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An Application of Growth Curve Analysis to the
Ammunition Stockpile Deterioration Model

SO YOUNG SOHN

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Abstract

Ammunition deterioration during storage has considerable economic consequences. A reliable prediction model for the ammunition deterioration rate is necessary for long-term procurement and maintenance planning. A random effect growth curve analysis is employed to formulate a prediction model for ammunition deterioration rates in terms of concurrent characteristics such as depot condition and vendor information. The resultant prediction model can be used to determine the appropriate time for reorder or renovation of ammunition before the quality reaches unacceptable levels. A two-stage analysis is used to estimate parameters involved in the prediction model. Necessary estimation methods are discussed. An example is given to illustrate the implementation procedure of the prediction model suggested in this paper.

Key Words: Random Effect Logistic Regression Model, Deterioration Rate, A Two-Stage Estimation

1. INTRODUCTION

Each year, the U.S. military services purchase millions of dollars worth of major caliber ammunition from several vendors to prepare for possible war. Ammunition lots purchased are stored in depots until they are used. In view of the fact that most ammunition is produced long before its ultimate consumption, it is important that the material be adequately stored and maintained to remain usable.

However, according to the GAO report (The Comptroller General (1982)), the military services are experiencing significant problems in maintenance of depot and renovation of ammunition. These problems cause ammunition to deteriorate faster than necessary maintenance can be performed and a serious backlog of ammunition in need of maintenance. In an attempt to prevent such undesirable phenomena, in this paper, a quantitative model that can relate a depot maintenance plan to the quality control of ammunition is considered.

The objective of this paper is to outline a prediction model for ammunition deterioration in terms of depot characteristics and vendor information. The goal is to provide estimation methods by which one can determine the appropriate time for reorder or renovation of ammunition before the quality reaches unacceptable levels. The suggested model can also be used for vendor control and examination of depot maintenance policy.

In order to formulate such a prediction model for ammunition deterioration rates, a random effect growth curve analysis is employed. The analysis of a random effect growth curve model has been extensively used in biomedical research (Rao (1965), Stiratelli et al. (1984), and Vonesh and Carter (1987)). An application of growth curve analysis to physical

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engineering is given in Sohn and Mazumdar (1991). In general, the goal of growth curve analysis is to estimate random growth rates of a time varying principal attribute in terms of concurrent individual characteristics which do not change over the experimental period. The analysis of growth curves is typically carried out by using a two-stage random effect model. The first stage consists of the within-individual regression model in which the within-individual growth rates for the serial observations are estimated for all individuals. In the second stage, estimated growth rates are related to a set of covariates representing concurrent individual characteristics.

In this paper, a growing pattern of defective proportion of an attribute of ammunition lot over time (deterioration rate) is viewed in analogy to a growth rate of a principal attribute of an individual experimental unit while vendor information and depot characteristics associated with each lot can be considered as concurrent individual characteristics. In order to predict deterioration rates of a certain type of ammunition in terms of depot characteristics and vendor information, a random effect logistic regression model (Stiratelli et al. (1984)) is employed assuming that the pattern of deterioration follows a logistic function. For estimation of parameters in a random effect logistic regression model, a two-stage estimation based on the method used in Korn and Whittemore (1979) is employed. An example is given to illustrate the implementation procedure of the random effect logistic model suggested.

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2. RELATED STUDIES

The most important deterioration processes occurring to ammunition during storage are corrosion of the metallic components and chemical and physical changes within the internal structure of the explosives, which may result in reduced stability and performance (Eckman (1974), Eriksen and Strømsøe (1978), and Wilson and Bird (1986)). The factors which have the greatest influence on these processes are temperature and humidity which are directly related to the location and condition of the depot. Even under similar conditions, different types of ammunition may degrade with different deterioration rates. In some instances, different vendors provide the same types of ammunition that, nonetheless, have varying deterioration rates. Most of the studies in the literature do not reveal the quantitative relationship between deterioration rates and depot condition, types of ammunition and vendor information.

For instance, Eriksen and Strømsøe (1978) conducted an extensive experiment covering a 9 year period to examine deterioration of six types of artillery ammunition (105 mm HE, 90 mm HE, 57 mm HE, 40 mm HE, hand grenades Mk2, and antitank mines M6A2) at three aboveground and three underground depot sites. Various attributes were observed: vacuum stability and water content of propellant, impact sensitivity of primers, burning rate of propellant, muzzle velocity, and the number of misfires and duds. Inspection frequencies varied from two to four times covering combinations of starting year (1968) and the subsequent 1st, 5th, 7th and 9th year after storage, depending upon the attributes inspected. Based on these repeated measurements, the contingency table and the analysis of variance were used to test if the levels of ammunition deterioration were significantly

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different in terms of the conditions of depot and the types of ammunition.

The results of their study are as follows: Overall, aboveground sites surrounded by inland climate performed better than underground depots that were old and technically in poor shape; the visual inspection revealed a considerable difference in corrosion attack between the ammunition types; water content both in the primer and the propellant charges varied significantly depending on the depot in which lots of ammunition were stored; most of the chemical analyses were of little interest and should be postponed to the end of trials. Their approach, however, did not provide a comprehensive model which related the level of deterioration of certain type of ammunition to depot condition. If the level of deterioration over time can be expressed as a function of depot conditions or/and vendor information, reorder time of a certain type of ammunition which reaches certain quality can be predicted in terms of given information at the depot. An application of growth curve analysis which facilitates such a prognostication is illustrated in the following section.

3. A GROWTH CURVE ANALYSIS FOR AMMUNITION DETERIORATION RATE

Consider a certain type of ammunition (say, 105mm HE). In this section, the assumptions used in the growth curve analysis to predict the deterioration rate of 105mm HE ammunition are summarized. Similar assumptions can be made for other types of ammunition.

Basic unit of purchase is a lot and as a result of acceptance sampling, the qualities of incoming ammunition lots would be homogeneous regardless of different sources of manufacturers. While several hundreds of such lots are stored in various locations, the quality of ammunition deteriorates with the passage of time. Deterioration rates may vary depending upon both the condition of depot in which ammunition lots are stored and manufacturers who provide the ammunition lots.

In order to inspect the pattern of deterioration of various lots over the time, it is assumed that a set of initially selected lots in various depots is marked and these lots are repeatedly inspected on a sampling basis without rectification over a certain time period. Lot size is assumed to be sufficiently large that it would not be depleted at the end of repeated sampling inspections. Modes of inspection can cover visual inspection, laboratory examination and functional trials (Eriksen and Strømsøe (1978)). The proportion of samples which do not satisfy a predetermined level of an attribute or the combinations of attributes can be used as a proxy for the accumulated level of deterioration at the time of inspection. For instance, among many other attributes of functional trials, the relative number of misfires and duds to the sample size observed at each inspection cycle can be considered as a measure for the level of deterioration of each lot at the

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time of inspection.

For each lot, the proportion of defective rounds is assumed to increase as time goes on following a logistic curve. The logistic model is well established as a basis for analyzing such a phenomenon when the response variable follows binomial distribution. The logistic model formed to describe the deteriorating pattern for each lot is called a within-individual model. Deterioration rate of a lot of ammunition, which represents a deteriorating pattern over time of a lot often differs from that of another lot. The model in which varying deterioration rates between lots are compared in relation to depot characteristics and vendor information associated with each lot is called the between-individual regression model. Depot conditions and vendor information associated with each lot of certain ammunition are assumed to remain constant during the experimental period while they would vary over different lots. Necessary notations for a two-stage random effect logistic model for ammunition deterioration rate is as follows:

NOTATIONS

i: index for individual lot, $i = 1, .., N$

N: total number of ammunition lots employed in the experiment

j: index for inspection, $j = 1, .., n_i$

n_i: the number of inspections to be done on a lot i over experimental period

t_{ij}: jth inspection time of a lot i

Y_{ij}: sample size of lot i at the jth inspection

x_{ijk}: a dichotomous variable indicating if each item k taken from the sample at the jth inspection of lot i is defective or not

X_{ij}: the number of defectives found in a sample size of Y_{ij}; $X_{ij} = \sum_{k=1}^{Y_{ij}} x_{ijk}$

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$p_{ij} = X_{ij}/Y_{ij}$: proportion defective at the jth inspection of a lot i

$\exp(\beta_0)/(1+\exp(\beta_0))$: initial proportion defective of ammunition lot

β_i: the deterioration rate of lot i (log odds ratio over the unit time interval for lot i)

$(z_{i1}, z_{i2}, .., z_{im})$: covariates indicating concurrent characteristics of lot i (e.g., dummy variables indicating different combinations of depot condition and vendor information of ammunition lot i)

$\gamma_1, .., \gamma_m$: regression coefficients for $(z_{i1}, .., z_{im})$

m: total number of covariates in the between-individual model

γ: a $p \times 1$ vector of γ_i's, $(\gamma_1, .., \gamma_m)'$

Z: an $N \times m$ matrix of covariates $(z_{i1}, .., z_{im})$

ϵ_i: random error of $log\beta_i \sim N(0, \sigma^2)$.

$\hat{p}_i^*$: estimated expected proportion defective of lot i at time t given $(z_{i1}, .., z_{im})$

$\hat{t}_i^*$: estimated expected time that quality of lot i reaches a predetermined level of proportion defective p given $(z_{i1}, .., z_{im})$

$\hat{\beta}_0, \hat{\beta}_i$: maximum likelihood estimators of β_0 and β_i

$log\hat{\beta}$: an $N \times 1$ matrix of $log\hat{\beta}_i$'s, $(log\hat{\beta}_1, .., log\hat{\beta}_N)'$

δ_i: estimation error associated with $log\hat{\beta}_i$, $\delta_i \sim N(0, \tau_i^2)$

τ_0^2, τ_i^2: variance of $\hat{\beta}_0$ and that of $\hat{\beta}_i$

$\hat{\tau}_0^2, \hat{\tau}_i^2$: maximum likelihood estimators of τ_0^2 and τ_i^2

$\hat{W}$: an $N x N$ diagonal matrix with diagonal elements, $1/(\hat{\sigma}^2 + \hat{\tau}_i^2/\hat{\beta}_i^2)$'s

$\hat{\gamma}$: maximum likelihood estimator for γ

Mathematical formulations regarding the random effect logistic model are as follows:

In the within-individual model, proportion defective of lot i at the jth

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inspection, p_{ij} is assumed to follow a logistic function of time t_{ij}:

For $i = 1, .., N$, and $j = 1, .., n_i$

$$p_{ij} = exp(\beta_0 + \beta_i t_{ij})/(1 + exp(\beta_0 + \beta_i t_{ij})) \tag{1}$$

or

$$log(p_{ij}/(1 - p_{ij})) = \beta_0 + \beta_i t_{ij} \tag{2}$$

In the between-individual model, deterioration rate of ammunition lot i, β_i, is represented as a function of covariates (e.g., depot condition, and vendor information) as well as a random error (ϵ_i):

$$\beta_i = f(\gamma_1 z_{i1} + .. + \gamma_m z_{im} + \epsilon_i) \tag{3}$$

where ϵ_i follows independent $N(0, \sigma^2)$. The proportion defective of ammunition lot would not decrease as time goes on. The following function is used to ensure positive deterioration rates, i.e., positive β_i's:

$$\beta_i = exp(\gamma_1 z_{i1} + .. + \gamma_m z_{im} + \epsilon_i) \tag{4}$$

or

$$log(\beta_i) = \gamma_1 z_{i1} + .. + \gamma_m z_{im} + \epsilon_i \tag{5}$$

After replacing β_i in (2) with that in (4), one could estimate regression parameters (β_0, $\gamma_1, .., \gamma_m$, and σ^2), by maximizing the marginal likelihood of the data based on the combined model. However, exact solutions for the normal equations based on the combined model are analytically and computationally infeasible (Stiratelli et al. (1984)). Indicating these problems, Stiratelli et al. (1984) discussed alternative ways of estimating parameters (EM algorithm and a two stage estimation). When β_i in (2) is replaced with

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that in (4), random error ϵ_i's appear as coefficients of the regression model. In EM algorithm they are treated as missing observations and are estimated by cycling back and forth between the E-step and the M-step until convergence is reached. When a two-stage estimation method is used, the estimation procedure for the within-individual model is separated from that for the between-individual model. For instance, in the within-individual regression model, deterioration rates of a set of individual ammunition lots are estimated for all experimental lots. In the between-individual regression model, the estimated deterioration rates are used as values of a dependent variable and they are regressed on depot characteristics and vendor information associated with each individual lot.

In general, a two-stage estimation method is computationally less intensive and analytically easier than the EM algorithm (Stiratelli et al. (1984)). In the following section, a two-stage estimation is described in order to formulate a prediction model for ammunition deterioration.

4. A TWO-STAGE ESTIMATION

First, in order to estimate deterioration rates in the within-individual model (1), the following likelihood function of x_{ijk} conditional on $\beta_0, \beta_1, .., \beta_N$ is formulated:

$$L = \prod_{i=1}^{N} \prod_{j=1}^{n_i} \prod_{k=1}^{Y_{ij}} p_{ij}^{x_{ijk}} (1 - p_{ij})^{1-x_{ijk}} = \prod_{i=1}^{N} \prod_{j=1}^{n_i} \binom{Y_{ij}}{X_{ij}} p_{ij}^{X_{ij}} (1 - p_{ij})^{Y_{ij}-X_{ij}} \qquad (6)$$

where p_{ij} is as in equation (1).

Maximum likelihood (ML) estimates, $\hat{\beta}_0, \hat{\beta}_1, .., \hat{\beta}_N$ are obtained by solving a set of normal equations of log likelihood function of (6) with respect to $\beta_0, \beta_1, .., \beta_N$. Estimated parameters ($\hat{\beta}_i$'s) are in turn assumed to be approximately normal with means equal to the true individual parameters (β_i's) and variances (τ_i^2's). Diagonal elements of the inverse of the negative information matrix evaluated at ($\hat{\beta}_0, \hat{\beta}_1, .., \hat{\beta}_N$) provide the estimated variances of ($\hat{\beta}_0, \hat{\beta}_1, .., \hat{\beta}_N$), which are ($\hat{\tau}_0^2, \hat{\tau}_1^2, .., \hat{\tau}_N^2$).

Secondly, resultant $\hat{\beta}_i$ replaces unobservable β_i in the between-individual model. This replacement, however, brings the estimation error δ_i to the equation (5):

$$\log(\hat{\beta}_i) = \gamma_1 z_{i1} + .. + \gamma_m z_{im} + \epsilon_i + \delta_i. \qquad (7)$$

where δ_i is statistically independent of ϵ_i. It is assumed that δ_i follows independent Normal distribution with mean 0 and variance τ_i^2/β_i^2 (var(δ_i) = var($\log\hat{\beta}_i \mid \beta_i$) $\simeq$ var($(\hat{\beta}_i - \beta_i)/\beta_i$) = τ_i^2/β_i^2). The var(δ_i) is approximated by $\hat{\tau}_i^2/\hat{\beta}_i^2$ and $\log\hat{\beta}_i$ is marginally distributed as independent N($\gamma_1 z_{i1} + , .., + \gamma_m z_{im}$, $\sigma^2 + \hat{\tau}_i^2/\hat{\beta}_i^2$). Standard normal theory can be used to estimate the between-individual model parameters γ_i's and σ^2 which maximize the following log likelihood function at the estimated τ_i^2 :

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$$LL = -\sum_{i=1}^{N} log(\sigma^2 + \hat{\tau}_i^2) - \sum_{i=1}^{N} (log\hat{\beta}_i - \sum_{l=1}^{m} \gamma_l z_{il})^2/(\sigma^2 + \hat{\tau}_i^2/\hat{\beta}_i^2). \qquad (8)$$

The resultant MLE of γ is

$$\hat{\gamma} = (Z'\hat{W}Z)^{-1}(Z'\hat{W}(log\hat{\beta})) \qquad (9)$$

and the estimated variance of $\hat{\gamma}$ is $(Z'\hat{W}Z)^{-1}$ where an $N \times N$ diagonal matrix, $\hat{W}$, consists of diagonal elements, $1/(\hat{\sigma}^2 + \hat{\tau}_i^2/\hat{\beta}_i^2)$'s. In order to be able to estimate γ, the number of lots (N) employed in the experiment must be lager than the number of covariates (m) to be used in the between-individual model. In addition, as N increases, accuracy related to the inference concerning γ would increase.

When these $\hat{\gamma}$'s replace γ's in (7), $log\hat{\beta}_i$ can be predicted in terms of $z_{i1}, .., z_{im}$:

$$\log(\hat{\hat{\beta}}_i) = \hat{\gamma}_1 z_{i1} + .. + \hat{\gamma}_m z_{im}. \qquad (10)$$

Use of (10) to quality control of ammunition, vendor control and depot maintenance plan is illustrated in the next section along with a numerical example.

5. IMPLIMENTATION

Unfortunately, actual data related to type, quantity, and depot condition of ammunition were omitted from the GAO report (The Comptroller General (1982)) to keep it unclassified. In the Norwegian study cited in the earlier part of this paper, repeated measurements on the deterioration of ammunition was not exposed for all the experimental lots. In fact, only the summary statistics for deterioration of several groups of ammunition were available, respectively. In order to illustrate implementation procedures for the methods suggested in this paper, a numerical example is given based on the simulated data generated partly based on the parameters used in the Norwegian study (Eriksen and Strømsøe (1974)) and the guideline of the U.S. Army Ammunition Surveillance Procedure (Supply Bulletin SB 742-1 (1988)).

Table 1 contains information regarding the series of the number of defective items (Y_{ij}) found in 20 ammunition lots (N). These lots are classified into four groups depending upon the vendor who provided ammunition and depot in which ammunition is stored: group 1 consists of 5 lots ($i = 1, .., 5$) which are provided by vendor A and stored in depot 1; group 2 consists of 5 lots ($i = 5, .., 10$) which are provided by vendor B and stored in depot 1; group 3 consists of 5 lots ($i = 11, .., 15$) which are provided by vendor A and stored in depot 2; and group 4 consists of 5 lots ($i = 16, .., 20$) which are provided by vendor B and stored in depot 2. All the lots are inspected annually based on sample size of 20 (Y_{ij}). Total number of inspections of each lot (n_i) varies over 4 different groups: $n_i = 7$ for $i = 1, .., 5$; $n_i = 8$ for $i = 6, .., 10$; $n_i = 9$ for $i = 11, .., 15$; and $n_i = 9$ for $i = 16, .., 20$.

Our goals are first to find the relationship between deterioration rates

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and concurrent characteristics of ammunition lot (depot condition and vendor information); secondly to determine if any group of ammunition causes significantly different deterioration rates from the others; and finally to predict the time for renovation of ammunition in terms of characteristics of depot and vendor.

In order to achieve the first goal, a two-stage estimation is employed. First of all, the ML estimates for parameters in the within-individual model ($\hat{\beta}_0$, $\hat{\beta}_1,..,\hat{\beta}_{20}$ and $\hat{\tau}_0^2$, $\hat{\tau}_1^2,..,\hat{\tau}_{20}^2$) are obtained based on (6). For this step, PROC LOGISTIC of a statistical package SAS (SAS (1989)) is used by applying the values of X_{ij}, Y_{ij}, t_{ij} in Table 2 to model (2). Resultant $\hat{\beta}_i'$s and $\hat{\tau}_i^{2'}$s are summarized in Table 2 while $\hat{\beta}_0$ turns out to be -4.9034 which indicates that the estimated initial proportion defective of ammunition, $(exp(\hat{\beta}_0)/(1 + exp(\hat{\beta}_0)))$, is about 0.74% regardless of sources of vendor. Sample patterns of the actual deterioration (Actual:X_{ij}/Y_{ij} verses t_{ij}) are given in Figure 1 along with those of the estimated deterioration (Fitted: $(exp(\hat{\beta}_0 + \hat{\beta}_i t_{ij})/(1 + exp(\hat{\beta}_0 + \hat{\beta}_i t_{ij}))$ verses t_{ij}).

The between-individual model is formed to relate deterioration rates to the ammunition characteristics. $log\hat{\beta}_i$ is used as dependent variable and the four dummy variables ($z_{i1},..,z_{i4}$) representing 4 combinations of depot and vendor characteristics are used as covariates without an intercept: $z_{i1} = 1$ for ammunition lots which belong to group 1, $z_{i1} = 0$ otherwise; $z_{i2} = 1$ for ammunition lots in group 2, $z_{i2} = 0$ otherwise; $z_{i3} = 1$ for ammunition lots in group 3, $z_{i3} = 0$ otherwise; $z_{i4} = 1$ for ammunition lots in group 4, $z_{i4} = 0$ otherwise. Given such ($z_{i1},..,z_{i4}$) as well as ($\hat{\beta}_i'$, and $\hat{\tau}_i^{2'}$s), an optimization package GAMS (GAMS, 1988) is applied to (8) in order to obtain $\hat{\gamma}_1,..,\hat{\gamma}_4$ and $\hat{\sigma}^2$. Estimates are summarized in Table 2 along with standard errors of

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$\hat{\gamma}_1, .., \hat{\gamma}_4$ obtained from the square root of the diagonal elements of $(Z'\hat{W}Z)^{-1}$

where a 20x4 covariate matrix $Z = \begin{pmatrix} 1_5 & 0 & 0 & 0 \\ 0 & 1_5 & 0 & 0 \\ 0 & 0 & 1_5 & 0 \\ 0 & 0 & 0 & 1_5 \end{pmatrix}$; $1_5 = (1,1,1,1,1)'$ is

a 5x1 unit vector; and $\hat{W}$ is a 20x20 diagonal matrix consisting of diagonal elements $1/(\hat{\sigma}^2 + \hat{\tau}_i^2/\hat{\beta}_i^2)$. Values of $\hat{\tau}_i^2$ and $\hat{\beta}_i^2$'s are given in Table 1 while $\hat{\sigma}^2 = 0.012$.

Estimated γ_l ($l = 1, .., 4$) is the weighted average of $log\hat{\beta}_i'$s which belong to the group l. A large γ_l indicates fast deterioration. For instance, the estimated average log deterioration rate of ammunition lots in group 2 (-0.8336) is lager than that of ammunition lots in group 1 (-0.9022). To test if any group characteristic causes significantly faster deterioration rate than another, a $(1 - \alpha)100\%$ confidence interval for $l'\gamma$ can be used:

$$l'\hat{\gamma} \pm t(\alpha/2, N - p)\sqrt{l'(Z'\hat{W}Z)^{-1}l} \tag{11}$$

where l is a 4x1 vector which contrasts a pair of two group characteristics; t is a student t distribution with degrees of freedom $N - p$.

In order to test if the difference in two expected log deterioration rates between group 1 and group 2 is significant, one can use $l' = (1, -1, 0, 0)$ to obtain a 95% confidence interval for $\gamma_1 - \gamma_2$:

$(1, -1, 0, 0)(-0.9022, -0.8336, -0.7184, -0.6567)'$

$\pm t(0.025, 20 - 4)\sqrt{(1, -1, 0, 0)(Z'\hat{W}Z)^{-1}(1, -1, 0, 0)'}$

where $(Z'\hat{W}Z)^{-1} = \begin{pmatrix} 0.00708 & 0 & 0 & 0 \\ 0 & 0.00473 & 0 & 0 \\ 0 & 0 & 0.00278 & 0 \\ 0 & 0 & 0 & 0.00226 \end{pmatrix}$.

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The resultant interval $(-0.0686 \pm 2.12(0.10870))$ contains 0 which implies that at $\alpha = 5\%$ vendor effect is not significant on the deterioration of ammunition stored in depot 1. Similar comparisons can be made by modifying l's. Complete analyses of 95% confidence intervals for other pairs are summarized in Table 3. Overall, significant differences are not found except for the difference between group 1 and group 4.

The resultant estimates from (10) can also be applied to the analyses of ammunition lots which are not used in the experiment. In order to predict the expected quality of a new ammunition lot i' at time t in terms of depot and vendor characteristics associated with lot i' the following $\hat{p}_i^*$ is used:

$$\hat{p}_i^* = exp(\hat{\beta}_0 + \hat{\hat{\beta}}_i t)/(1 + exp(\hat{\beta}_0 + \hat{\hat{\beta}}_i t)) \tag{12}$$

where $\hat{\hat{\beta}}_i = exp(\hat{\gamma}_1 z_{i'1} +,,+ \hat{\gamma}_m z_{i'm})$. For instance, the predicted proportion defective of ammunition lot i' provided by vendor A after 5-year storage in depot 1 is

$$\hat{p}_i^* = exp(-4.9034 + exp(-0.9022) \times 5)/(1 + exp(-4.9034 + exp(-0.9022) \times 5))$$

$$=0.0534. \tag{13}$$

In addition, one can estimate the expected time $\hat{t}_i^*$ when the quality of ammunition lot i' reaches a predetermined level p:

$$\hat{t}_i^* = [log(p/(1-p)) - \hat{\beta}_0]/exp(\hat{\gamma}_1 z_{i1} +,,+ \hat{\gamma}_m z_{im}). \tag{14}$$

Expected time $\hat{t}_i^*$ when the quality of ammunition lot i' provided by vendor A and stored in depot 1 reaches $p=0.5$ is estimated as

$$\hat{t}_i^* = [log(0.5/(1 - 0.5)) + 4.9034]/exp(-0.9022) = 12.1. \tag{15}$$

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For the four groups considered in this example, $\hat{p}_i^*$ when $t=5$ and $\hat{t}_i^*$ for $p = 0.5$ are given in Table 4.

6. SUMMARY AND DISCUSSION

A random effect logistic regression model was used to formulate a prediction model for ammunition deterioration rate in terms of depot and vendor information. It was assumed that once lots are selected for initial inspection, they will be inspected repeatedly over a certain time period without any rectification. The proportion defective was considered a measure of ammunition deterioration. Regardless of different sources of vendors, the initial quality of ammunition was assumed to be homogeneous as was its eventual deterioration with the passage of time. The log odds ratio due to unit time passage was defined as a deterioration rate which determines the shape of the logistic curve. Deterioration rates of different lots of ammunition often vary and their variation was assumed to be explained partly by a function of depot characteristics and vendor information associated with each lot.

A random effect logistic regression model was used to predict the deterioration rate of ammunition in terms of concurrent characteristics of the ammunition lot. A two-stage analysis was employed to estimate deterioration rates and relate them to the characteristics involved in ammunition lot. Using the approach of this paper, one can determine the appropriate time for reorder or renovation of ammunition before the quality reaches unacceptable levels when depot and vendor information associated with ammunition lot is available. By comparing the expected deterioration rates of ammunition stored in different locations one can improve depot maintenance policy. The similar approach can be used for vendor control.

In the two-stage model used in this paper, we considered a certain type of ammunition and its deterioration was compared in terms of vendor and

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depot characteristics. In order to compare deterioration rates of different types of ammunition, the following approach can be used: First, use a group of relatively homogeneous types of ammunition in the within-individual model, and secondly, classify them not only in terms of depot condition and vendor information but also by types of ammunition.

In order to increase the accuracy of resultant estimators of a two-stage estimation, sufficiently large number of inspection which captures the noticeable deterioration of ammunition would be necessary over many experimental lots. However, such data may not be readily available in reality. When a sufficient number of repeated measurements of actual data is not available, accelerated testing may be an alternative means of meeting the time and budgetary restrictions of the experiment. For instance, environmental tests under simulated climatic extremes are an integral part of normal engineering tests. These tests include accelerated depot conditions at the test site when the characteristics of the test item necessitate the gathering of such data. Nonetheless, when only a small number of data is available, resampling procedure such as bootstrap method (Dalal et al (1990)) can be used to make inferences concerning deterioration rates at the cost of heavy computation.

The measure of rates of ammunition deterioration considered in this paper was the proportion of defective items. When other variables such as muzzle velocity or water content are used as a measure of deterioration, modifications of the within-individual model may be necessary to take into account the different nature of the measure. However, the basic concept of a two-stage analysis still applies to any modified models.

A restriction used in the between-individual model was that a lot of

ammunition would be stored in the same depot in the experimental period. A prediction model that reflects changes in depot conditions associated with a lot of ammunition is under investigation and is left for further research.

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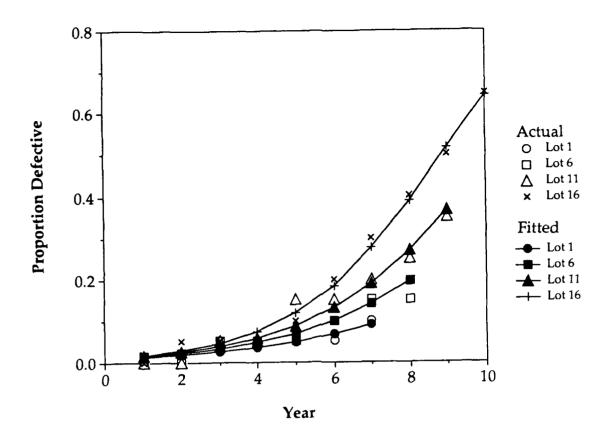
Figure 1. Deteriorating Patterns of Ammunition Quality

Table 1: Number of Defectives X_{ij} Observed in Sample Size of 20 at the jth Inspection of Lot i

	group 1					group 2				
					lot i					
t_{ij}	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	1	1	1	0	1	1	1	1	1	0
4	1	1	1	1	1	1	1	1	1	0
5	1	1	1	1	1	1	1	2	1	0
6	1	2	2	1	1	2	1	3	2	1
7	2	2	2	2	1	3	2	4	3	1
8	na	na	na	na	na	3	2	5	4	1
$\hat{\beta}_i$	0.3853	0.4490	0.4172	0.3671	0.3853	0.4446	0.3649	0.4969	0.4446	0.2310
$\hat{\tau}_i^2$	0.0071	0.0055	0.0063	0.0077	0.0071	0.0033	0.0045	0.0028	0.0033	0.0088

	group 3					group 4				
					lot i					
t_{ij}	11	12	13	14	15	16	17	18	19	20
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	1	1	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	2	1	1	1	1	1
5	2	1	2	1	3	2	2	2	2	2
6	3	2	2	2	5	4	3	3	4	2
7	4	3	3	2	7	6	4	4	5	4
8	5	4	4	2	10	8	6	6	8	5
9	7	5	6	5	13	10	8	8	10	7
10	na	na	na	na	na	13	10	10	13	9
$\hat{\beta}_i$	0.4884	0.4355	0.4535	0.4143	0.6290	0.5582	0.5009	0.5009	0.5536	0.4776
$\hat{\tau}_i^2$	0.0019	0.0022	0.0021	0.0023	0.0018	0.0014	0.0014	0.0014	0.0014	0.0014

Table 2: Fitting the Between-Individual Model

l	$\hat{\gamma}_l$	standard error($\hat{\gamma}_l$)
1	-0.9022	0.084186
2	-0.8336	0.068770
3	-0.7184	0.052690
4	-0.6567	0.047560

Table 3: 95% Confidence Intervals for the Difference between Expected Log Deterioration Rates of Two Groups

group l (vendor and depot)	-	group l' (vendor and depot)	(lower limit, upper limit)
1 (A and 1)	-	2 (B and 1)	(-0.2990, 0.1618)
1 (A and 1)	-	3 (A and 2)	(-0.3943, 0.0267)
1 (A and 1)	-	4 (B and 2)	(-0.4504, -0.0406)
2 (B and 1)	-	3 (A and 2)	(-0.2989, 0.0685)
2 (A and 1)	-	4 (B and 2)	(-0.3542, 0.0004)
3 (A and 2)	-	4 (B and 2)	(-0.2122, 0.0888)

Table 4: Predicted Proportion Defective at year 5 and Estimated Time for $p = 0.5$

$\hat{p}_l^*$ (%)	$\hat{t}_l^*$ (year)
5.34	12.1
6.12	11.3
7.83	10.1
9.02	9.5

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