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ANALYSIS OF RADOME INDUCED

CROSS POLARIZATION

by

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FORWORD

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ABSTRACT

Streamlined radomes will always introduce depolarization because of the fact that any material surface viewed at an oblique angle has different transmission coefficients for fields polarized parallel and perpendicular to the plane of incidence, a phenomenon known as divorce. The cross polarized fields are similar in nature to those responsible for Condon lobes of parabolic reflectors, and which are known to make those antennas susceptible to cross-polarization jamming. The relative strength of those lobes is directly related to the amount of divorce, and is quite sensitive to the phase difference of parallel and perpendicular transmission coefficients. A mere 20° of phase divorce has a comparable effect to 3 dB of amplitude divorce.

This paper presents a detailed analysis for general radome shapes, based on geometrical optics (ray tracing). Example results are presented for conical and tangent ogive radome geometries used in conjunction with a circular-aperture antenna that is mechanically scanned. The results show that when the antenna is scanned off axis, the cross-polarized pattern changes from a Condon lobe structure to a difference pattern in the direction orthogonal to the axis of scan, e.g.: under azimuth scanning the cross-polarized sum pattern resembles an elevation difference pattern, while the cross-polarized elevation difference resembles a sum beam. Hence, interference in the cross polarization will introduce a tracking error in the direction orthogonal to the scan plane.

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1. INTRODUCTION

Radomes used to protect antennas from the outside environment or to streamline the platform on which the antenna is installed are almost always designed to appear electrically invisible to the extent possible. The dominant concerns in radome design are maximizing transmissivity and minimizing boresight error. A third concern that has not received as much attention is radome-induced depolarization. The antenna beam will almost always pass through the radome at an oblique angle. Whenever it does, it is susceptible to depolarization simply because any material surface viewed obliquely presents different transmission coefficients to fields polarized perpendicular and parallel to the surface, a phenomenon sometimes referred to as "divorce".

The purpose of this analysis is to assess the degradation in antenna performance as a function of radome transmissivity. The derivation presented here, based on geometrical optics, is valid for most radome shapes and antenna scan directions. The results are applicable to any radome material, including multilayered dielectrics, as long as the transmissivity vs. angle of the flat surface is known.

This paper will first present a simplified analysis for a conical radome with an on-axis antenna beam, in order to illustrate some of the principles involved. Second, the complete analysis for arbitrary radome shapes and arbitrary scan directions is given, with illustrating examples for a tangent ogive radome. These analyses show that in general, a radome that has a significant level of divorce will generate cross polarized patterns that are remarkably similar to the "Condon lobes" observed with parabolic reflectors.

2. METHODOLOGY

2.1. Radome Analysis Techniques

There are two widely-accepted numerical methods for radome transmission/ reflection analysis. One method calculates the field distribution due to the antenna on the inner radome surface at a number of discrete points, and the far field as an integration over the outer surface, accounting for transmissivity effects [1],[2]. This is generally referred to as "surface integration." It is the most accurate, but also the most time-consuming. It is the method of choice when sidelobe levels must be known accurately. The second method is ray tracing, where rays emanating from the antenna are traced through the radome to an equivalent aperture whose field distribution is used as a secondary source for the far field [3]. Ray tracing is computationally faster than surface integration. Although it does not accurately predict sidelobe levels outside those nearest the main beam, it nonetheless accurately characterizes the properties of the main beam, and those of the difference channel beams used for monopulse tracking. Since these are the parameters of greatest interest here, the ray-tracing method was the one chosen for this study. Moreover, many of the geometrical derivations used in ray tracing will also be required for the surface integration, so many of the derivations that follow would be useful in developing a more general radome simulation based on surface integration.

2.2. Overview of the Analysis Method

Figure 1 illustrates an antenna scanned to an arbitrary angle inside a conical radome. The antenna aperture projects through the radome onto an equivalent aperture outside the radome. The resulting equivalent aperture has an amplitude and phase distribution that are distorted due to the radome's imperfect transmissivity properties. The far field pattern of that

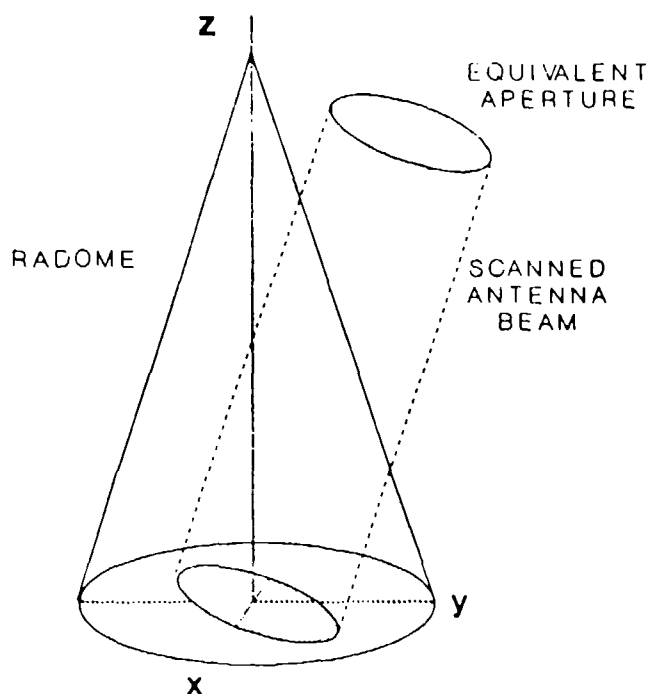


Figure 1. Generic Radome Geometry with Scanned Antenna

distorted aperture distribution gives the equivalent properties of the antenna plus the radome. Thus, the problem can be viewed as one of calculating the equivalent aperture's amplitude and phase. Another viewpoint that may be more natural from an optics perspective is to regard the illuminated portion of the radome as a "lens" having a transmissivity distribution. Removing the radome and multiplying the original antenna aperture distribution by that transmissivity gives an "equivalent" antenna that includes the radome effects.

The procedure begins by establishing a series of sample points on the antenna surface. If the antenna is an array, those sample points are simply the array elements. For each sample point, there is a ray that passes through the radome and terminates at a similar point in the equivalent aperture. We must first calculate the point where that ray passes through the radome, and the direction of the radome's inward surface normal at that intercept point. Then the incident field can be decomposed into components parallel and perpendicular to the plane of

incidence, which are acted upon separately by the two components of the transmissivity. Then the transmitted field components are recombined into components co-polarized and cross-polarized to the incident field. The details of this analysis method are given in section 3, but first, a simplified case is discussed to show the general concepts.

2.1. Conical Radome with Boresighted Antenna

To illustrate the effects of radome-induced de-polarization, we begin with the fairly generic problem of a conical radome over a circular-aperture antenna that looks out through the cone's apex. The actual antenna properties are not important as long as we can assume that antenna-radome interactions are negligible. Assuming also that the antenna is in the antenna's far field then all rays from the antenna aperture to the radome are parallel. This allows us to treat the radome's properties as an alteration of the antenna aperture distribution. In other words, we will construct an equivalent aperture in front of the radome onto which we will project the antenna's aperture distribution multiplied by the radome's transmittance function. This equivalent aperture is then used as a secondary source of radiation to predict the far field pattern properties of the antenna/radome combination.

2.3.1. Geometry Description

The transmittance function depends on the radome surface properties and on the incident wave polarization. For instance, the incident wave may have its electric field parallel to the surface, referred to as perpendicular polarization because E is perpendicular to the "plane of incidence". The plane of incidence, illustrated in Figure 2, is that plane containing the incident ray and the surface normal vector. Perpendicular polarization is sometimes referred to as TF_z .

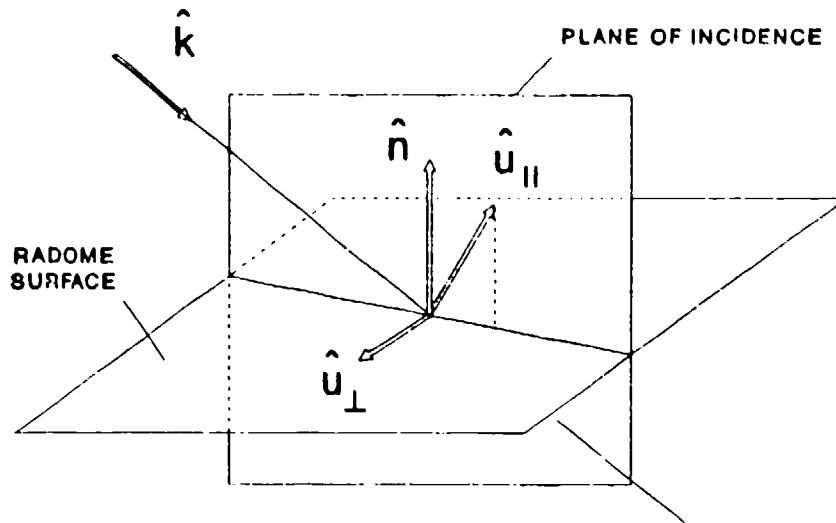


Figure 2. Plane of Incidence (Ray Fixed Coordinate System)

polarization when discussing flat surfaces whose z unit vector is in the normal direction. Alternately, the incident wave may be parallel polarized (TM_z), with $\mathbf{H}$ parallel to the surface and $\mathbf{E}$ parallel to the plane of incidence. Most often the wave must be written as a linear combination of the two polarizations.

Suppose that as in Figure 3a, the antenna, located in the x - y plane, is boresighted along the z axis, looking straight out the nose of the cone. Any incident ray is parallel to the z axis. The plane of incidence is always the constant- ϕ plane containing the ray's source location (in ρ, ϕ coordinates). The unit vectors $\hat{u}_{\parallel}$ and $\hat{u}_{\perp}$ are aligned with the electric field components in the ρ and ϕ directions, respectively.

Figure 3b shows the same situation viewed looking down onto the antenna along the radome axis, but also representing the antenna as an array of slots polarized in the y direction. Consider a ray emanating from a slot on the $+y$ axis: Its polarization is entirely parallel to its plane of incidence (the y - z plane). On the other hand, a ray from a slot on the $+x$ axis is polarized perpendicular to its plane of incidence (the x - z plane). If the radome has phase

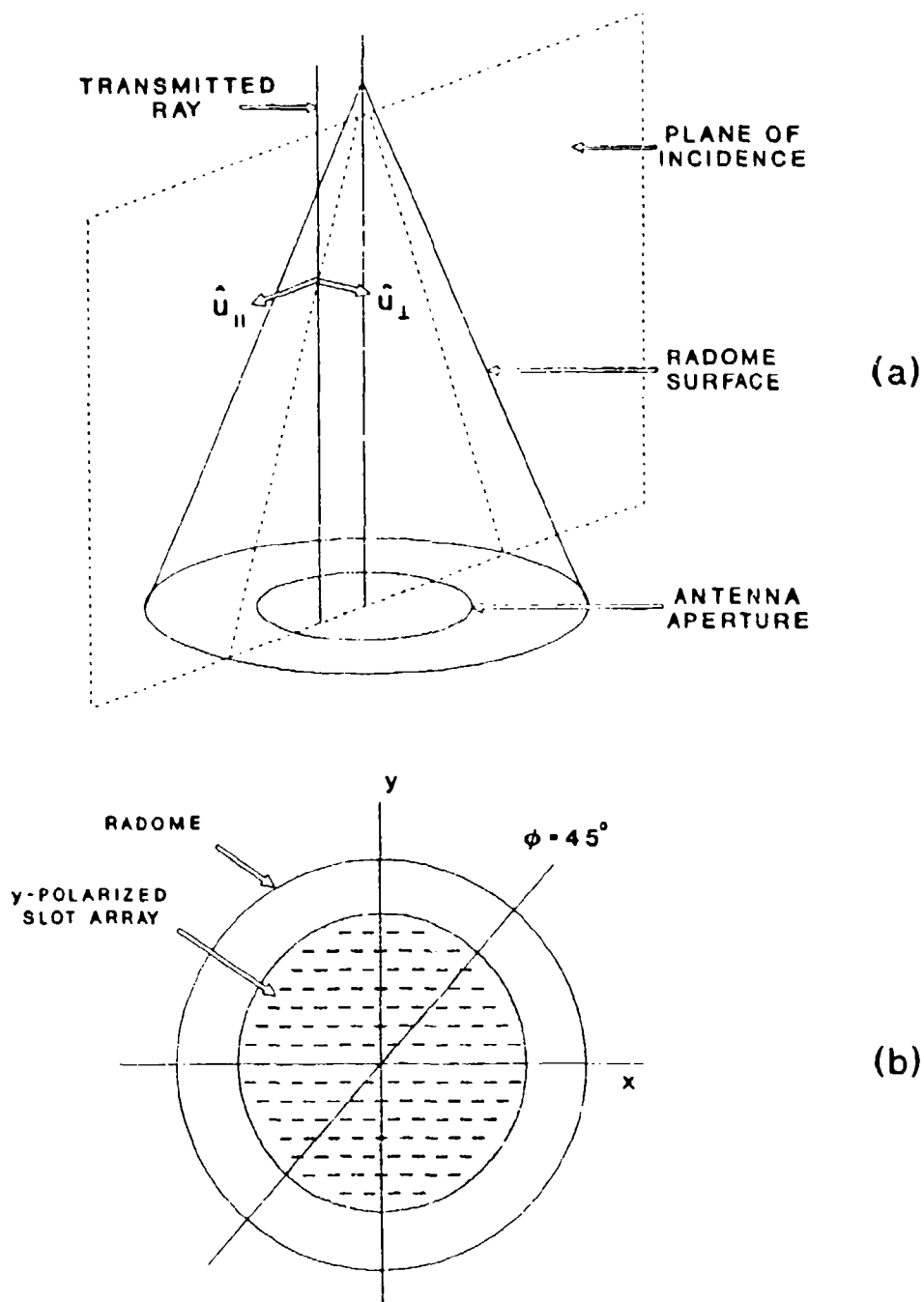


Figure 3. Conical Radome with Boresighted Antenna: (a) Perspective View Showing Plane of Incidence; (b) Nose-on View Illustrating Polarization of Rays from a Slot Array Antenna

divorce, then one ray will be delayed more than the other as they pass through the radome, but neither one will experience depolarization. Now consider a ray originating along the $\phi=45^\circ$ plane: Its polarization is evenly split between parallel and perpendicular. Amplitude divorce will cause one component to be attenuated more than the other, and the emerging ray's polarization will be different than that of the incident ray (but still linear polarization). Phase divorce will cause one component to be delayed relative to the other, and the emerging ray will have elliptical polarization. This effect is the cause of radome depolarization.

2.3.2. Transmission Calculation

Knowing the angle between the incident ray and the radome, we can find a transmissivity for each of the two polarizations. Let $\gamma_{\parallel}(\alpha)$ and $\gamma_{\perp}(\alpha)$ denote the complex transmissivity for the parallel and perpendicular components of a ray incident at the angle α from the radome's inward surface normal. For this special case of conical radome with boresighted antenna, every ray from the antenna has the same incidence angle α . Let the antenna be linearly polarized in the y direction. Upon reaching the radome's interior surface, the incident field decomposes into orthogonal components:

$$E_{\parallel}^i = E_y^i \hat{y} \cdot \hat{u}_{\parallel} ; E_{\perp}^i = E_y^i \hat{y} \cdot \hat{u}_{\perp} \quad (2.1)$$

then the transmitted components are

$$\begin{aligned} E_{\parallel}^t &= E_{\parallel}^i \gamma_{\parallel} \\ E_{\perp}^t &= E_{\perp}^i \gamma_{\perp} \end{aligned} \quad (2.2)$$

$$\begin{aligned} E_y^t &= E_{\parallel}^t \hat{u}_{\parallel} \cdot \hat{y} + E_{\perp}^t \hat{u}_{\perp} \cdot \hat{y} \\ E_x^t &= E_{\parallel}^t \hat{u}_{\parallel} \cdot \hat{x} + E_{\perp}^t \hat{u}_{\perp} \cdot \hat{x} \end{aligned} \quad (2.3)$$

Equations (2.1) through (2.3) are completely general, and apply to any radome geometry as long as the incident wave is y-polarized. Now, specializing them to the geometry in Figure 3:

$$\begin{aligned}\hat{y} \cdot \hat{u}_1 &= \hat{y} \cdot -\hat{\rho} = -\sin \phi \\ \hat{y} \cdot \hat{u}_\perp &= \hat{y} \cdot -\hat{\phi} = -\cos \phi\end{aligned}\tag{2.4}$$

$$\begin{aligned}E_1' &= -E_y^i \gamma_1 \sin \phi \\ E_\perp' &= -E_y^i \gamma_\perp \cos \phi\end{aligned}\tag{2.5}$$

$$E_y' = E_y^i (\gamma_1 \sin^2 \phi + \gamma_\perp \cos^2 \phi)\tag{2.6}$$

$$E_x' = E_y^i (\gamma_1 - \gamma_\perp) \sin \phi \cos \phi\tag{2.7}$$

From the last expression it is clear that a cross-polarized component is introduced any time there is a difference in either the amplitude or the phase of the transmission coefficients (divorce). That cross polarization component is maximum along the diagonals $\phi=45^\circ, 135^\circ$. The surface plot of Figure 4 shows the shape of this cross-polarized distribution (the original antenna distribution is a simple cosine taper). This is the amplitude distribution in the equivalent aperture located "in front" of the radome. Figure 5 is a contour plot of the magnitude. It is extremely important to note that each quadrant has constant phase, but *alternating quadrants are 180° out of phase*. Also note that whenever there is no divorce, there is no cross polarized component, and the co-polarized (y) component experiences only a constant phase shift. *The form of the cross polarization components shown above are identical to those experienced with parabolic reflectors and known to be the source of "Condon lobes" that made those antennas susceptible to cross-polarization jamming [4],[5:153-155], [6:337].*

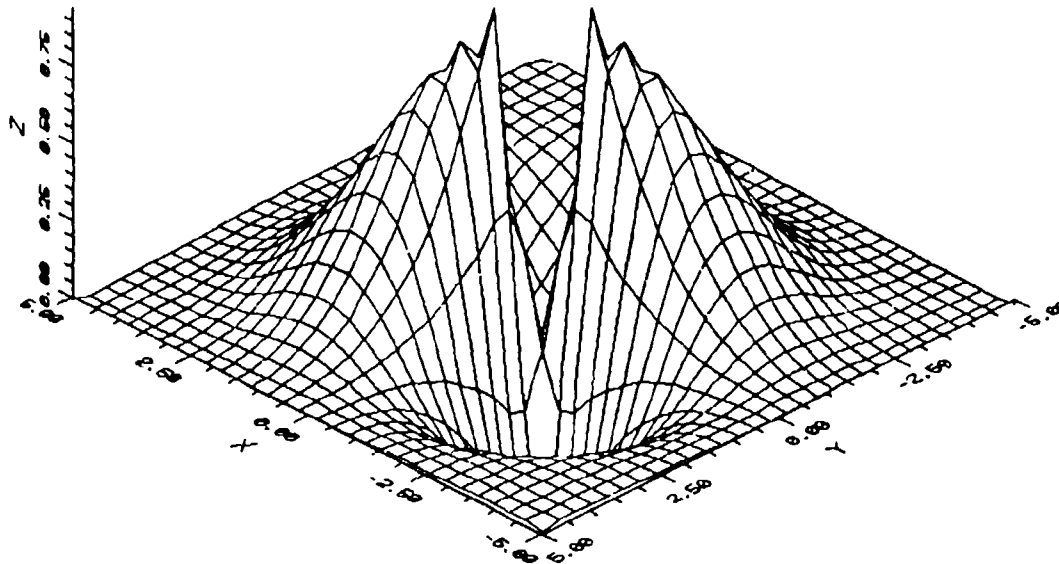


Figure 4. Surface Plot of Cross-Polarized Aperture Amplitude

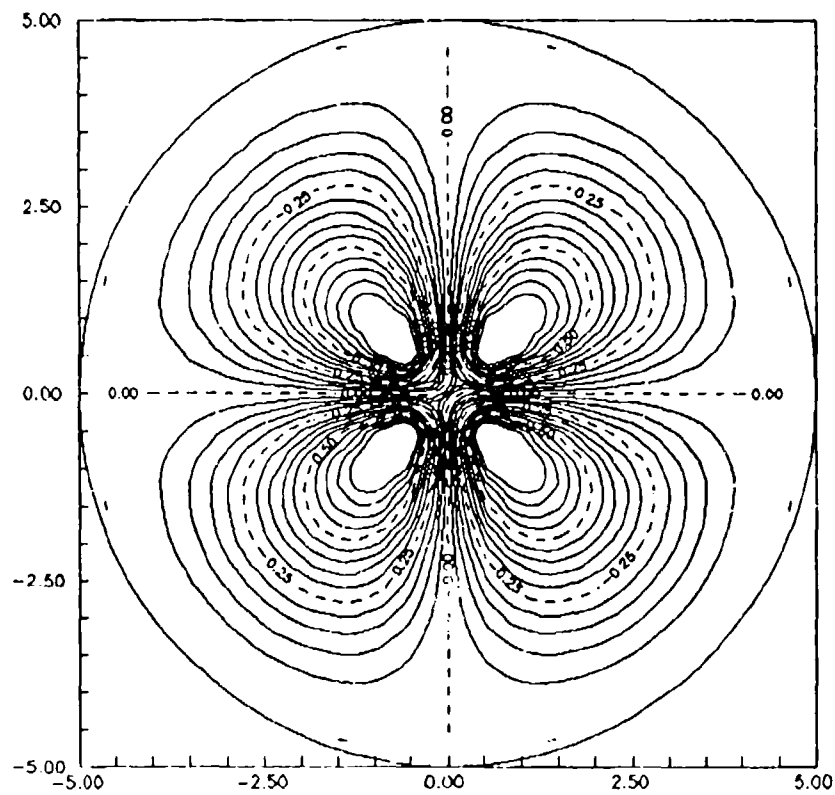


Figure 5. Contour Plot of Cross-Polarized Aperture Amplitude

2.3.3. Example Results

The equivalent aperture distribution is now treated as a source of radiation, as though it were itself an antenna. When there is no divorce, the far field patterns in the diagonal ($\phi=45^\circ$) plane are as shown in Figure 6. These patterns are normalized to a uniformly illuminated aperture with unit current amplitude. The beam peak is lower than 0 dB due to "taper loss". The sum pattern is due to a cosine amplitude distribution: $1 + \cos(\pi\rho/a)$, where a is the aperture radius. The azimuth difference pattern is formed by phase shifting the left side of the aperture ($x > 0$) relative to the right side by 180° , and setting the amplitude in the center of the antenna to zero. A monopulse tracking antenna will usually have an elevation difference as well, formed by phase shifting top and bottom halves of the antenna aperture. The difference lobes have equal magnitude, but 180° phase difference.

The form of the cross polarized distribution due to divorce suggests that we will not see any cross polarized pattern in either the $\phi=0^\circ$ or $\phi=90^\circ$ planes because of its anti-symmetry. For instance, measuring along the horizontal axis, the contributions to the far field from the top and bottom of the aperture are equal and opposite. The most pronounced effects are in the diagonal plane, where those contributions tend to reinforce each other. Thus, *measurements of antenna cross polarization must be taken in the diagonal scan plane, not the principal planes when the antenna is boresighted with the radome axis.*

Figures 7 and 8 are the sum pattern in the diagonal plane when the radome's E and H plane transmissivity have 15° and 30° phase divorce, respectively, and no amplitude divorce. The only effect on the co-polarized pattern is a slight rise in the sidelobes. But the cross-polarized term is significant because of its resemblance to the difference pattern. It actually has four main lobes, one in each quadrant with nulls in the principal azimuth and elevation planes.

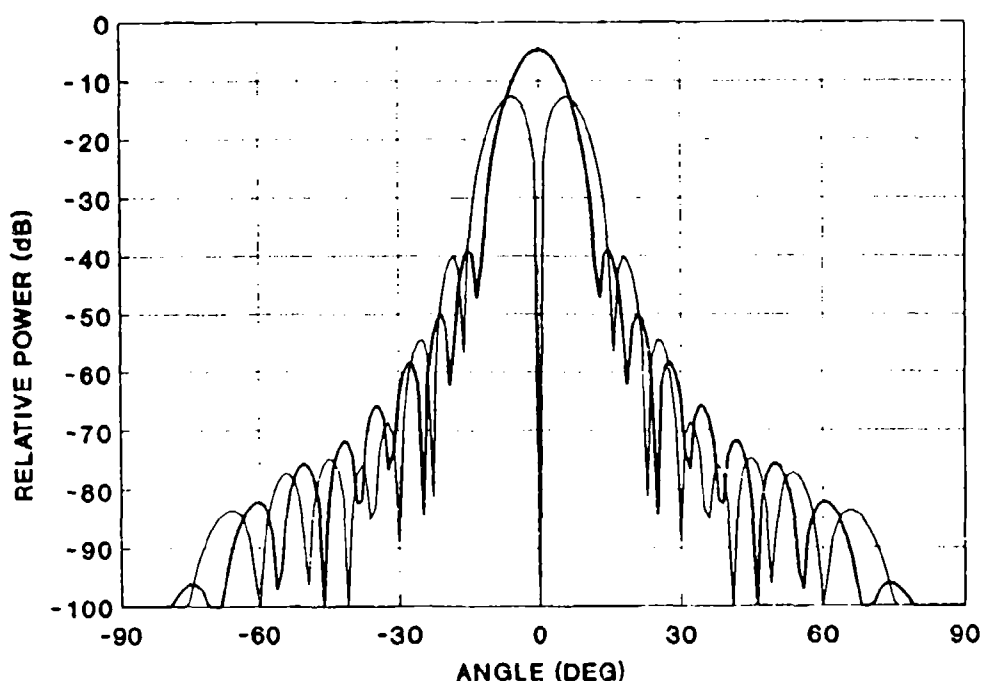


Figure 6. Sum and Azimuth-Difference Patterns (No Radome)

Figures 9 and 10 are the co- and cross- polarized difference patterns, also with 15° and 30° phase divorce. The cross-pol difference pattern is very similar to the co-pol and its main lobes are also phase shifted relative to each other. The effect this might have on a tracking system depends on whether those lobes have the same phase or opposite phase of the co-polarized difference pattern lobes.

Figure 11 shows contours (in dB) of the main beam to Condon lobe ratio as a function of both amplitude divorce (abscissa) and phase divorce (ordinate). It illustrates that the radome's cross-pol performance is, if anything, more sensitive to phase divorce than to amplitude divorce. For example, *20° of phase divorce is comparable to 3 dB of amplitude divorce!* (However, large amplitude divorce also implies strong internal reflections, which may pose more serious problems than depolarization.) Figure 11 may be used to specify maximum tolerances on the radome properties in order to permit a desired cross-polarization level.

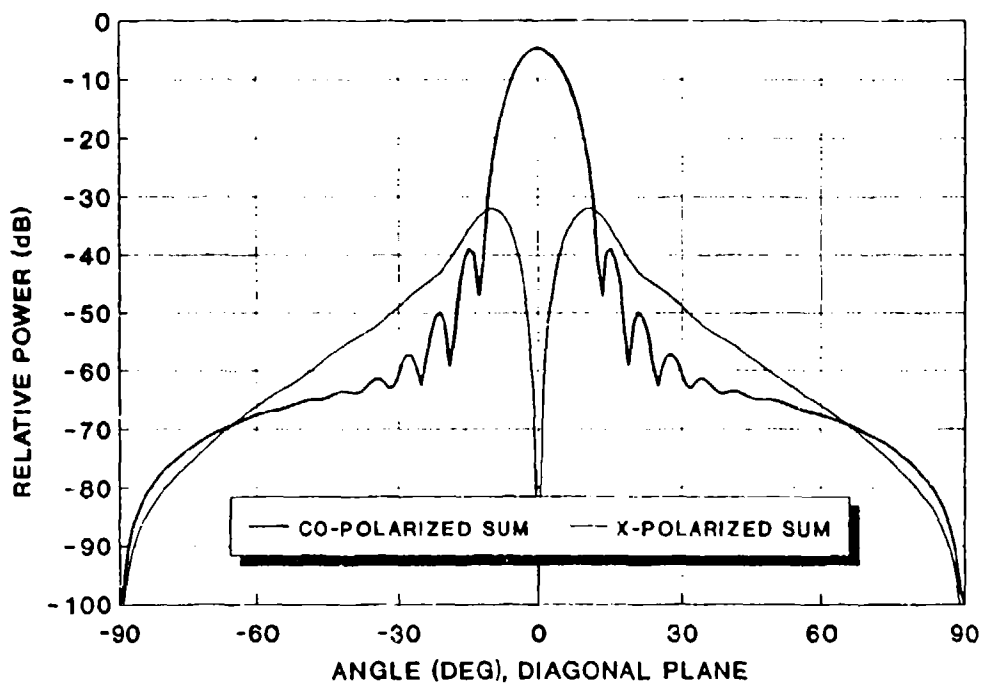


Figure 7. Co- and Cross-Polarized Patterns, 15° Phase Divorce

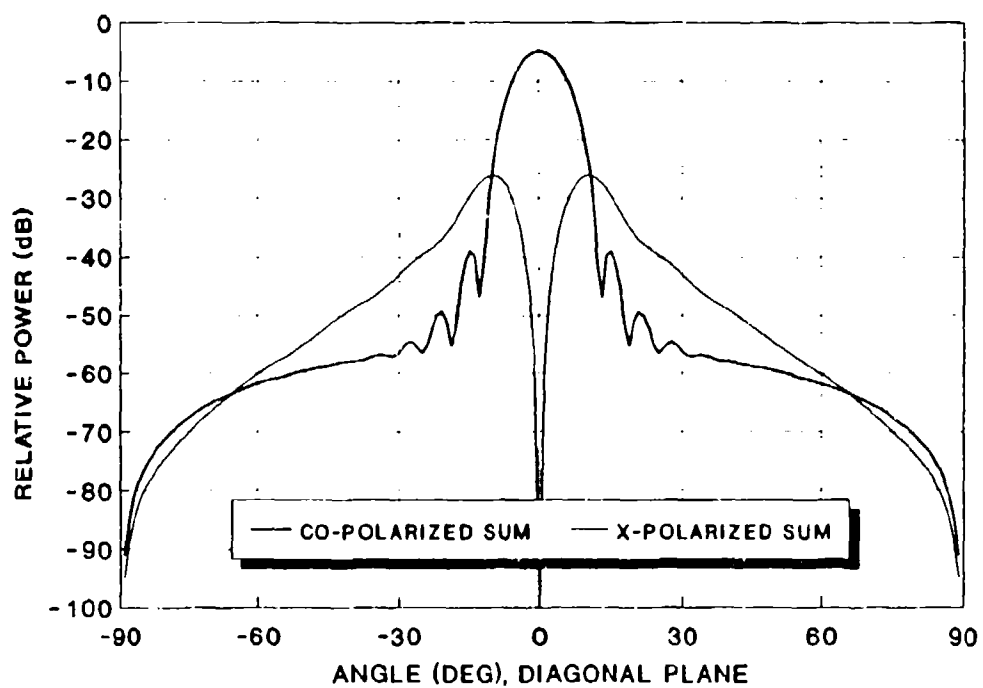


Figure 8. Co- and Cross-Polarized Patterns, 30° Phase Divorce

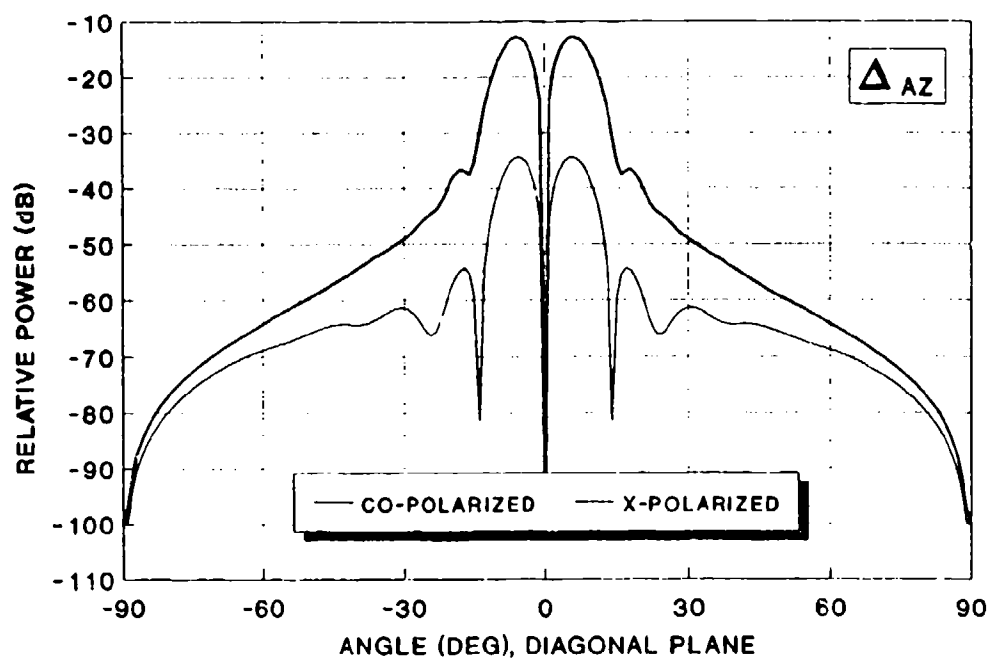


Figure 9. Co- and Cross-Polarized Difference Patterns, 15° Phase Divorce

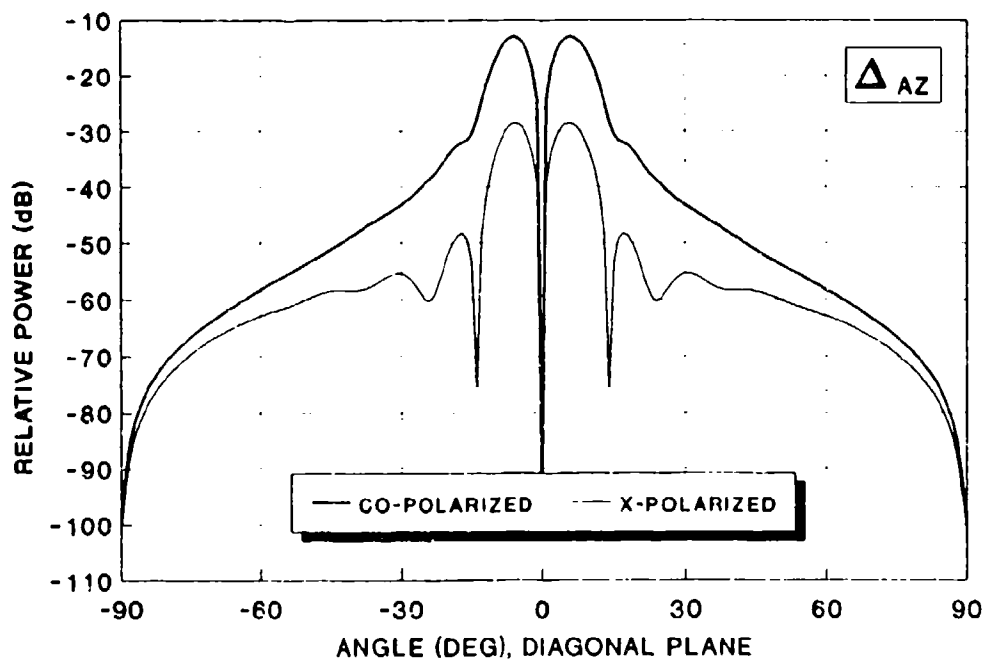


Figure 10. Co- and Cross-Polarized Difference Patterns, 30° Phase Divorce

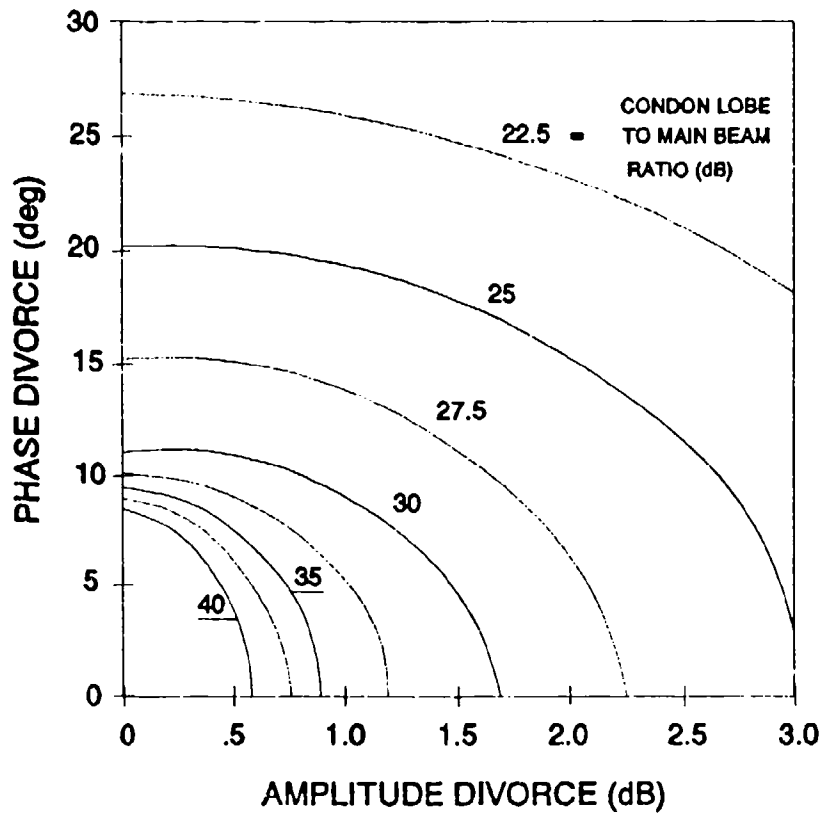


Figure 11. Contours of Constant Main Beam to Condon Lobe Ratio (dB) vs. Amplitude Divorce (dB) and Phase Divorce (deg)

In many respects, the cone shape is the worst for cross polarization when the antenna is not scanned because the incidence angle is very steep where the strongest part of the beam passes through. The boresight scan case is also not representative of a typical scenario because the depolarization at off-boresight scan angles might be less because the incidence angle is shallower. Thus it is clear that a more detailed analysis is necessary to generalize the above results to more practical situations. The following section extends this analysis method to a more realistic radome shape (tangent ogive), and with the antenna scanned away from nose-on incidence.

3. DETAILED ANALYSIS PROCEDURE

This section expands on the generic analysis procedure used for the above conical radome analysis. Analyses for off-axis scan are presented for both conical and tangent ogive radomes. The antenna is considered to be a circular flat-plate array that is mechanically scanned. The end product from this analysis is a general simulation program, RADOME2, described in the Appendix. A brief outline of the procedure is as follows:

- (1) Find locations of rotated antenna aperture sample points - this establishes the source and destination points for rays passing through the radome;
- (2) Find the points where the rays intersect the radome;
- (3) Decompose the incident field into polarization components;
- (4) Determine transmission coefficients;
- (5) Combine polarization components to find equivalent aperture distribution; and
- (6) Calculate the radiated fields.

3.1. Antenna Scanning - Coordinate Rotation

The antenna will be assumed to be an array whose elements are distributed over a circle of radius a in the $z=0$ plane. When the antenna is mechanically scanned to the angle θ_0 from the z axis in the ϕ_0 plane (polar coordinate angles), most elements will be shifted either up or down in z , and their x and y coordinates will also change.

In order to find the rotated element locations (x_r, y_r, z_r) in terms of the original locations (x, y, z) , we will view the antenna's scanning as a sequence of three coordinate transformations:

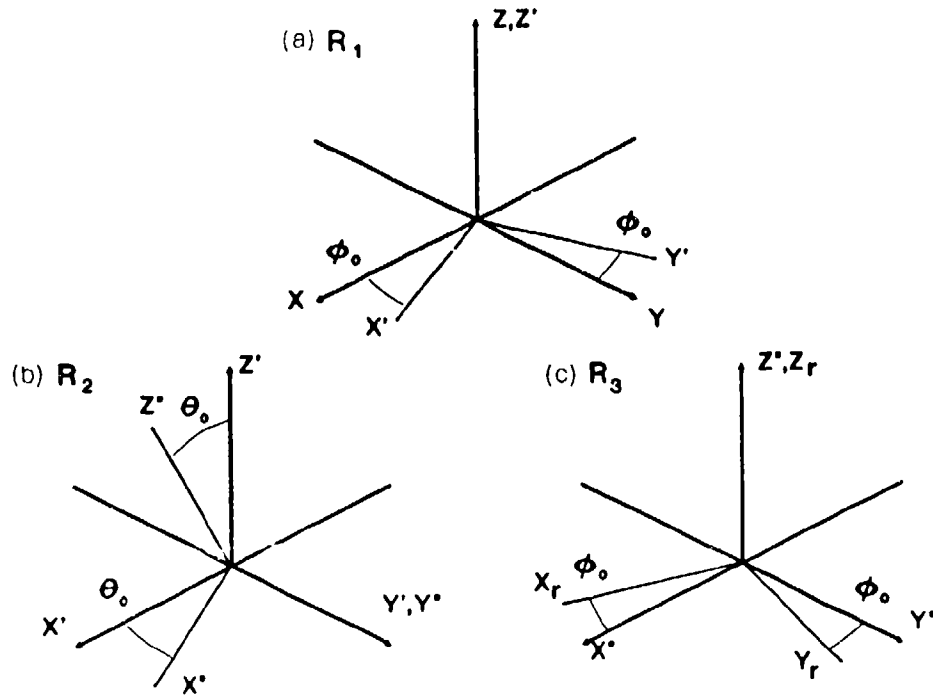


Figure 12. Sequence of Three Coordinate Rotations for Antenna Scan Calculation

- (1) Rotation by ϕ_0 about the z -axis, producing coordinates (x', y', z') ;
- (2) Rotation by θ_0 about the y' axis, resulting in (x'', y'', z'') ; and
- (3) Rotation by $-\phi_0$ about the z'' axis, resulting in (x_r, y_r, z_r) .

These coordinate rotations are illustrated in Figure 12. Each of these transformations may be characterized by a rotation matrix [7:199-200]:

$$R_1 = \begin{bmatrix} \cos \phi_0 & \sin \phi_0 & 0 \\ -\sin \phi_0 & \cos \phi_0 & 0 \\ 0 & 0 & 1 \end{bmatrix}; R_2 = \begin{bmatrix} \cos \theta_0 & 0 & -\sin \theta_0 \\ 0 & 1 & 0 \\ \sin \theta_0 & 0 & \cos \theta_0 \end{bmatrix}; R_3 = \begin{bmatrix} \cos \phi_0 & -\sin \phi_0 & 0 \\ \sin \phi_0 & \cos \phi_0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.1)$$

and the new coordinates are found through matrix multiplication:

$$\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = R_1 R_2 R_3 \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (3.2)$$

$$R_1 R_2 R_3 = \begin{bmatrix} \cos^2 \phi_0 \cos \theta_0 + \sin^2 \phi_0 & \cos \phi_0 \sin \phi_0 (1 - \cos \theta_0) & -\cos \phi_0 \sin \theta_0 \\ \cos \phi_0 \sin \phi_0 (1 - \cos \theta_0) & \sin^2 \phi_0 \cos \theta_0 + \cos^2 \phi_0 & \sin \phi_0 \sin \theta_0 \\ \cos \phi_0 \sin \theta_0 & -\sin \phi_0 \sin \theta_0 & \cos \theta_0 \end{bmatrix} \quad (3.3)$$

and using the fact that $z=0$, we finally obtain

$$\begin{aligned} x_r &= x(\cos^2 \phi_0 \cos \theta_0 + \sin^2 \phi_0) + y \cos \phi_0 \sin \phi_0 (1 - \cos \theta_0) \\ y_r &= x \cos \phi_0 \sin \phi_0 (1 - \cos \theta_0) + y(\sin^2 \phi_0 \cos \theta_0 + \cos^2 \phi_0) \\ z_r &= x \cos \phi_0 \sin \theta_0 - y \sin \phi_0 \sin \theta_0 \end{aligned} \quad (3.4)$$

Eqs. (3.4) give the coordinates of points on the rotated antenna. The ray from each point is described by the line joining the point (x_r, y_r, z_r) to a corresponding point (x_a, y_a, z_a) on the equivalent aperture:

$$\begin{aligned} x_a &= x_r + h \sin \theta_0 \cos \phi_0 \\ y_a &= y_r + h \sin \theta_0 \sin \phi_0 \\ z_a &= z_r + h \cos \theta_0 \end{aligned} \quad (3.5)$$

where h is the radome height measured from the $z=0$ plane to the tip. Finally, the ray is described by the parametric equation:

$$\bar{r}(t) = \bar{r}_r + \hat{x}(x_a - x_r)t + \hat{y}(y_a - y_r)t + \hat{z}(z_a - z_r)t \quad 0 \leq t \leq 1 \quad (3.6)$$

The next step is to find the point where this ray intersects the radome surface. The following will describe this calculation for a cone and for an ogive.

3.2. Ray Tracing - Intercept Points

3.2.1. Cone

The general form for an equation describing a three-dimensional surface is $g(x,y,z)=0$.

For a cone with base radius b and height h :

$$g(x,y,z) = x^2 + y^2 - b^2 \left(1 - \frac{z}{h}\right)^2 \quad (3.7)$$

The point where the ray (3.6) intercepts the radome is found by substituting

$x_r + t(x_a - x_r)$, $y_r + t(y_a - y_r)$ and $z_r + t(z_a - z_r)$ for x, y and z , respectively, in (3.7). The result is the following quadratic equation $g(t)=0$:

$$\begin{aligned} g(t) &= At^2 + Bt + C \\ A &= (x_a - x_r)^2 + (y_a - y_r)^2 - \frac{b^2}{h^2} (z_a - z_r)^2 \\ B &= 2[(x_a - x_r)x_r + (y_a - y_r)y_r + \frac{b^2}{h^2} (z_a - z_r)(h - z_r)] \\ C &= x_r^2 + y_r^2 - \frac{b^2}{h^2} (h - z_r)^2 \end{aligned} \quad (3.8)$$

After solving (3.8) for t , the coordinates of the intercept point (x_t, y_t, z_t) are found by substituting t into (3.6). Note that the quadratic gives two solutions for t , but the one with $0 \leq t \leq 1$ is the only correct choice.

We will also need to know the surface normal at the point of intersection. The surface normal anywhere on the surface is in the direction of the gradient of g :

$$\nabla g = \hat{x}(2x) + \hat{y}(2y) + \hat{z} \left[2 \frac{b^2}{h^2} \left(1 - \frac{z}{h}\right) \right] \quad (3.9)$$

The normal at the intercept point is simply (3.9) evaluated at (x_t, y_t, z_t) . The angle of incidence,

α , is found from $\cos(\alpha) = (\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}) / |\hat{\mathbf{k}}|$ where $\hat{\mathbf{k}}$ is the propagation vector ($\hat{\mathbf{k}} = \hat{\mathbf{r}}_a - \hat{\mathbf{r}}_r$) and $\hat{\mathbf{n}}$ is the surface normal.

3.2.2. Tangent Ogive

An ogive is simply a surface of revolution formed by two intersecting circular arcs. A tangent ogive is a special case in which the base is perpendicular to the sides everywhere they intersect. Then if the base of the ogive is joined to a cylinder with the same radius, both surfaces are tangent at the points where they join. A general equation for the tangent ogive with base b and height h is

$$\begin{aligned} g(x,y,z) &= x^2 + y^2 - (\sqrt{R^2 - z^2} - a)^2 = 0 \\ R &= \frac{h^2 + b^2}{2b} \\ a &= R - b \end{aligned} \quad (3.10)$$

Substituting the components of the parametric vector (3.6) into (3.10) produces the following expression $g(t)=0$ to be solved for t :

$$\begin{aligned} g(t) &= At^2 + Bt + C + 2a\sqrt{Dt^2 + Et + F} = 0 \\ A &= (x_a - x_r)^2 + (y_a - y_r)^2 - (z_a - z_r)^2 \\ B &= 2[(x_a - x_r)x_r + (y_a - y_r)y_r + (z_a - z_r)z_r] \\ C &= x_r^2 + y_r^2 + z_r^2 - (R^2 + a^2) \\ D &= -(z_a - z_r)^2 \\ E &= -2z_r(z_a - z_r) \\ F &= R^2 - z_r^2 \end{aligned} \quad (3.11)$$

Unfortunately, this is not a simple quadratic as in the case of the cone, so it must be solved iteratively using Newton's method:

$$t_{k+1} = t_k - \frac{g(t_k)}{g'(t_k)} \quad (3.12)$$

The final converged value for t is substituted into (3.6) to find the intercept coordinates (x_t, y_t, z_t) .

The surface normal is the gradient evaluated at (x_t, y_t, z_t) :

$$\nabla g = \hat{x}(2x) + \hat{y}(2y) + \hat{z}(2z) \left(1 - \frac{a}{\sqrt{R^2 - z^2}} \right) \quad (3.13)$$

3.3. Ray Fixed Coordinates - Parallel/Perpendicular Decomposition

The properties of the radome shell material are most conveniently characterized in terms of parallel and perpendicular transmissivities. The ray-fixed coordinate system, illustrated earlier in Figure 2, is used to express the incident and transmitted fields in terms of parallel and perpendicular components. Referring to that figure, the vector $\bar{k}$ is in the direction of propagation: $\bar{k} = \bar{r}_a - \bar{r}_r$. At the point this ray intersects the radome, the radome's surface normal is $\hat{n}$. The unit vector $\hat{u}_\perp$ is perpendicular to both $\bar{k}$ and $\hat{n}$ since both of the latter are in the plane of incidence. It is given by

$$\hat{u}_\perp = \frac{\bar{k} \times \hat{n}}{|\bar{k} \times \hat{n}|} \quad (3.14)$$

then the orthogonal unit vector $\hat{u}_\parallel$ is found from

$$\hat{u}_\parallel = \hat{u}_\perp \times \hat{k} \quad (3.15)$$

The three unit vectors $\hat{u}_\perp$, $\hat{u}_\parallel$ and $\hat{k}$ form the basis vectors for the ray fixed coordinate system.

The incident field is decomposed using

$$\begin{aligned} E_{\parallel}^i &= \bar{E}^i \cdot \hat{u}_{\parallel} \\ E_{\perp}^i &= \bar{E}^i \cdot \hat{u}_{\perp} \end{aligned} \quad (3.16)$$

3.4. Transmission & Radiation Calculation

The two components of incident electric field parallel and perpendicular to the plane of incidence are scalars. The transmitted field components are simply the product of those scalars and the corresponding complex transmissivities $\gamma_{\parallel}$ and $\gamma_{\perp}$. The transmitted field is

$$\begin{aligned} \bar{E}^t &= (E_{\parallel}^i \gamma_{\parallel}) \hat{u}_{\parallel} + (E_{\perp}^i \gamma_{\perp}) \hat{u}_{\perp} \\ &= E_{\parallel}^t \hat{u}_{\parallel} + E_{\perp}^t \hat{u}_{\perp} \end{aligned} \quad (3.14)$$

This transmitted field is resolved into co-polarized and cross-polarized components as follows: Let $\hat{e}^i$ be a unit vector in the direction of the incident electric field, then the unit vector in the incident magnetic field direction is $\hat{h}^i = -\hat{e}^i \times \hat{k}$. The transmitted components are

$$\begin{aligned} E_{CP}^t &= \bar{E}^t \cdot \hat{e}^i \\ E_{XP}^t &= \bar{E}^t \cdot \hat{h}^i \end{aligned} \quad (3.18)$$

These scalar components are regarded as the co- and cross-polarized excitations of the equivalent aperture.

The antenna's co-polarized radiation pattern is calculated from

$$E_{CP}(\theta, \phi) = \sum_{m=1}^N E_{CP}^t(m) g(\theta, \phi) e^{-jk(x_m \sin \theta \cos \phi + y_m \sin \theta \sin \phi)} \quad (3.19)$$

where $f(\theta, \phi)$ is the radiation pattern of an array element. The complex coefficients $E_{CP}^t(m)$ include the amplitude weighting for low sidelobes as well as beam steering phase shifts, if applicable. The subscript m denotes the index of the antenna element where the ray originated

and (x_m, y_m) are its coordinates *before scanning*. In other words, the pattern computation is always done relative to the antenna's boresight direction, not the radome's. The cross-polarized radiated field calculation is the same as (3.19) with subscripts XP.

This completes the analysis procedure derivation. The program RADOME2, described in the Appendix, incorporates the above equations (3.1) through (3.19). It is capable of arbitrary scan angles, cone and tangent ogive radome shapes with arbitrary height and base radius and mechanically or electronically scanned antennas. The initial antenna amplitude distribution is a Taylor distribution with user-specified sidelobe level. The radome transmissivity as a function of incidence angle is entered through a data file, and the program interpolates to find the transmissivity at the required incidence angles. The following section presents some of the results from the program.

4. CALCULATED RESULTS

4.1. Comparison of Conical and Ogival Radomes

The test case geometry is shown in Figure 13. The radome has a base radius of 7.5λ and a height of 30λ . The antenna has an aperture radius of 5λ . The radome shell has the transmissivity vs. angle shown in Figure 14. This is a typical characteristic for a multilayer radome: the amplitude is well matched out to the design angle (here 45°) after which the components begin to separate, while the phase begins to separate immediately away from normal incidence with the difference increasing nearly linearly with incidence angle. The initial antenna distribution is a Taylor distribution (specialized to a circular aperture [8]) with $\bar{n}=4$, meaning that the first 4 sidelobes on each side of the main beam are maintained at -40 dB relative to the main beam, and the remaining sidelobes are progressively lower.

It is clear from Figure 13 that when the antenna beam is directed through the radome apex, the incidence angles are lower for the ogive shape than for the cone shape. The result is that the ogive (Figure 15a) has much lower Condon lobes than the cone (Figure 15b). However, the on-axis scan is the worst case for the cone, and the incidence angle decreases rapidly as the antenna beam scans, so that in the range of 30° - 60° scan, its performance is comparable to that of the ogive.

4.2. Effects of Antenna Scanning

The final set of calculations involve scanning the antenna in the $\phi=0$ plane, that is in the azimuth plane corresponding to $y=0$. The antenna is vertically polarized, with electric field in the y direction. At 30° scan, the cross polarized aperture amplitude is as shown in Figure 16. It is now symmetric only about the x axis, unlike the on axis case (see Fig. 5), which was

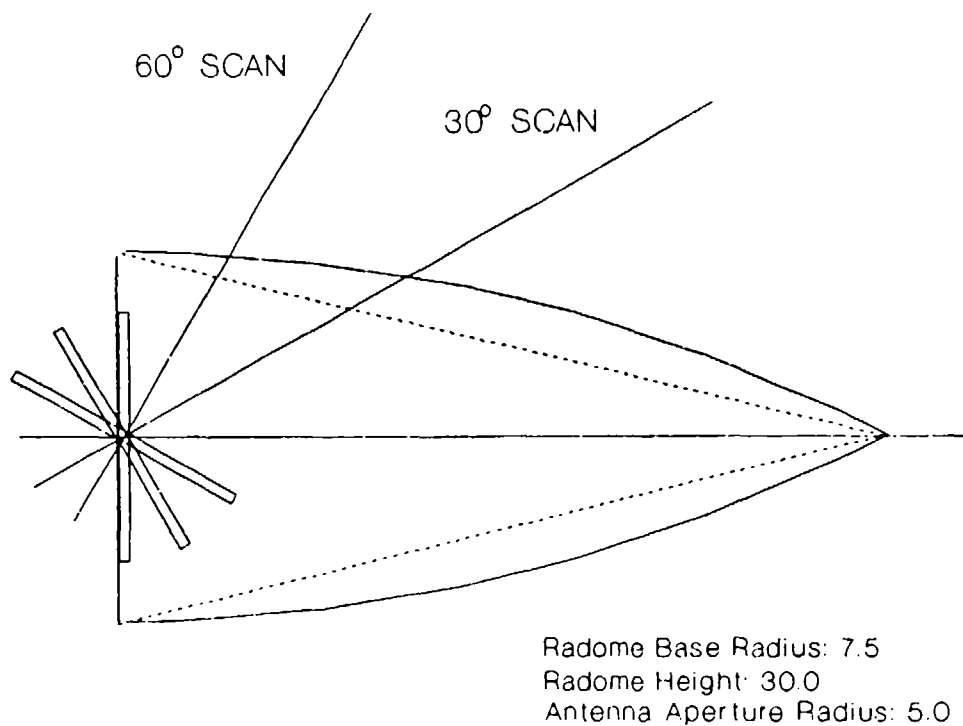


Figure 13. Test Case Geometries (Cone and Ogive)

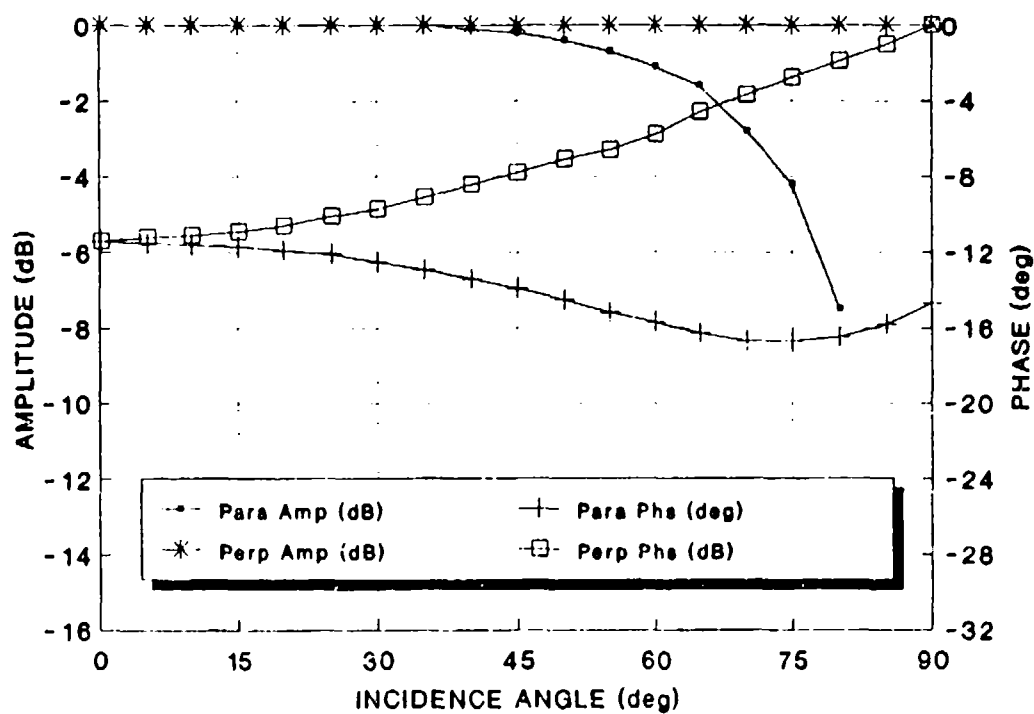


Figure 14. Surface Transmissivity vs. Angle for Test Case

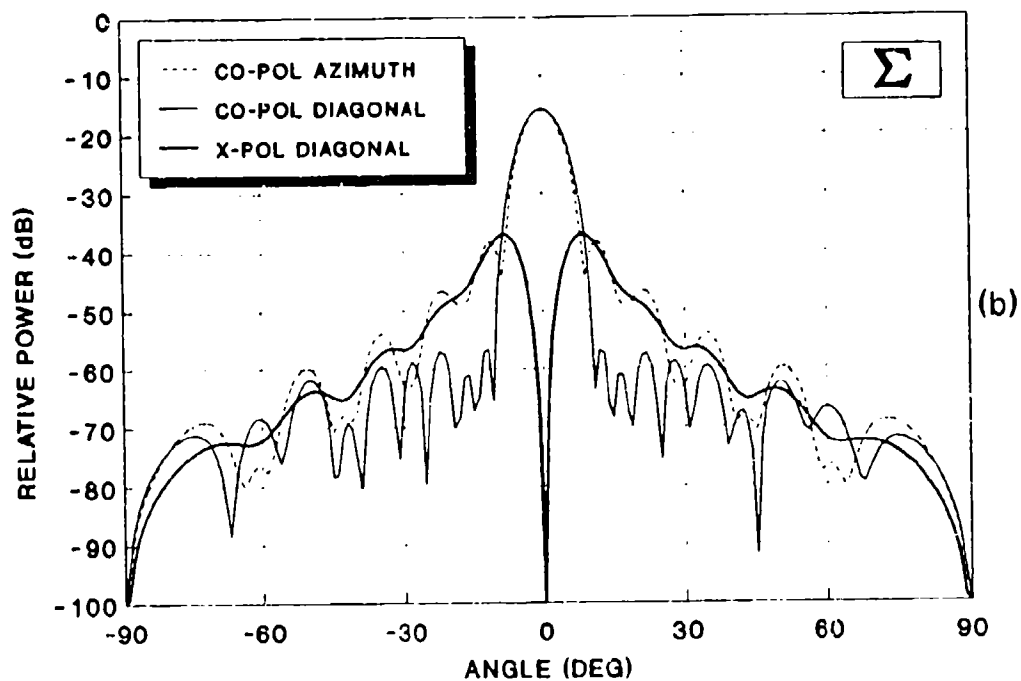
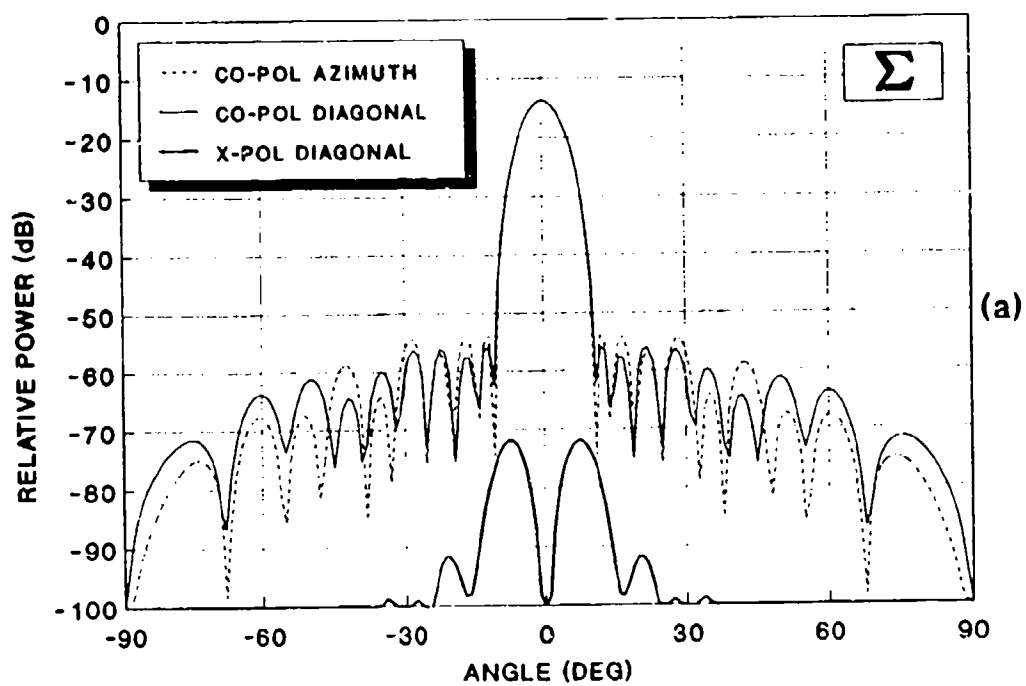


Figure 15. Co- and Cross-Polarized Patterns for On-Axis Scan:
 (a) Tangent Ogive Radome; (b) Conical Radome

symmetric about both axes. The cross-polarized phase, shown in Figure 17, is antisymmetric. The result of a symmetric amplitude and antisymmetric phase is, of course, a difference pattern. Thus, there are only two lobes in the cross-polarized pattern instead of four. Those lobes appear in the elevation plane as well as the diagonal plane, as shown in Figure 18a. The cross-polarized azimuth pattern is not quite zero because of a slight asymmetry in the element locations. Figure 18b shows two of the co-polarized patterns for amplitude comparison. Figures 19a and 19b are the same patterns, but with the antenna scanned to 60° . Scanning in the elevation plane has a similar effect, except that the cross-polarized patterns will have twin difference lobes in the azimuth pattern instead of the elevation pattern.

Last, Figure 20 shows the response of the two difference channels for the same case as Figure 18, i.e. 30° azimuth scan. The cross-polarized pattern in the elevation difference channel looks more like a sum beam. The azimuth difference channel, on the other hand, still has a null in the antenna's broadside direction.

The anticipated effect of this is that when the antenna is looking out through the left or right side, the jammer may cause an error in the elevation direction; and when the antenna is looking out through the top or bottom of the radome, a cross-polarized jammer may cause a tracking error in the azimuth direction. The exact nature of the tracking error and the J/S ratio required to defeat the monopulse tracking depends on several other factors, such as the receiver design and the nature of the jamming signal. Those are recommended topics for future study.

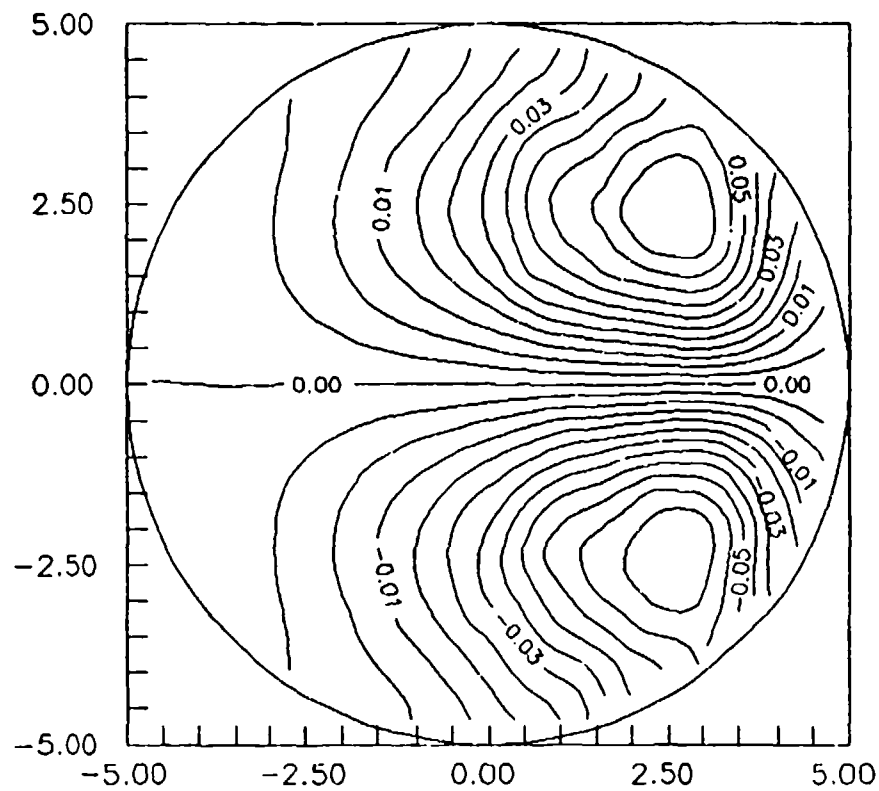


Figure 16. Cross-Polarized Aperture Amplitude Distribution at 30° Azimuth Scan, Ogive Radome

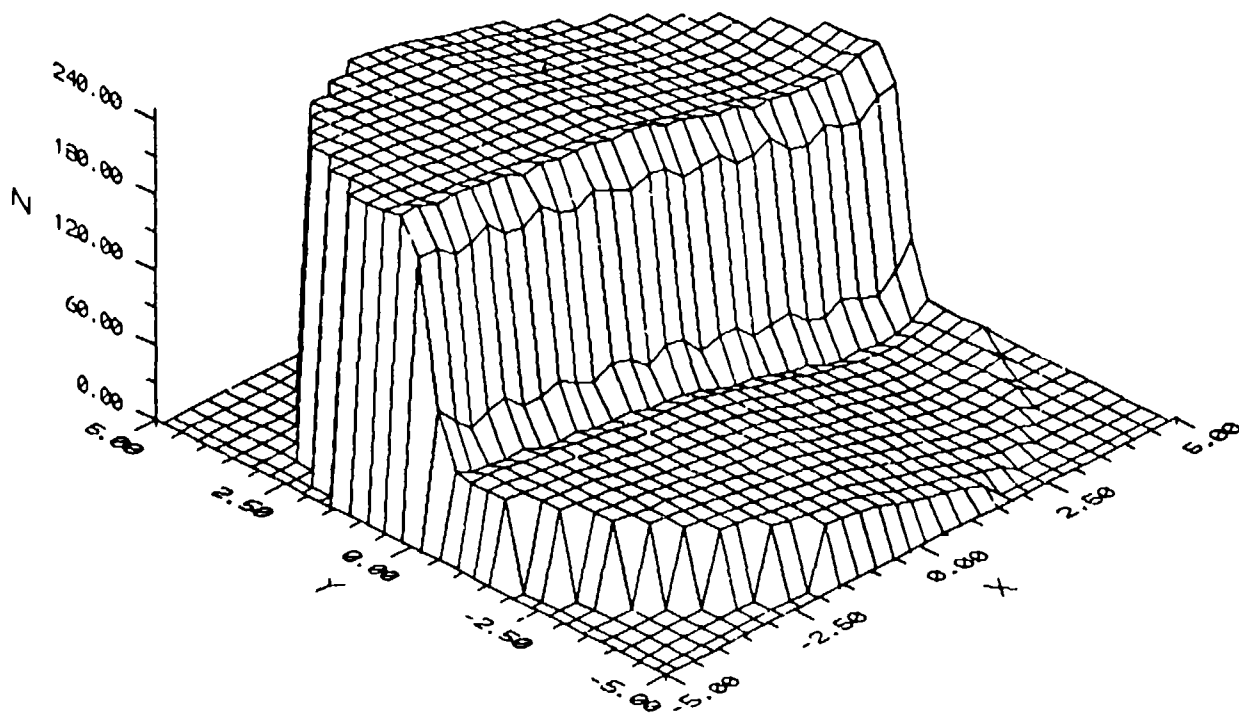


Figure 17. Cross-Polarized Aperture Phase at 30° Azimuth Scan, Ogive Radome

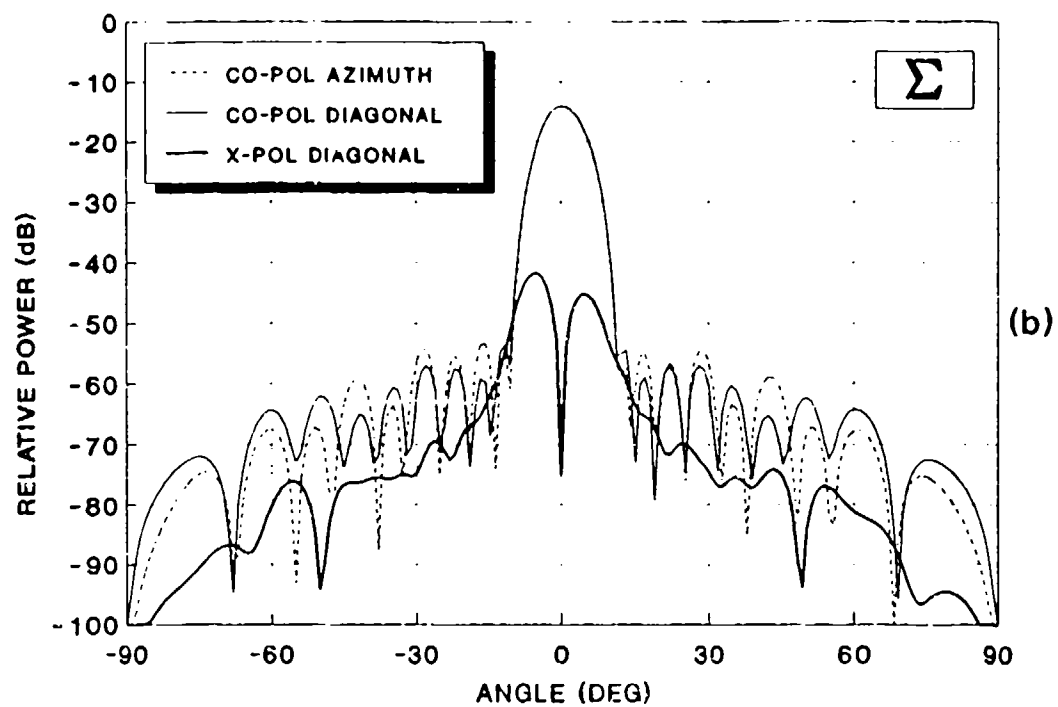
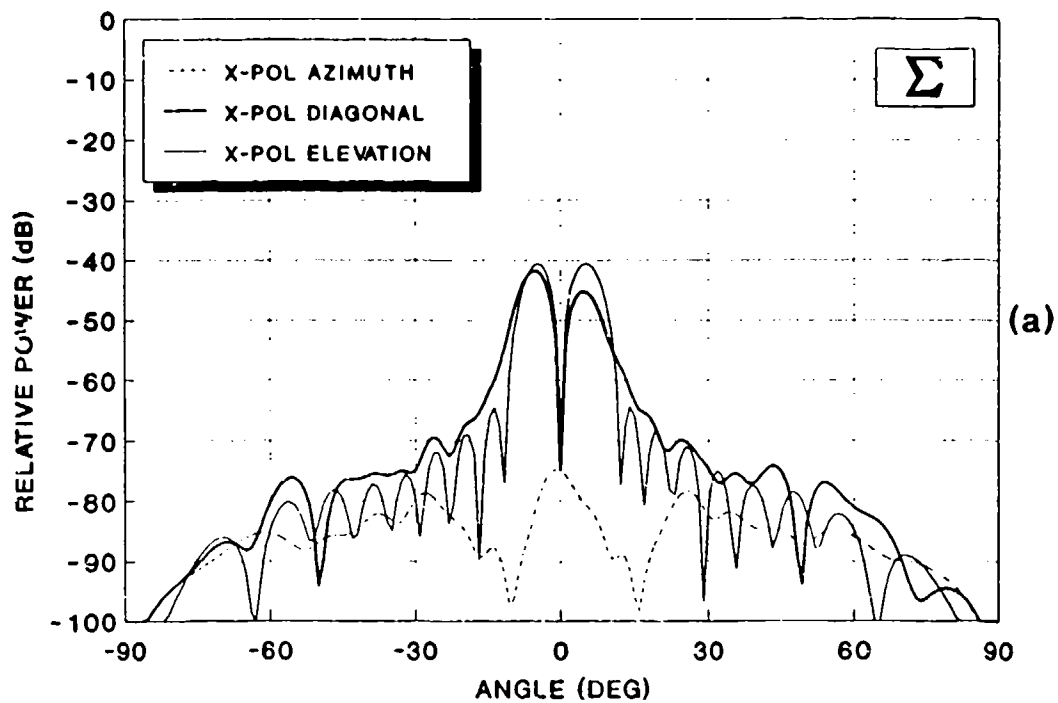


Figure 18. Antenna Patterns with Ogive Radome at 30° Azimuth Scan:
 (a) Cross-Pol Pattern Cuts; (b) Comparison of Co- and Cross-Pol

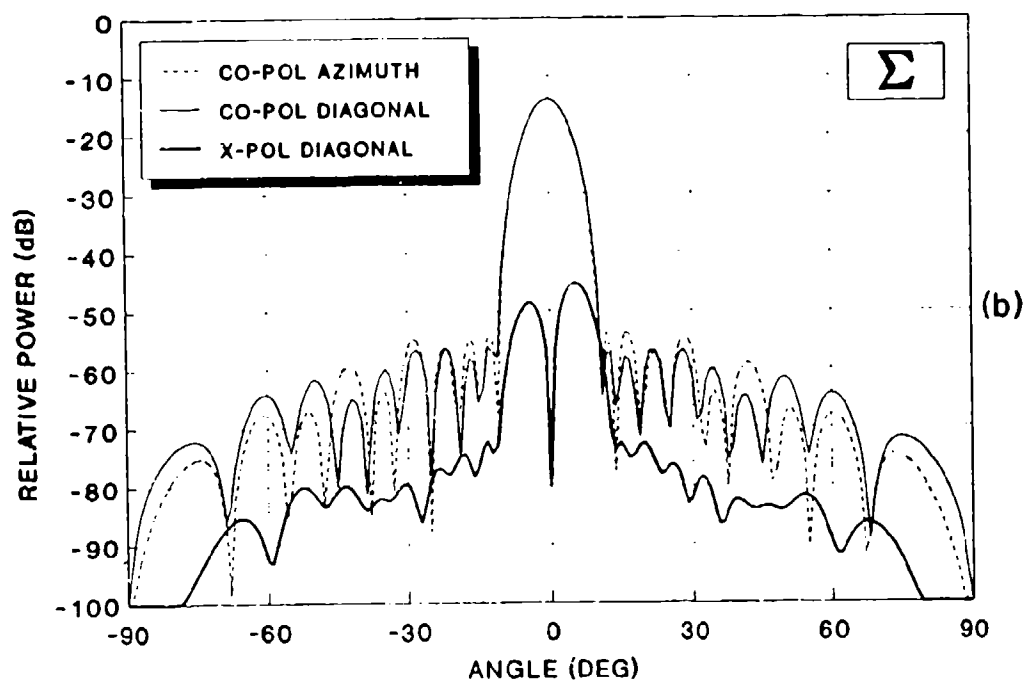
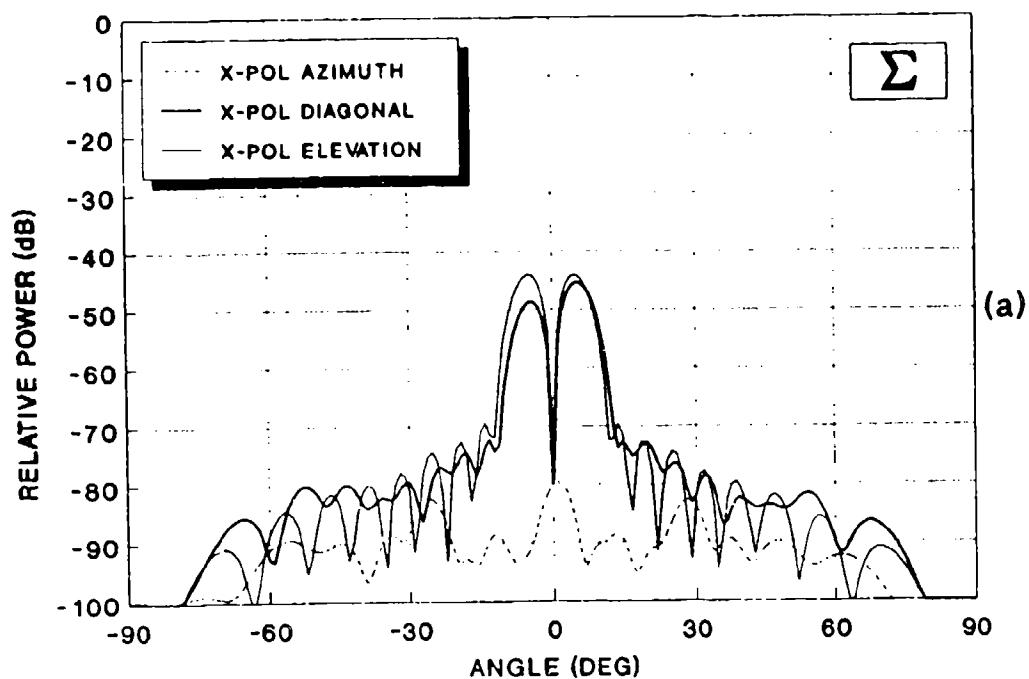


Figure 19. Ogive Radome Patterns at 60° Azimuth Scan: (a) Cross-Polarized Patterns; (b) Comparison of Co- and Cross-Polarized Patterns.

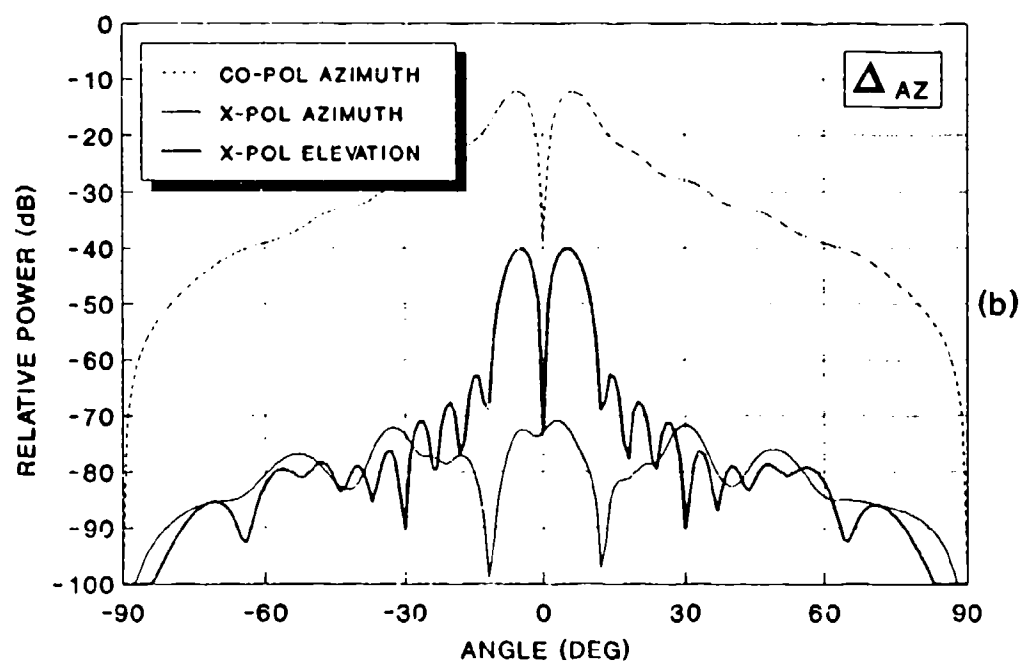
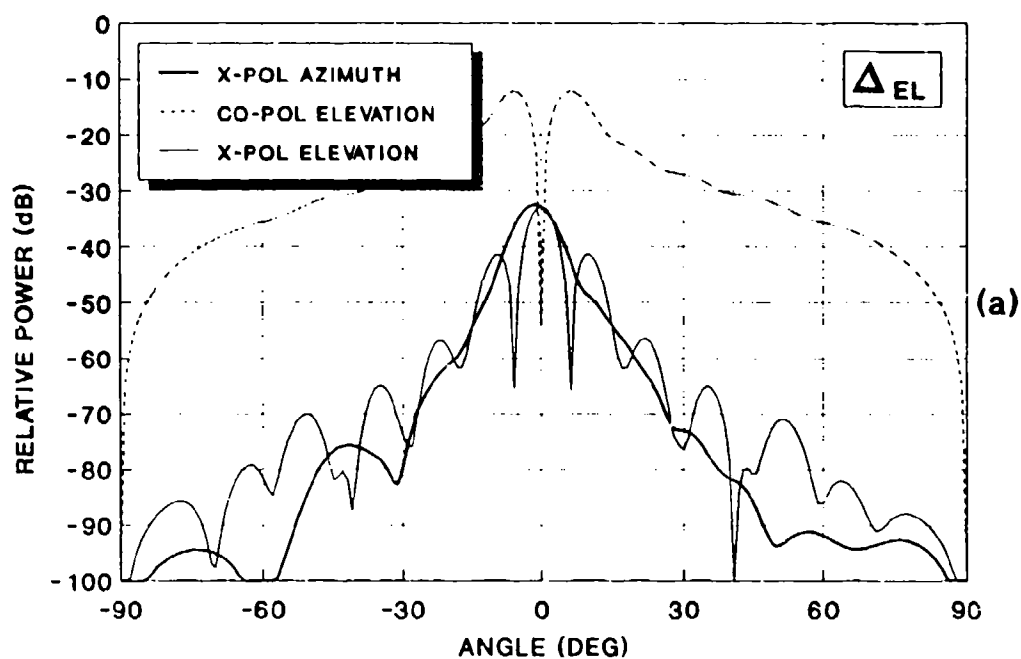


Figure 20. Antenna Patterns with Ogive Radome at 30° Azimuth Scan:
 (a) Elevation Difference Channel; (b) Azimuth Difference Channel

5. CONCLUSIONS AND RECOMMENDATIONS

Every radome will generate some depolarization simply because the antenna beam passes through it at oblique angles, and any material surface viewed obliquely has different parallel and perpendicular transmissivities (divorce). This report has shown that the cross polarized fields generate difference patterns due to their antisymmetric nature. An on axis beam will generate the classic Condon lobe pattern observed earlier in parabolic reflectors. When the antenna beam is scanned off axis, so that it looks out through the side of the radome, the cross polarized pattern has only two lobes, whose beamwidths and locations are identical to those of the monopulse difference pattern's main lobes, but their response is seen in the receiver's sum channel. The result is a susceptibility to cross-polarization jamming.

The relative strength of the cross-pol lobes was shown to be directly related to the transmissivity properties of the radome structure. The transmission coefficients for parallel and perpendicular polarizations are usually different in both amplitude and phase. It was shown that the phase divorce is of greater concern than the amplitude divorce, with a mere 10° phase difference causing more depolarization than 1 dB amplitude difference.

This study used a fairly simple geometrical-optics analysis. The results are expected to be reasonably accurate for angles near the main beam. It is recommended that these results be compared with measured data to validate their accuracy. The sidelobe levels shown are not expected to be accurate because they do not account for internal radome reflections. If good estimates of sidelobe levels are required, we recommend that the analysis method be modified to use surface integration.

This report identifies a *potential* problem. The extent of a system's vulnerability to

cross-polarization jamming depends on the receiver design. We recommend that the results presented here be used to simulate the tracking performance of a monopulse radar in order to determine how much radome divorce can be tolerated and still maintain a desired level of tracking accuracy.

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APPENDIX A **CODE DOCUMENTATION**

1. DESCRIPTION

The FORTRAN program RADOME2 is a transmission-only geometrical optics model of an antenna/radome combination. Its main purpose is to predict the cross-polarization that is generated when the radome has a significant level of divorce. Co-pol and cross-pol patterns are computed in each of three planes: azimuth, elevation and diagonal. Monopulse difference patterns (elevation or azimuth) may also be calculated. The radome shape may be conical or tangent ogive, either one with user-specified base radius and height. Alternatively, the radome may be removed entirely to observe the antenna properties alone. The antenna is a flat plate array with a circular aperture, and may be scanned mechanically or electronically. It incorporates a low-sidelobe Taylor amplitude distribution.

The code is not intended to provide accurate estimates of sidelobe levels. When the radome has significant transmission loss, internal reflections will be significant, and may dominate the sidelobe levels. Those internal reflections are not accounted for in RADOME2. See section 3 of this report for a detailed description of the theory and derivation for the geometrical optics analysis. This appendix is intended to provide instructions for using the code, as well as enough details of its structure to allow for future modifications.

1.1. Radome Description

The user may choose between two radome geometries: a right circular cone; and a tangent ogive. In both cases, only two independent variables need to be specified: the base

radius; and the "height," the length from base to tip. One may also choose no radome, in which case the antenna properties alone are computed.

1.2. Antenna Description

The antenna is a circular aperture, whose radius is less than that of the radome. Its center is always located at $(x,y,z)=(0,0,0)$, which is also the center of the radome base. The antenna elements are spaced in an equilateral triangular lattice. The maximum number of elements is 500. If the antenna is to be mechanically scanned, the element spacing may be chosen as anything less than 1 wavelength. If it is to be electronically scanned, the elements should be closer together to prevent grating lobes. For example, a spacing of about $.6\lambda$ is required to prevent grating lobes at scan angles of 60° .

The amplitude weighting is a Taylor distribution, which gives low sidelobes. The user must specify two parameters: SLL, the dB level of the highest sidelobe relative to the main beam; and NBAR, an integer, which is the number of sidelobes at the peak level. The pattern will have NBAR sidelobes on each side of the main beam at -SLL dB, and the remaining sidelobes will be progressively lower.

The user must specify whether the antenna is to be mechanically scanned, or electronically scanned. If electronic scanning is chosen, the main beam will be somewhat broader in the direction of scan. In either case, the antenna patterns are calculated relative to the antenna's surface normal. The antenna is y polarized, and the "azimuth" and "elevation" planes are taken as the x-z, and y-z planes, respectively.

Monopulse difference patterns may also be computed. The difference pattern is generated by phase shifting one side of the aperture by 180° relative to the other. The user may choose

either elevation difference or azimuth difference. Note that the program must be executed three times to generate all three monopulse components: Σ (sum); Δ_{AZ} ; and Δ_{EL} .

2. INPUTS AND OUTPUTS

RADOME2 reads two data files and generates four others. This section provides a detailed description of each file. Note that each of the output files is overwritten if it already exists.

The first input file, RADOME2.INP, contains information on the geometry and other problem parameters such as scan angles, user options, etc. It is an optional file: the user may choose instead to enter all the same data from the keyboard. The second input file contains the transmissivity data, and the filename is entered either from RADOME2.INP or from the keyboard.

The output file RADOME2.LOG is a lengthy "log" file containing calculations from each stage of the radome transmission calculation. The user specifies whether or not the file is to be written. Since it is intended primarily for validation and troubleshooting, it is normally not created. RADOME2.PAT is the output file for antenna radiation patterns. Its seven columns are the angle and co- and cross-pol components in each of three planes: azimuth, elevation and diagonal. RADOME2.APR is the "equivalent" aperture distribution, i.e. the original Taylor distribution modified by the radome transmission properties. It contains the element locations (original, unscanned x and y coordinates) and the co- and cross-pol element excitations as complex numbers. RADOME2.IPT contains the coordinates of where the ray from each antenna element intersects the radome surface. Finally, RADOME2.SNM contains the three components of the surface normal at the intersect point.

2.1. Geometry & Configuration File (Input)

The first input the program requires is whether the configuration data is to be entered from the keyboard or from a data file. If the user responds with 1 for data file, the program will retrieve all remaining inputs from the file RADOME2.INP. An example of this file is given below. Note that if keyboard input is chosen, the sequence of inputs is identical to those in the RADOME2.INP. File input is recommended, since most of the parameters remain unchanged from run to run.

In the input file, there may be any number of comment lines preceding the data, and the last comment line must have the character # in column 1. If there are no comments, that will be the first line. The data is unformatted, but the values must appear in the order shown in the example below.

```
Input file for program RADOME2
Line1: 1 denotes ogive radome, 0 denotes cone and 99 is no radome
Line2: 7.5 and 30.0 are the radome base radius and height, respectively, in wavelengths
Line3: 5 = antenna aperture radius, .5 = interelement spacing (wavelengths)
       40 = sidelobe level
       4 = NBAR
Line4: 30.0,0.0 = theta0, phi0
Line5: OPT2=0 for mechanical beam steering, 1 for electronic
       OPT3=0 for sum patterns, 1 for az. diff., 2 for el. diff.
       OPT4=1 enable log file, anything else disables logging
Line6: Data file for transmissivity values
#      (last comment line must begin with this character)
1
7.5, 30
5.0, 0.5, 40, 4
30.0, 0.0
0, 0, 1
div1.dat
```

The file may contain as many comment lines as desired, but they must all precede the data.

Input is unformatted

Figure A1. Example Input File RADOME2.INP

The scan angles θ_0 are ϕ_0 are spherical coordinate angles, with θ measured from the z

axis and ϕ measured from the x axis in the x-y plane. Thus ϕ_0 is the scan plane, where 0° is azimuth and 90° is elevation.

2.2. Transmissivity Data File (Input)

An example transmissivity data file is shown below. Like the file RADOME2.INP, the data is preceded by a line with # in the first column, and all lines preceding it are ignored.

```

File DIV1.DAT
Sample file with 1.0 dB and 10 deg divorce
at 60 degrees incidence

transmissivity vs. angle

                                The file may contain as many
                                comment lines as desired, but
                                they must all precede the data.

Angle      Para      Perp
#      Mag  Phase  Mag  Phase
# 19
0.1  0.0  -11.4  0.0  -11.4
5.0  0.0  -11.5  0.0  -11.2
10.0 0.0  -11.6  0.0  -11.1
15.0 0.0  -11.7  0.0  -10.9
20.0 0.0  -11.9  0.0  -10.6
25.0 0.0  -12.1  0.0  -10.1
30.0 0.0  -12.5 -0.0  -9.7
35.0 0.0  -12.9 -0.0  -9.1
40.0 -0.1 -13.4 -0.0  -8.4
45.0 -0.2 -13.9 -0.0  -7.8
50.0 -0.4 -14.5 -0.0  -7.1
55.0 -0.7 -15.2 -0.0  -6.6
60.0 -1.1 -15.7 -0.0  -5.8
65.0 -1.6 -16.3 -0.0  -4.6
70.0 -2.8 -16.7  0.0  -3.7
75.0 -4.2 -16.7  0.0  -2.8
80.0 -7.5 -16.5  0.0  -1.9
85.0 -15.0 -15.8  0.0  -1.0
90.0 -45.0 -14.7  0.0  -0.0

                                (last comment line must begin with #)
                                This is the number of values in the file

                                Input data is
                                unformatted

                                Magnitude values
                                are in dB

                                Phase values
                                are in degrees

```

Figure A2. Example Transmissivity Input File

The first data line is the number of angles that are tabulated in the file. For each angle, the program reads the angle in degrees, and the transmissivity values as magnitude in dB and phase in degrees.

The program will use this input data to create lookup tables for parallel and perpendicular transmissivities at 1° increments from 0° to 90°. Hence the input data may be coarsely sampled as long as transmissivity is a fairly smooth function of angle. The program uses Lagrange interpolation to create the lookup table from the input data. It will in fact extrapolate if the input data does not cover the entire range from 0° to 90°. Note that the radome material is assumed to be anisotropic, implying that its transmissivity is a function only of the angle from the normal.

2.3. Aperture Distribution File (Output)

Element Number	Coordinates in wavelengths*		Element Excitation			
	x	y	Co-polarized		Cross-polarized	
#			real	imag	real	imag
1	-.2500	-4.9796	.1164	-.0146	.1164	-.0088
2	.2500	-4.9796	.1134	-.0134	.1134	-.0083
3	-2.0000	-4.5466	.1223	-.0173	.1223	-.0070
4	-1.5000	-4.5466	.1254	-.0168	.1254	-.0076
5	-1.0000	-4.5466	.1312	-.0169	.1312	-.0085
6	-.5000	-4.5466	.1354	-.0162	.1354	-.0094
7	.0000	-4.5466	.1351	-.0151	.1351	-.0092
8	.5000	-4.5466	.1303	-.0134	.1303	-.0086
9	1.0000	-4.5466	.1223	-.0114	.1223	-.0078
10	1.5000	-4.5466	.1105	-.0083	.1105	-.0057
.	one line per array element					
.						
356	-.7500	4.5466	.1336	-.0163	.1336	.0091
357	-.2500	4.5466	.1362	-.0160	.1362	.0095
358	.2500	4.5466	.1335	-.0146	.1335	.0090
359	.7500	4.5466	.1270	-.0127	.1270	.0083
360	1.2500	4.5466	.1166	-.0099	.1166	.0068
361	1.7500	4.5466	.1047	-.0066	.1047	.0044
362	.0000	4.9796	.1144	-.0138	.1144	.0084

Figure A3. Example Output File RADOME2.APR

The figure above shows the content and format of the aperture distribution output file. The first column is the element number identifier. The next two columns are the x,y coordinates

of the unscanned antenna elements. The remaining columns are the effective element excitations accounting for the transmission properties of the radome. The values in this figure are those generated using the input files RADOME2.INP and DIV1.DAT shown earlier.

2.4. Far Field File (Ouput)

Observation Angle (deg)	Azimuth		Relative Power in dB			
	Co-pol	X-pol	Diagonal		Elevation	
	Co-pol	X-pol	Co-pol	X-pol	Co-pol	X-pol
-90	-100.000	-100.000	-100.000	-100.000	-100.000	-100.000
-89	-98.508	-100.000	-89.059	-100.000	-92.210	-100.000
-88	-92.239	-100.000	-83.002	-100.000	-86.185	-100.000
-87	-88.326	-98.287	-79.422	-98.014	-82.660	-100.000
-86	-85.319	-95.843	-76.847	-95.623	-80.161	-100.000
-85	-82.790	-93.976	-74.821	-93.819	-78.233	-100.000
.						
.						
-10	-40.983	-90.525	-38.877	-42.453	-37.424	-46.419
-9	-31.418	-85.915	-30.407	-39.914	-29.411	-41.904
-8	-25.167	-82.909	-24.557	-37.992	-23.838	-38.782
-7	-20.537	-80.283	-20.138	-36.688	-19.600	-36.649
-6	-16.953	-77.385	-16.685	-36.006	-16.277	-35.339
-5	-14.148	-74.536	-13.966	-35.976	-13.659	-34.800
-4	-11.972	-72.166	-11.850	-36.681	-11.625	-35.068
-3	-10.337	-70.443	-10.258	-38.304	-10.102	-36.302
-2	-9.185	-69.390	-9.140	-41.288	-9.042	-38.929
-1	-8.483	-68.985	-8.463	-47.062	-8.417	-44.381
0	-8.210	-69.195	-8.210	-69.195	-8.210	-69.195
1	-8.359	-69.969	-8.375	-47.930	-8.417	-44.515
2	-8.933	-71.202	-8.963	-42.734	-9.042	-38.985
.						
.						
85	-81.854	-93.488	-74.674	-92.974	-78.370	-100.000
86	-84.256	-95.402	-76.676	-94.749	-80.297	-100.000
87	-87.150	-97.884	-79.231	-97.130	-82.796	-100.000
88	-90.975	-100.000	-82.798	-100.000	-86.321	-100.000
89	-97.186	-100.000	-88.847	-100.000	-92.346	-100.000
90	-100.000	-100.000	-100.000	-100.000	-100.000	-100.000

Figure A4. Example Pattern Output File RADOME2.PAT

The listing shown above is the pattern file generated using the input files RADOME2.INP and DIV1.DAT shown earlier. The number of pattern values and the angle increment are fixed by a PARAMETER statement in the main program. The observation angle is always relative

to the antenna's surface normal, so the main beam appears at 0° , independent of the scan angle. The values in RADOME2.PAT may represent Σ (sum) Δ_{AZ} or Δ_{EL} , depending on the choice of OPT(3) in the input file. The figure above shows sum patterns. Values are in dB relative to a uniformly illuminated aperture. Thus, "taper loss" (the reduction in aperture efficiency due to a low-sidelobe distribution) and radome transmission loss are both accounted for, and that is the reason that the main beam peak is lower than 0 dB. Note that the patterns are "clipped" so that values below -100 dB are set to -100 dB.

2.5. Intercept Points and Surface Normals (Outputs)

The two figures on the following page are example outputs files RADOME2.SNM and RADOME2.IPT, which are the surface normals and intercept points, respectively, of rays originating at each antenna element and passing through the radome to an equivalent aperture outside the radome. The surface normals are unit vectors, directed inward. In the .SNM file, the second and third columns are the x,y coordinates of the antenna elements before scanning. In the .IPT file, the first column is the antenna element index, and the remaining three are the intercept coordinates in units of wavelengths.

2.6. Log File (Output, Optional)

An example of the log file RADOME2.LOG is shown on the following page. It contains detailed information about the radome transmission calculations made for each element. The last option in the input file is set to 1 if this file is to be created (if it does not already exist). The first block of information written is a verification of the configuration and geometry parameters. After that, a separate block is written for each element. If the meaning of this output data is not

Element Number	Element Coordinates in Wavelengths*		Components of Unit Surface Normal Vector		
	x	y	x	y	z
1	-.2500	-4.9796	-.4908	.5124	.7046
2	.2500	-4.9796	-.5242	.5313	.6655
3	-2.0000	-4.5466	-.3963	.4002	.8263
4	-1.5000	-4.5466	-.4273	.4151	.8032
5	-1.0000	-4.5466	-.4593	.4308	.7768
6	-.5000	-4.5466	-.4923	.4471	.7468
.					
.					
359	.7500	4.5466	-.5776	-.4905	.6525
360	1.2500	4.5466	-.6123	-.5085	.6055
361	1.7500	4.5466	-.6464	-.5264	.5523
362	.0000	4.9796	-.5074	-.5218	.6857

Figure A5. Example Output File RADOME2.SNM (Surface Normals)

Aperture Element Number	Ray-Radome Intercept Coordinates (in wavelengths)		
	x	y	z
1	4.7693	-4.9796	8.7607
2	4.9125	-4.9796	8.0087
3	4.5024	-4.5466	11.7984
4	4.6802	-4.5466	11.1064
5	4.8480	-4.5466	10.3970
6	5.0057	-4.5466	9.6701
.			
.			
359	5.3543	4.5466	7.7739
360	5.4747	4.5466	6.9824
361	5.5836	4.5466	6.1711
362	4.8423	4.9796	8.3872

Figure A6. Example Output File RADOME2.IPT (Intercept Points)

self-evident, see Section 3 of this report for a detailed explanation.

3. PROGRAM FLOW

The figure below shows the order in which the program executes. The boldface titles next to flowchart blocks indicate the important subroutines that assist those tasks. The most analytically detailed procedures are the top three blocks in the right column. Two of those

```

RADOME2 - LOG FILE
ANTENNA PARAMETERS:
APERTURE RADIUS = 5.00
SIDELOBE LEVEL = - 40. dB    NBAR = 4
SCAN ANGLES: THETA = 30.00 deg.    PHI = .00 deg.
RADOME PARAMETERS:
BASE RADIUS = 7.50    HEIGHT = 30.00
OPTIONS:   #1   #2   #3   #4
           1    0    0    1

POINT # 1 INTERSECT:  4.769 -4.980  8.761
SURFACE NORMAL:  -.491   .512   .705
INCIDENCE ANGLE:      68.601940
E-CROSS-K:  -.866   .000   .500
U-PERP:  -.477  -.835   .275
U-PARA:  -.723   .550   .417
Incident E components (para,perp):  .5504 -.8349
Incident H components (para,perp):  .8349 .5504
Co-pol trans. coeff. (real,imag):  .7422 -.2208
X-pol trans. coeff. (real,imag):  .9975 -.0702

The above 10 lines are repeated for each antenna element

```

Figure A7. Example Log File RADOME2.LOG

calculations are bypassed if there is no radome present. Further details of the actions taken within those blocks, as well as within the two subroutines RCCONE and TANGIVE, may be found in Section 3 of this report, as well as in the internal comments in the source listing that follows.

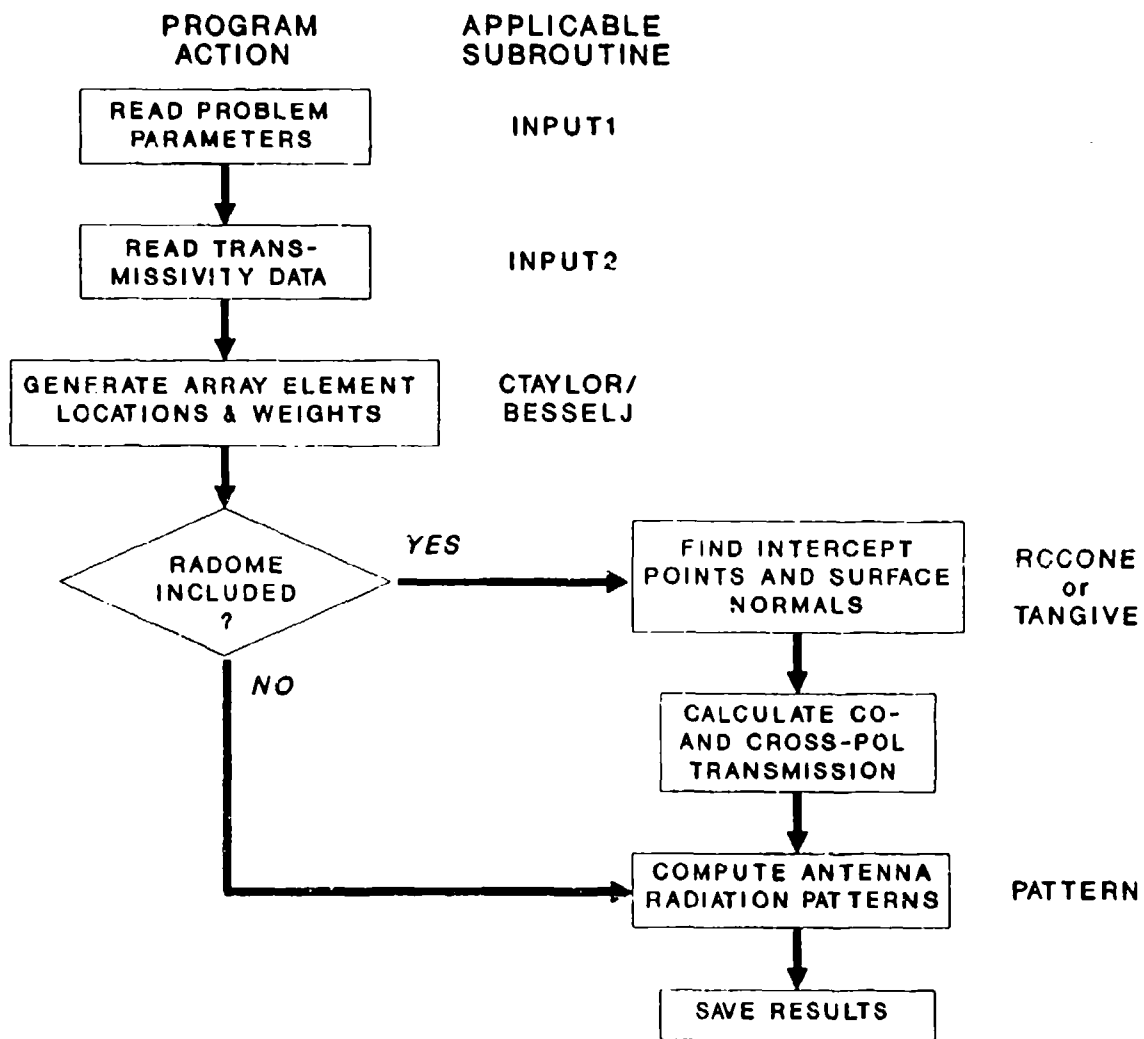


Figure A8. Flowchart for Program RADOME2

4. SOURCE CODE LISTING

```
C-----
C--                                RADOME2  V1.0                                --
C-----
C--                                Dan McGrath                                --
C--                                Air Force Institute of Technology            --
C--                                Department of Electrical and Computer Engineering --
C--                                Wright-Patterson AFB, OH  45433              --
C--
C--  Origination Date/ Last modified: 25 Jul 91 / 15 Oct 91                --
C--  Language/ Operating System: FORTRAN (Microsoft)/ MS-DOS              --
C-----
C
C  Purpose:
C    This program simulates the pattern performance of an array
C    antenna with a non-ideal radome. The antenna may be either phase
C    steered or mechanically scanned. The radome geometry may be either
C    a right-circular cone or a tangent ogive. The output consists of
C    the co- and cross-polarized azimuth and elevation radiation patterns.
C    An optional output is the co- or cross-polarized aperture amplitude
C    distribution. The antenna has a low-sidelobe Taylor distribution.
C    The radome may have a variable transmissivity vs. incidence angle,
C    which may be different in the parallel and perpendicular polarizations.
C
C  Input Files:
C    The user must specify an input file containing the
C    radome's parallel and perpendicular plane transmissivities
C    (in dB) versus incidence angle (in deg.). See the header
C    for routine INPUT2 for a detailed description of the file
C    format.
C    The problem parameters may be input from a data file
C    or manually from the keyboard. If the former option is
C    chosen, then the file RADOME2.INP must be in the current
C    directory. See subroutine INPUT1.
C
C  Output Files:
C    The output file RADOME2.APR will contain the effective
C    aperture distributions: the antenna element's
C    coordinates and the co- or cross- polarized amplitude
C    and phase (in degrees). See the header for routine
C    OUTPUT2 for a detailed description of the file format.
C    The file RADOME2.PAT will contain all four
C    radiation patterns: co- and cross-pol azimuth and
C    elevation patterns in dBi.
C    The files RADOME2.IPT and RADOME2.SNM will contain,
C    respectively, the x,y,z coordinates and the surface normal
C    vector components at the points where rays from the
C    antenna intersect the radome surface.
C    All four of the above output files, RADOME2.APR,
C    RADOME2.PAT, RADOME2.IPT and RADOME2.SNM will be over-
C    written if they already exist.
C
C  Subroutines:
C    CTAYLOR - Calculates a Taylor amplitude distribution for
C              a circular-aperture antenna
C    BESSELJ - Function subprogram for Bessel functions needed
C              for CTAYLOR
C    PATTERN - Calculates far field radiation pattern for an
C              array antenna
C    ANGLE   - Finds the polar angle of an x,y point
C    INPUT1  - Reads problem parameters
C    INPUT2  - Reads radome transmissivity file
```

```

C      TANGIVE - Calculates intersection point and surface
C              normal vector for a tangent ogive
C      RCCONE  - Calculates intersection point and surface
C              normal vector for a right circular cone
C      PHASE   - Finds phase angle of a complex number
C      ATANH   - Calculates hyperbolic arctangent (for CTAYLOR)
C-----
C-----

```

PROGRAM RADOME2

Variable Declarations

```

C-----
C      PARAMETER (MXEL=500,MXRW=41,MXFF=361,MXNTA=90)
C      MXEL = Maximum number of antenna array elements
C      MXRW = Maximum number of rows in the array
C      MXFF = Maximum number of far field pattern angles
C      MXNTA= Max number of angles in transmissivity tables
C      PARAMETER (PI=3.14159265359,DTR=PI/180.)
C      DTR = Degrees to radians conversion
C      PARAMETER (PT0=-90.,DT=1.0,NANG=181)
C      PT0 = Initial theta angle for pattern calculations
C      DT  = theta increment
C      NANG = number of pattern points
C      DIMENSION X(MXEL),Y(MXEL),Z(MXEL),R(MXEL),AMP(MXEL),PHS(MXEL)
C      X,Y,Z = Antenna element coordinates
C              (dimensions in wavelengths)
C      R = Radius to element from array center (wavelengths)
C      AMP,PHS = Scratch arrays for element amplitude and
C              phase for pattern calculation
C      COMPLEX A(MXEL),AC(MXEL),AX(MXEL)
C      A      = Array element excitation
C      AC,AX = Effective co-pol and cross-pol aperture
C              amplitudes accounting for radome effects
C      DIMENSION XIP(MXEL),YIP(MXEL),ZIP(MXEL),
+      XSN(MXEL),YSN(MXEL),ZSN(MXEL)
C      XIP,YIP,ZIP = Coordinates of the point where a ray
C              from the array element intercepts the radome
C      XSN,YSN,ZSN = Surface normal vector components at
C              the intercept point
C      DIMENSION FFCA(MXFF),FFCE(MXFF),FFCD(MXFF),
+      FFXA(MXFF),FFXE(MXFF),FFXD(MXFF)
C      FFCA = Co-polarized azimuth pattern in dBi
C      FFCE = Co-polarized elevation pattern
C      FFCD = Co-polarized diagonal plane pattern
C      FFXA = Cross-polarized azimuth pattern
C      FFXE = Cross-polarized elevation pattern
C      FFXD = Cross-polarized diagonal pattern
C      COMPLEX TCA(0:MXNTA),TXA(0:MXNTA)
C      TCA = Co-pol transmissivity amplitude (dB)
C      TXA = Cross-pol transmissivity amplitude (dB)
C      COMPLEX ETL,ETM
C      ETL,ETM are temporary variables used for
C      transmissivity calculations
C      INTEGER OPT(4),NBAR
C      OPT(1) = 0 for conical radome, 1 for tangent ogive
C              or 99 for no radome
C      OPT(2) = 0 for mechanical beam steering, 1 for
C              phase steering
C      OPT(3) = 0 to simulate sum channel patterns
C              = 1 for azimuth difference
C              = 2 for elevation difference
C      OPT(4) = 1 to save intermediate calculations in file
C              RADOME2.LOG
C      NBAR = Number of fixed-level sidelobes in antenna
C              pattern as determined by Taylor distribution

```

```

C      CHARACTER*20 FIL1
C      FIL1 = Input file for radome transmissivity
C      REAL RAD,TO,PO,BAS,HGT,SLL
C      RAD = Antenna radius in wavelengths
C      TO,PO = Spherical coordinate angles theta and phi
C              for scan direction (radome axis is theta=0)
C              in radians
C      BAS = Radome base radius in wavelengths
C      HGT = Radome height in wavelengths
C      DD = Array element spacing in wavelengths
C      SLL = Antenna design sidelobe level in dB
C              relative to main beam peak
C
C-----
C      Input and Array Element Calculations
C-----
C      Introduction message
C      WRITE(*,1)
1      FORMAT(//,1X,30('-'),/, ' ----- PROGRAM RADOME-2 -----',/,
+          1X,30('-'),//)
C
C      Get problem parameters
C
C      CALL INPUT1(RAD,TO,PO,FIL1,EAS,HGT,DD,SLL,NBAR,OPT)
C
C      Get radome transmissivity values from data file
C      and interpolate to create a lookup table
C
C      IF(OPT(1).NE.99) CALL INPUT2(FIL1,TCA,TXA)
C
C      Calculate the element locations for the unscanned antenna:
C
C      These are an equilateral-triangular lattice with
C      spacing DD in wavelengths between elements.
C      DD=.5/sin(60) is adequate to prevent grating lobes
C      for a phase-steered array, and DD=1/sin(60) is
C      adequate for a mechanically-steered array.
C
C      SIN60=SIN(PI/3.)
C      NEL=0
C      DY=DD*SIN60
C      XX=0
C      NX=RAD/DD+5
C      NY=NX/SIN60
C      DO 10 I=-NY,NY+1
C          XX=XX+.5
C          IF(XX.GE..75) XX=XX-1.
C          Y1=(I-.5)*DY
C          DO 10 K=-NX,NX
C              X1=(K+XX)*DD
C              RR=SQRT(X1**2+Y1**2)
C              IF(RR.GT.RAD) GO TO 10
C              NEL=NEL+1
C              IF(NEL.GT.MXEL) THEN
C                  WRITE(*,*) ' >>> Error: Too many array elements'
C                  GO TO 1111
C              END IF
C              X(NEL)=X1
C              Y(NEL)=Y1
C              Z(NEL)=0.0
C              R(NEL)=RR
10      CONTINUE
12      WRITE(*,12) NEL
C      FORMAT(' ..... Number of array elements= ',I4,' .....')

```

```

C Calculate the element amplitude weights
C
  DO 20 I=1,NEL
    NCALL=I-1
    RNORM=R(I)/RAD
    CALL CTAYLOR(SLL,RNORM,NBAR,GP,NCALL)
    A(I)=CMPLX(GP,0.)
C      Reverse amplitudes as appropriate for
C      azimuth or elevation difference
C      IF(OPT(3).EQ.1) THEN
C        IF(X(I).LT.0.) A(I)=CMPLX(-GP,0.)
C        IF(X(I).EQ.0.) A(I)=(0.,0.)
C      ELSE IF(OPT(3).EQ.2) THEN
C        IF(Y(I).LT.0.) A(I)=CMPLX(-GP,0.)
C        IF(Y(I).EQ.0.) A(I)=(0.,0.)
C      END IF
C      initialize co & cross pol distributions
    AC(I)=A(I)
    AX(I)=(0.0,0.0)
20  CONTINUE
C-----
C                               Antenna Scanning
C-----
C
C Calculate the array element phase shifts for electronic
C scanning to the spherical angle theta=T0, phi=P0.
C
C      Get sin, cos of scan angles
    ST0=SIN(T0)
    CT0=COS(T0)
    SP0=SIN(P0)
    CP0=COS(P0)
41  IF (OPT(2).EQ.0.OR.T0.EQ.0.) GO TO 65
    WK=2.*PI
    WKS=WK*ST0
    DO 50 I=1,NEL
      PHAS=WKS*(X(I)*CP0+Y(I)*SP0)
      AAMP=REAL(A(I))
      A(I)=CMPLX(AAMP*COS(PHAS),AAMP*SIN(PHAS))
50  CONTINUE
C      Begin recording to log file
65  IF(OPT(4).EQ.1) THEN
    OPEN(9,FILE='RADOME2.LOG')
    WRITE(9,91) RAD,SLL,NBAR,T0/DTR,P0/DTR,BAS,HGT,OPT
91  FORMAT(/,10X,' RADOME2 - LOG FILE ',/, ' ANTENNA ',
+ ' PARAMETERS:',/, ' APERTURE RADIUS =',F5.2,/, ' SIDELOBE ',
+ ' LEVEL = ',F4.0, ' dB      NBAR =',I2,/, ' SCAN ANGLES: ',
+ ' THETA =',F7.2, ' deg.    PHI =',F7.2, ' deg.',/, ' RADOME ',
+ ' PARAMETERS:',/, ' BASE RADIUS =',F5.2, ' HEIGHT =',F5.2,/,
+ ' OPTIONS:  #1  #2  #3  #4',/,12X,I2,3(3X,I2),/)
    END IF
    IF(OPT(1).EQ.99) GO TO 115
    WRITE(*,*) ' ..... Beginning Aperture Calculation ..... '
    WRITE(*,*)
C-----
C                               Projected Aperture Calculation
C      If there is no radome, this block is skipped
C-----
C Calculate the effective co- and cross-polarized aperture
C amplitude and phase distributions as follows:
C (1) Find intersection of ray from element I with radome
C (2) Find radome surface normal at intersection point
C (3) Interpolate from radome transmissivity table to
C      obtain para. & perp. pol. transm. coefficients
C (4) Apply coefficients to decomposed incident wave

```

```

C      polarization, then recombine
C
DO 100 I=1,NEL
  WRITE(*,'(1H+,5X,A,I4)') ' Element number: ',I
  In the case of mechanical antenna scanning,
  calculate the coordinates of the gimballed
  antenna array element
  IF(OPT(2).EQ.0.AND.TO.NE.0.) THEN
    skip this calculation if the scan angle TO=0
    XR=X(I)*(CTO*CPO**2+SPO**2)+Y(I)*CPO*SPO*(1-CTO)
    YR=X(I)*CPO*SPO*(1-CTO)+Y(I)*(CTO*SPO**2+CPO**2)
    ZR=-X(I)*STO*CPO-Y(I)*STO*SPO
  ELSE
    XR=X(I)
    YR=Y(I)
    ZR=Z(I)
  END IF
  Find coordinates of aperture points
61  XA=XR+HGT*STO*CPO
    YA=YR+HGT*STO*SPO
    ZA=ZR+HGT*CTO
  Find intersect points and surface normals
  IF(OPT(1).EQ.1) THEN
    CALL TANGIVE(XR,YR,ZR,XA,YA,ZA,BAS,HGT,
+      XIP(I),YIP(I),ZIP(I),XSN(I),YSN(I),ZSN(I))
  ELSE
    CALL RCCONE(XR,YR,ZR,XA,YA,ZA,BAS,HGT,
+      XIP(I),YIP(I),ZIP(I),XSN(I),YSN(I),ZSN(I))
  END IF
  IF(OPT(4).EQ.1) THEN
    WRITE(9,64) I,XIP(I),YIP(I),ZIP(I),XSN(I),YSN(I),ZSN(I)
64  FORMAT(' POINT #',I3,' INTERSECT:',3F7.3,/,
+      ' SURFACE NORMAL:',3F7.3)
  END IF
  Calculate incident polarization ("e") vector
  Note that the antenna is assumed to be y-pol-
  arized before scanning. It remains so if the
  steering is electronic.
  IF(OPT(2).EQ.0) THEN
    XP=CPO*SPO*(1-CTO)
    YP=CTO*SPO**2+CPO**2
    ZP=-STO*SPO
  ELSE
    XP=0.
    YP=1.0
    ZP=0.
  END IF
  Calculate incidence angle: the normalized
  dot product of the ray direction with the
  surface normal is the cosine of the
  incidence angle
  XRD,YRD,ZRD are the "k" vector
  RDMG=SQRT((XA-XR)**2+(YA-YR)**2+(ZA-ZR)**2)
  XRD=(XA-XR)/RDMG
  YRD=(YA-YR)/RDMG
  ZRD=(ZA-ZR)/RDMG
  AINC=ACOS(ABS(XSN(I)*XRD+YSN(I)*YRD+ZSN(I)*ZRD))*180./PI
  IINC=AINC
  IF(IINC.GT.90) IINC=180-IINC
  IF(OPT(4).EQ.1) WRITE(9,*) ' INCIDENCE ANGLE:',AINC
  Find cross product e x k
  XEK=(ZP*YRD-YP*ZRD)
  YEK=(XP*ZRD-ZP*XRD)
  ZEK=(YP*XRD-XP*YRD)
  Find vector normal to plane of incidence

```

```

C      (The notation convention uses last letter L or M
C      to denote perpendicular and parallel, respectively)
XUL=(ZSN(I)*YRD-YSN(I)*ZRD)
YUL=(XSN(I)*ZRD-ZSN(I)*XRD)
ZUL=(YSN(I)*XRD-XSN(I)*YRD)
C      and normalize it (this is "u-perp")
ULM=SQRT(XUL**2+YUL**2+ZUL**2)
XUL=XUL/ULM
YUL=YUL/ULM
ZUL=ZUL/ULM
C      Find vector in plane of incidence ("u-para")
C      (This one doesn't need normalization because it
C      is the cross product of unit vectors, and so it
C      must also be a unit vector)
XUM=(YUL*ZRD-ZUL*YRD)
YUM=(ZUL*XRD-XUL*ZRD)
ZUM=(XUL*YRD-YUL*XRD)
IF(OPT(4).EQ.1) THEN
  WRITE(9,66) XEK,YEK,ZEK,XUL,YUL,ZUL,XUM,YUM,ZUM
66  FORMAT('          E-CROSS-K:',3F7.3,/,
+         '          U-PERP:',3F7.3,/,
+         '          U-PARA:',3F7.3)
  END IF
C      Decompose incident wave into parallel and
C      perpendicular components
EIL=(XP*XUL+YP*YUL+ZP*ZUL)
EIM=(XP*XUM+YP*YUM+ZP*ZUM)
HIL=(XEK*XUL+YEK*YUL+ZEK*ZUL)
HIM=(XEK*XUM+YEK*YUM+ZEK*ZUM)
C      Calculate transmitted components
ETL=TXA(IINC)*EIL
ETM=TCA(IINC)*EIM
C      AC(I) represents the dot product of the transmitted
C      E field vector with the unit vector in the incident
C      E field direction
AC(I)=A(I)*(ETL*EIL+ETM*EIM)
C      AX(I) is the dot product of the transmitted field
C      E field vector with the unit vector in the incident
C      H field direction
AX(I)=A(I)*(ETL*HIL+ETM*HIM)
IF(OPT(4).EQ.1) THEN
  WRITE(9,71) EIM,EIL,HIM,HIL,TCA(IINC),TXA(IINC)
71  FORMAT(' Incident E components (para,perp):',2F7.4,/,
+         ' Incident H components (para,perp):',2F7.4,/,
+         ' Co-pol trans. coeff. (real,imag):',2F7.4,/,
+         ' X-pol trans. coeff. (real,imag):',2F7.4,/)
  END IF
100  CONTINUE
-----
C      Radiation Patterns
-----
115  WRITE(*,*) ' ..... Calculating Patterns ..... '
C
C      The pattern cuts must be made relative to the main
C      beam position. In the phase scanning case, it is
C      necessary to rotate the antenna by T0 in the (P0+180)
C      direction in order to bring the main beam back to the
C      theta,phi=0,0 direction:
IF(T0.EQ.0..OR.OPT(2).EQ.0) GO TO 119
C      Skip this if the scan direction is zero
C      or if the antenna is mechanically scanned
DO 117 I=1,NEL
  XR=X(I)*(CT0*CP0**2+SP0**2)+Y(I)*CP0*SP0*(1-CT0)
  YR=X(I)*CP0*SP0*(1-CT0)+Y(I)*(CT0*SP0**2+CP0**2)
  ZR=X(I)*ST0*CP0+Y(I)*ST0*SP0

```

```

      X(I)=XR
      Y(I)=YR
      Z(I)=ZR
117  CONTINUE
C    Co-pol patterns
119  DO 120 I=1,NEL
      AMP(I)=CABS(AC(I))
      PHS(I)=PHASE(AC(I))*180./PI
120  CONTINUE
      CALL PATTERN(X,Y,Z,AMP,PHS,NEL,PTO,DT,NANG, 0.0,FFCA,1.0,1)
      CALL PATTERN(X,Y,Z,AMP,PHS,NEL,PTO,DT,NANG,45.0,FFCD,1.0,1)
      CALL PATTERN(X,Y,Z,AMP,PHS,NEL,PTO,DT,NANG,90.0,FFCE,1.0,1)
C    Cross-pol patterns
      IF(OPT(1).EQ.99) GO TO 132
      AMPMX=0.
      DO 130 I=1,NEL
        AMP(I)=CABS(AX(I))
        IF(AMP(I).GT.AMPMX) AMPMX=AMP(I)
        PHS(I)=PHASE(AX(I))*180./PI
130  CONTINUE
      IF(AMPMX.EQ.0) THEN
        WRITE(*,*) ' !!! The cross-pol amplitude is zero; cross- '
        WRITE(*,*) '      pol patterns will not be calculated.'
      END IF
C      If the cross-pol amplitude is zero or if there is
C      no radome, then there is no cross polarized pattern
132  IF(AMPMX.EQ.0.OR.OPT(1).EQ.99) THEN
      DO 135 I=1,NANG
        FFXA(I)=0.0
        FFXD(I)=0.0
        FFXE(I)=0.0
135  CONTINUE
      ELSE
        CALL PATTERN(X,Y,Z,AMP,PHS,NEL,PTO,DT,NANG, 0.0,FFXA,1.0,1)
        CALL PATTERN(X,Y,Z,AMP,PHS,NEL,PTO,DT,NANG,45.0,FFXD,1.0,1)
        CALL PATTERN(X,Y,Z,AMP,PHS,NEL,PTO,DT,NANG,90.0,FFXE,1.0,1)
      END IF
C-----
C                                Output
C-----
C
      IF(OPT(4).EQ.1) CLOSE(9)
C  Intersection points
      OPEN(11,FILE='RADOME2.IPT')
      DO 210 I=1,NEL
        WRITE(11,211) I,XIP(I),YIP(I),ZIP(I)
210  CONTINUE
211  FORMAT(2X,I4,3(2X,F10.4))
      CLOSE(11)
C
C  Surface normal components
      OPEN(12,FILE='RADOME2.SNM')
      DO 220 I=1,NEL
        WRITE(12,213) I,X(I),Y(I),XSN(I),YSN(I),ZSN(I)
213  FORMAT(2X,I4,5(2X,F10.4))
220  CONTINUE
      CLOSE(12)
C
C  Aperture distributions
      OPEN(13,FILE='RADOME2.APR')
      DO 230 I=1,NEL
        IF(OPT(1).EQ.99) AX(I)=(0.0,0.0)
        WRITE(13,231) I,X(I),Y(I),CABS(AX(I)),PHASE(AX(I)),
+ CABS(AC(I)),PHASE(AC(I))
230  CONTINUE

```

```

231    FORMAT(2X,I4,6(2X,F10.5))
      CLOSE(13)
C
C  Far field patterns
      OPEN(14,FILE='RADOME2.PAT')
      DO 240 I=1,NANG
        II=I-91
        WRITE(14,241) II,FFCA(I),FFXA(I),FFCD(I),
+          FFXD(I),FFCE(I),FFXE(I)
240    CONTINUE
241    FORMAT(2X,I4,6(2X,F10.5))
      CLOSE(14)
1111   STOP
      END
C-----
C                               End of main program RADOME2
C-----

C=====
C                               SUBROUTINE INPUT1
C  This routine handles all input and error checking of inputs.  It
C  offers the user the option of entering all parameters from the
C  keyboard, or from the data file RADOME2.INP.
C  The input file format is as follows:
C  Comment lines: the file may contain a header with any number of
C  comment lines, the last of which contains a # in column 1.
C  Data records are in free format, containing the following:
C  1:  OPT(1)          (Radome type option)
C  2:  RAD,DD,SLL,NBAR (Antenna radius, element spacing,
C                      sidelobe level and Taylor NBAR parameter)
C  3:  T0,PO          (Scan angles theta,phi)
C  4:  OPT(2),OPT(3),OPT(4) (Options)
C  5:  FIL1          (Radome transmissivity data file if OPT1<>99)
C=====

      SUBROUTINE INPUT1(RAD,T0,PO,FIL1,BAS,HGT,DD,SLL,NBAR,OPT)
      INTEGER OPT(4)
      CHARACTER*20 FIL1
      CHARACTER*1 CH1
      PI=3.141592 359
      WRITE(*, '(A\)' ) ' Enter 0 for keyboard input, 1 for data file:'
      READ(*,*) IKEY
      IF(IKEY.EQ.1) GO TO 51
      WRITE(*,20)
20    +   FORMAT(' Choose the radome type:',/, ' 0 for cone,'
+     ' 1 for ogive or 99 for no radome')
      READ(*,*) OPT(1)
      IF(OPT(1).NE.99) THEN
        WRITE(*,21)
21    +   FORMAT(1X, ' Enter the following radome parameters:',/,
+     '   BAS = base radius in wavelengths',/, '   HGT = height',
+     '   in wavelengths')
        WRITE(*, '(A\)' ) ' Enter BAS,HGT'
        READ(*,*) BAS,HGT
      END IF
      WRITE(*,22)
22    +   FORMAT(1X, ' Enter the following antenna parameters:',/,
+     '   RAD = aperture radius in wavelengths',/,
+     '   DD = interelement spacing in wavelengths',/,
+     '   SLL = peak sidelobe level, e.g. 40',/,
+     '   NBAR = Taylor parameter',/, '   T0,PO = ',
+     '   theta and phi scan angles (deg)')

      WRITE(*, '(A\)' ) ' Enter RAD,DD,SLL,NBAR,T0,PO: '

```

```

      READ(*,*) RAD,DD,SLL,NBAR,TO,PO
      TO=TO*PI/180.
      PO=PO*PI/180.
      WRITE(*,23)
23    FORMAT(1X,' Choose the following options:',/,
+      ' OPT2 = 0 for mechanical scanning, or 1 for electronic',/,
+      ' OPT3 = 0 for sum patterns, =1 for az. diff., =2',
+      ' for el. diff.',/, ' OPT4 = 1 to save intermediate',
+      ' calculations in a log file')
      WRITE(*,'(A)') ' Enter OPT2, OPT3, OPT4: '
      READ(*,*) OPT(2),OPT(3),OPT(4)
      IF(OPT(1).NE.99) THEN
        WRITE(*,*) ' What is the name of the data file containing'
        WRITE(*,'(A)') ' the transmissivity data? '
        READ(*,'(A)') FIL1
      END IF
      RETURN
C----- This block for file input -----
51    OPEN(4,FILE='RADOME2.INP',STATUS='OLD')
C      Bypass comment lines (if any) in input file
55    READ(4,'(A1)') CH1
      IF(CH1.NE.'#') GO TO 55
      READ(4,*) OPT(1)
      IF(OPT(1).NE.99) READ(4,*) BAS,HGT
      READ(4,*) RAD,DD,SLL,NBAR
      READ(4,*) TO,PO
      TO=TO*PI/180.
      PO=PO*PI/180.
      READ(4,*) OPT(2),OPT(3),OPT(4)
      IF(OPT(1).NE.99) READ(4,'(A)') FIL1
      RETURN
END

C=====
C      SUBROUTINE INPUT2
C      This subroutine reads a data file containing radome surface
C      transmissivity information and uses interpolation to create a
C      lookup table.
C      The input data file may contain any number of comment lines
C      preceding the actual data. The symbol # must be in the first
C      column of the last comment line (which may or may not contain
C      any comment). The first data record is the number of angles for
C      which data is available, NANG. The next NANG lines contain the
C      incidence angle in degrees, the co-pol transmissivity in dB,
C      the co-pol transmission phase in deg, the cross-pol transmissivity
C      in dB, and the cross-pol transmission phase in deg. Zero degrees
C      incidence is the surface normal direction. It is assumed that
C      the radome material is isotropic, that is it has the same value
C      of transmissivity at the angle theta in any phi plane.
C      Three-point Lagrange interpolation is used to fill in the
C      lookup table entries between 0 and 90 degrees. 0 degrees is
C      normal incidence. Note that Lagrange interpolation can also be
C      used to extrapolate. If the data file does not contain values
C      all the way out to 90 degrees from normal, the table values will
C      be extrapolated out to 90 degrees. Unfortunately, this extrapo-
C      lation may not be accurate, especially if the last angle given is
C      much less than 90 degrees. Note also that the phases in the input
C      data file must be continuous, without "wrapping" at +/-180.
C      The output data is complex values ("gamma").
C=====

      SUBROUTINE INPUT2(FIL1,TC,TX)
      COMPLEX TC(91),TX(91)
C      TCA,TXA = co- and cross-pol transmission coefficients
C      (sometimes denoted gamma-para/perp)

```

```

      REAL DCA(91),DCP(91),DXA(91),DXP(91),DANG(91)
C      DCA,DCP,DXA,DXP are scratch arrays for reading the
C      values from the data file. DANG is an array with
C      the angle values.
      CHARACTER*20 FIL1
      CHARACTER*1 CH1
C      Read the data file
      OPEN(10,FILE=FIL1,STATUS='OLD')
C      Bypass comment lines
      READ(10,'(A1)') CH1
      IF(CH1.NE.'#') GO TO 5
C      Get the number of data points
      READ(10,*) NANG
C      Read the data
      DO 10 I=1,NANG
        READ(10,*) DANG(I),DCA(I),DCP(I),DXA(I),DXP(I)
C      Convert amplitudes from dB to volts
        DCA(I)=10.**(DCA(I)/20.)
        DXA(I)=10.**(DXA(I)/20.)
10    CONTINUE
      CLOSE(10)
C      Begin interpolation
      N=3
      DO 20 I=0,90
        J=I+1
        X=I
        IF(X.GT.DANG(N).AND.N.LT.NANG) N=N+1
        X0=DANG(N-2)
        X1=DANG(N-1)
        X2=DANG(N)
        S0=(X-X1)*(X-X2)/(X0-X1)/(X0-X2)
        S1=(X-X0)*(X-X2)/(X1-X0)/(X1-X2)
        S2=(X-X0)*(X-X1)/(X2-X0)/(X2-X1)

        TCA=DCA(N-2)*S0+DCA(N-1)*S1+DCA(N)*S2
        TCP=DCP(N-2)*S0+DCP(N-1)*S1+DCP(N)*S2
        TXA=DXA(N-2)*S0+DXA(N-1)*S1+DXA(N)*S2
        TXP=DXP(N-2)*S0+DXP(N-1)*S1+DXP(N)*S2
C      Convert phase to radians
        DTR=3.141592 359/180.
        TCP=TCP*DTR
        TXP=TXP*DTR
C      Save as complex
        TC(J)=CMPLX(TCA*COS(TCP),TCA*SIN(TCP))
        TX(J)=CMPLX(TXA*COS(TXP),TXA*SIN(TXP))
20    CONTINUE
      RETURN
      END

C=====
C      SUBROUTINE TANGIVE
C      This subroutine finds the point of intersection of a ray
C      with a tangent ogive radome and the surface normal vector at the
C      point of intersection.
C      The inputs are X0,Y0,Z0, the coordinates of the ray's initial
C      point, XA,YA,ZA, the coordinates of the ray's destination point
C      on the effective aperture outside the radome, and the radome base
C      (BB) and height (H).
C      The point where the ray intersects the radome is XI,YI,ZI.
C      The components of the surface normal vector are XSN,YSN,ZSN.
C      All dimensions are in wavelengths.
C      Newton's method is used to solve for the intersection point
C      to an accuracy specified by the variable TOL.
C=====

```

```

SUBROUTINE TANGIVE(XO,YO,ZO,XA,YA,ZA,BB,H,XI,YI,ZI,XSN,YSN,ZSN)
PARAMETER(TOL=1.0E-5,MXITER=50)
C      No more than MXITER iterations will be performed
C      to attempt to find t such that g(t)<=TOL
R=(H**2+BB**2)/2./BB
RSQ=R**2
AA=R-BB
C
DX=XA-XO
DY=YA-YO
DZ=ZA-ZO
DXSQ=DX**2
DYSQ=DY**2
DZSQ=DZ**2
C      Calculate coefficients of parametric equation
C      g(t)= At^2+Bt+C+ 2a*SQRT(Dt^2+Et+F)
A=DXSQ+DYSQ+DZSQ
B=2*(XO*DX+YO*DY+ZO*DZ)
C=XO**2+YO**2+ZO**2-RSQ-AA**2
D=-DZSQ
E=-2*ZO*DZ
F=RSQ-ZO**2
C
C      Solve for intersection using Newton's method
C
T=.5
DO 10 I=1,MXITER
  TSQ=T**2
  GG=SQRT(D*TSQ+E*T+F)
  GT=A*TSQ+B*T+C+2.*AA*GG
C      GT is the function g(t)
  GP=2*A*T+B+AA*(2*D*T+E)/GG
C      GP is its first derivative, g'(t)
  TM=T
  T=TM-GT/GP
  IF(GT.LT.TOL) GO TO 20
10 CONTINUE
C
C      Calculate intersection coordinates
C
20 XI=XO+T*(XA-XO)
  YI=YO+T*(YA-YO)
  ZI=ZO+T*(ZA-ZO)
C      Some quantities become undefined if the ray passes
C      directly through the tip, so to avoid that situation
C      a small increment will be added to XI,YI
  IF(XI.EQ.0..AND.YI.EQ.0.) THEN
    XI=XI+.0001
    YI=YI+.0001
  END IF
C      Find surface normal vector and normalize it; note that
C      it is the INWARD norward, hence the sign changes
  XSN=-2*XI
  YSN=-2*YI
  ZSN=-2*ZI*(1.-2*AA/SQRT(RSQ-ZI**2))
  SNM=SQRT(XSN**2+YSN**2+ZSN**2)
  XSN=XSN/SNM
  YSN=YSN/SNM
  ZSN=ZSN/SNM
  RETURN
END

```

```

C=====
C               SUBROUTINE RCCONE
C   This subroutine finds the point of intersection of a ray
C   with a right-circular cone radome, and the surface normal vector
C   at the point of intersection.
C   The inputs are X0,Y0,Z0, the coordinates of the ray's initial
C   point, XA,YA,ZA, the coordinates of the ray's destination point
C   on the effective aperture outside the radome, and the radome base
C   (BB) and height (H).
C   The point where the ray intersects the radome is XI,YI,ZI.
C   The components of the surface normal vector are XSN,YSN,ZSN.
C   All dimensions are in wavelengths.
C=====
      SUBROUTINE RCCONE(X0,Y0,Z0,XA,YA,ZA,BB,H,XI,YI,ZI,XSN,YSN,ZSN)
      BSQ=BB**2
      HSQ=H**2
      DX=XA-X0
      DY=YA-Y0
      DZ=ZA-Z0
      DXSQ=DX**2
      DYSQ=DY**2
      DZSQ=DZ**2
C       Calculate quadratic coefficients
      A=DXSQ+DYSQ+DZSQ*BSQ/HSQ
      B=2*(DX*X0+DY*Y0+DZ*(H-Z0)*BSQ/HSQ)
      C=X0**2+Y0**2-(H-Z0)**2*BSQ/HSQ
C       Solve quadratic for intersection point T; the solution
C       chosen is the one inside  ZA > z > Z0
      S=SQRT(B**2-4.*A*C)
      T1=(-B+S)/2/A
      T2=(-B-S)/2/A
      Z1=Z0+T1*DZ
      IF(T1.GE.0..AND.T1.LE.1.) THEN
        T=T1
      ELSE
        T=T2
      END IF
C       Calculate intersection coordinates
20     XI=X0+T*DX
      YI=Y0+T*DY
      ZI=Z0+T*DZ
C       Some quantities become undefined if the ray passes
C       directly through the tip, so to avoid that situation
C       a small increment will be added to XI,YI
      IF(XI.EQ.0..AND.YI.EQ.0.) THEN
        XI=XI+.0001
        YI=YI+.0001
      END IF
C       Find surface normal vector and normalize it; the sign
C       is changed because it is to be the INWARD normal
      XSN=-2*XI
      YSN=-2*YI
      ZSN=-2*BSQ*(1.-ZI/H)/H
C       normalization
      SNM=SQRT(XSN**2+YSN**2+ZSN**2)
      XSN=XSN/SNM
      YSN=YSN/SNM
      ZSN=ZSN/SNM
      RETURN
      END

```

```

C=====
C ROUTINE PATTERN : DAN MCGRATH : SEP 87 : MS FORTRAN
C This subroutine calculates the radiation pattern of an array
C antenna whose elements have arbitrary locations, amplitudes and
C phases.
C Inputs: X,Y,Z = Arrays containing the coordinates of the antenna
C             elements (in wavelengths)
C           N     = Number of antenna elements
C           A     = Array containing element amplitudes (voltage)
C           P     = Array containing element phases (deg)
C           PHI   = The spherical coordinate phi angle for the
C                   pattern cut
C           DT    = The angle increment in degrees
C           TO    = Initial theta angle in degrees
C           M     = Number of pattern points to be computed
C           EPE   = Element pattern factor: antenna elements are
C                   assumed to have a cos^EPE factor (0 for isotropic)
C           NORM  = Normalization option: If NORM=0 pattern will be
C                   normalized so that its peak value is 0 dB. If
C                   NORM=1 pattern is normalized to N times a
C                   unit amplitude (dBi pattern).
C=====
C SUBROUTINE PATTERN(X,Y,Z,A,P,N,TO,DT,M,PHI,FFP,EPE,NORM)
C DIMENSION X(N),Y(N),Z(N),A(N),P(N),FFP(M)
C PARAMETER (PI=3.141592 359,DTR=PI/180.,FLOOR=-100)
C FLOOR is the minimum dB level - calculated values
C that are less than FLOOR will be set to FLOOR
C SP=SIN(PHI*DTR)
C CP=COS(PHI*DTR)
C FMX=0.
C AMX=0.
C WRITE(*,*)
C Find Maximum Element Amplitude
C DO 10 J=1,N
10 IF(ABS(A(J)).GT.AMX) AMX=ABS(A(J))
C Calculate the pattern at M angles
C DO 100 I=1,M
C TH=(TO+DT*(I-1))*DTR
C ST=SIN(TH)
C CT=COS(TH)
C evaluate element pattern factor
C IF(EPE.EQ.0.) THEN
C EP=1.0
C ELSE
C IF(CT.EQ.0.) GO TO 55
C IF(CT.NE.0.) EP=ABS(CT)**EPE
C END IF
C EX1=ST*CP*2.*PI
C EX2=ST*SP*2.*PI
C EX3=CT*2.*PI
C SR=0.0
C SI=0.0
C .....for each angle, sum over the array.....
C DO 50 J=1,N
C IF(A(J).EQ.0) GO TO 50
C AMP=A(J)*EP
C ARG=P(J)*DTR-X(J)*EX1-Y(J)*EX2-Z(J)*EX3
C SR=SR+AMP*COS(ARG)
C SI=SI+AMP*SIN(ARG)
50 CONTINUE
55 FFP(I)=SR**2+SI**2
C NAL=M-I
C WRITE(*, '(1H+,5X,I4,A)') NAL, ' POINTS LEFT'
100 IF(FFP(I).GT.FMX) FMX=FFP(I)

```

```

C      Convert to dB and normalize
      IF(NORM.NE.0.AND.NORM.NE.1) FMX=1.0
      IF(NORM.EQ.1) FMX=FLOAT(N)**2
      IF(FMX.EQ.0.) FMX=1.0E-10
      DO 150 I=1,M
C          set minimum power to prevent overflow
          IF(FFP(I).LT.1.0E-10) FFP(I)=1.0E-10
          FFP(I)=10.*ALOG10(FFP(I)/FMX)
          IF(FFP(I).LT.FLOOR) FFP(I)=FLOOR
150      CONTINUE
      RETURN
      END

```

```

C=====
C  Function PHASE finds the angle argument of a complex number,
C  defined as follows:
C      Real,Imaginary      Definition
C      0      <0      -pi/2
C      0      >0      pi/2
C      <0      0      -pi
C      >=0      0      0
C      >0      /=0      arctan(im/re)
C      <0      /=0      arctan(im/re)+pi
C=====

```

```

      FUNCTION PHASE(CC)
      COMPLEX CC
      PI=3.141592 359
      CR=REAL(CC)
      CI=AIMAG(CC)
      IF(CR.EQ.0.) THEN
          IF(CI.EQ.0.) THEN
              PHASE=0.
          ELSE
              IF(CI.GT.0.) PHASE=PI/2.
              IF(CI.LT.0.) PHASE=-PI/2.
          END IF
      ELSE
          ARG=CI/CR
          PHASE=ATAN(ARG)
          IF(CR.LT.0.) THEN
              IF(PHASE.LE.0.) THEN
                  PHASE=PHASE+PI
              ELSE
                  PHASE=PHASE-PI
              END IF
          END IF
      END IF
      RETURN
      END

```

```

C=====
C==      SUBROUTINE CTAYLOR      May 1984      FORTRAN      ==
C==      by Dan McGrath, Rome Air Development Center (RADC/EEA)      ==
C==      This subroutine calculates the coefficients for a Taylor      ==
C==      amplitude distribution for a circular aperture array antenna.      ==
C==      The Taylor distribution holds a number of sidelobes nearest      ==
C==      the main beam to a specified level, and the remainder roll off      ==
C==      gradually.  NBAR is the number of fixed-level sidelobes, and      ==
C==      their amplitude relative to the main beam peak is specified by      ==
C==      the variable SLLDB, the sidelobe level in dB, e.g. SLLDB=40      ==
C==      for first NBAR sidelobes 40 dB lower in relative power than      ==
C==      the main beam.  SLLDB must be a positive number!  NBAR must      ==
C==      be less than 10.      ==
C==      The subroutine must be called repeatedly, once for each      ==
C==      array element.  R is the element's radius normalized to the      ==
C==      aperture radius.  The calculated amplitude weight is returned      ==
C==      in the variable GP.  NCALL is a flag used to prevent repeated      ==

```

```

C== overhead calculations: it must be zero on the first call, and ==
C== should be nonzero on successive calls. ==
C== This routine uses the function subprogram BESSELJ. ==
C== Reference: "Microwave Scanning Antennas," Vol. I, ==
C== by R.C. Hansen, Academic Press, 1966. ==
C=====

```

```

SUBROUTINE CTAYLOR(SLLDB,R,NBAR,GP,NCALL)
DIMENSION RJOUM(0:9),UN(0:9),FUM(0:9)
DOUBLE PRECISION BESSELJ
IF(NCALL.NE.0) GO TO 50
C      The following setup block only needs
C      to be executed on the first call
C
C      UN's are the roots of J1(pi*u)
DATA(UN(L),L=0,9)/0.0,1.2196699,2.2331306,3.2383154,
+ 4.2410628,5.2427643,6.2439216,7.2447598,8.2453948,9.2458927/
PI=3.141592 359
ETA=10.0**(SLLDB/20)
DO 10 I=0,9
  X=PI*UN(I)
10  RJOUM(I)=BESSELJ(0,X)
  FUM(0)=1.0
  A=ATANH(SQRT(1.-1./ETA**2))/PI
  SIGMSQ=UN(NBAR)**2/(A**2+(NBAR-.5)**2)
  DO 30 M=1,9
    PRNUM=1.0
    PRDEN=1.0
    DO 20 N=1,NBAR-1
C      The denominator product excludes the N=M term
      IF(N.NE.M) PRDEN=PRDEN*(1.-UN(M)**2/UN(N)**2)
      PRNUM=PRNUM*(1.-UN(M)**2/(SIGMSQ*(A**2+(N-.5)**2)))
20  CONTINUE
    FUM(M)=-RJOUM(M)*PRNUM/PRDEN
30  CONTINUE
C      After the first call, calculations may start here
50  SUM=0.0
  P=R*PI
  DO 60 M=0,NBAR-1
    SDEN=RJOUM(M)**2
    X=P*UN(M)
    SNUM=FUM(M)*BESSELJ(0,X)
    SUM=SUM+SNUM/SDEN
60  CONTINUE
  GP=2.0*SUM/PI**2
  RETURN
END

```

```

C=====
C== FUNCTION BESSELJ Jun 83-Aug 84 FORTRAN4/5 ==
C== by Boris Tomasic and Dan McGrath ==
C== Rome Air Development Center, RADC/EEA ==
C== This function subprogram finds the value of the ==
C== Bessel function of the first kind, JN(X) of order N ==
C== and argument X. ==
C== 0 <= N <= 50 ; 0.0 <= X <= 100.0 ==
C=====

```

```

DOUBLE PRECISION FUNCTION BESSELJ (N,X)
DOUBLE PRECISION FACT(300),TRM1,TRM2,S,SUM,CC

  IF (X.GT.10.0) GO TO 100
  IF(X.EQ.0.0.AND.N.EQ.0) SUM=1.0
  IF(X.EQ.0.0.AND.N.EQ.0) GO TO 60
C      For small arguments, ascending series formula is used

```

```

FACT(1)=1.0
DO 10 I=2,N+41
10 FACT(I)=FACT(I-1)*FLOAT(I-1)
   SUM=(X/2.)**N/FACT(N+1)
   DO 50 K=1,40
   TRM1=(X/2.)**(N+K)/FACT(K+1)
   TRM2=(X/2.)**K/FACT(N+K+1)
   S=TRM1*TRM2*(-1)**K
   SUM=SUM+S
50 IF(DABS(S).LT.1.0E-10.AND.I.GT.15) GO TO 60
60 BESSELJ=SUM
   RETURN
C   For arguments larger than 10. a recursive
C   difference equation solution is used. Ref.
C   Abramowitz & Stegun pg. 385
100 CONTINUE
   IF(X.LT.40) L=INT(16.1*(1.+X**2*4.6875E-5)*SQRT(X)+30)
   IF(X.GE.40) L=INT(1.5*X+83.)
   L1=L+1
   FACT(L1)=0.
C   In this block FACT does not represent factorials
   FACT(L)=1.0D-60
   DO 130 I=2,L
   II=L-I+1
130 FACT(II)=2.*FLOAT(II)/X*FACT(II+1)-FACT(II+2)
   SUM=0.
   DO 140 II=3,L,2
140 SUM=SUM+FACT(II)
   CC=1./(FACT(1)+2.*SUM)
   BESSELJ=CC*FACT(N+1)
   RETURN
END

C=====
C==                FUNCTION ATANH                ==
C==      This is the hyperbolic arctangent function. It is ==
C== calculated using the identity ==
C==      ATANH(x) = .5 * ln[(1+x)/(1-x)] ==
C== The argument is restricted to |x| < 1 ==
C=====
      FUNCTION ATANH(X)
      R=(1+X)/(1-X)
      ATANH=.5*ALOG(R)
      RETURN
      END

```