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THESIS

PROBABILISTIC STRENGTH-LIFE MODEL
FOR GRAPHITE FIBERS UNDER STRESS

by

Nigel Ian Gardener

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Thesis Advisor: Prof. E. M. Wu

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Probabilistic Strength-Life Model for Graphite Fibers Under Stress

by

Nigel Ian Gardener
Lieutenant Commander, United States Navy
B.A., The Citadel, 1978

Submitted in partial fulfillment of the requirements for
the degree of

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Author: ______________________
Nigel Ian Gardener

Approved by: ______________________
Edward M. Wu, Thesis Advisor

____
Michael R. Gorman, Second Reader

____
Daniel J. Collins, Chairman,
Department of Aeronautics and Astronautics

ABSTRACT

The work done in this investigation is part of a continuing program designed to develop probabilistic strength and life models for filament composite materials. In this particular experiment, groups of single fibers from two chemically identical different production spools of graphite were loaded to identical sustained tensile loads in order to determine their life statistics while under stress. While previous work has produced models of composite reliability as a function of strength, this work develops a model which will be necessary to predict the reliability of composite fibers as a function of time. Parametric and non-parametric methods were used to quantify the relationship between fiber reliability as a function of stress and as a function of time.

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I. INTRODUCTION

While still in its infant stages, the use of composite materials in manufacturing is steadily increasing, particularly in high reliability, low accessibility environments such as aircraft fuselages and satellite fuel tanks. High strength to weight ratios, combined with minimal maintenance requirements and extreme resistance to corrosion make composites ideally suited for structures previously built exclusively of steel or aluminum. However, because composite usage has such a short history, there is no significant data base which would allow accurate reliability predictions.

Through an ongoing program of research and experimentation, The Advanced Composites Laboratory at the Naval Postgraduate School at Monterey, CA, is defining a probabilistic model which is based on the failure mechanism of composites in tension. Small scale designs which exploit fiber strength under tension, have accurately verified the Local Load Sharing model. More recent work is concentrating on the application of this model to large scale structures. With this in mind, it is important to understand that a structure must not only be predictable (reliable) in strength, but also in life. Because of their non-homogeneity, the study of composite materials must be further broken down to the study of not only individual constituents, but also of their inter-relationships with each other. For our purposes here, all work is done exclusively with graphite/ epoxy composite.; therefore, a proper understanding of the fiber, the matrix, *and the interaction between them*, is essential to formulating an accurate model of composite reliability.

The primary objective of this project was to examine a large sample space of graphite fibers under different load conditions, and formulate a model of their reliability in life. The fiber life results would then be applied to composite life experiments now on-going designed to predict the life of composite structures. With data from these two projects in hand, one could then ostensibly isolate the solution to the fiber/ matrix interface problem and produce a realistic life prediction model.

II. BACKGROUND

A. STRENGTH DETERMINATION OF FIBER REINFORCED COMPOSITES

Fiber Reinforced Composite materials generally consist of high strength fibers bonded together in a ductile matrix. In these materials the fibers carry essentially all of the stresses imposed on the structure while the matrix serves as the primary vehicle for transferring load between adjacent fibers, known as the Local Load Sharing model. In this model, the local load carried by a bundle of fibers is carried equally until the weak fibers begin to rupture. When a fiber breaks, its load is transferred to the adjacent fibers through the matrix. Rosen has modeled the load transfer by the existence of an ineffective length at the end of each (failed) fiber [Ref. 1]. In this region, the longitudinal stress (σ) diminishes to zero while shear stresses (τ) rise to a maximum. As these ineffective lengths accumulate, loads continuously transfer from the weak (failed) fibers to the strong ones. If enough fiber failure sites are in close proximity to each other, the matrix begins to crack or de-bond from the fibers. When this occurs, the matrix can no longer transfer the load and the sample fails catastrophically. Thus composite strength can be modeled as a function of fiber strength, the ability of the matrix to transfer load, and the interface between the fiber and matrix. The mathematical model which predicts the probability of failure of the composite (in tension) from the statistics of fiber strength and the effective length has been established by Harlow and Phoenix [Ref 2 and 3].

These three quanta are recognized as building blocks in composite strength determination. Matrix optimization is guided by this model. From a given assortment of matrix materials, one may determine which has the most suitable characteristics for stiffness and transferring shear stresses. Fiber strength statistics are determined through loading to destruction measurement. However, it is not feasible to directly measure and quantify the molecular level interface region. This can be accomplished by measuring the strength of a composite of known fiber and matrix. the fiber strength statistics and composite strength statistics can then be used simultaneously to solve for the ineffective length parameter.

B. LIFE DETERMINATION OF FIBER REINFORCED COMPOSITES

Decades of experience with homogeneous materials support the correlation between strength and life: that is, the larger the factor of safety for a given structure, the longer it can be expected to last. However, Coleman's theory of time dependence in the mechanical breakdown of fibers [Ref. 4] implies that the strength/life relation in load bearing fibers may not be so straightforward. Using the "weakest link hypothesis" first developed by Pierce in 1926, Coleman concluded that fiber life was instead a function of the statistical distribution of these "weak links" in time dependent strength. The time dependent strength of the fiber, termed breaking kinetics, is the extreme value of fiber life under a given load history. This is fundamental to the modeling of composite life.

Only when fiber life statistics are available can the model be applied to composite structures. Hence, the purpose for this work.

The theory proposed is, the strength of a specific fiber can be related to the life of that same fiber. The dichotomy of course is, once a fiber has been broken in the strength test, another molecularly identical sample is not available for the life test. The alternative is to obtain a set of *statistically identical* fibers and test a portion of them in strength and another portion in life.

This is being accomplished by using two different spools of AS-4 graphite fiber, AS4-008 and AS4-019. A statistically meaningful number of samples have been taken from these two spools to estimate the respective parameters of the Weibull Distribution. The estimated parameters of these two different spools are different, suggesting the intrinsic strength of these two spools are from two different populations.

III. FILAMENT LIFE TESTING METHOD

A. TEST SPECIMENS

Test samples were single strands of Hercules Magnamite AS4 cut to two inch gauge lengths. Each sample consisted of one fiber filament mounted on a paper cardboard with a cutout for handling. The cardboard mount was severed with a hot wire prior to load application. Specific handling and mounting procedures are described in Appendix A and in Reference 5. Once mounted, the fiber diameters were measured using a laser diffraction device also following the procedures described in Reference 5.

B. TEST EQUIPMENT DESCRIPTION

The single filament samples were arranged in groups of 64 at four different sustained load levels by dead weights. The elapsed time between the startup of load and ending of stress-rupture at each station was triggered by an infra-red light switch. Test equipment is described in Appendix B. A polymethyl methacrylate (PMMA) cover was used to protect the samples from ambient air disturbances and ultraviolet (UV) radiation. It is known that UV light degrades the strength of some fibers; therefore, PMMA was chosen as a shield and is analyzed in Appendix C. Once loaded, the fibers were continuously monitored by a dedicated PC (personal) class computer. Software programs are reproduced in Appendix D.

C. TESTING PROCEDURE

1. Strength Test

Fiber strength data were acquired previously and are covered in References 5 through 7.

2. Life Test

During the diameter measurement process, every fifth fiber was tested to failure in strength using a calibrated load cell. This provided the link between the two groups needed to verify that the samples tested in life were statistically identical to those tested previously in strength. The actual loading of the samples on the board was carefully controlled and, as much as possible, were all done by the same operator. The loading procedure is also described in Appendix A.

There were four levels of load set up on the board, 14.17 g. (Level 1), 13.15 g. (Level 2), and 12.20 g. (Level 3) and 10.96 g. (Level 4). Each load level had 124 stations, 64 for each spool. This allowed a total of 512 life samples to be measured at sustained loads. Level 4 was planned but postponed. Instead those stations were also loaded to 14.17 g. to gain additional measurements at that level. The sample board alternated rows with 008 and 019 fiber to give 4 rows of each. After each fiber was loaded, the date and time of the load were recorded in the lab notebook and also in a software spreadsheet. If a fiber failed on loading, its failure stress was recorded. After loading, each station was actively monitored by an AT computer which recorded stress failures as they occurred. Exact failure information was periodically retrieved and recorded in the notebook and spreadsheet. The spreadsheet automatically computed realized data for stress ruptures, early censor (Type Iv) stress ruptures, fail on load stresses, and existing lifetimes for each fiber type at each load level. It would then arrange the desired data in a form easily transported to other programs for analysis or presentation.

IV. RESULTS

A. TEST DATA

1. Overview

The data presented herein will be used to identify a suitable strength-life model for fiber filaments. A probabilistic formulation of the strength-life model is a joint distribution of the random variables, stress (applied), σ,and elapsed time, t, denoted $f(\sigma,t)$. The parameters of this joint distribution are determined by three classes of realized data. The first is data realized in strength over negligible time. The second is data realized in life for a given stress history, in this experiment, constant, sustained stress. The third is data realized in strength given a survival in time history. The first two cases, that is, strength and life data, will be presented in this investigation. To examine variability in production, two spools of AS-4 graphite fibers were treated as two distinct populations, designated as spool 008 and 019. The division of fibers from these two spools is traced out in Figures 1 and 2. The entire population of either spool was denoted as $\{D\}$. This population is divided into strength test samples yielding data, $\{D_s\}$, and life test samples yielding data, $\{D_t\}$. The respective sub-division of these two data sets is described in the following two sections.

2. Strength Data

The strength data set comes from work done previously using the same 008 and 019 fiber populations [Refs. 5 through 7]. The sources of this data are summarized in Table 1.

TABLE 1. SOURCES OF FIBER STRENGTH DATA

DATA SET	NAME	REF. NO.
$\{D_{s1}\}$	Dr. Edward M. Wu Mr. Glen Nypiak	6
$\{D_{s2}\}$	Lt. David Bell, USN	5
$\{D_{s3}\}$	LCDR Carl Englebert, USN	7
$\{D_{s4}\}$	Mr. Jim Nageotte	7

3. Life Data

Within the life data set, $\{D_t\}$, not all of the samples were realized in life. Some of the samples were tested to assure the life samples were statistically the same as the strength samples and are denoted as $\{D_{ts}\}$ (Figures 1 and 2). Some samples failed before they reached the target sustained load level. Thus, this was part of the life test subset but was realized in strength and is denoted as $\{D_{t1s}\}$, $\{D_{t2s}\}$, and $\{D_{t3s}\}$. Based on kinetic theory [Ref. 2], it is generally believed that the load supported by a fiber increases as the loading rate increases; therefore, failures in strength would be hypothetically eliminated if the fibers could be loaded instantaneously. While this not physically possible, the data from samples which failed on loading can be accounted for by treating the life samples conditionally to strength greater than the sustained load. Data obtained from this experiment is summarized in Tables 2 through 4. Ruptures occurring on loading, prior to the desired load are identified as "Fail on Load". Stress ruptures occurring in time but caused by other than intrinsic stimulus internal to the sample are "Type Iv" censored. This data is statistically treated as greater than or equal to the realized life. The censorings are chance events as denoted by the variable time at censor. Stress ruptures in time are "Exact" and fibers still under load at the end of the experiment are "Type I" censored. This data is statistically treated as greater than or equal to the time the data was interpreted.

TABLE 2. RESULTS OF LOAD LEVEL 1

Type of Failure	AS4-008	AS4-019
Fail on Load $\{D_{t\,1s}\}$	53	57
Type Iv Censor $\{D_{t\,1v}\}$	9	7
Exact $\{D_{t\,1t}\}$	15	20
Type I Censor $\{D_{t\,1c}\}$	116	110

TABLE 3. RESULTS OF LOAD LEVEL 2

Type of Failure	AS4-008	AS4-019
Fail on Load $\{D_{t2s}\}$	26	13
Type Iv Censor $\{D_{t2v}\}$	4	1
Exact $\{D_{t2t}\}$	9	6
Type I Censor $\{D_{t2c}\}$	62	63

TABLE 4. RESULTS OF LOAD LEVEL 3

Type of Failure	AS4-008	AS4-019
Fail on Load $\{D_{t3s}\}$	10	11
Type Iv Censor $\{D_{t3v}\}$	1	3
Exact $\{D_{t3t}\}$	3	1
Type I Censor $\{D_{t3c}\}$	62	63

B. DATA ANALYSIS

1. Analysis as a Function of Stress

a. Overview

Before proceeding with an analysis of life based on strength, it must first be confirmed that the two sets of data, $\{D_s\}$ and $\{D_t\}$, are indeed statistically from the same population. As can be seen in Figures 1 and 2, each spool has yielded two sets of samples. The first set, $\{D_s\}$, was tested entirely in strength. Within the second set, $\{D_t\}$, every fifth fiber was tested in strength in order to link the two sample sets $\{D_s\}$ and $\{D_t\}$.

b. Non-Parametric Interpretation

Non-parametric data analysis was conducted by plotting the cumulative failures in a weakest link (Weibull Probability) coordinate. This required the data to be ordered and a rank assigned. For this experiment, Expected Rank was used and is defined: the i^{th} realization of the ordered x_i is

$$\frac{i}{N+1} \quad \text{for } i = 1,2,3 \ldots N$$

and x_i is the ordered data (from weakest to strongest) in a set of N samples. An ambiguity arises when one considers the inclusion of censored data. Because the point is censored, its exact strength, and therefore its exact rank, cannot be precisely determined. Furthermore, because the data is ranked, this uncertainty perturbs the rank of each subsequent data point greater than the censored data. For this reason, censored data was omitted from the graphical representation. However, censored data did not affect the Likelihood calculations and was therefore included in the parametric interpretation.

Once the data has been ranked, it is linearized by using the weakest link transformation. This method linearizes the ranked data and allows a visual observation of the lower tail characteristics. The weakest link transformation is defined as:

$$F^*(\sigma) = \ln(-\ln(1-F(\sigma)))$$

where $F(\sigma)$ is the empirical rank of the data in question. Recalling that each fiber is modeled as a chain of independent links, the strength of a given length of fiber must be the same as the strength of the weakest link in that chain, hence $F^*(\sigma)$ can be further defined as the probability of failure of the weakest link of a given gauge length of fiber.

Strength data, $\{D_s\}$, from these two spools (008 and 019) is plotted in Figures 3 and 4. This set of fibers had previously been confirmed to have come from the same set of like materials and gauge lengths (Ref. 9). Figures 5 and 6 depict the F* plot of the set D_{t_s} overlaid with the set D_s. The close proximity of the two sets of data indicates they too are of the same fiber and gauge length. A cursory run-test on the ranked set of merged data shown in Figures 7 and 8 shows acceptable levels of statistical clumping and completes the non-parametric identicalness test of the two sets.

c. Parametric Interpretation

A parametric analysis of the same data was also conducted by using the Weibull distribution for both strength and life:

$$F(x) = 1 - \exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\}$$

where α and β are the shape and location parameters respectively. Given a data set, these parameters may be obtained by using the Maximum Likelihood Estimator (MLE) derived in Appendices E and F. The attribute of this method is there is no requirement to order the data which eliminates the process of ranking which allows the inclusion of any censored data points. Also associated with the MLE is the concept of confidence interval. The confidence region of the parameters α and β can be obtained from the likelihood contour plots. The confidence region is the confidence, stated as a percentage, that a selected α and β are the actual α and β. Parameters chosen at a very low confidence interval have a low probability of matching the actual parameters, but when they do, they are highly accurate. A very high confidence interval, while allowing more room for error, is not very precise and is of little practical value.

Figures 9 and 10 show a three dimensional representation of the Likelihood for the D_s data set along with various confidence region contours. The 5 % and 1 % confidence region contours are shown in Figures 11 and 12. The shape parameter, α, and location parameter, β, for this data set can be determined from Figures 3 and 4 graphically by the slope of the line of data points and the value of the X axis at $F^* = 0$ respectively. Alternately, the estimate can also be based on the calculus MLE derived in Appendix E.

In order to show that $\{D_s\}$ is identical to $\{D_{ts}\}$, the likelihood ratio can be used. The likelihood ratio is graphically represented by the volume of the intersection of the two likelihoods at a given confidence interval, divided by the volume of the same confidence interval of the accepted baseline data set, $\{D_s\}$ or, symbolically

$$\frac{V(D_s) \bigcap V(D_{ts})}{V(D_s)}$$

: $V(D_s)$ and $V(D_{ts})$ are the volumes of the likelihood contours within the desired confidence interval. Figures 13 and 14 show overlay plots of the Likelihood contours of N_s and N_{ts} at 5% and 1 % confidence intervals. By examining these figures, it can be determined that the likelihood ratios for 008 and 019 at the 5 % confidence interval are estimated to be .75 and .70 respectively, which is deemed to be acceptable for this confidence interval.

The two sets of data can now be merged and the tested in life fibers are confirmed to be of the same population as the tested in strength fibers. As a last point of confirmation, Figures 15 and 16 overlay this merged data set $\{D_s+D_{ts}\}$ with the original set, $\{D_s\}$ showing they are essentially identical, as do the contour overlay plots shown in Figures 17 and 18.

A distinction can now be made concerning those fibers which failed on load ($\{D_{t1s}\}$, $\{D_{t2s}\}$, $\{D_{t3s}\}$). Although intended to be measured in time, these fibers actually failed in strength prior to reaching the target loads, σ_1, σ_2, and σ_3 respectively; therefore, this set of data can be considered as realized in strength to be included in the merged strength data set $\{D_s+D_{ts}\}$. The Expected Rank of the exact failed in strength data was presented as before with the exception that, the samples which reached the target load in life test were considered Type I strength censored at the stress rupture load, This is the equivalent to stating its strength would be greater than the stress rupture load. As with *all* non-parametric analyses, these censored points were not presented in the F* domain because of the ambiguity in rank. By treating censored data in this manner, the graphical representation of the middle and upper tails remains unaffected, while the resolution of the lower tail is improved. Fail on loading data, $\{D_{t1s}\}$, $\{D_{t2s}\}$, and $\{D_{t3s}\}$, is presented in Figures 19 through 24 and can be analyzed in the same manner as the set $\{D_{ts}\}$ with similar conclusions being drawn. Thus, the set of fibers tested in time can now be considered statistically identical to the set of fibers tested in stress.

The final step in strength analysis was to analyze the entire population of strength data and determine a shape (α) and location (β) parameter for the Weibull Probability of Failure Function which would be necessary to determine the Likelihood in Life. Results of non-parametric analysis of the set $\{D\}$ in strength are shown in Figures 25 and 26. Because of the data censored above 12.20, 13.15, and 14.17 grams, a value for α and β can only be estimated from the set $\{D_s + D_{ts}\}$; therefore, a parametric analysis in the form of the Likelihood contours of the set $\{D\}$ is shown in Figures 27 and 28.

2. Analysis as a Function of Time

Since the life of a fiber is dependent on the fiber's surviving the loading process, ruptures during loading must be considered in determining the Likelihood of the fiber in life. The reliability of the fiber in strength will be necessary to determine its reliability in life. Therefore, equation (1) in Appendix E must be modified to include the fibers reliability in strength:

$$R(\sigma) = \exp\left\{-\left(\frac{\sigma_c}{\beta_\sigma}\right)^{\alpha_\sigma}\right\}$$

and the Likelihood in the time domain becomes:

$$L = \left[1 - \exp\left\{-\left(\frac{\sigma_c}{\beta_\sigma}\right)^{\alpha_\sigma}\right\}\right]^k \left[\alpha_t^{m_1} \beta_t^{m_1 \alpha_t} \prod_{i=1}^{m_1} x_i^{\alpha_t - 1} \exp\left\{-\beta_t^{-\alpha_t} x_i^{\alpha_t}\right\}\right]$$

$$\left[\prod_{i=m_1}^{m} \exp\left\{-\left(\frac{\hat{x}_i}{\beta_t}\right)^{\alpha_t}\right\}\right]\left[\exp\left\{-\left(\frac{x_m}{\beta_t}\right)^{\alpha_t}\right\}\right]^{(n-m)}$$

where σ_c is the target stress and k is the number of left censored fibers. By determining estimates of α_σ and β_σ from Figure 27 for Spool 008 and evaluating the Likelihood contour over a set range of α_t and β_t, the Likelihoods for each of the three load levels can be plotted and are shown in Figures 29 through 31 which were developed using an estimate of 5.0 for α_σ and 18.0 for β_σ. A similar procedure was followed for Spool 019.

Although not investigated during this analysis, an alternative method of calculating the Likelihood would be to evaluate the fail on loading data as censored in time rather than stress. In order to do this, one must first assign a time, t_0, at which the sample is considered to have attained the target stress. All samples which fail on loading can be considered to have a life less than or equal to t_0, or:

$$F(t) = \left[1 - \exp\left\{-\left(\frac{t_0}{\beta_t}\right)^{\alpha_t}\right\}\right]$$

Under this interpretation, the Likelihood can be calculated using:

$$F(t) = \left[1 - \exp\left\{-\left(\frac{t_0}{\beta_t}\right)^{\alpha_t}\right\}\right]^k \left[\alpha_t^{m_1}\beta_t^{m_1\alpha_t}\prod_{i=1}^{m_1} x_i^{\alpha_t - 1}\exp\left\{-\beta_t^{-\alpha_t}x_i^{\alpha_t}\right\}\right]$$

$$\left[\prod_{i=m_1}^{m}\exp\left\{-\left(\frac{\hat{x}_i}{\beta_t}\right)^{\alpha_t}\right\}\right]\left[\exp\left\{-\left(\frac{x_m}{\beta_t}\right)^{\alpha_t}\right\}\right]^{(n-m)}$$

3. Construction of the Strength-Life Model

For a life distribution given a sustained stress level, σ_i, the Weibull model used was:

$$F(t|\sigma = \sigma_i) = 1 - \exp\left\{-\left(\frac{t}{\beta_t}\right)^{\alpha_t}\right\}$$

Using this model, the Likelihood contours for the life data realized up to the current time were plotted and are shown in Figures 29 through 31.The flatness of the contours is the result of an extremely broad range of β which is caused by the elapsed time (at the time of data interpretation) being much smaller than the underlying location parameter of life, β. Therefore, only very tentative estimates of β are possible at this time. The shape and location parameters are visually estimated to be:

Stress Level, σ_i	$\hat{\alpha}_t$	$\hat{\beta}_t$
14.17 g.	.15	1.0E+11
13.15 g.	.15	1.0E+12
12.20 g.	.15	5.0E+13

The estimated life at different levels of sustained stress is shown in Figure 32 evaluated at three levels of probability of failure, 0.001, 0.01, and 0.1. The figure can be considered as a graphical representation of the Strength-Life Model for AS4 graphite under sustained stress. The expected life of the fiber, evaluated at the desired probability of failure at a given stress level, can be obtained directly from the model.

V. CONCLUSIONS AND RECOMMENDATIONS

The Strength-Life Model, once verified, can be a useful and important tool for predicting fiber reliability using strength statistics. In design, it allows accurate determination of maximum design stress for an expected life. In maintenance and repair, it facilitates the calculation of the remaining life of an existing structure. It also predicts stress levels at which an existing structure may be operated that can increase or decrease its remaining life.

The following items are recommended for follow-on research:.

1. Application and verification of the model using larger scale composite samples.

2. Mathematical formulation to predict life based solely on strength.

3. The life test be continued to improve the estimation of the parameters of the model.

APPENDIX A

PROCEDURES.

Fiber Handling. Fiber handling procedures are specific and are in effect continuously.

1. Unless required for the immediate task at hand, all fibers and filaments shall be stored in dark, secure containers.
2. Fibers and filaments are fragile. Any fiber or filament that is dropped or otherwise subjected to extreme stress, shall be discarded.
3. Whenever possible, all samples should be handled by one end only so as not to exert any tension on the sample.
4. Use of fluorescent lights shall be kept to a minimum.

Filament Mounting. Filaments were prepared according to the following guidelines:

1. Strip off a random length of fiber from the spool, sever, and discard.
2. Cut the next 12 - 15 cm. of fiber from the spool and tape one end to the glass plate.
3. Pour a small puddle of ethyl alcohol on the glass plate to float the free end of the bundle. The filaments will begin to float apart. Gently fan out the filaments floating in the alcohol and let stand until all the

alcohol has evaporated.

4. Write the correct serial numbers on the carriers to be loaded.

5. Using a small piece of transparent tape on the end of a modelling knife, lift the free end of **one** filament and slide an empty carrier under it.

6. While maintaining filament alignment with the two small holes in the carrier, lower the raised filament end and tape it to the **glass plate**.

7. Use two small pieces of tape to secure the filament to the carrier.

8. After the filament is secured between the two pieces of tape, carefully cut the filament to free it and the carrier from the glass plate and place the filament/carrier assembly on the cold curing plate.

9. Repeat steps 5-8 eight more times. After the ninth filament has been mounted, discard the remains of the bundle.

NOTE

In order to maintain standard curing times and temperatures, do not activate the heat element in the hot plate until the plate is full and do not remove the samples until the plate has been turned off and has cooled.

10. Activate the hot plate with a setting of 200° F.

11. Thoroughly mix a small amount of epoxy (Devcon 2 Ton).

12. Reduce hot plate temperature to 150° F.

13. Place one dot of epoxy at each end of the elongated hole in each carrier to permanently secure both ends of the filaments to their respective carriers. Also, place a dot on the outboard ends of the two small holes to re-enforce them.

14. When the last filament has been epoxied, turn off the hot plate and let cool.

15. Store mounted filaments in designated containers.

Filament Loading.

Loader Start up

1. Turn on power supplies for load cell and elevator.
2. Turn on HP-85 and attached Data Acquisition Unit.
3. Insert program tape into HP-85 and type LOAD "QUIK4".
4. Type RUN.
5. Select "k2" (INPUT) key and enter the date.
6. With load cell assembled and unladen, select "k1" (ADJ B) key.

This sets the bias to 0. To check, select "k3" (WEIGH). If a weight within .003 g of 0.0g is not obtained, re-select ADJ B until satisfactory. Accurate readings should also be verified using calibrated weights.

Filament Loading

1. Remove Plexiglass cover from sample rack.

2. Without disturbing adjacent filaments, lower stabilizer bolts as far as possible on stations to be loaded.

3. Adjust the spring so that the flag will operate the optical trigger without rubbing against it.

4. Remove the weight from the station to be loaded.

5. Using tweezers, grasp the desired filament by one end of it's carrier and hang it on the hook attached to the flag.

6. Using tweezers, hang the weight on the bottom hole of the carrier.

7. Verify the flag does not rub against the trigger.

8. Lower the load cell elevator far enough that it will fit onto the rack without disturbing the hung filament.

9. Without anything touching the top of the load cell, press "ADJ B" on the HP-85.

10. When the screen has stopped scrolling, raise the elevator until the compliance spring is completely retracted and the paper carrier is carrying exactly zero load.

11. Verify that the PC records the station as unloaded as the flag activates the optical trigger.

12. Select "LOAD" on the HP-85 and answer the cues:

"ENTER THE SAMPLE #"

"ENTER THE STATION #"

13. Select "CONT"

14. Verify the weight of the vial shown by the computer is correct for the station being loaded.

15. Ensure the direction switch on the elevator is in the "down"

position and the power switch "off".

16. Check the setting on the hot wire power supply and turn it on. The wire should glow a dull red, not bright.

17. Without touching the filament or disturbing adjacent filaments, carefully burn through both sides of the paper carrier at the bottom of the slot.

18. Turn off the hot wire and return it to it's holder.

19. Select "CONT" on the HP-85 and move the elevator power switch to "on". Monitor the elevator as it lowers to ensure it does not bottom out prior to being unloaded.

20. Verify the PC records the station being loaded as the flag blocks the optical trigger.

NOTE. If the filament breaks before it is completely loaded, stop the elevator, record the mass displayed by the HP-85, and start again from step 24.

21. When the weight hangs free, the HP-85 will beep twice. When this happens, stop the elevator. After two minutes, the HP-85 will beep twice again, allowing you to continue.

22. Lower the elevator until it can be withdrawn without disturbing the filament.

23. Once the elevator has been withdrawn, carefully, **without disturbing the filament**, adjust the stabilizer screw so that it is centered on the vial without touching it and has a vial/stabilizer gap of

not more than 1 mm.

24. Load a sheet of paper into the plotter and select "shift k2" (PLOT) on the HP-85, answering the cues as required.

25. Record filament load information in Fiber Life Data Book and file load plot in the plot binder.

26. Repeat from step 2 as necessary.

27. When all desired stations have been loaded, verify all stabilizer bolts are positioned correctly and replace Plexiglass cover.

28. Verify PC is reporting the correct status for the stations loaded.

APPENDIX B

EQUIPMENT DESCRIPTION

Data for the experiment was collected by suspending fiber samples on the individual stations (512 total) of a large sample board. Load and fail status was actively monitored by individual infrared switchesthrough ribbon wire which fed the data to an interface box communicating with a PC-AT. (Figure B-1).

The sample board was made up of 512 stations arranged in 8 rows and 64 columns. Each station consisted of a mounting point, a high compliance spring (k=0.635 mm/g), an optical switch and trigger, a filament sample, a weight, and an adjustable stabilizer screw. The board was constructed of aluminum channel stock and fiberboard. When not being loaded, the stations were covered with a protective sheet of 1/4" PMMA.

The trigger was a wire paper clip with the inner loop removed. The top half of this outer loop was then wrapped in black vinyl electricians tape. The fiber sample, mounted in its paper carrier, was hung on this trigger which in turn hung from the bottom of the spring. With the weight attached to the bottom of the fiber carrier, the black tape blocked the optical switch. When the filament broke, the weight dropped away while the spring retracted the trigger allowing light to pass through the paper clip thus closing the optical switch, which was detected and recorded by the PC.

The weights were plastic vials filled with lead shot and were loaded to 3 weight levels: 12.20, 13.15, and 14.17 g. Two rows of AS-4 graphite, batch no. 008, and two rows of AS-4 batch no. 019 were loaded at 14.17 g., one row of 008 and one row of 019 were loaded at 13.15, and one row of 008 and one row of 019 were loaded at 12.20 g. It was decided to double the number of high weighted samples in order to obtain more data points prior to censoring the experiment.

Filaments were permanently mounted in paper carriers to facilitate a reliable and convenient method of storage and handling. Once assembled in it's proper station, the paper carrier was severed using a hot wire instead of scissors to reduce the chance of sending a shock through the filament. [Ref. 3]

Each of the 512 optical switches was connected to a custom designed interface box. This box contained a power supply for the optical switches as well as a multiplexing circuit which fed status information to a MetraByte I/O card in the PC. The board was divided into 64 8-bit words and software was written to cycle through each of the words and to report any changes in the value of the word. For example, if stations 1-8 are all loaded the PC reads a value of 0000 0000. If station 2 fails, the next time this byte is read, it would show a value of 0000 0010. The PC would then record the failure and continue monitoring the board. A backup power supply was connected to the interface box and PC in case of a filament failure during a power outage. PC software is reproduced in Appendix D.

Start times for each filament were also recorded by the PC when a bit change from 1 to 0 was detected. However, consistently accurate start times were highly dependent on a function of individual spring compliance, precise trigger construction , loader motor speed, and masses of the weights. In order to eliminate this variability, the loader software used to program the HP-85 also incorporated a timing mechanism. As the load cell was lowered, the HP-85 monitored the weight on the cell. When zero weight was detected, a beep sounded and a two minute timer was started. The load cell was left under the sample until expiration of the two minute time period. Should a sample have failed during the first two minutes of life, its lifetime would be recorded to the nearest 1/10 second. After two minutes, such precise record keeping was not necessary and timing was turned over to the PC.

The HP-85 was programmed to read the load cell approximately 5 times per second during the loading sequence and could therefore provide accurate data on the loads as they were applied. After loading, this information was sent to a plotter and a record of each load made for analysis. If a filament ruptured any time during the load sequence, up to the expiration of the two minute time period, the HP-85 would record the load on the filament at the time of failure or, if already loaded, the life of the filament up to the first two minutes.

Filaments were loaded by using a 150 g. capacity Sensotec load cell mounted to an electrically controlled, hydraulically actuated elevator. The electric motor, Western Gear model P5B24R3 incorporated a built in reduction gear and was run at 7.5 volts giving a start to finish load run of about 30 seconds yielding 170-220 data points as the load on the filament increased. The motor was mechanically connected to a disposable 5 cc hypodermic syringe which acted as the master cylinder. The slave cylinder was an identical syringe with the load cell mounted on a platform secured to the plunger. This arrangement was in turn mounted to a Plexiglass platform which provided a stable base during loading. The slave and master were connected with 3 feet of 1/8" ID vinyl tubing; the hydraulic fluid was tap water.

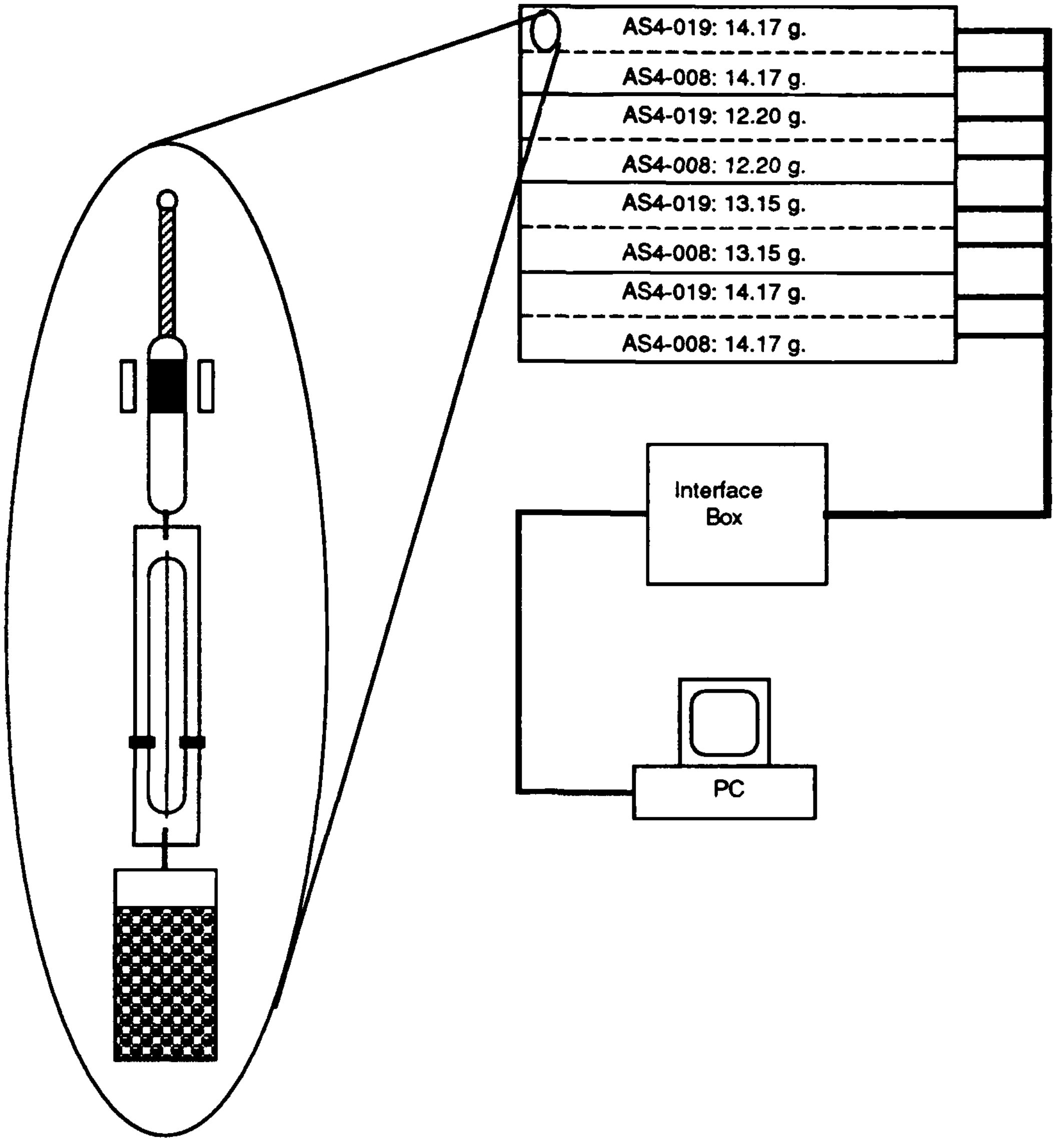

Figure B-1. Diagram of Fiber Life Testing Apparatus.

APPENDIX C

ULTRAVIOLET (UV) LIGHT SHIELDING

The life of certain fibers, (Kevlar for example) are known to be affected by exposure to UV radiation. To assure these life test experiments were performed in an inert environment, an investigation into the presence of UV light was conducted.

The sample rack was assembled in a secure laboratory environment on a basement level with windows facing Northeast. The rack itself was mounted on the Southeast wall and therefore, could not receive any direct sunlight. Windows were covered by opaque commercial grade drapery. Room lighting was fluorescent overhead with plastic covers. The sample racks were covered with 1/4 inch Polymethyl methacrylate (PMMA) Plexiglas. Light from the two sources was filtered twice before reaching the samples. The indirect sunlight passed through standard grade soda lime window glass and then through the PMMA rack covers. Overhead lighting was filtered through a plastic optical diffuser and then through the PMMA rack cover.

Tests for the intensity of 300-400 nm UV light were conducted using a 100 Watt mercury lamp at a distance of 30 ft. from the spectrometer. All measurements were made at the surface of the sample material. Results of the spectrum analysis are shown in Figures C-1 through C-3. To explore materials for improved UV shielding, analysis was also done on

Polycarbonate (Lexan) and a UV filtering grade of PMMA (Acrylite OP-2). Results from these two materials are shown in Figures C-4 and C-5. The improvement in shielding appeared to be a small range of an additional 25 nm to that already provided by the PMMA and was therefore not deemed to be significant.

Based on these measurements, the amount of UV light reaching the actual fiber samples was judged to be insufficient to affect life testing.

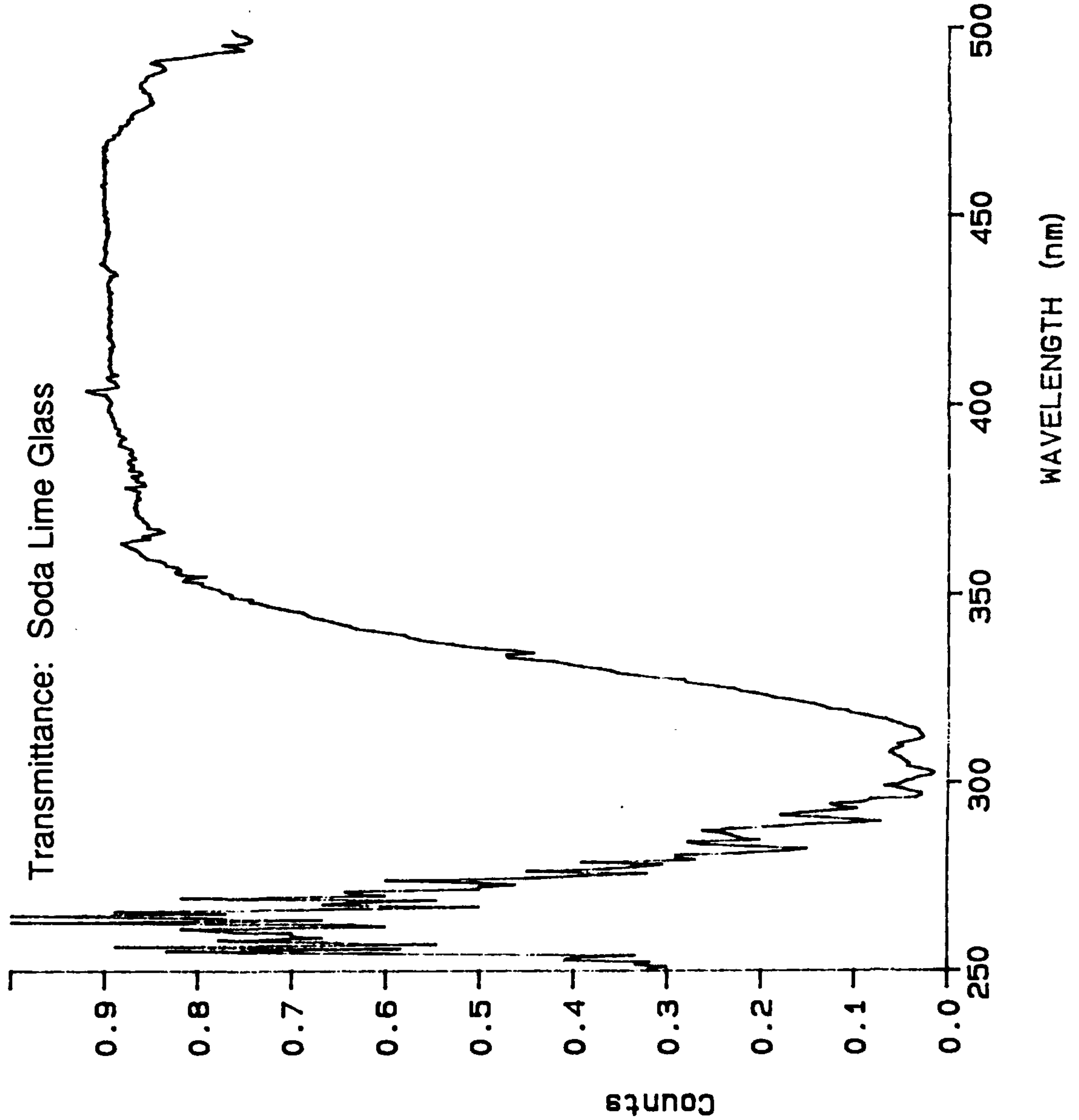

Figure C-1. Results of UV Transmittance test for Soda Lime Glass.

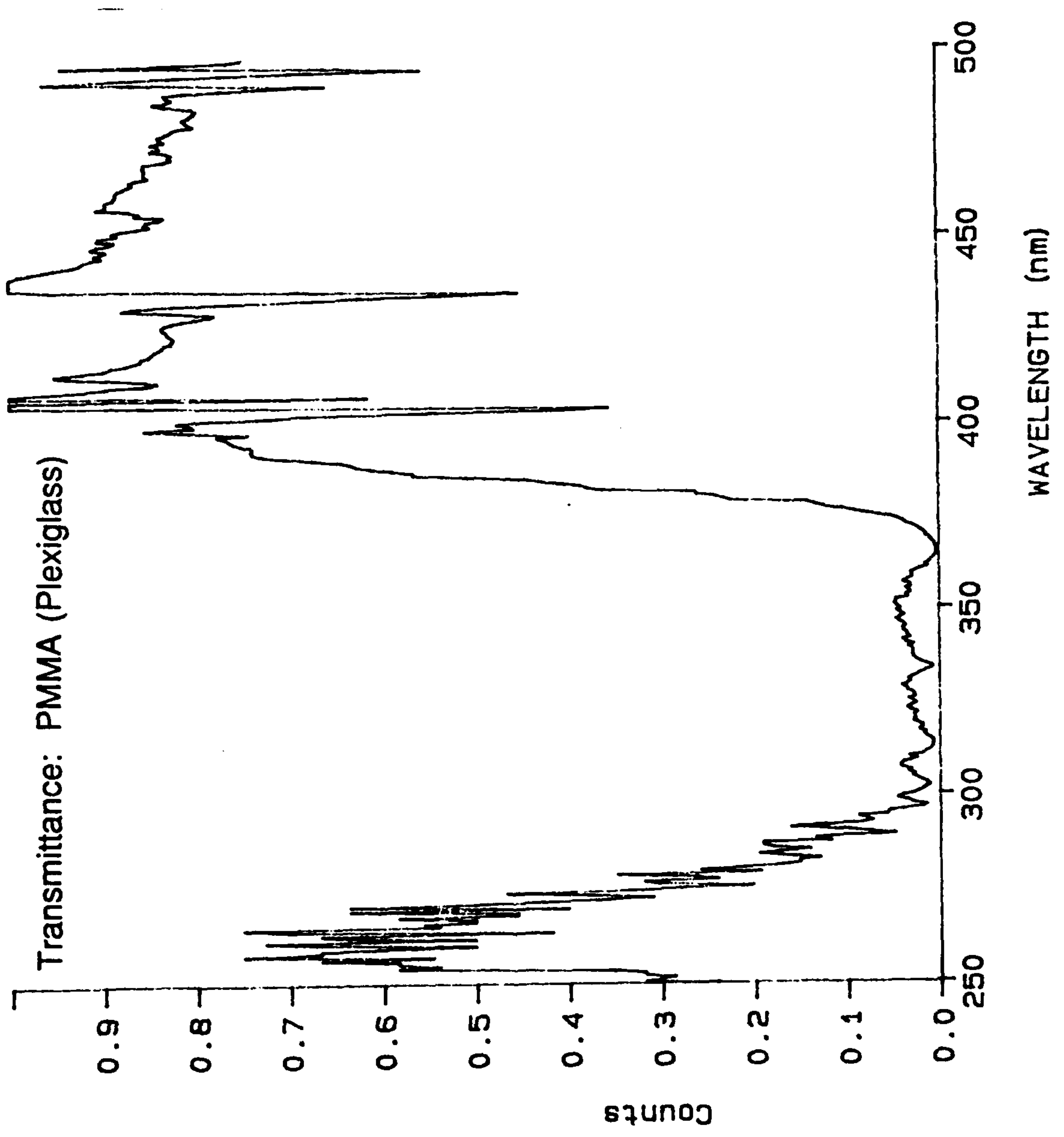

Figure C-2. Results of UV Transmittance test for PMMA.

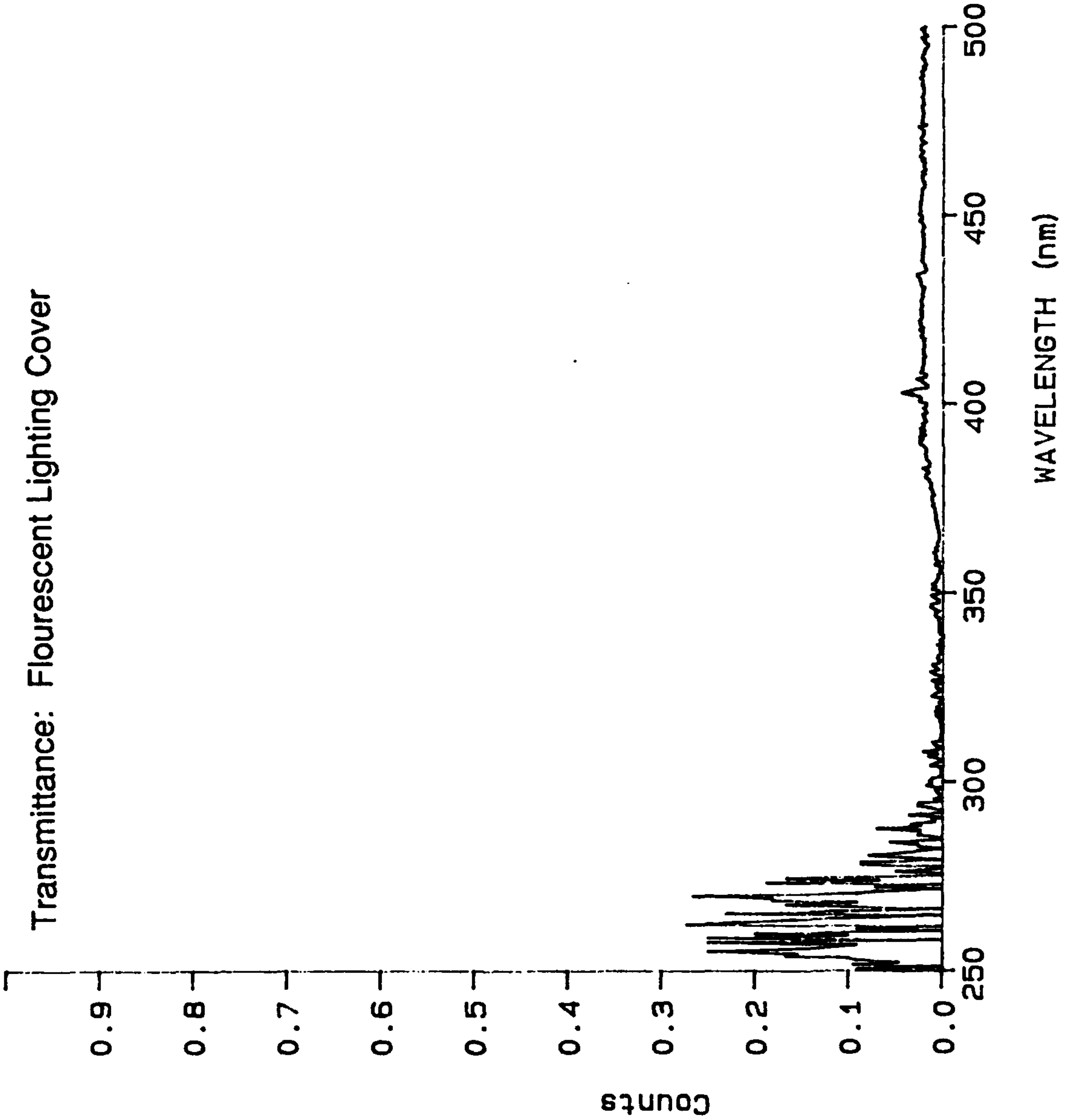

Figure C-3. Results of UV Transmittance test for Installed Flourescent Lighting Cover.

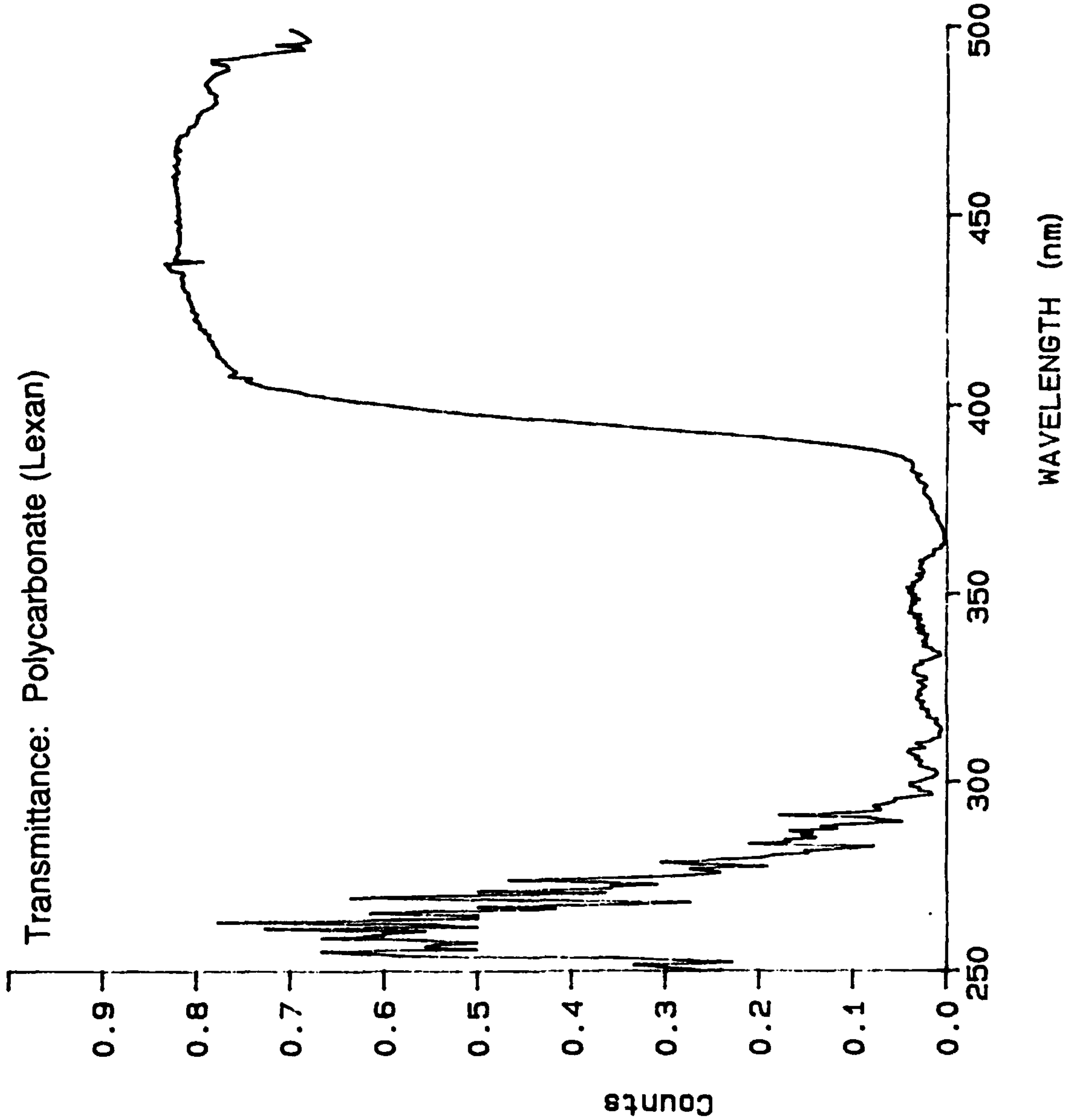

Figure C-4. Results of UV Transmittance test for Polycarbonate.

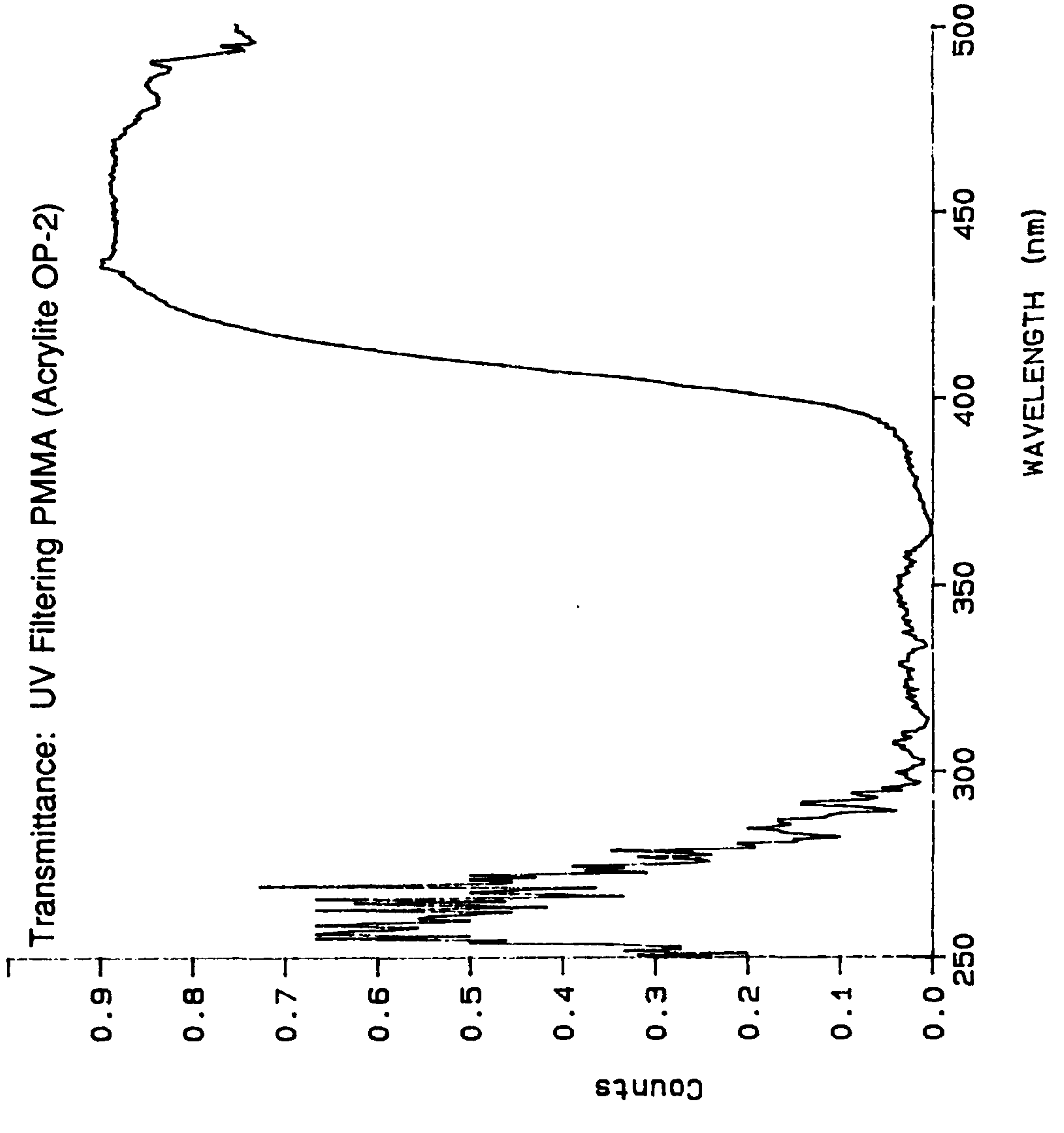

Figure C-5. Results of UV Transmittance test for UV Filtering PMMA.

APPENDIX D

<u>MAIN PROGRAM:</u>

```
     '** Filamod7 **
     ' Jim Nageotte Nov 13, 1990

Start:
     DIM newrd%(64), OldRd%(64), row%(8),      * Dimension Variables
     Status$(2), StarTime$(512)                *
     row%(1) = 1: row%(2) = 2: row%(3) = 4:    * Declare constants
     row%(4) = 8: row%(5) = 16: row%(6) = 32:
     row%(7) = 64: row%(8) = 128
     a% = 0

     OUT &H303, &H10 '                         * Reset I/O Card

     TYPE Status2                              * Declare file type
          OldRd AS INTEGER                     * for status file
     END TYPE
     DIM Stat AS Status2

     TYPE Times                                * Declare file type
          StarTime AS STRING * 16              * for Start times file
     END TYPE
     DIM Tim AS Times

     CLS
     PRINT "Ver 11.13.90"
     GOSUB Gettime
     PRINT "Power up at "; TIME$; " on ";      * Print start up message
     DATE$                                     * & current date on screen.
     PrintStat = INP(&H3BD)    '               * Read printer status.
                                               * 223 indicates that the
     IF PrintStat = 223 THEN   '               * printer is ready.
        LPRINT "Power up at "; TIME$;
        " on "; DATE$                          * Print start up message
        LPRINT : LPRINT                        * only if printer is ready.
     END IF

     OPEN "Status.Dat" FOR RANDOM              * Open Status File for
     AS #1 LEN = 2      '                      * random access.
     FOR idx = 1 TO 64 '                       * Read previous status
        GET #1, idx, Stat                      * from disk file.
        OldRd%(idx) = Stat.OldRd
     NEXT idx
     CLOSE #1                                  * Close Status File.

     OPEN "Times.Dat" FOR RANDOM AS #2         * Open Start Times File
     LEN = 16      '                           * and read start times
     FOR i = 1 TO 512                          * for all 512 stations.
       GET #2, i, Tim
       StarTime$(i) = Tim.StarTime
     NEXT i
```

```
CLOSE #2                                      * Close Start times file.

DO
    GOSUB Checksw                                 * Read 8 bit status words.
    FOR i = 1 TO 64                               * until something changes.

    IF newrd%(i) = OldRd%(i) THEN GOTO Same
    a0% = OldRd%(i): a1% = newrd%(i)              * Compares Status read from
    FOR j = 1 TO 8: jdx% = 9 - j                  * disk to that read from
       b0% = FIX(a0% / row%(jdx%))                * the switches. If they are
       b1% = FIX(a1% / row%(jdx%))                * not equal it looks more
       IF b0% = b1% THEN GOTO Nextbit             * closely at the switches.
       Station = (i - 1) * 8 + jdx%
        IF b1% < 0 OR b1% > 1 THEN                * 0 or 1 are the only valid
          PRINT "Error in Status b1%"             * values for b1%.
          BEEP
        END IF
        IF b1% = 1 THEN                           *"1" indicates "Loaded"
         PRINT "Station #"; Station;              * Print loaded message to
         "Loaded  "; RdTime$; " on ";             * screen.
         RdDate$
         GOSUB Store                              * Store new Start time to
                                                  * disk file.
         PrintStat = INP(&H3BD)                   * Check printer Status.
         IF PrintStat = 223 THEN                  * If printer is ready
           LPRINT "Station #";                    * then print loaded
           Station; "   Loaded ";                 * message to printer.
           LPRINT RdTime$; " on ";
           RdDate$
         END IF
         BEEP
        END IF

     IF b1% = 0 THEN                              * "0" indicates failed.
      GOSUB Elapsed                               * Find elapsed time.
      PRINT:PRINT "Station #"; Station;           * Print failure information
      "  "; StarTime$(Station);"to";PRINT         * to screen.
      LEFT$(RdTime$, 5); " ";RdDate$:PRINT
      " "; YEARS; "Years, "; DayT; "Days ";
      PRINT Min; " Minutes": PRINT
      PrintStat = INP(&H3BD)                      * Check printer Status.
      IF PrintStat = 223 THEN                     * If printer is ready,
        LPRINT " ": LPRINT "Station #";           * then print failure
        Station;"    ";                           * information to printer.
        StarTime$(Station);" to ";
        LEFT$(RdTime$, 5);" "; RdDate$
        LPRINT " ";YEARS; "Years, "; DayT; "Days ";
        LPRINT Min; "Minutes"
        LPRINT " "
      END IF
```

```
     END IF



Nextbit:
     a0% = a0% - b0% * row%(jdx%): a1% = a1% - b1% * row%(jdx%)
     NEXT j

     OldRd%(i) = newrd%(i)
     OPEN "Status.dat" FOR RANDOM AS
     #1 LEN = 2                             * Update disk Status file.
     Stat.OldRd = OldRd%(i)
     PUT #1, i, Stat
     CLOSE
Same:
     NEXT i

     LOOP
     END                                    * End of loop that compares
                                            * old status to new and end
                                            * of main program.
```

SUBROUTINES:

Check Switch compares the current status to the status read off the disk file. It loops until there is a change. It then sets a flag and returns with the current time and date to start checking each of the 512 stations and determine what type of change has taken place. By checking the status in 8 bit words and in such a small loop, we keep the time required to check all switches and the response time to a minimum.

```
Checksw:
     DO
     GOSUB Gettime
     a% = 0: idx = 0: RdTime$ = TIME$       * Get current time.
     RdDate$ = DATE$                        * Get current date.
     FOR i = 1 TO 8: OUT &H301, row%(i)     * Select Row to read.
       FOR j = 0 TO 7: OUT &H302, j         * Select word to read.
         idx = idx + 1
         newrd%(idx) = INP(&H300)
           IF newrd%(idx) <> OldRd%(idx)    * Set Flag if Status has
             THEN a% = 1                    * changed.
       NEXT j
     NEXT i
     LOOP UNTIL a% = 1                      * Loop until Status has
     BEEP                                   * changed.
     RETURN
```

Elapsed Time calculates the time between the Start time read from disk and the current time. It calls two other sub routines to do this, Julian and Minutes. Julian is called twice, the first time it converts the Start time to a Julian date and the second time it converts the current date to a Julian date. The conversion is necessary to allow subtraction to be performed on the dates. Having arrived at the number of Years and Days before the fiber ruptured, we call the subroutine Minutes to subtract the times involved and return with the correct number of minutes. An adjustment to the number of days may be made based on the number of minutes.

```
Elapsed:
     ConvDate$=RIGHT$(StarTime$(Station),10) * Select Start Date.
     GOSUB Julian                            * Convert to julian date.
     StartYr = Year
     StartDy = Days
     ConvDate$ = RdDate$                     * Select Current Date.
     GOSUB Julian                            * Convert to julian date.
     StopYr = Year
     StopDy = Days
     YEARS = StopYr - StartYr                * Subtract years.
     DayT = StopDy - StartDy                 * Subtract days.
     IF DayT < 0 THEN
          DayT = DayT + 365
          YEARS = YEARS - 1
     END IF
     GOSUB Minutes                           * Calculate minutes.
     IF Min < 0 THEN
          Min = Min + 1440
          DayT = DayT - 1
     END IF
     RETURN

Julian:
     Mon = VAL(LEFT$(ConvDate$, 2))         * Get month, day and year
     Day = VAL(MID$(ConvDate$, 4, 2))       * from the date string.
     Year = VAL(RIGHT$(ConvDate$, 2))
     SELECT CASE Mon
          CASE 1
               Days = Day
          CASE 2
               Days = Day + 31
          CASE 3
               Days = Day + 59
          CASE 4
               Days = Day + 90                * Calculate total days.
          CASE 5
               Days = Day + 120
          CASE 6
               Days = Day + 151
          CASE 7
               Days = Day + 181
```

```
        CASE 8
             Days = Day + 212
        CASE 9
             Days = Day + 243
        CASE 10
             Days = Day + 273
        CASE 11
             Days = Day + 304
        CASE ELSE
             Days = Day + 334
    END SELECT
    RETURN
```

Store simply opens the Start Time file on disk, a random access file, and stores only the new time without reading or re-writing the unaffected times. The file is then closed and the routine exited.

```
Store:
     OPEN "Times.Dat" FOR RANDOM AS #2 LEN = 16
     Tim.StarTime = LEFT$(RdTime$, 5) + " " + RdDate$
     PUT #2, Station, Tim
     CLOSE #2
     StarTime$(Station) = Tim.StarTime
     RETURN
```

Minutes recovers the values of Minutes and Hours from the time strings and converts them to Minutes only for ease of mathematical operations in the time keeping.

```
Minutes:
     HrSt$ = LEFT$(StarTime$(Station), 2)
     Min$ = MID$(StarTime$(Station), 4, 2)
     Hrs = VAL(HrSt$)
     MinSt = VAL(Min$)
     MinS = Hrs * 60 + MinSt
     Hrf$ = LEFT$(RdTime$, 2)
     MinF$ = MID$(RdTime$, 4, 2)
     Hrf = VAL(Hrf$)
     MinFn = VAL(MinF$)
     MinF = Hrf * 60 + MinFn
     Min = MinF - MinS
     RETURN
```

Gettime reads I/O ports to read the time directly from the system clock. It then converts these values to a string that resembles the "TIME$" function. The program then uses this string called "Rtime$" for Real Time String.

```
Gettime:
     Sec$ = LTRIM(STR$(INP(&H2C0) AND &HF))
     Secs$ = LTRIM(STR$(INP(&H2C1) AND &HF))
     Minute$ = LTRIM(STR$(INP(&H2C2) AND &HF))
     Minutes$ = LTRIM(STR$(INP(&H2C3) AND &HF))
     Hour$ = LTRIM(STR$(INP(&H2C4) AND &HF))
     Hours$ = LTRIM(STR$(INP(&H2C5) AND &HF))
     Rtime$ = "00:00:00"
     MID$(Rtime$, 1) = Hours$
     MID$(Rtime$, 2) = Hour$
     MID$(Rtime$, 4) = Minutes$
     MID$(Rtime$, 5) = Minute$
     MID$(Rtime$, 7) = Secs$
     MID$(Rtime$, 8) = Sec$
     RETURN
```

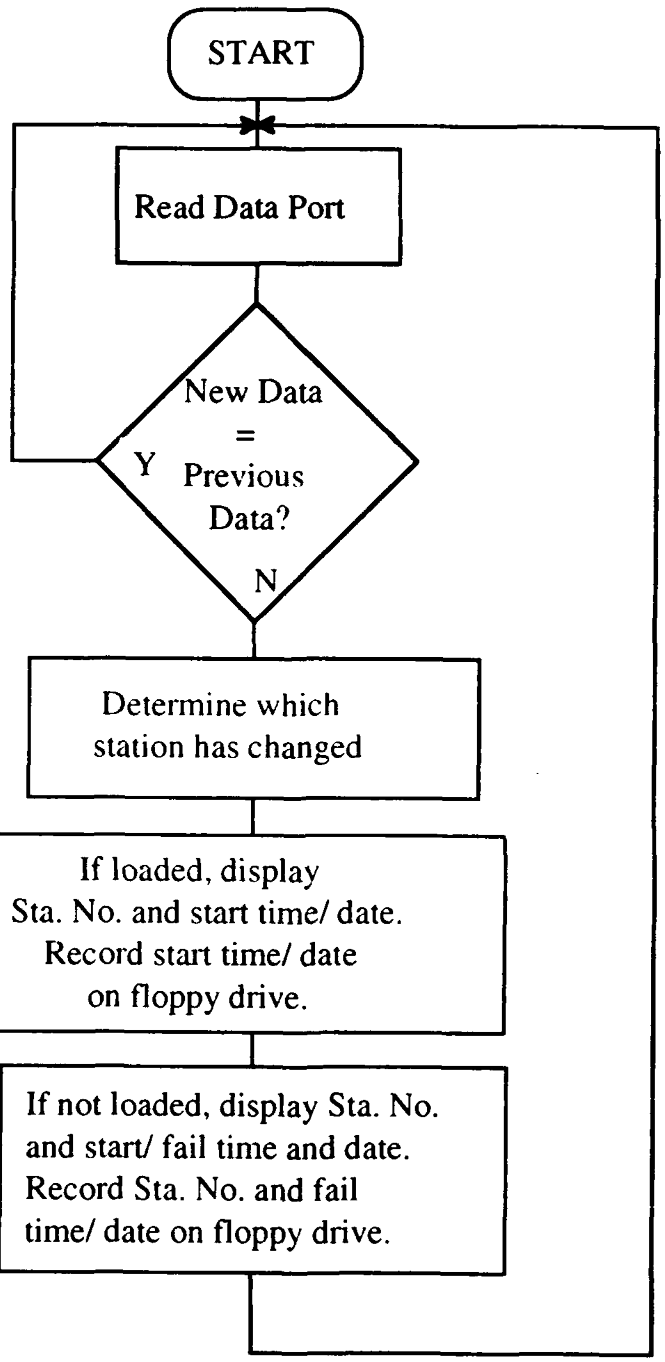

Figure D-1. Flowchart for Life Test Monitoring Program.

APPENDIX E

I. LIKELIHOOD DERIVATION FOR EXACT DATA

For Exact Data:

$$L_{exact} = f(x_i, \alpha, \beta) = d/dx\ F(x_i, \alpha, \beta)$$

$$:F(x,\alpha,\beta) = 1 - \exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\}$$

Taking the derivative with respect to x:

$$f(x) = \frac{\partial F}{\partial x} = -\exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\} \cdot \left(\frac{1}{\beta^{\alpha}}\alpha x^{\alpha-1}\right)$$

$$= \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\}$$

The Likelihood for all data, x_i, =

$$L(x_i) = \prod_{i=1}^{n} f(x_i, \alpha, \beta)$$

$$= \prod_{i=1}^{n} \left[\frac{\alpha}{\beta} \left(\frac{x}{\beta} \right)^{\alpha-1} \mathbf{exp} \left\{ - \left(\frac{x_i}{\beta} \right)^{\alpha} \right\} \right] \qquad (1)$$

Rearranging,

$$= \prod_{i=1}^{n} \left[\alpha\beta^{-1} \beta^{(1-\alpha)} \mathbf{exp} \left\{ - \left(\frac{x_i}{\beta} \right)^{\alpha} \right\} \right]$$

$$= \prod_{i=1}^{n} \left[\alpha\beta^{-\alpha} x_i^{(\alpha-1)} \mathbf{exp} \left\{ - \left(\frac{x_i}{\beta} \right)^{\alpha} \right\} \right]$$

$$= \alpha^{n} \beta^{-n\alpha} \prod_{i=1}^{n} x_i^{(\alpha-1)} \mathbf{exp} \left\{ - \left(\frac{x_i}{\beta} \right)^{\alpha} \right\}$$

$$= \alpha^{n} \beta^{-n\alpha} \prod_{i=1}^{n} x_i^{(\alpha-1)} \mathbf{exp} \left\{ - \beta^{-\alpha} \sum_{i=1}^{n} x_i^{\alpha} \right\} \qquad (2)$$

Let $\Lambda = \ln L = \ln f(x_i,\alpha,\beta)$

$$= \ln\left[\alpha^n\beta^{-n\alpha}\prod_{i=1}^{n} x_i^{(\alpha-1)} \exp\left\{-\beta^{-\alpha}\sum_{i=1}^{n} x_i^{\alpha}\right\}\right]$$

$$= n\ln\alpha - n\alpha\ln\beta + \sum_{i=1}^{n}\ln\left[x_i^{(\alpha-1)} \exp\left\{-\beta^{-\alpha}\sum_{i=1}^{n} x_i^{\alpha}\right\}\right]$$

$$= n\ln\alpha - n\alpha\ln\beta + (\alpha-1)\ln\sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\ln\left[\exp\left\{-\beta^{-\alpha}\sum_{i=1}^{n} x_i^{\alpha}\right\}\right]$$

$$= n\ln\alpha - n\alpha\ln\beta + (\alpha-1)\ln\sum_{i=1}^{n} x_i - \beta^{-\alpha}\sum_{i=1}^{n} x_i^{\alpha} \qquad (3)$$

To find the maximum Likelihood, (L_{max}), set the derivative of Λ with respect to α and β, to zero:

$$\frac{\partial\Lambda}{\partial\alpha} = \frac{\partial\Lambda}{\partial\beta} = 0$$

Taking the partial derivative of eq. (3) with respect to α and β and setting it to zero yields:

$$\frac{\partial \Lambda}{\partial \alpha} = \frac{n}{\alpha} - n \ln \beta + \sum_{i=1}^{n} \ln x_i + \beta^{-\alpha} \ln \beta \sum_{i=1}^{n} x_i^{\alpha} - \beta^{-\alpha} \sum_{i=1}^{n} x_i^{\alpha} \ln x_i = 0 \qquad (4)$$

$$\frac{\partial \Lambda}{\partial \beta} = -\frac{n\alpha}{\beta} + \frac{\alpha}{\beta^{(\alpha+1)}} \sum_{i=1}^{n} x_i^{\alpha} = 0 \qquad (5)$$

Rearranging eq. (5):

$$\frac{n\alpha}{\beta} = \frac{\alpha}{\beta^{(\alpha+1)}} \sum_{i=1}^{n} x_i^{\alpha}$$

$$n = \frac{1}{\beta^{\alpha}} \sum_{i=1}^{n} x_i^{\alpha} \qquad (6)$$

$$\beta = \left(\frac{1}{n} \sum_{i=1}^{n} x_i^{\alpha} \right)^{\frac{1}{\alpha}} \qquad (7)$$

Now replace the n in the 2nd term of eq.(4), with eq. (6):

$$\frac{n}{\alpha} - \left(\frac{1}{\beta^{\alpha}}\sum_{i=1}^{n} x_i^{\alpha}\right)\ln\beta + \sum_{i=1}^{n}\ln x_i + \beta^{-\alpha}\ln\beta\sum_{i=1}^{n} x_i^{\alpha} - \beta^{-\alpha}\sum_{i=1}^{n} x_i^{\alpha}\ln x_i = 0$$

and cancel with the 4th term:

$$\frac{n}{\alpha} + \sum_{i=1}^{n}\ln x_i - \beta^{-\alpha}\sum_{i=1}^{n} x_i^{\alpha}\ln x_i = 0$$

From eq.(6), substitute

$$n\,\beta^{\alpha} = \sum_{i=1}^{n} x_i^{\alpha}$$

and divide both sides by n yielding:

$$\frac{1}{\alpha} + \frac{1}{n}\sum_{i=1}^{n}\ln x_i - \frac{\sum_{i=1}^{n} x_i^{\alpha}\ln x_i}{\sum_{i=1}^{n} x_i^{\alpha}} = 0$$

Rearranging to set the right side equal to α:

$$\frac{1}{\alpha} = \frac{\sum_{i=1}^{n} x_i^{\alpha} \ln x_i}{\sum_{i=1}^{n} x_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n} \ln x_i$$

,or

$$\alpha = \frac{1}{\dfrac{\sum_{i=1}^{n} x_i^{\alpha} \ln x_i}{\sum_{i=1}^{n} x_i^{\alpha}} - \dfrac{1}{n}\sum_{i=1}^{n} \ln x_i} \qquad (8)$$

APPENDIX F

I. LIKELIHOOD DERIVATION FOR EXACT DATA WITH TYPE I AND TYPE I-V CENSORING

For Exact data which has both Type I and Type I-v (Variable: Cause of failure not quantifiable in this experiment) censoring:

$$\text{Likelihood } L = \left[\prod f(x_i,\alpha,\beta)\right]\prod\left[1-F(\hat{x}_i,\alpha,\beta)\right]\left[1-F(x_j,\alpha,\beta)\right]^{(n-m)}$$

Exact Type I-v Type I

$$: f(x_i,\alpha,\beta)=\frac{\partial}{\partial t}1-\exp\left\{-\left(\frac{x_i}{\beta}\right)^{\alpha}\right\}$$

$$: 1-F(\hat{x}_i,\alpha,\beta)=\exp\left\{-\left(\frac{\hat{x}_i}{\beta}\right)^{\alpha}\right\}$$

$$: 1-F(x_j,\alpha,\beta)=\exp\left\{-\left(\frac{x_j}{\beta}\right)^{\alpha}\right\}$$

: $n \triangleq$ The total number of fibers

$m \triangleq$ The total number of fibers that failed

$m_l \triangleq$ The number of fibers failing in life

$m - m_l \triangleq$ The number of fibers subject to Type I – v censoring

Therefore,

$$L = \left[\alpha^{m_l} \beta^{-m_l \alpha} \prod_{i=1}^{m_l} x_i^{\alpha-1} \exp\left\{ -\beta^{-\alpha} x_i^{\alpha} \right\} \right]$$

$$\left[\prod_{i=m_l}^{m} \exp\left\{ -\left(\frac{\hat{x}_i}{\beta} \right)^{\alpha} \right\} \right] \left[\exp\left\{ -\left(\frac{x_j}{\beta} \right)^{\alpha} \right\} \right]^{(n-m)} \qquad (1)$$

Let $\Lambda \equiv \ln L$

$$= \ln\left[\alpha^{m_l} \beta^{-m_l \alpha} \prod_{i=1}^{m_l} x_i^{\alpha-1} \exp\left\{ -\beta^{-\alpha} x_i^{\alpha} \right\} \right] +$$

$$\ln\left[\prod_{i=m_l}^{m} \exp\left\{ -\left(\frac{\hat{x}_i}{\beta} \right)^{\alpha} \right\} \right] + \ln\left[\left[\exp\left\{ -\left(\frac{x_j}{\beta} \right)^{\alpha} \right\} \right]^{(n-m)} \right]$$

$$= m_l \ln\alpha - m_l \alpha \ln\beta + (\alpha - 1) \sum_{i=1}^{m_l} \ln x_i - \beta^{-\alpha} \sum_{i=1}^{m_l} x_i^{\alpha} -$$

$$\beta^{-\alpha} \sum_{i=m_l}^{m} \hat{x}_i^{\alpha} - (n-m)\left(\frac{x_j}{\beta} \right)^{\alpha} \qquad (2)$$

To find the maximum likelihood of L,

Set $\frac{\partial \Lambda}{\partial \alpha} = \frac{\partial \Lambda}{\partial \beta} = 0$

Taking the partial derivative of eq.(2) with respect to α yields:

$$\frac{\partial \Lambda}{\partial \alpha} = \frac{m_1}{\alpha} - m_1 \ln \beta + \sum_{i=1}^{m_1} \ln x_i - \left[\beta^{-\alpha} \sum_{i=1}^{m_1} x_i^{\alpha} \ln x_i - \beta^{-\alpha} \ln \beta \sum_{i=1}^{m_1} x_i^{\alpha}\right] -$$

$$\left[\beta^{-\alpha} \sum_{i=m_1}^{m} \hat{x}_i^{\alpha} \ln \hat{x}_i - \beta^{-\alpha} \ln \beta \sum_{i=m_1}^{m} \hat{x}_i^{\alpha}\right] - (n-m)\left(\frac{x_j}{\beta}\right)^{\alpha} \ln\left(\frac{x_j}{\beta}\right) = 0 \qquad (3)$$

Taking the partial derivative of eq.(2) with respect to β yields:

$$\frac{\partial \Lambda}{\partial \beta} = -m_1 \alpha \beta^{-1} + \alpha \beta^{-\alpha-1} \sum_{i=1}^{m_1} x_i^{\alpha} + \alpha \beta^{-\alpha-1} \sum_{i=m_1}^{m} \hat{x}_i^{\alpha} + \alpha \beta^{-\alpha-1} (n-m) x_j^{\alpha} = 0$$

Multiplying through by $\frac{\beta^{\alpha+1}}{\alpha}$,

$$-m_1 \beta^{\alpha} + \sum_{i=1}^{m_1} x_i^{\alpha} + \sum_{i=m_1}^{m} \hat{x}_i^{\alpha} + (n-m) x_j^{\alpha} = 0$$

Now divide by m_l and rearrange to isolate β:

$$\beta^{\alpha} = \frac{1}{m_l}\left(\sum_{i=1}^{m_l} x_i^{\alpha} + \sum_{i=m_l}^{m} \hat{x}_i^{\alpha} + (n-m)x_j^{\alpha}\right)$$

$$\beta = \left[\frac{1}{m_l}\left(\sum_{i=1}^{m_l} x_i^{\alpha} + \sum_{i=m_l}^{m} \hat{x}_i^{\alpha} + (n-m)x_j^{\alpha}\right)\right]^{\frac{1}{\alpha}} \qquad \textbf{(4)}$$

To solve for α, first multiply eq. (3) by β^{α} and rearrange:

$$\beta^{\alpha}\left(\frac{m_l}{\alpha} - m_l \ln\beta + \sum_{i=1}^{m_l} \ln x_i\right) + \ln\beta \sum_{i=1}^{m_l} x_i^{\alpha} + \ln\beta \sum_{i=m_l}^{m} \hat{x}_i^{\alpha} + (n-m)x_j^{\alpha} \ln\beta =$$

$$\sum_{i=1}^{m_l} x_i^{\alpha} \ln x_i + \sum_{i=m_l}^{m} \hat{x}_i^{\alpha} \ln \hat{x}_i + (n-m)x_j^{\alpha} \ln x_j$$

Now gather the $\ln\beta$ terms,

$$\beta^{\alpha}\left(\frac{m_l}{\alpha} - m_l \ln\beta + \sum_{i=1}^{m_l} \ln x_i\right) + \ln\beta\left(\sum_{i=1}^{m_l} x_i^{\alpha} + \sum_{i=m_l}^{m} \hat{x}_i^{\alpha} + (n-m)x_j^{\alpha}\right) =$$

$$\sum_{i=1}^{m_l} x_i^{\alpha} \ln x_i + \sum_{i=m_l}^{m} \hat{x}_i^{\alpha} \ln \hat{x}_i + (n-m)x_j^{\alpha} \ln x_m$$

Divide both sides by m_l and rearrange to bring α to the left hand side:

$$\frac{1}{\alpha}=\frac{1}{m_l\beta^{\alpha}}\left\{\left[\sum_{i=1}^{m_l}x_i^{\alpha}\ln x_i+\sum_{i=m_l}^{m}\hat{x}_i^{\alpha}\ln\hat{x}_i+(n-m)x_j^{\alpha}\ln x_j\right]-\underbrace{\left[\sum_{i=1}^{m_l}x_i^{\alpha}+\sum_{i=m_l}^{m}\hat{x}_i^{\alpha}+(n-m)x_j^{\alpha}\right]}_{a}\ln\beta\right\}+$$

$$\ln\beta+\frac{1}{m_l}\sum_{i=1}^{m_l}\ln x_i \tag{5}$$

Obtain from eq. (4),

$$m_l\beta^{\alpha}=\sum_{i=1}^{m_l}x_i^{\alpha}+\sum_{i=m_l}^{m}\hat{x}_i^{\alpha}+(n-m)x_j^{\alpha} \tag{6}$$

and substitute for (a) in eq. (5):

$$\frac{1}{\alpha}=\frac{1}{m_l\beta^{\alpha}}\left\{\left[\sum_{i=1}^{m_l}x_i^{\alpha}\ln x_i+\sum_{i=m_l}^{m}\hat{x}_i^{\alpha}\ln\hat{x}_i+(n-m)x_j^{\alpha}\ln x_m\right]-\underbrace{\left[m_l\beta^{\alpha}\right]}_{a}\ln\beta\right\}+$$

$$\ln\beta+\frac{1}{m_l}\sum_{i=1}^{m_l}\ln x_i$$

Cancel the lnβ terms and invert both sides:

$$\alpha = \left[\frac{\left[\sum_{i=1}^{m_1} x_i^{\alpha} \ln x_i + \sum_{i=m_1}^{m} \hat{x}_i^{\alpha} \ln \hat{x}_i + (n-m) x_j^{\alpha} \ln x_j \right]}{m_1 \beta^{\alpha}} + \frac{1}{m_1} \sum_{i=1}^{m_1} \ln x_i \right]^{-1}$$

Finally, eliminate β by substituting eq. (6) for the denominator:

$$\alpha = \left[\frac{\left[\sum_{i=1}^{m_1} x_i^{\alpha} \ln x_i + \sum_{i=m_1}^{m} \hat{x}_i^{\alpha} \ln \hat{x}_i + (n-m) x_j^{\alpha} \ln x_j \right]}{\sum_{i=1}^{m_1} x_i^{\alpha} + (n-m) x_j^{\alpha} + \sum_{i=m_1}^{m} \hat{x}_i^{\alpha}} + \frac{1}{m_1} \sum_{i=1}^{m} \ln x_i \right]^{-1} \qquad (7)$$

APPENDIX G

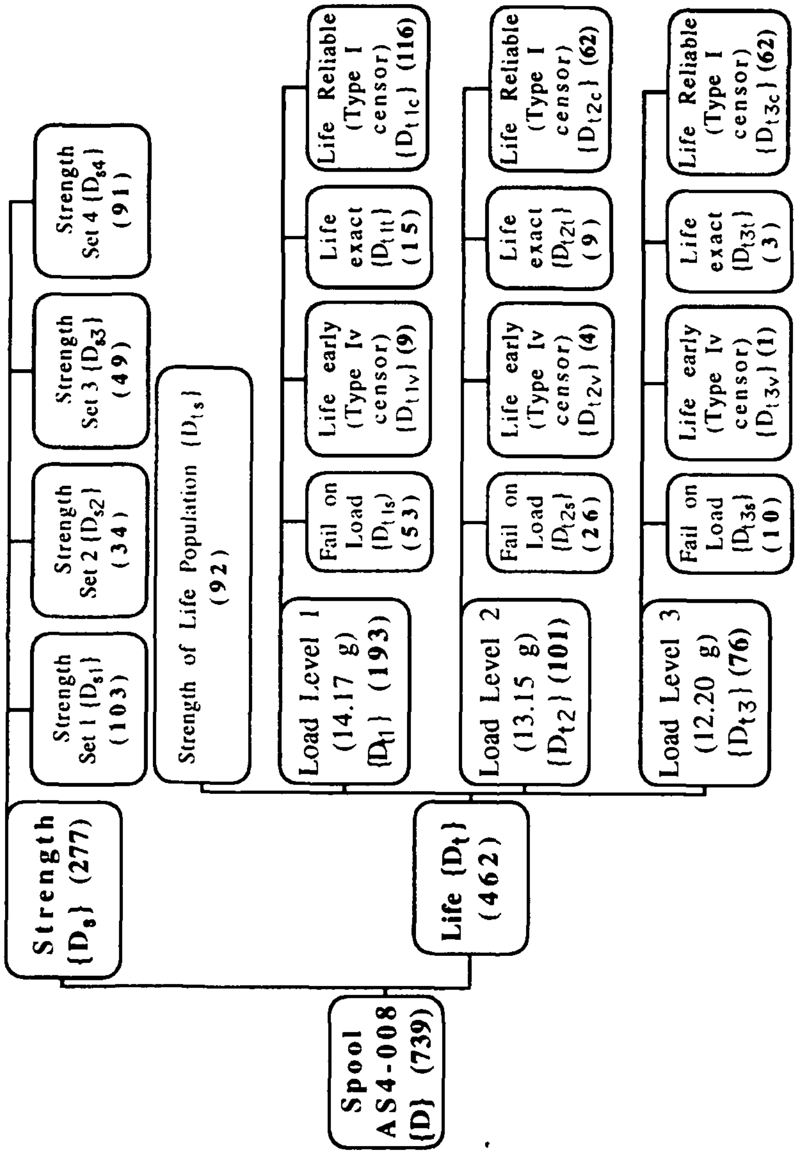

Figure 1. Relationship of Fiber sample sets from Spool AS4-008 population.

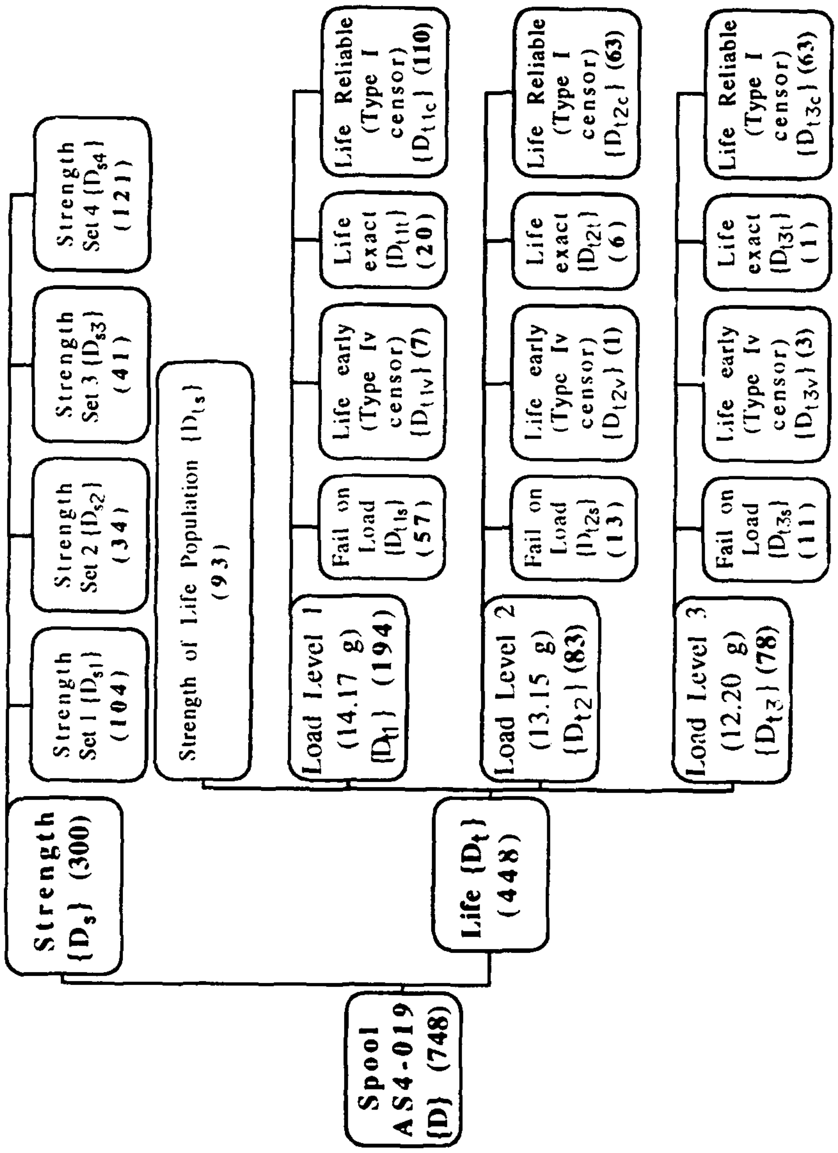

Figure 2. Relationship of Fiber sample sets from Spool AS4-019 population.

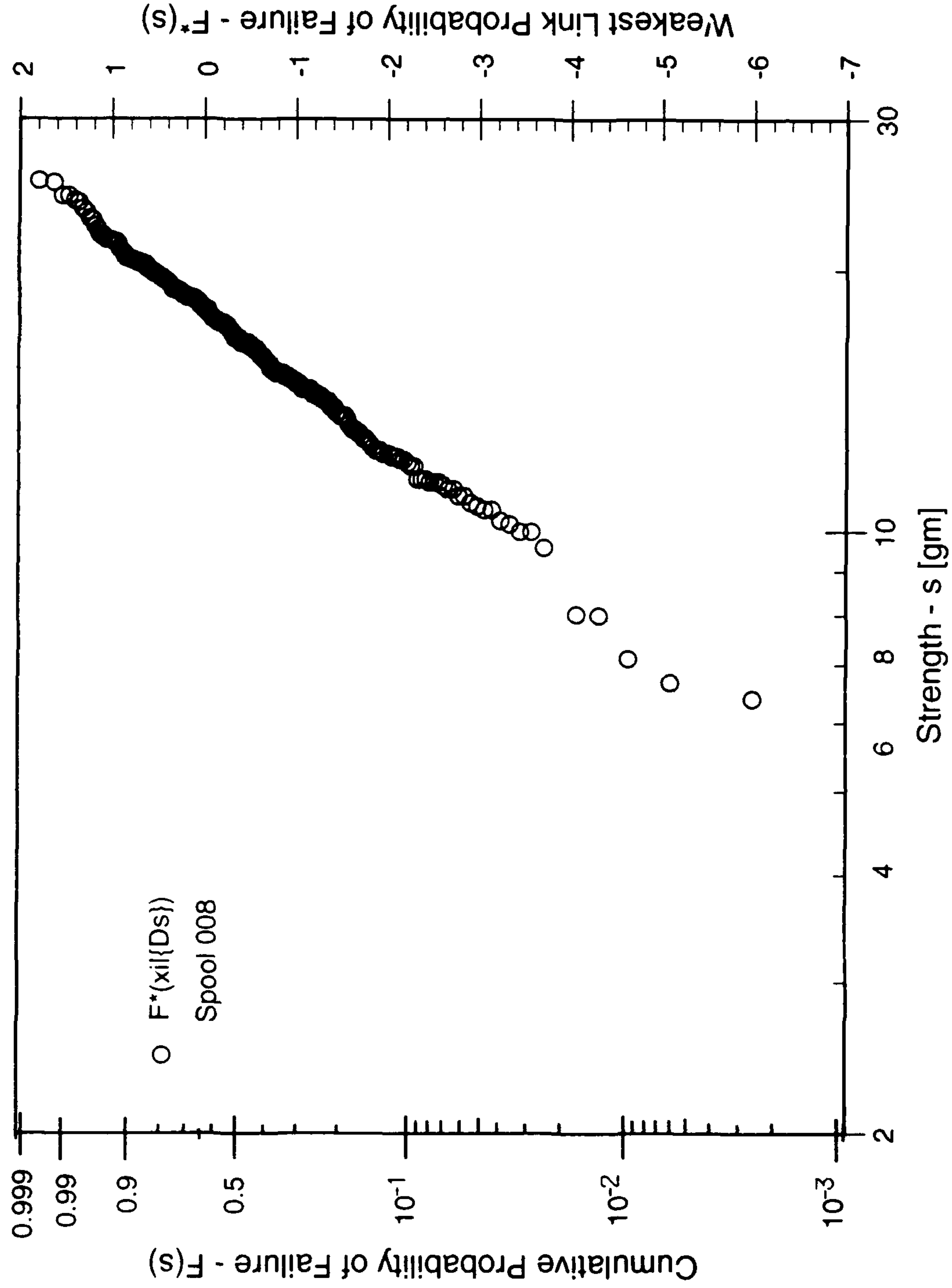

Figure 3. Expected Rank of 008 strength set $\{D_S\}$

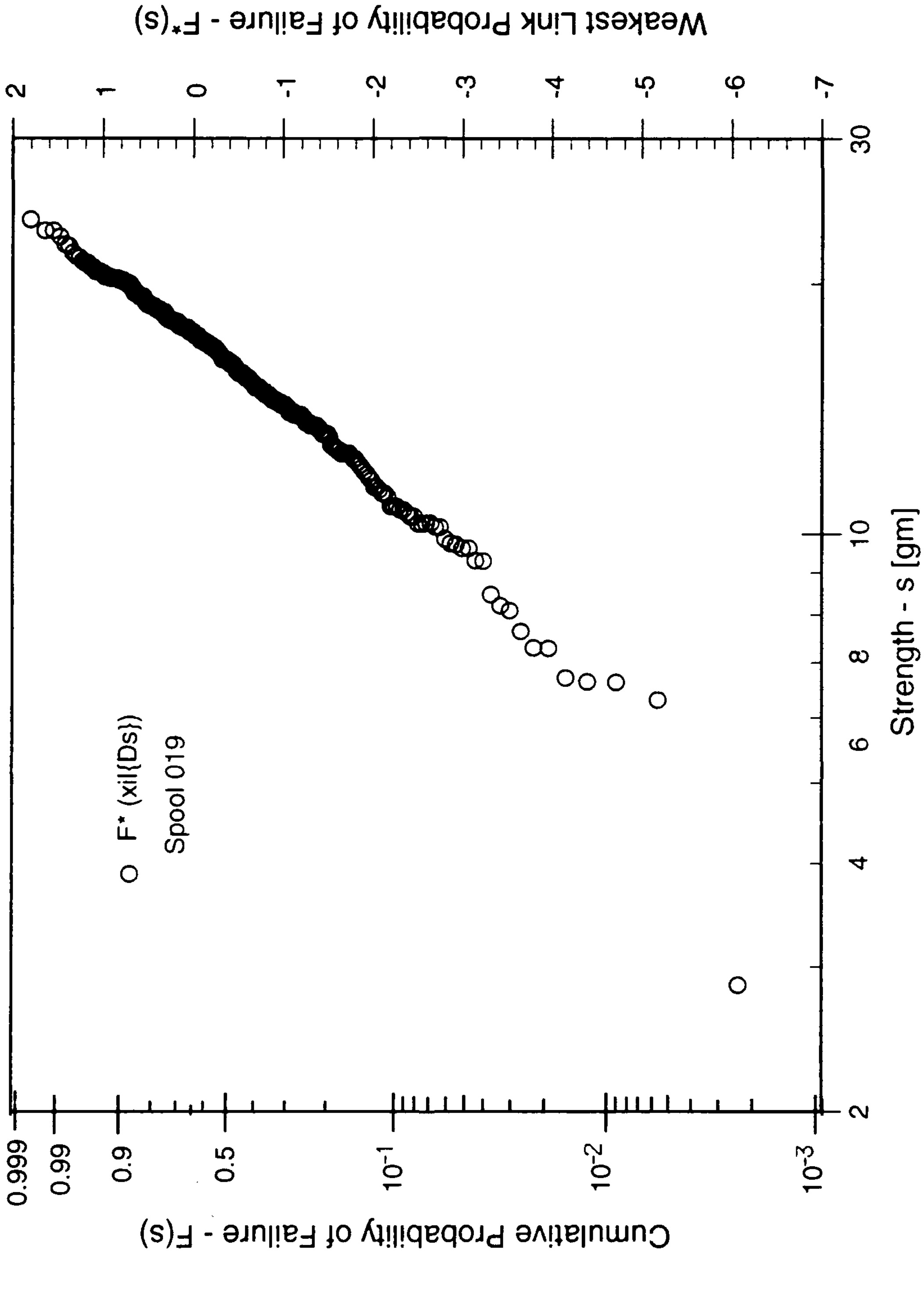

Figure 4. Expected Rank of 019 strength set $\{D_S\}$

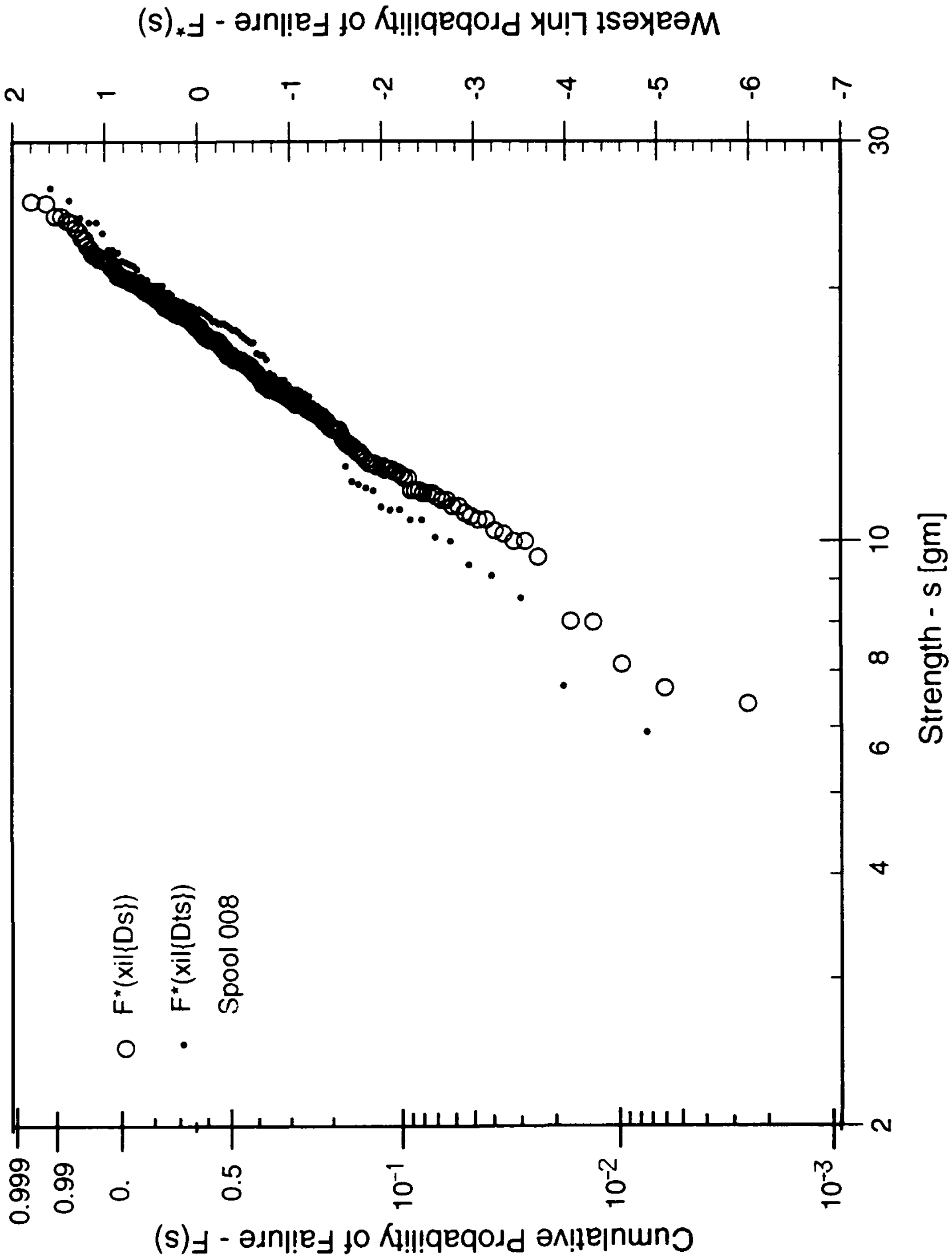

Figure 5. Expected Rank of 008 strength sets $\{D_S\}$ and $\{D_{tS}\}$

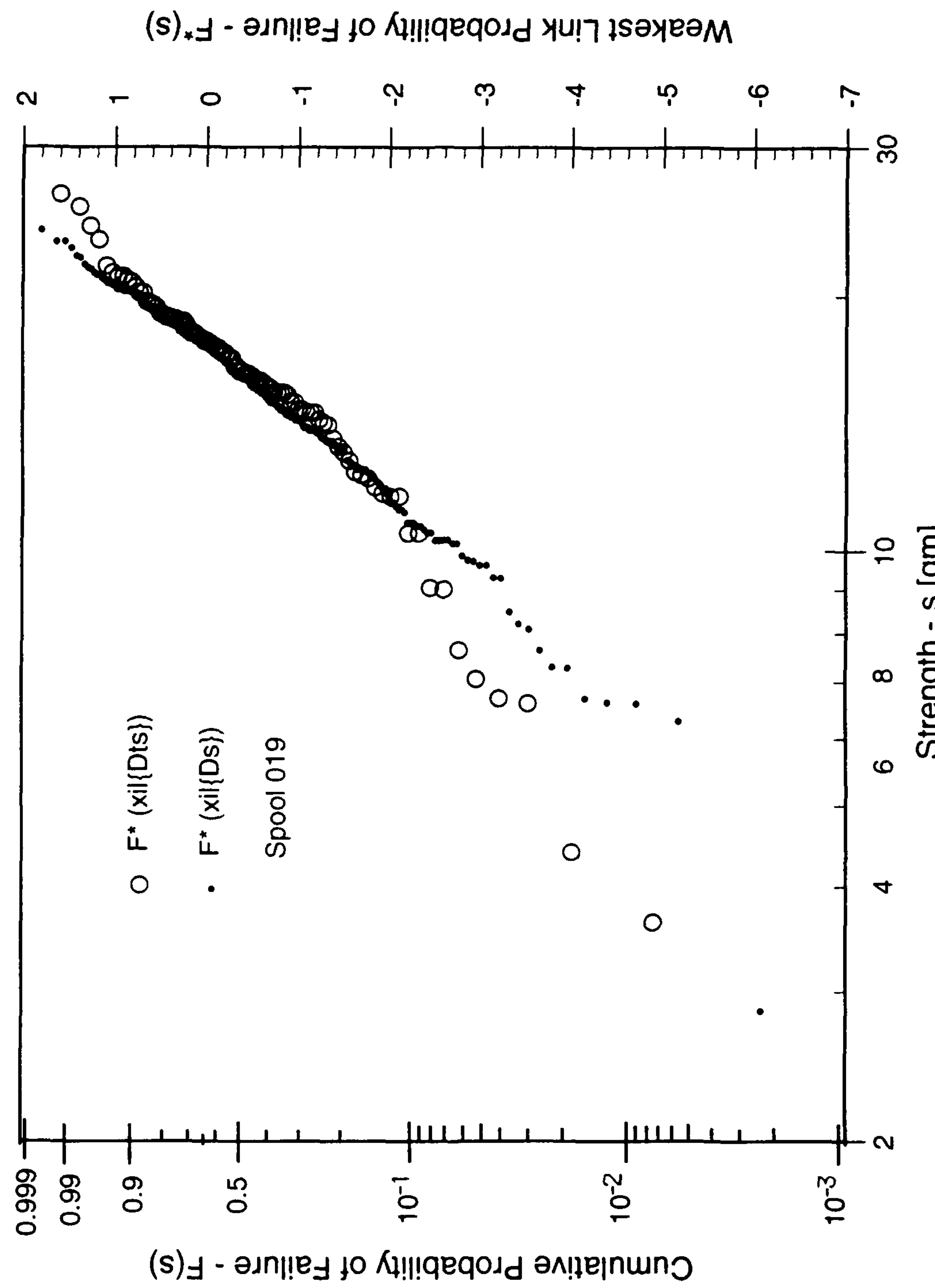

Figure 6. Expected Rank of 019 strength sets $\{D_s\}$ and $\{D_{ts}\}$

	A	B	C
1	AS4-008		
2			
3			
4			
5			
6	xi (Ds|Ns)	xi (Dts|Nts)	F*({Ds+Dts}|N
7		5.909	-6.277309565
8	6.393		-5.382913936
9	6.666		-4.917448539
10		6.711	-4.600324093
11	7.12		-4.359337455
12	7.997		-4.164810785
13	8.017		-4.001618953
14		8.54	-3.860998686
15	8.9		-3.737415596
16		9.085	-3.627150043
17		9.364	-3.52758132
18	9.58		-3.436793307
19	10.037		-3.35334254
20	10.04		-3.276114499
21		10.047	-3.204230672
22		10.136	-3.136986293
23	10.234		-3.073807343
24	10.263		-3.014220084
25		10.551	-2.957828979
26		10.564	-2.904300374
27	10.588		-2.853350225
28	10.592		-2.804734725
29	10.654		-2.758243047
30	10.776		-2.713691648
31		10.909	-2.670919757
32		10.929	-2.629785756
33	10.973		-2.590164254
34		10.987	-2.551943695
35	10.998		-2.515024401
36	11.169		-2.479316935
37	11.199		-2.444740751
38	11.279		-2.41122305

Figure 7a. Expected Rank data of merged 008 strength set $\{D_S+D_{tS}\}$

	A	B	C
39	11.378		-2.378697815
40	11.4		-2.347104998
41	11.417		-2.316389819
42	11.451		-2.286502166
43	11.483		-2.25739608
44	11.485		-2.229029309
45		11.548	-2.20136292
46		11.598	-2.174360956
47		11.694	-2.147990145
48		11.808	-2.122219637
49	11.869		-2.097020773
50	11.901		-2.072366884
51	12.043		-2.04823311
52	12.123		-2.024596241
53	12.141		-2.00143457
54	12.151		-1.978727772
55	12.18		-1.956456785
56	12.2		-1.934603706
57	12.252		-1.913151704
58	12.3		-1.89208493
59	12.327		-1.871388446
60		12.334	-1.851048155
61	12.389		-1.831050739
62	12.401		-1.811383603
63	12.439		-1.792034822
64	12.505		-1.772993096
65	12.55		-1.754247705
66	12.664		-1.735788473
67	12.793		-1.717605728
68	12.825		-1.699690271
69	12.858		-1.682033346
70	13.01		-1.664626612
71	13.043		-1.647462114
72	13.051		-1.630532264
73	13.1		-1.613829816
74	13.237		-1.597347846
75	13.28		-1.581079732
76		13.408	-1.56501914

Figure 7b. Expected Rank data of merged 008 strength set $\{D_S+D_{tS}\}$ (cont.)

	A	B	C
77		13.474	-1.549160005
78	13.486		-1.533496514
79		13.583	-1.518023096
80	13.585		-1.502734408
81	13.604		-1.48762532
82	13.63		-1.472690904
83	13.633		-1.457926429
84	13.67		-1.443327343
85	13.68		-1.428889267
86	13.85		-1.41460799
87	13.86		-1.400479453
88		13.897	-1.386499749
89	13.92		-1.37266511
90		14.001	-1.358971904
91	14.1		-1.345416626
92	14.103		-1.331995894
93	14.129		-1.318706441
94		14.176	-1.305545112
95	14.2		-1.292508859
96	14.245		-1.279594733
97	14.286		-1.266799883
98	14.29		-1.25412155
99	14.34		-1.241557064
100	14.36		-1.229103839
101	14.4		-1.216759369
102	14.44		-1.204521228
103	14.454		-1.192387062
104	14.56		-1.18035459
105	14.588		-1.168421598
106	14.6		-1.156585938
107	14.6		-1.144845525
108	14.6		-1.133198334
109	14.614		-1.121642399
110	14.62		-1.110175808
111	14.71		-1.098796705
112	14.779		-1.087503283
113	14.79		-1.076293787
114	14.83		-1.065166507

Figure 7c. Expected Rank data of merged 008 strength set $\{D_S+D_{tS}\}$

(cont.)

	A	B	C
115	14.881		-1.05411978
116	14.892		-1.04315199
117	14.9		-1.032261558
118		14.935	-1.021446952
119	14.95		-1.010706674
120	14.973		-1.000039269
121		14.985	-0.989443316
122	15.025		-0.978917429
123	15.05		-0.968460258
124	15.09		-0.958070486
125		15.103	-0.947746825
126	15.106		-0.937488023
127	15.107		-0.927292852
128	15.15		-0.917160117
129	15.153		-0.907088649
130	15.17		-0.897077306
131		15.174	-0.887124971
132	15.193		-0.877230555
133	15.209		-0.867392991
134		15.239	-0.857611235
135	15.241		-0.847884268
136	15.27		-0.838211091
137	15.29		-0.828590727
138	15.31		-0.81902222
139	15.372		-0.809504633
140		15.379	-0.80003705
141	15.41		-0.790618572
142	15.411		-0.781248319
143	15.567		-0.771925429
144	15.59		-0.762649056
145		15.59	-0.753418372
146		15.632	-0.744232563
147		15.634	-0.735090833
148	15.657		-0.725992399
149	15.724		-0.716936494
150		15.761	-0.707922364
151	15.781		-0.698949269
152	15.851		-0.690016484

Figure 7d. Expected Rank data of merged 008 strength set $\{D_S+D_{tS}\}$ (cont.)

	A	B	C
153		15.853	-0.681123295
154	15.891		-0.672269002
155	15.899		-0.663452915
156	16		-0.654674359
157	16.031		-0.645932667
158	16.099		-0.637227187
159	16.18		-0.628557274
160	16.186		-0.619922295
161	16.19		-0.61132163
162	16.24		-0.602754663
163	16.27		-0.594220794
164	16.272		-0.585719427
165	16.28		-0.577249978
166	16.39		-0.568811872
167	16.41		-0.560404541
168	16.43		-0.552027425
169	16.45		-0.543679974
170	16.456		-0.535361644
171	16.471		-0.5270719
172	16.48		-0.518810214
173	16.482		-0.510576063
174	16.487		-0.502368933
175		16.493	-0.494188317
176	16.509		-0.486033714
177	16.561		-0.477904628
178	16.58		-0.469800571
179	16.65		-0.461721059
180	16.65		-0.453665616
181	16.66		-0.445633769
182	16.664		-0.437625052
183		16.684	-0.429639003
184		16.696	-0.421675168
185	16.7		-0.413733093
186		16.774	-0.405812333
187	16.813		-0.397912445
188	16.87		-0.390032992
189	16.927		-0.382173539
190	17		-0.374333657

Figure 7e. Expected Rank data of merged 008 strength set $\{D_s+D_{ts}\}$ (cont.)

	A	B	C
191	17.007		-0.366512921
192	17.105		-0.358710907
193	17.11		-0.350927199
194	17.24		-0.343161382
195		17.288	-0.335413042
196	17.293		-0.327681773
197		17.305	-0.319967169
198	17.34		-0.312268827
199	17.34		-0.304586348
200	17.34		-0.296919335
201	17.352		-0.289267393
202	17.353		-0.281630131
203	17.36		-0.274007158
204	17.36		-0.266398087
205		17.36	-0.258802533
206	17.372		-0.251220111
207	17.374		-0.243650439
208	17.442		-0.236093137
209	17.45		-0.228547827
210	17.46		-0.221014131
211	17.47		-0.213491672
212		17.517	-0.205980076
213	17.57		-0.198478968
214		17.575	-0.190987976
215	17.593		-0.183506728
216	17.6		-0.17603485
217	17.66		-0.168571974
218	17.69		-0.161117727
219	17.78		-0.153671739
220		17.798	-0.146233641
221	17.831		-0.138803061
222		17.85	-0.13137963
223		17.942	-0.123962977
224	17.97		-0.116552732
225	17.98		-0.109148523
226	17.98		-0.101749978
227	18		-0.094356725
228	18.009		-0.08696839

Figure 7f. Expected Rank data of merged 008 strength set $\{D_S+D_{tS}\}$ (cont.)

	A	B	C
229		18.038	-0.079584599
230	18.064		-0.072204977
231		18.112	-0.064829146
232		18.161	-0.057456727
233	18.183		-0.050087343
234	18.204		-0.04272061
235		18.208	-0.035356146
236		18.227	-0.027993566
237	18.24		-0.020632481
238		18.254	-0.013272503
239		18.286	-0.005913238
240	18.33		0.001445707
241	18.351		0.008804732
242	18.405		0.016164235
243	18.421		0.023524621
244	18.46		0.030886298
245		18.464	0.038249675
246	18.49		0.045615167
247	18.49		0.052983192
248	18.51		0.060354172
249	18.553		0.067728535
250	18.566		0.07510671
251	18.586		0.082489134
252	18.599		0.089876248
253	18.602		0.097268498
254	18.607		0.104666334
255		18.616	0.112070215
256	18.631		0.119480602
257	18.67		0.126897966
258	18.672		0.134322781
259		18.678	0.141755531
260	18.693		0.149196704
261	18.739		0.156646797
262	18.75		0.164106316
263		18.768	0.171575771
264	18.77		0.179055685
265	18.85		0.186546586
266		18.863	0.194049014

Figure 7g. Expected Rank data of merged 008 strength set $\{D_S+D_{tS}\}$ (cont.)

	A	B	C
267	18.892		0.201563515
268	18.914		0.209090648
269	18.93		0.216630981
270		18.944	0.224185092
271	18.95		0.231753572
272	18.96		0.239337022
273		18.971	0.246936055
274	18.995		0.254551297
275	19.003		0.262183388
276	19.01		0.269832981
277		19.1	0.277500743
278		19.145	0.285187356
279	19.15		0.292893517
280	19.156		0.300619941
281		19.198	0.308367359
282	19.27		0.316136519
283		19.3	0.323928187
284	19.304		0.33174315
285		19.338	0.339582214
286	19.36		0.347446207
287	19.397		0.355335978
288		19.411	0.363252398
289		19.419	0.371196363
290	19.502		0.379168796
291	19.52		0.387170644
292	19.544		0.395202882
293	19.559		0.403266514
294	19.579		0.411362574
295	19.62		0.419492128
296	19.653		0.427656275
297		19.686	0.435856149
298	19.715		0.444092922
299	19.77		0.452367801
300	19.786		0.460682038
301	19.79		0.469036924
302		19.805	0.477433797
303		19.83	0.48587404
304	19.85		0.494359088

Figure 7h. Expected Rank data of merged 008 strength set $\{D_S+D_{tS}\}$ (cont.)

	A	B	C
305	19.882		0.502890427
306	19.939		0.511469599
307	19.983		0.520098203
308	20.03		0.528777902
309	20.078		0.537510422
310	20.08		0.54629756
311		20.154	0.555141184
312		20.161	0.564043241
313		20.192	0.573005759
314		20.212	0.582030855
315		20.241	0.591120736
316	20.259		0.600277711
317	20.292		0.60950419
318	20.31		0.618802698
319	20.341		0.628175878
320	20.369		0.6376265
321	20.377		0.647157472
322	20.417		0.656771847
323	20.424		0.666472832
324	20.471		0.676263806
325	20.49		0.686148325
326	20.54		0.696130141
327		20.542	0.706213212
328		20.547	0.716401726
329	20.56		0.726700111
330	20.588		0.73711306
331	20.62		0.747645553
332	20.661		0.75830288
333	20.683		0.769090668
334	20.684		0.780014914
335		20.708	0.791082018
336	20.85		0.80229882
337	20.956		0.813672644
338		21.056	0.825211346
339	21.062		0.836923369
340	21.185		0.848817803
341		21.258	0.860904457
342		21.37	0.873193937

Figure 7i. Expected Rank data of merged 008 strength set $\{D_S+D_{tS}\}$ (cont.)

	A	B	C
343		21.453	0.885697741
344	21.54		0.898428359
345	21.56		0.911399399
346	21.575		0.924625728
347		21.59	0.938123629
348	21.613		0.951911004
349	21.674		0.966007591
350	21.69		0.980435233
351	21.72		0.995218203
352	21.89		1.010383575
353	21.906		1.025961696
354	21.95		1.041986739
355		22.142	1.058497395
356	22.254		1.075537728
357		22.31	1.093158236
358		22.327	1.111417205
359	22.525		1.130382434
360	22.86		1.150133476
361	22.964		1.17076459
362		23.258	1.192388692
363	23.442		1.215142758
364	23.56		1.239195351
365		23.952	1.264757405
366	23.991		1.292098101
367		24.034	1.321569089
368	24.149		1.353642988
369		24.342	1.388977764
370	24.384		1.428531448
371	24.405		1.473784057
372	25.341		1.527217406
373	25.4		1.593533139
374		25.515	1.683656356
375		26.418	1.837091063

Figure 7j. Expected Rank data of merged 008 strength set $\{D_S+D_{tS}\}$ (cont.)

	E	F	G
1	AS4-019		
2			
3			
4			
5			
6	xi (Ds|Ns)	xi (Dts|Nts)	F*({Ds+Dts}|N
7	2.847		-5.97508053
8		3.626	-5.280660273
9		4.398	-4.87391938
10	6.3		-4.584958802
11	6.607		-4.36053401
12		6.607	-4.176928467
13	6.628		-4.021491042
14	6.7		-3.886670131
15		6.7	-3.767594793
16		7.066	-3.660939176
17	7.284		-3.564331084
18	7.292		-3.476018971
19	7.643		-3.394672689
20		7.643	-3.319258291
21	8.094		-3.24895613
22	8.208		-3.18310544
23	8.47		-3.121165758
24		9.011	-3.062689378
25		9.048	-3.007301271
26	9.284		-2.954684156
27	9.3		-2.904567225
28	9.62		-2.856717479
29	9.624		-2.810933011
30	9.734		-2.767037699
31	9.753		-2.724877002
32	9.865		-2.684314566
33	10.219		-2.645229481
34	10.226		-2.607514029
35	10.27		-2.571071837
36	10.28		-2.535816334
37	10.342		-2.501669464
38	10.342		-2.468560608

Figure 8a. Expected Rank data of merged 019 strength set $\{D_s+D_{ts}\}$

	E	F	G
39		10.476	-2.436425665
40	10.486		-2.405206277
41	10.494		-2.374849157
42		10.494	-2.345305525
43	10.615		-2.316530606
44	10.663		-2.288483209
45	10.7		-2.261125352
46	10.772		-2.234421935
47	10.775		-2.208340459
48	10.802		-2.182850774
49	11.1		-2.157924855
50	11.21		-2.133536612
51	11.231		-2.109661709
52	11.3		-2.086277417
53	11.4		-2.063362471
54	11.448		-2.040896948
55		11.593	-2.018862156
56	11.61		-1.997240535
57		11.623	-1.976015569
58	11.686		-1.955171699
59		11.693	-1.934694259
60	11.812		-1.9145694
61		11.884	-1.894784036
62	11.928		-1.875325788
63	12.017		-1.856182932
64	12.098		-1.837344355
65	12.163		-1.818799513
66		12.201	-1.800538394
67	12.31		-1.782551477
68		12.318	-1.76482971
69	12.33		-1.74736447
70		12.404	-1.730147543
71	12.44		-1.713171094
72	12.469		-1.696427648
73	12.487		-1.679910065
74	12.5		-1.66361152
75	12.505		-1.647525488
76	12.507		-1.631645723

Figure 8b. Expected Rank data of merged 019 strength set $\{D_S+D_{tS}\}$ (cont.)

	E	F	G
77	12.614		-1.615966245
78	12.626		-1.600481324
79	12.678		-1.585185464
80	12.69		-1.570073395
81	12.768		-1.555140055
82	12.8		-1.540380586
83		12.805	-1.525790316
84		13.073	-1.511364754
85	13.09		-1.497099581
86	13.18		-1.482990638
87	13.193		-1.469033921
88	13.196		-1.455225571
89	13.205		-1.44156187
90	13.267		-1.428039232
91		13.348	-1.414654197
92	13.36		-1.401403425
93	13.424		-1.388283692
94	13.457		-1.375291882
95	13.484		-1.362424984
96	13.491		-1.349680088
97	13.5		-1.337054379
98	13.51		-1.324545132
99	13.562		-1.31214971
100	13.58		-1.29986556
101	13.616		-1.287690209
102		13.647	-1.27562126
103	13.71		-1.26365639
104	13.762		-1.251793346
105	13.834		-1.240029943
106	13.86		-1.228364061
107	13.871		-1.21679364
108	13.872		-1.205316683
109	13.889		-1.193931249
110	13.892		-1.182635452
111	13.91		-1.171427459
112	13.98		-1.160305488
113	13.99		-1.149267807
114	14.002		-1.13831273

Figure 8c. Expected Rank data of merged 019 strength set $\{D_s+D_{ts}\}$ (cont.)

	E	F	G
115	14.01		-1.127438617
116	14.069		-1.116643872
117		14.085	-1.105926941
118	14.16		-1.095286311
119		14.213	-1.084720508
120	14.27		-1.074228096
121	14.271		-1.063807675
122	14.305		-1.053457881
123		14.328	-1.043177384
124	14.332		-1.032964885
125	14.346		-1.022819119
126	14.353		-1.012738851
127	14.37		-1.002722875
128	14.416		-0.992770013
129	14.434		-0.982879117
130	14.481		-0.973049063
131	14.485		-0.963278755
132	14.528		-0.953567122
133	14.54		-0.943913114
134		14.579	-0.934315709
135		14.586	-0.924773904
136	14.596		-0.915286721
137		14.648	-0.9058532
138	14.649		-0.896472405
139	14.676		-0.887143417
140	14.678		-0.877865339
141		14.681	-0.868637291
142	14.69		-0.859458411
143		14.697	-0.850327856
144	14.702		-0.841244799
145	14.767		-0.832208431
146	14.825		-0.823217957
147	14.83		-0.814272599
148	14.855		-0.805371595
149	14.929		-0.796514195
150	14.95		-0.787699666
151	14.97		-0.778927288
152	15.003		-0.770196353

Figure 8d. Expected Rank data of merged 019 strength set $\{D_S+D_{tS}\}$ (cont.)

	E	F	G
153		15.02	-0.761506169
154		15.031	-0.752856053
155	15.037		-0.744245339
156	15.041		-0.735673368
157	15.05		-0.727139496
158	15.06		-0.71864309
159	15.18		-0.710183527
160	15.2		-0.701760194
161		15.26	-0.693372491
162	15.278		-0.685019826
163	15.287		-0.676701617
164	15.356		-0.668417292
165		15.356	-0.660166289
166	15.385		-0.651948053
167		15.392	-0.64376204
168		15.396	-0.635607713
169	15.4		-0.627484544
170	15.41		-0.619392013
171		15.419	-0.611329608
172	15.52		-0.603296823
173	15.521		-0.595293163
174	15.522		-0.587318137
175	15.596		-0.579371261
176	15.602		-0.571452061
177	15.615		-0.563560065
178	15.632		-0.555694812
179		15.632	-0.547855844
180	15.64		-0.540042711
181	15.65		-0.532254968
182	15.744		-0.524492176
183		15.744	-0.516753902
184	15.841		-0.509039717
185		15.841	-0.501349199
186	15.852		-0.49368193
187		15.852	-0.486037498
188	15.872		-0.478415495
189		15.886	-0.470815517
190		15.892	-0.463237168

Figure 8e. Expected Rank data of merged 019 strength set $\{D_S+D_{tS}\}$ (cont.)

	E	F	G
191	15.951		-0.455680051
192	15.954		-0.448143779
193	15.96		-0.440627964
194	15.96		-0.433132227
195	16.084		-0.425656188
196	16.096		-0.418199474
197		16.096	-0.410761715
198	16.145		-0.403342544
199	16.18		-0.395941599
200	16.19		-0.388558519
201	16.19		-0.381192948
202		16.199	-0.373844532
203		16.22	-0.366512921
204	16.242		-0.359197766
205	16.248		-0.351898724
206	16.248		-0.344615452
207		16.266	-0.337347611
208	16.29		-0.330094863
209		16.33	-0.322856874
210	16.35		-0.315633311
211	16.369		-0.308423844
212	16.452		-0.301228146
213		16.452	-0.294045889
214	16.48		-0.286876749
215		16.508	-0.279720405
216	16.6		-0.272576535
217	16.617		-0.265444821
218	16.685		-0.258324945
219	16.7		-0.25121659
220	16.703		-0.244119442
221	16.735		-0.237033188
222	16.77		-0.229957515
223	16.78		-0.222892112
224	16.82		-0.215836669
225	16.841		-0.208790876
226	16.858		-0.201754426
227		16.858	-0.194727009
228	16.877		-0.18770832

Figure 8f. Expected Rank data of merged 019 strength set $\{D_S+D_{tS}\}$ (cont.)

	E	F	G
229	16.88		-0.180698052
230	16.913		-0.173695898
231		16.921	-0.166701553
232	16.95		-0.159714712
233	16.96		-0.152735069
234	17.03		-0.14576232
235	17.047		-0.138796158
236		17.063	-0.13183628
237	17.082		-0.124882379
238	17.11		-0.117934151
239	17.133		-0.11099129
240	17.149		-0.104053488
241		17.17	-0.09712044
242		17.175	-0.090191837
243	17.22		-0.083267372
244	17.244		-0.076346736
245	17.252		-0.069429618
246	17.32		-0.062515708
247	17.34		-0.055604693
248	17.34		-0.048696261
249		17.362	-0.041790096
250	17.38		-0.034885882
251	17.38		-0.027983303
252		17.426	-0.021082037
253	17.501		-0.014181765
254		17.501	-0.007282163
255	17.508		-0.000382906
256	17.528		0.006516332
257	17.54		0.013415882
258	17.545		0.020316076
259	17.57		0.027217246
260	17.607		0.034119732
261		17.621	0.041023871
262	17.677		0.047930008
263	17.682		0.054838487
264	17.686		0.061749658
265	17.704		0.068663873
266	17.71		0.075581488

Figure 8g. Expected Rank data of merged 019 strength set $\{D_S+D_{tS}\}$ (cont.)

	E	F	G
267	17.72		0.082502862
268	17.72		0.089428358
269		17.725	0.096358345
270		17.729	0.103293193
271	17.757		0.110233279
272		17.784	0.117178982
273	17.802		0.124130689
274	17.81		0.131088789
275	17.848		0.138053677
276		17.889	0.145025754
277	17.91		0.152005427
278	17.955		0.158993106
279	17.98		0.16598921
280	17.98		0.172994164
281	17.996		0.180008397
282		18	0.187032348
283	18.03		0.19406646
284	18.03		0.201111186
285		18.053	0.208166985
286	18.056		0.215234324
287	18.081		0.22231368
288	18.118		0.229405535
289	18.13		0.236510382
290	18.146		0.243628725
291		18.146	0.250761074
292	18.166		0.257907952
293	18.173		0.265069889
294	18.217		0.27224743
295		18.229	0.279441128
296	18.412		0.286651549
297	18.452		0.293879271
298	18.47		0.301124884
299	18.495		0.308388991
300	18.5		0.31567221
301	18.513		0.322975173
302		18.513	0.330298526
303	18.561		0.33764293
304	18.601		0.345009064

Figure 8h. Expected Rank data of merged 019 strength set $\{D_s+D_{ts}\}$ (cont.)

	E	F	G
305	18.619		0.352397622
306	18.62		0.359809316
307		18.652	0.367244876
308	18.7		0.374705053
309	18.7		0.382190615
310	18.707		0.389702352
311		18.726	0.397241076
312		18.74	0.404807622
313	18.75		0.412402847
314	18.75		0.420027633
315	18.75		0.42768289
316		18.83	0.435369553
317	18.836		0.443088586
318		18.84	0.450840982
319	18.883		0.458627765
320		18.92	0.466449994
321	18.93		0.474308759
322	18.932		0.482205188
323		18.932	0.490140445
324	18.945		0.498115736
325	19		0.506132305
326		19.018	0.514191443
327	19.046		0.522294485
328		19.066	0.530442816
329	19.093		0.538637872
330	19.26		0.546881141
331	19.32		0.55517417
332	19.327		0.563518565
333	19.33		0.571915995
334	19.336		0.580368198
335	19.35		0.588876981
336		19.406	0.597444228
337	19.433		0.606071901
338	19.475		0.614762049
339	19.507		0.623516809
340		19.507	0.632338414
341	19.52		0.641229199
342		19.551	0.650191609

Figure 8i. Expected Rank data of merged 019 strength set $\{D_S+D_{tS}\}$ (cont.)

	E	F	G
343		19.664	0.659228202
344	19.693		0.668341661
345	19.77		0.677534801
346	19.84		0.686810578
347	19.962		0.696172099
348	19.99		0.705622633
349	20.02		0.715165624
350	20.105		0.724804703
351	20.12		0.734543703
352	20.128		0.744386675
353	20.15		0.754337905
354	20.191		0.764401931
355	20.2		0.774583572
356		20.208	0.784887941
357		20.242	0.79532048
358	20.243		0.805886983
359	20.251		0.816593634
360	20.269		0.827447039
361	20.27		0.838454269
362	20.31		0.849622905
363	20.32		0.86096109
364	20.331		0.87247759
365	20.352		0.884181854
366	20.4		0.8960841
367	20.406		0.908195392
368	20.44		0.920527746
369		20.503	0.933094241
370	20.594		0.945909155
371	20.627		0.958988117
372	20.665		0.972348291
373	20.67		0.986008583
374	20.725		0.999989899
375		20.824	1.014315437
376	20.86		1.029011046
377		20.86	1.044105659
378	20.928		1.059631811
379	20.961		1.07562628
380		21.105	1.092130875

Figure 8j. Expected Rank data of merged 019 strength set $\{D_S+D_{tS}\}$ (cont.)

	E	F	G
381		21.12	1.109193421
382	21.18		1.126868998
383	21.23		1.145221526
384	21.27		1.164325803
385		21.34	1.184270189
386	21.46		1.205160173
387	21.58		1.227123219
388		21.729	1.250315479
389	21.82		1.274931321
390	22.246		1.301217244
391	22.33		1.329492854
392	22.836		1.360183769
393	23.177		1.39387574
394	23.191		1.431409074
395		23.287	1.474056351
396	23.89		1.523892344
397		24.158	1.584681715
398		25.54	1.664532681
399		26.393	1.787810166

Figure 8k. Expected Rank data of merged 019 strength set $\{D_S+D_{tS}\}$ (cont.)

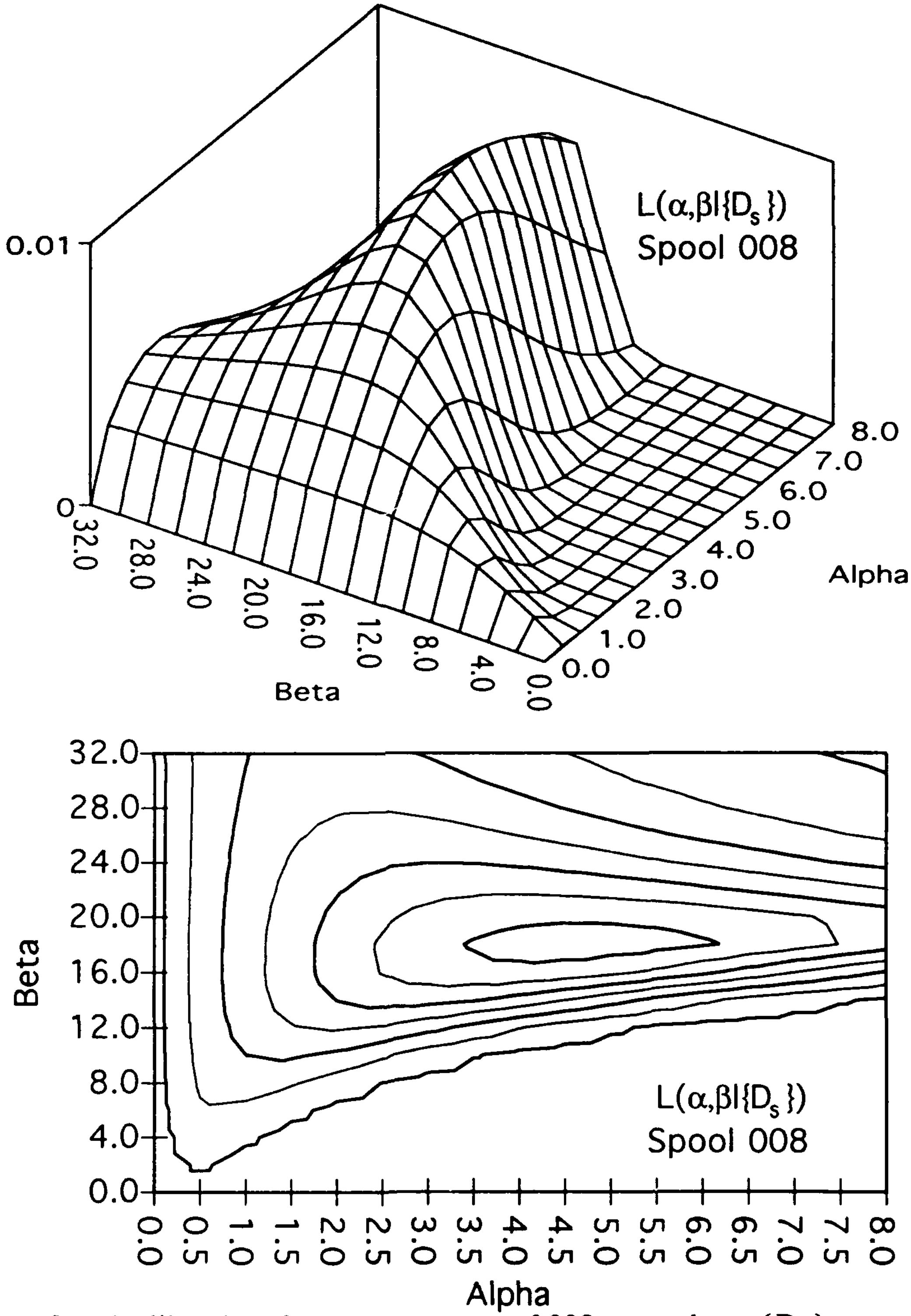

Figure 9. Likelihood surface and contours of 008 strength set $\{D_S\}$

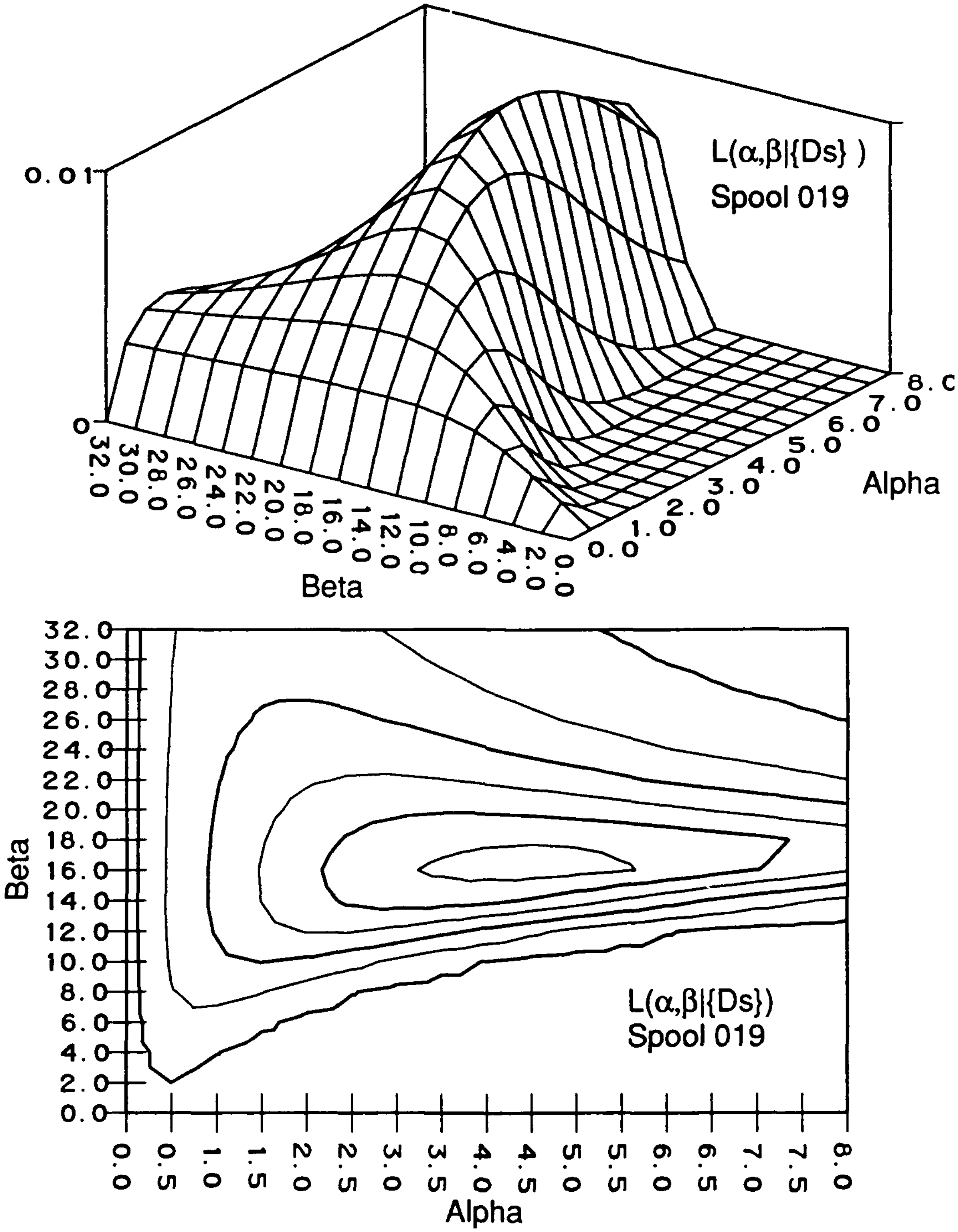

Figure 10. Likelihood surface and contours of 019 strength set $\{D_S\}$

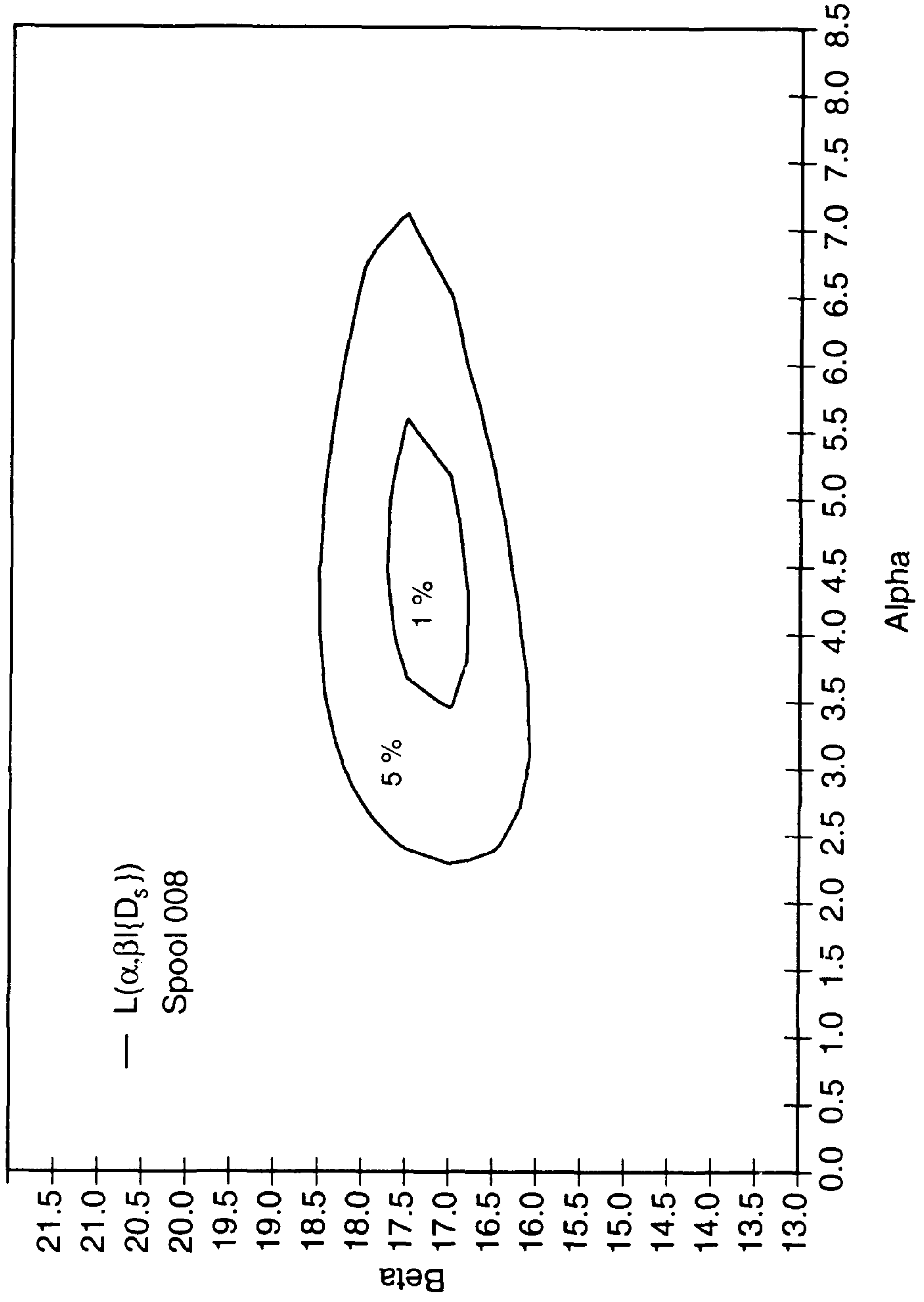

Figure 11. 5 % and 1 % confidence intervals for 008 strength set $\{D_S\}$

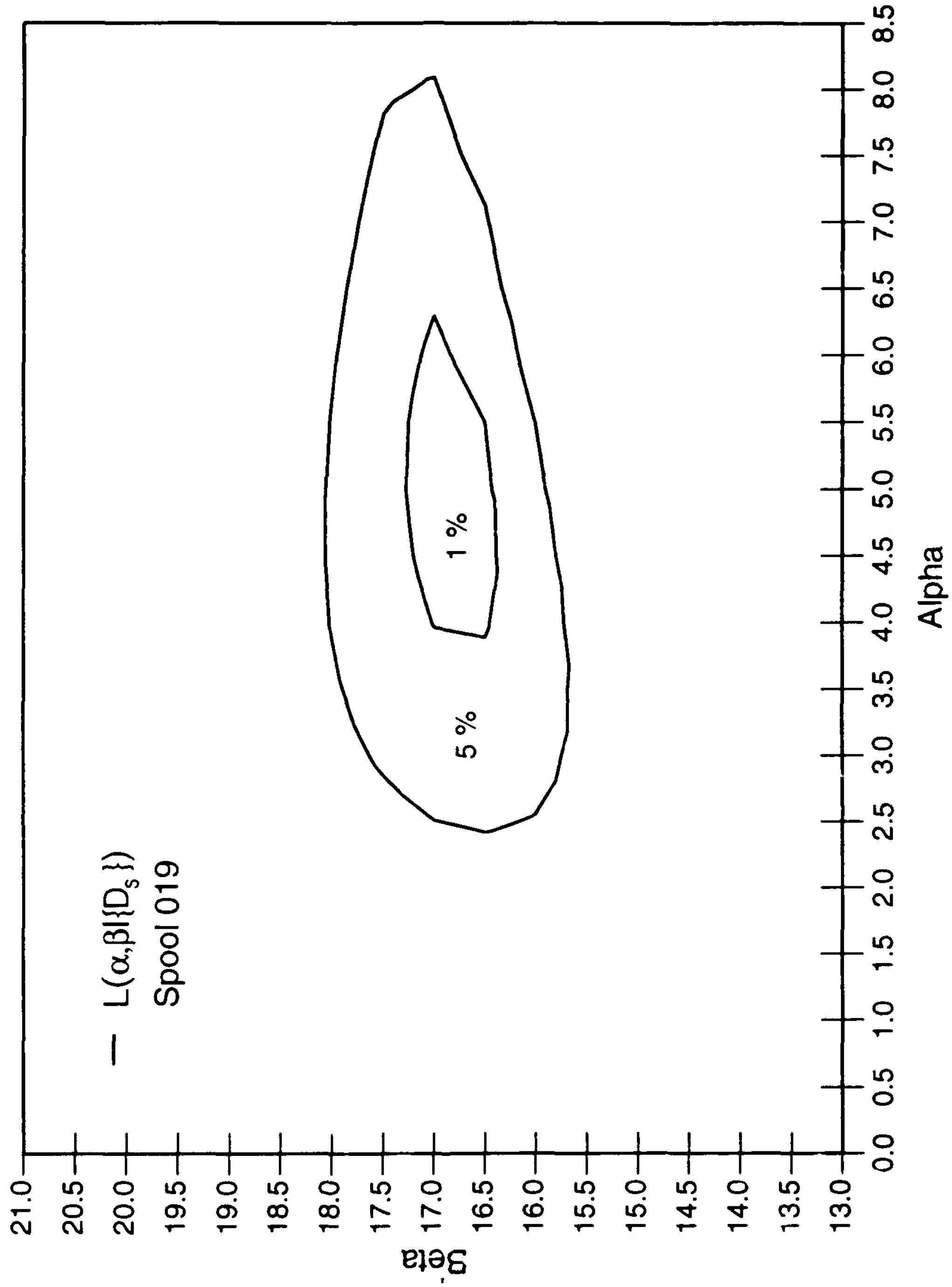

Figure 12. 5 % and 1 % confidence intervals for 019 strength set $\{D_S\}$

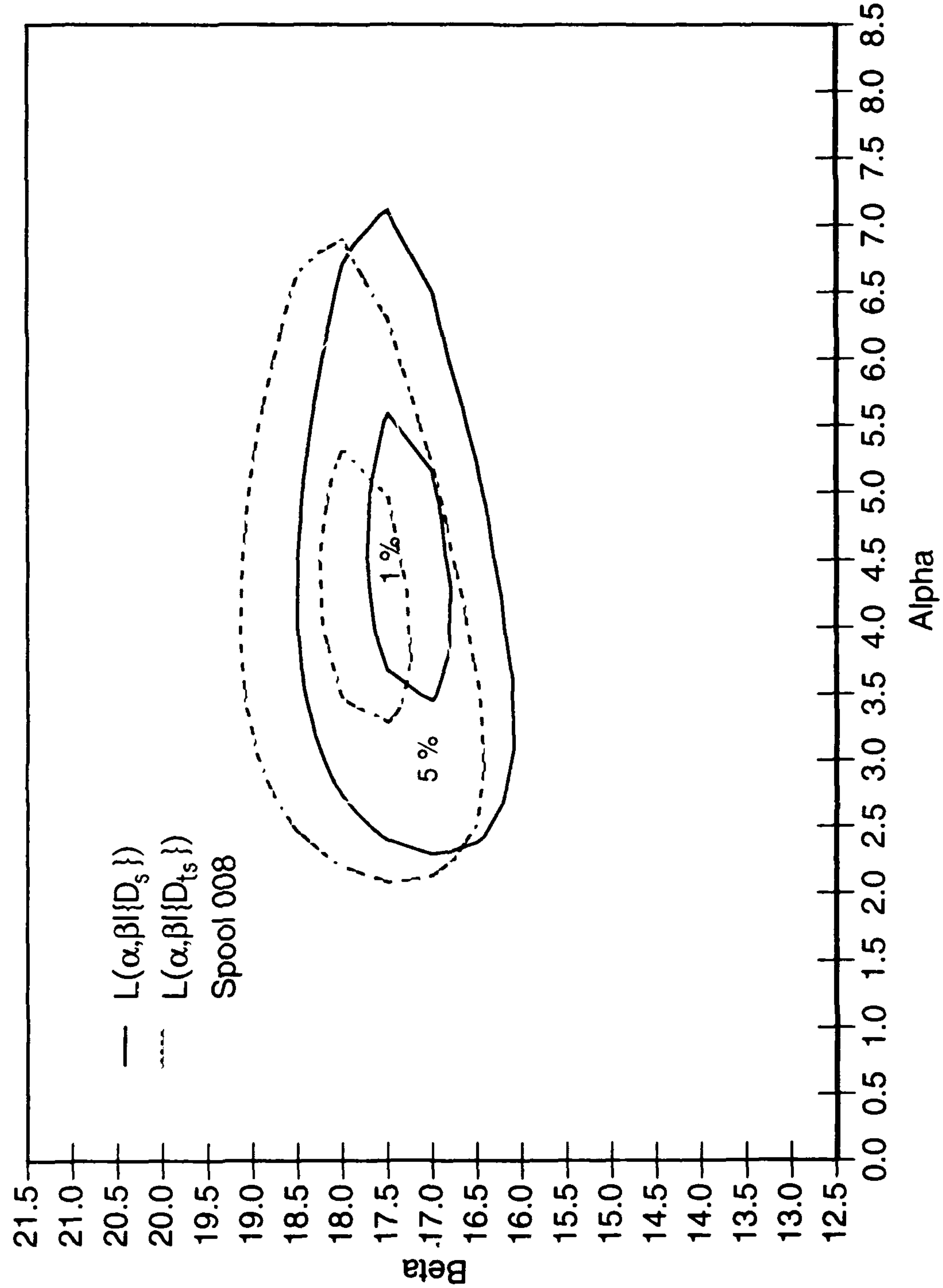

Figure 13. 5 % and 1 % confidence intervals for 008 strength sets $\{D_S\}$ and $\{D_{tS}\}$

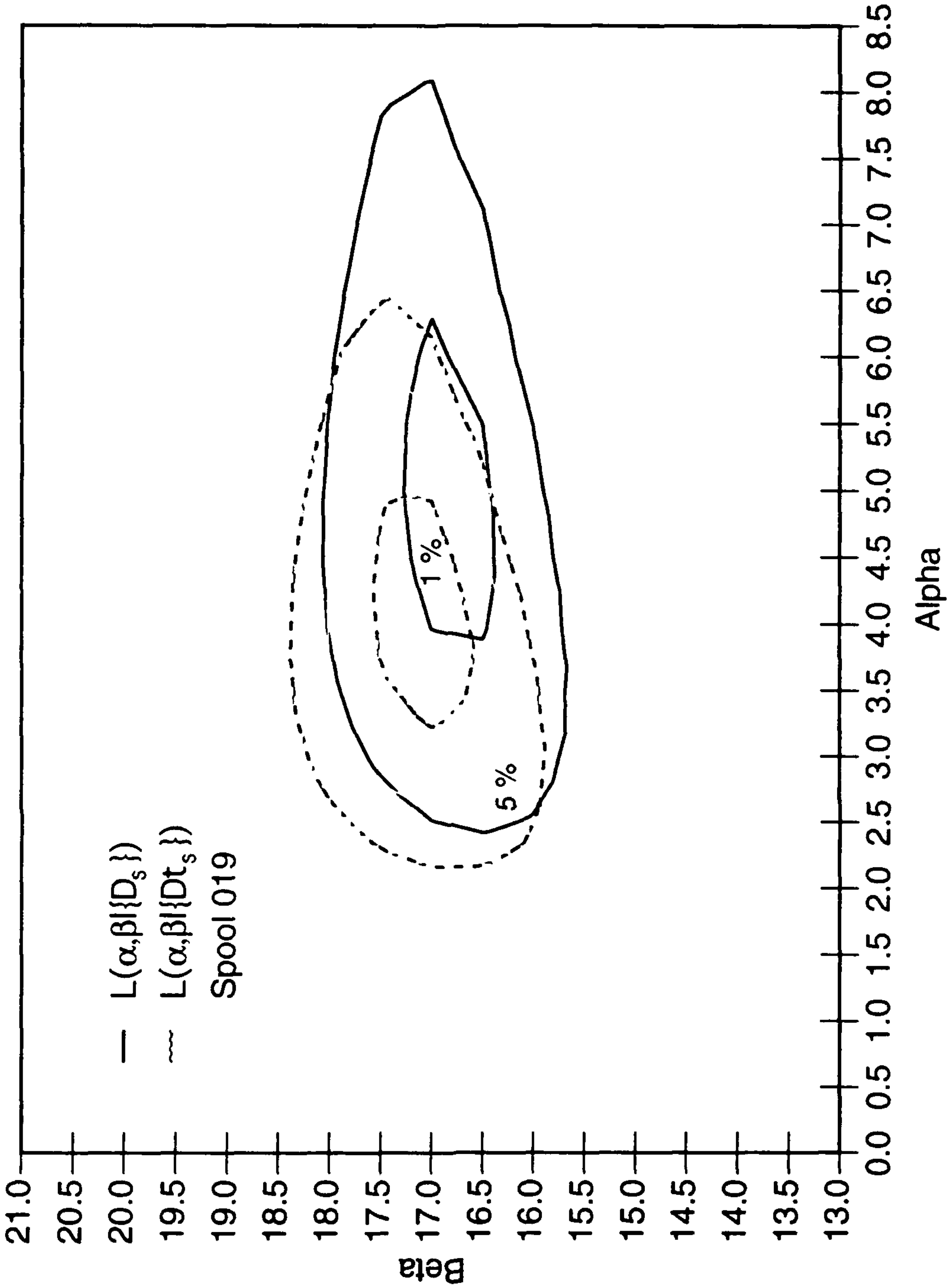

Figure 14. 5 % and 1 % confidence intervals for 019 strength sets {D_S} and {D_{tS}}

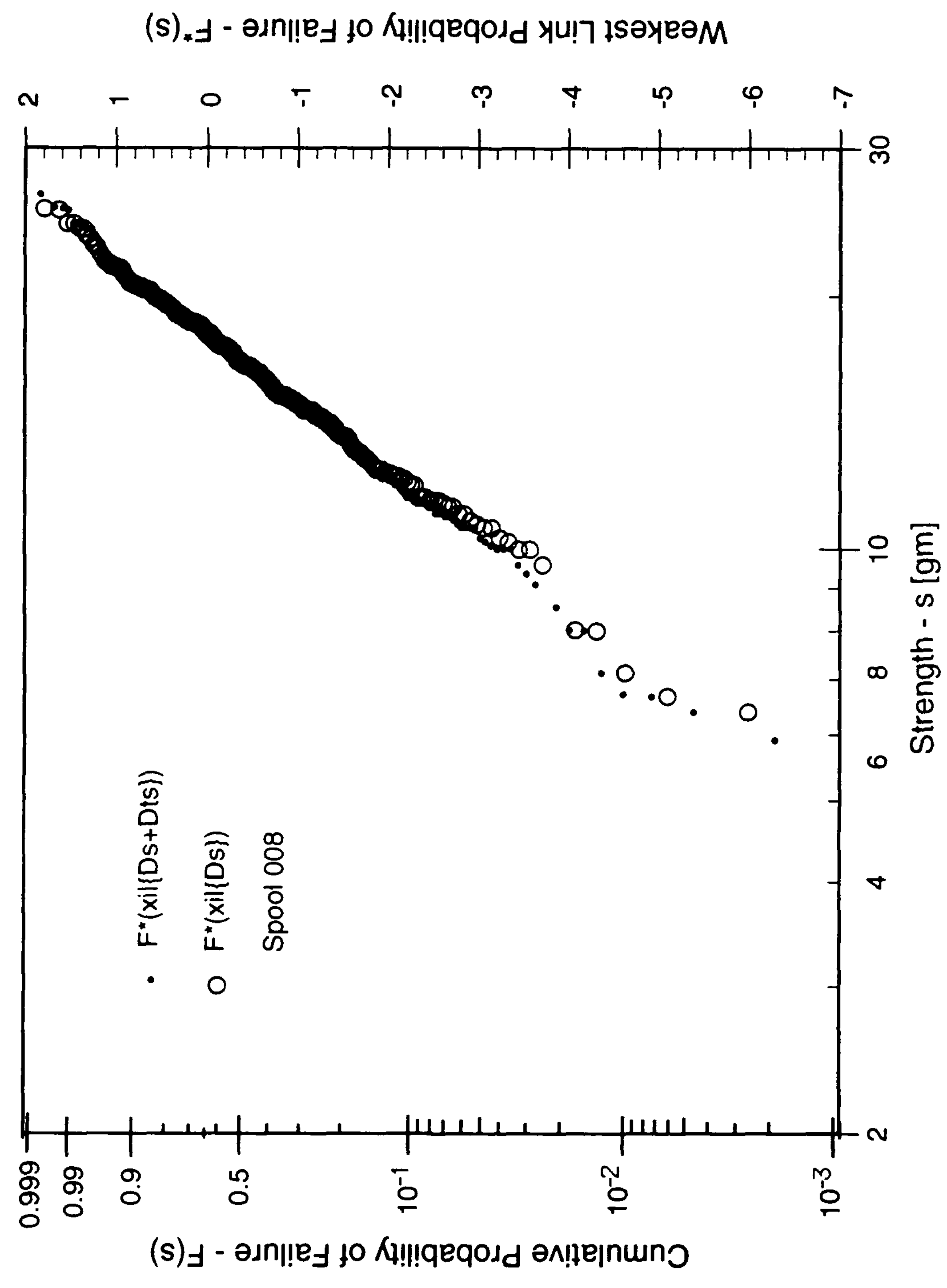

Figure 15. Expected Rank of 008 strength sets $\{D_s + D_{ts}\}$ and $\{D_s\}$

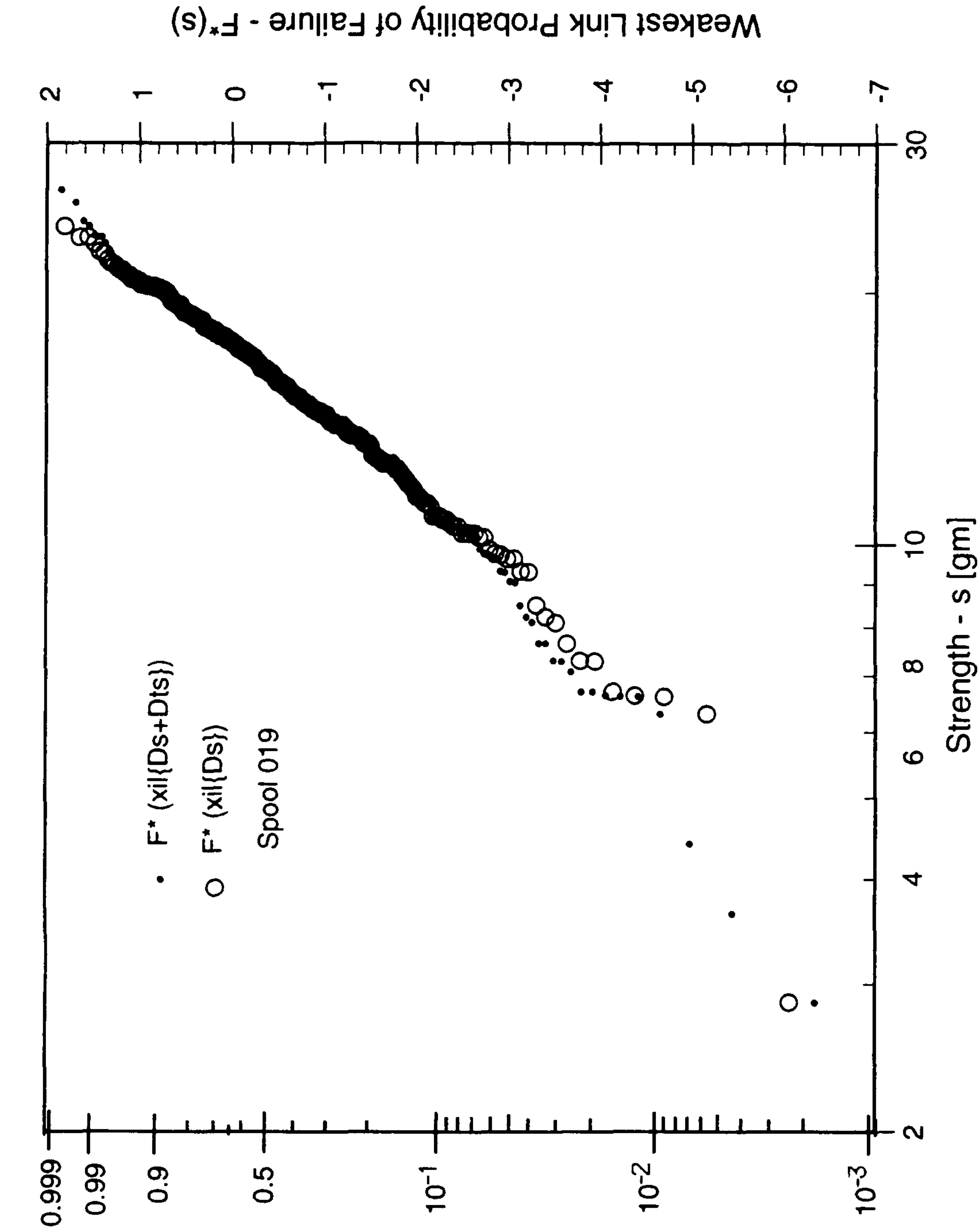

Figure 16. Expected Rank of 019 strength sets $\{D_s + D_{ts}\}$ and $\{D_s\}$

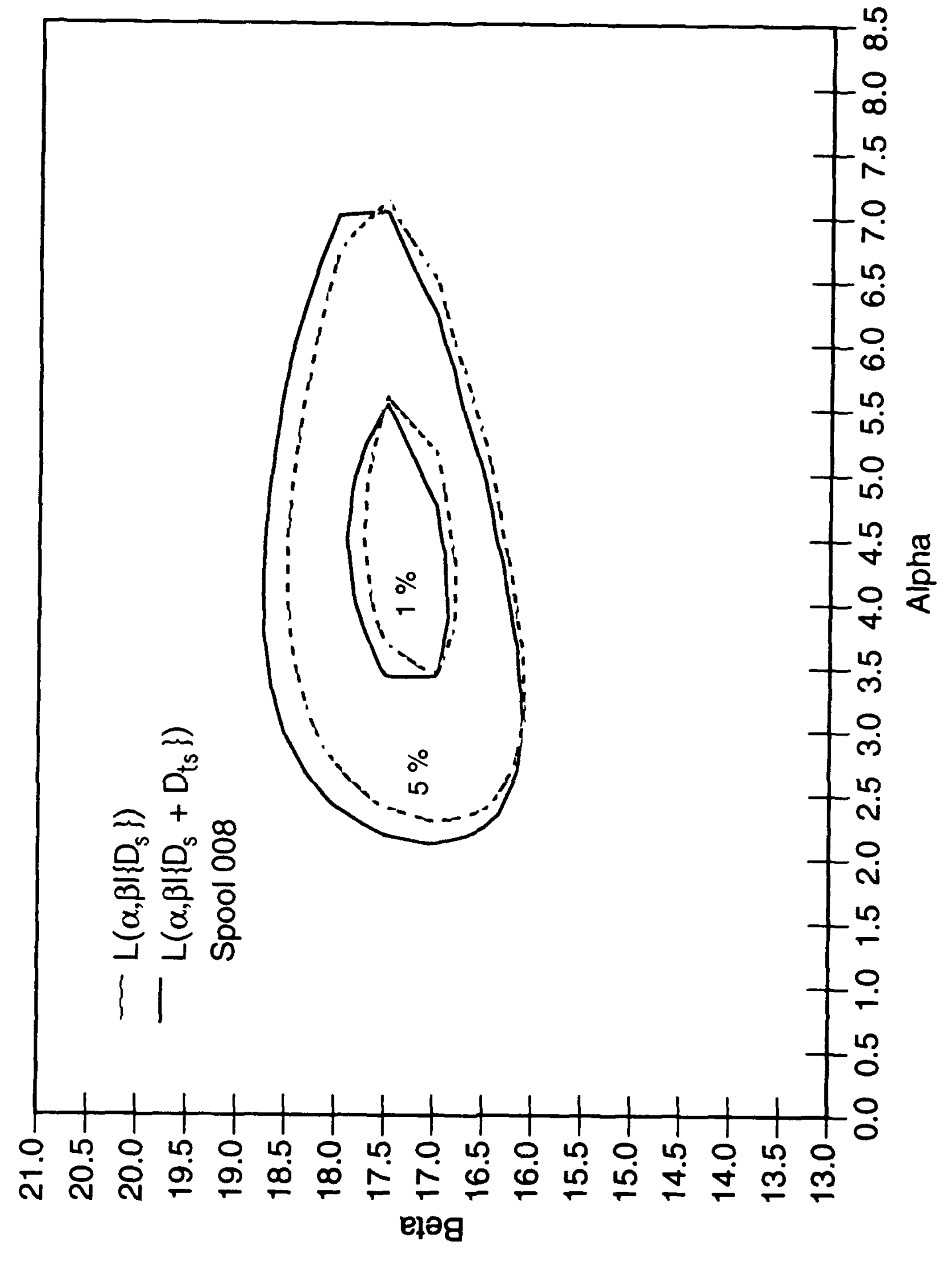

Figure 17. 5 % and 1 % confidence intervals for 008 strength sets $\{D_S\}$ and $\{D_S + D_{tS}\}$

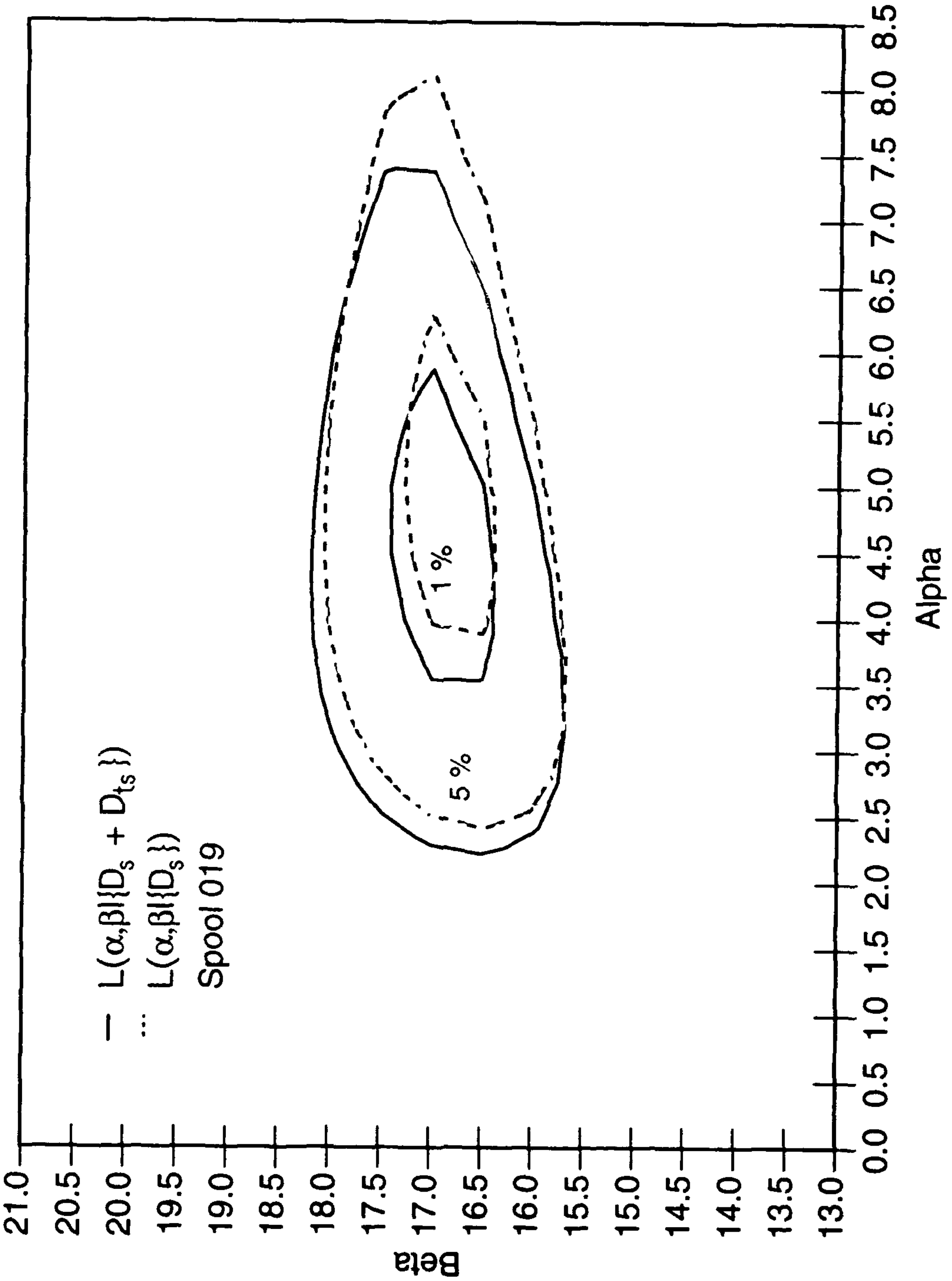

Figure 18. 5 % and 1 % confidence intervals for 019 strength sets $\{D_S\}$ and $\{D_S + D_{tS}\}$

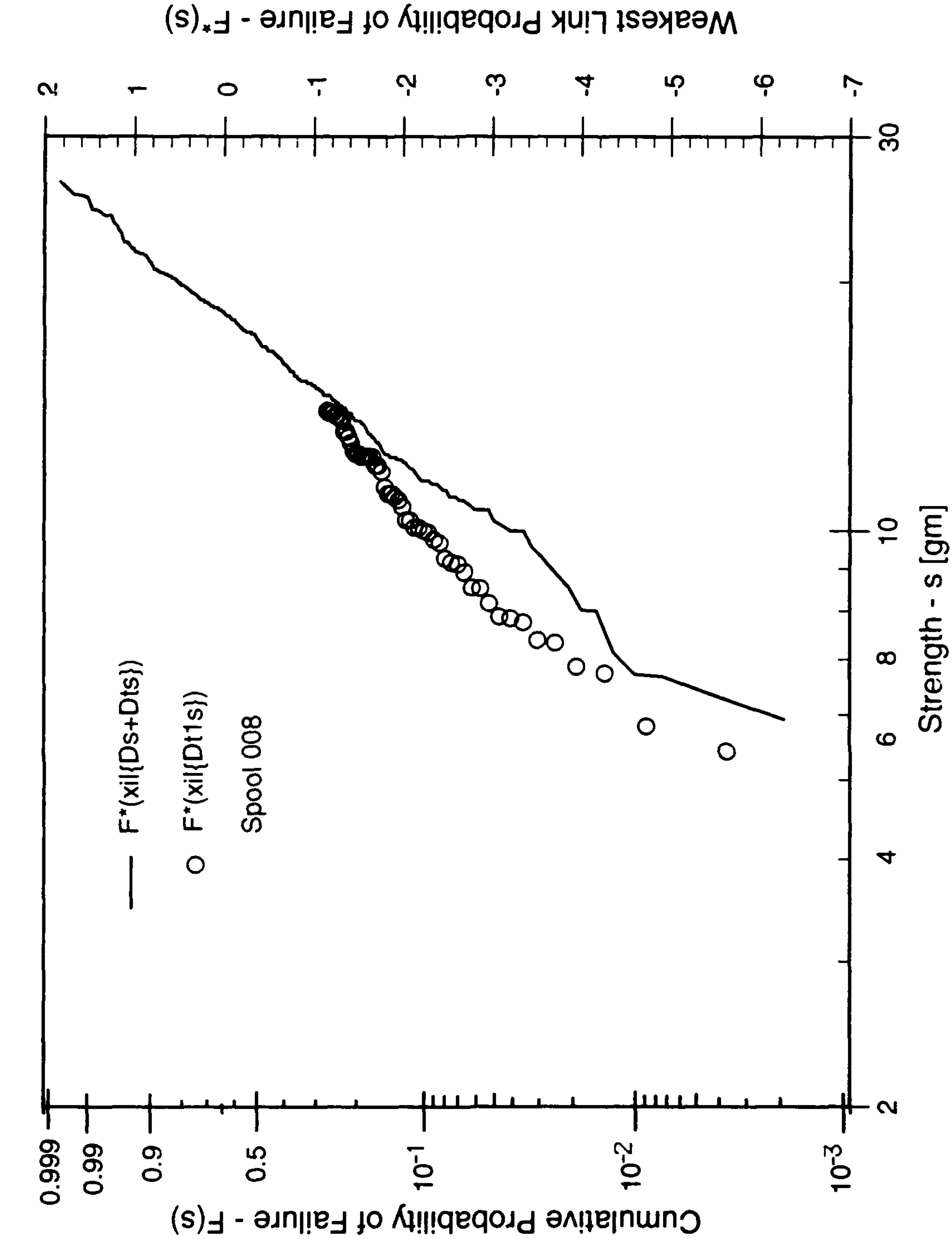

Figure 19. Expected Rank of 008 strength sets $\{D_s + D_{ts}\}$ and $\{D_{tls}\}$

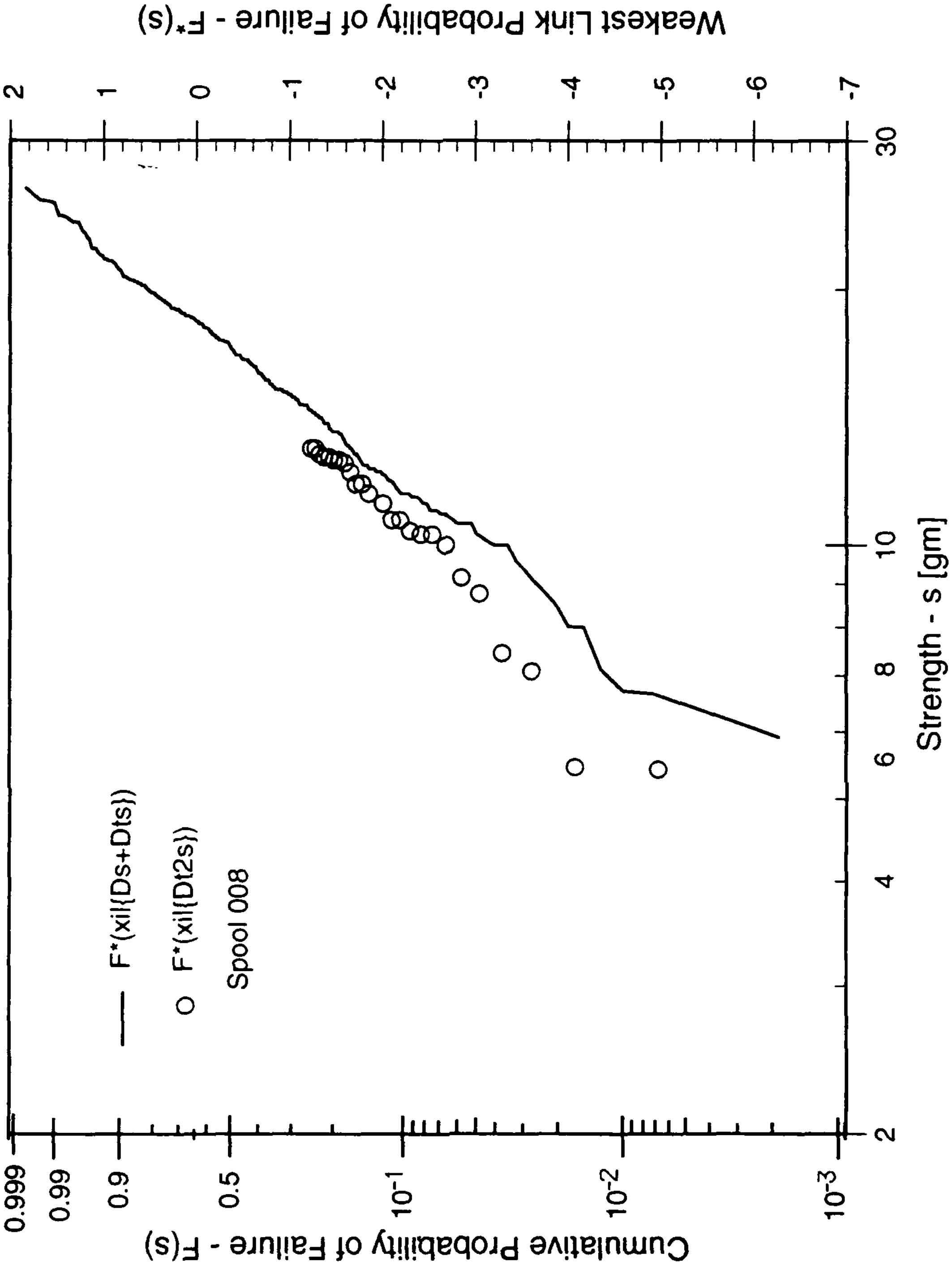

Figure 20. Expected Rank of 008 strength sets $\{D_s + D_{ts}\}$ and $\{D_{t2s}\}$

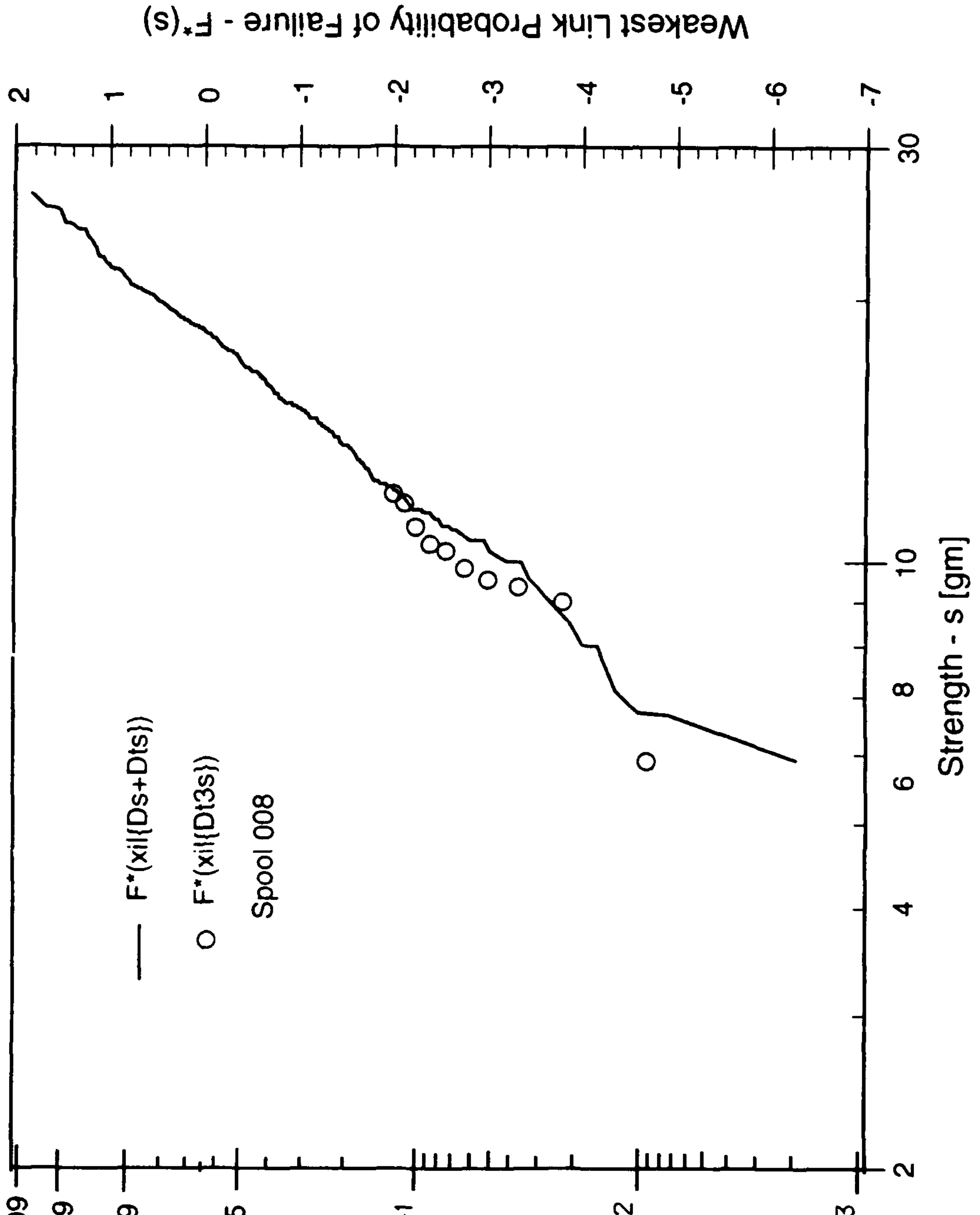

Figure 21. Expected Rank of 008 strength sets $\{D_s + D_{ts}\}$ and $\{D_{t3s}\}$

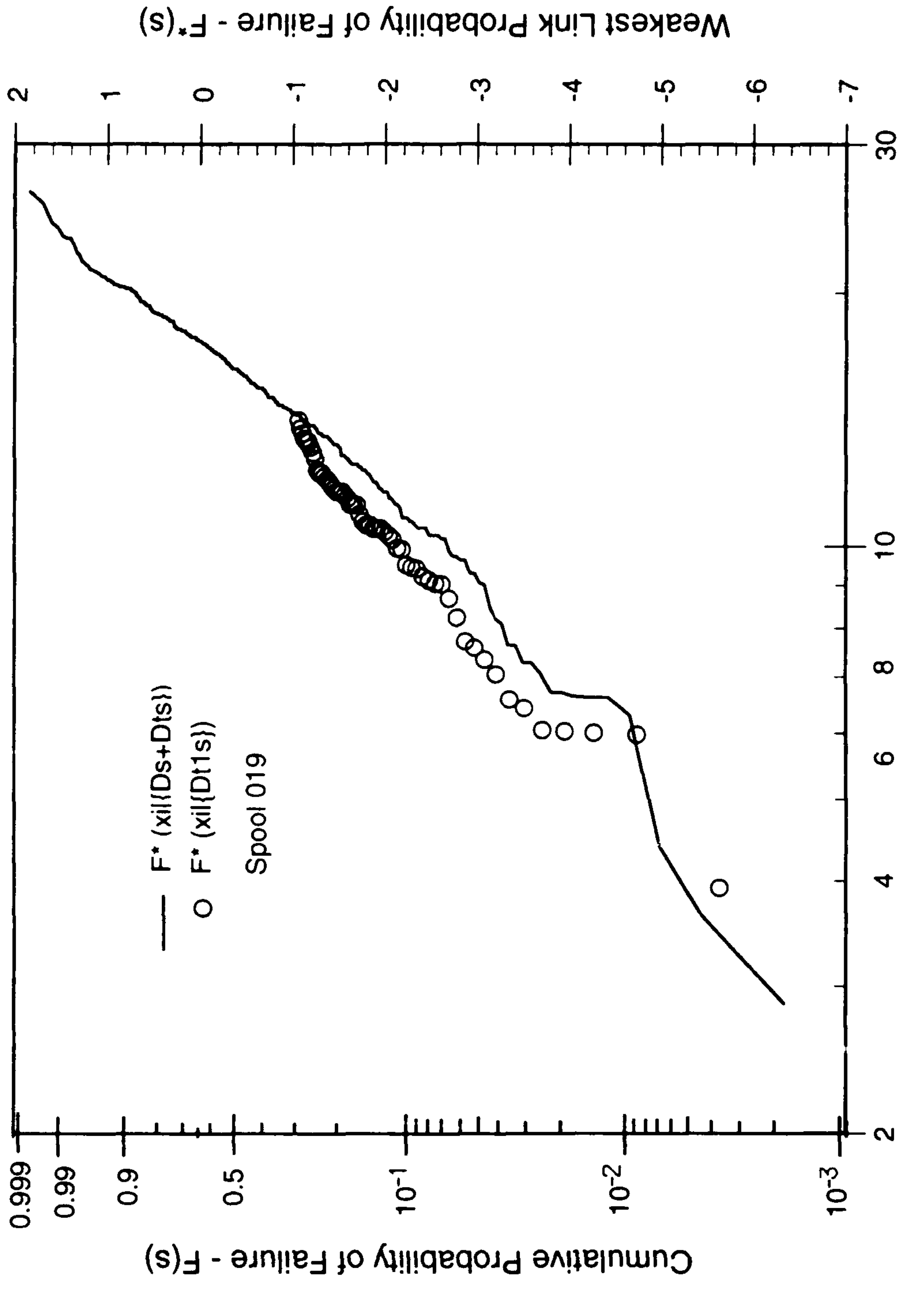

Figure 22. Expected Rank of 019 strength sets $\{D_s + D_{ts}\}$ and $\{D_{t1s}\}$

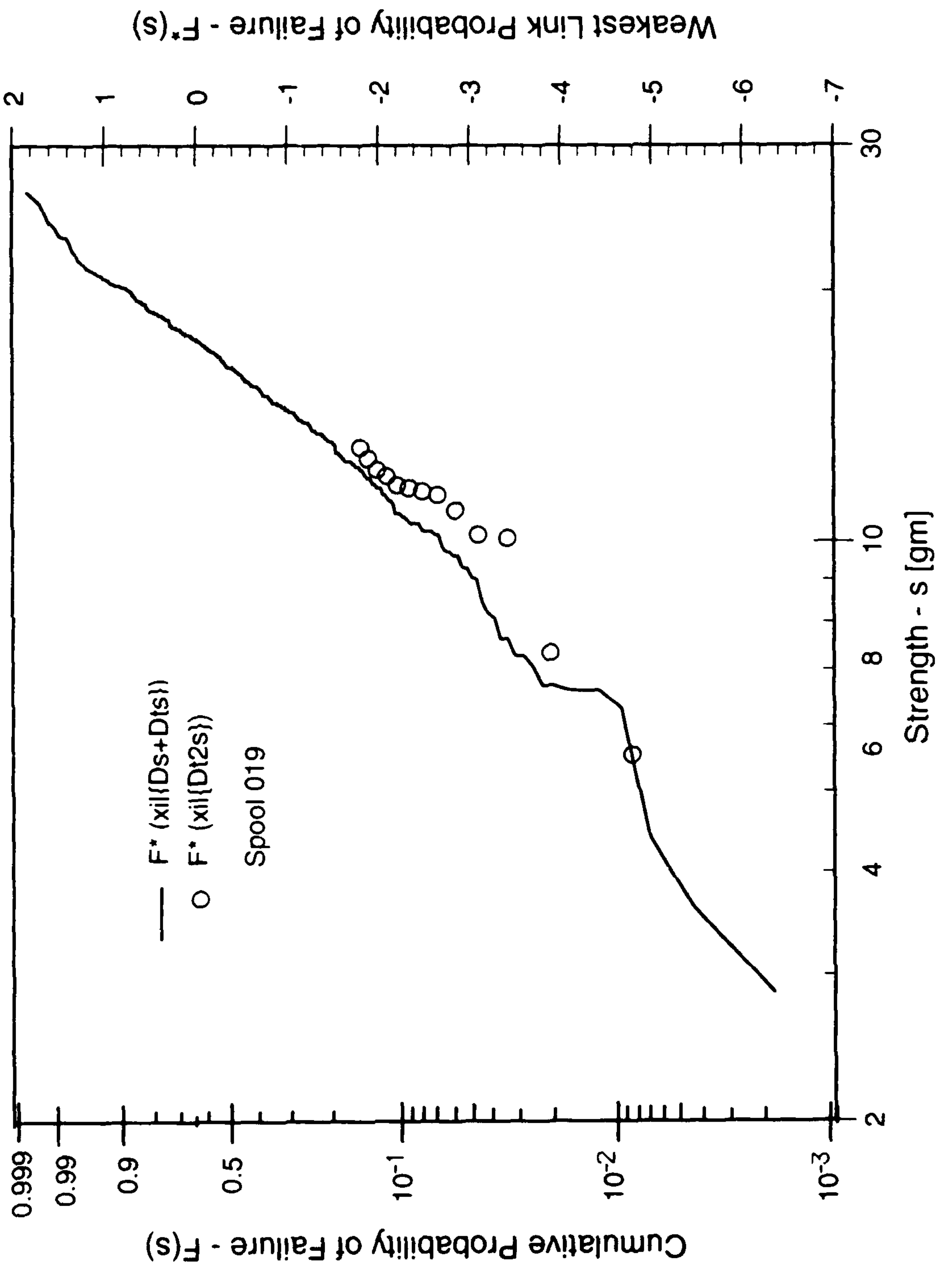

Figure 23. Expected Rank of 019 strength sets $\{D_s + D_{ts}\}$ and $\{D_{t2s}\}$

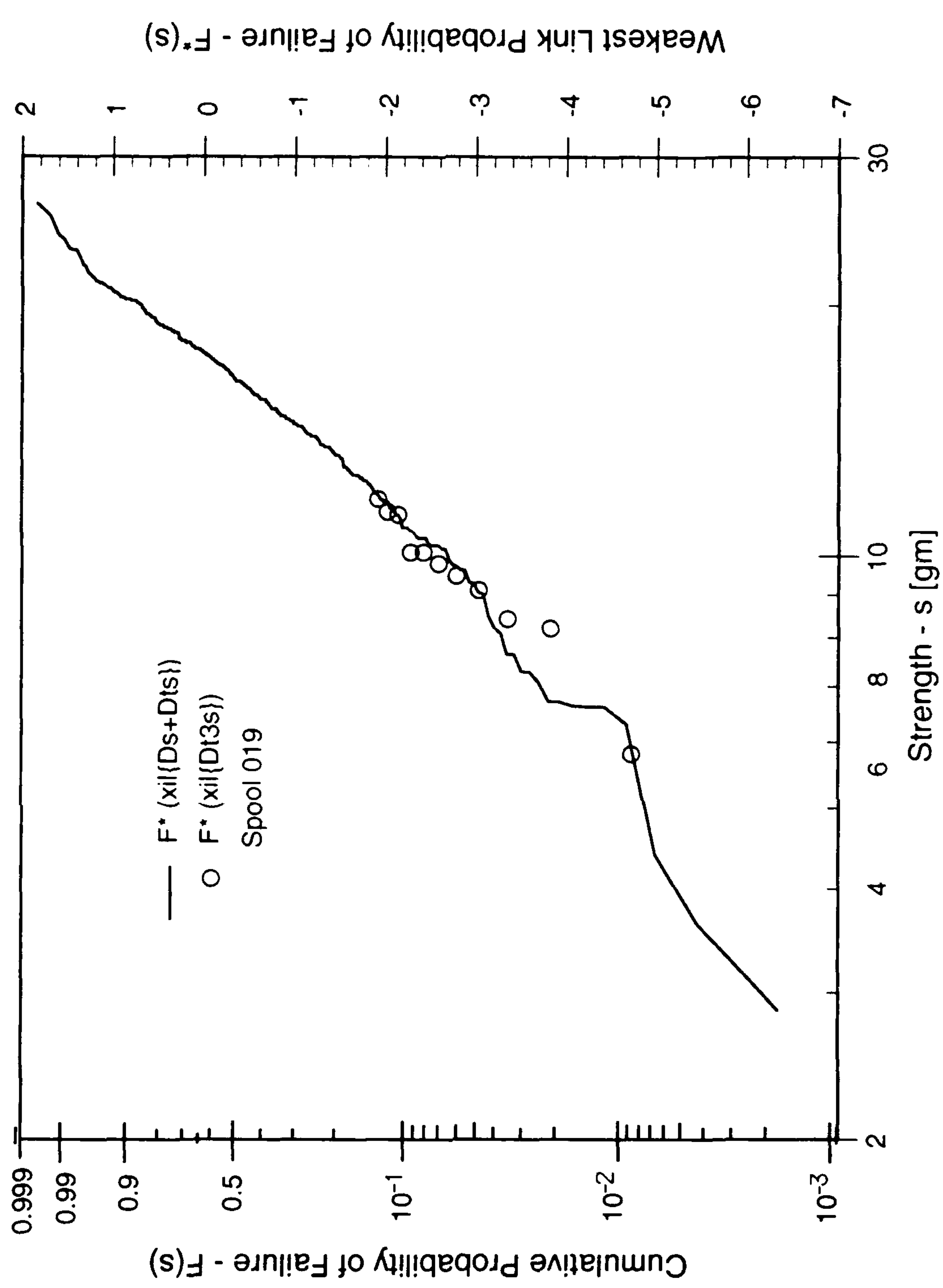

Figure 24. Expected Rank of 019 strength sets $\{D_s + D_{ts}\}$ and $\{D_{t3s}\}$

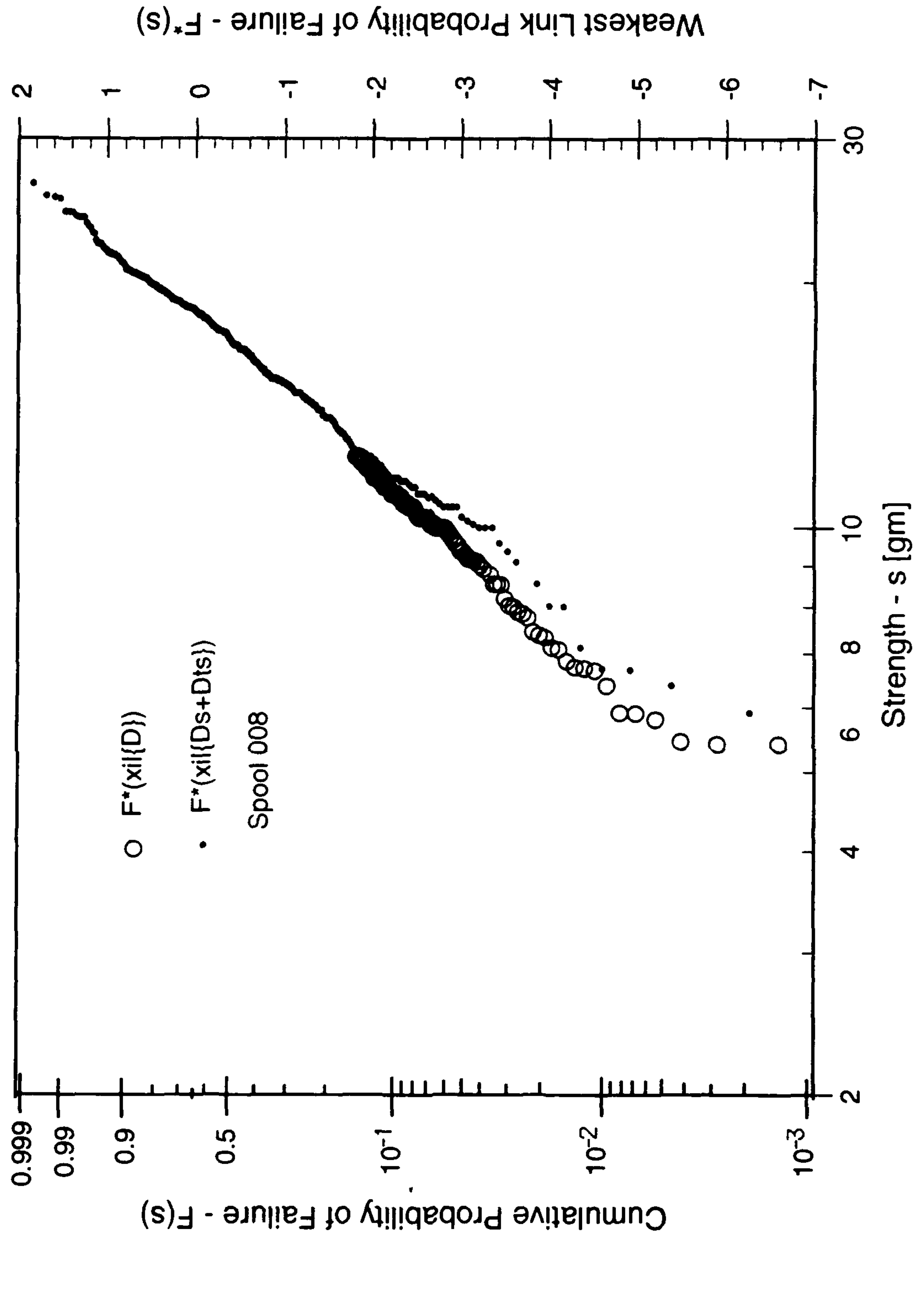

Figure 25. Expected Rank of 008 strength sets $\{D_s + D_{ts}\}$ and $\{D\}$

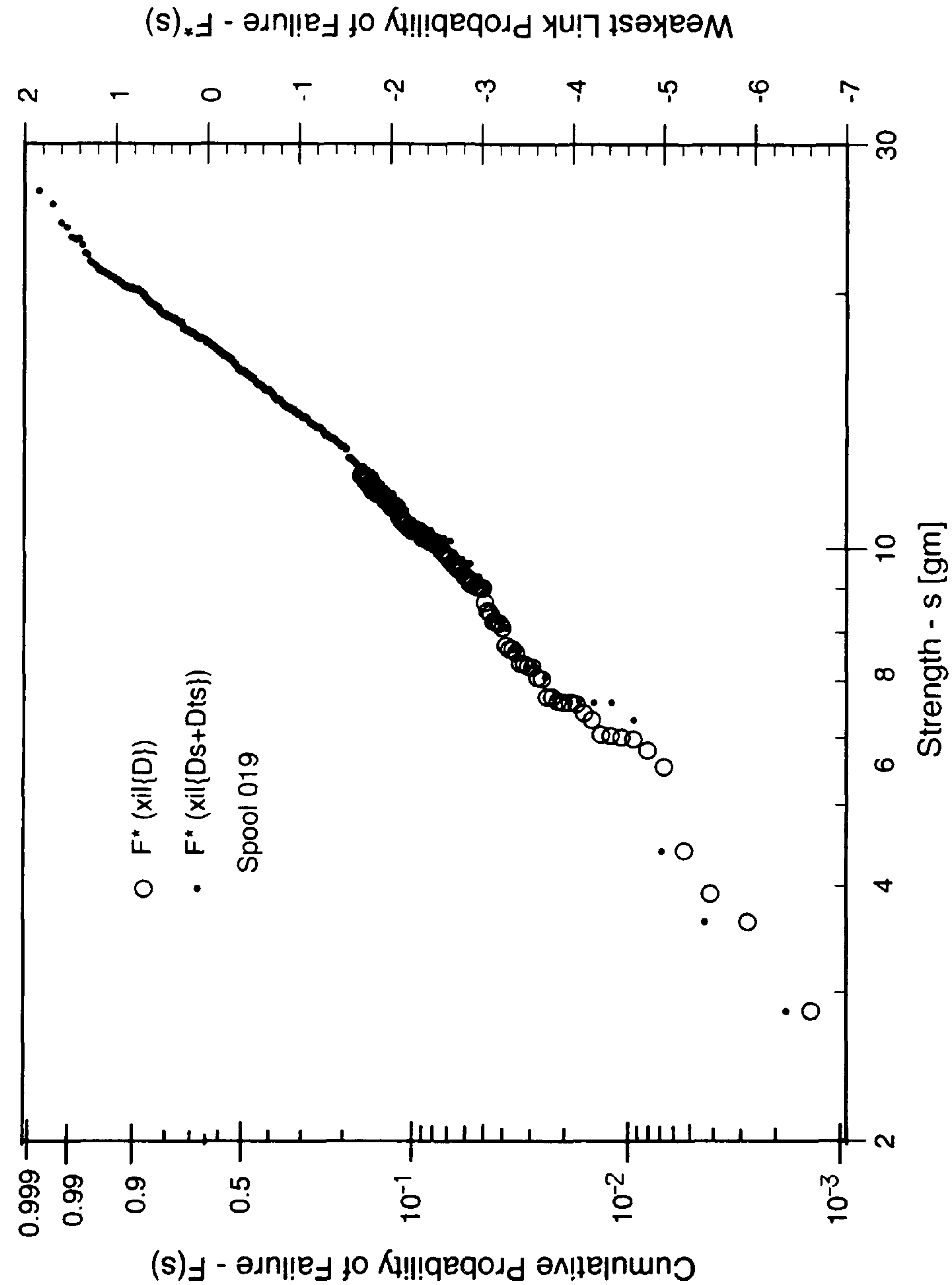

Figure 26. Expected Rank of 019 strength sets $\{D_s + D_{ts}\}$ and $\{D\}$

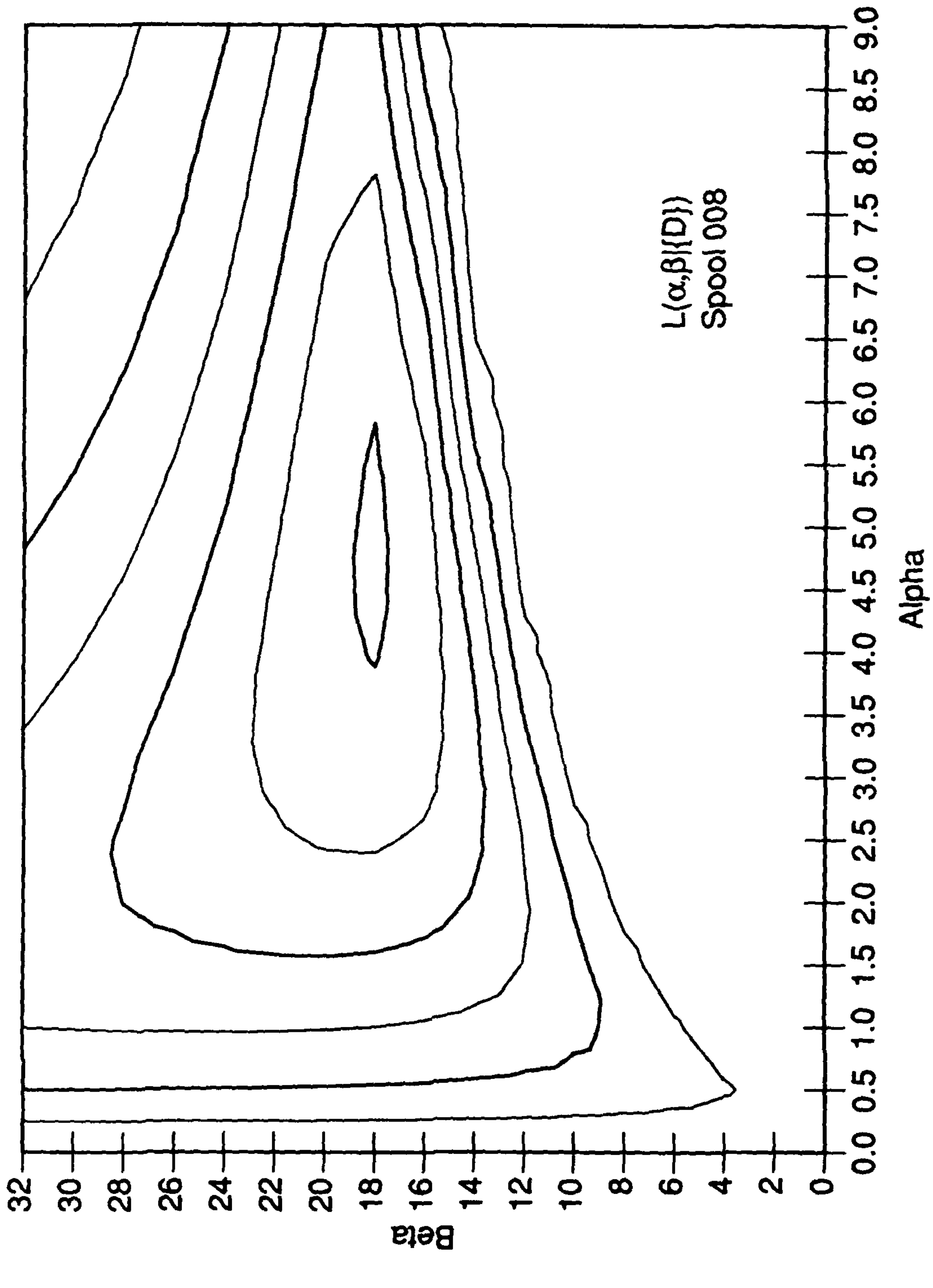

Figure 27. Likelihood contours of 008 strength set {D}

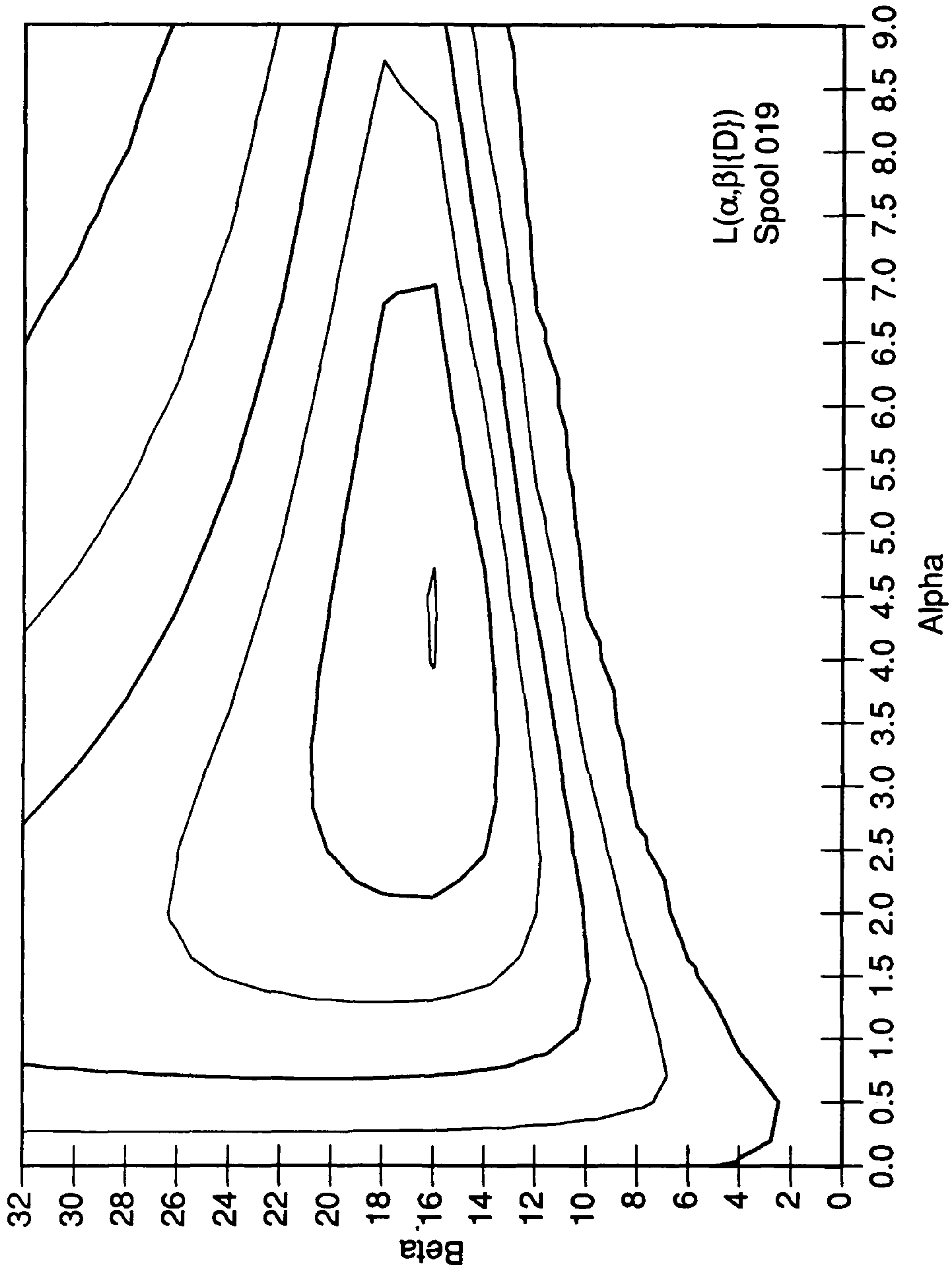

Figure 28. Likelihood contours of 019 strength set {D}

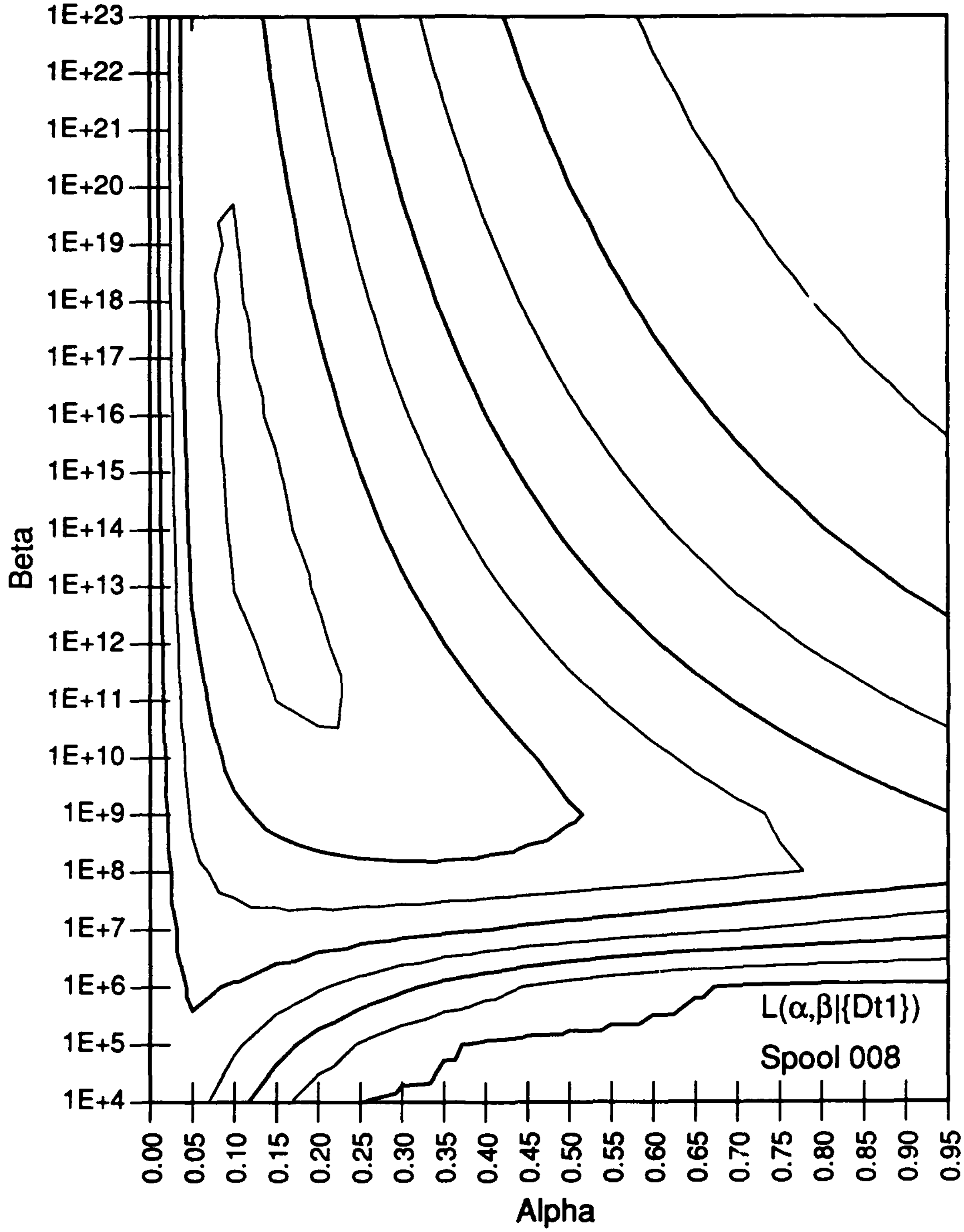

Figure 29. Likelihood contours of 008 life set $\{D_{t1}\}$

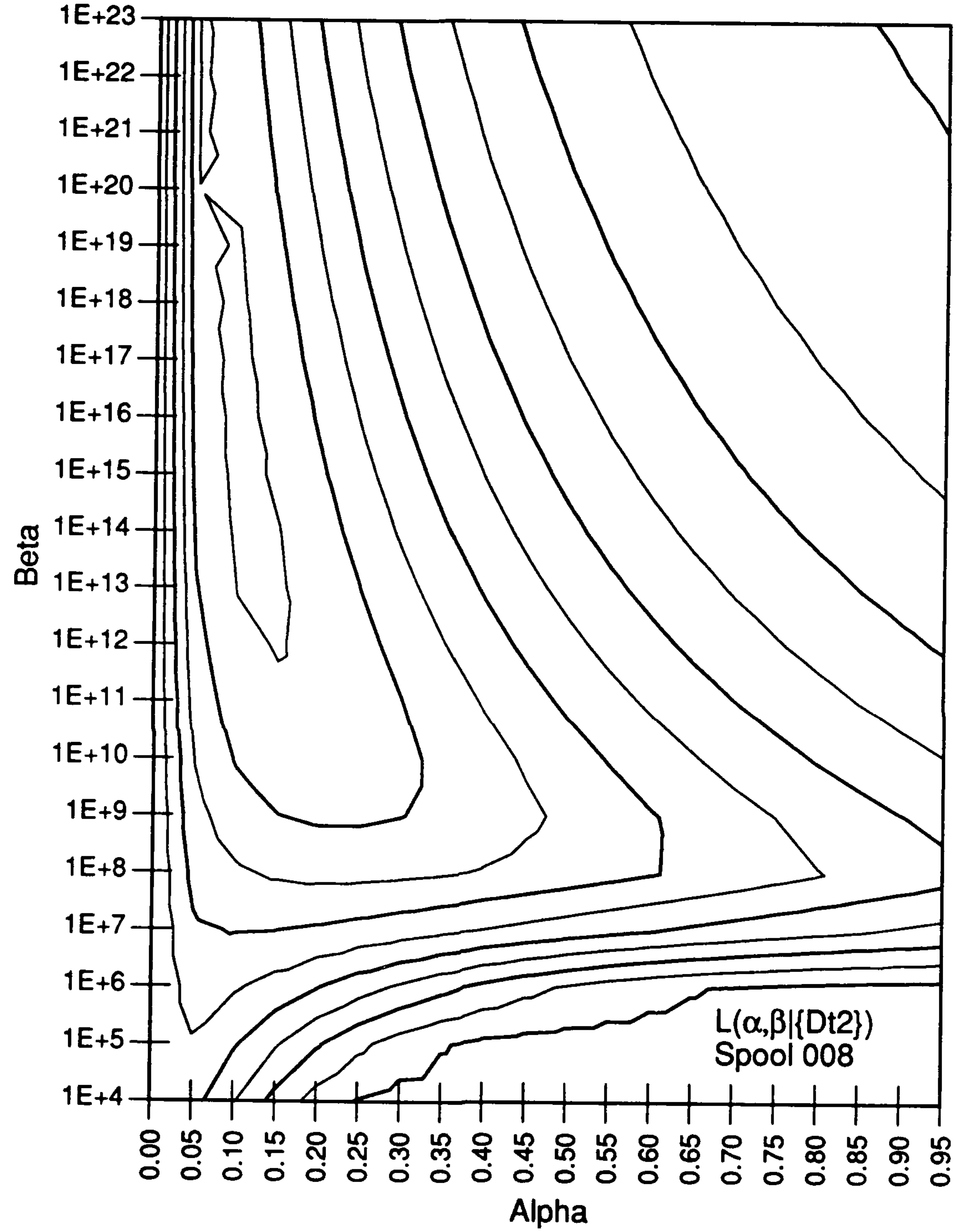

Figure 30. Likelihood contours of 008 life set $\{D_{t2}\}$

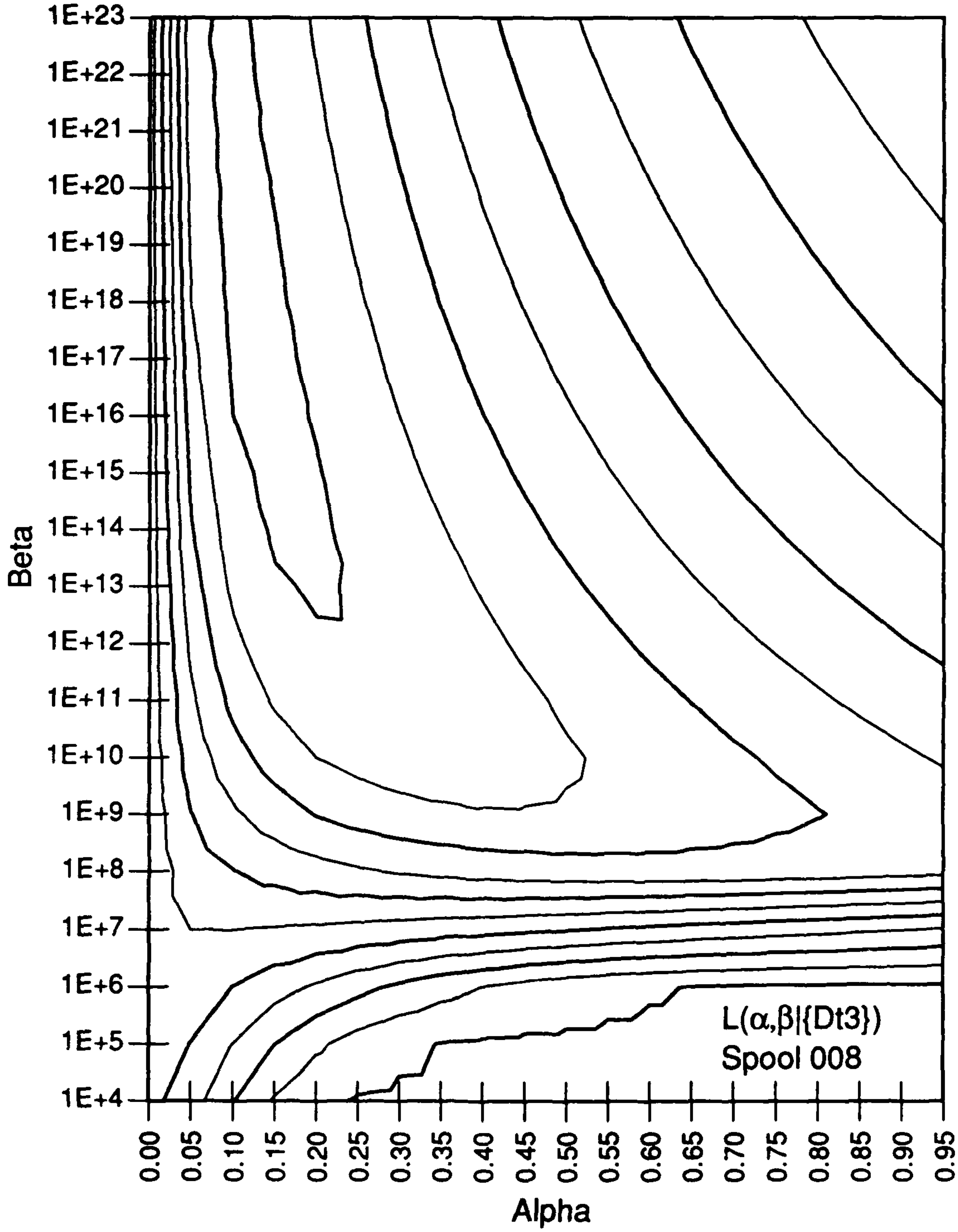

Figure 31. Likelihood contours of 008 life set $\{D_{t3}\}$

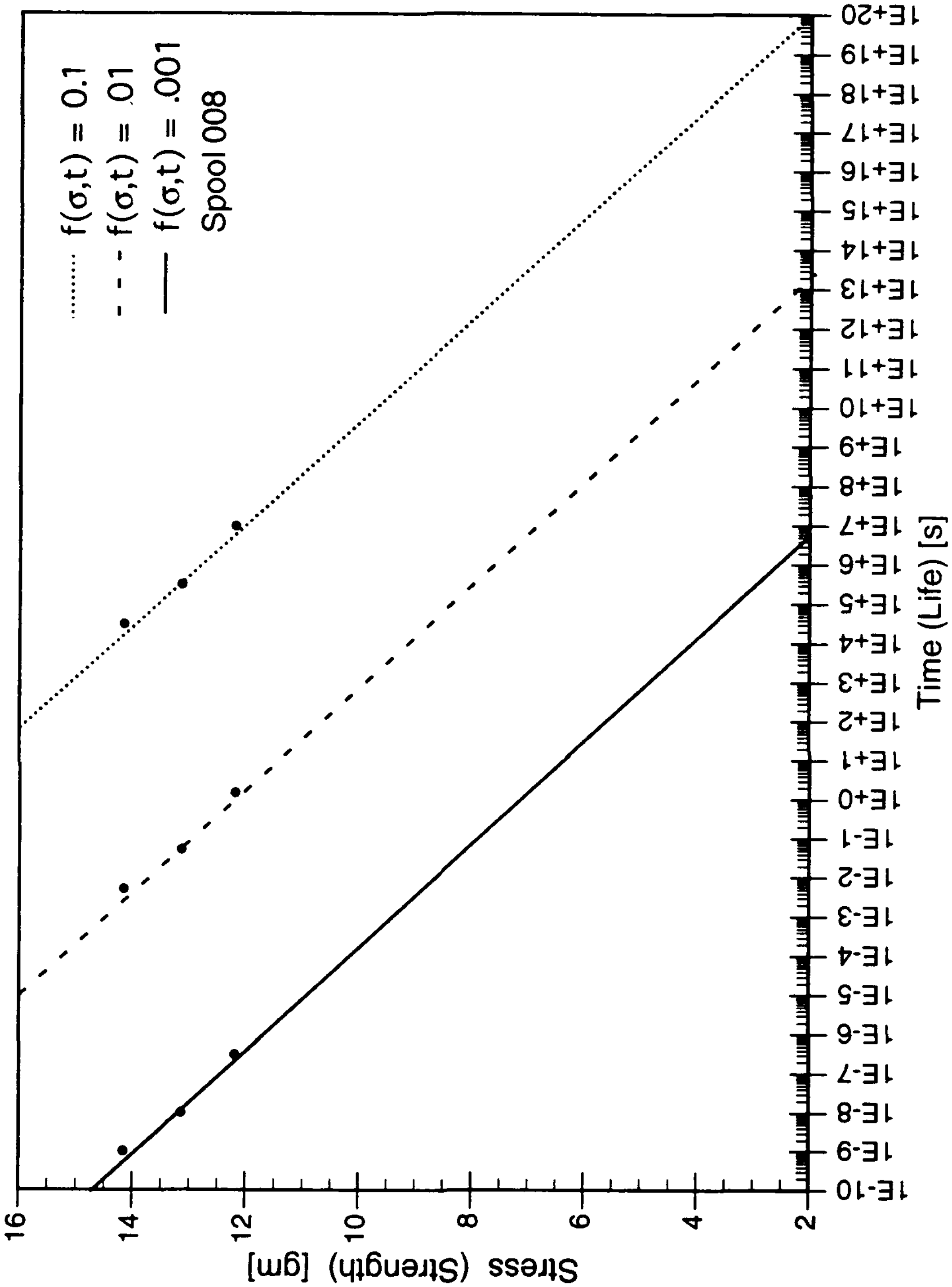

Figure 32. Strength-Life Model for AS4-008

LIST OF REFERENCES

1. Rosen, B. W., Tensile *Failure of Fibrous Composites*, Journal of American Institute of Aeronautics and Astronautics, vol 2, no. 11, pp. 1985-1991, November, 1964

2. Harlow, D.G. and Phoenix, S.L., *The Chain of Bundles Probability Model for the Strength of Fibrous Materials I: Analysis and Conjectures*, Journal of Composite Materials. vol 12, pp. 195-214. 1978

3. Harlow, D.G. and Phoenix, S.L., *The Chain of Bundles Probability Model for the Strength of Fibrous Materials II: A Numerical Study of Convergence,* Journal of Composite Materials. vol 12, pp. 314-334. 1978

4. Coleman, Bernard D., "Statistics and Time Dependence of Mechanical Breakdown in Fibers," *Journal of Applied Physics*, vol. 29, no. 6,pp. 968-983, June 1958.

5. Bell, D. K., *Composite Reliability Enhancement via Preloading*, Master's Thesis, Naval Postgraduate School, Monterey, CA, June, 1987.

6. Test data by Edward M. Wu and Glenn Nypiak, Lawrence Livermore National Laboratory, 1983

7. Englebert, C. R., *Statistical Characterization of Graphite Fiberfor Prediction of Composite Structure Reliability*, Master's Thesis, Naval Postgraduate School, Monterey, CA, June, 1990.

8. Johnson, Eric P., *Composite Strength Statistics from Fiber Strength Statistics*, Master's Thesis, Naval Postgraduate School, Monterey, CA, June, 1991.

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