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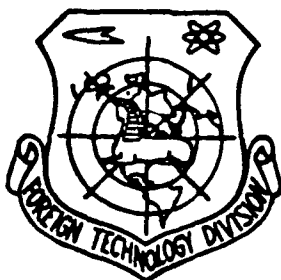


METHODS OF GUIDANCE FLIGHT DYNAMICS

by

Zhu Wenxuan

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METHODS OF GUIDANCE FLIGHT DYNAMICS

Zhu Wenxuan

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SUMMARY Setting up and solving ballistic type rocket kinetic equations in inertial coordinate systems with directions which do not vary, it is possible not to consider dragging inertial or centrifugal forces or coriolis inertial forces and set up inertial guidance on a convenient and simple, quick and clear foundation. This is different from what has been published in the past, where studies have been done of various types of flight mechanics problems in coordinate systems which turn, following along with the earth. We illucidate the initial take off conditions as rockets leave the surface of the earth, and analyze, in detail, the air or atmospheric effects associated with air being pulled along by the gravity of the earth as it turns and the treatment methods for aerodynamic forces on rockets moving in an inertial space. We derive a formula for calculating the geographical location of movement parameters, used for rockets in inertial coordinate systems, and a formula for dynamic coordinate system movement parameters.

KEY TERMS Inertial Guidance, Coordinates, Flight Mechanics, Kinetic Equation

As far as the literature^(1,2,3,4) and publications associated with the kinetics and kinematics of domestic and foreign studies of lifting rockets and other similar types of spacecraft are concerned, although all start out through classical mechanics, in the end, however, they all infer the handling of aerodynamic forces in coordinate systems which turn together along with the earth as well as various similar types of forces and motion parameters. Moreover, it is necessary to consider centrifugal inertial forces and coriolis inertial forces as well as other similar types of virtual forces. Speaking in terms of the researchers who have handled guidance and navigation systems for rockets and other similar types of spacecraft, if one analyzes motion characteristics of inertial instruments, it becomes even more complicated. In many years of research and practical application, we took rocket kinetics problems and solved them in an unmoving inertial space, obtaining satisfactory results.

I. SEVERAL COORDINATE SYSTEMS^[3]

1. The Earth Centered Coordinate System E, X_e, Y_e, Z_e .

The origin point E is placed at the geometrical center of the earth. The EX_e axis in the plane of the equator, from the center of the earth E, points toward the point of intersection of the prime meridian and the equator and is positive in direction. The EY_e axis, in the plane of the equator, is perpendicular to the EX_e axis and is positive in direction toward the outside. The EZ_e axis, along the axis of the earth's autorotation, forms a clockwise orthogonal coordinate system with EX_e and EY_e . This is as shown in Fig.1.

2. Guidance Coordinate System O_o, X_o, Y_o, Z_o

The origin point O_o is the launch point of the rocket on the earth's surface. The O_oX_o axis, within the horizontal plane that passes through O_o , points toward the direction of the flight and is positive. The included angle or angle of offset between it and the due north direction of the meridian group tangent is the azimuth angle. The O_oY_o axis follows along a plumbline upward and is positive. The O_oZ_o axis, in the horizontal plane that passes through O_o , forms a clockwise orthogonal coordinate system with O_oX_o and O_oY_o . At the instant that the rocket leaves the launch pad (time $t=0$ sec), one takes the direction $O_oX_oY_oZ_o$ is pointing in inertial space and firmly fixes it invariable. It then forms an inertial coordinate system. See Fig.1.

3. Launch Coordinate System or Dynamic Coordinate System OXYZ.

At the instant the rocket takes off ($t=0^s$), the origin points O and O_o , the OX axis and O_oX_o , the OY axis and O_oY_o , and the OZ axis and O_oZ_o , are all superimposed on each other. After the rocket leaves the ground and takes off, OXYZ is firmly connected to the surface of the earth. Moreover, it turns together with the earth. See Fig.1.

4. Spacecraft Coordinate System $O_1X_1Y_1Z_1$

The origin point O_1 is positioned at the center of mass of the rocket body. The O_1X_1 axis runs along the main axis of inertia of the rocket, points toward the nose, and is positive. O_1Z_1 , in the secondary plane, is perpendicular to the $X_1O_1Y_1$ main plane of symmetry. This is as shown in Fig.2.

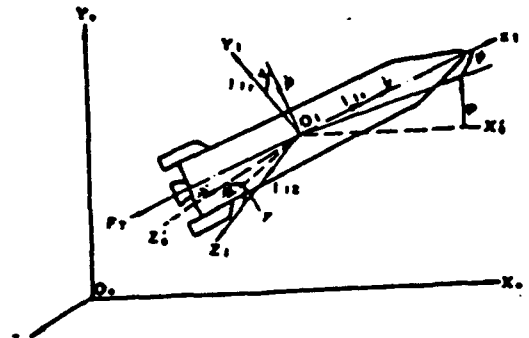


Fig.1 The Relationships of Such Coordinate Systems as $EX_eY_eZ_e$, $OX_oY_oZ_o$, and $OXYZ$

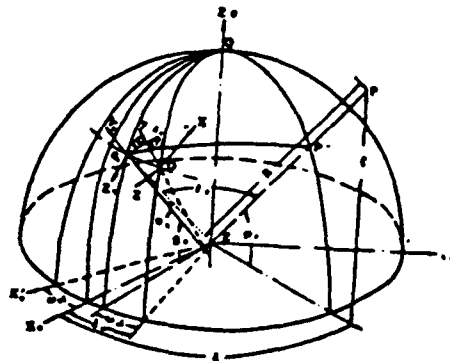


Fig.2 The Relationship of $O_o X_o Y_o Z_o$ and $O_1 X_1 Y_1 Z_1$

5. Airflow Coordinate System $O_2X_2Y_2Z_2$

The origin point O_2 is coincident with the center of mass of the rocket body O_1 . The axis O_2X_2 is in line with the air flow velocity vector V_d corresponding to the rocket. The O_2Y_2 axis, in the main rocket plane $X_1O_1Y_1$, goes upward and is positive. The O_2Z_2 axis forms a clockwise rectangular coordinate system with the O_2X_2 and O_2Y_2 axes. If one takes the $O_2X_2Y_2Z_2$ coordinate system and rotates it about the O_2Y_2 axis through the lateral slide angle β , and, again, rotates it about the O_1Z_1 axis through the angle of attack α , then it is coincident with O, X, Y, Z . See Fig.3.

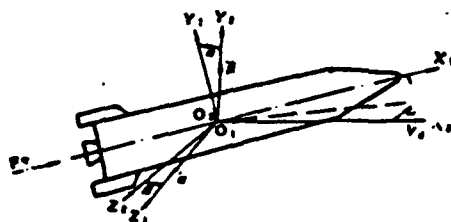


Fig.3 The Relationships of O_1, X_1, Y_1, Z_1 and O_2, X_2, Y_2, Z_2

The unit vectors on the axes of each coordinate system are taken as a unit 1, adding the corresponding coordinate system's lower corner subscripts to express that. For example, l_{ox}, l_{oy} , and l_{oz} are the unit vectors of the guidance or inertial coordinate system O, X, Y, Z on the three axes O, X, Y , and O, Y, Z .

II. KINETIC EQUATIONS

Taking out the influences of other celestial bodies, one only considers the effects of earth's gravity, G_e . The high speed flow of combustion gases sprayed out by the rocket engines produces as an effect the thrust vector F_T on the rocket. Its magnitude is related

to the amount of mass consumed as fuel in a unit time $\dot{G}=dG/dt$ (kg/s), the engine jet tube characteristics, and the height h at which the rocket flies off the surface. Normally, this can be expressed as [1,2,3]

$$F_r = \dot{G} \cdot g_e \cdot I_b + S_e P_e \left(1 - \frac{P}{P_e}\right) \quad (2.1)$$

On the basis of the principles of momentum, the rocket's instantaneous acceleration vector in the inertial coordinate system $O_O X_O Y_O Z_O$, $V_O = dV_O/dt$, satisfies the equation below

$$\dot{G}(t) \cdot V_O = F + G_e \quad (2.2)$$

Force vector F is the vector sum of thrust F_r , control force F_K , and the pneumatic force F_q . $g_e = 9.80665$ (m/s²) and is the mass conversion constant. I_b is the engine's specific impulse. S_e (unclear) is the engine combustion gas flow aperture area. P/P_O is the ratio of the gas pressure and the surface atmospheric pressure P_O , which varies with the height h from which rockets fly off the surface. If one takes a certain vector, such as F , and uses a line matrix to express it, its symbols and the vector symbols are the same. One then has $F = [F_X F_Y F_Z]^T$. The upper right superscript T expresses transposed positions. F_X , F_Y , and F_Z are the components of F on the $O_O X_O$, $O_O Y_O$, and $O_O Z_O$ axes. When using the acceleration of gravity $g = [g_X g_Y g_Z]^T$ and comparative force or perceived acceleration $\dot{W} = [\dot{W}_X \dot{W}_Y \dot{W}_Z]^T$ to express equation (2.2), one obtains

$$\dot{V}_O = \dot{W} + g \quad (2.3)$$

In this 其中 $g = \frac{1}{G} G_e$, $\dot{W} = \frac{1}{G} F$. The force vector

$$F = F_r + F_K + F_e \quad (2.3.1)$$

The thrust force F_T and control force F_K are normally given in cubic coordinate systems. Moreover, the aerodynamic force Q_2 in the gas flow coordinate system $O_2X_2Y_2Z_2$ can be transformed into $O_1X_1Y_1Z_1$ and expressed. In this way, the comparative force components on the main inertial O_1X_1 , O_1Y_1 , and O_1Z_1 axes are respectively

$$\dot{W}_{x1} = \frac{1}{G}(F_{1rx} + F_{1Kx} + Q_{1x}) \quad (2.4.1)$$

$$\dot{W}_{y1} = \frac{1}{G}(F_{1ry} + F_{1Ky} + Q_{1y}) \quad (2.4.2)$$

$$\dot{W}_{z1} = \frac{1}{G}(F_{1rz} + F_{1Kz} + Q_{1z}) \quad (2.4.3)$$

The comparative force vector $\dot{W}_1 = [\dot{W}_{x1} \ \dot{W}_{y1} \ \dot{W}_{z1}]^T$. The relationship between the unit vectors l_0 and l_1 of $O_0X_0Y_0Z_0$ and $O_1X_1Y_1Z_1$ can be used to link up or associate the orthogonal transformation matrix D , which is nothing else than

$$l_1 = D \cdot l_0 \quad (2.5)$$

Because of this, the comparative force vector W in the guidance coordinate system $O_0X_0Y_0Z_0$ is capable of being aided by D_1 using the comparative force vector $\dot{W}_1$ in $O_1X_1Y_1Z_1$ to express as

$$\dot{W} = D \cdot \dot{W}_1 \quad (2.6)$$

Gauging or measurement assemblies used in the guidance and stability systems of such types of carrier or delivery craft as rockets are, it goes without saying, gyroscope stabilized platforms or sensitively connected inertial assemblies. The various elements of the orthogonal transformation matrix D , d_{ij} ($i=1,2,3$, $j=1,2,3$), are all capable of being arrived at. For example, in Fig.2, rotating the pitch angle φ around the axis O_0Z_0 , and again, rotating the yaw angle ψ around the transitional axis OY'_1 , and, finally, revolving the roll angle γ around the axis O_1X_1 , in that case, d_{ij} is then possible to make use of to obtain the sine and cosine functions of Euler angles φ , ψ , and γ .

Taking rockets, when they act as rigid bodies, the Euler equations for the revolved center of mass O_1 are normally set up in a cubic coordinate system^[6]:

$$J_x \dot{\omega}_{1x} + (J_z - J_y) \omega_{1z} \omega_{1y} = M_{1x} \quad (2.7.1)$$

$$J_y \dot{\omega}_{1y} + (J_x - J_z) \omega_{1x} \omega_{1z} = M_{1y} \quad (2.7.2)$$

$$J_z \dot{\omega}_{1z} + (J_y - J_x) \omega_{1y} \omega_{1x} = M_{1z} \quad (2.7.3)$$

In the equations, J_x , J_y , and J_z are the instantaneous momenta of rotation of the rocket about the main axes of momentum O_1X_1 , O_1Y_1 , and O_1Z_1 . M_{1x} , M_{1y} , M_{1z} are component moments of force along the three axes discussed above. ω_{1x} , ω_{1y} , ω_{1z} are components of angular acceleration for rotation of the rocket about O_1X_1 , O_1Y_1 , and O_1Z_1 . The angular acceleration vector is

$$\omega_1 = [\omega_{1x} \ \omega_{1y} \ \omega_{1z}]^T \quad (2.8)$$

This article only illucidates movements of the center of mass. It does not discuss the kinetic equations for motions around center of mass.

III. INITIAL CONDITIONS AND AERODYNAMIC FORCES

In the inertial coordinate system $O_0 X_0 Y_0 Z_0$, as far as study of the flight kinetics of rockets and other similar carrier or delivery vehicles is concerned, the most important thing is to elucidate the methods of handling the effects of aerodynamic forces with which the atmosphere acts on rockets in high speed flight as it rotates along with the earth, and, on the foundation of discussions with fellow workers, in conjunction with the carrying out of theoretical analysis, to obtain correct solutions.

Observed in inertial space, rockets, before they take off, rotate together along with the earth. They are dragged along by the force of the earth's gravity and connected firmly to the surface of the earth. However, they already possess a dragging or centrifugal linear acceleration. This velocity vector V_{00} is the initial velocity taking off from earth under the effect of the thrust of the engines. The magnitude of V_{00} is determined by the vertical distance of the launch point O_0 from the earth's axis of autorotation. In the same way, the atmosphere, which wraps around the earth, is also pulled firmly by the force of the earth's gravity and follows the earth in its rotation. Observed from inertial space, this atmosphere also possesses similar linear acceleration.

Looking from the revolving earth, the rocket, before it takes off, is not moving. The atmosphere is stationary. The rocket's movement velocity relative to the atmosphere is zero. As a result of this, there are no aerodynamic or pneumatic force effects on the stationary rocket. This is saying nothing else than that, if, when one calculates the magnitude of the effects of aerodynamic or pneumatic forces on the rocket in inertial space, it is necessary to take the dragging or centrifugal velocity associated with the rocket rotating along with the earth and the velocity of the atmosphere moving together with the earth and subtract them from each other, one obtains the rocket's velocity relative to atmospheric movements as zero, making the aerodynamic forces borne by the rocket, when it is erect on the launch pad and has not taken off yet, to be zero. If one takes the initial absolute velocity V_{00} of the rocket before it takes off, as observed in inertial space, and takes it as the movement velocity relative to the atmosphere, calculating out the effects of

pneumatic forces on the rocket, in that case, the rocket erected on the launch pad will just be pushed over onto the ground. However, this does not correspond to the facts.

After the rocket, under the effects of thrust F_T , leaves the launch pad and takes off, the absolute velocity of the motion in inertial space gets greater and greater. The height of the separation from the earth's surface is also able to continuously increase. The magnitude of the aerodynamic forces that it exerts at a given point P separated from the center of the earth E is also determined by the velocity of the atmosphere corresponding to its being pulled around by the earth. It is only in the inertial space $O_0 X_0 Y_0 Z_0$ that, as far as the rocket's velocity relative to the atmosphere is concerned, it is appropriate to use this relative air flow velocity to calculate the effects of pneumatic or aerodynamic forces on the rocket.

The components of the earth's rotational velocity vector ω_e on the various individual axes of the inertial or guidance coordinate system $O_0 X_0 Y_0 Z_0$ are:

$$\Omega_x = \omega_e \cdot \cos B_0 \cdot \cos A_0 = \omega_e \cdot b_x \quad (3.1.1)$$

$$\Omega_y = \omega_e \cdot \sin B_0 = \omega_e \cdot b_y \quad (3.1.2)$$

$$\Omega_z = \omega_e \cdot (-\cos B_0 \cdot \sin A_0) = \omega_e \cdot b_z \quad (3.1.3)$$

In these, B_0 is the geographical latitude of the rocket launch point O_0 . A_0 is the azimuth angle of $O_0 X_0$. In $O_0 X_0 Y_0 Z_0$, the absolute velocity of the air at the place P(R) where the rocket is located is nothing else than the dragging or centrifugal velocity V_e . It can be expressed as

$$V_e = \omega_e \times R = \begin{bmatrix} 1_{ex} & 1_{ey} & 1_{ez} \\ \Omega_x & \Omega_y & \Omega_z \\ R_x & R_y & R_z \end{bmatrix} \quad (3.2)$$

The magnitude of the radius vector from the point P toward the center of the earth, $R=[R_x R_y R_z]^T$, is

$$R=\sqrt{(X_0+R_{0x})^2+(Y_0+R_{0y})^2+(Z_0+R_{0z})^2} \quad (3.3)$$

In this, $R_1=[R_{0x}, R_{0y}, R_{0z}]^T$ is the radius vector toward the center of the earth for the initial location of the rocket O_0 in $O_0X_0Y_0Z_0$. From the origin point O_0 , the position vector pointing toward P is $\xi=[X_0 Y_0 Z_0]^T$.

Given initial conditions, that is to say, the instant of rocket take off is $t=0$ sec and the location relative to the $O_0X_0Y_0Z_0$ system is zero, that is, $X_0=Y_0=Z_0=0$ and $R=R_1$, the initial velocity V_{00} is nothing else than the dragging or centrifugal velocity of point O_0 , that is,

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$$V_{00}=V_{01,00}=V_{01,00} \quad (3.4)$$

In this way, the variable coefficient differential equation (2.3) is then capable of using numerical value methods for its solution. The absolute velocity of the rocket V_0 at a certain instant as well as the position ξ and R can all be obtained.

The rocket's velocity relative to the motion of the earth is also nothing else than the motion velocity V_d relative to the atmosphere and is the difference between the absolute velocity V_0 and the dragging or centrifugal velocity $V_{e(unclear)}$

$$V_d=V_0-V_{e(unclear)} \quad (3.5)$$

The aerodynamic drag force X_q and the relative velocity V_d are mutually opposite in direction, that is, they are mutually opposite to the direction of O_2X_2 . The aerodynamic or pneumatic lift (normal pneumatic lift) Y_q is positive in the direction along O_2Y_2 . The lateral pneumatic force Z_q is opposite in direction to O_2Z_2 . They are respectively used in the several forms set out below for calculations

$$X_r = -C_x \cdot q \cdot S_c \quad (3.6)$$

$$Y_r = C_y^a \cdot q \cdot S_c \cdot a \quad (3.7)$$

$$Z_r = -C_z^{\beta} \cdot q \cdot S_c \cdot \beta \quad (3.8)$$

In this, C_x , C_y^a , and C_z^{β} (unclear) are, respectively, the aerodynamic or pneumatic drag coefficient of the rocket, the aerodynamic lift coefficient, and the aerodynamic or pneumatic laterally directed force coefficient. As far as impact pressure is concerned, one uses the form

$$q = \frac{1}{2} \rho \cdot V^2 \quad (3.9)$$

to calculate it. The air density ρ changes along with changes in the height h of the rocket as it flies off the surface. S_c is the reference cross section surface area.

From Fig.3 one can see that the airflow coordinate system $O_2 X_2 Y_2 Z_2$ goes through a rotation through the lateral slide angle β and the angle of attack a to arrive at $O_1 X_1 Y_1 Z_1$. The transformation between the unit vectors of the two coordinate systems uses the matrix E and is expressed as

$$l_i = E \cdot l_i \quad (3.10)$$

Obviously, E is a third order orthogonal matrix. Its various elements are functions of a and β :

$$E = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} = \begin{bmatrix} \cos \beta \cdot \cos a & \sin a & -\sin \beta \cdot \cos a \\ -\cos \beta \cdot \sin a & \cos a & \sin \beta \cdot \sin a \\ \sin \beta & 0 & \cos \beta \end{bmatrix} \quad (3.11)$$

The angle of attack α and the lateral slide angle β are calculated by the use of the formulae set out below:

$$\alpha = \sin^{-1}(-V_{1r}/V_1) \quad (3.12)$$

$$\beta = \sin^{-1}(V_{1z}/\sqrt{V_{1x}^2 + V_{1z}^2}) \quad (3.13)$$

In the cubic coordinate system $O_1X_1Y_1Z_1$, the relative velocity $V_1 = [V_{1x} V_{1y} V_{1z}]^T$ is obtained by V_d in the $O_0X_0Y_0Z_0$ system going through transformation. It is capable of being expressed as

$$V_1 = D^T V_d \quad (3.13)$$

Obviously, $V_1 = V_d$ are invariant. As far as the aerodynamic or pneumatic force $Q_2 = [X_q Y_q Z_q]^T$ in the airflow coordinate system $O_2X_2Y_2Z_2$ is concerned, it is possible to make use of the matrix E to transform it into the cubic coordinate system $O_1X_1Y_1Z_1$, that is,

$$Q_1 = E \cdot Q_2 \quad (3.14)$$

Control force vectors F_K are always given in the $O_1X_1Y_1Z_1$ coordinate system. Practically speaking, their handling is different because of different types of servomechanisms. Up to now, we have already, in $O_1X_1Y_1Z_1$, completely obtained the thrust force F_{1T} , the aerodynamic or pneumatic force Q_1 , and the control force F_K . It is possible, from equations (2.4) and (2.6), to calculate out the comparative forces for changes over time, $\dot{W}$ and $\dot{W}_{t(\text{unclear})}$.

When one takes the earth to be acting as a revolving, symmetrical ellipsoid sphere, it is possible to only give consideration to the gravitaional force potential's second degree harmonic coefficient J_2 quantity^[5]. The components of the gravitational acceleration g , which the rocket receives pointing toward the vicinity of the center

of the earth and which are in directions along $O_0 X_0$, $O_0 Y_0$, and $O_0 Z_0$ are:

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$$g_v = g_c \cdot \left\{ -\frac{R_z^2}{R^3} \cdot \frac{R_z}{R} + J_1 \frac{R_z^2}{R^3} \cdot \left[\frac{R_z}{R} \cdot \left(5 \frac{\xi^2}{R^2} - 1 \right) - 2b \cdot \frac{\xi}{R} \right] \right\} \quad (3.15)$$

$(v = X, Y, Z)$

In this, the constant of gravitational force is

$$g_c \cdot R_c^2 = fM, \quad (3.16)$$

R_c is the earth's equatorial radius. g_c is the acceleration of gravity on the equator. The vertical distance that the rocket's real time position $P(R)$ is off the plane of the equator is

$$\xi = b_x(X_0 + R_{0x}) + b_y(Y_0 + R_{0y}) + b_z(Z_0 + R_{0z}) \quad (3.20)$$

The point P 's geocentric latitude φ_x is calculated from the equation

$$\varphi_x = \sin^{-1} \left(\frac{\xi}{R} \right) \quad (3.21)$$

$2b_v \sin \varphi_x$ in g_v does not exist in OXYZ when one is studying the motion equations for the center of mass of rockets.

The application of the various expressions set out above is a condition for the complete solution of the kinetic equation (2.3). It is then possible to precisely determine, in changing situations, such motion parameters as the absolute velocity V_0 and the position R or ξ .

IV. KINETIC EQUATIONS FOR THE UNPOWERED SECTION

As far as the rocket or its released nose cone is concerned, in

the section of the flight after the propulsion system is shut off and there are no thrust force effects or attitude angle controls, it is possible to take them to be acting as a point mass or material point particle. It is not necessary to consider the angle of attack α or the lateral slide angle β . The only aerodynamic force there is is drag. Moreover, it is opposite in direction to V_d ,

$$X_d = -C_{xt} \cdot q \cdot S_t \quad (4.1)$$

The comparative force is

$$W_b = \frac{S_t}{2G_t} \cdot C_{xt} \cdot \rho \cdot V_d^2 \quad (4.2)$$

In this, C_{xt} , S_t , and G_t are, respectively, the nose cone's aerodynamic drag coefficient, reference cross section surface area, and its mass. Taking W_b and analyzing it along the three axes of the inertial coordinate system $O_O X_O Y_O Z_O$, one then has kinetic equations similar to (2.3);

$$\dot{V}_{dx} = -W_b \frac{V_{dx}}{V_d} + g_x \quad (4.3.1)$$

$$\dot{V}_{dy} = -W_b \frac{V_{dy}}{V_d} + g_y \quad (4.3.2)$$

$$\dot{V}_{dz} = -W_b \frac{V_{dz}}{V_d} + g_z \quad (4.3.3)$$

In this, the form expressing the acceleration of gravity $g = [g_x g_y g_z]^T$ is the same as (3.15). The initial conditions for equation (4.3) are the motion parameters $V_O(tf)$ and $\xi(tf)$ for the instant when the thrust stops or the instant tf when the nose cone and the rocket separate.

In situations in which the flight of the nose cone reentering the atmosphere has attitude angle controls, the aerodynamic force Q_2 contains the drag force X_q , the lift force Y_q , and the lateral force Z_q . It is necessary to make use of the handling methods of 3. Moreover, one must do transformations into the coordinate system (for example, $O_5X_5Y_5Z_5$) firmly connected to the "horse's head" cone. After solving Euler equations similar to (2.7), one obtains the transformation matrix D . In this way, the aerodynamic or pneumatic relative forces are also capable of being accurately handled in $O_0X_0Y_0Z_0$.

V. MOTION PARAMETERS OF THE REVOLVING EARTH

The preceding several sections studied methods of handling aerodynamic forces in inertial space and the problems of solving kinetic equations. However, are the results in line with solutions of kinetic equations in the dynamic coordinate system $OXYZ$? In using transform methods, from $V_0 = [V_{0x} V_{0y} V_{0z}]^T$ and $\xi = [X_0 Y_0 Z_0]^T$, one obtains dynamic coordinate system motion parameters, and, in conjunction with that, after one makes a comparison, one, then, has naturally dispelled misgivings. Referring to Fig.1, going through the transformations described below, it is possible to search out the relationship between $O_0X_0Y_0Z_0$ and $OXYZ$.

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The first step is to rotate $O_0X_0Y_0Z_0$ counterclockwise in a positive direction around O_0Y_0 an azimuth angle A_0 forming $O_0X_0'Y_0Z_0'$. The tangent line O_0N associated with the meridian line of O_0X_0' and the point O_0 is duplicated and points north. O_0Z_0' , within the horizontal plane at point O_0 , is perpendicular to the meridian plane and points east.

The second step is to rotate $O_0X_0'Y_0Z_0'$ counterclockwise around O_0Z_0' an angle equal to the latitude B_0 forming $O_0X_0''Y_0'Z_0'$. O_0Y_0' is parallel to the plane of the equator. O_0X_0'' is parallel to the earth's axis of autorotation EZ_e .

The third step is, in a situation in which the three axes of $O_0 X_0'' Y_0'' Z_0''$ are maintained in an unchanging direction, to take O_0 and translate it to the center of the earth E forming $EX_0'' Y_0'' Z_0''$.

The fourth step is to take $EX_0'' Y_0'' Z_0''$ and rotate it counterclockwise through an angle $\omega_e t$ around EX_0'' , causing $X_0'' Y_0''$ to rotate into the meridian plane of point O.

The fifth, sixth, and seventh steps are, respectively, the inverse processes of the first, second, and third steps. Finally, one then takes $O_0 X_0 Y_0 Z_0$ and transforms it to a position duplicating OXYZ. Moreover, one obtains the transformation matrix B. It has already been demonstrated that the matrix B is orthogonal. The inverse matrix B^{-1} and the transformed position matrix B^T are equal,

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \quad (5.1)$$

This is capable of being written to become the equation below:

$$B = B_4 + \cos \omega_e t (I - B_4) + \sin \omega_e t \cdot B_5 \quad (5.2)$$

In the equation, I is a third order unit matrix. B_4 and B_5 are also both third order matrices:

$$B_4 = \begin{bmatrix} b_x b_x & b_x b_y & b_x b_z \\ b_y b_x & b_y b_y & b_y b_z \\ b_z b_x & b_z b_y & b_z b_z \end{bmatrix} \quad (5.3)$$

$$B_5 = \begin{bmatrix} 0 & -b_z & b_y \\ b_z & 0 & -b_x \\ -b_y & b_x & 0 \end{bmatrix} \quad (5.4)$$

The relative velocity V in the dynamic coordinate system $OXYZ$ is calculated from the equation

$$V = B \cdot V_d \quad (5.5)$$

In this, $V_d = [V_{dx} V_{dy} V_{dz}]^T$ is the relative velocity expressed by the use of (3.5) in $O_0X_0Y_0Z_0$. The relative location $= [XYZ]^T$ in $OXYZ$ is also capable of being expressed by the use of matrix B as

$$\eta = B \cdot R - R_1 \quad (5.6)$$

The magnitude r of the distance from the center of the earth r of the current position P of the rocket is

$$r = \sqrt{(X + R_{0x})^2 + (Y + R_{0y})^2 + (Z + R_{0z})^2} \quad (5.7)$$

Obviously, the invariant inequalities $V^2 = V_d^2 = V_1^2$ and $r^2 = R^2$ are all established. Because of this, it is accurate to make use of the height $h = R - R_1$ by which the rocket leaves the surface of the earth in the inertial coordinate system $O_0X_0Y_0Z_0$ to handle the atmospheric constants ρ/ρ_0 and p/p_0 .

The expressions (5.5) and (5.6) for the velocity V and the position η in the dynamic coordinate system $OXYZ$, arrived at from going through transformations of the parameters V_d and ξ in the inertial space $O_0X_0Y_0Z_0$, are important results. They clearly show that, in $O_0X_0Y_0Z_0$, solving kinetic equations is capable of satisfying various types of requirements. For example, during rocket test flights, ground optical transit theodolites, radar, or other similar tracking and measuring equipment requires V and η to act as a reference basis. In these two types of coordinate system, the results from simultaneously solving rocket kinetic equations clearly demonstrate the accuracy of (5.5) and (5.6), verifying, in the coordinate system $O_0X_0Y_0Z_0$, that solving the rocket kinetic equations is correct.

As far as the geographical latitude B_1 and the geographical longitude λ of the position P for the current location of the rocket at the instant t is concerned, it is also possible to make use of the position R in $O_0 X_0 Y_0 Z_0$ to accurately calculate it out. Referring to Fig.1, the geographical latitude φ_x can be obtained from

$$\sin \varphi_x = \zeta / R \quad (6.1)$$

In this, R is the distance of the point P from the center of the earth E. ζ is the vertical height of the point P off the plane of the equator. The geographical latitude is

$$B_1 = t_e^{-1}(t_e \varphi_x / (1 - e^2)) \quad (6.2)$$

In reality, this only has meaning on the surface of the earth. However, φ_x has significance everywhere. The line connecting the center of the earth E and P intersects with the surface of the earth at point P'. In such a case, the distance of the point P' from the center of the earth is

$$R^1 = R_e \sqrt{1 - e^2} / \sqrt{1 - e^2 \cdot \cos^2 \varphi_x} \quad (6.3)$$

As far as the rocket's reaching point P in space at the current instant t is concerned, in terms of the take off point O_0 's inertial space span or tensile angle, J_1 (unclear) is the voyage or course angle. Using the numerical product of the two vectors from the points O_0 and P pointing toward the center of the earth E, it is expressed as

$$J_1 = \cos^{-1} \left[\frac{R_1}{R} + \frac{X_0 R_{0x} + Y_0 R_{0y} + Z_0 R_{0z}}{R_1 R} \right] \quad (6.4)$$

After going through the time t at the earth's prime meridian, in inertial space, it turns from EX_e' to EX_e , that is, about the axis of autorotation of the earth, EZ_e turns through the angle $\omega_e t$. In the same way, $OXYZ$ also, from the originally duplicate location at $O_0X_0Y_0Z_0$, rotates through the same angle $\omega_e t$. From Fig.1, one can see that the difference in longitude of the point P relative to point O_0 in inertial space is

$$\Delta l = (\lambda - \lambda_0 + \omega_e t). \quad (6.5)$$

On the spherical surface triangle EO_0QP' , respectively making use of cosine theorems and sine theorems, it is possible to solve for an expression for geographical longitude λ . From the cosine theorem of the course or angle of travel J_1 (unclear), one has

$$\begin{aligned} \cos J_1 = & \cos\left(\frac{\pi}{2} - \varphi_{s0}\right) \cdot \cos\left(\frac{\pi}{2} - \varphi_s\right) - \sin\left(\frac{\pi}{2} - \varphi_{s0}\right) \\ & \cdot \sin\left(\frac{\pi}{2} - \varphi_s\right) \cdot \cos \Delta l \end{aligned} \quad (6.6)$$

Because of this, one obtains the formula to calculate the geographical longitude λ of the rocket's current position P

$$\cos(\lambda - \lambda_0 + \omega_e t) = (\cos J_1 - \sin \varphi_{s0} \cdot \sin \varphi_s) / (\cos \varphi_{s0} \cdot \cos \varphi_s) \quad (6.7)$$

In this, λ_0 is the latitude of the O_0 point at the instant of the rocket's take off, $t = 0$ sec. In order to accurately determine the quadrant of λ , on the same spherical surface triangle EO_0QP' , one makes use of the cosine theorem. It is then possible to solve the expression $\sin(\lambda - \lambda_0 + \omega_e t)$. From Fig.1, it is possible to see that λ_0 is also the actual geographical longitude of the origin point O of the dynamic coordinate system $OXYZ$. The geographical longitude of initiation points or origins O_0 and O , $\varphi_{xc} = B_0$

74. In this,

$$\mu_e = a_e \cdot \sin 2B_e \quad (6.8)$$

This is perpendicular deviation. However,

$$a_e = (R_e - R_p) / R_e \quad (6.9)$$

is the rate of flattening of the earth. R_e and R_p are, respectively, the equatorial radius and the polar radius of the earth. The azimuth angle of the current position P is

$$f_1 = A_e + f_1 \quad (6.10)$$

In this, f_1 is the angle of offset or deviation relative to the main plane of the flight $X_O Y_O Z_O$ for the current time position of the rocket P, that is, the included angle between $O_O P$ and $X_O O_P O_O$. Obviously,

$$f_1 = \sin^{-1} \left(\frac{Z_e + R_{ee}}{\sqrt{(X_e + R_{ee})^2 + (Z_e + R_{ee})^2}} \right) \quad (6.11)$$

From this, it is possible to obtain a precise determination of the quadrant of λ .

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CONCLUSIONS

The practical realization of the study and design of the inertial guidance systems of our carrier rockets clearly demonstrates that, in the inertial coordinate system $O_O X_O Y_O Z_O$, which is illustrated by this article, the solutions of rocket kinetics and the derivations of various individual formulae are correct. This is particularly true for the motion parameters V and η as well as geographical coordinates B and λ and is completely in line with the results obtained in the dynamic coordinate system $OXYZ$ from solving kinetic equations.

Making use of the inverse matrix $B^{-1}=B^T$, taking carrier rockets' actual flight course track parameters and transforming them into $O_0X_0Y_0Z_0$, the carrying out of checks and tests on the errors of inertial instrumentation and the analysis of other errors in guidance systems must, of necessity, be feasible.

There are a number of representative works^{1,2,3,4} and treatises which, in the dynamic coordinate system OXYZ, which rotates following the earth, set up and solve kinetic equations of rockets, handle various types of problems associated with flight force mechanics, calculate aerodynamic forces with relative ease, make observations and measurements as well as tracking rocket motions without the need for transformations. With regard to the extreme importance of motions and errors associated with the study of carrier rocket inertial guidance systems in inertial space as well as inertial instrumentation, the methods studied in this article have value. In particular, writing them down into an article is convenient for use as a reference. In conjunction with that, they have been even more deeply discussed with coworkers.

In the process of the studies in this article, we were aided by Li Zhentao, Wang Guangmin, and other similar comrades. We have opted for the use of several formulae associated with rocket kinetic equations set up by them in the OXYZ system. We wish to express our sincere thanks to them!

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