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Final Report

Prof. John C. Strikwerda

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## Finite Difference Methods for Incompressible Viscous Flow in Scientific Computing

### Statement of Research Objectives.

The research of Prof. Strikwerda discussed in the original proposal centered around three topics in computational fluid dynamics. These were: schemes for the time-dependent Navier-Stokes equations, improved methods for the steady Navier-Stokes equations, and domain decomposition methods. In spite of the funding being at a much lower level than requested and for a rather limited time, progress on these topics was significant.

### Summary of Significant Results.

Prof. Strikwerda continued the development of his domain decomposition code. Various modifications to the iterative methods were tested. Several new features were added, including different boundary conditions and improved interpolation methods. The current code handles various types of boundary conditions. The domains that are used by the code are rectangles, disks, annuluses, half-disks, and half-annuluses.

Mr. Dongho Shin was supported as a research assistant by this ARO grant for the summer and part of the Fall of 1991. He has been working on developing fast solvers for the Stokes and Navier-Stokes equations. He has shown that the pressure equation method developed by himself and Prof. Strikwerda for the Stokes equations on simple domain shapes converges in a number of iterations that is independent of grid size. Thus the total amount of work is proportional to the number of grid points, which is optimal. The method uses the conjugate gradient and multigrid algorithms. A paper on this method has been submitted to the SIAM Journal on Scientific and Statistical Computing.

Mr. Shin has also proved several theorems relating to the regularity of the solutions of the method. He has also extended the method to the Navier-Stokes equations and employed the method with domain decomposition. Mr. Shin will be getting his Ph.D. degree this summer, and these results will be part of his thesis.

Fourth-order accurate methods for the Stokes and Navier-Stokes equations on rectangular domains have also been developed and tested. In using domain decomposition, the higher order accurate methods can lead to great efficiency by requiring fewer grid points in rectangular domains. It has been found that the domains with coarser grids and higher-order accurate schemes combine well with the standard second-order accurate methods of neighboring domains.

During the Fall of 1991, a CM-5 computer was installed at the Computer Sciences Department. Prof. Strikwerda has a student, Zhan Deng, who has begun to program on the CM-5. Miss Deng plans to program the domain decomposition method on the CM-5 as part of her doctoral research.

Prof. Strikwerda has also continued collaborative work with two other researchers. Prof. A. Mukherjee of the University of Pennsylvania and Prof. Strikwerda have worked on using partial differential equation models to analyze local area network congestion control protocols. This work has resulted in a paper which was accepted for the SIGCOMM'91 conference. There were only 28 papers accepted out of 128 submitted. Work is continuing on related topics.

Prof. Strikwerda also has been continuing his collaboration with Mr. John Considine of the U.S.D.A. Forest Products Laboratory on an analysis of deformation of paper under pressure. This work has further applications can be applied to other membrane problems. Work is planned on analyzing the wrinkling of membranes under stress. Several publications are planned.

### **Publications**

This list is of publications of Prof. Strikwerda for the period March 1991 to April 1992. the period covered by this grant.

1. Analysis of Dynamic Congestion Control Protocols--A Fokker-Planck Approximation (with A. Mukherjee), conference proceedings of the SIGCOMM'91 Conference.
2. An Analysis of the Burst Test Geometry. A New Approach. (with John M. Considine). Proceedings of the 1991 International Paper Physics Conference, (1991) pp. 579-584.
3. Fast Solvers for Finite Difference Approximations for the Stokes and Navier-Stokes Equations, (with Dongho Shin). Computer Sciences Tech. Report #1034. August 1991. Submitted to SIAM J. on Scientific and Statistical Computing.
4. Cardinal Series Interpolation to Nonuniform Grids. Computer Sciences Tech. Report #1075. February 1992. Submitted to J. Approx. Theory.

### **Supported Personnel**

Prof. John C. Strikwerda, Principal Investigator.

Mr. Dongho Shin, Ph.D. degree (Mathematics) expected July 1992.

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