7

Rule:____________
For every unit increase in x, there is a ______ unit ________ in f(x)

(8)

x	y
11	0
15	4
25	14

Rule:____________
For every unit increase in x, there is a ______ unit ________in y.

(9)

z	f(z)
0	10
10	14
20	18

Rule:____________
For every unit increase in z, there is a ______ unit ______ in f(z)

(10)

z	f(z)
2	3
10	3
15	3

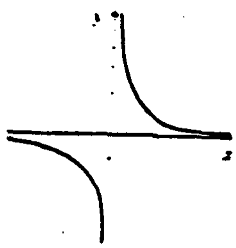
Rule:____________
For every unit increase in z, there is a ______ unit______ in f(z).

WORKSHEET III

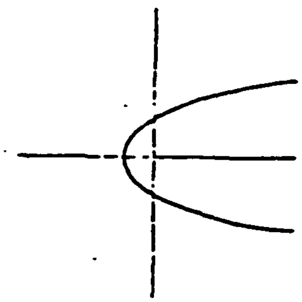
1. Sketch the graph of each rule given on Worksheet II, and note the slope and intercepts.

2. Which of the following graphs represent functions. Write yes or no and justify.

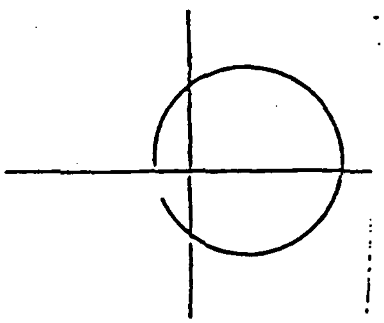
(a)

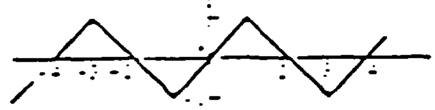


(b)



(a) ______________________ (b) ______________________





(c) ______________________ (d) ______________________

3. Discuss the direction of the line if the slope of that line

a) is negative

____

____

____

b) is positive

____

____

____

c) is 0

____

____

____

d) does not exist

____

____

____

WORKSHEET IV

A SEQUENCE IS A FUNCTION

NAME______________________________ DATE__________________

NUMBER SEQUENCES

A number sequence is a succession of numbers arranged according to some definite pattern. Any number in this sequence is related to its preceding number according to a definite plan. In the relatively simple sequence, 3, 5, 7, 9, 11 .... two is added to each number in the sequence. Other relationships may involve the processes of subtraction, multiplication, division, squaring, extracting roots and, in the case of more difficult problems, it may involve a combination of these processes. For example, in the sequence 1, 2, 2, 5, 3, 10, 4, 17, 5, the second number in each pair is the square of the first number plus 1. Thus, 2 is $1^2 + 1$, 5 is $2^2 + 1$, 10 is $3^2 + 1$, etc. Similarly, in the sequence 3, 13, 53, 213 .... one is added to the product of the preceding number and 4.

PRACTICE EXERCISES

Find the next number in each of the following sequences

1. 1, 3, 5, 7, 9 . . .
2. 3, 3, 6, 6, 9, 9, 12 . . .
3. 6, 11, 16, 21, 26 . . .
4. −9, −6, −3, 0 . . .
5. 36, 39, 39, 43, 43, 48, 48 . . .
6. $28\frac{1}{8}, 21\frac{1}{8}, 14\frac{1}{8}, 7\frac{1}{8}$ . . .
7. 5, 7, 11, 17, 25 . . .
8. −26, −20, −14, −8 . . .
9. 18, 26, 27, 35, 36, 44 . . .
10. 5, 6, 8, 11, 15, . . .
11. $7\frac{1}{2}, 6\frac{3}{4}, 6, 5\frac{1}{4}, 4\frac{1}{2}$ . . .
12. $9\frac{3}{4}, 9\frac{5}{8}, 9\frac{1}{2}, 9\frac{3}{8}$ . . .
13. 2.52, 3.02, 3.52, 4.02 . . .
14. 5.3, 6.4, 7.5, 8.6, 9.7 . . .
15. 11, 28, 79, 232 . . .
16. 5, 8, 9, 12, 13, 16 . . .
17. 7, 8, 10, 13, 17, 22 . . .
18. 79, 77, 75, 73, 71 . . .
19. 79, 74, 70, 67, 65 . . .
20. 21, 20, 18, 15, 11, 6 . . .
21. 13, 12, 10, 7 . . .
22. −5, 10, −20, 40, −80 . . .
23. −2, −4, −8, −16, −32 . . .
24. 1, 4, 9, 16, 25, 36, 49 . . .
25. 5, 10, 17, 26, 37 . . .
26. 2, 6, 14, 30, 62 . . .
27. 1782, 594, 198, 66 . . .
28. 4752, 792, 132 . . .
29. 99, 88, 77, 66, 55 . . .
30. 100, 81, 64, 49, 36 . . .
31. 5, 2, 5, 4, 5, 6, 5 . . .
32. 48, 24, 12, 6 . . .
33. 80, 2, 40, 2, 20, 2 . . .
34. 5, 8, 24, 27, 81, 84 . . .
35. 3, 5, 10, 12, 24, 26 . . .
36. 5, 6, 8, 11, 15, 20 . . .
37. 7, 14, 14, 21, 21, 28, 28 . . .
38. 3, 6, 5, 8, 7, 10, 9 . . .
39. 6, 10, 13, 17, 20, 24 . . .
40. 12, 16, 13, 17, 14, 18 . . .
41. 5, 6, 8, 11, 15, 20, 26 . . .
42. $\frac{1}{15}$, 0.2, 0.6, 1.8 . . .
43. 65, 60, 56, 53, 51 . . .
44. 256, 16, 4, 2 . . .
45. 400, 361, 324, 289, 256 . . .
46. 121, 144, 169, 196 . . .
47. 512, 343, 216, 125, 64 . . .
48. 2, 5, 15, 18, 54 . . .
49. 16, 256, 15, 225, 14, 196, 13 . . .
50. 4, 64, 5, 125, 6 . . .

NAME ______________________________

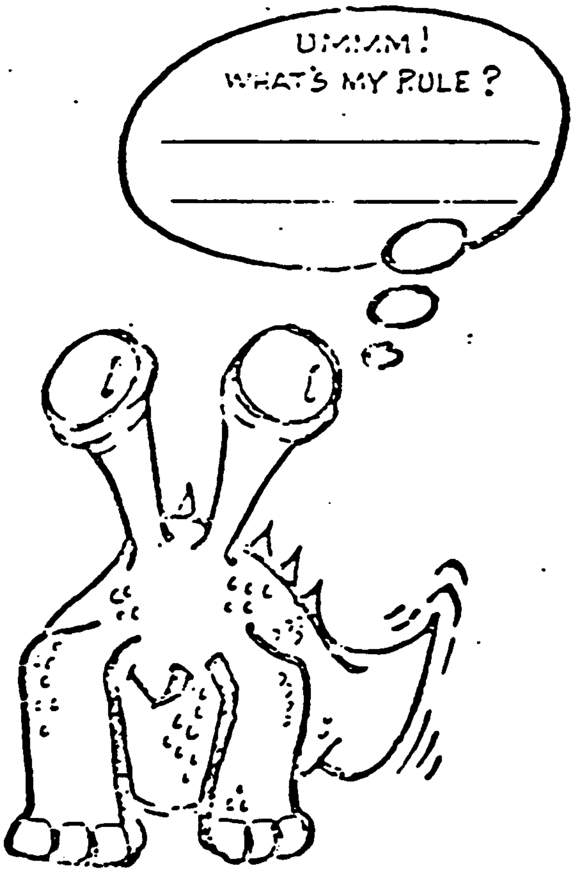
All of these are escams.

3 → 4 | 9 → 64 | 4 → 9

5 → 16 | 21 → 400

Make escams out of these:

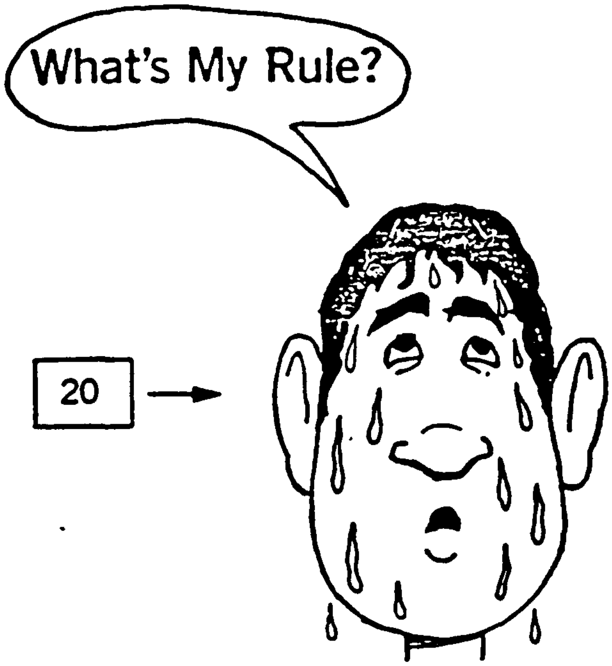
1. 6 → ☐
2. 1 → ☐
3. 10 → ☐
4. 2 → ☐
5. 12 → ☐
6. 20 → ☐



Make your own escams!

◯ → ☐

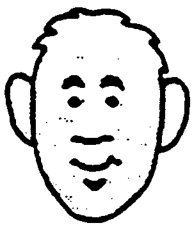
◯ → ☐

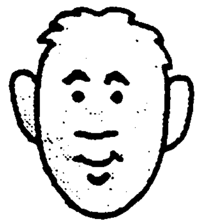


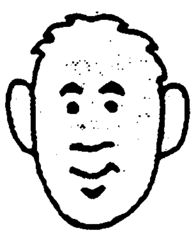
20 → → 7

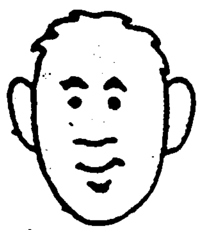
13 →  → 0

23 →  → 10

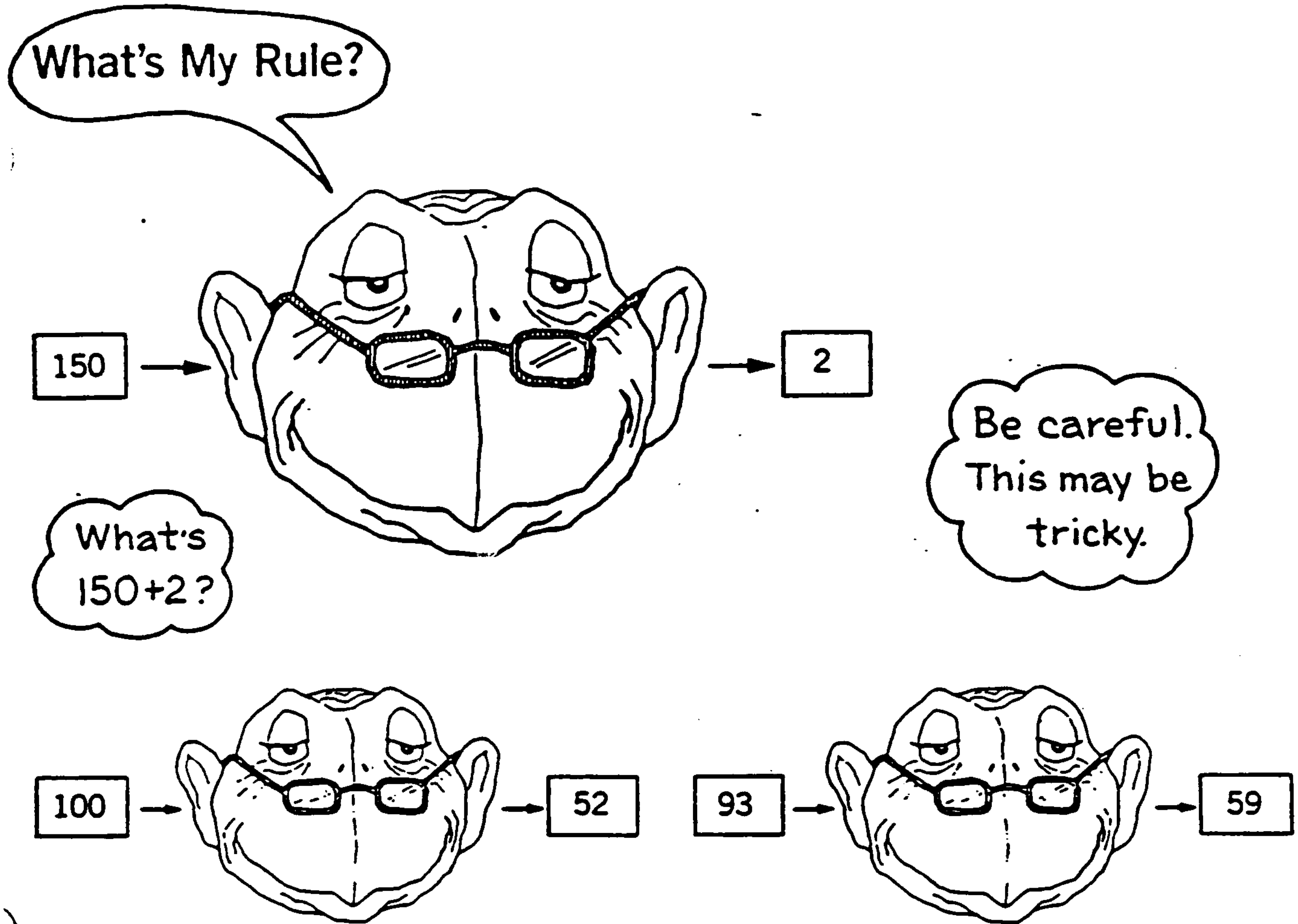
1. 14 →  → ☐

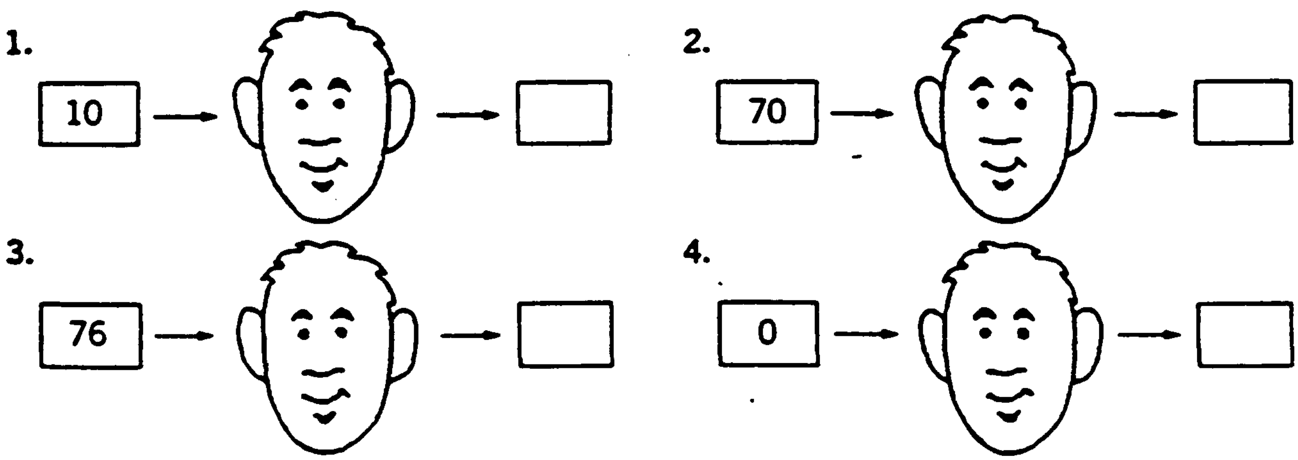
2. 20 →  → ☐

3. 100 →  → ☐

4. 992 →  → ☐

The answer to 4 is the height in meters of the world's highest waterfall.





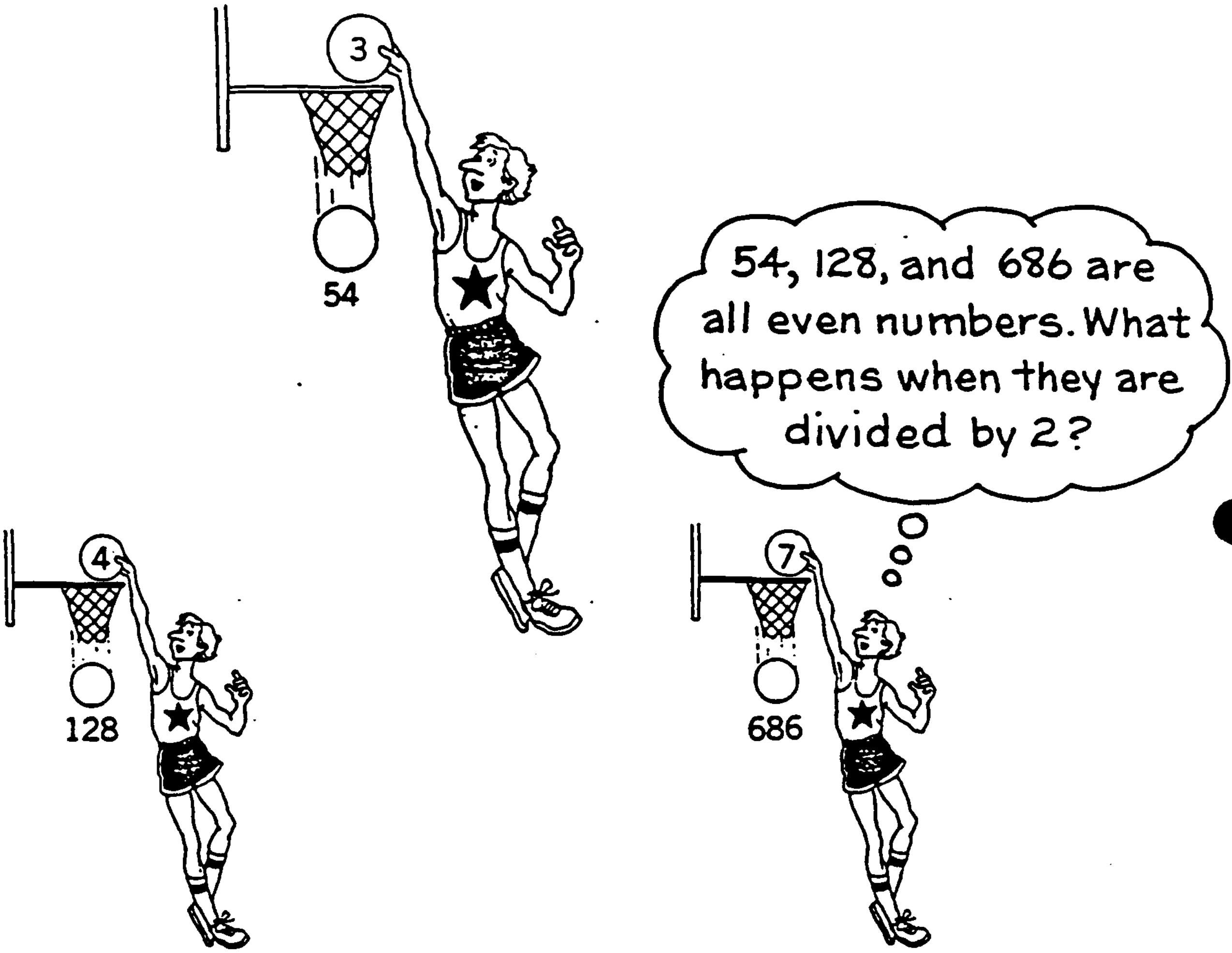
The answer to 4 is the oldest recorded age of any animal (a tortoise).

What's My Rule?

1.	3→□→39,	8→□→104,	11→□→143,	100→□→	______
2.	5→□→13,	11→□→ 19,	37→□→ 45,	1000→□→	______
3.	9→□→ 6,	14→□→ 11,	98→□→ 95,	800→□→	______
4.	5→□→14,	12→□→ 7,	2→□→ 17,	19→□→	______
5.	3→□→13,	9→□→ 37,	15→□→ 61,	100→□→	______
6.	1→□→10,	6→□→ 0,	4→□→ 4,	5→□→	______
7.	4→□→17,	8→□→ 37,	17→□→ 82,	400→□→	______
8.	3→□→21,	13→□→ 91,	23→□→161,	103→□→	______
9.	8→□→46,	2→□→ 10,	14→□→ 82,	50→□→	______
10.	3→□→34,	18→□→ 19,	23→□→ 14,	30→□→	______
11.	5→□→14,	8→□→ 23,	10→□→ 29,	1000→□→	______
12.	1→□→87,	7→□→ 69,	15→□→ 45,	20→□→	______
13.	2→□→29,	8→□→ 77,	10→□→ 93,	500→□→	______
14.	0→□→58,	9→□→ 49,	45→□→ 13,	50→□→	______
15.	5→□→15,	2→□→ 9,	17→□→ 39,	500→□→	______
16.	4→□→ 3,	11→□→ 31,	20→□→ 67,	1010→□→	______

To check your answers, use the Answer List for Review Problems.

What Comes Out?



1. **In:** 1

Out: ☐

2. **In:** 5

Out: ☐

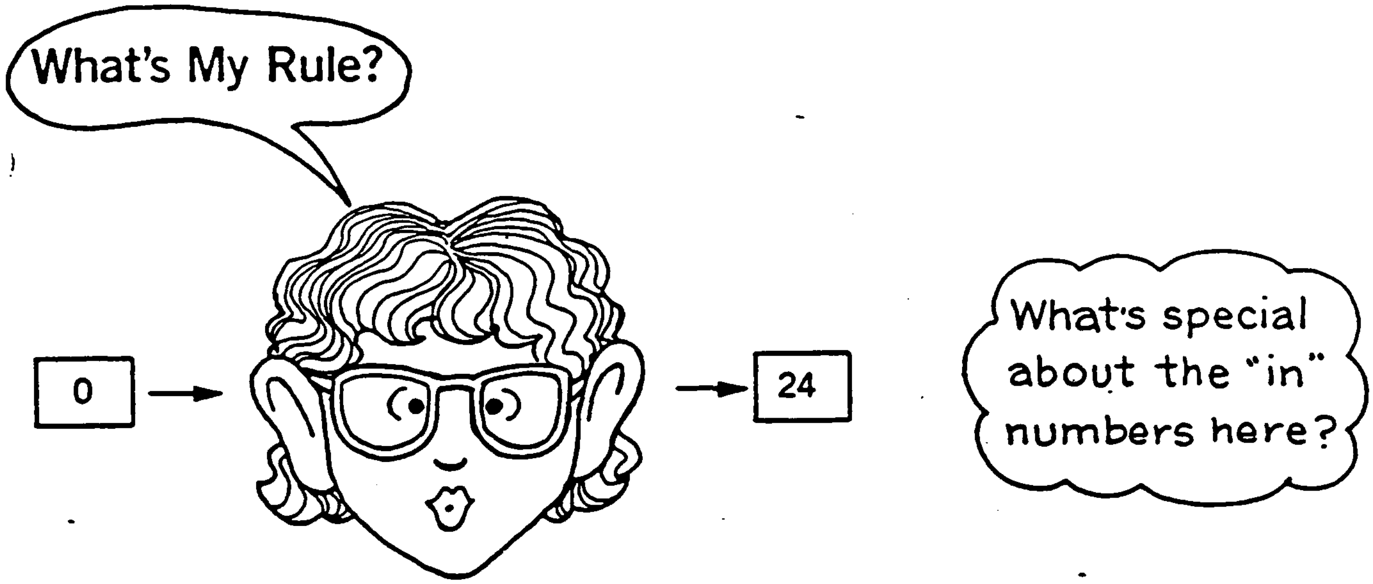
3. **In:** 21

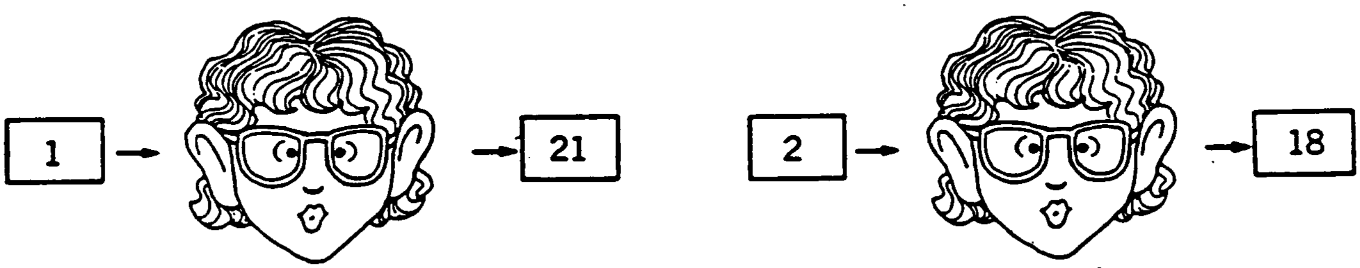
Out: ☐

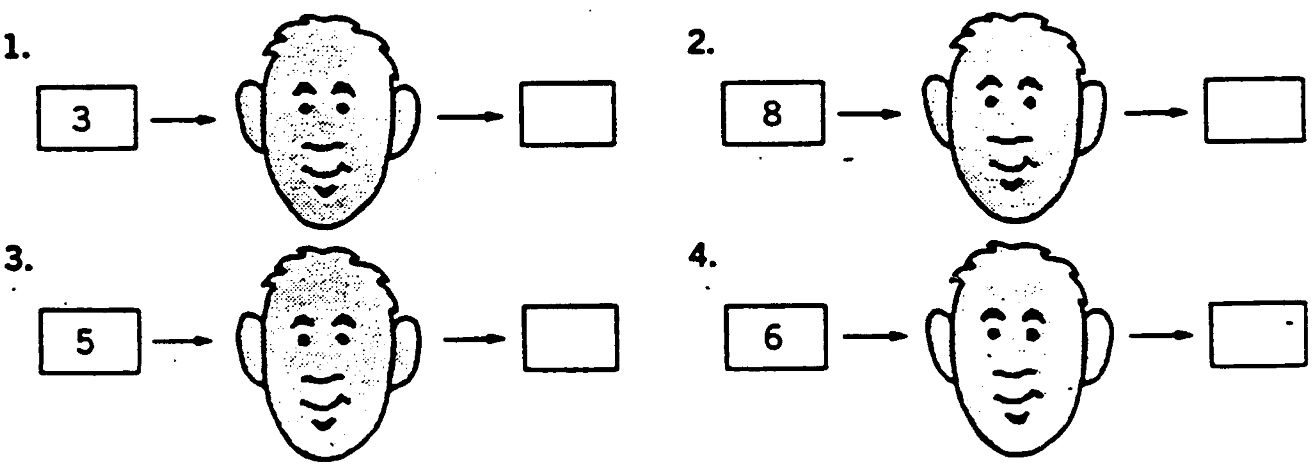
4. **In:** n

Out: ☐

The answer to 4 is an expression involving n.







The answer to 4 is equal to both the sum and the product of the same three numbers.

What Comes Out?

1. In: 5

   Out: ☐

2. In: 10

   Out: ☐

3. In: 17

   Out: ☐

4. In: n

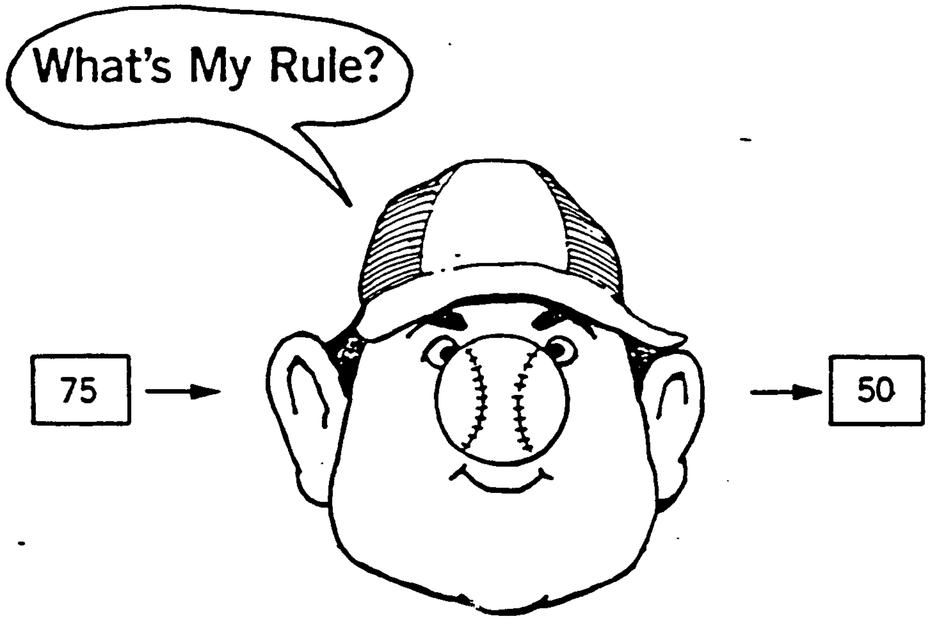
   Out: ☐

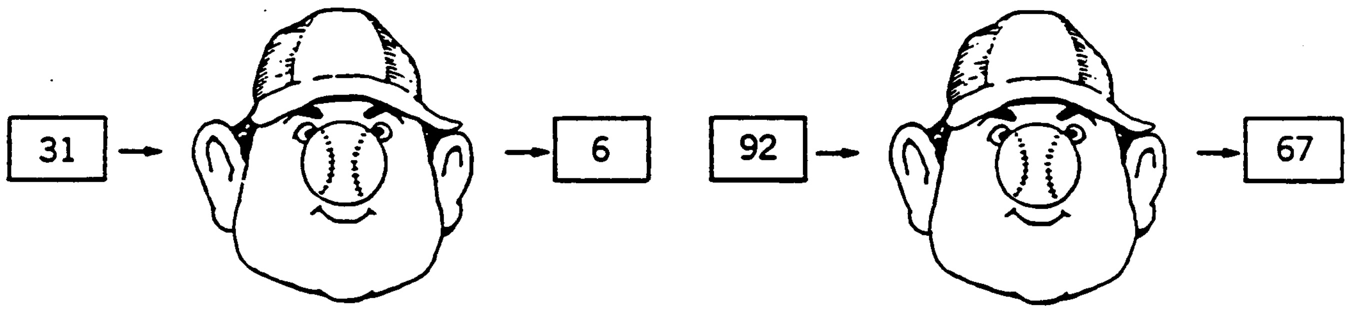
The answer to 4 is an expression involving n.

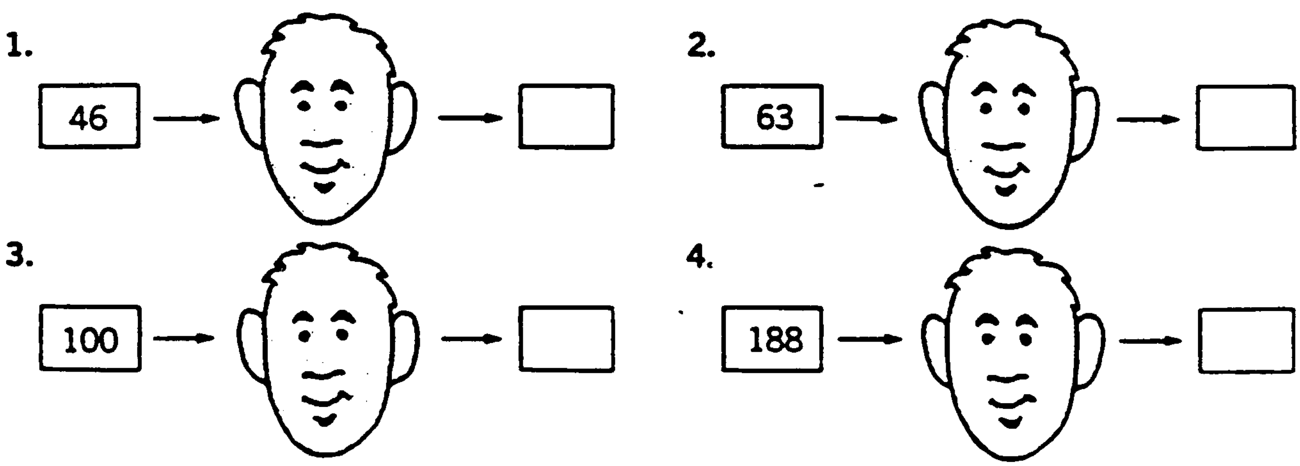
What Comes Out?

1.	$0 \to \square \to 16$,	$16 \to \square \to 32$,	$8 \to \square \to 24$,	$n \to \square \to$	______
2.	$0 \to \square \to 0$,	$1 \to \square \to 1$,	$8 \to \square \to 64$,	$n \to \square \to$	______
3.	$10 \to \square \to 1$,	$25 \to \square \to 16$,	$18 \to \square \to 9$,	$n \to \square \to$	______
4.	$1 \to \square \to 10$,	$2 \to \square \to 13$,	$3 \to \square \to 18$,	$n \to \square \to$	______
5.	$12 \to \square \to 60$,	$36 \to \square \to 36$,	$4 \to \square \to 68$,	$n \to \square \to$	______
6.	$8 \to \square \to 8$,	$7 \to \square \to 23$,	$6 \to \square \to 36$,	$n \to \square \to$	______
7.	$8 \to \square \to 104$,	$7 \to \square \to 91$,	$0 \to \square \to 0$,	$n \to \square \to$	______
8.	$5 \to \square \to 125$,	$2 \to \square \to 8$,	$3 \to \square \to 27$,	$n \to \square \to$	______
9.	$5 \to \square \to 27$,	$2 \to \square \to 12$,	$3 \to \square \to 17$,	$n \to \square \to$	______
10.	$1 \to \square \to 3$,	$4 \to \square \to 66$,	$10 \to \square \to 1002$,	$n \to \square \to$	______
11.	$1 \to \square \to 4$,	$4 \to \square \to 25$,	$9 \to \square \to 60$,	$n \to \square \to$	______
12.	$2 \to \square \to 5$,	$3 \to \square \to 24$,	$7 \to \square \to 340$,	$n \to \square \to$	______
13.	$3 \to \square \to 91$,	$30 \to \square \to 10$,	$12 \to \square \to 64$,	$n \to \square \to$	______
14.	$3 \to \square \to 9$,	$1 \to \square \to 3$,	$2 \to \square \to 5$,	$n \to \square \to$	______
15.	$3 \to \square \to 26$,	$1 \to \square \to 2$,	$2 \to \square \to 8$,	$n \to \square \to$	______
16.	$6 \to \square \to 36$,	$5 \to \square \to 68$,	$3 \to \square \to 92$,	$n \to \square \to$	______

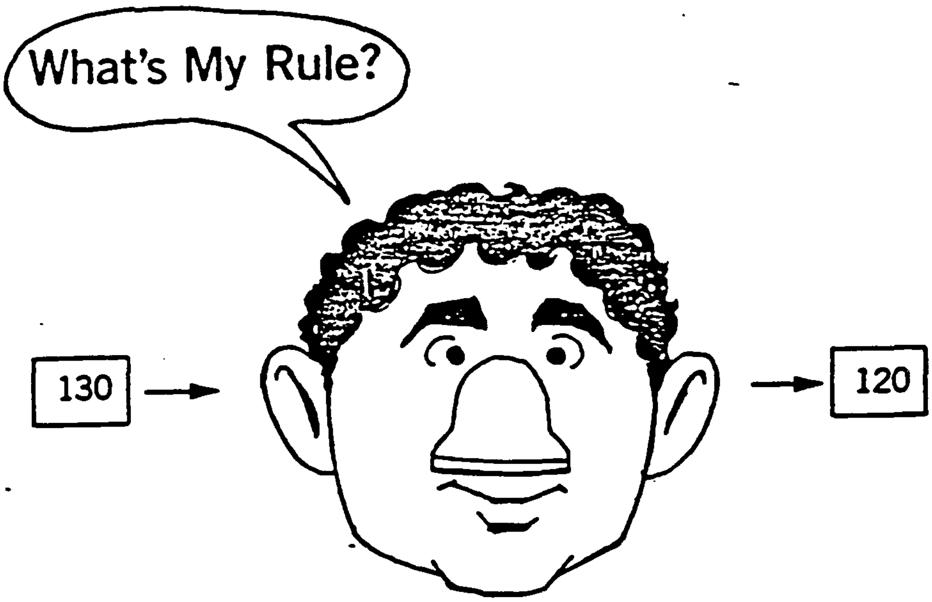
To check your answers, use the Answer List for Review Problems.



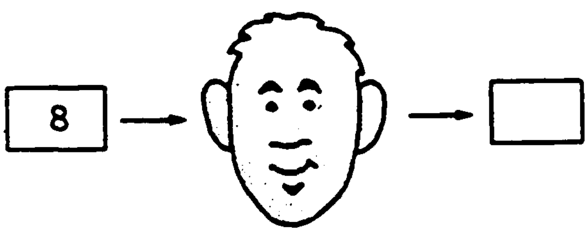




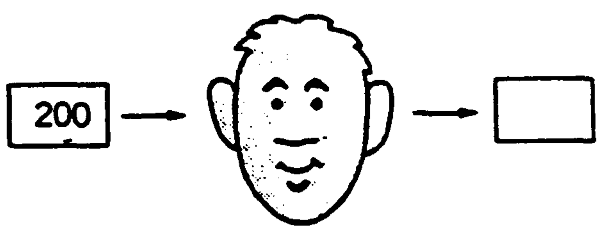
The answer to 4 is the speed in kilometers per hour of the fastest recorded pitch in baseball.



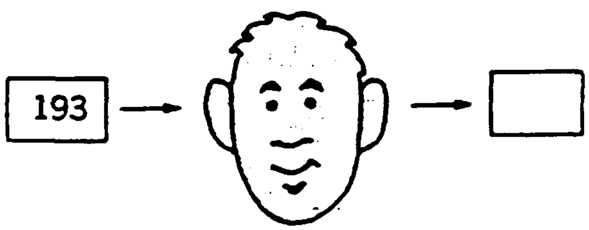
1.



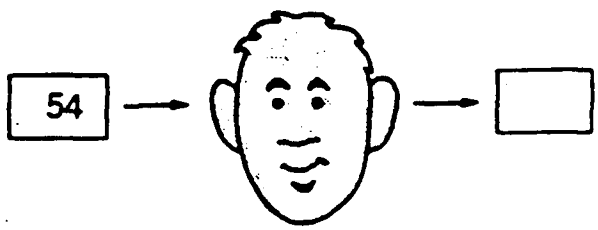
2.



3.



4.



The answer to 4 is the weight in metric tons of the heaviest bell in the world.

What's My Rule?

Activity 21

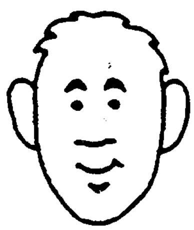
3 →  → 974

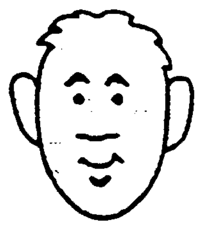
What's special about the "in" numbers here?

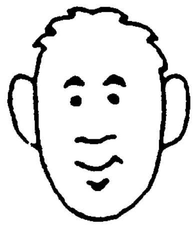
13 →  → 884

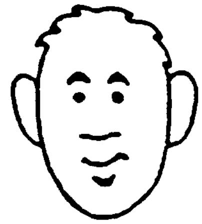
23 →  → 794

Try to guess what number comes out. If you need a hint, look up the answer for 1 on Answer List 1, for 2 on Answer List 2, for 3 on Answer List 3, and for 4 on Answer list 4.

1. 43 →  → ☐

2. 93 →  → ☐

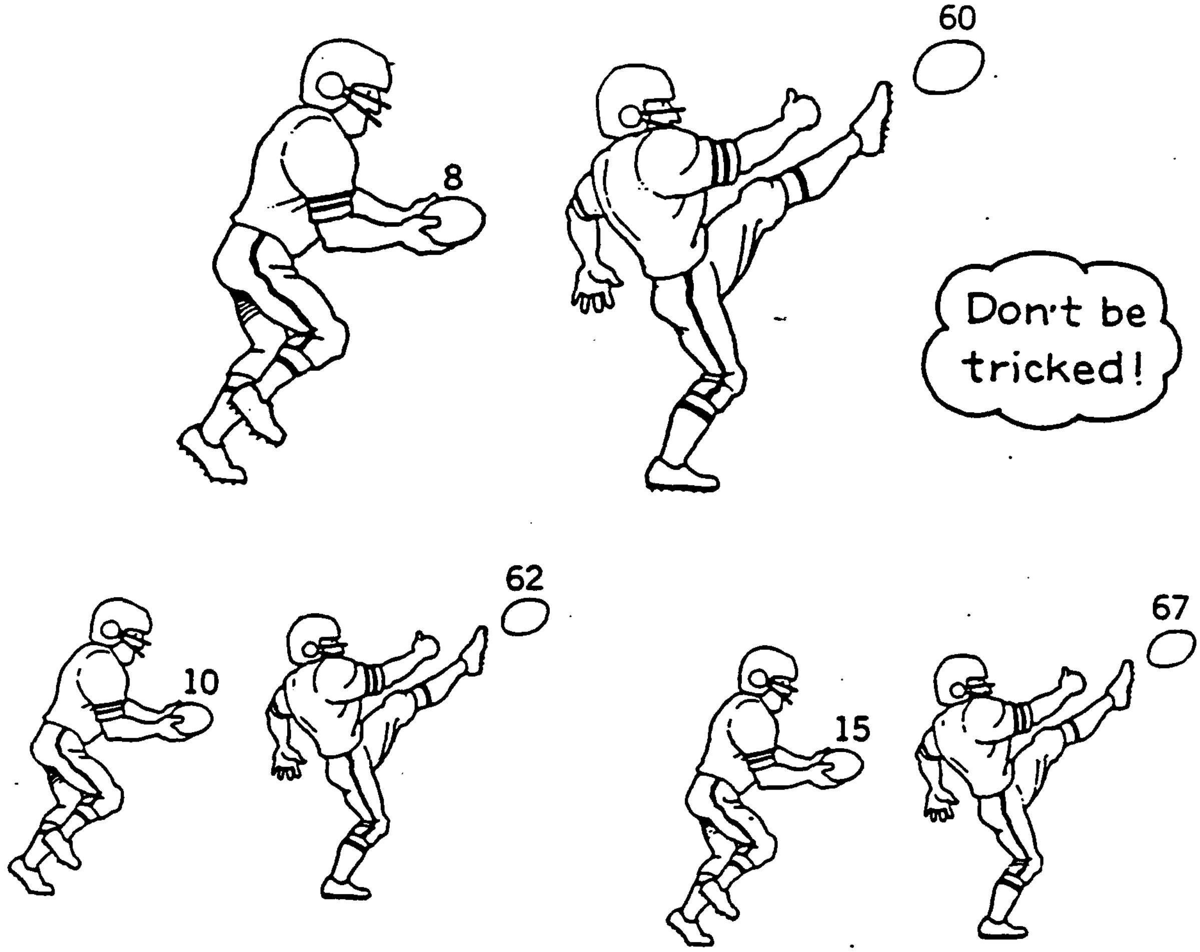
3. 100 →  → ☐

4. 68 →  → ☐

The answer to 4 is the weight in kilograms of the heaviest Indian tiger.

What Comes Out?

Activity 25



Try to guess what number comes out. If you need a hint, look up the answer for **1** on Answer List 1, for **2** on Answer List 2, for **3** on Answer List 3, and for **4** on Answer list 4.

1. In: 3

Out: ☐

2. In: 12

Out: ☐

3. In: 100

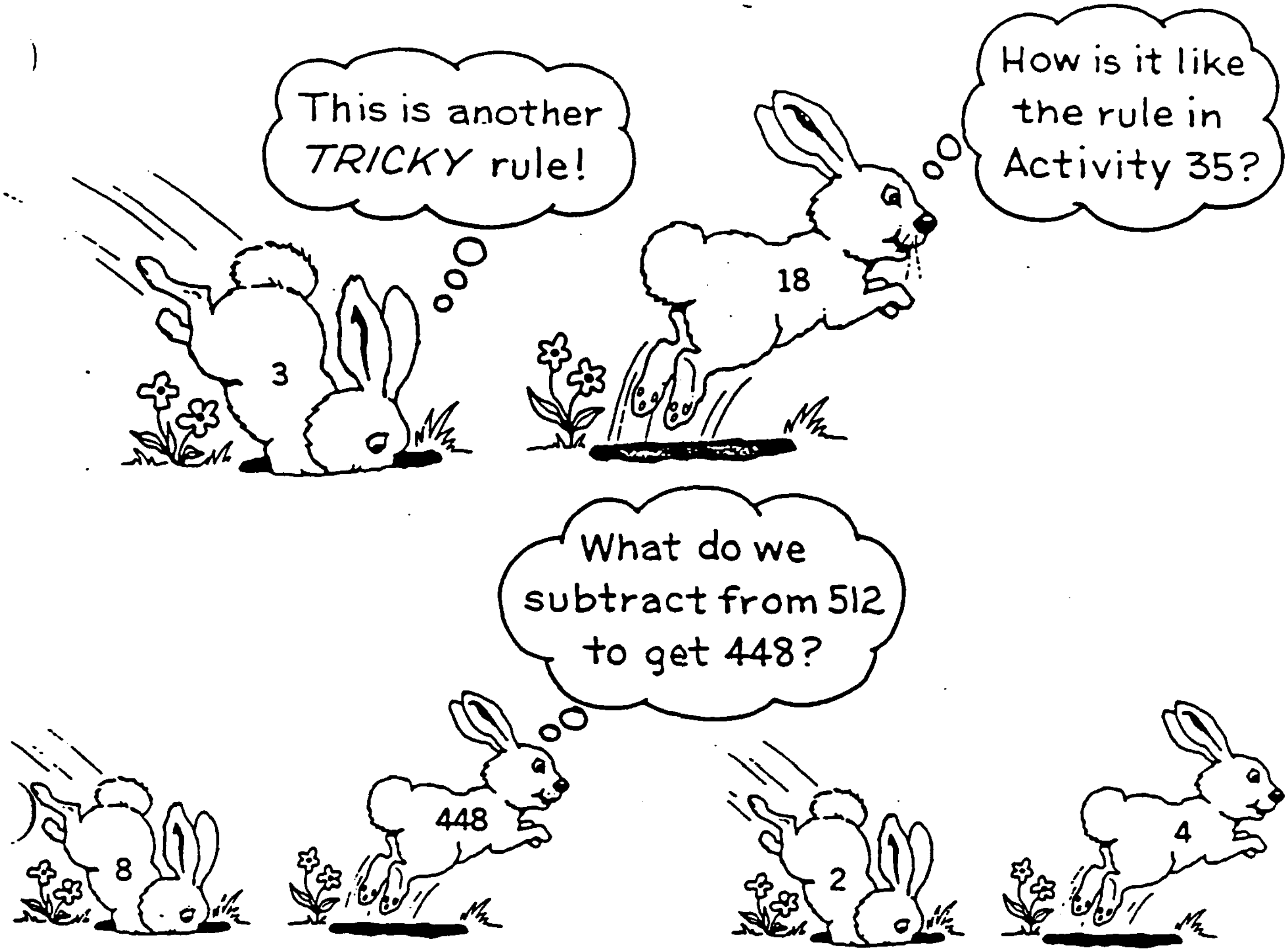
Out: ☐

4. In: n

Out: ☐

The answer to **4** is an expression involving n.

What Comes Out?



Try to guess what number comes out. If you need a hint, look up the answer for **1** on Answer List 1, for **2** on Answer List 2, for **3** on Answer List 3, and for **4** on Answer list 4.

1. In: 4

Out: ☐

2. In: 6

Out: ☐

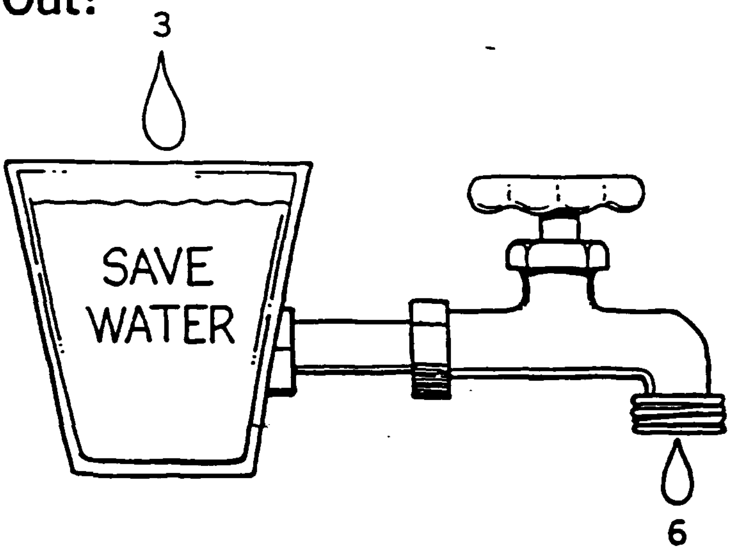
3. In: 1

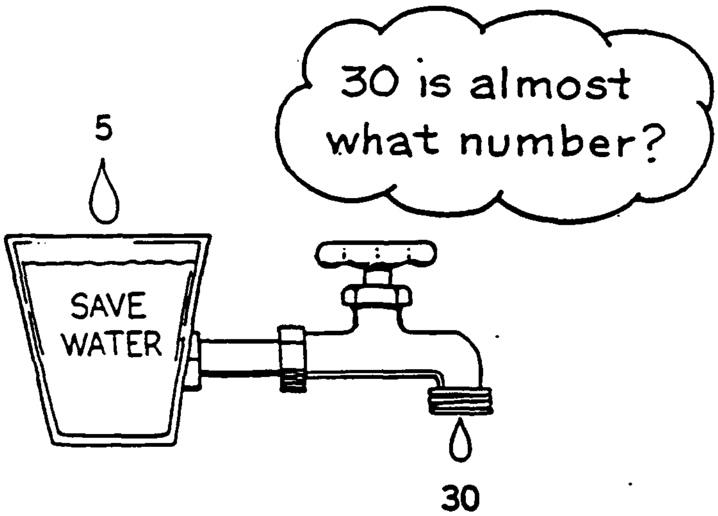
Out: ☐

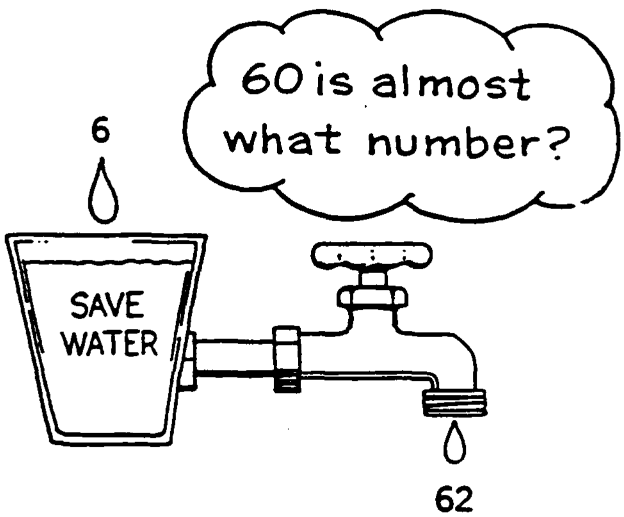
4. In: n

Out: ☐

The answer to **4** is a slightly complicated expression involving n.







Try to guess what number comes out. If you need a hint, look up the answer for 1 on Answer List 1, for 2 on Answer List 2, for 3 on Answer List 3, and for 4 on Answer list 4.

1. In: 2

Out: ☐

2. In: 1

Out: ☐

3. In: 4

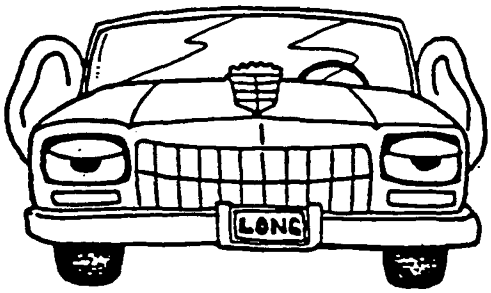
Out: ☐

4. In: n

Out: ☐

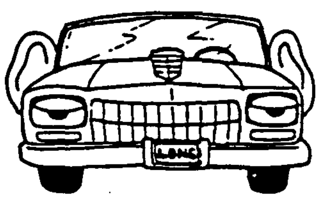
The answer to 4 is an expression involving n.

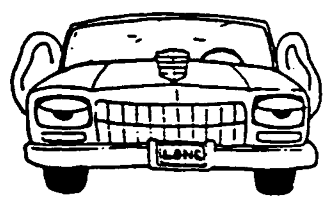
What's My Rule?

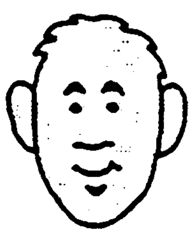
3 → 

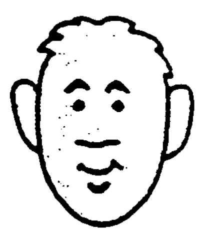
→ 39

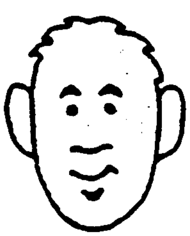
This may be very difficult.

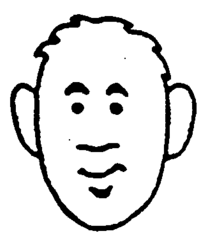
4 →  → 34

0 →  → 54

1. 6 →  → ☐

2. 8 →  → ☐

3. 10 →  → ☐

4. 9 →  → ☐

The answer to 4 is the length in meters of the longest car (a Cadillac).

PROBLEM SOLVING

1. At a party for mathematicians the host announces the following:

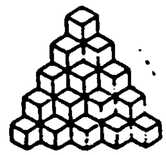
   I have three daughters. The product of their ages is 72. The sum of their ages is the same number as our house number. How old are my daughters?

   The guests confer, go outside to look at the house number, and then return to say that there is insufficient information to solve the problem. Thereupon the host adds this statement:

   My oldest daughter loves chocolate pudding.

   With this new bit of information the guests are able to determine the ages of the three daughters. What are these ages?

2. The tower to the right is made of 35 cubes in 5 layers. How many cubes are needed to form a similar tower with 10 layers?



3. What is my number?

   (a). It is a two-digit number.
   (b). It is a multiple of 6.
   (c). The sum of the digits is 9.
   (d). The ten's digit is one-half of the unit's digit.

4. The table to the right defines a binary operation on the set {a,b,c} if it is completed with elements of that set. How many binary operations can be defined on the set {a,b,c}?

+	a	b	c
a	___	___	___
b	___	___	___
c	___	___	___

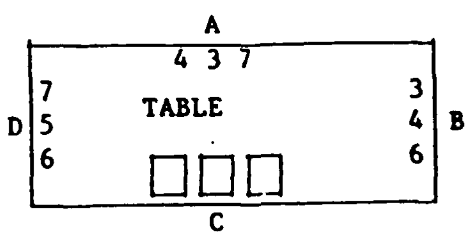
5. To play this game two decks of cards are needed: One deck contains the cards 3 through 7 of each suit of a bridge deck; the second deck has twelve question cards.

   To play this game, select groups of four players and place the two decks of cards in the middle of the group. After the decks have been shuffled, each player draws three cards, face down, from the bridge deck which he/she then (without seeing his numbers) props up in front of him/her for the other players to see. When play begins each player can see the number combinations of all the players except his own.

   The player who begins, draws the top card from the second deck, reads the question aloud for all players to hear and answers the question in accordance with the three combinations which he can see, before putting the card on the bottom of the deck.

   The following represents a record of play among players A, B, C and D with C's hand unknown to the reader. Answers of successive players are given as they occurred in actual play. The problem is to determine the implications of the answers; thus guessing the three numbers C had.

NO. OF PLAY	PLAYER	QUESTION	ANSWER
1	D	How many cards have numbers that are multiples of 2?	Three
2	A	How many 7's can you see?	Two
3	B	Do you see more odd or even numbers?	More odd
4	D	What is the sum of the numbers you can see?	Forty-two



Devising a Plan

a. Looking for a Pattern -

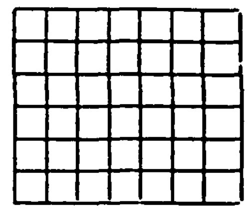
When the famous German mathematician Karl Gauss was a child, his teacher required the students to find the sum of the first 100 natural numbers. The teacher expected this problem to keep the class occupied for a considerable amount of time. Gauss gave the answer almost immediately. Can you?

b. Making a Table or Graph -

How many ways are there to make change for a quarter using only dimes, nickels, and pennies?

c. Using a Special or Simpler Case -

Using the existing lines in a square array of squares to form squares, how many different squares are there?



d. Identifying a Subgoal -

Kasey Kassion, a disk jockey for a 24-hour radio station, announces and plays each week' top forty rock songs on the radio all week. Suppose he decides to play the top song 40 times, the number two song 39 times, the number three song 38 times, and so on. If each song takes 4 minutes to play, how much time is left for other songs, commercials, news breaks, and other activities?

e. Using a Related Problem -

Find the sum of $1 + 4 + 7 + 10 + 13 + \ldots + 3004$.

f. Working Backwards -

Charles and Cynthia play a game called NIM. Each has a box of matchsticks. They take turns putting 1, 2, or 3 matchsticks in a common pile. The person who is able to add a number of matchsticks to the pile to make a total of 24 wins the game. What should be Charles' strategy to be sure he wins the game?

g. Writing Equations -

As he grew older, Abraham De Moivre, a mathematician who helped in the development of probability, discovered one day that he has begun to require 15 minutes more sleep each day. Based on the assumption that he required 8 hours of sleep on date A and that from date A he had begun to require an additional 15 minutes of sleep each day, he predicted when he would die. The predicted date of death was the day when he would require 24 hours of sleep. If this indeed happened, how many days did he live from date A?

h. Using a Diagram or Model -

It is the first day of class for the course in mathematics for elementary school teachers, and there are 20 people present in the room. To become acquainted with one another, each person shakes hands just once with everyone else. How many handshakes take place?

i. Guessing and Checking -

Marques, a fourth grader, said to Mr. Treacher, "I'm thinking of a number less that or equal to 1000. Can you guess my number?" Mr. Treacher replied, "Not only can I guess your number, but I can guess it in no more than ten questions, provided that your answers to my questions are yes or no and are truthful." How could Mr. Treacher have been so positive about the maximum number of questions he would have to ask?

JULY—A

NCTM

1 Given two equilateral triangles, find a third one whose area is the sum of the other two.

2 One circle divides a plane into two regions, two circles into four regions, three circles into ___ regions, four circles into ___ regions, and n circles into ___ regions.

3 1845b **Georg Cantor,** founded set theory and contributed to analysis. Invented the Cantor set.

4 One square determines two regions, but two congruent squares can determine ten regions. Three squares determine ___ regions. Four congruent squares determine ___ regions. So n squares determine ___ regions.

5 **6** The figures below contain one, three, and six rectangles, respectively.

How many rectangles are in ? ___

How many rectangles are in ? ___

How many rectangles are possible in a string of n squares? ___

7 What other numbers complete the pattern?

$5^2 - 5 = 4^2 + 4$

$7^2 - 7 = 6^2 + 6$

8 **9** There are three rectangles in . There are nine in and eighteen in . How many are in ? ___

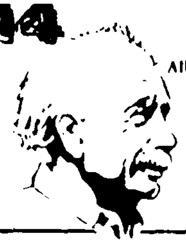
If n is the number of stacks (columns) of rectangles that comprise each figure, then what is the total number of rectangles (of all sides) in each figure? ___

10 1748b **John Playfair,** worked in Euclidean geometry; found substitute for Euclid's parallel postulate.

11 1780b **August Leopold Crelle,** advocated learning mathematics as a way of developing rational thinking; discovered Abel and Jacobi.

12 **13** There are six rectangles in . There are eighteen in . How many rectangles are in ? ___

If n is the number of stacks (columns) of rectangles in each figure, then what is the total number of rectangles (of all sizes) in each figure? ___

14 
1879b **Albert Einstein,** stimulated interest in Riemannian geometry through his theory of relativity.

15 The Fibonacci sequence is 1, 1, 2, 3, 5, 8, ... Plot $1 \cdot x + 1 \cdot y = 2$ and $1 \cdot x + 2 \cdot y = 3$. Where do the resulting lines intersect? Pick three other consecutive elements in the sequence, such as a, b, and c in $ax + by = c$, and see if they go through the same point.

16 1750b **Caroline Herschel,** astronomer, found three new nebulae and eight comets.


17 Two two-digit numbers can be formed with the digits 4 and 2, namely, 24 and 42. We know that $24 + 42 = 66$. Make two other two-digit numbers and add them together. What number divides all the sums?

18 1690b **Christian Goldbach,** conjectured that all even numbers except 2 can be represented as the sum of two primes. What do you think?

$7 + 3 = 10$
$7 + 5 = 12$
$7 + 7 = 14$
...

19 Return to the problem for March 17. How does the sum of the two single digits compare to the sum of the two two-digit numbers? (For example, compare $4 + 2 = 6$ with $24 + 42 = 66$.)

20 How many triangles are in each shape?

$n = 1$ $n = 2$ $n = 3$ $n = 4$

How many more △s are in the figure for $n = 2$ compared to $n = 1$? For $n = 3$ compared to $n = 2$? What pattern can you find in the differences?

21 1884b **George David Birkhoff,** first major American mathematician, contributed in many areas, ergodic theorem.


22 Find the decimal values to ten places for 2/7, 3/7, 4/7, 5/7, and 6/7. What patterns do you notice in these values?

$\frac{1}{7} = 0.1428571428$5

$\frac{2}{7} = ?$

23 
1882b **Amalie Emmy Noether,** leader in the development of modern algebra.

24 1809b **Joseph Liouville,** attempted to classify all algebraic functions, first proof of existence of transcendental numbers. A Liouville number is an irrational number x such that for any integer n, there is a rational number p/q such that $q > 1$ and $|x - p/q| < \frac{1}{q^n}$.

25 1798b **Christof Gudermann,** worked with spherical geometry, saw value of using infinite series in calculus.

26 
1773b **Nathaniel Bowditch,** wrote about astronomy, worked as an actuary. His book *The American Practical Navigator* is still available.

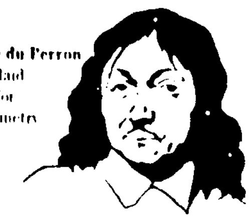
27 1857b **Karl Pearson,** founded field of statistics. Introduced chi square test, Pearson's coefficient.


28 Continue this pattern for 4 more steps and then compare your result to $\sqrt{2}$. Try beginning with a fraction other than 3/2.

$\frac{3}{2} \to \frac{3+2(2)}{3+2} = \frac{7}{5} \to \frac{7+2(5)}{7+5} = \frac{17}{12} \to$

29 Find three consecutive integers that add to 75, three that add to 99, and three that add to 150. If n is a number that is the sum of three consecutive numbers, what divisors must it have?

30 1890b **Stefan Banach,** one of the originators of functional analysis, developed theory of topological spaces. Banach algebra, Banach space.

31 1596b **René du Perron Descartes,** laid foundation for analytic geometry.


JULY — B

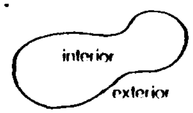
1 In every numeration base b is a three digit integer that is $(b+1)$ times the sum of its digits. In base ten it is 198, since
$198 = (10+1)(1+9+8)$
What numbers have this property for bases two through nine?

2 A fair coin with faces 0 and 1 is tossed repeatedly to see whether the ordered triple 111 or 011 turns up first. What is the probability that 111 will turn up first?

3 1777b. **Louis Poinsot,** established theory of regular star polygons

4 Find three consecutive binomial coefficients in the ratio 1 to 2 to 3. That is,
$\binom{n}{k}:\binom{n}{k+1}:\binom{n}{k+2} = 1:2:3$

5 1838b. **Camille Jordan,** worked in algebra and group theory; "Jordan curve theorem," "Jordan canonical form."



6 The right triangle with sides of length 5, 12, and 13 has the property that its area is equal to its perimeter. Find another right triangle with sides of integral length that has this property.

7 1871b. **Émile Borel,** contributed to analysis and probability

8 1888b. **Richard Courant,** worked in physics, mini-max theory of eigen values, founded Courant Institute at New York University

9 The first few factorials greater than 1 are clearly not perfect squares: $2! = 2$; $3! = 6$; $4! = 24$; $5! = 120$. Can any factorial greater than 1 be a perfect square?

10 1875b. **Issai Schur,** stated that every matrix is unitarily similar to a triangular matrix

11 Guess by what rule the following equalities are composed, then make up another such equality of your own.
$12 \times 42 = 21 \times 24$
$13 \times 62 = 31 \times 26$

12 For each odd positive integer n, show that
$1 + 9 + 9^2 + 9^3 + \cdots + 9^n$
is composite. For example,
$1 + 9 = 10 = 2 \times 5;$
$1 + 9 + 9^2 + 9^3 = 820 = 10 \times 82$

13 1876b. **Erhard Schmidt,** worked on integral equations and Hilbert space theory

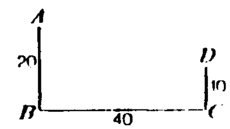
14 Gil is 17 years older than Sheila. If his age is written after hers, the result is a four-digit perfect square. The same statement could be made 13 years from now. Find Sheila's present age.

15 1850b. **Sofya Korvina Krukovskaya** (Sonya Kovalevski), researcher in differential equations and algebra

16 Find the sum of the series
$\frac{1}{2} + \left(\frac{1}{3}+\frac{2}{3}\right) + \left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right) + \left(\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}\right) + \cdots + \left(\frac{1}{100}+\frac{2}{100}+\frac{3}{100}+\cdots+\frac{99}{100}\right)$

17 At precisely what time between one and two o'clock is the minute hand exactly over the hour hand?

18 Timothy must get from point A to point D and must touch some point between B and C along the way. (For example, he could go from A to B to D, or A to C to D, or A to the midpoint between B and C and then to D.) What is the shortest length of such a journey satisfying these conditions?



19

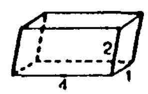
20 The ancient Egyptians had a rule that told them when, given two unit fractions with the second denominator twice the first, the sum of the two unit fractions would itself be a unit fraction. For example $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ is not a unit fraction, but $\frac{1}{3} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$ is. What was this necessary and sufficient rule?

21

22 1874b. **Leonard Dickson,** known for his text on history of number theory

23 1862b. **David Hilbert,** in 1900 posed 23 unsolved problems that stimulated mathematics research

Hilbert parallelotope



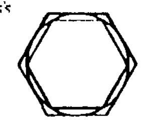
24 1798b. **Karl von Staudt,** worked in projective geometry
1916 Browning, Montana, a record variation in one day, 44°F to −56°F. What is the record difference in degrees?

25 1736b. **Joseph Louis Lagrange,** astronomer, contributed new ideas for solving equations with complex variables

26 1862b. **E. H. Moore,** developed general theory of limits.
1978 **Peter Dowdeswell** ate 13 raw eggs without shells in 2.2 seconds. How fast did he eat 1 egg?

27 1832b. **Charles Dodgson** (Lewis Carroll), logician, geometer, advanced voting theory, wrote about Alice

28 1540b. **Ludolph van Ceulen,** computed π (Ludolph's number) to 35 places using Archimedes's method



29 In bowling, a perfect score of 300 can be obtained in only one way: getting twelve strikes in a row. Can any other scores be obtained in only one way?

30 Friday the thirteenth has a reputation as being an unlucky day. Is it possible to have a year in which there are no Friday the thirteenths?

31 How many digits does the number $(9)^{(9^9)}$ have when written in base nine?

JULY — C

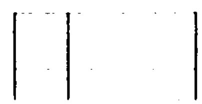
NCTM

1
What percentage of the area of a circle is enclosed by an inscribed isosceles triangle one of whose sides is the diameter of the circle?

2
Dave computed an answer to be 22.5. However, in the last step of the computation he multiplied by 0.3 instead of dividing by 0.3. Assuming that Dave computed correctly, what is the correct answer?

3
About how many times will your heart beat during this month?

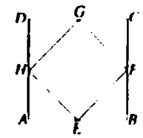
4
How many rectangles do you see in this figure?



5
What is the area of a square whose diagonal is one unit longer than the length of its side?

6
1631b. **Pierre Herigone,** first person to use the symbol ∠ for angle.

7
The area of square *ABCD* is 100 cm². *E* and *F* are the midpoints of sides on which they lie. What is the area of square *EFGH*?



8
Two more people are ahead of me in line than are behind me. There are three times as many people in line as there are people behind me. How many people are ahead of me in line?

9
1746b. **Gaspard Monge,** worked in descriptive geometry.

10
The first U.S. transcontinental railroad was completed on this date in the year _ _ 2 _ _.
The sum of the digits in the year is 24.
The ones and tens digits are multiples of 3.
The hundreds digit is one less than the ones digit.

11
1659. The division symbol, ÷, was first used by Johann Rahn.

12
Continue the pattern:

$1^3 = 1^2 - 0^2$
$2^3 = 3^2 - 1^2$
$3^3 = 6^2 - 3^2$
$4^3 =$
$5^3 =$

13
1753b. **Lazare Nicolas Marguerite Carnot,** politically active and mathematically concerned with the efficiency of machines.

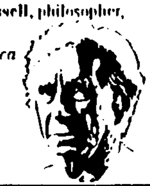
14
When the length of each side of a square is increased by 5 units, the area of the square is 2¼ times the area of the original square. What is the area of the original square?

15
What percentage of the states in the U.S. have names that begin with a vowel?

16
1718b. **Maria Gaetana Agnesi,** authored *Analytic Institutions,* a study of algebra, geometry, and calculus.

17
What is the karat rating of a gold ring that is 58 percent gold?

18
1872b. **Bertrand Russell,** philosopher, logician, coauthor of *Principia Mathematica.*

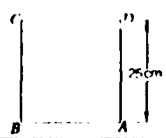


19
A "prime day" is a day such that both the *month* and the *day* are prime numbers. How many prime days occur in 1984? (Example: 2/2/81 is the first prime day.)

20
What is the value of a metric ton of nickels?

21
1858b. **Edouard Goursat,** contributor to analysis; "Cauchy Goursat theorem."

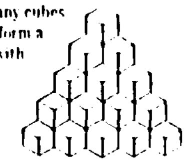
22
What are the dimensions of the square *EFGH* you must cut from the square *ABCD* so that the area of square *ABCD* is reduced by 25 percent?



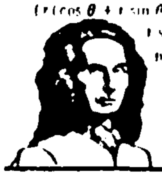
23
What three consecutive whole numbers have a product of 29,760?

24
Which pairs of two digit consecutive numbers have squares that differ by a perfect square?

25
This tower is made of 35 cubes in 5 layers. How many cubes are needed to form a similar tower with 10 layers?



26
1667b. **Abraham De Moivre,** proved $(r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta)$, formulated notion of the normal distribution curve.



27
Three straws are chosen from a set of nine straws whose lengths are 1 inch, 2 inches, 3 inches, ..., 9 inches. What is the probability that the three straws, placed end to end, will form a triangle?

28
Choose any number, multiply by 2, add 5, multiply by 5, subtract 25, and divide by 10. Compare your result to the original number.

29
Choose any number, multiply by 3, add 8, add your original number, divide by 4, and subtract your original number. Compare your result to the original number. Write an equation using *n* for some number that describes the steps in this problem and in the problem for 28 May.

30
Try to write each integer from 1 through 32 as a sum of two or more consecutive positive integers. Does a pattern occur in the numbers that cannot be represented in this way?

31
Square the numbers between 10 and 20. How many palindromes do you get? (A palindrome reads the same forward as backward.) How many palindromes are between 20² and 30²? What would you expect for the squares between 30² and 40²? Check it out.

JULY — D

1
Sam and Susie are siblings. Sam has as many brothers as sisters. Susie has twice as many brothers as sisters. How many boys and how many girls are in the family?

2
A grandfather clock takes 30 seconds to strike six. How long does it take to strike twelve?

3
Generalize the following pattern and prove it:

$$(1\tfrac{1}{2}) \times 3 = 1\tfrac{1}{2} + 3,$$
$$(1\tfrac{1}{3}) \times 4 = 1\tfrac{1}{3} + 4,$$
$$(1\tfrac{1}{4}) \times 5 = 1\tfrac{1}{4} + 5$$

4
1582 date changed to 15 October by **Pope Gregory XIII** to bring calendar reform

5
1781 b. **Bernhard Bolzano,** contributed to the theory of logic and function. A Bolzano function cannot be integrated.

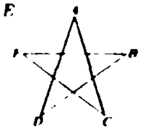
6
1831 b. **Richard Dedekind,** worked on the theory of numbers; developed concepts of ring and field

7
In the following multiplication problem, all the digits 0 through 9 have been used once, and only once. Can you fill in the blanks to make it true?

```
   [] 0 2
 ×   3 []
[] 5 [] [] []
```

8
A worker's salary is reduced by p percent. By what percent would this salary then have to be raised to bring it back to the original amount?

9
In the following figure, find the sum $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E$.



10
Which is larger, 2^1 or 1^2? 3^2 or 2^3? 4^3 or 3^4? 7^6 or 6^7? ... ?
Can you predict the larger of *any* two positive integers that follow this pattern?

11
In the following multiplication problem each letter stands for a different digit. Find the digits.

```
  ABCDE
×     4
-------
  EDCBA
```

12
If a faucet drips at the rate of a drop every 2 seconds and it takes 575 drops to fill a 100 ml bottle, how many liters of water are wasted in a week? A month? A year?

13
A horse is tethered to a rope at one corner of a square corral (outside the corral) that is 10 feet on each side. The horse can graze at a distance of 18 feet from the corner of the corral where the rope is tied. What is the total grazing area for the horse?

14
What are the next four letters in this sequence?
O, T, T, F, F, S, S, __, __, __, __

15
1608 b. ***Evangelista Torricelli,*** Galileo's student, developed the barometer

16
At least two of ways of representing 20 can be found using three 3s and standard mathematical symbols. Find one.

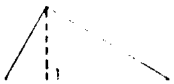
17 **18**

The gold was missing. The thief was either the butler, the maid, or the cook. During the investigation, each made the following statement:
Butler: The maid stole the gold.
Maid: That is true!
Cook: I did not steal the gold.
As it happened, at least one of them lied and at least one of them told the truth. Who stole the gold?

19
Fill in the missing spaces in this multiplication problem.

```
        [] [] [] 2
      ×    3 [] [] []
      ---------------
       [] 8 [] 2
      [] 0 []
   [] 9 4 []
   ---------------
   3 [] [] 4 [] 2
```

20
The sides and height of a triangle are four consecutive whole numbers. What is the area of the triangle?



21
A student was thinking of evaluating $(9)^{(9^9)}$. Before starting, however, she wisely decided to approximate the size of her answer. She made a calculation based on being able to write five digits per inch. How long did she find her answer to be to the nearest mile?

22
Each letter in the sum stands for one and only one digit. Find the correct digit.

```
  TICK
  TOCK
  TICK
  TOCK
     A
 -----
 CLOCK
```

23
In a classroom the 25 seats are arranged in a square of 5 rows, with 5 desks in each row. The teacher announces that students are to change their seats by going either to the seat in front, in back, or to the left or right. Can this order be carried out? Prove your answer.

24
1858 **Richard Dedekind** produced his definition of continuity

25
1811 b. **Evariste Galois,** laid foundation for modern algebra

26
Three tired and hungry men had a bag of apples. When they were asleep one of them awoke, ate 1/3 of the apples, and went back to sleep. Later, a second man awoke, ate 1/3 of the remaining apples, and went back to sleep. Finally, the third man awoke and ate 1/3 of the remaining apples, leaving 8 apples in the bag. How many apples were in the bag originally?

27

28
In each of the following, each letter stands for a single digit. The codes are independent of each other. What does each letter stand for?
a. Y + Y + Y = MY
b. XXX + B = BAAA
c. MA + A = AM.
d. ON + ON + ON + ON = GO

29
1675 **Gottfried Wilhelm von Leibniz** first used the $\int$ and d in a letter explaining the process of integration

30
Generalize the following pattern and prove it:

$$(4\tfrac{1}{2}) \cdot 3 = (4\tfrac{1}{2}) - 3$$
$$(5\tfrac{1}{3}) \cdot 4 = (5\tfrac{1}{3}) - 4,$$
$$(6\tfrac{1}{4}) \cdot 5 = (6\tfrac{1}{4}) - 5$$

31
1815 b. **Karl Weierstrass,** founded modern theory of functions

COMPUTER SCIENCE

Summary - Week 1

June 24, 1991

Lecture Topics: Unit I - Introduction to Algorithm Development through Karel the Robot Graphics

Monday, June 24 — **Tour Facilities**; Discuss **Hardware** and **History** while in the large mainframe area, a terminal lab and the class PC lab

Tuesday, June 25 — **Lecture: Introduction**
The Computer Model

Karel the Robot Graphics to Demonstrate Algorithm Development

Wednesday, June 26 — **Lab Assignment #1 Karel Get Paper**

Objective: Intro to the Use of the Computer
Intro to Turbo Pascal
Intro to Algorithms

Thursday, June 27 — **Lecture (1/2): Structured Algorithm Development**

Lab Assignment #1B Pick Up Groceries

Objective: Algorithm Structure and Simple Procedures (without parameters)

Friday, June 28 — **Lecture (1/2): Top Down Design**

Examples such as planning a school party

Lab Assignment #1C Mail From the Box

Objective: Further Structured Algorithm Development using More than One Procedure

Karel the Robot - Introduction

Initially there will be 4 basic objectives. They are to familiarize you with:

- Karel's environment and capability.
- Creating, editing and saving Karel's environment.
- Using executable programs.
- Creating, editing, saving and executing Karel's programs.

Karel's Environment and Capability

Karel lives in a world of 19 *avenues* (north and south) and 19 *streets* (east and west). This world is permanently bounded by inpenetrable *walls*. Other walls can be placed between any two *intersections*. This world can also have objects called *beepers* which can be placed on any intersection. Karel is a *robot* which can occupy any intersection in the environment.

Karel understands 5 primitive commands:

- *MoveRobot*
- *TurnLeft*
- *PickBeeper*
- *PutBeeper*
- *TurnOff*

If Karel tries to move forward and his path is blocked, or if he tries to pick up a beeper and no beeper is present then an *error shutdown* occurs (a tone will be emitted until *enter* is pressed).

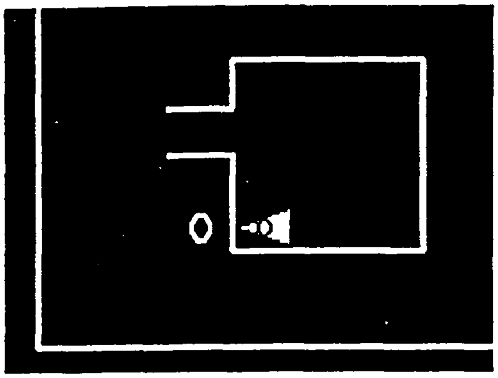
Creating, Editing and Saving Karel's Environment

The program *EditGrid.exe* allows the user to create, edit and save Karel's environment. The program allows you 4 modes:

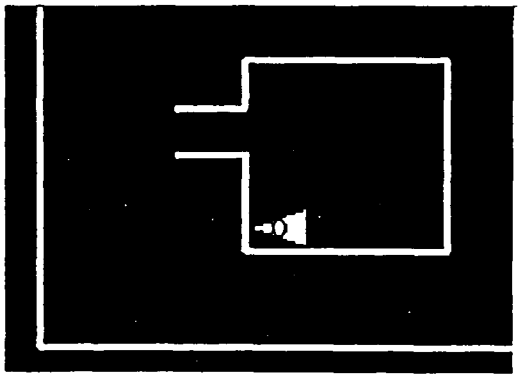
- Edit Beepers
- Edit Walls
- Edit Robot
- Save File

Introduction to Computer Science I
Laboratory #1

A) *Karel* is lying in his bed when the *paper* is delivered. Write and execute a program for *Karel* so that he retrieves his *paper* and returns to bed. (Examine the pre and post conditions illustrated below in creating your data file and writing your program.)

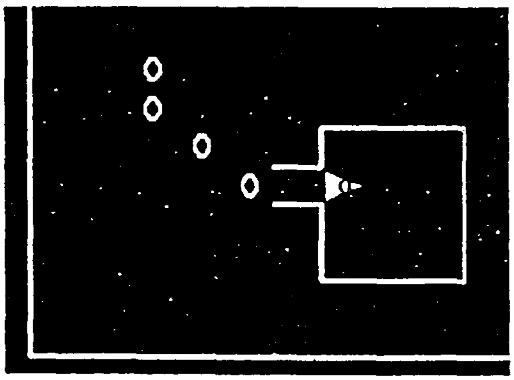


Pre

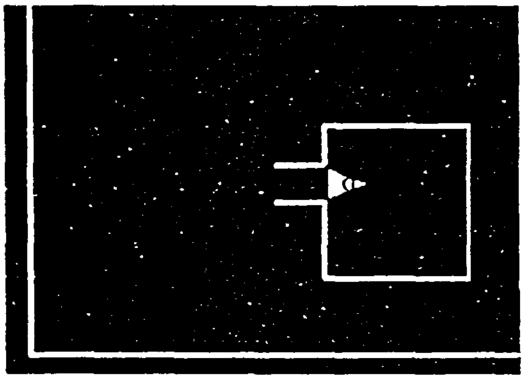


Post

B) Returning home from the grocery store *Karel* drops some *items* outside his home. Write a program for *Karel* that retrieves the dropped items and returns to the house. (Examine the pre and post conditions illustrated below in creating your data file and writing your program.)

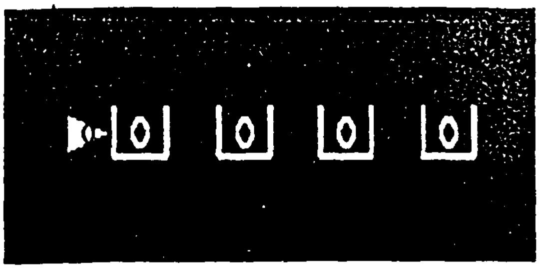


Pre

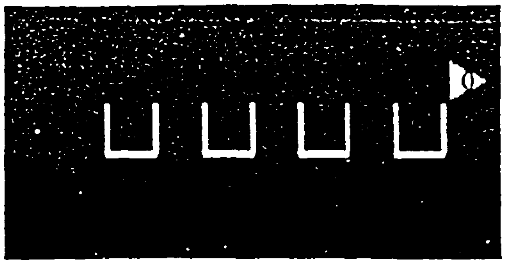


Post

C) *Karel* must remove the *mail* from four *mailboxes*. Write a program for *Karel* that retrieves the *mail*. (Examine the pre and post conditions illustrated below in creating your data file and writing your program.)

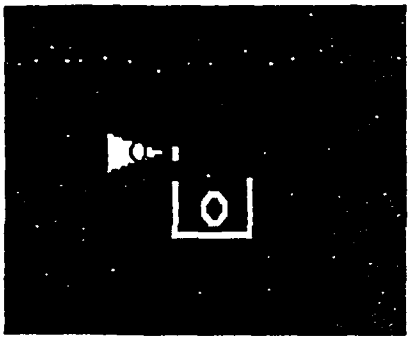


Pre

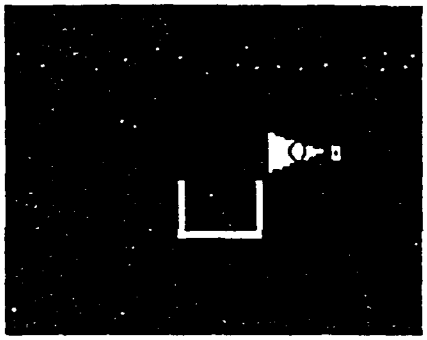


Post

Define and use a new instruction - *GetMailFromBox*. The pre and post conditions for this instruction are illustrated below.



Pre



Post

Top Down Design

Top down design is a very valuable approach to the design of programs. One way to begin to understand this concept is to examine the structure of a Karel program to pick up 3 books left on some stairs (see Fig. 1):

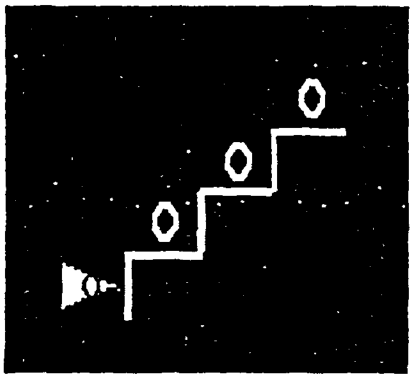


Figure 1

```
program Pick_Up_Books;
uses robot;

procedure TurnRight;
begin
   TurnLeft;
   TurnLeft;
   TurnLeft
end;

procedure ClimbStair;
begin
   TurnLeft;
   MoveRobot;
   TurnRight;
   MoveRobot
end;

begin

    ClimbStair;
    PickBeeper;

    ClimbStair;
    PickBeeper;

    ClimbStair;
    PickBeeper;

    TurnOff

end.
```

In examining this program we must distinguish between its *syntactical* structure and its *logical* structure. The program's *syntactical* structure is viewed sequentially from top-to-bottom and left-to-right. Thus first the procedures are specified and next the main program is specified. It is important to understand that this *syntactical* structure is for a compiler - not humans! The *logical* structure of the program is for humans.

Logically we begin with the problem (see Fig. 2). Next we move to the main program body; then the ClimbStair definition; and finally the TurnRight definition. This logical structure is said to be constructed top down. To make this more concrete note procedure ClimbStair. This procedure defines ClimbStair in terms of TurnRight. ClimbStair is said to be a higher order abstraction than TurnRight (just as TurnRight is a higher order abstraction than TurnLeft). The logical structure begins with the highest order abstractions and moves down. The syntactical structure begins with the lowest order abstractions and moves up. Our approach to the design of programs will be top down, i.e. following the logical structure.

Problem: Pick beepers off stairs.

Main

ClimbStair;
PickBeeper;
ClimbStair;
PickBeeper;
ClimbStair;
PickBeeper;
TurnOff

ClimbStair

TurnLeft;
MoveRobot;
TurnRight;
MoveRobot

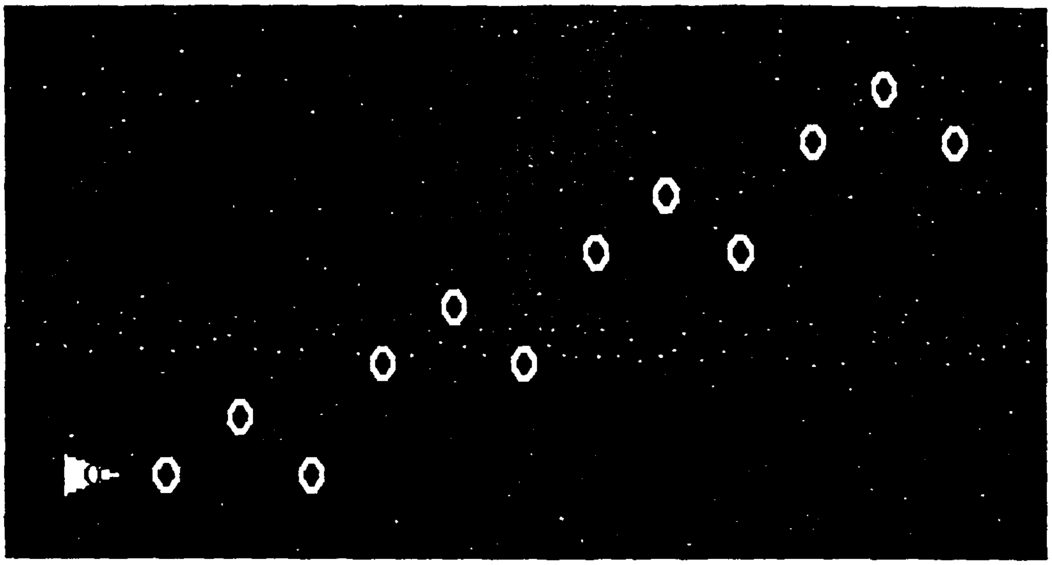
TurnRight

TurnLeft;
TurnLeft;
TurnLeft

Figure 2

Laboratory #2

Given the 12 beepers in the figure below that are arranged into 4 groups of triangles (3 beepers each). Write a program for Karel to pick up these beepers using top-down design.



SUMMARY - WEEK 2

July ~~2~~ 1, 1991

Lecture Topics: **Unit I - Intro to Decision and Control Structures (within the context of Karel the Robot Graphics)**

Monday, July 1 — **Lab Assignment #2 Triangles**

Objective: Top Down Design
Simple Procedure Development

Tuesday, July 2 — **Lecture: Making Decisions**

Boolean Variables
IF..THEN and IF..THEN..ELSE

Classroom discussion: The Hurdle Problem
How to decide where to move

Wednesday, July 3 — **Lecture (1/2) Repetition - Loops**

WHILE Loops
Classroom discussion: Allowing for various sizes to the hurdle problem or the harvest problem

Lab Assignment #3 A. Harvest problem
Objective: More Complex Top Down Design

Lab Assignment #3 B. The Partial Harvest Problem
Objective: Simple decisions

Thursday, July 4 — **Holiday**

Friday, July 5 — **Lab Assignment # 4A. Find Key in a Room**
Objective: More complex decisions and control

Lab Assignment #4B. Find Key in a Room of any Size
Objective: While loops in problems

Lab Assignment #4

For Friday July 5 and Monday July 8

Use data file at Options..Parameters.. Cell.dat

A) Karel locks himself in a rectangular cell and accidently loses the key. Write a program for Karel to search the cell and retrieve the key. (see Figure 1 below) Karel can be at any intersection facing any direction when the program is initiated. **Do not use WHILE-DO loops for this problem.**

Hint: Face north and go to north wall. Next go to northwest corner and face south. Now systematically search the cell column by column.

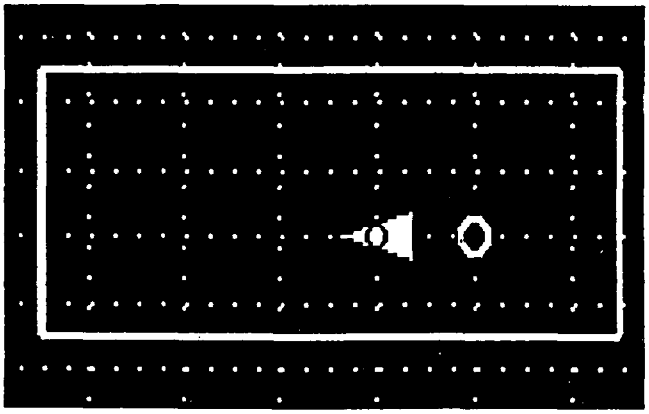


Figure 1

B) Write a program for the problem above where the cell can be of any size. **You should use WHILE-DO to solve this problem.**

Summary - Week 3

July 8, 1991

Lecture Topics: Unit II - Pascal Language and Simple Algorithms
Unit III - Pascal Decision and Control Structures

Note: Units II and III are combined in transition from algorithms and labs stated in the context of Karel the Robot to more traditionally stated lab problems.

Monday, July 8 **Lab Assignment #4B Key in a Room (Any Size)**

Objective: Using While loops to allow for indefinite conditions

Complete any lab assignments to date

Tuesday, July 9 **Lecture: Numeric and Character Variables**

Readln Writeln

Integer Expressions and Assignment

Classroom Discussion: Assignment Statements; Numeric variables in counting loops

HOMEWORK: Worksheet Unit II-1; Plan/write Lab #5

Wednesday, July 10 **Lab Assignment #5 Mountain**

Objective: Review decisions and looping control

Thursday, July 11 **Lecture: Boolean Variables and Expressions**

Relational Operators > >= < <= = <>

Boolean Operators **AND OR**

HOMEWORK: Worksheet Unit III; Plan/write Lab #6

Friday, July 12 **Lab Assignment #6 Easter Sunday**

Objective: Use of Integer expressions, Assignment, Readln and Writeln

HOMEWORK: Plan and begin writing Lab #7

(Monday, July 15) **Lab Assignment #7 Quiz**

Objective: Use of decision making

Computer Science Worksheet Unit II-1
July 9, 1991

1. Recall the rules for naming variables. Look at the examples in your notes. Mark the following as valid or invalid variable names.

MySchool	______	Time	______	Distance	____
B	______	Number	______	ok	-----
1C	______	A1	______	$Yes	______

2. Choose meaningful variable names for the following values. Indicate type **INTEGER** or **REAL** after the **Colon**. Place a **semicolon** after the type at the end of the statement. In other words, use proper syntax for declaring the variable. For example:

```
MySchoolCode: INTEGER;
Rate: REAL;
Count: INTEGER;
```

your year of birth ____________________:__________;

your grade point average ____________________:__________;

Now, you fill in the proper punctuation for the answers.

your class size ________________________________

Joe's batting average ________________________________

your home room number ________________________________

number of girls in the class __________________________

3. Recall the order of operations:

first	*	multiply
	DIV	integer divide - the quotient only
	MOD	the remainder only after division
	/	real number division
last	+	add
	-	subtract

The rules are the same as algebra. As in algebra, ( ) parentheses may change the order of operations.
Evaluate the following expressions, where the values are:
A is 2 B is 6 C is 1 D is 8 E is 9

X := A * B + C; ____________   X := C + B DIV A; ______

X := B + E MOD A; _________   X := (E + C) DIV B; _____

X := (A + B) * (C - D); _______   X := A + B * C; _________

Computer Science Worksheet: Unit II-2

Arithmetic Expressions and Program Order

1. What is the value of Sum after the following program segment is executed?

```
VAR  A, B, C, Sum: INTEGER;

A := 2;
B := 8;
C := B DIV A;
Sum := A + C;          value of Sum _________
```

2. Recall our classromm discussion. Trace the values of the variables in the following program segment. Use a ? if the value is unknown at the time.

a.

	A	B	C
VAR A,B,C,N: INTEGER;	______	______	______
A := 12;	______	______	______
N := 4;	______	______	______
B := A * N;	______	______	______
C := A DIV N;	______	______	______
A := C;	______	______	______
B := A + C;	______	______	______

b. Assume the same variable declarations

N := 2;	______	______	______
B := N * N;	______	______	______
A := 25;	______	______	______
C := A MOD B + N;	______	______	______
B := A + C DIV N;	______	______	______
WRITELN `A, B, C);	______	______	______

Lab 6 Friday, July 11, 1991

Objective: to write and run a standard Pascal program
to use arithmetic expressions
to use assignment statements
to use simple decisions

Call this program EasterSunday

Let us suppose we want to plan our spring vacation and we know it occurs during the week after Easter. We need to figure our the date for Easter next year.

The date for any Easter Sunday can be computed in the following manner. Please note that all variables are type INTEGER.

Let A be Year MOD 19

Let B be Year MOD 4

Let C be Year MOD 7

Let D be (19 * A + 24) MOD 30

Let E be (2 * B + 4 * C + 6 * D + 5) MOD 7

Then the date for Easter Sunday is **March (22 + D + E)**. We can see that this can also generate a date in April. We need to check if the sum of (22 + D + E) is greater than 31, the number of days in March.

Write a program that uses the year 1991 for a convenient test. Then test it with other years. Then run it with confidence for year 1992.

After all is finished, can you run this program in a loop for years 1990 to 1995?

HINTS:

Be careful with MOD and DIV operators.
You only have one decision at the end of the program.
Do not try to make other variable names besides the single letter as shown; here it is appropriate. Of course we need a variable for Year.

LAB 7 (Monday, July 15, 1991)

Lab Unit III - Decisions and Control

Write and run these programs:

Objective: Use of string variables
Decision making using IF.....THEN
Appreciation of computer word processing

Write a program to produce a form letter (or other form text such as a greeting card). The program must have at least two variable values (such as name, grade, age, or occasion) and at least one decision to provide alternate print messages such as "son" or "daughter".

Objective: Decision making in programs
Appreciation of programs for interactive computer aided instruction

Design a four question multiple-choice quiz for one of your classmates to take at the terminal. The quiz questions will be "asked" by the terminal and the answer will be received by the terminal. The quiz will also be graded by the program. The quiz may be about material that you have studied in this class (what about from your almanac ?) or about other material. Try to think about your own experiences with computer aided instruction (CAI). Be sensitive to interactive responses. The computer should congratulate a correct answer and provide help for an incorrect answer.

Summary - Week 4

July 15, 1991

Lecture Topics: Unit III - Review Pascal Decision and Control; Accumulation

Unit IV - Program Structure; Procedures with Parameters

Monday, July 15 **Lab Assignment #7 (Standard Pascal)**

A. Letter or greeting card (Class discussion)

Objective: to use variables, READLN, WRITELN
to make simple decisions

B. Multiple Choice Quiz

Objective: to use decisions
to understand accumulation

Tuesday, July 16 **Lecture: More Boolean Logic; NOT Boolean negation**

Homework: Worksheet Unit III-1

Wednesday, July 17 **Lab: Continue problems assigned to date**

Thursday, July 18 **Lecture: Program Structure** using

1) Robot example - Trail Problem
2) "standard" example - Quiz Problem

Introduction to Procedures with Parameters

Homework: Worksheet Unit III-2

Friday, July 19 **Lab:**

1) **Complete Quiz Problem from class discussion**
2) **Continue problems assigned to date**
3) **Choose further work from the Lab Problem set on pages 53-62**

PROJECT PROBLEM SET

Choose programs to develop from the following sets of problems. There is one set of problems defined in terms of robot tasks. There is another set of problems defined in "standard" terms. You may choose as many problems as you are able to finish from either set. The problems are listed in approximate degree of difficulty. Complete descriptions of the problems follow this list.

ROBOT LAB PROBLEMS

8. Room with or without a door

9. Tunnel

a. count the number of moves, left and right turns

10. a. Draw Filled Right Triangle

b. Diamond

"STANDARD" LAB PROBLEMS

11. Rolls of Coins

12. Opinion Poll

13. HI-LO Game

14. Coin Flips

15. Tic-tac-toe Game

16. Secret Massage Decoder

Laboratory Exercise #8

Write a program for *Karel* to exit a rectangular through the doorway if a door exists (Figure 1). If no doorway exists (Figure 2), *Karel* should turn himself off after discovering this.

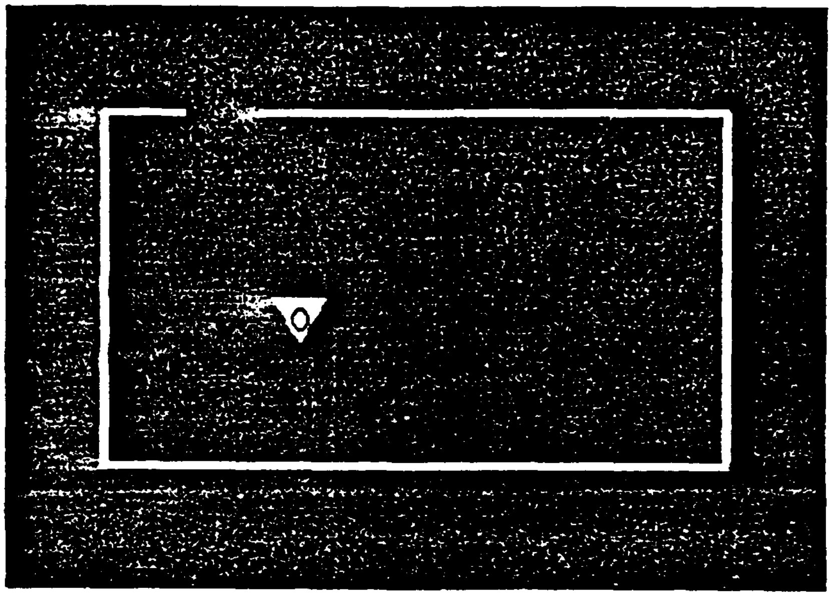


Figure 1

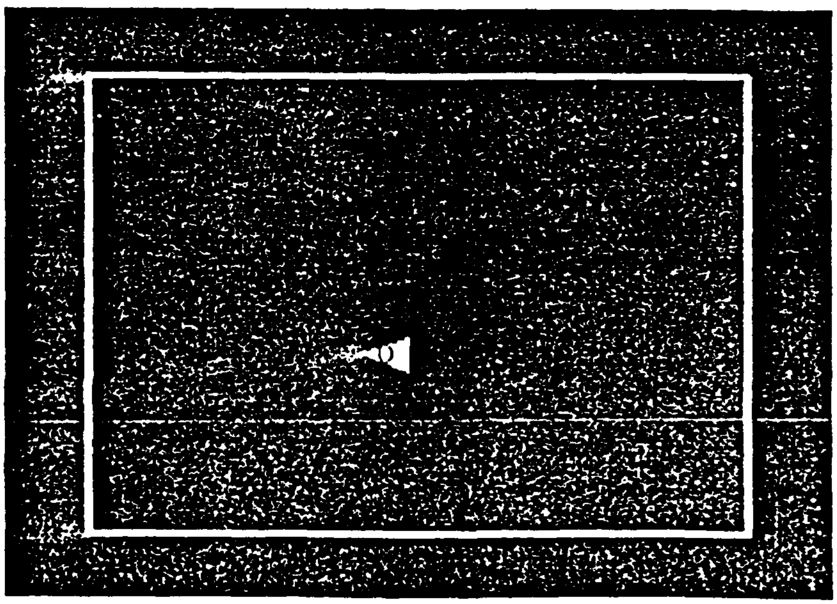
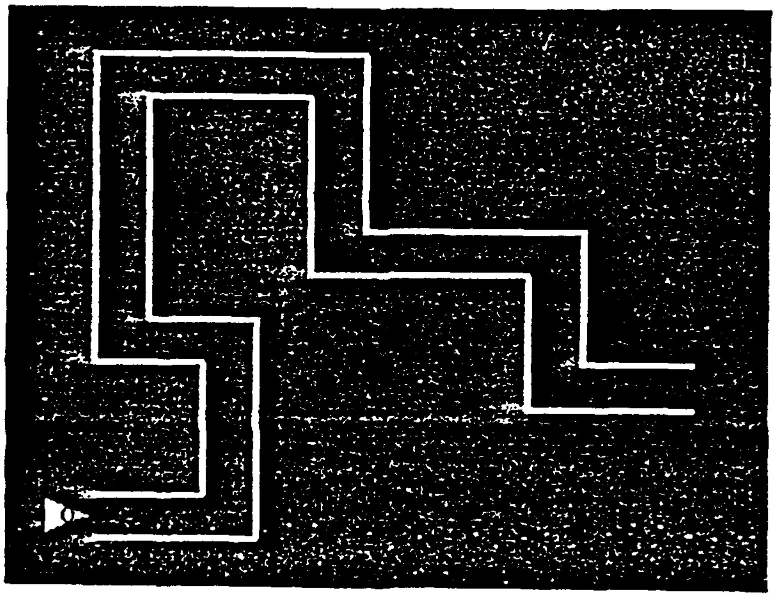


Figure 2

Laboratory # 7 ## 9a

Given the *Tunnel* problem, i.e. move *Karel* through the tunnel. Write a program that displays the total number of left turns, number of right turns and the number of moves. For example the program might display:

> There were 4 left turns.
> There were 4 right turns.
> There were 38 moves..



Summary - Week 5

July 22, 1991

Lecture Topics: Unit IV - Procedures with Parameters

Unit V - Introduction to Arrays and Data Structures

Monday, July 2 — Continue problems assigned to date

Tuesday, July 23 — Lecture: Procedures with Parameters (Cont'd)

Arrays and Tables

Wednesday, July 24 — Lab: Hi-LO Game;

Objective: by discovery method
Continue problems assigned to date

Thursday, July 25 — Lecture: Introduction to Data Structures; Searching and Sorting (with emphasis on binary properties)

Friday, July 26 — Lab: The Great Sorting Race

Objective: to appreciate time differences in the choice of algorithms

Complete any problems and projects assigned to date

Lab Exercise

Wednesday, July 24, 1991

HILO GAME

Today we will play a little guessing game called HILO. It is somewhat like "The Price Is Right". The computer will ask you to guess a number that falls within a certain range, let us say between 1 and 10. The player has a certain number of tries to guess the correct number.

In our game the range of numbers to guess will become larger and larger. Your task, of course, is to try to win all of the time.

Steps to take:

1. At the command prompt

    C:\ROBOT>

2. Type

    HILO

    This will start the game.

3. Play the game as instructed.

4. On this paper, write down the number of tries you needed to win at each level.

Level 1. Range 1..10 ________________

Level 2. Range 1..50 ________________

Level 3. Range 1..100 ________________

Level 4. Range 1..1000 ________________

5. What was your winning strategy? (Answer in the space below)

Lab Exercise

Friday, July 26, 1991

The Great Sorting Race

Objective: *to appreciate time differences in the choice of algorithms*

Each of you has a program file called SORT on you disk. This file will sort (arrange in order from low to high) an array of almost 1000 numbers. However, as you know from our previous discussions, the algorithm (or precise method) which is chosen to perform a task can greatly effect the time to finish.

Algorithms which are based on binary strategy, as we observed in the HILO game, are faster than most others. In the task of sorting data, we must compare items to each other and we must move the items into new positions. The choice of algorithm can make a considerable difference in the number of comparisons of data elements and the number of movements of these data elements. When the number of items (data elements) is small, the time difference is not a factor, but as the number of items grows larger the difference in time will become much, much larger.

Your computers have various sort algorithms. Let us see which particular ones are the fastest in this race. Like any race, we will all start together, by the series of instructions at "Get Ready", "Get Set" and "GO". Take a minute to review these instructions before we actually start. Your lab instructor will be the starter for the race. You will also record your start time in minutes and seconds and your finish time in minutes and seconds - just to be sure. Are there any questions before we start?

Get Ready: get to the command prompt

C:\ROBOT>

Get SET: Type SORT
DO NOT hit the enter key yet!!!!!

GO: **Hit the Enter Key now!**

Start time: ________ minutes ________ seconds

Finish time: ________ minutes ________ seconds

Do you think that your particular algorithm was based on binary strategy ? _________(yes/no) Why?

Computer Science Survey - July 26, 1991

Please answer the questions to help us evaluate this course. There are three or four possible answers for each question. Please put a mark in the column which applies to your answer. Some of the questions relate to before this class and others relate to after this class; please answer appropriately. These questions are more detailed than those asked orally at the beginning of the class.

BEFORE THIS CLASS

A. Did you use the computer for the following:

	Never	*Every Month*	*Every Week*	*One Term*
1.Games?	|	|	|	|
2.Computer aided learning?	|	|	|	|
3. LOGO ?	|	|	|	|
4. BASIC Programming Language ?	|	|	|	|
5. Another computer related class ? Please describe	|	|	|	|

IF YOU USED A COMPUTER BEFORE THIS CLASS, ANSWER THIS GROUP.

B. Did you understand the following:

	Did not understand	*Understood a little*	*Understood a lot*
1. Procedures or subroutines ?	|	|	|
2. Variables ?	|	|	|
3. Decisions - IF Statements ?	|	|	|
4. Boolean Logic ?	|	|	|
5. Loops - WHILE or FOR Statements ?	|	|	|

ALL ANSWER THIS GROUP.
AFTER THIS CLASS,

C. Do you understand the following:

	Do not understand	*Understand a little*	*Understand a lot*
1. Procedures or subroutines ?	|	|	|
2. Variables ?	|	|	|
3. Decisions - IF Statements ?	|	|	|
4. Boolean Logic ?	|	|	|
5. Loops - WHILE or FOR Statements ?	|	|	|

D. Compared to each other, which kinds of problems did you like better ? Circle your choice.

ROBOT "STANDARD" Problems

E. Do you think you will take a computer course sometime before you graduate from high school? Circle your choice.

YES NO

F. School ______________________________

G. Grade in September ____________________

H. Do you have access to a computer after this course ? Circle your choice.

YES NO

Where ? ______________________________

STATISTICS AND OR

I. LINE PLOTS

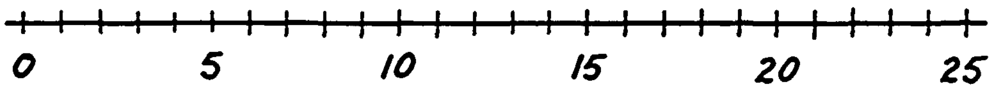
The 1984 Winter Olympics were held in Sarajevo, Yugoslavia. The table below lists the total number of gold, silver, and bronze medals won, by country.

Country	Total Medals	Country	Total Medals
Austria	1	Italy	2
Canada	4	Japan	1
Czechoslovakia	6	Liechtenstein	2
Finland	13	Norway	9
France	3	Sweden	8
Germany, East	24	Switzerland	5
Germany, West	4	USSR	25
Great Britain	1	United States	8
		Yugoslavia	1

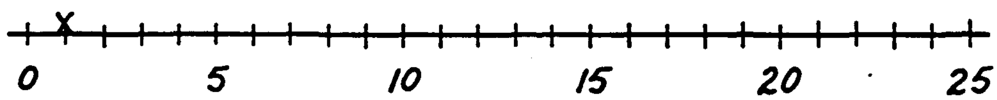
Source: *The World Almanac and Book of Facts*, 1985 edition.

Let's make a *line plot* of these data. First, make a horizontal line.

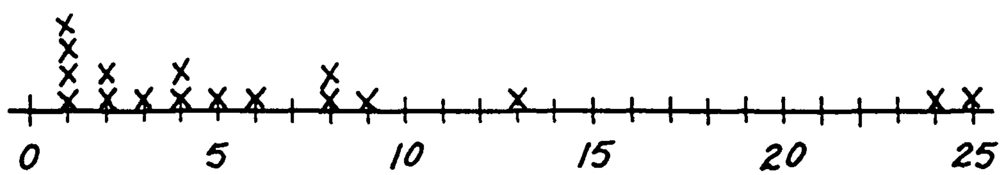
Then, put a scale of numbers on this line using a ruler. Since the smallest number of medals is 1 and the largest is 25, the scale might run from 0 to 25 as shown below.



The first country, Austria, won one medal. To represent Austria, put an X above the line at the number 1.



Continuing this way with the other countries, we can complete the line plot as shown below.



From a line plot, features of the data become apparent that were not as apparent from the list. These features include:

- *Outliers* — data values that are substantially larger or smaller than the other values
- *Clusters* — isolated groups of points
- *Gaps* — large spaces between points

It is also easy to spot the largest and smallest values from a line plot. If you see a cluster, try to decide if its members have anything special in common. For example, in the previous line plot the two largest values form a cluster. They are the USSR and East Germany — both eastern European countries. These two values are quite a bit larger than the rest, so we could also consider these points to be outliers.

Often, we would like to know the location of a particular point of interest. For these data, we might want to know how well the United States did compared to the other countries.

Discussion Questions

1. How many countries won only one medal?
2. How many countries won ten or more medals?
3. Do the countries seem to fall into clusters on the line plot?
4. Describe how the United States compares with the other countries.
5. In this book, you will often be asked to "describe what you learned from looking at the plot." Try to do this now with the plot of medal winners, then read the following sample.

> Seventeen countries won medals in the 1984 Winter Olympics. Two countries, the USSR with 25 and East Germany with 24, won many more medals than the next country, Finland, with 13. The remaining countries were all clustered, with from 1 to 9 medals each. The United States won 8 medals, more than 11 countries but not many in comparison to the leaders. One noticeable feature about these 17 countries is that, with the exception of the United States, Canada, and Japan, they are all in Europe.
>
> The list does not say how many countries did not win any medals. This might be interesting to find out.

Writing descriptions is probably new to you. When you look at the plot, jot down any observations you make and any questions that occur to you. Look specifically for outliers, clusters, and the other features we mentioned. Then organize and write your paragraphs as if you were composing them for your English teacher. The ability to organize, summarize, and communicate numerical information is a necessary skill in many occupations and is similar to your work with science projects and science laboratory reports.

Application 2

Causes of Death

The United States Public Health Service issues tables giving death rates by cause of death. These are broken down by age group, and the table below is for people 15-24 years of age. It gives death rates per 100,000 population for 16 leading causes of death. As an example, a death rate of 1.7 for leukemia means that out of 100,000 people in the United States aged 15-24, we can expect 1.7 of them will die annually from leukemia.

Cause of Death	Death Rate (per 100,000 people aged 15-24 per year)
heart diseases	2.9
leukemia	1.7
cancers of lymph and blood other than leukemia	1.0
other cancers	3.6
strokes	1.0
motor vehicle accidents	44.8
other accidents	16.9
chronic lung diseases	0.3
pneumonia and influenza	0.8
diabetes	0.3
liver diseases	0.3
suicide	12.3
homicide	15.6
kidney diseases	0.3
birth defects	1.4
blood poisoning	0.2

Source: National Center for Health Statistics, Monthly Vital Statistics Report, August 1983.

1. Of 100,000 people aged 15-24, how many would you expect to die annually from pneumonia and influenza?
2. Of 1,000,000 people aged 15-24, how many would you expect to die annually from pneumonia and influenza?
3. Suppose there are 200,000 people, and 3 die from a certain cause. What is the death rate per 100,000 people?
4. Of 250,000 people aged 15-24, about how many would you expect to die annually from motor vehicle accidents?
5. Construct a line plot of these data. To avoid crowding when plotting the X's, round each death rate to the nearest whole number.
6. Which cause of death is an outlier?

Application 1

Rock Albums

The following list of the top 10 record albums in the first five months of 1985 is based on *Billboard* magazine reports.

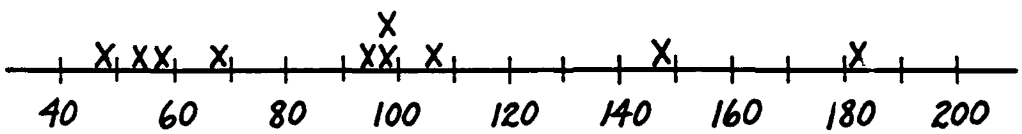
Artist	Title	Total Points
Bruce Springsteen	"Born in the U.S.A."	183
Madonna	"Like a Virgin"	149
Phil Collins	"No Jacket Required"	108
John Fogerty	"Centerfield"	97
Wham!	"Make It Big"	97
Soundtrack	"Beverly Hills Cop"	93
Tina Turner	"Private Dancer"	69
Prince	"Purple Rain"	59
Foreigner	"Agent Provocateur"	54
USA for Africa	"We Are the World"	49

Source: *Los Angeles Times*, May 25, 1985.

The total points were calculated by giving 10 points for each week an album was number 1 on the *Billboard* charts, 9 points for each week it was number 2, 8 points for each week it was number 3, and so forth.

1. If a record was number 1 for 3 weeks, number 2 for 5 weeks, and number 3 for 2 weeks, how many total points would it have?
2. How many points does a record earn by being number 5 for 1 week?
3. If a record was number 4 for 3 weeks and number 5 for 1 week, how many total points would it have?
4. Find two ways for a record to earn 25 points.
5. There were about 21 weeks in the first five months of 1985. Find a way for "Born in the U.S.A." to earn 183 points in these 21 weeks.

The following line plot was constructed from these data.



6. Which record(s) is an outlier?
7. Do the records seem to cluster into more than one group?
8. List the records in the lowest group.
9. List the records in the next lowest group.
10. Write a description of what you learned from studying this plot.

VI. SCATTER PLOTS

The table below gives the box score for the first game of the 1985 National Basketball Association Championship series.

Los Angeles Lakers 114, Boston Celtics 148

LOS ANGELES

	Min	FG-A	FT-A	R	A	P	T
Worthy	37	8-19	4-6	8	5	1	20
Rambis	22	4-6	0-0	9	0	2	8
Jabbar	22	6-11	0-0	3	1	3	12
Magic Johnson	34	8-14	3-4	1	12	2	19
Scott	30	5-14	0-0	2	0	2	10
Cooper	24	1-5	2-2	2	2	3	4
McAdoo	21	6-13	0-0	3	0	5	12
McGee	15	4-7	4-5	2	2	1	14
Spriggs	15	4-7	0-2	3	4	1	8
Kupchak	16	3-3	1-2	2	1	3	7
Lester	4	0-1	0-0	0	1	0	0
Totals	240	49-100	14-21	35	28	23	114

Shooting field goals, 49.0%, free throws, 66.7%

BOSTON

	Min	FG-A	FT-A	R	A	P	T
Bird	31	8-14	2-2	6	9	1	19
McHale	32	10-16	6-9	9	0	1	26
Parish	28	6-11	6-7	8	1	1	18
Dennis Johnson	33	6-14	1-1	3	10	1	13
Ainge	29	9-15	0-0	5	6	1	19
Buckner	16	3-5	0-0	4	6	4	6
Williams	14	3-5	0-0	0	5	2	6
Wedman	23	11-11	0-2	5	2	4	26
Maxwell	16	1-1	1-2	3	1	0	3
Kite	10	3-5	1-2	3	0	1	7
Carr	4	1-3	0-0	1	0	1	3
Clark	4	1-2	0-0	1	3	0	2
Totals	240	62-102	17-25	48	43	17	148

Shooting field goals, 60.8%, free throws, 68.0%

Key for table

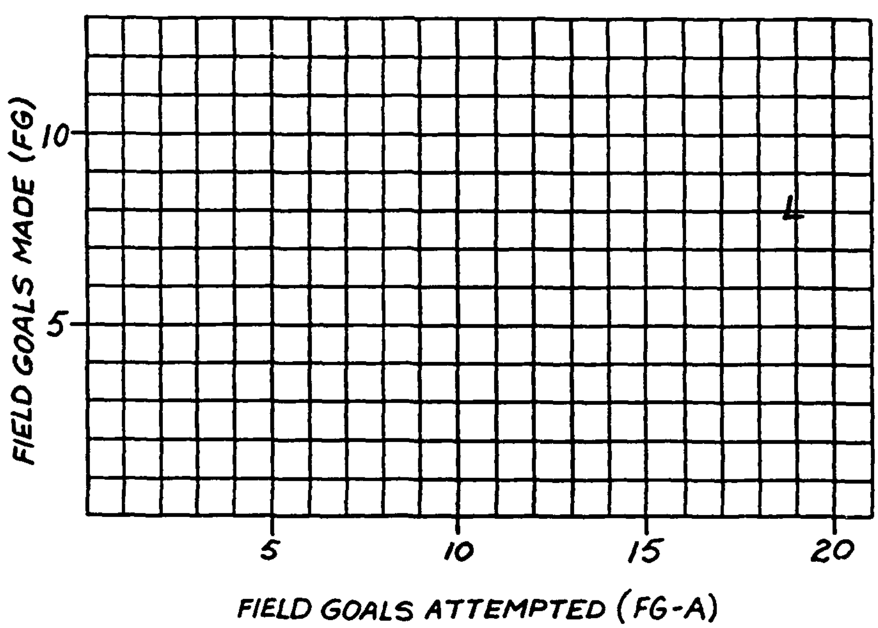
Min	Minutes played
FG-A	Field goals made - field goals attempted
FT-A	Free throws made - free throws attempted
R	Rebounds
A	Assists
P	Personal fouls
T	Total points scored

Source: *Los Angeles Times*, May 28, 1985.

Discussion Questions

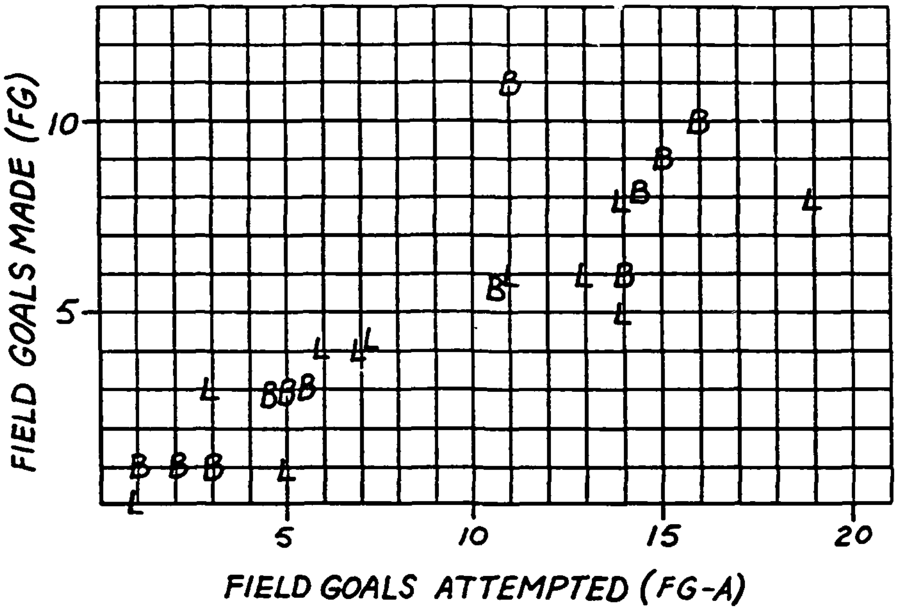
1. How many rebounds did Kevin McHale make?
2. Which player played the most minutes?
3. Which player had the most assists?
4. How many field goals did James Worthy make? How many did he attempt? What percentage did he make?
5. Five players are on the court at one time for each team. Determine how many minutes are in a game.
6. Which team made a larger percentage of free throws?
7. How is the T (total points scored) column computed? Verify that this number is correct for Magic Johnson and for Kevin McHale. (Caution: Some of the field goals for other players were three point shots.)

Do you think that the players who *attempt* the most field goals are generally the players that *make* the most field goals? Of course! We can see this from the box score. To further investigate this question, we will make a *scatter plot* showing field goals made (FG) and field goals attempted (FG-A). First, set up a plot with field goals attempted on the horizontal axis and field goals made on the vertical axis.



Worthy, the first player, attempted 19 field goals and made 8 of them. The L on the preceding plot represents Worthy. The L is above 19 and across from 8. We used an L to show that he is a Los Angeles player.

The completed scatter plot follows. Each B stands for a Boston player and each L for a Los Angeles player.



As we suspected, this plot shows that players who attempt more field goals generally make more field goals, and players who attempt few field goals make few field goals. Thus, there is a *positive* association between field goals attempted and field goals made.

However, we can see much more from this plot. First, a player who makes every basket will be represented by a point on the line through the points (0, 0), (1, 1), (2, 2), (3, 3), and so forth. Second, the players who are relatively far below this line were not shooting as well as the other players. Finally, we can observe the relative positions of the two teams in this plot.

Discussion Questions

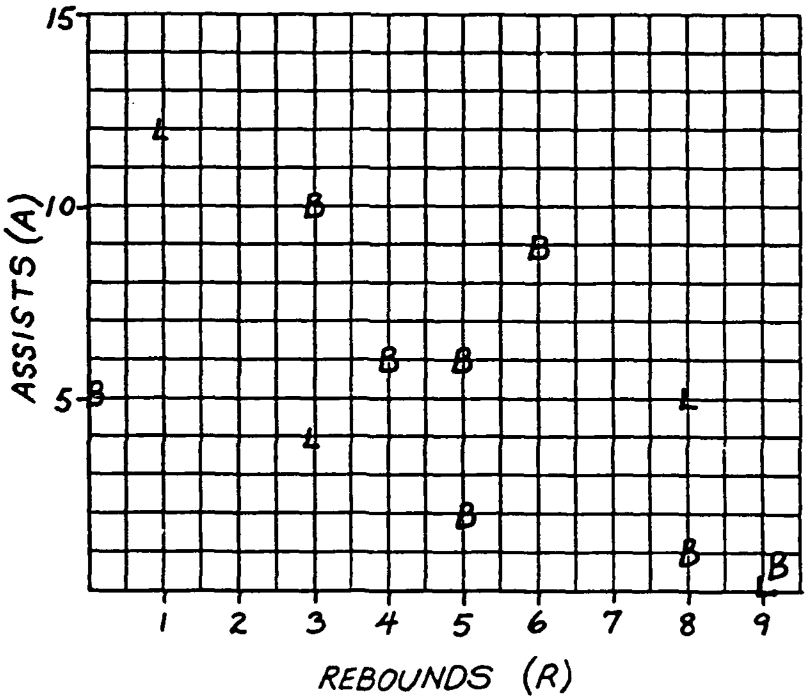
1. Using the scatter plot, find the points that represent the three perfect shooters.
2. Why are all the points below a diagonal line running from lower left to upper right?
3. Is there a different pattern for Los Angeles and Boston players?
4. Which three Laker players were not shooting very well that game?
5. Suppose a player attempts 9 field goals. About how many would you expect him to make?
6. Write a brief description of the information conveyed by this scatter plot. Then read the following sample discussion. Did you notice any information not listed in this sample discussion?

In this plot, we were not surprised to see a positive association between the number of field goals attempted and the number of field goals made. There were three players, two from Boston and one from Los Angeles, who made all the field goals they attempted. One of these Boston players was truly outstanding as he made eleven out of eleven attempts. The Laker players who attempted a great number of field goals generally did not make as many of them as did the Celtics who attempted a great number of field goals. This could have been the deciding factor in the game.

The points seem to cluster into two groups. The cluster on the upper right generally contains players who played over 20 minutes and the one on the lower left contains players who played less than 20 minutes.

An assist is a pass that leads directly to a basket. A player is credited with a rebound when he recovers the ball following a missed shot. Do you think that players who get a lot of rebounds also make a lot of assists? It is difficult to answer this question just by looking at the box score.

To answer this question, we will make a scatter plot showing rebounds (R) and assists (A). This plot includes all players who made at least four rebounds or four assists.



This plot shows that players who get *more* rebounds generally have *fewer* assists, and players who get *fewer* rebounds have *more* assists. Thus, there is a *negative* association between rebounds and assists.

Discussion Questions

1. Do the players who get the most rebounds also make the most assists?

2. Suppose a player had 7 rebounds. About how many assists would you expect this player to have?

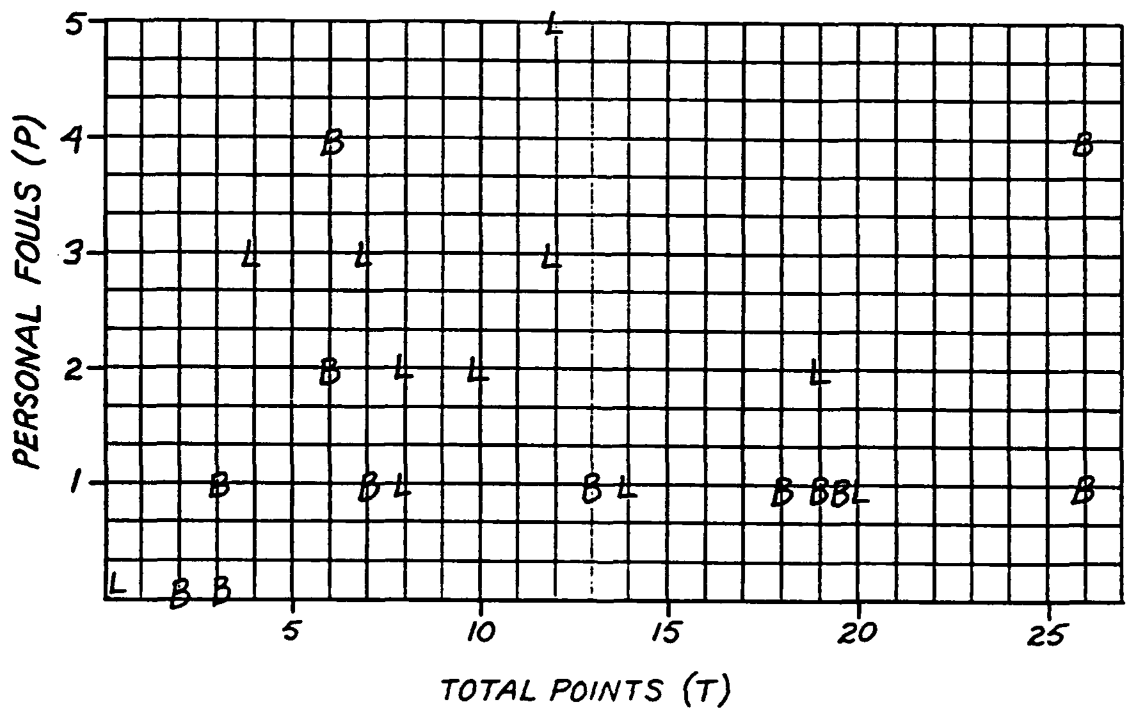
3. Is there a different pattern for Boston players than for Los Angeles players?

4. Why do you suppose players who get a lot of rebounds do not make a lot of assists?

5. If you were the coach and you wanted a player to make more assists, would you instruct him to make fewer rebounds?

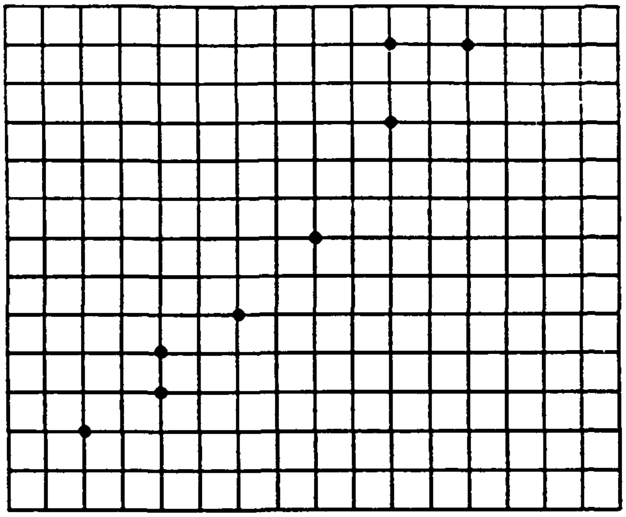
6. Why didn't we include players who would have been in the lower left-hand corner of this plot?

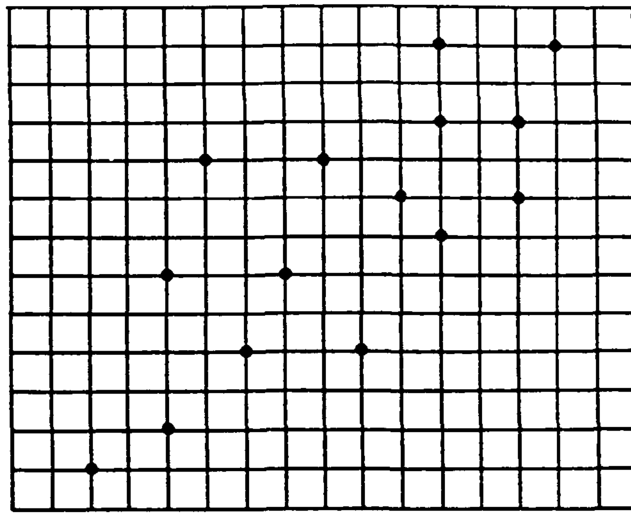
The following scatter plot shows total points and personal fouls for all players.



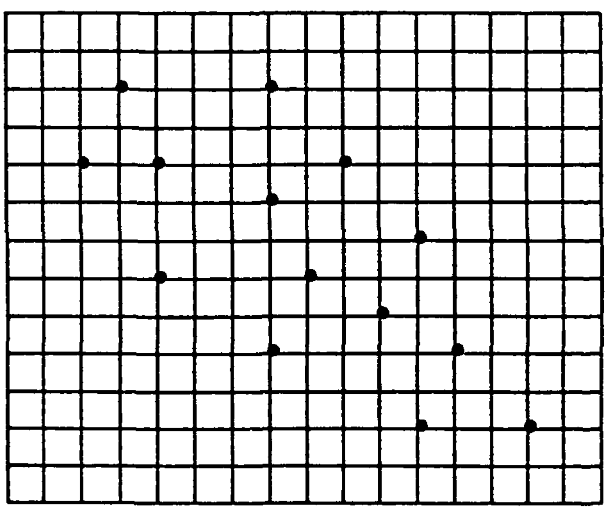
This plot shows *no association* between total points scored and the number of personal fouls committed.

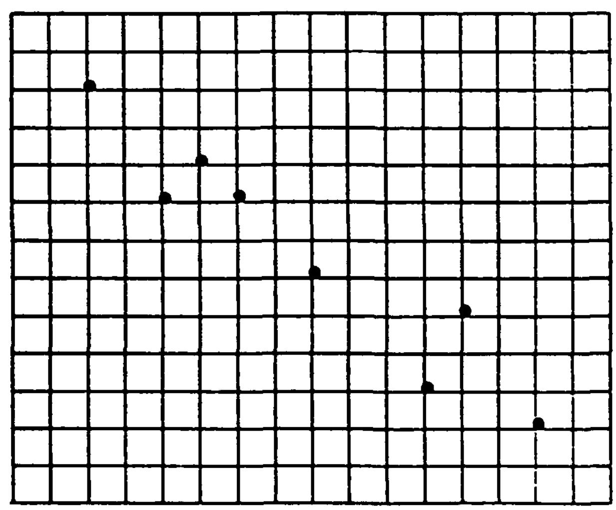
In summary, the following scatter plots show *positive association.*





The following scatter plots show *negative association.*





Application 22

Walk-around Stereos

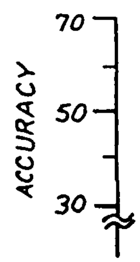
The following table lists 22 "walk-around stereos," each with its price and overall score. The overall score is based on "estimated overall quality as tape players, based on laboratory tests and judgments of features and convenience." A "perfect" walk-around stereo would have a score of 100. Consumers Union says that a difference of 7 points or less in overall score is not very significant.

Ratings of Walk-around Stereos

Brand and Model	Price	Overall Score
AIWA HSP02	$120	73
AIWA HSJ02	180	65
JVC CQ1K	130	64
Sanyo MG100	120	64
Sony Walkman WM7	170	64
Sanyo Sportster MG16D	70	61
Toshiba KTVS1	170	60
JVC CQF2	150	59
Panasonic RQJ20X	150	59
Sharp WF9BR	140	59
Sony Walkman WM4	75	56
General Electric Stereo Escape II 35275A	90	55
KLH Solo S200	170	54
Sanyo Sportster MG36D	100	52
Koss Music Box A2	110	51
Toshiba KTS3	120	47
Panasonic RQJ75	50	46
Sears Cat. No. 21162	60	45
General Electric Great Escape 35273A	70	43
Sony Walkman WMR2	200	41
Sony Walkman WMF2	220	38
Realistic SCP4	70	37

Source: *Consumer Reports Buying Guide*, 1985.

1. Which walk-around stereo do you think is the best buy?
2. A scatter plot will give a better picture of the relative price and overall score of the walk-around stereos. Make a scatter plot with price on the horizontal axis. You can make the vertical axis as follows:



The ≈ lines indicate that part of the vertical axis is not shown, so that the plot is not too tall.

3. Which stereo appears to be the best buy according to the scatter plot?
4. Is there a positive, negative, or no association between price and overall score?
5. Given their overall scores, which walk-around stereos are too expensive?

Application 25

Speeding

The following table shows average freeway speeds as recorded by highway monitoring devices in California. The newspaper gave no explanation why the average speed is missing for 1971 and 1973.

Year	Average Highway Speed in Miles per Hour
1970	59
1971	—
1972	61
1973	—
1974	55
1975	56
1976	57
1977	57
1978	57
1979	58
1980	56
1981	57
1982	57

Source: *Los Angeles Times*, May 22, 1983.

1. Construct a plot over time of the average speeds.
2. Can you guess what year the 55 miles per hour speed limit went into effect?
3. Some people think drivers are ignoring the 55 miles per hour speed limit. Do you think your plot shows that this is the case?
4. The fatalities in California per 100 million miles driven are shown in the following table. Construct a plot over time of these data.

Year	Fatalities per 100 Million Miles
1970	3.8
1971	3.2
1972	3.2
1973	3.0
1974	2.2
1975	2.2
1976	2.3
1977	2.4
1978	2.6
1979	2.5
1980	2.5
1981	2.4
1982	2.1

Source: *Los Angeles Times*, May 22, 1983.

5. Was there a decrease in fatalities when the 55 miles per hour speed limit took effect?
6. Another way to display these data is with a scatter plot of fatalities against speed. Construct such a plot. Place the values for speed on the horizontal axis. Plot the last two digits of the year instead of a dot.
7. What do you learn from the plot in question 6?
8. Why is the plot in question 6 the best one?

VII. LINES ON SCATTER PLOTS

The 45° Line

In the last section we interpreted scatter plots by looking for general relationships of positive, negative, and no association. We also looked for clusters of points that seemed special in some way. This section shows how interpretations of scatter plots are sometimes helped by adding a straight line to the plot. Two different straight lines are used. One is the 45° line going through the points (0, 0), (1, 1), (2, 2), and so forth. The second type is a straight line that is fitted to go through much of the data.

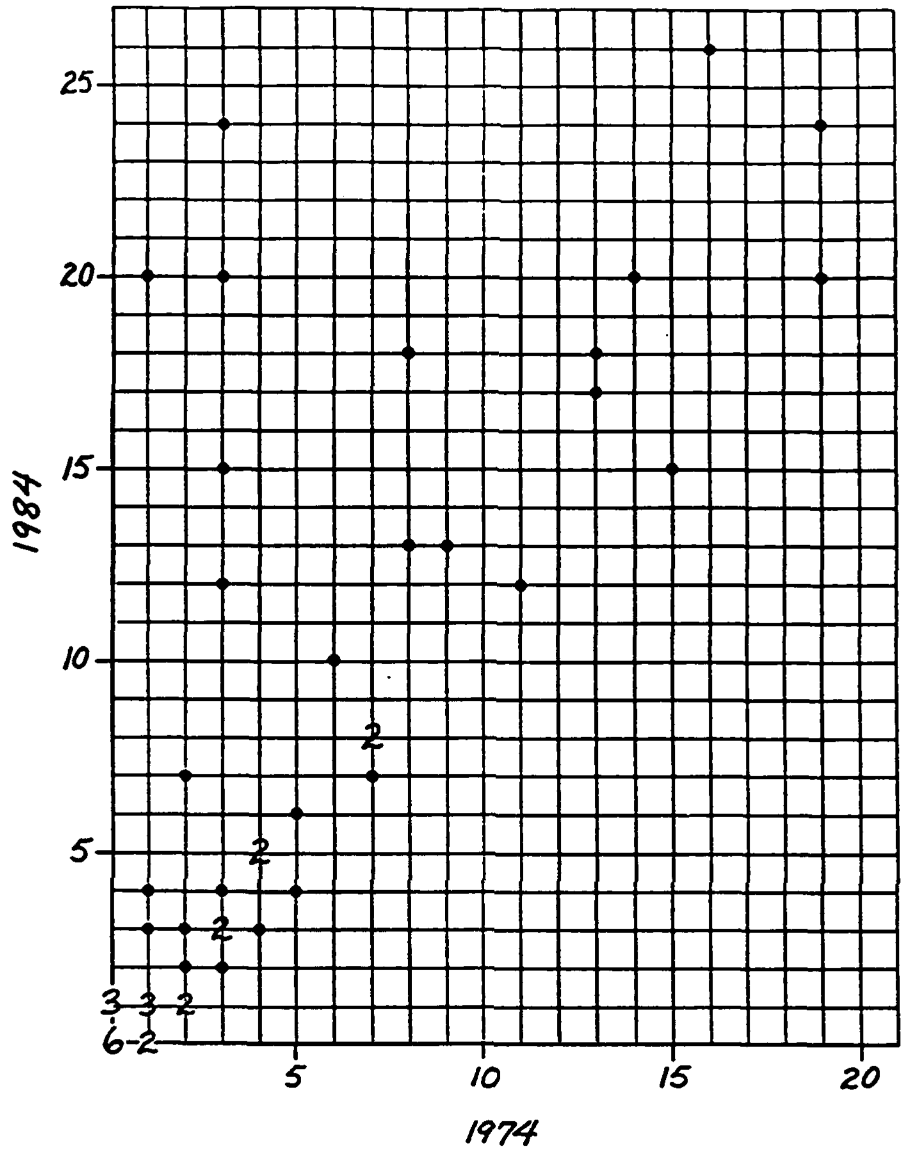
This table lists the number of black state legislators for each state in 1974 and 1984.

Number of Black State Legislators

	1974	1984		1974	1984
Alabama	3	24	Montana	0	0
Alaska	2	1	Nebraska	1	1
Arizona	2	2	Nevada	3	3
Arkansas	4	5	New Hampshire	0	0
California	7	8	New Jersey	7	7
Colorado	4	3	New Mexico	1	0
Connecticut	6	10	New York	14	20
Delaware	3	3	North Carolina	3	15
District of Columbia	n/a	n/a	North Dakota	0	0
Florida	3	12	Ohio	11	12
Georgia	16	26	Oklahoma	4	5
Hawaii	0	0	Oregon	1	3
Idaho	0	0	Pennsylvania	13	18
Illinois	19	20	Rhode Island	1	4
Indiana	7	8	South Carolina	3	20
Iowa	1	1	South Dakota	0	0
Kansas	5	4	Tennessee	9	13
Kentucky	3	2	Texas	8	13
Louisiana	8	18	Utah	0	1
Maine	1	0	Vermont	0	1
Maryland	19	24	Virginia	2	7
Massachusetts	5	6	Washington	2	3
Michigan	13	17	West Virginia	1	1
Minnesota	2	1	Wisconsin	3	4
Mississippi	1	20	Wyoming	0	1
Missouri	15	15	Total	236	382

Source: Joint Center for Political Studies.

The scatter plot of the 1984 number against the 1974 number follows:



A striking feature of the plot is that the points all seem to lie above an (imaginary) diagonal line. Another feature is that there are many points in the lower left-hand corner. In fact, several states sometimes lie at exactly the same point. For example, Arkansas and Oklahoma both lie at (4, 5). To show this, we placed a 2 at (4, 5).

Discussion Questions

1. Place a ruler on the plot next to the line going through (0, 0), (10, 10), (20, 20), and so forth. For states on this line, the 1984 and 1974 numbers of black legislators are equal. How many points are exactly on this line?

2. If a point is above this line, the number of black legislators in that state in 1984 is larger than the number of black legislators that state had in 1974. Name three states for which this statement is true.

3. How many points fall below this line? What can we say about these states? What is the maximum (vertical) distance any of these is below the line? What does this mean in terms of the number of black legislators in 1974 and 1984?

4. Again, consider states above this line, those where the number of black legislators was larger in 1984 than in 1974. What are the names of the 7 or so states that lie farthest above the line? What do these states have in common?

5. The number of black legislators has generally increased from 1974 to 1984. Does this mean that the percentage of legislators who are black has necessarily increased? (Hint: Is the total number of legislators in a state necessarily the same in 1984 as in 1974?)

In summary, this 45° line (sometimes called the $y = x$ line) divides the plot into two regions. We should try to distinguish the characteristics of the points in the two regions. In this plot the top region contains states where the number of black legislators in 1984 is larger than it was in 1974. Most of the states lie in this region. The points in this region that are farthest from the line are those where the number has increased the most from 1974 to 1984. These states turn out to be states in the deep south. There are only a few points slightly below the 45° line, where the number of black legislators was greater in 1974 than in 1984. These are all states that had only 5 or fewer black legislators in 1974. Almost half the states are in the lower left-hand corner, with 5 or fewer in both years. Two states, Illinois and Maryland, had relatively large numbers in both years.

It would have been helpful to plot each state's abbreviation (such as NY for New York) instead of a dot. However, there wasn't room to do this for the states in the lower left corner.

Application 28

Smoking and Heart Disease

The following table lists 21 countries with the cigarette consumption per adult per year and the number of deaths per 100,000 people per year from coronary heart disease (CHD).

Country	Cigarette Consumption per Adult per Year	CHD Mortality per 100,000 (ages 35-64)
United States	3900	257
Canada	3350	212
Australia	3220	238
New Zealand	3220	212
United Kingdom	2790	194
Switzerland	2780	125
Ireland	2770	187
Iceland	2290	111
Finland	2160	233
West Germany	1890	150
Netherlands	1810	125
Greece	1800	41
Austria	1770	182
Belgium	1700	118
Mexico	1680	32
Italy	1510	114
Denmark	1500	145
France	1410	60
Sweden	1270	127
Spain	1200	44
Norway	1090	136

Source: *American Journal of Public Health.*

1. In which country do adults smoke the largest number of cigarettes?
2. Which country has the highest death rate from coronary heart disease?
3. Which country has the lowest death rate from coronary heart disease?
4. If we want to predict CHD mortality from cigarette consumption, which variable should be placed on the horizontal axis of a scatter plot?
5. a) Make a scatter plot of the data.
   b) Draw two vertical lines so there are seven points in each strip.
   c) Place an X in each strip at the median of the cigarette consumption and the median of the CHD mortality.
   d) Do the three X's lie close to a straight line?
   e) Draw in the fitted line.

6. a) Which three countries lie the farthest vertical distance from the line?

   b) How many units do they lie from the line?

   c) Considering the cigarette consumption, are these countries relatively high or low in CHD mortality?

7. If you were told that the adults in a country smoke an average of 2500 cigarettes a year, how many deaths from CHD would you expect?

8. If you were told that the adults in a country smoke an average of 1300 cigarettes a year, how many deaths from CHD would you expect?

9. (For class discussion) Sometimes strong association in a scatter plot is taken to mean that one of the variables *causes* the other one. Do you think that a high CHD death rate could cause cigarette consumption to be high? Could high cigarette consumption cause the CHD death rate to be high? Sometimes, though, there is not a causal relationship between the two variables. Instead, there is a hidden third variable. This variable could cause both of the variables to be large simultaneously. Do you think that this might be the situation for this example? Can you think of such a possible variable?

10. (For students who have studied algebra.) Choose two points on the fitted line, and from them find the equation of the line. Express it in the form $y = mx + b$, where y is mortality from coronary heart disease per 100,000 people (aged 35-64) per year, and x is cigarette consumption per adult per year. Using this equation, how many additional deaths per 100,000 people tend to result from an increase of 200 in cigarette consumption? What number of cigarettes per year is associated with one additional death from CHD per 100,000 people per year?

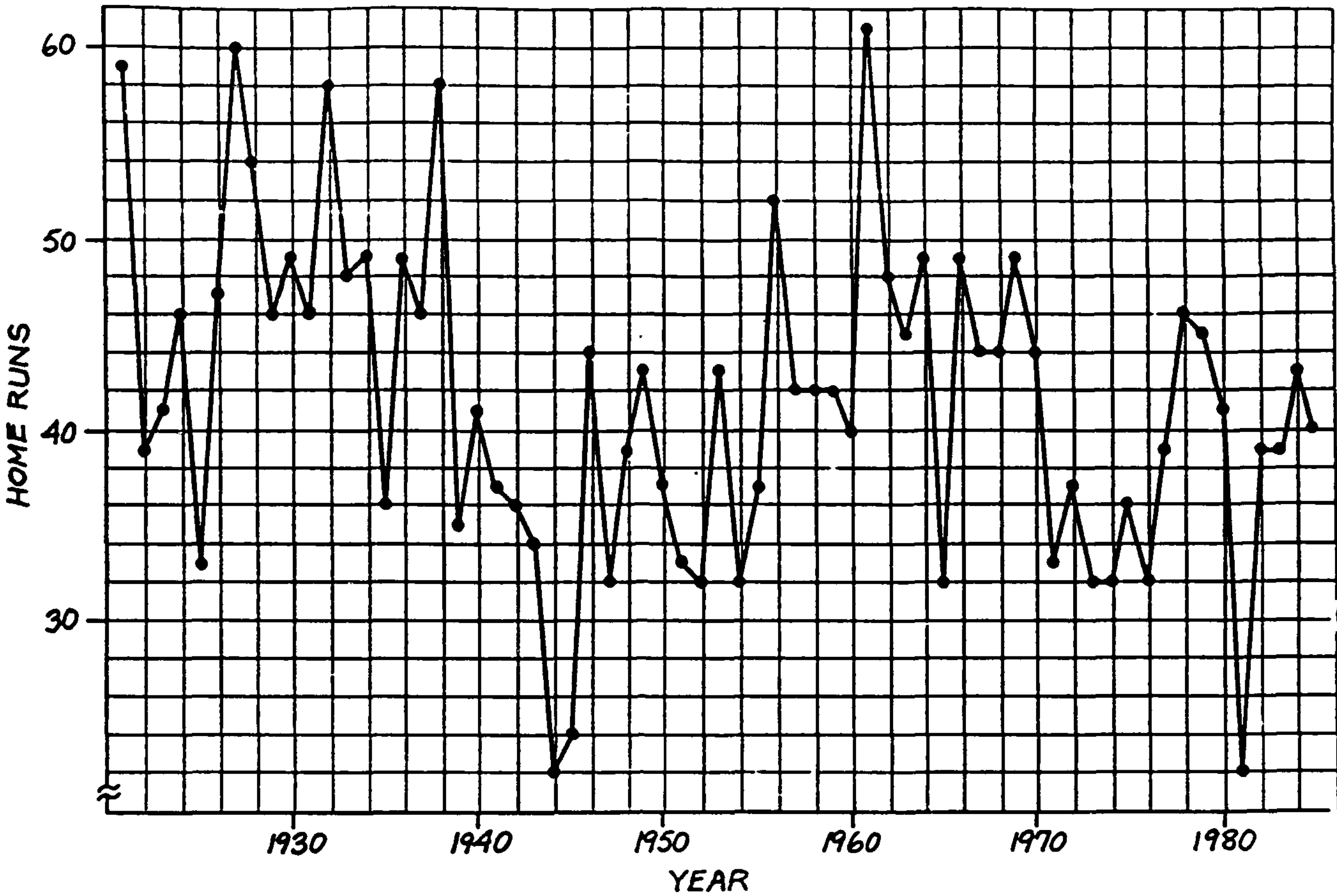
VIII. SMOOTHING PLOTS OVER TIME

The following table lists the American League home run champions from 1921 to 1985.

Year	American League	HR	Year	American League	HR
1921	Babe Ruth, New York	59	1957	Roy Sievers, Washington	42
1922	Ken Williams, St. Louis	39	1958	Mickey Mantle, New York	42
1923	Babe Ruth, New York	41	1959	Rocky Colavito, Cleveland	42
1924	Babe Ruth, New York	46		Harmon Killebrew, Washington	
1925	Bob Meusel, New York	33	1960	Mickey Mantle, New York	40
1926	Babe Ruth, New York	47	1961	Roger Maris, New York	61
1927	Babe Ruth, New York	60	1962	Harmon Killebrew, Minnesota	48
1928	Babe Ruth, New York	54	1963	Harmon Killebrew, Minnesota	45
1929	Babe Ruth, New York	46	1964	Harmon Killebrew, Minnesota	49
1930	Babe Ruth, New York	49	1965	Tony Conigliaro, Boston	32
1931	Babe Ruth, New York	46	1966	Frank Robinson, Baltimore	49
	Lou Gehrig, New York		1967	Carl Yastrzemski, Boston	44
1932	Jimmy Foxx, Philadelphia	58		Harmon Killebrew, Minnesota	
1933	Jimmy Foxx, Philadelphia	48	1968	Frank Howard, Washington	44
1934	Lou Gehrig, New York	49	1969	Harmon Killebrew, Minnesota	49
1935	Jimmy Foxx, Philadelphia	36	1970	Frank Howard, Washington	44
	Hank Greenberg, Detroit		1971	Bill Melton, Chicago	33
1936	Lou Gehrig, New York	49	1972	Dick Allen, Chicago	37
1937	Joe DiMaggio, New York	46	1973	Reggie Jackson, Oakland	32
1938	Hank Greenberg, Detroit	58	1974	Dick Allen, Chicago	32
1939	Jimmy Foxx, Boston	35	1975	George Scott, Milwaukee	36
1940	Hank Greenberg, Detroit	41		Reggie Jackson, Oakland	
1941	Ted Williams, Boston	37	1976	Graig Nettles, New York	32
1942	Ted Williams, Boston	36	1977	Jim Rice, Boston	39
1943	Rudy York, Detroit	34	1978	Jim Rice, Boston	46
1944	Nick Etten, New York	22	1979	Gorman Thomas, Milwaukee	45
1945	Vern Stephens, St. Louis	24	1980	Reggie Jackson, New York	41
1946	Hank Greenberg, Detroit	44		Ben Oglivie, Milwaukee	
1947	Ted Williams, Boston	32	1981	Bobby Grich, California	22
1948	Joe DiMaggio, New York	39		Tony Armas, Oakland	
1949	Ted Williams, Boston	43		Dwight Evans, Boston	
1950	Al Rosen, Cleveland	37		Eddie Murray, Baltimore	
1951	Gus Zernial, Chicago-Philadelphia	33	1982	Gorman Thomas, Milwaukee	39
1952	Larry Doby, Cleveland	32		Reggie Jackson, California	
1953	Al Rosen, Cleveland	43	1983	Jim Rice, Boston	39
1954	Larry Doby, Cleveland	32	1984	Tony Armas, Boston	43
1955	Mickey Mantle, New York	37	1985	Darrell Evans, Detroit	40
1956	Mickey Mantle, New York	52			

Source: *The World Almanac and Book of Facts*, 1985 edition.

From this list it is difficult to see any general trends in the number of home runs through the years. To try to determine the general trends, we will make a scatter plot over time of the number of home runs hit by the champions and connect these points.



This scatter plot looks all jumbled up! It is impossible to see general trends because of the large fluctuations in the number of home runs hit from year to year. For example, 58 home runs were hit in 1938 compared to only 35 the next year. This variation gives the plot a sawtooth effect. The highs and lows, not the overall pattern, capture our attention. To remove the large fluctuations from the data, we will use a method called *smoothing*.

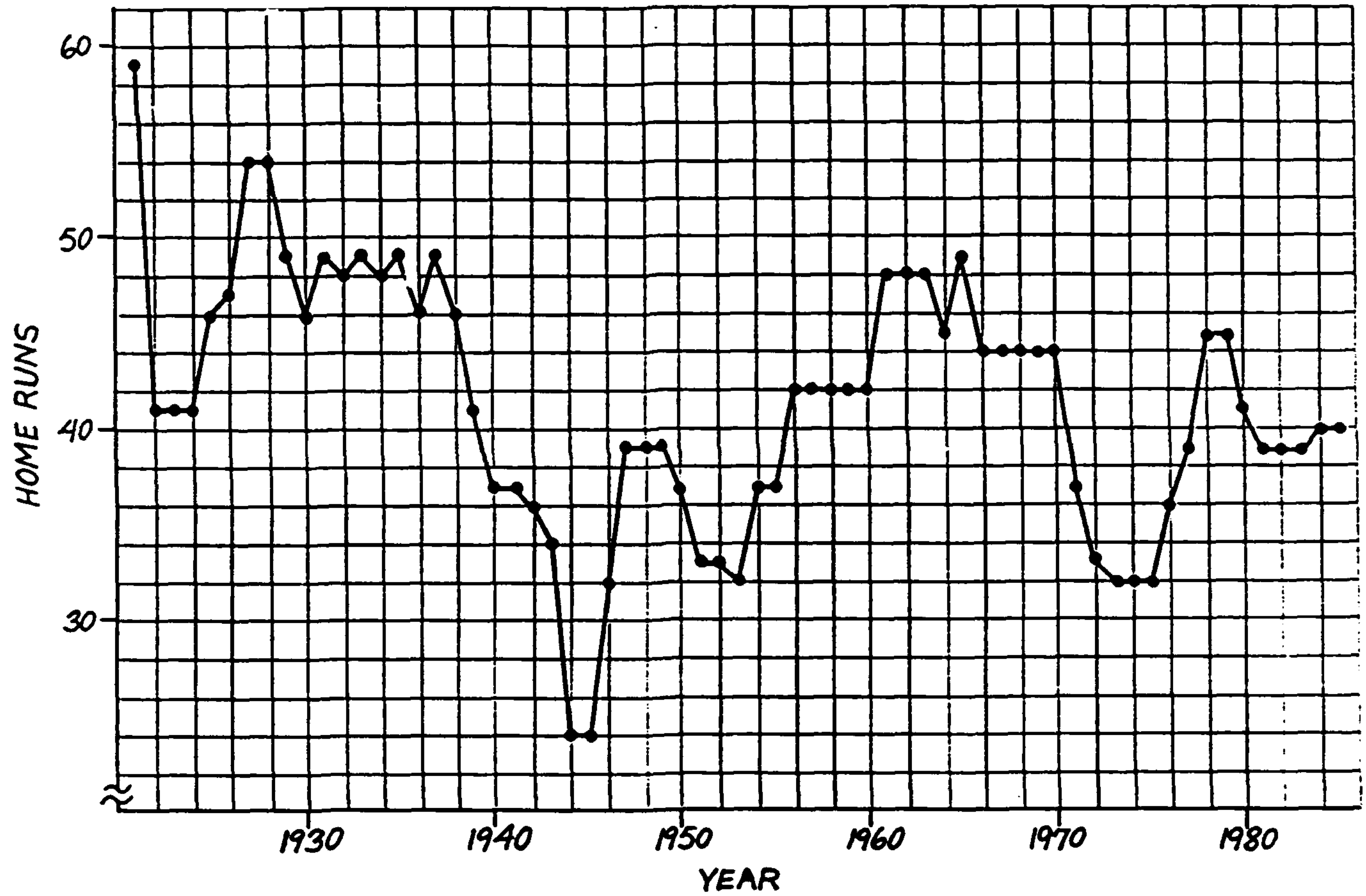
To illustrate, the smoothed version of the first ten years of the home run champions' data follows.

Year	Home Runs	Smoothed Values
1921	59	59
1922	39	41
1923	41	41
1924	46	41
1925	33	46
1926	47	47
1927	60	54
1928	54	54
1929	46	49
1930	49	46
1931	46	

To find the smoothed value for 1924, for example, the 46 home runs for that year are compared to the number of home runs for the year before, 41, and the number of home runs for the following year, 33. The median of the three numbers, 41, is entered into the smoothed values column.

For the first and last years, just copy the original data into the smoothed values column.

The plot of the connected smoothed values follows. Notice what has happened to the large fluctuation between 1938 and 1939. Since this plot is smoother than the previous one, we can see general trends better, such as the drop in the number of home runs in the 1940's.



Discussion Questions

1. Complete the smoothed value column through 1940 for the next ten American League home run champions.

2. Study the smoothed plot of the American League home run champions.

   a. What happened around 1940 that could have affected the number of home runs hit?

   b. Did the increase in the number of games from 154 to 162 in 1961 have an effect on the number of home runs hit?

3. Study the following rule changes. Do any of them seem to have affected the number of home runs hit by the champions?

   1926 — A ball hit over a fence that is less than 250 feet from home plate will not be counted as a home run.

   1931 — A fair ball that bounces over a fence will be counted as a double instead of a home run.

   1959 — New ballparks must have a minimum distance of 325 feet down the foul lines and 400 feet in center field.

   1969 — The strike zone is decreased in size to include only the area from the armpit to the top of the knee.

   1969 — The pitcher's mound is lowered, giving an advantage to the hitter.

   1971 — All batters must wear helmets.

4. In 1981 there was a strike that shortened the season. Can this be seen in the original data? In the smoothed values?

5. Since they were not smoothed, the endpoints may appear to be out of place. The number of home runs hit in 1921 seems too high. Can you determine a better rule for deciding what to write in the smoothed values column for the endpoints?

6. Imagine a curve through the smoothed values. Try to predict the number of home runs hit in 1986.

7. Some students feel that smoothing is not a legitimate method. For example, they do not like changing the original 33 home runs in 1925 to 46 home runs on the plot of smoothed values. Write a description of the trends that are visible in the smoothed plot that are not easily seen in the original plot. Try to convince a reluctant fellow student that smoothing is valuable. Then study the following answer. Did you mention features we omitted?

The original plot of the time series for home runs gives a very jagged appearance. There were values that were quite large for two years in the 1920's, two years in the 1930's, and also in 1961. Extremely low values occurred in the mid-1940's and in 1981. Using this plot, it is difficult to evaluate overall trends. However, the values in the 1940's and early 1950's seem lower than the values in the late 1920's and 1930's.

We get a stronger impression of trends from the smoothed plot of the home run data. In particular, for the years from 1927 to 1935, the values are generally higher than at any other time before or since. The only period that was nearly comparable was in the early 1960's. The original data show that the champions causing the earlier values to be large were Babe Ruth, Jimmy Foxx, and Lou Gehrig. In the 1960's, it was Roger Maris and Harmon Killebrew. These players clearly were outstanding home run hitters!

Application 35

Birth Months

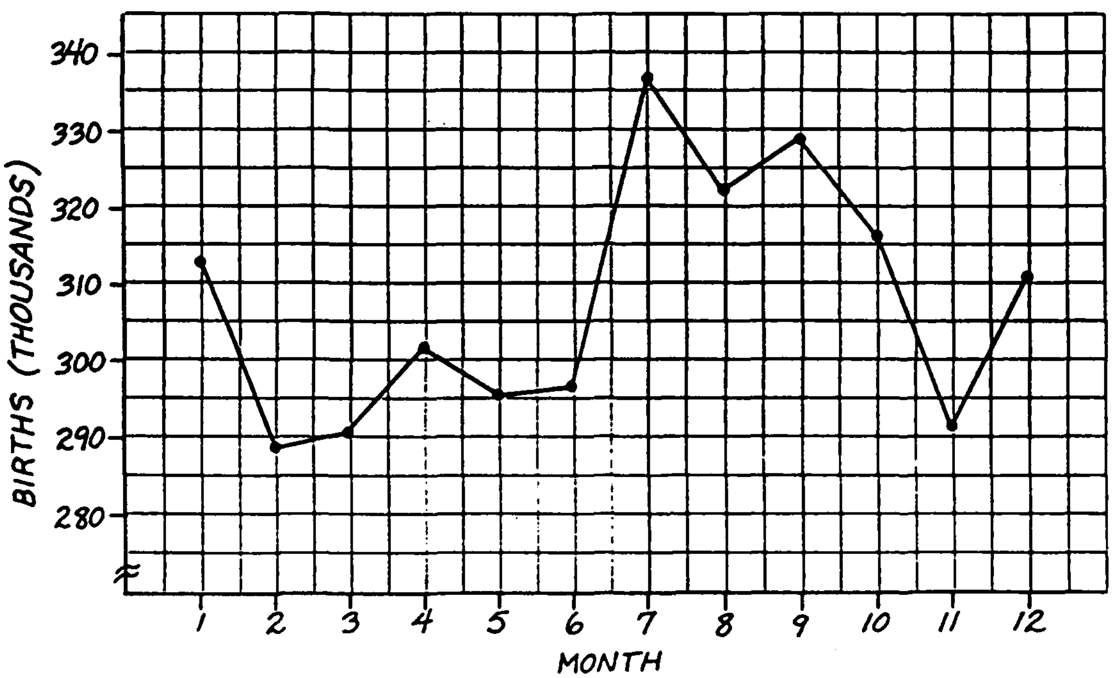
The following table gives the number of babies born in the United States for each month of 1984. The numbers are in thousands.

Month	Births (thousands)	Smoothed Values
January	314	
February	289	
March	291	
April	302	
May	296	
June	297	
July	336	
August	323	
September	329	
October	316	
November	292	
December	311	

Source: National Center for Health Statistics.

1. How many babies were born in May 1984?
2. In which month were the most babies born?

The time series plot for these data is given as follows. This plot is a good candidate for smoothing because of the sawtooth effect. This appearance is an indication that some points are unusually large or small.



3. Copy and complete the "Smoothed Values" column.

4. Make a scatter plot of the smoothed values.

5. What is the general trend in the number of babies born throughout the year?

Application 36

Olympic Marathon

The following table shows the winning times for the marathon run (slightly more than 26 miles) in the 1896-1984 Olympics. The times are rounded to the nearest minute.

Year	Winner Name, Country	Time		Time in Minutes	Smoothed Values
1896	Loues, Greece	2 hours	59 minutes	179	
1900	Teato, France	3	0	180	
1904	Hicks, U.S.A.	3	29	209	
1908	Hayes, U.S.A.	2	55	175	
1912	McArthur, South Africa	2	37	157	
1920	Kolehmainen, Finland	2	33	153	
1924	Stenroos, Finland	2	41	161	
1928	El Ouafi, France	2	33	153	
1932	Zabala, Argentina	2	32	152	
1936	Son, Japan	2	29	149	
1948	Cabrera, Argentina	2	35		
1952	Zatopek, Czechoslovakia	2	23		
1956	Mimoun, France	2	25		
1960	Bikila, Ethiopia	2	15		
1964	Bikila, Ethiopia	2	12		
1968	Wolde, Ethiopia	2	20		
1972	Shorter, U.S.A.	2	12		
1976	Cierpinski, East Germany	2	10		
1980	Cierpinski, East Germany	2	11		
1984	Lopes, Portugal	2	9		

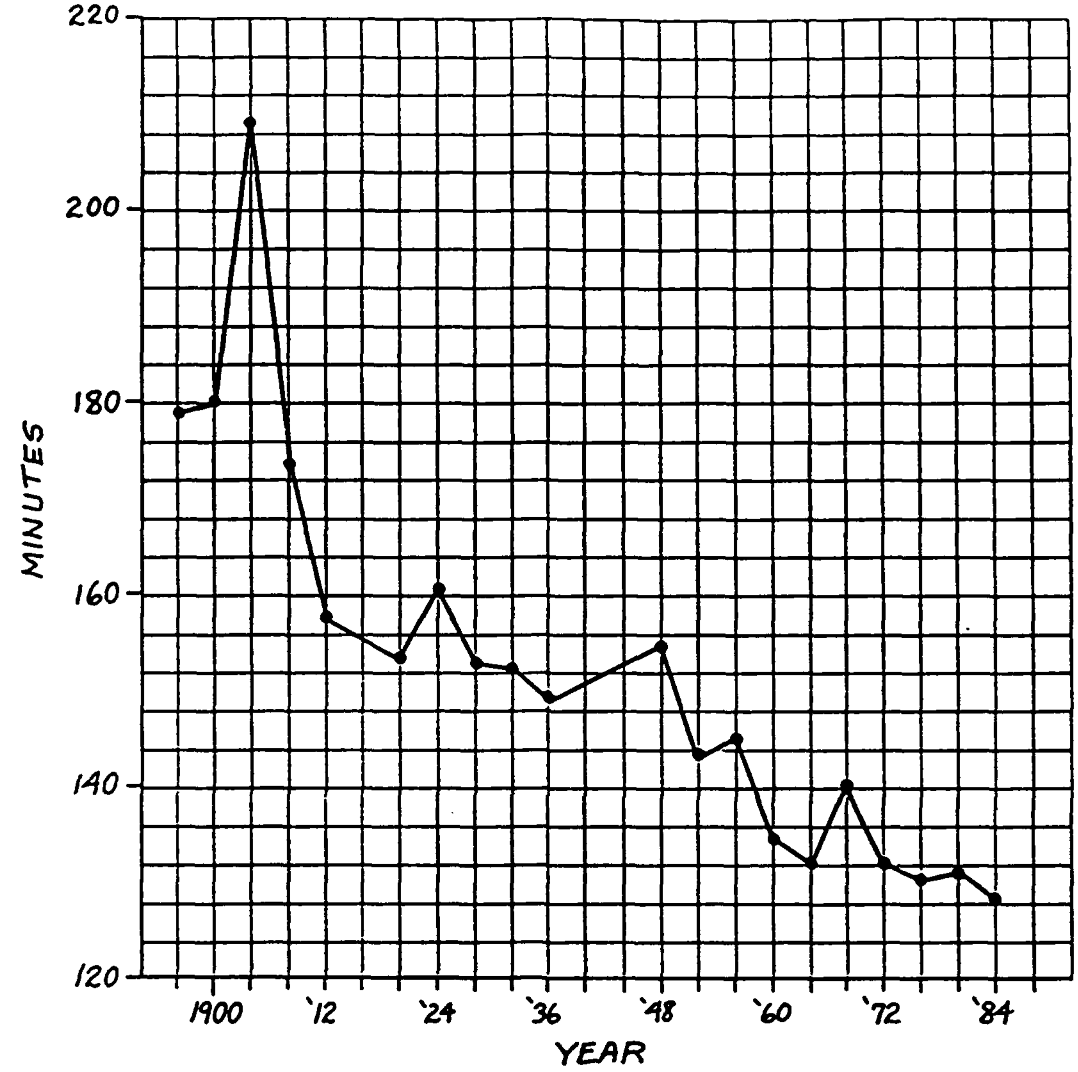
Source: *The World Almanac and Book of Facts*, 1985 edition.

1. The first Olympic women's marathon was not held until 1984. The winner was Joan Benoit of the United States with a time of 2 hours 25 minutes. What was the first year that a Olympic men's marathon winner was able to beat this time?

2. Find the three years when the Olympics were not held. Why were the Olympics not held in these years?

3. Complete the second to the last column of the previous table by converting each time to minutes. The first ten are done for you.

A plot over time with year on the horizontal axis and time in minutes on the vertical axis is shown as follows:



4. What trends do you see in this plot?
5. On the time series plot, which year is farthest from the general trend?
6. Complete the last column of the previous table by smoothing the "time in minutes" column.
7. Construct a plot over time for the smoothed values.
8. Study your plot over time for the smoothed values.
   a. When did the largest drop in time occur?
   b. What do you predict for the winning time in the 1988 Olympic marathon?
   c. Describe the patterns shown on your plot in a short paragraph.

APPENDIX D

A Summer Program in Mathematics and Computer Science
Closing Activities
July 26, 1991

Introduction.. Professor Bernis Barnes
Director of Program

Career Awareness Forum.................................. Professor Gail Finley
Assistant Director of Program
Moderator

Participants.. Dr. Elvira Doman
Physiology & Biochemistry
Associate Program Director
National Science Foundation

Miss Judith Richardson
Computer Science
Director
Pre-Engineering Program
D. C. Public Schools

Dr. John Alexander
Mathematics
University of D. C.

Dr. J. Arthur Jones
Mathematics
President
Futura Technologies, Inc.

Question/answer Period

Remarks.. Dr. Marc J. Lipman
Office of Naval Research
Department of the U.S. Navy

Dr. Philip L. Brach
Dean
College of Physical Science,
Engineering & Technology
University of D. C.

Presentation of Certificates.................................. Prof. Barnes, Dr. Lipman, Dean Brach

This is to certify that

---

has successfully completed the ONR-UDC
Summer Program in Mathematics and Computer Science
for Academically Oriented Students held at the
University of the District of Columbia,
24 June through 26 July, 1991.

President ***University of the District of Columbia***	***Chief of Naval Research***
Project Director, ***Summer Program in Mathematics*** ***and Computer Science***	***Scientific Officer*** ***Math- Science Division*** ***Office of Naval Research***

APPENDIX E

Demographic Data

Name ______________________________

Home Address ______________________________

Street Address

____

City Zip Code

Telephone Number ______________________________

School (1988-1989) ______________________________

School (1989-1990) ______________________________

Date of Birth ______________________________

Place of Birth ______________________________

Are you a U.S. Citizen ________

Intended Occupation ______________________________

Parents/Guardians ______________________________

Address (if different from yours) ______________________________

____

Telephone Number (if different from yours) ______________________________

Occupation of Mother ______________________________

Occupation of Father ______________________________

University of the District of Columbia
College of Physical Science Engineering and Technology

Department of Mathematics
4200 Connecticut Avenue, N.W.
Washington, D.C. 20008

Telephone (202) 282-3171

STUDENT QUESTIONNAIRE

Summer Program in Mathematics and Computer Science

This evaluation is designed to help improve the summer program based on your experience. Please answer each question honestly and according to the directions. Feel free to make comments to clarify your response in the space below each question.

I. ENCIRCLE THE RESPONSE OF YOUR CHOICE:

1. Has this program helped to increase your appreciation of mathematics and computer science? YES NO

   Comments (How? or Why not?):

2. Has this program helped to increase your understanding of mathematics and computer science? YES NO

   Comments (How? or Why not?):

3. Has this program helped to increase your awareness of career opportunities in mathematics based fields. YES NO

   Comments(How? or Why not?):

4. Will you be able to perform better in mathematics when you return to school as a result of this experience? YES NO

   Comments (How? or Why not?):

5. Has this program experience inspired you to pursue the more challenging math courses in high school? YES NO

   Comments (which ones? or why not?):

6. Did you learn to reason more clearly this summer? YES NO

   Comments (How can you tell?)

7. Was this program what you expected it to be? YES NO

   Comments (How or why not?):

II. ENCIRCLE THE ANSWER THAT BEST DESCRIBES YOUR OPINION:

8. The subject matter in this program was

too easy — just right — too difficult

9. The size of the classes was

too small — just right — too large

10. Class periods were

too short — just right — too long

11. Five weeks was

too short — just right — too long

III. RATE THE ITEMS BELOW BY THE FOLLOWING SCALE:

a. poor
b. fair
c. good
d. excellent
e. exceptional

________	Teachers	Comments: ________________________
________	Assignments	________________________
________	Films & videos	________________________
________	Trips	________________________
________	Eating facilities	________________________
________	Computer Facilities	________________________

IV. COMMENTS:

13. What did you like most about the program?

14. What did you like least about the program?

15. If you would recommend this program to a friend, what would you tell him/her?

CAREER CHOICE

____