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**Research in Nonparametric
Incomplete Rankings Procedures,
Small Sample Asymptotics, and
Statistical Decision Theory**

Douglas A. Wolfe, L. Mark Berliner, and Michael A. Fligner
Department of Statistics

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a. Papers Submitted to Refereed Journals

1. Berliner, L.M., "Discussion: What is the likelihood function?" Accepted for publication in Statistical Decision Theory and Related Topics IV.
2. Berliner, L.M., "Issues in Bayesian process design." Accepted for publication in Proceedings of the Korean Statistics Conference.
3. Berliner, L.M. and Hill, B. M., "Bayesian nonparametric survival analysis." Accepted for publication in Journal of the American Statistical Association.
4. Dean, A.M., Aubuchon, J.C., and Wolfe, D.A., "A note on exchangeability of random variables." Submitted to Journal of the Royal Statistical Society, Series B.
5. Fligner, M.A., "Small sample asymptotics." Accepted for publication in Journal of Educational Statistics.
6. Fligner, M.A. and Verducci, J., "A nonparametric test for judges' bias in an athletic competition." Accepted for publication in Journal of the Royal Statistical Society, Series C.
7. Fligner, M.A. and Verducci, J., "Multistage ranking models." Accepted for publication in Journal of the American Statistical Association.
8. Juang, S. and Wolfe, D.A., "Nonparametric changepoint procedures for repeated measures data." Accepted for publication in Communications in Statistics, Series A, Theory and Methods.
9. Wolfe, D.A., "Testing nonparametric null hypotheses that are composite: problems and possible solutions." Accepted for publication in Proceedings of the Korean Statistics Conference.

b. Papers Published in Refereed Journals

1. Berger, J.O. and Berliner, L.M., "Robust Bayes and empirical Bayes analysis with ϵ -contaminated priors." Annals of Statistics, 14, 461-486, 1986.
2. Berliner, L.M., "Bayesian control in mixture models." Technometrics, 29, 455-460, 1987.
3. Costello, P.S. and Wolfe, D.A., "A new nonparametric approach to the problem of agreement between two groups of judges." Communications in Statistics, Series B, Simulation and Computation, 14(4), 791-805, 1985.
4. Costello, P.S. and Wolfe, D.A., "An extreme null distribution approach to the problem of paired comparisons." Communications in Statistics, Series A, Theory and Methods, 16(8), 2327-2341, 1987.
5. Damico, J.A. and Wolfe, D.A., "Extended tables of the exact distribution of a rank statistic for all treatments multiple comparisons in one-way layout designs." Communications in Statistics, Series A, Theory and Methods, 16(8), 2343-2360, 1987.

6. Fairley, D. and Fligner, M.A., "Linear rank statistics for the ordered alternative problem." Communications in Statistics, Series A, Theory and Methods, 16(1), 1-16, 1987.
7. Fligner, M.A., "Pairwise versus joint ranking: another look at the Kruskal-Wallis statistic." Biometrika, 72(3), 705-709, 1985.
8. Fligner, M.A. and Fairley, D., "Stochastic equivalence of ranking methods." Communications in Statistics, Series A, Theory and Methods, 15(6), 1855-1864, 1986.
9. Fligner, M.A. and Verducci, J., "Distance based ranking models." Journal of the Royal Statistical Society, Series B, 48(3), 359-369, 1986.
10. Fligner, M.A. and Verducci, J., "A class of concordance measures." Communications in Statistics, Series A, Theory and Methods, 16(5), 1479-1504, 1987.
11. Wolfe, D.A. and Schechtman, E., "Multiple changepoints problem-nonparametric procedures for estimation of the points of change." Communications in Statistics, Series B, Simulation and Computation, B14(3), 615-631, 1985.

d. Books (and sections thereof) Published

1. Wolfe, D.A., "Placement statistics." Appeared as a section in *Encyclopedia of Statistical Sciences*, Volume 6, 1985, 751-754, John Wiley & Sons, Inc., New York. (Edited by Samuel Kotz and Norman L. Johnson.)

g. Invited Presentations at Topical or Scientific/Technical Society Conferences

1. Berliner, L.M., "Bayesian control of mixture models." Presented at the ORSA-TIMS Annual Meeting, Atlanta, Georgia, November 5, 1985.
2. Berliner, L.M., "Discussion: What is the likelihood function?". Presented at the Purdue Symposium on Statistical Decision Theory, Purdue University, June 6, 1986.
3. Berliner, L.M., "Bayesian control of mixture models." Presented at the Symposium on Bayesian Statistics, University of Warwick, England, July 11, 1986.
4. Berliner, L.M., "Bayesian process design." Presented at the Korean Statistics Conference, Seoul, Korea, August 27, 1987.
5. Wolfe, D.A., "Testing nonparametric composite null hypotheses: application to binomials and several two-way layouts." Presented at the Annual Meeting of the Statistical Society of Canada, University of Guelph, Ontario, Canada, May 15, 1984.
6. Wolfe, D.A., "Testing nonparametric null hypotheses that are composite: problems and possible solutions." Presented at a meeting of the Southwest Michigan Chapter of the American Statistical Association, Kalamazoo, Michigan, March 14, 1985.
7. Wolfe, D.A., "Grouped chi-square tests for the two-way layout." Presented at the Third Annual Ohio Statistics Conference, Columbus, Ohio, September 19, 1986.
8. Wolfe, D.A., "Testing nonparametric null hypotheses that are composite: problems and possible solutions." Presented at the Korean Statistics Conference, Seoul, Korea, August 28, 1987.

h. Contributed Presentations at Topical or Scientific/Technical Society Conferences

1. Berliner, L.M. and Goel, P.K., "Bayesian inference with partial prior information." Presented at the NBER-NSF Conference on Bayesian Inference in Econometrics, Ohio State University, May 16, 1986.
2. Fligner, M.A. and Verducci, J., "A class of concordance measures." Presented at The One Hundred Forty-Sixth Annual Meeting of the American Statistical Association, Chicago, Illinois, August 19, 1986.
3. Wolfe, D.A., "Distribution-free grouped chi-square tests for the two-way layout." Presented at the Sino-American Statistics Conference, Beijing, China, September 1, 1987.

i. Honors / Awards / Prizes

1. Wolfe, D.A., Selected as a Fellow of the American Statistical Association, 1986.

j. Technical Reports Published or Non-Refereed Journals

1. Berliner, L.M. and Goel, P.K., "Incorporating partial prior information: ranges of posterior probabilities." Ohio State University Statistics Department Technical Report No. 357, 1986.

Abstracts for Papers Published or Accepted for Publication
in Refereed Journals

D. A. Wolfe

1. Costello, P.S. and Wolfe, D.A., "A new nonparametric approach to the problem of agreement between two groups of judges." Communications in Statistics, Series B, Simulation and Computation, 14(4), 791-805, 1985.

The problem of agreement between two or more groups of judges has received considerable attention during the past decade, beginning with the papers by Schucany & Frawley (1973, Psychometrika) and Hollander & Sethuraman (1978, Biometrika). Two very different definitions of agreement have emerged in the literature. The first states that two groups of judges agree if they use the same probability distribution to select their rank vectors, while the second defines agreement in terms of correlation coefficients. We utilize the first definition in developing a new approach to solving this agreement problem, relying heavily on computer-generated tables.

2. Costello, P.S. and Wolfe, D.A., "An extreme null distribution approach to the problem of paired comparisons." Communications in Statistics, Series A, Theory and Methods, 16(8), 2327-2341, 1987.

David (1963, *The Method of Paired Comparisons*) and Davidson & Farquhar (1976, Biometrics) contain extensive bibliographies of proposed approaches to problems involving paired comparisons. However, each of the discussed methods that is based on a hypothesis test relies heavily on the assumption that all paired comparisons are made independently. In this paper we eliminate this assumption and develop a new procedure based on an adaptation of a statistic considered by Kendall & Babington Smith (1940, Biometrika). We show that their original test procedure substantially underestimates the true significance level if the comparisons are not made independently. Our modification utilizes the approach developed in Costello & Wolfe (1985) for the problem of agreement between two groups of judges and relies heavily on computer-generated tables.

3. Damico, J.A. and Wolfe, D.A., "Extended tables of the exact distribution of a rank statistic for all treatments multiple comparisons in one-way layout designs." Communications in Statistics. Series A. Theory and Methods, 16(8), 2343-2360, 1987.

In this paper we present the exact null distribution of a rank statistic used in making all treatments multiple comparisons in one-way layout designs.

If we let X_{ij} ($i=1, \dots, n_j$; $j=1, \dots, k$) denote the i th observation from the j th population, then we are dealing with independent random samples of sizes $n_1, \dots, n_k$ from populations 1, ..., k , respectively. Letting

$$R_{.j} = \left[\sum_{i=1}^{n_j} r_{ij} / n_j \right], \quad j=1, \dots, k,$$

where r_{ij} is the combined samples rank of X_{ij} among all $N = (n_1 + \dots + n_k)$ observations, and $N^* = [\text{least common multiple of } n_1, \dots, n_k]$, we define

$$R^* = N^* \max\{ |R_{.u} - R_{.v}|, u=1, \dots, k-1; v=u+1, \dots, k\}.$$

Tables are given for the upper tail probabilities of the exact null distribution of R^* for most useful combinations of $k=3$, $1 \leq n_1 \leq n_2 \leq n_3 \leq 6$ and $k=4$, $1 \leq n_1 \leq n_2 \leq n_3 \leq n_4 \leq 6$.

4. Juang, S.H. and Wolfe, D.A., "Nonparametric changepoint procedures for repeated measures data". To appear in Communications in Statistics. Series B. Simulation and Computation.

Let $(X_{i1}, \dots, X_{in})$, $i=1, \dots, m$, be m mutually independent, identically distributed, continuous n -dimensional random vectors. The setting where $(X_{i1}, \dots, X_{ir})$ is an exchangeable vector and $(X_{i,r+1}, \dots, X_{in})$ is an exchangeable vector but the complete vector $(X_{i1}, \dots, X_{in})$ is not exchangeable for $i=1, 2, \dots, m$, and some unknown integer r , $1 \leq r \leq (n-1)$, is referred to as a changepoint problem for repeated measures data with at most one change. In this paper we propose three nonparametric procedures to test for non-exchangeability in such data resulting from a change in location. Simulated small-sample null distributions for the three test statistics are obtained and asymptotic approximations for those null distributions are provided. When a change is detected to be significant by one or more of these tests, estimators for the changepoint r and the magnitude of the change are proposed and studied.

5. Wolfe, D.A., "Testing nonparametric null hypotheses that are composite: problems and possible solutions". To appear in Proceedings of the Korean Statistics Conference.

Often in hypothesis testing settings the proper null hypothesis is inherently composite. When working with a particular assumed underlying distributional model (e.g., the normal distribution), tests for such composite null hypotheses are often based on the associated likelihood ratio statistics. However, this approach is generally not appropriate for nonparametric problems. In this paper we discuss some of the possible alternative approaches that are available to nonparametricians in such settings. We illustrate these techniques in the two-sample binomial problem and in testing for agreement between two groups of judges.

6. Wolfe, D.A. and Schechtman, E., "Multiple changepoints problem-nonparametric procedures for estimation of the points of change." Communications in Statistics, Series B, Simulation and Computation, B14(3), 615-631, 1985.

In this paper we consider several procedures for estimating the sites of multiple changepoints in the distribution of a sequence of independent, continuous observations. Each of these estimation schemes is based on the statistic proposed by Schechtman (1982, Communications in Statistics, Theory and Methods) for testing hypotheses about a single changepoint, in conjunction with a variety of techniques for detecting local maxima in a sequence of data. The results of an extensive Monte Carlo investigation are presented and used to provide guidelines for selecting a particular estimation procedure for a specific application.

L.M. Berliner

1. Berger, J.O. and Berliner, L.M., "Robust Bayes and empirical Bayes analysis with ϵ -contaminated priors." Annals of Statistics, 14, 461-486, 1986.

For Bayesian analysis, an attractive method of modelling uncertainty in the prior distribution is through use of ϵ -contamination classes, i.e., classes of distributions which have the form $\pi = (1-\epsilon)\pi_0 + \epsilon q$, π_0 being the base elicited prior, q being a "contamination," and ϵ reflecting the amount of error in π_0 that is deemed possible. Classes of contaminations that are considered include (i) all possible contaminations, (ii) all symmetric, unimodal contaminations, and (iii) all contaminations such that π is unimodal.

Two issues in robust Bayesian analysis are studied. The first is that of determining the range of posterior probabilities of a set as π ranges over the ϵ -contamination class. The second, more extensively studied, issue is that of selecting, in a data dependent fashion, a "good" prior distribution (the Type-II maximum likelihood prior) from the ϵ -contamination class, and using this prior in the subsequent analysis. Relationships and applications to empirical Bayes analysis are also discussed.

2. Berliner, L.M., "Bayesian control in mixture models." Technometrics, 29, 455-460. 1987.

The problem is to choose values for independent variables in a regression mixture model with the intent of controlling the output toward a specified target value. This problem is posed as a Bayesian decision problem. The calculation of Bayes rules is shown to require solution of a quadratic programming problem. An example based on an article by Snee (1981) concerning gasoline blends is discussed.

3. Berliner, L.M. and Hill, B. "Bayesian nonparametric survival analysis." Journal of the American Statistical Association, 83, 772-784 (with discussion). 1988.

This article considers a Bayesian nonparametric approach to a (right) censored data problem. Although the results are applicable to a wide variety of such problems, including reliability analysis, the discussion centers on medical survival studies. We extend the posterior distribution of percentiles given by Hill (1968) to obtain predictive posterior probabilities for the survival of one or more new patients, using data from other individuals having the same disease and given the same treatment. The analysis hinges on three assumptions: (a) The new patients and the previous patients are all deemed to be exchangeable with regard to survival time. (b) The posterior prediction rule, in the case of no censoring or ties among (say n) observed survival times, assigns equal probability of $1/(n+1)$ to each of the $n+1$ open intervals determined by these values. (c) The censoring mechanisms are "noninformative." Relationships with other Bayesian approaches are discussed, as well as comparisons with the product limit rule of Kaplan and Meier (1958). To enhance the value of these comparisons, we analyze some examples of Kaplan and Meier (1958) and Gehan (1965).

4. Berliner, L.M. "A Bayesian approach to process design." In *Recent Developments in Statistics and Their Applications*. J. Klein and J. Lee (Eds.) Freedom Academy Press, Seoul, 21-37. 1989.

Substantial interest has been shown in many disciplines in process design. We will begin with a critical review of some pertinent literature with emphasis on "Taguchi's Methods." We then consider some of the mathematical and statistical structures underlying process design with emphasis on a Bayesian approach to the problem.

5. Berliner, L.M. and Goel, P.K. "Incorporating partial prior information: Ranges of posterior probabilities". In *Bayesian and Likelihood Methods in Statistics and Econometrics*. S. Geisser, et. al.(Eds.), 397-406. 1990.

This paper considers the incorporation of partial prior information concerning an unknown parameter in statistical inference and decision making problems. Such prior information is combined with experimental information to produce ranges of posterior probabilities concerning the parameter of interest. An example from Martz and Waller (1983) is pursued in depth.

M. A. Fligner

1. Fligner, M. A., "Pairwise versus joint ranking: Another look at the Kruskal-Wallis statistic". Biometrika, 72,705-709, 1985.

A test statistic for the k-sample problem location problem is constructed by appropriately combining all pairwise two-sample Wilcoxon tests. The result is an analogue of the Kruskal-Wallis statistic in the sense that the Pitman asymptotic relative efficiency between the two is one. However, the approximate Bahadur efficiency of the analogue relative to the Kruskal-Wallis statistic is shown to be greater than or equal to one at every alternative.

2. Fligner, M. A. and Fairley, D. A., "Stochastic equivalence of ranking methods". Communications in Statistics. Series A. Theory and Methods, 15(6), 1855-1864.

The asymptotic behavior of linear rank statistics for comparing the locations of two populations, where the observations are ranked jointly with other populations, is considered. Under certain conditions, the asymptotic behavior of these statistics does not depend on which other populations are included in the ranking. In particular, the difference of a pair of these statistics, with the same score function, but based on two different rankings, converges to zero in probability under Pitman alternatives and Chernoff-Savage conditions on the scores and underlying distributions.

3. Fligner, M. A. and Verducci J., "Distance based ranking models". Journal of the Royal Statistical Society. Series B. 48(3), 359-369, 1986.

A class of ranking models is proposed for which the probability of a ranking decreases with increasing distance from a modal ranking. Some special distances decompose into a sum of independent components under the uniform distribution. These distances lead to multiparameter generalizations whose parameters may be interpreted as information at various stages in the ranking process.

4. Fligner M. A. and Verducci, J., "A class of concordance measures". Communications in Statistics. Series A. Theory and Methods. 16(5), 1479-1504, 1987.

When a judge ranks items, the ranks assigned to the items are jointly distributed random variables. The class of concordance measures based on the marginal distributions of these variables is characterized by bi-affine similarity functions that measure how well pairs of judges tend to agree. The class contains the population versions of several familiar indices of concordance, including Kendall's W.

5. Fligner, M. A., "Small Sample Asymptotics". To appear in the Journal of Educational Statistics.

An approach for modifying the results of asymptotic theory to improve the performance of statistical procedures in small to moderate sample sizes is described in the context of hypothesis testing. The method is illustrated by a series of examples.

6. Fligner, M. A. and Verducci, J. "A nonparametric test for judges' bias in an athletic competition". To appear in the Journal of the Royal Statistical Society. Series C.

A panel of judges views the performance of contestants in random order. The judges independently score each performance on a fixed scale. Do scores from the panel of judges reflect a bias toward an a priori ranking ranking of the contestants, such as a tendency to rank the earlier contestants lower than the latter? A nonparametric test based on Kendall's tau-distance is developed to answer this question.

7. Fligner M. A. and Verducci, J. "Multistage ranking models". To appear in Journal of the American Statistical Association.

Suppose that a sample of people independently examines a fixed set of k items and then ranks these items according to personal judgement. The process of ranking the items is decomposed into $k-1$ stages. Various probability models are adopted at each stage, and properties of the resulting models for randomly sampled rankings are investigated.