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BY

M. A. STEPHENS

TECHNICAL REPORT NO. 454

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ON WATSON'S ANOVA FOR DIRECTIONS

by M.A. Stephens

Simon Fraser University

"One of the things which has impressed me in statistical work is how often desperate approximations seem to work well". -- Professor Geoffrey S. Watson, Louvain, August 1989.

1. Introduction

In the summer of 1989, there was a satellite meeting of the I.S.I. held in Louvain, Belgium. The theme was on Statistics in the Earth Sciences, and one session focussed on directional data and related problems. A feature of the session was a talk by Professor G.S. Watson (Geof), and in reviewing his own contributions, he made the comment quoted at the head of this article. The felicitous phrase "desperate approximations" led me to reflect on one of his most remarkable approximations, the ANOVA technique which he introduced for the analysis of samples of directional data.

This was given in two papers in 1956 (Watson, 1956; Watson and Williams, 1956). The methodology is intuitive, straightforward to apply, and uses well-known distributions, and it has been much used by practical workers in geology, biology, and other areas of science ever since.

Not long after Watson's ANOVA was introduced, I was fortunate to be a Ph.D. student under his supervision at Toronto. He introduced me to this field of directional data analysis, which might then be said to be in its infancy, and of which he is a pioneer. In my thesis (Stephens, 1962a), I produced nomograms and approximations for some conditional tests already suggested by Watson and

Williams; and also examined, and continued in subsequent papers in the 1960's, the accuracy of the ANOVA technique.

Essentially it is remarkably accurate in three dimensions, where usually the vectors are tightly clustered, but less so in two dimensions, where often data gives widely dispersed vectors. In my early papers some corrections were suggested for the χ^2 approximations which are used in the methodology; they were designed to keep the simplicity of the approximations. The F-statistics in the ANOVA were also used to give approximations to the conditional test statistics. In later work (Stephens, 1982) a use for the technique, applied to p -dimensional vectors, was suggested in a sociological context where student activity patterns were reduced to eight-dimensional unit vectors. More usefully, in the field of directional data, a multiway extension of Watson's ANOVA, corresponding to a nested analysis, was also developed.

In this article, in tribute to Professor Watson, these subjects are discussed again, with particular emphasis on the intuitive appeal and the practicality of the ANOVA method. The basic ANOVA technique is given in Sections 2 and 3, followed by an illustration of the multiway ANOVA in Section 4, here applied to directional data in three dimensions.

In Sections 5 and 6 the χ^2 approximations are re-examined, including some which arise in the multiway layout, and corrections are suggested for general p -dimensional vectors; once again the special role of $p=3$, data on the sphere, is evident. Finally some remarks are made, in Section 7, on interesting connections between Watson's ANOVA and the conditional tests mentioned above.

2. The ANOVA technique.

Watson's ANOVA is developed for one or more samples of unit vectors, denoting directions, which are assumed to come from the von Mises distribution in two or three (more generally, in p) dimensions. Suppose a random sample of N unit vectors is $OP_i, i=1, \dots, N$ with O at the centre of a circle or sphere and P_i on the circumference or surface, and suppose the vectors are drawn from a p -dimensional von Mises distribution. Without loss of generality we can suppose the modal vector OA to point north, and θ_i is the angle between OA and OP_i . Then the density per unit area of θ is proportional to $\exp(\kappa \cos \theta)$, where κ is a clustering parameter: large κ indicates that the OP_i are tightly clustered around OA . In analysing a sample, the important statistics are $\underline{R} = \sum_i OP_i$, its length R , and the component X on OA , when this is known or hypothesized. The likelihood ratio statistic for a test that OA is the modal vector gives the approximate distribution

$$2\kappa(R-X) \sim \chi_{p-1}^2 \quad (1)$$

This should be good for all κ , if N is large enough.

For large κ , when the θ_i will be small, Watson showed that, in p dimensions,

$$2\kappa(N-X) \sim \chi_{N(p-1)}^2 \quad (2)$$

and suggested

$$2\kappa(N-R) \sim \chi_{(N-1)(p-1)}^2 \quad (3)$$

From this it was a short step to the analogy with the analysis of variance:

$$2\kappa(N-X) = 2\kappa(N-R) + 2\kappa(R-X) \quad (4)$$

with approximate distributions

$$\chi_{Nr}^2 = \chi_{(N-1)r}^2 + \chi_r^2 \quad (5)$$

where here and henceforth we write r for $p-1$. If one were analysing s samples, the corresponding identity is, using R_i, X_i, N_i for sample i , and with $\sum N_i = N$, and $\sum_i R_i = R$;

$$2\kappa(N-R) = 2\kappa(N_1-R_1) + 2\kappa(N_2-R_2) \dots + 2\kappa(R_1+R_2+\dots+R_s - R) \quad (6)$$

with approximate distributions

$$\chi_{(N-1)r}^2 = \chi_{(N_1-1)r}^2 + \chi_{(N_2-1)r}^2 + \dots + \chi_{(s-1)r}^2 \quad (7)$$

3 Exact and approximate tests.

Many useful techniques flow from equations (1) to (7). Firstly, tests and confidence intervals for κ can be based on (2) and (3). Secondly, the ANOVA analogy would imply that the last term in (4) and (6) is independent of the previous terms on the right-hand side; thus, from (4), we have

$$(N-1) \frac{R-X}{N-R} \sim F_{r,(N-1)r} \quad (8)$$

Equation (8) is used to give a test that OA is a given unit vector $\underline{a}$; first one finds $\underline{R}$, which is the MLE of the modal vector, and then its component X on $\underline{a}$; then $(N-1)(R-X)/(N-R)$ is referred to the F distribution in (8).

For s samples, (6) gives a test that they all have the same modal vector, based now on $R_1 + R_2 + \dots + R_s - R = d_1$; the appropriate test becomes

$$\frac{(N-s)(R_1+R_2+\dots+R_s-R)}{(s-1)\{N-(R_1+R_2+\dots+R_s)\}} \sim F_{(s-1)r,(N-s)r} \quad (9)$$

We note here the intuitive appeal of the tests, remarked in the introduction. For one sample, if OA is the correct modal vector, one expects $\underline{R}$ to point nearly along OA , and so R and X will be similar in size: the ANOVA test is based on the statistic $R-X$. If several populations all have the same (unknown) modal vector, one would expect all the resultants $\underline{R}_1, \underline{R}_2, \dots, \underline{R}_s$ to point roughly in the same direction, that of the overall resultant $\underline{R}$, obtained by pooling all samples into one large sample: thus for four samples, say, the picture should look like Figure 1.

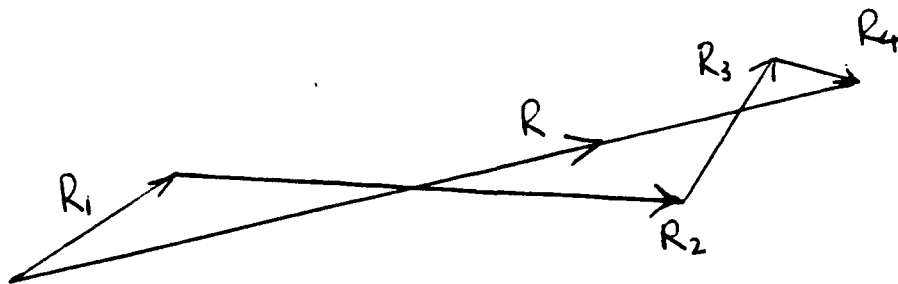


Fig. 1

Then the sum of the lengths of the $\underline{R}_i$ will be not much more than R , the length of $\underline{R}$; the test statistic in ANOVA depends on the difference $R_1 + R_2 + \dots + R_s - R$.

We now consider the χ^2 approximations for $N-X$ and $N-R$. For $p=3$, data on a sphere, the expressions for the distributions of X and R can be written in exact, if clumsy, closed forms, and with the help of computers (slightly younger than directional data) exact percentage points for R and X could be found. Tables of points have been published and also tables for multisample tests (Stephens, 1967, 1969b). In the present context, the important conclusion from this work is that, for 3-dimensional vectors, Watson's approximations (2) and (3) are very accurate, and these lead to accurate tests based on the F -distributions (8) and (9), provided κ is greater than about 5; in the main application at that time, to paleomagnetism studies in geology, κ is usually much greater than 5. Thus the end result of this work was entirely self-defeating - it showed my tables to be unnecessary, at least for the application then intended! Furthermore, the simplicity of tests (8) and (9) was much appreciated by practical statisticians, who, of course, are at home with χ^2 and F ; and in particular (9) makes unnecessary extensive tables which would otherwise have to be produced for various values of s , and combinations of $N_1, N_2, \dots, N_s$.

4. A multiway layout

Watson's ANOVA can be extended to a multiway layout (Stephens, 1982). This will be illustrated with an example involving directional data in 3 dimensions. Suppose the sets of unit vectors, with end-points on the unit sphere, are divided into a two-way layout, with I rows and J columns, and let cell (i,j) be in the i -th row and j -th column. Let N_{ij} be the number of vectors in cell (i,j) with $\underline{R}_{ij}$, length R_{ij} , being the resultant of these vectors. Let $\underline{R}_{i.}$ be the resultant, of length $R_{i.}$, of all vectors in row i , and similarly let $\underline{R}_{.j}$, length $R_{.j}$, be the resultant of all vectors in column

j. Finally suppose N is the total number of vectors, $N = \sum_{i,j} N_{ij}$, and let $R_{..}$, be the resultant of all the vectors. As before, write $r=p-1$. We now write the following identity:

$$2\kappa(N-R_{..}) = 2\kappa \sum_{j=1}^I (N_{1j}-R_{1j}) + 2\kappa \sum_{j=1}^I (N_{2j}-R_{2j}) + \dots + 2\kappa \sum_{j=1}^I (N_{Ij}-R_{Ij}) \\ + 2\kappa \left(\sum_{j=1}^I R_{1j}-R_{1.} \right) + \dots + 2\kappa \left(\sum_{j=1}^I R_{Ij}-R_{I.} \right) + 2\kappa \left(\sum_{i=1}^I R_{i.}-R_{..} \right);$$

collecting terms, we have

$$2\kappa(N-R_{..}) = 2\kappa \sum_{i=1}^I \sum_{j=1}^I (N_{ij}-R_{ij}) + 2\kappa \left(\sum_{j=1}^I R_{1j}-R_{1.} \right) + \dots + 2\kappa \left(\sum_{j=1}^I R_{Ij}-R_{I.} \right) \\ + 2\kappa \left(\sum_{i=1}^I R_{i.}-R_{..} \right) \quad (10)$$

with corresponding distributions, for large κ :

$$\chi^2_{(N-1)r} = \chi^2_{(N-I)r} + \chi^2_{(I-1)r} + \dots + \chi^2_{(I-1)r}.$$

The terms may be arranged in a variance component table as in Table 1. A final column, 'mean component', may be added, giving the value of the variance component divided by its degrees of freedom. In the table, the factor 2κ has been omitted in the formulas under "value". To test the null hypothesis H_0 : that there is no difference between rows, we calculate the quotient

$$Z_1 = \frac{(N-IJ) \left(\sum_{i=1}^I R_{i.} - R_{..} \right)}{(I-1) \left(N - \sum_{i=1}^I \sum_{j=1}^J R_{ij} \right)} \quad (11)$$

which, on H_0 , has an F distribution with $(I-1)r$ and $(N-IJ)r$ degrees of freedom. The null hypothesis is rejected for a significantly large value of Z_1 . Similarly, to test the

Table 1. Two-way analysis of variance for resultant vectors.

Variance component	Value	Degrees of freedom
Between rows	$\sum_{i=1}^I R_{i.} - R_{..}$	$(I-1)r$
Between cols within row 1	$\sum_{j=1}^J R_{1j} - R_{1.}$	$(J-1)r$
.	.	.
.	.	.
.	.	.
Between cols. within row I	$\sum_{j=1}^J R_{Ij} - R_{I.}$	$(J-1)r$
Within groups	$N - \sum_{i=1}^I \sum_{j=1}^J R_{ij}$	$(N-IJ)r$
Total	$N - R_{..}$	$(N-IJ)r$

null hypothesis H_0 : there is no difference between columns within row i , the quotient

$$Z_2 = \frac{(N-IJ) \left(\sum_{j=1}^J R_{ij} - R_{i.} \right)}{(J_i-1) \left(N - \sum_{i=1}^I \sum_{j=1}^J R_{ij} \right)} \quad (12)$$

is calculated, and compared with the F distribution with $(J-1)r$ and $(N-IJ)r$ degrees of freedom. The table thus allows an analysis of the type usually called a nested analysis of variance, and can be adapted to situations where the number of columns varies with the row (Stephens, 1982). Fig. 2 shows the way the resultants are subdivided.

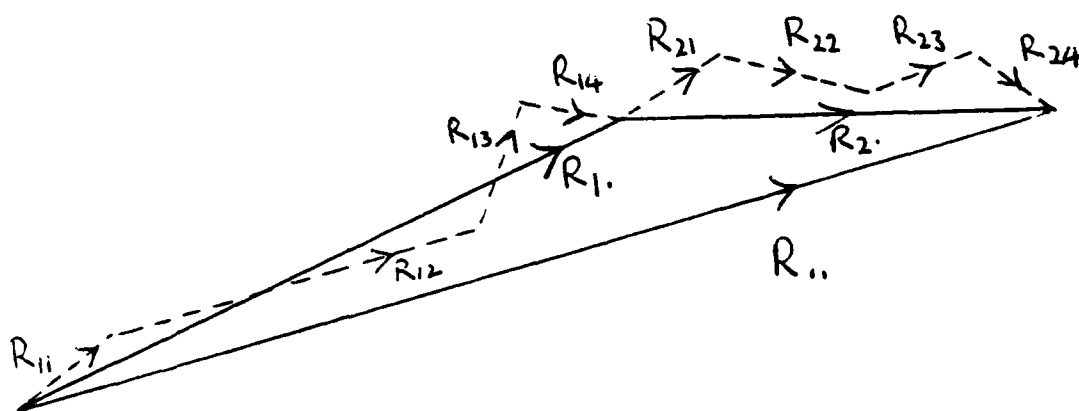


Fig. 2

Example. The data to hand consists of two samples of coal-cleat attitudes, kindly supplied by Professor Richard Alldredge of Washington State University, Department of Agriculture, and in turn taken from Jeran and Mashey (1970). The two samples, of sizes 69 and 55, will be put into 2 rows of a 2 way layout, and to illustrate the methodology, they have each been divided into 4 sub-groups, where

the vectors appeared to be clustered, and these are placed in the columns. The table of N_{ij} and resultant lengths is in Table 2; subscripts inside cells are omitted.

		Columns				
		1	2	3	4	Total
Row 1	N	19	19	16	15	69
	R	18.93	18.87	5.93	14.95	$R_1 = 68.37$
Row 2	N	21	20	7	7	55
	R	20.89	19.92	6.97	6.97	$R_2 = 54.22$

The overall resultant of all 124 vectors has length 84.15 .

These results lead to an ANOVA table as follows.

Between rows:	38.45	2	$Z_1 = 8907$
Between columns within row 1	0.39	6	$Z_2 = 30.1$
Between columns within row 2	0.52	6	$Z_2 = 40.06$
Within groups	0.50	232	

The test statistic Z_1 , to test for differences between rows, would be compared with $F_{2,232}$ and the test statistics Z_2 with $F_{6,232}$; clearly there is a difference between the two sites where samples were taken (rows) and also between columns, as one might expect since the subgroups are chosen to be apparent clusters in the data. In this analysis we can expect the F-tests to be accurate because the κ values in each cell are very large, indicated by how close R is to N in each cell.

5. Two dimensions.

In applications to directional problems in two dimensions - for example, in examining flights of birds, or movement of insects reacting to light - the vectors are typically more widespread, indicating a lower value of κ . The exact distributions of X and of R are intractable; but moments can be found, and accurate percentage points for these statistics were given by fitting Pearson curves using these moments. (Stephens, 1969a). These showed that the χ^2 approximations (2) and (3) were not as accurate in two dimensions as in three. It seemed a good idea to keep the χ^2 form of the approximation, and this could be done by changing κ to γ given by

$$\frac{1}{\gamma} = \frac{1}{\kappa} + \frac{3}{8\kappa^2} \quad (13)$$

This approximation was found as a compromise between two methods (Stephens, 1969a) and works well for κ as low as 2. (Of course, when κ becomes very small, it might be preferred first to test for randomness ($\kappa=0$) of the given vectors, and except for very large samples this will often not be rejected.)

6. Higher dimensions.

In Stephens (1982) an application of the von Mises distribution was made, to the analysis of data in the form of proportions of a continuum such as time, weight, volume, or income. The particular application was the following. Suppose y_i = proportion of a student's day spent in activity i , $i=1, \dots, p$, and let $x_i = \sqrt{y_i}$. Clearly the x_i are components of a vector of length 1, in p dimensions. In the application,

groups of student activity vectors were modelled as samples from 8-dimensional von Mises populations. In my thesis, and later in two Technical Reports (Stephens, 1962a, 1962d, 1980) distribution theory was given for the von Mises distribution in p dimensions. In particular, the χ^2 approximations (2) and (3) hold good for large κ , as before. The student activity patterns were therefore analysed using the multiway layout given above. It was also assumed that if the approximation worked better for $p=3$ than it did for $p=2$, it would surely work even better for higher p . This piece of induction turned out to be rather rash: Ulrich (1984) has since shown that it takes very high κ to work well in higher dimensions. (This would not invalidate the analysis in Stephens (1982), since vectors of the type discussed will certainly come from populations with very high κ). It seems fascinating that the sphere should be the special case where the approximations are excellent, so we now again examine the χ^2 approximations to see if they can be improved for $p > 3$. As before, we try to find γ so that

$$2\gamma(N-X) \sim \chi^2_{N(p-1)}, \quad 2\gamma(N-R) \sim \chi^2_{(N-1)(p-1)}$$

The method used will be to equate the mean of $2\gamma(N-X)$ to $N(p-1)$, since this mean can be found exactly. In p dimensions, $E(X/N) = I_{p/2}(\kappa)/I_{q/2}(\kappa)$, where $I_v(\kappa)$ is the Bessel function of the second kind of order v , and $q = p - 2$. Thus the new approximation gives

$$\frac{1}{\gamma} = \frac{2}{p-1} \left(1 - \frac{I_{p/2}(\kappa)}{I_{q/2}(\kappa)} \right) \quad (14)$$

An asymptotic expansion exists for $I_v(\kappa)$:

$$I_v(\kappa) = \frac{e^\kappa}{\sqrt{2\pi\kappa}} \left(1 - \frac{4v^2-1}{8\kappa} + \frac{(4v^2-1)(4v^2-9)}{2!(8\kappa)^2} - \frac{(4v^2-1)(4v^2-9)(4v^2-25)}{3!(8\kappa)^3} \dots \right)$$

A little algebra will then give

$$\frac{1}{\gamma} = \frac{1}{\kappa} - \frac{p-3}{4\kappa^2} - \frac{p-3}{4\kappa^3} + \frac{(p-3)(p+3)(p-7)}{64\kappa^4} + \frac{(p-3)(p^2-4p-9)}{16\kappa^5} + O(\kappa^{-6}) \quad (15)$$

We see at once that no correction of higher order than $1/\kappa$ is available for the special case $p=3$. This can be verified by direct examination of the mean $E(X/N)$ which reduces, for $p=3$, to $\coth \kappa - \frac{1}{\kappa}$, and $1/\gamma$ then equals $1/\kappa$ + a term which tends to zero faster than any power of κ (Stephens, 1969a). Further, for $p=2$, the second order correction to $1/\gamma$ is positive, but for $p \leq 4$ it is negative. Watson (1988) has himself returned to these questions, and has shown that, as $p \rightarrow \infty$, κ must be proportional to $\sqrt{p}$ in order for the approximate distributions to take hold. Studies are underway to use these results to provide χ^2 approximations for $p \geq 4$, and also to adapt the F-tests to give greater accuracy for small κ . Other problems of practical interest include investigating the accuracy of the multiway procedures for $p \geq 4$, and also the important question of the robustness of all the ANOVA techniques when the samples show different κ -values, although all are large.

7. Conditional tests and ANOVA.

We conclude by investigating the connection between Watson's ANOVA and conditional tests for the modal vector which were introduced by Watson and Williams (1956). These are based on the fact that, for any dimension, the distribution of R , given X , is independent of κ . A test for the modal vector, for example, is then based on whether $R > R_\alpha$, where R_α is the critical value of R at level α , for the value of X obtained by projecting R on the hypothesized modal

vector. These tests may be compared with the ANOVA tests for large κ ; we shall concentrate on the case $p = 3$, for which, again, closed form expressions can be found for the distribution of R , given X . (Watson and Williams, 1956). These authors noted that the conditional test, when X , and therefore R , are close to N , gives almost exactly the same result as the unconditional F-test (8); and both were equivalent to an expression for the confidence interval for the modal vector given by Fisher (1953), where the nuisance parameter κ is eliminated fiducially. The exact expressions for $P(R > R_\alpha | X)$ were used to obtain R_α for given X (and N) and significance levels $\alpha = 0.10, 0.05$, and 0.01 , and were published in the form of nomograms (Stephens, 1962c, reproduced in Biometrika Tables for Statisticians, Vol. 2, 1972, and in Mardia, 1972); the F-approximation was again used to provide an approximation for R_α . In fact, when X is greater than $N-2$, the exact value of R_α , for the conditional test, is given by solving Watson's unconditional test statistic set equal to the critical value of F , and holding X constant: that is

$$\frac{R_\alpha - X}{N - R_\alpha} = \frac{F_{2, 2N-2}(\alpha)}{N-1} \quad (16)$$

gives $R_\alpha | X$. For smaller X , but reasonably large, the equality becomes a good approximation. The similar equality on the circle is never exact, but the idea of using the F-ratio in this way to obtain R_α , was the basis of a test given in Stephens (1969b).

The technique expressed by (16) is not, of course, usually valid to obtain a critical value of the conditional distribution of a statistic such as R , given X , from the unconditional distribution of a function of both R and X . It is therefore interesting to see why it works here. The basis is the following characterisation of the Gamma distribution: if x and y are independent Gammas with the same scale parameter, then y/x is independent of $x + y$. Writing here $y = 2\kappa(R-X)$ and

$x = 2\kappa(N-R)$, and accepting Watson's χ^2 and independence approximations, we then have $(R-X)/(N-R)$ independent of $N-X$, and therefore independent of X . Then equation (16) follows. The exactness of (16) for large X on the sphere, and its accuracy when used as an approximation for smaller X , together with the similar results for the circle, compel us once again to appreciate the accuracy of Watson's original χ^2 approximations. For workers in several fields of science, they were successful enough to give the first techniques for the analysis of directional data -- desperate though they may have been!

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Watson's ANOVA technique for analyzing unit vectors in p dimensions has been usually used with directional data, with $p=2$ or 3 . It is shown how the one-way ANOVA can be extended to a multiway layout, and an example is given using coal-cleat directions. The accuracy of the techniques is reexamined, and it is shown why this can be expected to be greater for $p=3$ than for $p=2$, and possibly also for $p > 3$. Finally, conditional tests for directions are compared with the unconditional ANOVA test. It is shown that a characterization of the gamma distribution permits the conditional significance points to be found from the unconditional points using an argument which, in general, is valid.