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Technical Memorandum

UNWRAPPING TWO-DIMENSIONAL PHASE DATA IN CONTOUR MAPS

WILLIAM H. CARTER

THE JOHNS HOPKINS UNIVERSITY ■ APPLIED PHYSICS LABORATORY

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A computer algorithm has been discovered for unwrapping two-dimensional phase data and producing perfect contour maps without holes or dark bands of erroneous contour lines. The method employs local, temporary unwrapping within single grid squares of rectangular sampled data and the addition of extra contour levels outside the 0 to 2 π range to fill holes.

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ABSTRACT

A computer algorithm has been discovered for unwrapping two-dimensional phase data and producing perfect contour maps without holes or dark bands of erroneous contour lines. The method employs local, temporary unwrapping within single grid squares of rectangular sampled data and the addition of extra contour levels outside the 0 to 2 π range to fill holes.

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1. INTRODUCTION

In optics, electromagnetic theory, digital image processing, and many other areas of science and technology, it is often necessary to study two-dimensional phase data. To visualize such data, contour plots are usually used.[1-5] However, because phase data are usually only known modulo 2π, straightforward contour mapping of these data by any conventional computer contour-mapping program will give rise to dark bands of contour lines where the phase data have 2π phase jumps, as shown in Figure 1. (Ref. 1 [Figs. 3, 5, and 7] and Ref. 2 [Figs. 2, 4, 6, and 8] also show examples of dark bands in contour maps of phase data.) These bands are unattractive; worse, they obscure the data. Removal of the dark bands without redrawing the maps by hand is not straightforward, because two-dimensional phase data can not generally be unwrapped. A method that uses local unwrapping to get around this problem and draw perfect contour maps entirely by computer is presented in this report.

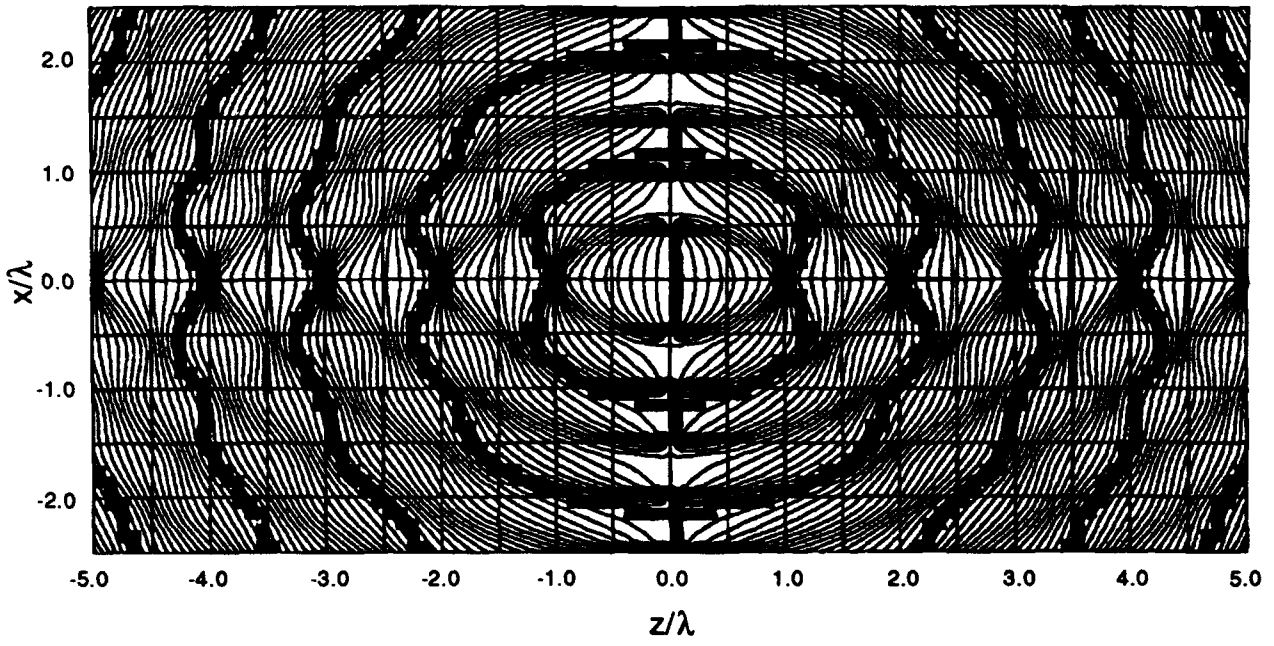

Figure 1 Contour map of the phase of the spherical scalar wave field over the region near the origin. (See Ref. 2 [Eq. 10], with a = 1, for a precise definition of this field). With the phase represented only by values in the range from 0 to 2π (modulo 2π), as in this figure, all of the contour lines with levels from 0 to 2π are drawn between neighboring data samples bounding the phase jumps. This gives rise to confusing heavy dark bands. The contour lines in this figure are 20° apart.

[1]W. H. Carter, "Band-limited Angular Spectrum Approximation to a Scalar Dipole Field," *Opt. Commun.* 2, 142-148, Figures 3, 5, and 7 (1970).

[2]W. H. Carter, "Anomalies in the Field of a Gaussian Beam Near Focus," *Opt. Commun.* 7, 211-218, Figures 2, 4, 6, and 8 (1973).

[3]W. H. Carter, "Band-Limited Angular-Spectrum Approximation to a Spherical Scalar Wave Field," *J. Opt. Soc. Am.* 65, 1054-1058, Figures 2 and 5 (1975).

[4]W. H. Carter, and T. J. Wieting, "The Field Distribution of a Focused Gaussian Beam Reflected at 45° from a Conducting Plane and Its Effects in Plasma-Ignition Experiments," *J. Appl. Phys.* 54, 519-523, Figure 6 (1983).

[5]W. H. Carter, A. Gamliel, and E. Wolf, "Coherence Properties of Fields Generated by a Lambertian Source," Annual Optical Society of America Meeting, Boston, Mass. (Nov. 1990).

2. UNWRAPPING BY HAND

The dark bands of extraneous contour lines appearing in phase contour maps like Figure 1 must be removed to clearly visualize the phase data. The original method used to remove the bands was simple and straightforward, but it was extremely laborious.

To remove the bands of extraneous contour lines, the contour maps of the same phase data were plotted twice, with the phase data represented in the range from $-\pi$ to π in one plot and those from 0 to 2π in the other. In both plots the point of origin was defined as the zero phase reference. Thus, in the two maps of the same data the regions covered by the bands were π apart. The maps were then traced by hand to remove the bands. Regions of data obscured by bands in one map were clear in the other map, so tracing the good contour lines was straightforward, but involved a great deal of time and effort. The phase contour maps in Refs. 3 and 4 were cleaned of bands using this method, and the results are perfectly satisfactory.

3. UNWRAPPING BY COMPUTER

The expense of the many hours of draftsman's time required to remove the bands by hand provided the motivation to develop a totally computerized method for plotting contour maps of phase data with no bands.

The most obvious approach, simply unwrapping the phase data before plotting, proved unsuccessful. This was attempted by first unwrapping the phase data along the first column of the two-dimensional array of sampled data and then using these modified data to unwrap the data along each row starting at this column. When these data were plotted as a contour map, it was immediately clear that two-dimensional phase data can not be globally unwrapped over a two-dimensional domain that includes zeros in the field amplitude. At isolated points with zero values in the field amplitude, the phase is singular. The phase shift along any path circling around one of these singularities changes by 2π for each circuit. Thus, if two-dimensional phase data are unwrapped along rows in one dimension and then a contour map is plotted from the data, dark bands will trail out behind each zero along the row. Unwrapping along columns instead of rows only moves the bands so that they trail out after the singularities along the columns instead of the rows. The problem is basic. Phase data can not be unwrapped around points of zero field. Inspection of the properly unwrapped phase data in Figure 5 (see also Refs. 3 and 4) clearly shows many such points of phase singularity where the phase contour lines all come together at the point.

The solution to this difficulty, it was found, is only to unwrap the phase data locally, and temporarily, within each of a series of squares in the contour-plotting computer program, just before the contour lines are mapped through that particular square. This method can be described more precisely by "walking through" the procedure using the diagram in Figure 2.

In Figure 2 a rectangular array of sampled two-dimensional phase data is represented by an array of numbered squares. The corners of each square represent the spatial location of four nearest-neighbor phase samples over the two-dimensional plane. The original value of the phase data (to three decimal places) is indicated in Figure 2 by the small number at each sample point. The squares are also numbered 0 through 59 for easy reference. Contour lines at 2.80, 3.14, and 0.34 are shown interpolated through the data grid.

The unwrapping method is performed as follows: Beginning examination of the contour map in the bottom left-hand corner of square 0, we find the data value at the first point is 2.70. The points at the other corners of the square are then tested to see if they differ from 2.70

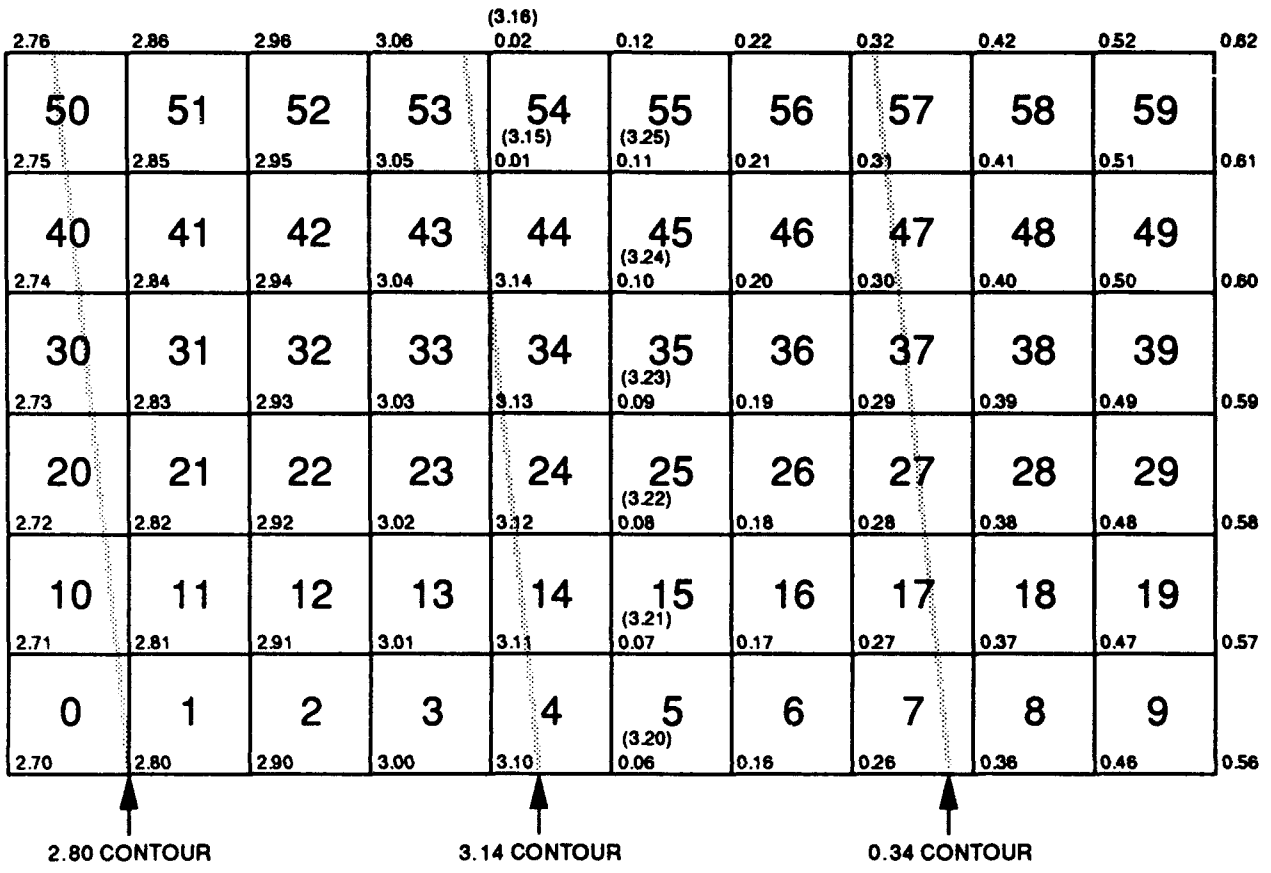

Figure 2 Diagram of the computer algorithm for unwrapping two-dimensional phase data in contour maps. The grid represents the array of rectangularly sampled data. The two-dimensional phase data are known at each line crossing in this grid, and the phase data value is indicated to three decimal places by the small number. The numbers in parentheses are the locally changed data values described in section 3. Contour lines at 2.80, 3.14, and 0.34 are interpolated through the data grid about 20° apart. No missing lines occur as a result of the local unwrapping, provided the phase data vary slowly relative to the contour line spacing, as shown here.

by more than π. As shown in the appendix, if any of the samples at the corners of this square differ from 2.70 by more than π, then 2π phase jumps surely exist between some of the data values along the path bounding the square. These jumps must be removed by adding or subtracting 2π from the points until they are all within π of the local phase reference, 2.70. In square 0 the points do not differ by more than π, so no unwrapping is required. Next, the line bounding each side of the square is tested to see if one of the contour lines is at a level between or at the phase data value bounding the line, as given by the numbers at the ends of the line at the corners of square 0. We find that two lines have data that bound 2.80, and we use linear interpolation along each of these two lines to find the point of intersection of this contour with the line. Finally, the 2.80 contour line is drawn between the two points through square 0, as shown in Figure 2.

The procedure is repeated in exactly the same manner with squares 1 through 3 without detecting any need for phase unwrapping. The lower left-hand data sample in each square is always used as the reference when testing for a need to unwrap. When we get to square 4, however, we find that the four data values at the corners of this square differ by more than π. Therefore, we unwrap the phase data for this square locally (and only while processing this square) by taking the data value in the lower left-hand corner of the square as the reference, and adding to or subtracting from the data values at the other corners of this square by 2π until the data at all four

corners differ by no more than π. The result of this data modification is shown in Figure 2 by the small numbers in parenthesis located just above the original data values. Using the new (temporarily) unwrapped data, we proceed as before to test for contour lines passing through the square and find that there is one contour at level 3.14 that should pass through square 4 from bottom to top. We use linear interpolation, as before, using the modified data at the upper and lower sides bounding the square to locate the contour line and, finally, we draw it through the square. If the data had not been modified, we would have had all contour lines with values 0.006 through 3.11 passing through square 4 and making a part of a dark band. Local unwrapping has deleted the band as intended.

When we move to square 5, we forget the changes that we made in the data in square 4 and start fresh. We use the original data value of 0.06 (not the unwrapped value 3.20 used in square 4) from the lower left-hand corner of square 5 to test the other corners for a need for unwrapping. We find that no unwrapping is needed in this square and that none of the contour lines crosses it.

We proceed through the rest of this row without any need for unwrapping, but discover a contour line crossing in square 7.

After finishing the first row, we go to square 10, which begins the second row. We test for a need for unwrapping using the data value 2.71, and find it unnecessary in this square. We find, however, that the 2.80 contour line continues on through this square and we plot it. Although it saves some time in the program to remember the coordinates of the contour line intercept with the bottom side of square 10—having calculated them before as the coordinates of the intercept with the top side of square 0—this is not essential.

Proceeding in this manner through all 59 squares, taking them in numerical order, we obtain a perfect contour map with no dark bands. In contouring these data, we find that we have to unwrap the phase data only in squares 4, 14, 24, 34, 43, 44, and 53. In all other squares the data can be used as given to determine the path of the contours.

This approach works perfectly so long as the data vary slowly from sample to sample relative to the interval between contour levels. If this condition does not hold, contour lines become discontinuous and are sometimes absent altogether. The result is a contour map with obvious holes, as shown in Figure 3.

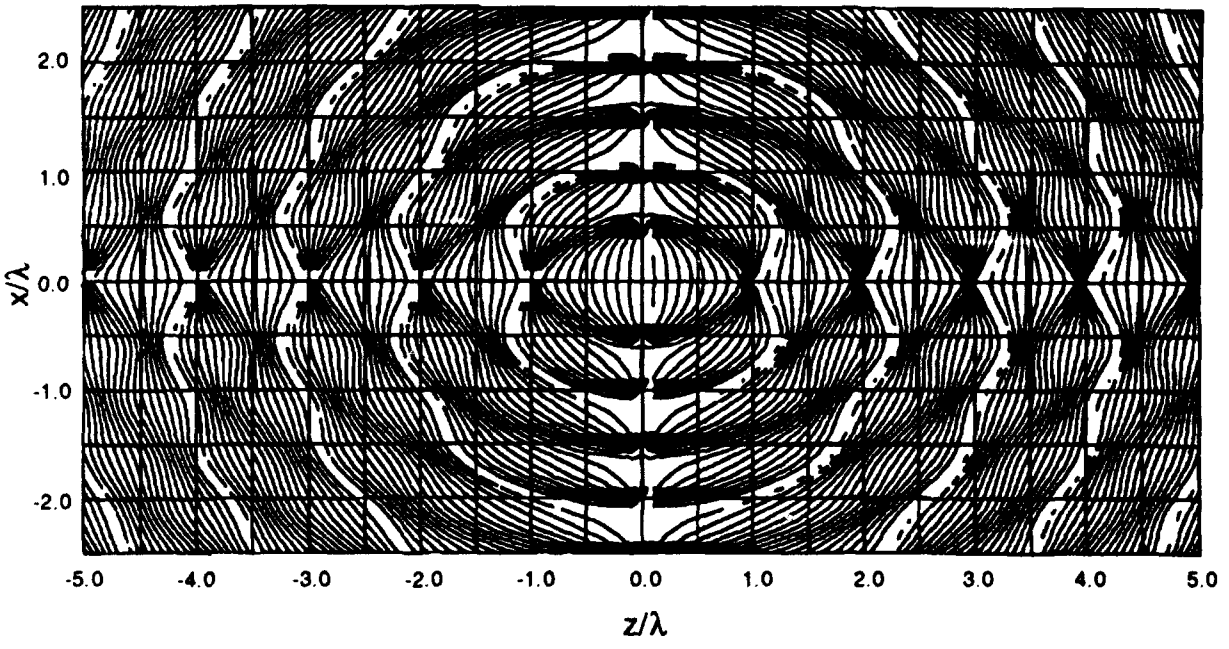

Figure 3 Contour map of the same phase data as in Figure 1 after they have been unwrapped by the computer method described in section 3. Only contour lines 20° apart in the range from 0 to 360° have been plotted. Note the missing lines in the plot, which are caused by the local unwrapping effect described in section 3.

4. MISSING CONTOUR LINES

The origin of the holes in the contour map shown in Figure 3 can best be explained by reference to the diagram in Figure 4, which is an enlargement of a part of the data grid from Figure 3.

The procedure described in section 3 was used to interpolate contour lines, which are only 0.04 radians apart, through this region. Because of the small interval between the contour levels relative to the rate at which the data vary between nearest-neighbor samples, the contour lines do not completely cover the phase data. Obvious holes appear in the map. This usually occurs, not because the contour levels are ridiculously close together, as is the case in Figure 4, but rather because the computer is unable to hold enough data samples to sample the phase data as densely as in Figure 4. For example, Figure 3 shows a contour map of the same data as in Figure 1, plotted using the local unwrapping procedure described in section 3, with contours 20° apart and spanning the range from 0 to 2π. Large holes appear in the map because the data are sparsely sampled.

Studying Figure 4, we see that contour lines 0.04 and 0.08 both end abruptly at the boundary between squares 44 and 54 because the data in square 44 had to be unwrapped, whereas the data in square 54 did not. Thus the contour lines at these levels do properly end as shown in Figure 4. Clearly, however, the physical locus of constant phase does not change at this boundary. Only the phase label used to describe it changes. A 0.04 contour in square 54 labels the same physical phase as a 3.18 contour in square 44. All we need to do to fill the hole in this map is to add two extra contour lines at 3.18 and 3.22. The 3.18 and 3.22 lines will be indistinguishable from the 0.04 and 0.08 lines in the resulting map, which is exactly what we want. Do not label the contour lines with their levels. Labeling the contour lines runs into trouble, since the phase data can not be globally unwrapped in two dimensions for very fundamental reasons, It is only important to know that the phase contour lines are physically 20° apart and that the zero phase reference is at the origin.

Thus, to fill up the holes in the contour map, we need only add extra contour lines with levels above and below the 0 to 2π range. A perfect contour map that has been treated in this manner is shown in Figure 5. These are exactly the same phase data shown in Figures 1 and 3, except the data have been locally unwrapped as described in section 3 and five extra contour lines have been added above and below the 0 to 2π range to fill in the holes. The number of lines was determined experimentally for these data and a 20° contour interval. More contours may be necessary if the data sampling is even sparser.

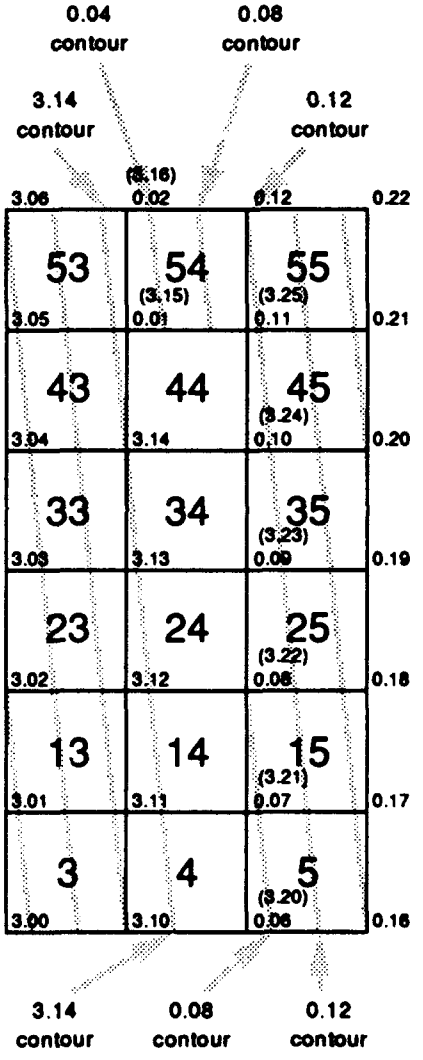

Figure 4 Diagram showing missing contour lines due to the effect of local unwrapping. This figure is an enlargement of a section of the data grid in Figure 2. The data and grid square numbers are the same, but the contour levels are only 0.04 radians apart. Because the phase data do not vary slowly relative to the interval between contour levels, missing lines occur. Extra contour levels exactly 2π greater (or less) than the missing contours must be provided to fill in the holes in the map, as described in section 3.

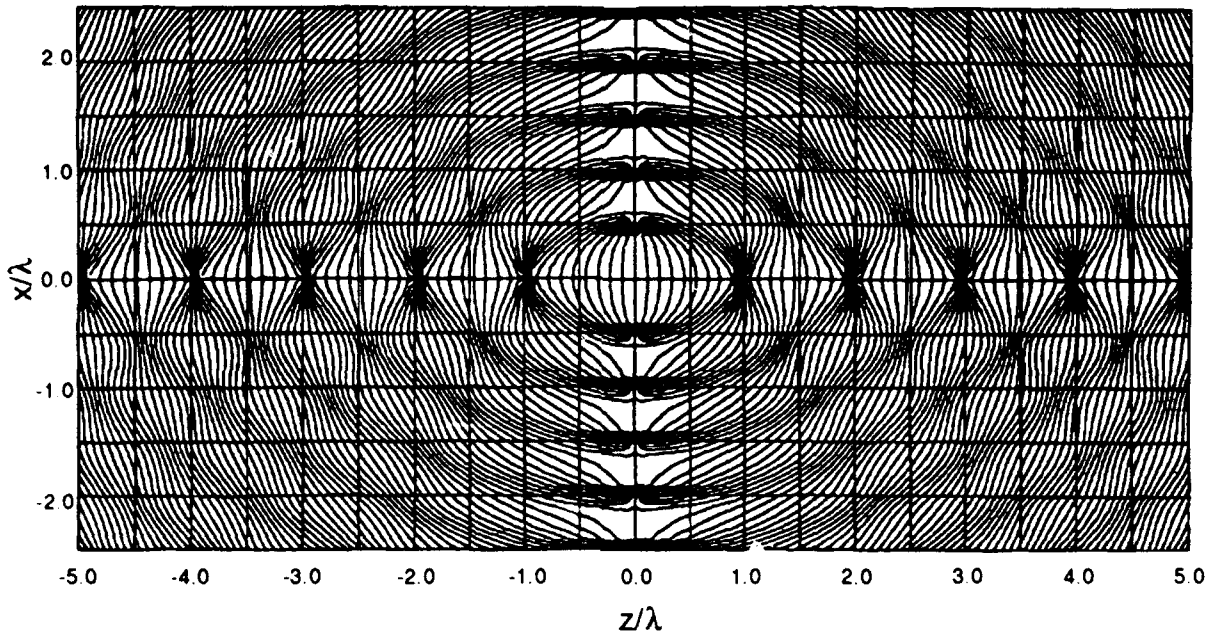

Figure 5 Contour map of the same phase data shown in Figures 1 and 3 after they have been unwrapped by the computer method described in section 3. In this case, equally spaced contour lines have been drawn 20° apart in the range from −100° to 440°. Note that the extra lines fill the holes in the plot shown in Figure 3 and yield a perfect contour map of this data.

5. CONCLUSIONS

A method for locally unwrapping phase data has been discovered that may also have more general applications. The method involves locally unwrapping the phase within a single square of the data grid just before the contour levels are interpolated through the square, and then starting fresh with unmodified data in each subsequent square of the data grid. Extra contour lines must sometimes be added outside of the range 0 to 2π to fill in holes that can appear in the maps. The method has been tested and appears to produce perfect contour maps of the phase data that has been considered.

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[2]W. H. Carter, "Anomalies in the Field of a Gaussian Beam Near Focus," *Opt. Commun.* **7**, 211-218, Figs. 2, 4, 6, and 8 (1973).

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[4]W. H. Carter, and T. J. Wieting, "The Field Distribution of a Focused Gaussian Beam Reflected at 45° from a Conducting Plane and Its Effects in Plasma-Ignition Experiments," *J. Appl. Phys.* **54**, 519-523, Fig. 6 (1983).

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APPENDIX: SAMPLING CRITERION FOR PHASE DATA

Consider a one-dimensional string of sampled phase data, which is stored (modulo 2π) with values in the range from 0 to 2π. Assume that the original phase function represented by this data changed by no more than θ between sample points.

We begin the proof with the simple case where the sampled function increases uniformly by θ between four samples in sequence. The four phase samples that arise can be represented on a phasor diagram, as shown in Figure A1. The number near the arrow for each phasor indicates the order of the sample that it represents in the sequence. Thus, the first sample is defined (by choice of phase reference) to have a value of 0°, the second sample has a value of θ, the third sample has a value of 2θ, and the fourth sample (which wraps around through 2π) has a value of $\theta - 2\theta'$; where θ' is the complementary angle to θ (i.e., $\theta + \theta' = 180°$). From the diagram it is clear that although the phase difference between the first three samples is just θ, the phase difference between the third and fourth samples is $\phi = \pi + \theta'$, because the fourth sample has wrapped around through 2π from the first sample at 0°. If θ' is positive (as it must be if $\theta < \pi$), then $\phi > \pi$. If θ' is negative (as it must be if $\theta > \pi$), then $\phi < \pi$. Thus, provided the original function varies by no

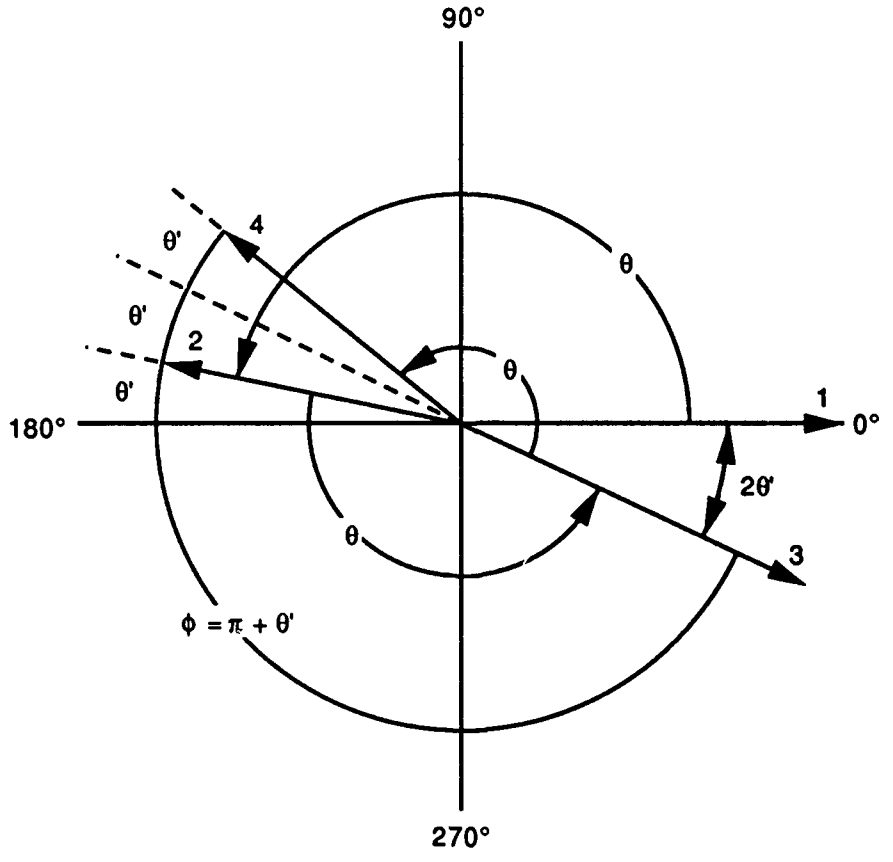

Figure A1 Phasor diagram showing validity of sampling criterion for four sequential phase samples. Note that if the sampling criterion defined in this appendix is followed, four samples of a uniformly increasing phase function will differ in phase by more than π between nearest-neighbor samples if, and only if, a phase jump has taken place between the values.

more than $\theta < \pi$ between samples, a phase difference of more than π between data values dependably indicates that a phase jump has taken place between the samples.

Next, we consider the more general situation where the phase interval between the sample points in the original phase function, as before, is always less than θ, but we make no assumption that four values in a row have the same phase difference. Let samples one and two in Figure A1 be arbitrary and consider only samples three and four, as shown in the phasor diagram in Figure A2. The angle α between the third sample and 0° must now be arbitrary, so that the third sample may have any value. The other phase relationships shown in Figure A2 (which were obtained from Figure A1) remain valid for any α. There are four possible cases, which can be analyzed by inspection of Figure A2. First, if $\theta < \pi$, as shown in Figure A2, and $\alpha > 0$ (so that a phase jump appears in the data between the two samples), then clearly the phase difference in the data recorded from 0 to 2π will be $\phi > \pi$. Second, if $\theta < \pi$ and $\alpha < 0$ (so that no phase jump appears between the samples), then the phase difference in the data will be $\theta < \pi$. Third, if $\theta > \pi$ and $\alpha > 0$, then the phase difference between the samples will be $\phi < \pi$. Finally, if $\theta > \pi$ and $\alpha < 0$, then the difference between the samples will be $\theta > \pi$.

We have proven a theorem. If a sampling criterion is defined such that a one-dimensional phase function is always sampled so the nearest-neighbor sampling points are always less than π apart in phase value, then the difference between any two nearest-neighbor sampled values recorded modulo 2π in the range from 0 to 2π will always be less than π if no phase jump has occurred between them and will always be greater than π if such a phase jump has occurred. Since this one-dimensional

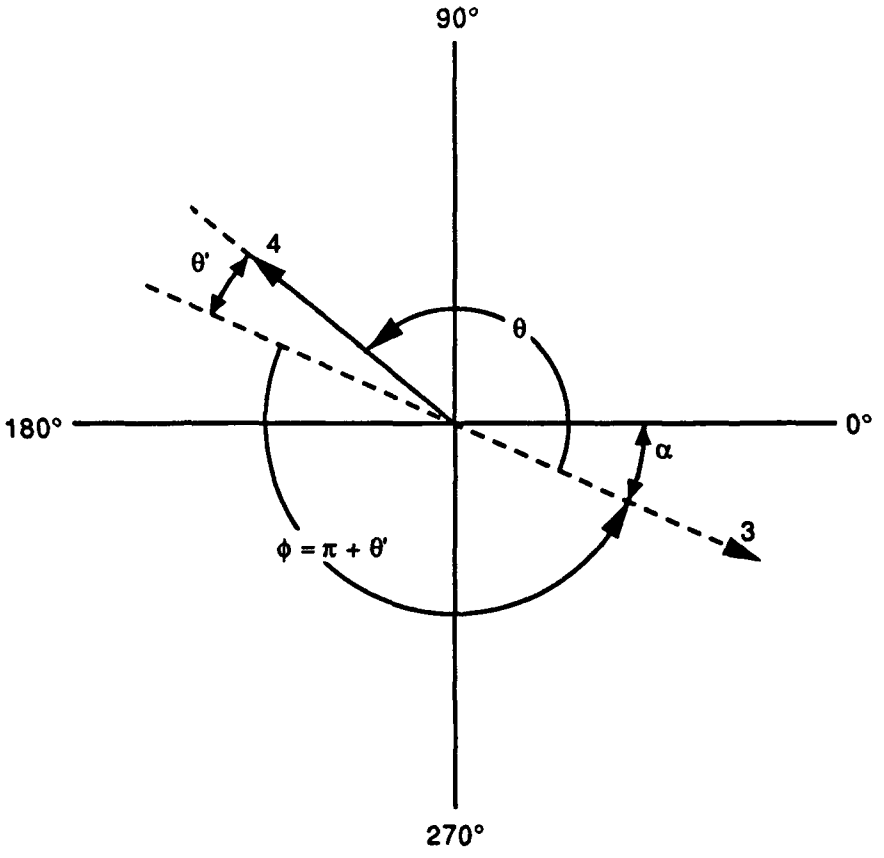

Figure A2 Phasor diagram showing validity of sampling criterion for two sequential phases samples with an arbitrary phase function. Note that if the sampling criterion defined in this appendix is followed, any two samples of an arbitrary phase function will differ in value by more than π between nearest-neighbor samples if, and only if, a phase jump has taken place between the values.

sampling theorem can be applied to the data along the path around each grid square shown in Figure 2, the method used in the algorithm described in section 3 to detect a need for phase unwrapping within a single grid square should always work, provided the defined sampling criterion holds.

This criterion is possibly related to the Nyquist criterion for unambiguous sampling of real data. The Nyquist criterion applies to data that can be represented by a real function $f(x)$ of a real variable x. It states that if $f(x)$ has a Fourier spectrum that vanishes above some frequency ω_0, then $f(x)$ can be exactly determined over some domain D of x from samples $f(x_i)$ taken at constant intervals $\Delta = x_{i+1} - x_i$ over D, provided that $\Delta \leq \pi/\omega_0$ (see, for example, J. W. Goodman, *Introduction to Fourier Optics*, McGraw-Hill, New York, 1968). Thus, the samples must be taken twice within each period of the frequency ω_0. The sampling theorem derived in this appendix applies to data that can be represented by the phase $\phi(x)$ of some complex function $g(x)$ of a real variable x. It states that this phase data can be unwrapped from samples recorded modulo 2π, provided that $\phi(x)$ varies by no more than π between the samples. Thus again, we must sample twice per period. The data being sampled is different. The definition of a period is different. What is required from the samples is different. But the requirement that two samples must be taken per period seems too similar to be a complete coincidence.

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