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## THESIS

SCHEDULING THE PEACETIME ROTATION  
OF PAKISTAN ARMY UNITS

by

Shafqat Baig

September, 1992

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| <p>Since Pakistan has greatly varying climates and terrains, the Pakistan Army rotates its units between locations so that no unit endures inequitable hardship or enjoys unfair advantage. Army peacetime policy specifies strict constraints on unit rotations, including restriction on: the length of a unit's stay in any location, the number of units moving at any time, and the allowable replacements for any moving unit. Scheduling rotations manually in accordance with these rules, as is currently practiced, is extremely difficult and time-consuming. This thesis presents an integer programming model that finds feasible, minimum-cost schedules for planning horizons of up to eight years. The model also ensures that the units are positioned at the end of the planning horizon so that feasible schedules exist for future planners. The model is implemented with commercially available software: the GAMS algebraic modelling language and the XA and OSL optimizers. Schedules are obtained for realistic test problems in less than an hour on a 486/33 personal computer.</p> |       |                              |                                                                                                  |                                                          |                             |
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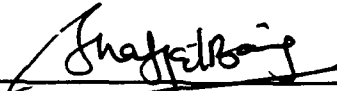
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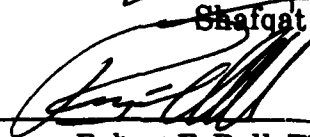
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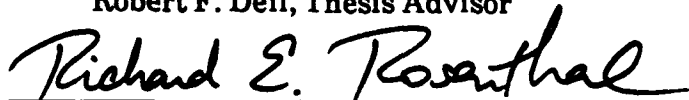
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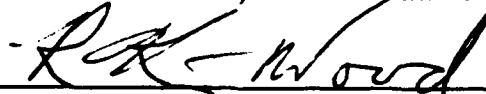
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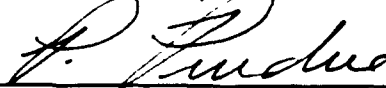
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## ABSTRACT

Since Pakistan has greatly varying climates and terrains, the Pakistan Army rotates its units between locations so that no unit endures inequitable hardship or enjoys unfair advantage. Army peacetime policy specifies strict constraints on unit rotations, including restrictions on: the length of a unit's stay in any location, the number of units moving at any time, and the allowable replacements for any moving unit. Scheduling rotations manually in accordance with these rules, as is currently practiced, is extremely difficult and time-consuming. This thesis presents an integer programming model that finds feasible, minimum-cost schedules for planning horizons of up to eight years. The model also ensures that the units are positioned at the end of the planning horizon so that feasible schedules exist for future planners. The model is implemented with commercially available software: the GAMS algebraic modelling language and the XA and OSL optimizers. Schedules are obtained for realistic test problems in less than an hour on a 486/33 personal computer.

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## **I. BACKGROUND**

This thesis develops an integer programming model to help the Pakistan Army determine an optimal peacetime unit rotation schedule. The goal of the schedule is to equitably distribute unit assignments among locations that are classified by different degrees of desirability. Scheduling is currently done manually, which is extremely difficult and time-consuming. Manual schedules have also positioned units at the end of the planning horizon so that no feasible schedules exist for future planners. This thesis presents an integer programming model that schedules yearly unit rotations for up to eight years. The model minimizes unit movement cost, adheres to policy restrictions, and ensures future feasibility when sufficient starting conditions exist.

### **A. PEACETIME ROTATION OF UNITS**

The Pakistan Army classifies military locations into peace areas (PAs), semi-hard areas (SHAs) and hard areas (HAs). This classification accounts for Pakistan's diverse terrain, ranging from desert to lofty mountains, and temperatures, ranging from below freezing to above 100 degrees. These categories also account for available facilities at the locations and the proximity to major metropolitan areas. To ensure that personnel serve equally in all three areas, unit



personnel are rotated between locations during peacetime on a regular basis.

#### **B. CLASSIFICATION OF UNITS**

Military units in the Pakistan Army are broadly classified according to operational roles (strike and defensive) and according to the type of equipment the unit operates (supplied by either Eastern or Western Bloc nations). These classifications place each unit into one of the four mutually exclusive operational categories:

1. Strike mission using Eastern bloc equipment,
2. Strike mission using Western bloc equipment,
3. Defensive role using Eastern bloc equipment,
4. Defensive role using Western bloc equipment.

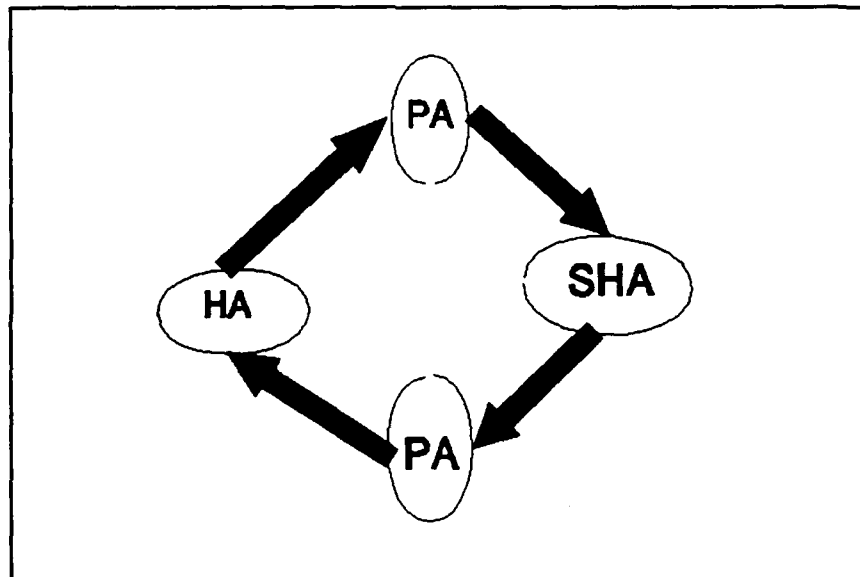
Further classification exists based on a unit's functional role (Armor, Artillery, Engineers, Infantry, Signals, Supply, etc). Only units of the same operational and functional role can replace each other, so the overall Army rotation problem naturally divides into separate rotation problems for units with the same operational and functional role.

#### **C. ROTATION POLICY**

The General Headquarters of the Pakistan Army has established a rotation policy, whose salient features are as follows.

1. Each unit's tenure requirement varies by location classification as follows:

- a. Peace Area (PA) 5-7 years,
  - b. Semi-hard Area (SHA) 2-4 years,
  - c. Hard Area (HA) 1-3 years.
2. A unit can move only if replaced by a unit of the same operational and functional role.
  3. If a unit at location A moves to location B, then a unit at location B must move to location A in the same year. This policy, referred to as "mutual replacement," simplifies transfer of operational and administrative responsibilities at both locations.
  4. Only personnel move. Equipment remains at its present location.
  5. No more than one unit can move from the same brigade in the same year. A brigade is composed of three units of similar operational and functional role. Not all units belong to brigades.
  6. Units that do not form part of a brigades (such as Engineers, Signals, Reconnaissance and Support Battalions) fall under direct control of the division.



**Figure 1.** Each unit must rotate through locations in the indicated order and stay in each position for a prescribed length of time.

There can be no more than one such unit move from the same location in the same year.

7. An individual unit must rotate according to the cycle of locations: PA -> SHA -> PA -> HA -> PA, as shown in Figure 1.
8. There is no restriction placed on which SHA or HA location a unit is transferred to. However, a unit must not return to its previous PA location.

#### **D. CURRENT SCHEDULING METHOD**

Currently, peacetime rotation schedules are developed manually by planners at the General Headquarters of the Pakistan Army. A five year schedule is developed on a yearly basis. The current method suffers from the following drawbacks:

1. The units may not be positioned at the end of the planning horizon in a way that ensures feasible schedules exist for future planners.
2. The schedule requires hundreds of man-hours to develop.
3. It is difficult to evaluate proposed policy changes.
4. The schedule may not be developed impartially.
5. The schedules may not be optimal, i.e., excess cost may be incurred by conducting more moves or transferring units over greater distances than necessary.

#### **E. THE NEED FOR A COMPUTER MODEL**

The drawbacks listed above for the current scheduling process necessitate a computer model to assist with the scheduling. In addition to the above factors, the model will enable "what-if" questions to be more easily answered. This thesis develops an integer programming model to overcome these difficulties.

## **F. THESIS OUTLINE**

Chapter II develops the integer program including detailed discussion of the constraints. Chapter III covers model refinement and sufficient conditions for future feasibility. Computational experience with realistic test problems forms Chapter IV. Chapter V contains conclusions and recommendations for implementation of the model.

## **II. PEACETIME ROTATION MODEL**

### **A. MODEL CLASSIFICATION**

The integer programming model introduced in this chapter captures the Pakistan Army's peacetime rotation policy described in Chapter I. The model ensures that tenure limitations at various locations are observed, individual units rotate according to the prescribed order (Figure 1), and yearly limits on moves within the same brigade or location are not exceeded. Mutual replacement of units is enforced by decomposing the problem of scheduling the Pakistan Army into the following four subproblems for each functional role:

1. Strike mission using Eastern bloc equipment,
2. Strike mission using Western bloc equipment,
3. Defensive role using Eastern bloc equipment,
4. Defensive role using Western bloc equipment.

The model formulation follows after introduction of appropriate notation.

### **B. THE PAKISTAN ARMY PEACETIME ROTATION MODEL**

#### **1. Indices**

$l, l'$  locations,  
 $t, t'$  time periods.

## 2. Given Data

|               |                                                                              |
|---------------|------------------------------------------------------------------------------|
| $l(i)$        | the initial location for unit $i$ ,                                          |
| $\min_l$      | minimum stay allowed at location $l$ ,                                       |
| $\max_l$      | maximum stay allowed at location $l$ ,                                       |
| $MCOST_{ll'}$ | movement cost of a unit from $l$ to $l'$ ,                                   |
| $Stay_{il}$   | number of years unit $i$ has been at location $l$ at the start of the model, |
| $PAL$         | set of peace area locations,                                                 |
| $HAL$         | set of hard area locations,                                                  |
| $SHAL$        | set of semi-hard area locations.                                             |

## 3. Derived Data

The derived data that follows is used to enforce the rotation policy. Chapter III describes how information necessary to develop these sets can be obtained.

|            |                                                                                                       |
|------------|-------------------------------------------------------------------------------------------------------|
| $S_{ll't}$ | set of all units possible to move from location $l$ to $l'$ at time $t$ ,                             |
| $F_{illt}$ | set of all possible locations that unit $i$ could move from at time $t$ if arriving at location $l$ , |
| $T_{illt}$ | set of all possible locations unit $i$ could move to at time $t$ if currently at location $l$ .       |

## 4. Decision Variables

|             |                                                                                     |
|-------------|-------------------------------------------------------------------------------------|
| $X_{ill't}$ | 1 if unit $i$ moves from location $l$ to location $l'$ at time $t$ and 0 otherwise. |
|-------------|-------------------------------------------------------------------------------------|

## 5. Formulation

The model minimizes the total cost associated with all scheduled moves, while ensuring all policy constraints are satisfied. This model is valid for time horizons up to 6 years. For time periods greater than 6 and less than or equal to 15 years, the model needs an additional constraint, which is discussed in Chapter IV.

The objective function is

$$\text{MINIMIZE} \quad \sum_i \sum_l \sum_{l'} \sum_t \text{MCOST}_{ll'} X_{ill't}$$

The constraints are as follows.

- 1) The unit's first move must be completed so that the minimum and maximum tenure requirements are satisfied at the unit's initial location.

$$\sum_{t=\min_i - \text{stay}_{il}}^{\max_i - \text{stay}_{il}} \sum_{l' \in T_{ill't}} X_{ill'(i)l't} = 1 \quad \forall i \quad (1)$$

- 2) The mutual replacement policy must be followed.

$$\sum_{i \in S_{l't}} X_{ill'l't} = \sum_{i \in S_{ll't}} X_{illl't} \quad \forall ll't \quad (2)$$

- 3) No more than one unit may move from the same location in the same year.

$$\sum_{l'} \sum_{i \in S_{ll't}} X_{illl't} \leq 1 \quad \forall lt \quad (3)$$

If two (or more) brigades have the same geographical location, the model breaks this location into separate brigade locations. In this manner constraint (3) also enforces the restriction that no more than one unit can move from the same brigade.

- 4) The number of times a unit enters location  $l$  up to time  $t$  must be less than or equal to the number of times it leaves the location up to time  $t + \max_i$ . These constraints, which can also be formulated in a noncumulative fashion, ensure that a unit leaves a

location in a certain year only if it arrived at that location in an appropriate earlier year. The cumulative form of constraint empirically provided better computational performance.

$$\sum_{t'=1}^t \sum_{l' \in F_{ilt'}} X_{ill't'} \leq \sum_{t'=1+\min_l}^{t+\max_l} \sum_{l' \in T_{ilt'}} X_{ill't'} \quad \forall i, t \quad (4)$$

- 5) A unit leaving location  $l$  at time  $t$  must have arrived at location between  $t-\max_l$  and  $t-\min_l$  years ago.

$$\sum_{l' \in T_{ilt}} X_{ill't} \leq \sum_{t'=t-\max_l}^{t-\min_l} \sum_{l' \in F_{ilt'}} X_{ill't'} \quad \forall i, t \quad (5)$$

Constraints (4) and (5) control the tenure requirements for all moves taking place after the first move. These constraints can not prevent a unit from moving away from the same location more than once in the last few time periods. Constraints (6) - (9) eliminate this problem.

- 6) Unit  $i$  is allowed at most one move from any PA to any HA during the planning horizon.

$$\sum_{l \in PAL} \sum_{l' \in HAL} \sum_t X_{ill't} \leq 1 \quad \forall i \quad (6)$$

- 7) Unit  $i$  is allowed at most one move from any PA to any SHA during the planning horizon.

$$\sum_{l \in PAL} \sum_{l' \in SHAL} \sum_t X_{ill't} \leq 1 \quad \forall i \quad (7)$$

- 8) Unit  $i$  is allowed at most one move from any HA to any PA during the planning horizon.

$$\sum_{l \in HAL} \sum_{l' \in PAL} \sum_t X_{ill't} \leq 1 \quad \forall i \quad (8)$$

- 9) Unit  $i$  is allowed at most one move from any SHA to any PA during the time horizon.

$$\sum_{l \in SHAL} \sum_{l' \in PAL} \sum_t X_{ill't} \leq 1 \quad \forall i \quad (9)$$

- 10) The total number of moves from PAs to HAs should be equal to the total number of moves from SHAs to PAs.



$$\sum_i \sum_{l \in PAL} \sum_{l' \in HAL} \sum_t X_{ill't} = \sum_i \sum_{l \in SHAL} \sum_{l' \in PAL} \sum_t X_{ill't} \quad (10)$$

This constraint helps position units appropriately at the end of the planning horizon, as discussed further in Chapter III.

### III. MODEL REFINEMENT

#### A. VARIABLE REDUCTION

The Pakistan Army contains as many as 87 units with the same operational and functional role spread over 30 locations. Solving one of these problems for eight time periods using the model described earlier could require over 600,000 binary variables. Fortunately, characteristics of the problem can be exploited to identify many impossible unit movements and eliminate the corresponding variables. The parameter  $OK_{ill't}$  is defined for this purpose. It has value 1 if and only if it is possible for unit  $i$  to move from location  $l$  to  $l'$  at time  $t$ . The sets  $S_{ll't}$ ,  $T_{ill't}$ ,  $F_{ill't}$  used in Chapter II are generated from this parameter. The steps to form  $OK_{ill't}$  are as follows.

1. The location tenure limits and the unit rotation policy coupled with a unit's past location, current location, and length of stay at the current location can be combined to eliminate combinations of moves in certain years. For example, a unit that has moved from a HA and has been at a PA for three years can not possibly move for two more years and then only to a SHA. This type of situation is treated below where  $OPA_i$  is the most recent PA location for unit  $i$  at the initial time period and  $Stay = \text{Minimum} \{Stay_{il}, \min_l\}$  for the unit's initial location  $l$ .

a. For any unit currently located at a PA and having previously served in a SHA,

$$OK_{ill't} = \begin{cases} 1 & \text{if } l \in PAL, & l' \in HAL, & 6 \leq t+Stay \leq 8, \\ 1 & \text{if } l \in HAL, & l' \in (PAL \setminus OPA_i), & 7 \leq t+Stay \leq 11, \\ 1 & \text{if } l \in (PAL \setminus OPA_i), & l' \in SHAL, & 12 \leq t+Stay \leq 18, \\ 0 & \text{otherwise} \end{cases}$$

where the symbol " $\setminus$ " denotes set difference.

b. For any unit currently located at a PA and having previously served in a HA,

$$OK_{ill't} = \begin{cases} 1 & \text{if } l \in PAL, & l' \in SHAL, & 6 \leq t+Stay \leq 8, \\ 1 & \text{if } l \in SHAL, & l' \in (PAL \setminus OPA_i), & 8 \leq t+Stay \leq 12, \\ 1 & \text{if } l \in (PAL \setminus OPA_i), & l' \in HAL, & 13 \leq t+Stay \leq 19, \\ 0 & \text{otherwise} \end{cases}$$

c. For any unit currently located at SHA,

$$OK_{ill't} = \begin{cases} 1 & \text{if } l \in SHAL, & l' \in (PAL \setminus OPA_i), & 3 \leq t+Stay \leq 5, \\ 1 & \text{if } l \in (PAL \setminus OPA_i), & l' \in HAL, & 8 \leq t+Stay \leq 12, \\ 1 & \text{if } l \in HAL, & l' \in PAL, & 9 \leq t+Stay \leq 15, \\ 0 & \text{otherwise} \end{cases}$$

d. For any unit currently located at HA.

$$OK_{ill't} = \begin{cases} 1 & \text{if } l \in HAL, & l' \in (PAL \setminus OPA_i), & 2 \leq t+Stay \leq 4, \\ 1 & \text{if } l \in (PAL \setminus OPA_i), & l' \in SHAL, & 7 \leq t+Stay \leq 11, \\ 1 & \text{if } l \in SHAL, & l' \in PAL, & 9 \leq t+Stay \leq 15, \\ 0 & \text{otherwise} \end{cases}$$

These relationships follow directly from the tenure and rotation policies. For example, suppose unit  $i$  is currently located at a PA and its old location is a HA (case b). This unit can move to a SHA after completing 5 to 7 years of stay at the PA. This unit would therefore be eligible to move when  $t+Stay$  is equal to 6, 7 or 8 years. If the unit arrives at SHA in year 6 (8), it could leave in years 8, 9 or 10 (10, 11

or 12) to a different PA, resulting in possible moves in years 8 through 12.

2. The mutual rotation policy also helps eliminate many variables. Consider a unit at location A that can move to location B at time  $t$ . Such a move can be scheduled only if another unit is eligible to move from location B to location A at the same time. Therefore, any  $OK_{ill't}$  that was 1 after step 1 is changed to 0 unless:

$$\sum_{i' \neq i} OK_{i'l't} \geq 1 .$$

For example, if unit  $i$  can move from location A to location B at  $t = 1, 2, 3$ , and unit  $i'$  can move from B to A only at  $t = 3$ , and no other units are available, then it is infeasible for unit  $i$  to move from A to B at  $t = 1, 2$ , so  $OK_{iAB1} = OK_{iAB2} = 0$ .

3. Extending the idea of step 2 to future moves, consider a unit that can move from location A to location B at time  $t$ . Such a move can only take place at time  $t$  if another unit is eligible to replace it at location B between  $t + \min_g$  and  $t + \max_g$ . Therefore, any  $OK_{ill't}$  that remains 1 after the first two steps is changed to 0 unless:

$$\sum_{i' \neq i} \sum_{l'} \sum_{t'=t+\min_l}^{t+\max_l} OK_{i'll't'} \geq 1 .$$

## B. CONDITIONS FOR FUTURE FEASIBILITY

The model needs to ensure that units are positioned at the end of the planning horizon so that feasible schedules exist for future planners. Sufficient conditions are developed to ensure that units are properly positioned. These conditions are established from characteristics of Pakistan Army units with similar operational and functional roles. These conditions are explained with the help of Figure 2.

Figure 2 shows a rotation schedule (repeatable indefinitely) that satisfies all restrictions of the rotation policy outlined in Chapter I. This figure contains 4 PAs, 1 HA, and 1 SHA locations where each location has 1 unit. The

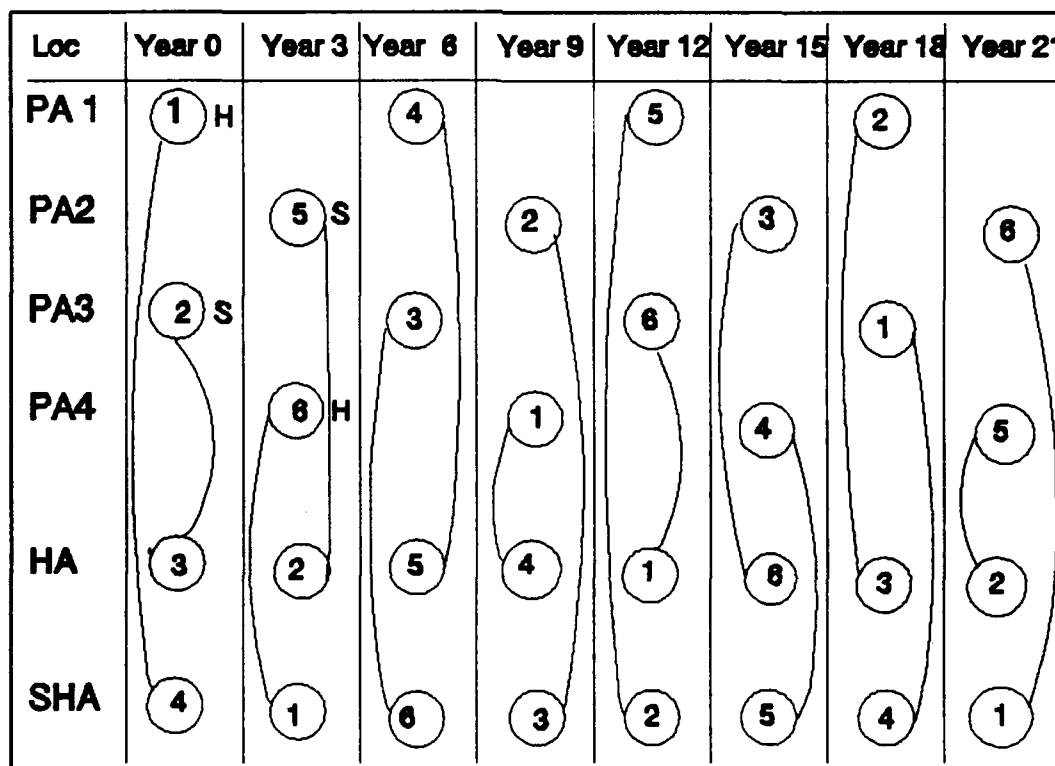


Figure 2. A six unit feasible rotation pattern that can be extended indefinitely.

circles contain unique unit identifiers and the arcs between the circles indicate units exchanging locations. In case of units 1, 2, 5, and 6, initially located at PAs, their last area is shown with H (for HA) or S (for SHA).

It is possible to add units in multiples of 6 (4 PA units, 1 HA unit, 1 SHA unit) up to the maximum of three at each location. Each 6 additional units (1 at each existing location) could be feasibly added to the above schedule by allowing moves in similar 3 year increments starting at year 1 for the first 6 units and at year 2 for the second 6. New locations with units following the same pattern could also be added.

The composition of the units in Figure 2 obey the following conditions.

**CONDITION 1.** The total number of units in PAs is twice the number of units in HAs and SHAs.

This condition results from the tenure requirements. A unit located at a HA must leave the location and switch with a unit at a PA every 1 to 3 years. This results in at least 2 replacements at every HA every 2 to 6 years. Similarly, each unit at a SHA must leave the location and switch with a unit at a PA every 2 to 4 years which results in at least 2 replacements every 4 to 8 years. There are therefore at least 4 units (twice the number in HAs and SHAs) moved from PAs to SHAs and HAs every 2 to 8 years.

**CONDITION 2.** The number of units in HAS and SHAs is equal.

This condition follows from the individual unit rotation cycle. Every unit located at a HA (SHA) goes to a PA and replaces a unit at a SHA (HA) after 6 to 10 (7 to 11) years. This provides a common time frame of 7, 8, 9 or 10 years where every unit currently at a HA (SHA) could complete its tenure requirement at both its HA (SHA) and subsequent PA and have an eligible unit rotation to a SHA (HA). The example in Figure 2 uses a common 9 year interval.

**CONDITION 3.** Half the units in PAs previously served in SHAs and half previously served in HAS.

This condition is caused by the mutual replacement policy. Each unit that rotates from a PA to a HA (SHA) must have previously served in a SHA (HA). Assuming this condition is initially satisfied, constraint (10) is imposed to ensure that the condition is maintained at the end of the model's planning horizon.

The three conditions above are not necessary to guarantee the existence of a feasible solution in the future. However, as proven in the Appendix, if the conditions are satisfied for the six years preceding the model and are enforced throughout the model, then future feasibility is guaranteed. Unfortunately, past rotation decisions did not always satisfy Condition 3. Therefore, the test problems of Chapter IV are

solved with constraint (10) instead of the more restrictive form of the constraint

$$\sum_i \sum_{l \in PAL} \sum_{l' \in HAL} X_{ill't} = \sum_i \sum_{l \in SHAL} \sum_{l' \in PAL} X_{ill't} \quad \forall t$$

which, when combined with the original constraints, would guarantee feasible rotations in the future. Constraint (10) empirically allowed feasible rotation schedules with ending conditions that allowed future feasible rotations.



#### IV. COMPUTATIONAL EXPERIENCE

The model is implemented in the General Algebraic Modeling System, GAMS, [Ref. 1] and solved using XA [Ref. 2] and OSL [Ref. 3]. All computational results are obtained using a 486/33 personal computer with 16 megabytes of RAM.

##### A. TEST PROBLEMS

Four test problems, described below, each of which corresponds to a specific but unspecified operational and functional role, are solved. These test problems are representative of the number of units and locations found in actual Pakistan Army data in size and composition. However,

TABLE 1. NATURE AND SIZE OF TEST PROBLEMS

| Problem Type                                  | Problem Size                                    | Generation Time<br>(seconds) |
|-----------------------------------------------|-------------------------------------------------|------------------------------|
| <u>INFANTRY 1</u><br>87 units<br>30 locations | 2,562 rows<br>12,242 columns<br>74,571 nonzeros | 524                          |
| <u>INFANTRY 2</u><br>72 units<br>24 locations | 1,599 rows<br>5,747 columns<br>35,037 nonzeros  | 236                          |
| <u>ARTILLERY</u><br>54 units<br>21 locations  | 1,088 rows<br>3,141 columns<br>18,530 nonzeros  | 152                          |
| <u>ENGINEERS</u><br>36 units<br>19 locations  | 913 rows<br>2,194 columns<br>14,829 nonzeros    | 114                          |

hypothetical data was used in place of real data due to the sensitivity of the information. Table 1 provides a

description of these problems for a six year planning horizon. The table includes each problem's name, the number of units and locations considered, and the time GAMS took to generate (but not solve) the corresponding model. Each problem satisfies conditions 1, 2, and 3 from Chapter III for the initial time period. Experimentation with starting conditions not satisfying these 3 conditions often resulted in infeasibility.

#### **B. INCREASING THE PLANNING HORIZON**

The Pakistan Army requires a five-year rotation schedule for planning purposes. Any scheduling further into the future can be accomplished by solving a series of problems with five year horizons (using the ending conditions of one problem as the beginning of another) or ideally by increasing the planning horizon considered by the model. Unfortunately, as Figure 3 shows for the Artillery problem, the planning horizon is the driving factor in problem size. A planning horizon of at most eight years is recommended since an increase in problem size can quickly exceed available memory and significantly increase computational requirements.

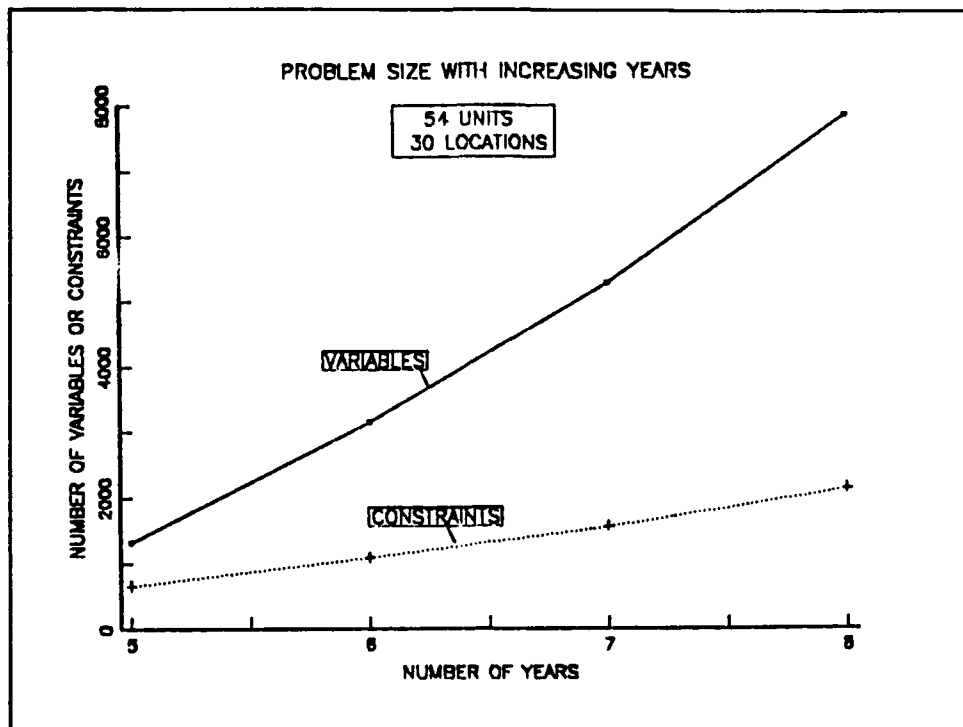


Figure 3. Sensitivity of problem size to the number of time periods.

### C. COMPUTATIONAL RESULTS

The test problems of Table 1 were solved for integer solutions with a 10% optimality tolerance, using both the XA and OSL solvers. Table 2 demonstrates these results and highlights OSL's superior performance.

XA was unable to solve the linear programming relaxation of the 8 year Artillery problem apparently due to cycling. Table 2 does not report the integrality gap which is zero in 5 of the 8 problems tested and less than 3% in the other cases.

TABLE 2. COMPARISON OF SOLVE TIME

| Problem Type                 | Solver | Solution Time (seconds) | Iterations |
|------------------------------|--------|-------------------------|------------|
| <u>INFANTRY 1</u><br>$t = 6$ | XA     | 3,575                   | 37,919     |
|                              | OSL    | 2,338                   | 7,062      |
| <u>INFANTRY 2</u><br>$t = 6$ | XA     | 1,568                   | 21,647     |
|                              | OSL    | 452                     | 3,178      |
| <u>ARTILLERY</u><br>$t = 6$  | XA     | 95                      | 2,519      |
|                              | OSL    | 173                     | 2,196      |
|                              | XA     | 679                     | 14,969     |
|                              |        | 211                     | 1,998      |
|                              | XA     | N O S O L               | U T I O N  |
|                              |        | 455                     | 3,642      |
| <u>ENGINEERS</u><br>$t = 6$  | XA     | 165                     | 4,735      |
|                              | OSL    | 105                     | 1,394      |
|                              | XA     | 2,855                   | 54,769     |
|                              |        | 344                     | 3,381      |
|                              | XA     | 8,950                   | 105,918    |
|                              |        | 1,627                   | 11,212     |

A six year planning horizon only allows the model to plan at most one PA move for each unit and the sets  $S_{ill't}$ ,  $F_{illt}$  and  $T_{illt}$  are used to prevent any unit from moving back to its last PA (a policy restriction described in Chapter I). Planning horizons greater than six years require an additional constraints to enforce this restriction since a 7 year time horizon could allow a unit to leave a SHA, complete its tenure at both a PA and HA and be eligible to move back to a PA. The

addition of constraints (11) ensures adherence to the policy for up to 15 years but they were not needed in the

$$\sum_{l' \in HAL} \sum_t X_{il'l't} + \sum_{l' \in SHAL} \sum_t X_{il'l't} \leq 1 \quad \forall i, l \in PA \quad (11)$$

test problems. These constraints were never violated in the optimal solution to the current model.

It is not surprising that constraints (11) are not violated since the objective was found empirically to cause the number of moves undertaken to be as few as possible. This observation coupled with constraint (10) resulted in almost all units staying in HAs for 2 or 3 years.

#### D. MODEL OUTPUT

Two reports are developed to display solutions. An example of the partial output is shown below in Table III. The first report shows unit P4 moving from location LHR2 to JMR21 in 1996 and then moving from location JMR21 to OKA7 in year 1998. It also shows unit P5 moving from LHR3 to GLT17 and from GLT17 to OKA9 in years 1994 and 1995 respectively.

The second report shows the mutual unit replacements on a yearly basis. For example, the entry (FY96.LHR2 .JMR21.P4 .B5 1) reports in 1996 unit P4 moves from LHR2 to JMR21 and unit B5 moves from JMR21 to LHR2.

**TABLE 3. OUTPUT OF REPORTS**

---

Report 1 (Individual unit moves)

INDEX 1 = P4

|             | FY96  | FY98  |
|-------------|-------|-------|
| LHR2 .JMR21 | 1.000 |       |
| JMR21.OKA7  |       | 1.000 |

INDEX 1 = P5

|             | FY94  | FY95  |
|-------------|-------|-------|
| LHR3 .GLT17 | 1.000 |       |
| GLT17.OKA9  |       | 1.000 |

Report 2 (Yearly mutual replacement)

|                           |   |
|---------------------------|---|
| FY93.LHR1 .JMR21.P1 .FF7  | 1 |
| FY94.BWP12.MUR15.AK4.B11  | 1 |
| FY94.BWP13.MUR14.AK5.B10  | 1 |
| FY95.GWA6 .QTA18.S2 .AK8  | 1 |
| FY96.LHR2 .JMR21.P4 .B5   | 1 |
| FY97.GWA6 .QTA18.B6 .P12  | 1 |
| FY98.GLT16.MTN22.P9 .AK11 | 1 |

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## **V. CONCLUSION AND RECOMMENDATIONS**

### **A. CONCLUSION**

Computational experience with the model demonstrates that feasible unit rotations can be developed for the desired time frame. Furthermore, these results can be obtained in less than 1 hour using an OSL solver on a personal computer.

Experience with the model has shown that the ability to feasibly satisfy rotation policy is very sensitive to the unit's initial conditions. If the sufficient conditions of Chapter III are satisfied for the six years preceding the model and are enforced throughout the model, then the feasibility of future schedules is guaranteed.

During the course of this study, GAMS has proven to be a useful tool for model development. The inherent feature of GAMS "dollar operator" used for exception handling in equations [Ref. 1:p. 92] was extremely useful to keep the number of variables and constraints down to a manageable level.

The capabilities and limitations of the model are summarized below:

#### **1. Capabilities**

- a. Obtains a feasible solution for all realistically sized data sets in under 1 hour using a personal computer.

- b. Preserves starting conditions at the end of the model's planning horizon which has empirically ensured future feasibility. A guarantee of future feasibility can be ensured if the conditions of Chapter III are satisfied for the six years preceding the model's planning.
- c. Changes can easily be incorporated and their effect seen.

## **2. Limitations**

- a. The model is very sensitive to starting conditions for future feasibility.
- b. The model has been built for a specific set of Army Policies. If the policies change, the model may require substantial redevelopment.

## **B. RECOMMENDATIONS**

- 1. Enforce Half the Units in PAs to Have Previously Served in SHAs and Half to Have Previously Served in HAs for Each Year of Planning Horizon**

This stipulation will allow guaranteed feasible rotations to be planned by the model after 6 years.

- 2. Tenure at HA**

The minimum tenure limit of 1 year at HAs should be changed to two years. Rotating every year is more costly and can easily cause condition 3 of Chapter III to be violated. This suggests a 1 year stay at HAs is difficult to enforce given all other policy restrictions.



### **3. Large Size Problems**

The largest size problem that should be considered for a 6 year planning horizon (based on a 16 megabyte RAM) is the INFANTRY 1 problem. If it is desired to run such problems for longer time periods, the units could be divided into mutually exclusive groups using arbitrary categories to further differentiate units. Rotations only within these restricted groupings could than be planned.

### **4. Manual Planning**

Figure 2 while used for illustration purposes also suggests an effective manner to manually plan unit rotations by dividing units into groups of six. This can be a useful aid for scheduling a small number of units.

## APPENDIX

### THEOREM

If conditions 1, 2 and 3 of Chapter III are satisfied for six years, they are sufficient to guarantee a future feasible rotation schedule.

### PROOF

Condition 3 stipulates that half the units in PAs must have previously served in HAs and half in SHAs. This status is maintained, on a yearly basis, only if each year a unit moves from a HA to a PA, a unit also moves from a SHA to a PA. The units at HAs and SHAs can therefore be separated into pairings which satisfy one of the following three cases.

**CASE 1.** A unit has been at a HA for 1 year and another unit has been at a SHA for 1 year.

**CASE 2.** A unit has been at a HA for 2 years and another unit has been at a SHA for 2 years.

**CASE 3.** A unit has been at a HA for 3 years and another unit has been at SHA for 3 years.

It is shown for each case that it is possible for the units at the HA and SHA to feasibly rotate in the same year with a unit currently at a PA location. Any six units satisfying conditions 1, 2 and 3 for the last 6 years and feasibly rotating can be used. Without loss of generality, the six units are numbered according to Figure 2 (i.e., unit

3 is at HA). For clarity, the unit identifiers are bold faced. Also, as in Figure 2, year 0 is considered the first year available to change a unit's location.

#### CASE 1

Unit 3 has stayed 1 year at a HA.

This implies unit 3 replaced unit 6, 1 year ago and unit 6 replaced unit 1, 2, 3 or 4 years ago.

Unit 4 has stayed 1 year at a SHA.

This implies unit 4 replaced unit 5, 1 year ago and unit 5 replaced unit 2, 3, 4 or 5 years ago.

Condition 3 ensures that each year a move from a HA to a PA occurs, a move from a SHA to a PA also occurs. Therefore, we need only consider unit 6 (5) replacing unit 1 (2), 3 or 4 years ago.

These conditions provide the following rotation eligibilities:

Unit 4 is eligible to move in years 1, 2 or 3 and

Unit 1 is eligible to replace unit 4

in years 2, 3 or 4 if unit 1 was replaced 3 years ago, or  
in years 1, 2 or 3 if unit 1 was replaced 4 years ago.

Unit 3 is eligible to move in years 0, 1 or 2 and

Unit 2 is eligible to replace unit 3

in years 2, 3 or 4 if unit 2 was replaced 3 years ago, or  
in years 1, 2 or 3 if unit 2 was replaced 4 years ago.

Therefore, it is feasible for both unit 3 and unit 4 to rotate in year 2.

## CASE 2

Unit 3 has stayed 2 years at a HA.

This implies unit 3 replaced unit 6, 2 years ago and unit 6 replaced unit 1, 3, 4 or 5 years ago.

Unit 4 has stayed 2 years at a SHA.

This implies unit 4 replaced unit 5, 2 years ago and unit 5 replaced unit 2, 4, 5 or 6 years ago.

Condition 3 ensures that each year a move from a HA to a PA occurs, a move from a SHA to a PA also occurs. Therefore, we need only consider unit 6 (5) replacing unit 1 (2), 4 or 5 years ago.

These conditions provide the following rotation eligibilities:

Unit 4 is eligible to move in years 0, 1 or 2 and

Unit 1 is eligible to replace unit 4

in years 1, 2 or 3 if unit 1 was replaced 4 years ago, or  
in years 0, 1 or 2 if unit 1 was replaced 5 years ago.

Unit 3 is eligible to move in years -1, 0 or 1 and

Unit 2 is eligible to replace unit 3

in years 1, 2 or 3 if unit 2 was replaced 4 years ago, or  
in years 0, 1 or 2 if unit 2 was replaced 5 years ago.

Therefore, it is feasible for both unit 3 and unit 4 to rotate in year 1.

### CASE 3

Unit 3 has stayed 3 years at a HA.

This implies unit 3 replaced unit 6, 3 years ago and unit 6 replaced unit 1, 4, 5 or 6 years ago.

Unit 4 has stayed 3 year at a SHA.

This implies unit 4 replaced unit 5, 3 years ago and unit 5 replaced unit 2, 5, 6 or 7 years ago.

Condition 3 ensures that each year a move from a HA to a PA occurs, a move from a SHA to a PA also occurs. Therefore, we need only consider unit 6 (5) replacing unit 1 (2), 5 or 6 years ago.

These conditions provide the following rotation eligibilities:

Unit 4 is eligible to move in years -1, 0 or 1 and

Unit 1 is eligible to replace unit 4

in years 0, 1 or 2 if unit 1 was replaced 5 years ago, ▯

in years -1, 0 or 1 if unit 1 was replaced 6 years ago.

Unit 3 is eligible to move in years -2,-1 or 0 and

Unit 2 is eligible to replace unit 3

in years 0, 1 or 2 if unit 2 was replaced 5 years ago, ▯

in years -1, 0 or 1 if unit 2 was replaced 6 years ago.

Therefore, it is feasible for both unit 3 and unit 4 to rotate in year 0.

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3. Optimization Subroutine Library (OSL), *Guide and Reference*, IBM Corporation, 1991.

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