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**ANALYTICAL SOLUTION OF  
ELASTIC-PLASTIC THICK-WALLED  
CYLINDERS WITH GENERAL HARDENING**

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MARCH 1992



**US ARMY ARMAMENT RESEARCH,  
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## TABLE OF CONTENTS

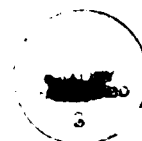
|                           | <u>Page</u> |
|---------------------------|-------------|
| ABSTRACT .....            | 1           |
| INTRODUCTION .....        | 1           |
| BASIC EQUATIONS .....     | 1           |
| GENERAL HARDENING .....   | 4           |
| MATERIAL PROPERTIES ..... | 6           |
| NUMERICAL RESULTS .....   | 7           |
| REFERENCES .....          | 7           |

### TABLES

|                                             |   |
|---------------------------------------------|---|
| 1. ELASTIC CONSTANTS FOR THREE METALS ..... | 6 |
| 2. PLASTIC CONSTANTS FOR THREE METALS ..... | 6 |

### LIST OF ILLUSTRATIONS

|                                                                                      |    |
|--------------------------------------------------------------------------------------|----|
| 1a. Effective stress-strain diagram .....                                            | 8  |
| 1b. Effective stress-plastic strain diagram .....                                    | 8  |
| 2. Variations of stresses and plastic hoop strains in a steel tube .....             | 9  |
| 3. Variations of stresses and plastic hoop strain in a copper tube .....             | 10 |
| 4. Variations of stresses and plastic hoop strain in an aluminum alloy tube .....    | 11 |
| 5. Comparisons of stresses and plastic hoop strain in a partially-plastic tube ..... | 12 |
| 6. Comparisons of stresses and plastic hoop strain in a fully-plastic tube .....     | 13 |



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# ANALYTICAL SOLUTION OF ELASTIC-PLASTIC THICK-WALLED CYLINDERS WITH GENERAL HARDENING

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**ABSTRACT.** This paper presents an analytical solution for the elastic-plastic behavior of thick-walled cylinders under internal pressure. The general hardening law employed in this investigation is a piecewise linear representation of arbitrary stress-strain curves in uniaxial form. Closed-form analytical solutions are developed for the stresses, the elastic and plastic strains, and the displacements by using Tresca's yield criterion and its associated flow rule. Experimental data obtained from cylinders made of either SAE 1045 steel, OFHC copper, or aluminum alloy 1100 are used to determine the material constants. Numerical results for partially-plastic and fully-plastic cylinders are presented for the radial distributions of plastic hoop strain, radial, and tangential stresses.

**1. INTRODUCTION.** Of all the available elastic-plastic solutions, the problem of pressurized thick-walled cylinders has received the greatest attention. This is because of the symmetric nature of the problem and its practical importance to pressure vessels and the autofrettage process of gun tubes. Many solutions for this problem have been reported over the last four decades [1-3]. Analytical solutions can be obtained only when simplifying assumptions are made regarding material properties. Bland [2] developed analytical solutions for materials with linear hardening properties. Recently, Megahed [3] considered a nonlinear hardening law  $\sigma = Y + A \cdot \epsilon_p^n$  in uniaxial form and developed an approximate solution for any value of the strain-hardening exponent  $n$ . Closed-form analytical solutions for the plastic hoop strain can be obtained only for four particular values ( $n = 1, 1/2, 1/3$ , and  $1/4$ ), and the integral has to be evaluated numerically for  $n = 1/3$  and  $1/4$ .

The general hardening law employed in this investigation is a piecewise linear representation of actual stress-strain curves in uniaxial form. A finite number of straight lines can represent arbitrary curves with greater accuracy than other representations [4]. The problem is formulated in a manner similar to [2,3] by using Tresca's yield criterion and the associated flow rule. Closed-form analytical solutions are developed for the stresses, the elastic and plastic strains, and the displacements.

**2. BASIC EQUATIONS.** Consider a long thick-walled cylinder, internal radius  $a$  and external radius  $b$ , that is subjected to internal pressure  $p$  causing partial plastification. Assuming small strain and no body forces in the axisymmetric state of generalized plane-strain, the radial and tangential stress,  $\sigma_r$  and  $\sigma_\theta$ , must satisfy the equilibrium equation

$$r(d\sigma_r/dr) = \sigma_\theta - \sigma_r \quad (1)$$

and the corresponding strains,  $\epsilon_r$  and  $\epsilon_\theta$ , are given in terms of the radial displacement,  $u$ , by

$$\epsilon_r = du/dr, \quad \epsilon_\theta = u/r \quad (2)$$

Total strains are decomposed into elastic and plastic components and the strain-stress relations are

$$\epsilon_r = [\sigma_r - \nu(\sigma_\theta + \sigma_z)]/E + \epsilon_r^p \quad (3a)$$

$$\epsilon_\theta = [\sigma_\theta - \nu(\sigma_r + \sigma_z)]/E + \epsilon_\theta^p \quad (3b)$$

$$\epsilon_z = [\sigma_z - \nu(\sigma_r + \sigma_\theta)]/E + \epsilon_z^p \quad (3c)$$

where  $E$  and  $\nu$  are elastic constants. Subject to  $\sigma_\theta \geq \sigma_z \geq \sigma_r$ , Tresca's criterion states that yielding occurs when

$$\sigma_\theta - \sigma_r = \sigma(\bar{\epsilon}^p) \quad (4)$$

where the yield stress  $\sigma$  is a function of plastic strain  $\epsilon^p$ . The associated flow rule states that

$$d\epsilon_\theta^p = -d\epsilon_r^p \geq 0 \quad \text{and} \quad d\epsilon_z^p = 0 \quad (5)$$

Hence, from Eq. (3c)

$$\sigma_z = \nu(\sigma_r + \sigma_\theta) + E\epsilon_z \quad (6a)$$

and the total axial force on any section is

$$F = 2\pi\nu a^2 p + \pi E(b^2 - a^2)\epsilon_z \quad (6b)$$

There are three cases of importance: first, plane-strain,  $\epsilon_z = 0$ ; second, a tube with open ends,  $F = 0$ ; and third, a tube with closed ends,  $F = \pi a^2 p$ . In the latter two cases, substitution into Eq. (6b) determines  $\epsilon_z$ . Since  $\epsilon_z$  is now known, Eqs. (3a) and (3b) are inverted in order to express stresses in terms of total strains and plastic hoop strain as follows:

$$\sigma_r = \hat{E}[\nu\epsilon_\theta + (1-\nu)\epsilon_r + (1-2\nu)\epsilon_\theta^p + \nu\epsilon_z] \quad (7a)$$

$$\sigma_\theta = \hat{E}[\nu\epsilon_r + (1-\nu)\epsilon_\theta - (1-2\nu)\epsilon_\theta^p + \nu\epsilon_z] \quad (7b)$$

where  $\hat{E} = E/[(1+\nu)(1-2\nu)]$ . Substitution of Eqs. (7a) and (7b) into Eqs. (1) and (2) yields the following differential equation:

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} = -\frac{1-2\nu}{1-\nu} \left[ \frac{2\epsilon_\theta^p}{r} + \frac{d\epsilon_\theta^p}{dr} \right] \quad (8)$$

Integrating with respect to  $r$  leads to

$$du/dr + u/r = -\left(\frac{1-2\nu}{1-\nu}\right)(2J + \epsilon_\theta^p) + 2C \quad (9a)$$

where

$$J = \int_a^r \epsilon_\theta^p r^{-1} dr \quad (9b)$$

Integrating again yields the analytical solution

$$U = -\left(\frac{1-2\nu}{1-\nu}\right)rJ + Cr + D/r \quad (10)$$

where C and D are integration constants to be determined from boundary conditions;  $\sigma_r = -p$  at  $r = a$  and  $\sigma_r = 0$  at  $r = b$ .

Upon substitution of the resulting values of C and D into the expressions of displacement, radial and hoop stresses, the following distributions are obtained:

$$\sigma_r = -p - \bar{E}J + (P + \bar{E}J_0)(1-a^2/r^2)/(1-a^2/b^2) \quad (11a)$$

$$\sigma_\theta = -p - \bar{E}(J + \epsilon_\theta^P) + (p + \bar{E}J_0)(1+a^2/r^2)/(1-a^2/b^2) \quad (11b)$$

$$Eu/r = (1+\nu)(1-2\nu)\sigma_r + 2(1-\nu^2)(a^2/r^2)(p + \bar{E}J_0)/(1 - \frac{a^2}{b^2}) - \nu E\epsilon_z \quad (11c)$$

where  $\bar{E} = E/(1-\nu^2)$ , and  $J_0$  is the value of the integral J at the plastic front,  $r = \rho$ , i.e.,

$$J_0 = \int_a^\rho \epsilon_\theta^P r^{-1} dr = \int_1^{\rho/a} \epsilon_\theta^P \xi^{-1} d\xi \quad (11d)$$

Note that  $\epsilon_\theta^P = 0$  and  $J = J_0$  throughout the outer elastic zone defined by  $\rho < r < b$ . At the plastic front, the Tresca effective stress  $\bar{\sigma} = \sigma_\theta - \sigma_r = Y$ , where Y is the initial yield stress and also  $\epsilon_\theta^P = 0$ . Therefore, using Eq. (11b) to provide  $\bar{\sigma}$ , one readily obtains

$$P + \bar{E}J_0 = \frac{\sigma_0}{2} \frac{\rho^2}{a^2} \left(1 - \frac{a^2}{b^2}\right) \quad (12)$$

Using Eq. (7), the distributions of  $\sigma_r$ ,  $\sigma_\theta$ , and u can be written in simpler forms as follows:

$$\sigma_r = -p - \bar{E}J + \frac{Y}{2} (\rho^2/a^2 - \rho^2/r^2) \quad (13a)$$

$$\sigma_\theta = \sigma_r + Y(\rho^2/r^2 - \frac{\bar{E}}{\sigma_0} \epsilon_\theta^P) \quad (13b)$$

$$Eu/r = (1+\nu)(1-2\nu)\sigma_r + Y(1-\nu^2)\rho^2/r^2 - \nu E\epsilon_z \quad (13c)$$

It is obvious from Eq. (13b) that Tresca effective stress  $\bar{\sigma}$  is simply given by

$$\bar{\sigma} = Y(\rho^2/r^2 - \frac{\bar{E}}{\sigma_0} \epsilon_\theta^P) \quad (14)$$

Therefore, if the radial variation of plastic hoop strain is known, the integral J and all field quantities can be determined.

3. **GENERAL HARDENING.** The general hardening law employed in this investigation is a piecewise linear representation. Arbitrary stress-strain curves in uniaxial form can be approximated by a finite number of straight lines [4]. The straight line through the origin is given by the relation

$$\bar{\sigma} = E\bar{\epsilon} \quad (15)$$

where  $E$  is Young's modulus. All of the other straight lines are given by the relation

$$\bar{\sigma} = (1-m_i)\sigma_{0i} + m_i E\bar{\epsilon} \quad (16)$$

where  $\sigma_{0i}$  is the stress at the intersection of the two straight lines given by Eqs. (15) and (16), and  $m_i E$  is the slope of the straight lines given by Eq. (16). Let  $\sigma_i, \epsilon_i$  be the stress and strain at the intersection of two straight lines with slope  $m_{i-1}E$  and  $m_i E$  as shown in Figure 1a. Then

$$\sigma_i = (1-m_{i-1})\sigma_{0i-1} + m_{i-1}E\epsilon_i = (1-m_i)\sigma_{0i} + m_i E\epsilon_i$$

which leads to  $\epsilon_i$  and  $\sigma_i$  in terms of  $\sigma_{0i}$  and  $m_i$

$$E\epsilon_i = [(1-m_i)\sigma_{0i} - (1-m_{i-1})\sigma_{0i-1}]/(m_i-1-m_{i-1}) \quad (17a)$$

and

$$\sigma_i = [m_{i-1}(1-m_i)\sigma_{0i} - m_i(1-m_{i-1})\sigma_{0i-1}]/(m_i-1-m_{i-1}) \quad (17b)$$

Eq. (16) can be written also as a function of effective plastic strain  $\bar{\epsilon}^p$  as shown in Figure 1b.

$$\bar{\sigma} = \sigma_{0i} + h_i E\bar{\epsilon}^p \quad \bar{\epsilon}_i^p \leq \bar{\epsilon}^p \leq \bar{\epsilon}_{i+1}^p \quad (18)$$

where  $h_i = m_i/(1-m_i)$ .

Since  $\epsilon_\theta^p = -\epsilon_r^p$  and  $\epsilon_z^p = 0$ , the effective plastic strain  $\bar{\epsilon}^p$  is determined as  $2\epsilon_\theta^p/\sqrt{3}$ , and hence, Eq. (14) is rewritten in terms of the plastic strain in the tube as

$$\sigma = Y(\rho^2/r^2 - \frac{\sqrt{3}}{2} \frac{E}{Y} \bar{\epsilon}^p) \quad (19)$$

A comparison between the expressions for effective stresses provided by Eqs. (18) and (19) yields the following explicit equation for  $\bar{\epsilon}^p$ :

$$\frac{E}{Y} \bar{\epsilon}^p = (\rho^2/r^2 - C_i)/b_i \quad (20)$$

which is valid in  $\bar{\epsilon}_i^p \leq \bar{\epsilon}^p \leq \bar{\epsilon}_{i+1}^p$  and  $r_i \geq r \geq r_{i+1}$  and

$$b_i = (\sqrt{3}/2)/(1-\nu^2) + h_i, \quad C_i = \sigma_{0i}/\sigma_0 \quad (21)$$

The values of  $r_i$  and  $r_{i+1}$  can be determined by

$$\begin{aligned} r_i &= \rho [b_i \frac{\epsilon}{Y} \bar{\epsilon}_i^p + C_i]^{-1/2} \\ r_{i+1} &= \rho [b_i \frac{\epsilon}{Y} \bar{\epsilon}_{i+1}^p + C_i]^{-1/2} \end{aligned} \quad (22)$$

If  $\bar{\epsilon}_1^p = 0$  and  $\sigma_{01} = Y$ , then  $r_1 = \rho$ . This is true for most materials. Since  $r_i \geq a$ , the calculation of  $r_i$  for  $i = 1, 2, \dots, m$  should stop when the above relation is violated, i.e.,  $r_{m+1} < a$ . Let us define  $V_i$  for  $i = 1, 2, \dots, m$  by the following integral:

$$\begin{aligned} V_i &= \int_{r_{i+1}}^{r_i} \frac{\epsilon}{Y} \bar{\epsilon}^p \frac{dr}{r} = \frac{1}{b_i} \int_{r_{i+1}}^{r_i} \left( \frac{\rho^2}{r^2} - C_i \right) \frac{dr}{r} \\ &= \frac{1}{2b_i} \left[ \left( \frac{\rho}{r_{i+1}} \right)^2 - \left( \frac{\rho}{r_i} \right)^2 - 2C_i \ln(r_i/r_{i+1}) \right] \end{aligned} \quad (23)$$

Then

$$\begin{aligned} V &= \int_a^r \frac{\epsilon}{Y} \bar{\epsilon}^p \frac{dr}{r} = \int_a^{r_m} \frac{\epsilon}{Y} \bar{\epsilon}^p \frac{dr}{r} + V_{m-1} + \dots + V_{i+1} + \int_{r_{i+1}}^r \frac{\epsilon}{Y} \bar{\epsilon}^p \frac{dr}{r} \\ &= \frac{1}{2b_m} \left[ \left( \frac{\rho}{a} \right)^2 - \left( \frac{\rho}{r_m} \right)^2 - 2C_m \ln \frac{r_m}{a} \right] + V_{m-1} + \dots + V_{i+1} \\ &\quad + \frac{1}{2b_i} \left[ \left( \frac{\rho}{r_{i+1}} \right)^2 - \left( \frac{\rho}{r} \right)^2 - 2C_i \ln(r/r_{i+1}) \right] \end{aligned} \quad (24)$$

and the maximum value of  $V$  is

$$V_0 = \int_a^{\rho} \frac{\epsilon}{Y} \bar{\epsilon}^p \frac{dr}{r} = \frac{1}{2b_m} \left[ \left( \frac{\rho}{a} \right)^2 - \left( \frac{\rho}{r_m} \right)^2 - 2C_m \ln \frac{r_m}{a} \right] + \sum_{i=1}^{m-1} V_i \quad (25)$$

The integrals  $V_i$  ( $i=1, 2, \dots, m-1$ ),  $V$ , and  $V_0$  given analytically by Eqs. (23), (24), and (25) can be easily evaluated. The integral  $J$  is related to the integral  $V$  by

$$J = \frac{\sqrt{3}}{2} \frac{Y}{\epsilon} V \quad (26)$$

All field quantities  $u$ ,  $\epsilon_r$ ,  $\epsilon_\theta$ ,  $\sigma_r$ ,  $\sigma_\theta$ ,  $\sigma_z$ , and  $\epsilon_\theta^p$  can now be calculated.

4. MATERIAL PROPERTIES. Test members were made from three different metals as follows: SAE 1045 steel, OFHC copper, and aluminum alloy 1100 [4]. The values of the elastic constants ( $E$  and  $\nu$ ) for the three metals are shown in Table 1. The values of the constants ( $\sigma_{0i}$ ,  $m_i$ ,  $\sigma_i$ ,  $\epsilon_i$ ) approximating the plastic portion of the stress-strain diagram for three metals are shown in Table 2.

TABLE 1. ELASTIC CONSTANTS FOR THREE METALS

| Material            | $E$ , Ksi | $\nu$ |
|---------------------|-----------|-------|
| SAE 1045 steel      | 30,000    | 0.29  |
| OFHC copper         | 16,000    | 0.35  |
| aluminum alloy 1100 | 10,250    | 0.33  |

TABLE 2. PLASTIC CONSTANTS FOR THREE METALS

| Straight Line       | $\sigma_{0i}$ , Ksi | $m_i$   | $\sigma_i$ , Ksi | $\epsilon_i$ , % |
|---------------------|---------------------|---------|------------------|------------------|
| SAE 1045 Steel      |                     |         |                  |                  |
| 1                   | 43.4                | 0.05083 | 43.4             | 0.145            |
| 2                   | 54.0                | 0.02858 | 66.924           | 1.687            |
| 3                   | 80.0                | 0.00847 | 90.638           | 4.453            |
| 4                   | 95.0                | 0.00309 | 103.542          | 9.532            |
| 5                   | 111.0               | 0.00128 | 122.280          | 29.745           |
| OFHC Copper         |                     |         |                  |                  |
| 1                   | 2.50                | 0.17125 | 2.50             | 0.016            |
| 2                   | 3.25                | 0.07063 | 3.686            | 0.059            |
| 3                   | 4.00                | 0.03125 | 4.553            | 0.136            |
| 4                   | 5.37                | 0.01991 | 7.000            | 0.765            |
| 5                   | 8.40                | 0.01313 | 14.151           | 2.790            |
| 6                   | 21.0                | 0.00450 | 27.484           | 9.137            |
| 7                   | 39.0                | 0.00078 | 42.757           | 30.350           |
| Aluminum Alloy 1100 |                     |         |                  |                  |
| 1                   | 8.0                 | 0.67024 | 8.0              | 0.078            |
| 2                   | 11.0                | 0.32683 | 11.942           | 0.135            |
| 3                   | 13.0                | 0.09561 | 13.557           | 0.184            |
| 4                   | 14.7                | 0.01590 | 15.007           | 0.332            |
| 5                   | 16.1                | 0.00210 | 16.310           | 1.131            |

Each of the stress-strain diagrams can be approximated by a finite number of straight lines with extreme accuracy. The error introduced by the approximation is less than 1 percent for all cases.

**5. NUMERICAL RESULTS.** Typical results for the analytical solution are presented first by means of prescribing a plastic front and determining the corresponding plastic hoop strain and radial and hoop stresses in the tube. A tube with  $b/a = 2$  is employed, and the plastic front is prescribed at  $p/a = 1.0, 1.2, 1.4, 1.6, 1.8,$  and  $2.0$ . Figure 2 illustrates the stresses and plastic hoop strains obtained using the material constants for SAE 1045 steel. Figures 3 and 4 present similar results for OFHC copper and aluminum alloy 1100, respectively. Figure 5 shows a comparison of stresses and plastic hoop strains for three partially-plastic tubes at  $p/a = 1.6$ . Figure 6 presents a similar comparison for three fully-plastic tubes at  $p/a = 2.0$ . Future work related to the results obtained here will look into the elastic-plastic behavior of the tube during pressure release. The influence of phenomena such as the Bauschinger effect on residual stresses should be modelled [5,6].

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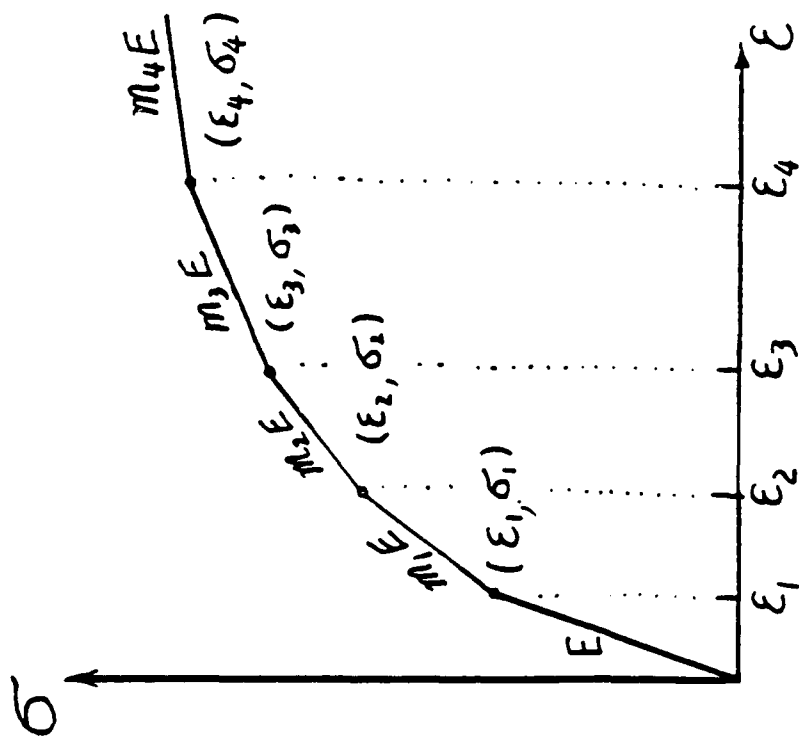


Fig. 1a. Effective stress-strain diagram.

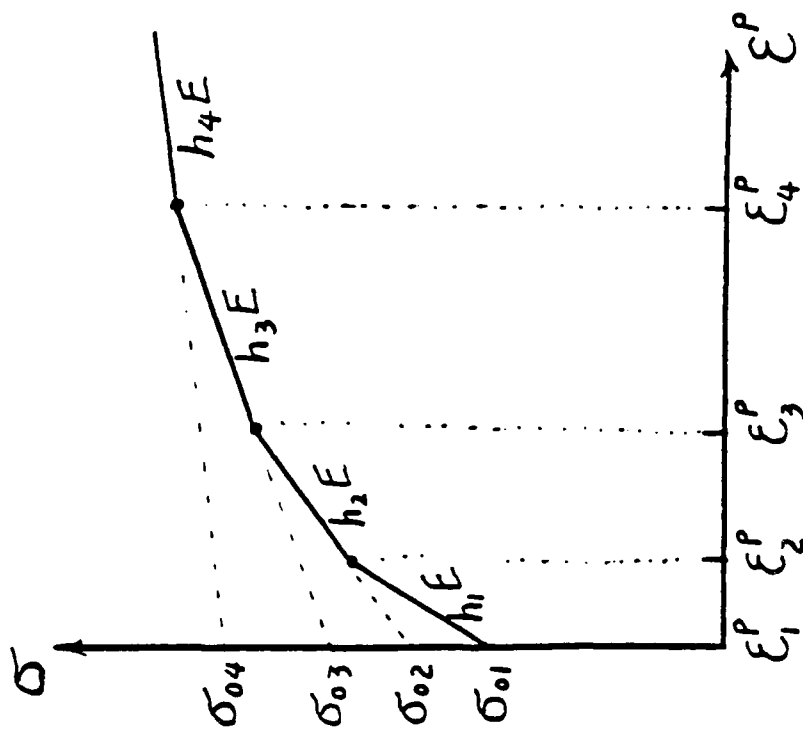


Fig. 1b. Effective stress-plastic strain diagram.

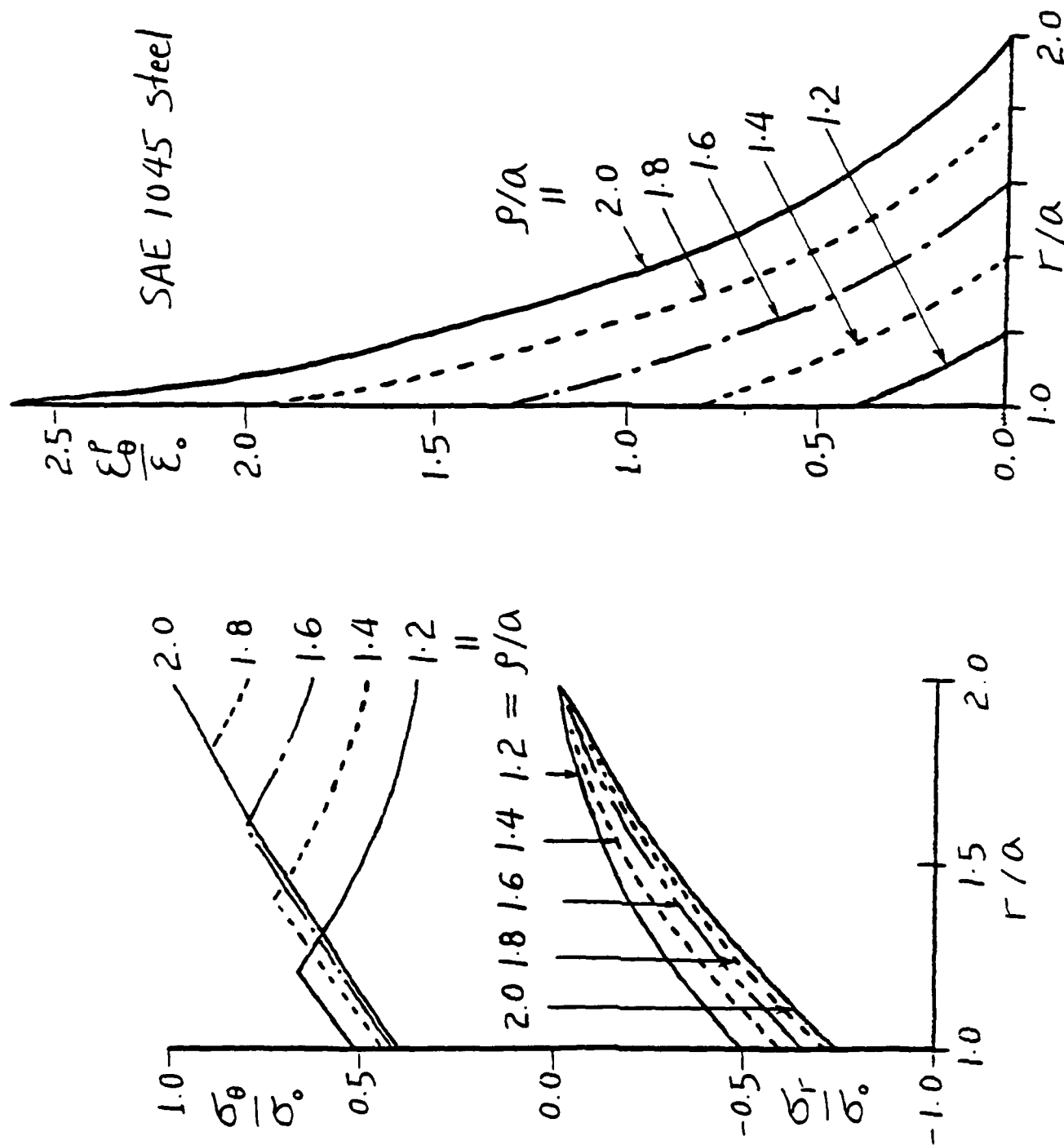


Fig. 2. Variations of stresses and plastic hoop strains in a steel tube.

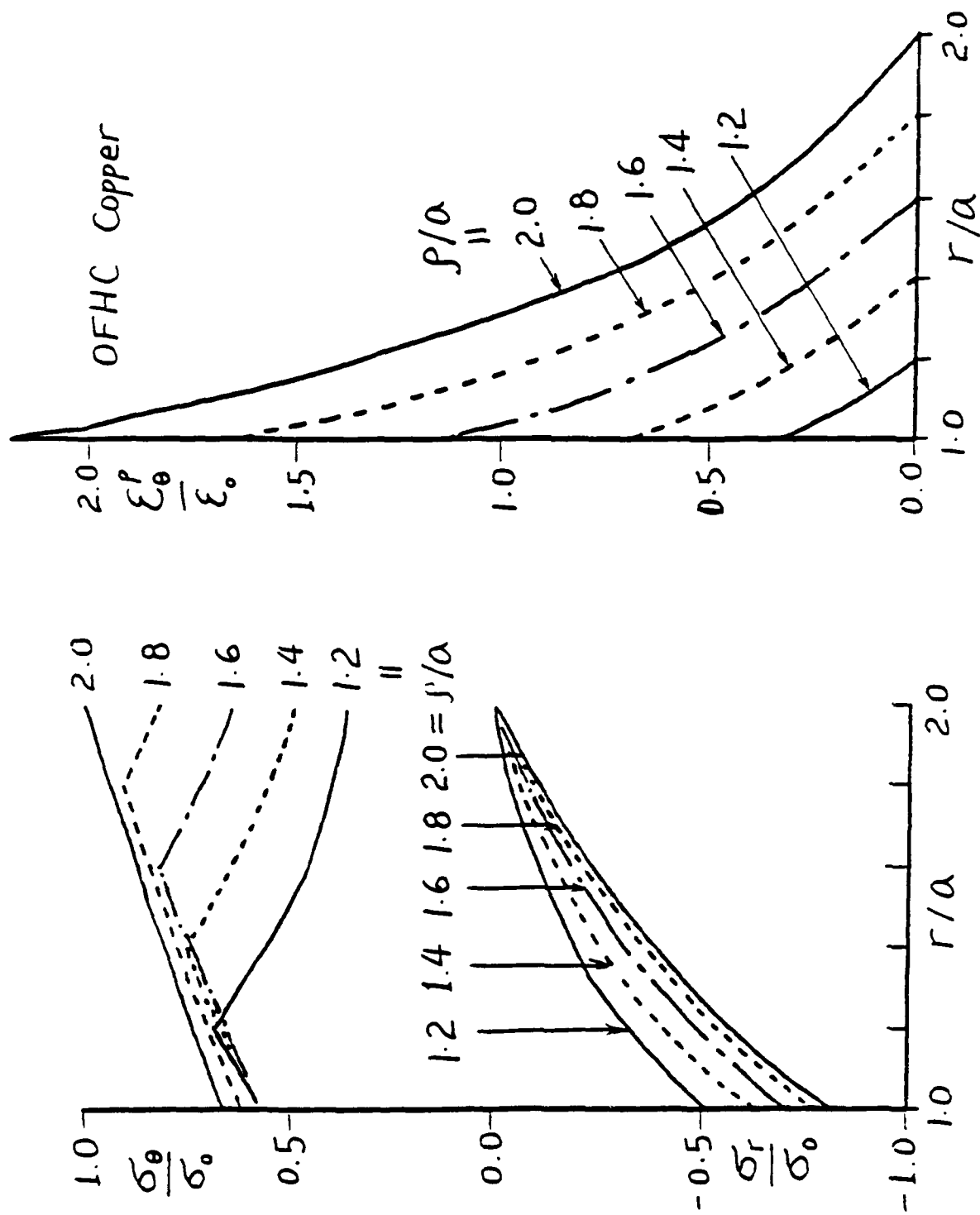


Fig. 3. Variations of stresses and plastic hoop strain in a copper tube.

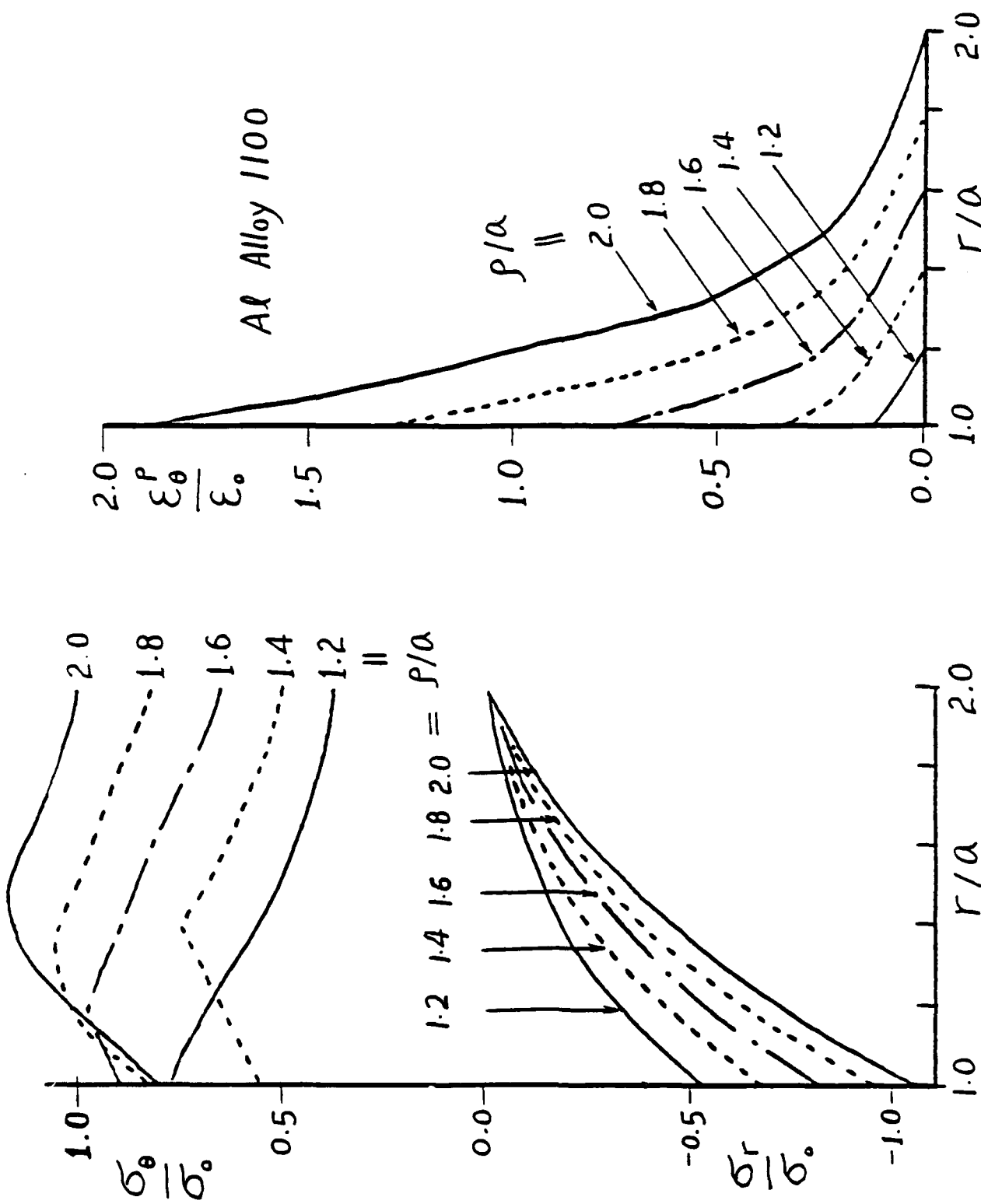


Fig. 4. Variations of stresses and plastic hoop strain in an aluminum alloy tube.

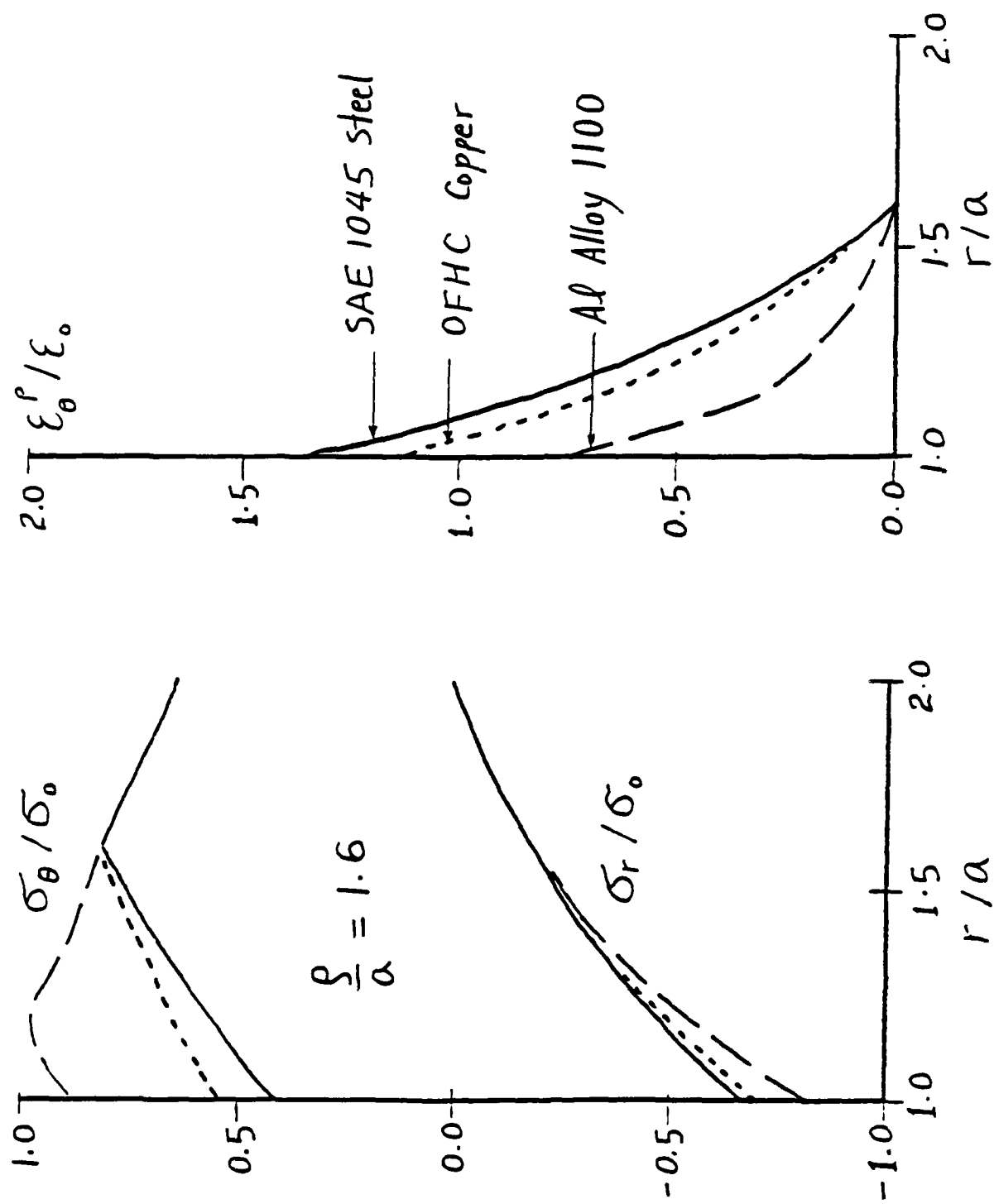


Fig. 5. Comparisons of stresses and plastic loop strain in a partially-plastic tube.

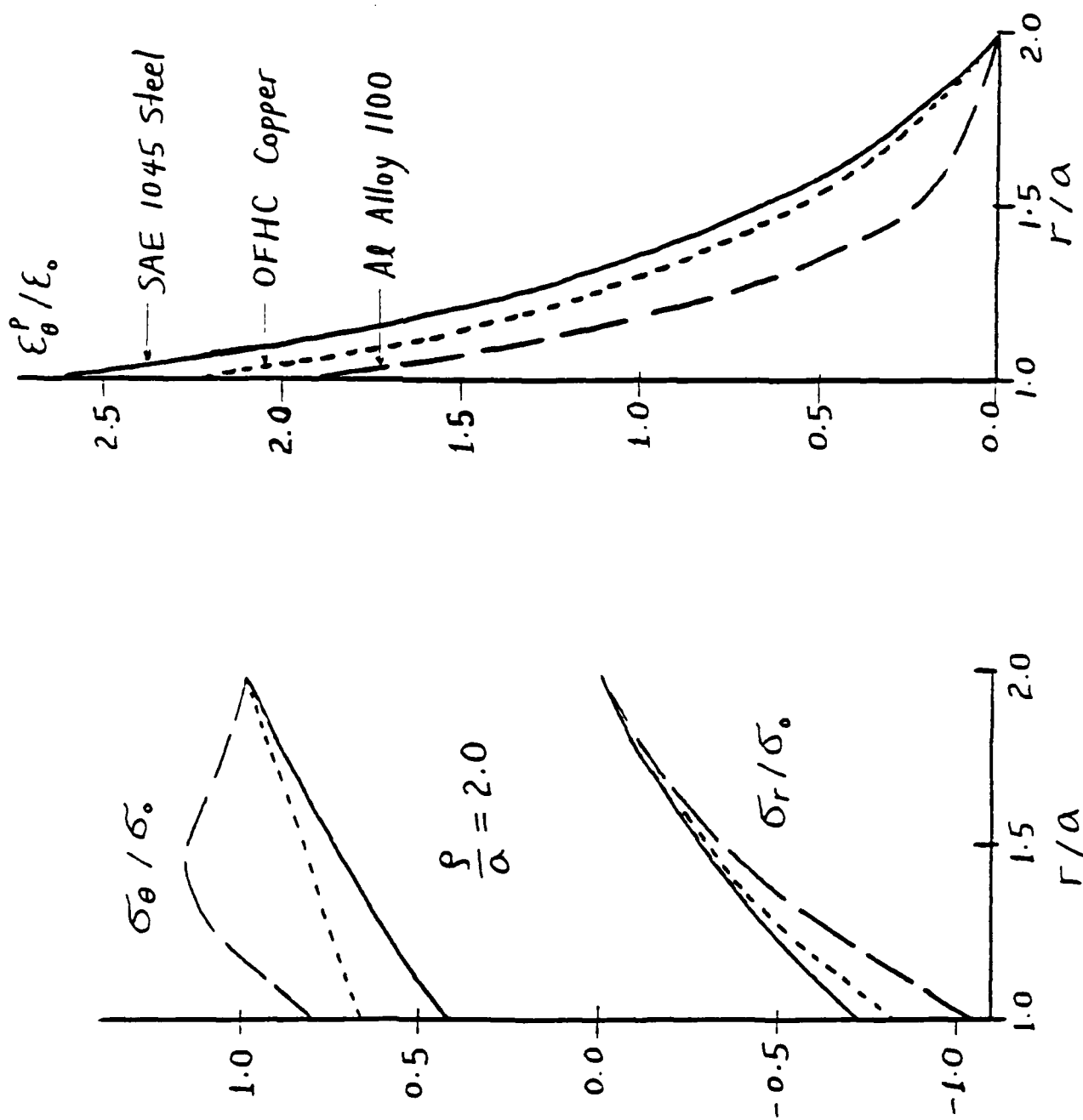


Fig. 6. Comparisons of stresses and plastic hoop strain in a fully-plastic tube.

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