ELECTRON TRANSIT TIME IN CENTRAL VIRTUAL ANODE WELLS†

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EMC2–0291–03

† This work performed under Contract No. MDA–972–90–C–0006 for the Defense Advanced Research Projects Agency, Defense Sciences Office.

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1. INTRODUCTION

In the computation of electron residence time within the core cavity of Polywell^{tm}/HEPS devices, it is necessary to determine the average time of transit of the electrons in the system across the system. This is needed in the EKXL code as a parameter in the basic code algorithm relating electron edge densities at successive time increments within each time interval computation. The time intervals are those periods over which the succession of linked steady–state solutions are obtained for particle density and potential distributions. An analogous parameter is required for the algorithm relating ion edge density dependence, as well.

For ions, this parameter can be calculated readily from the data available in each code computation; simply summing the ratio of spatial grid width to local grid average radial speed, over the complete grid range characterizing the problem. This works effectively, even though the ions are specified with a square function energy distribution, because all of the ions go to or through the central core of the system.

This is not the case for the electrons, however, as their radial and transverse energy spreads, coupled with the square distribution used to model these spreads, forces them to experience (in the computer code computation) a wide range of transit times. Some electrons are reflected by the negative potential well at mid–range radii, while others travel completely through the system core region. Once inside the electron–current–induced outer potential well the electrons travel inward with the ion stream. Those at the top of the energy spread will pass under the well (in the case of a central virtual anode, which is the potential distribution for most cases of interest), and will exhibit long transit times. Those at the mean energy will not reach small radii but will be reflected and return to the outer boundary with short transit times.

In principle it is possible to calculate the "average" of the transit time for all electrons by using code–generated data, just as for the case of the ions, above. However, in practice this has not been found to be a viable method, for a variety of reasons deriving from the numerical mechanics utilized in the basic computer code operations. In an earlier note the average electron transit time was derived from simple analytic formulae for the transit of electrons of any given energy through potential wells of simple shape. These included simple power–law potential wells, with well shape given by

$$e\phi(r) = E_o[1 - (r/R)^m] = E_o(1 - \langle r \rangle^m) \tag{1}$$

over the range $r_o < r < R$, and constant at $E \approx E_o$ inside the convergence–limited core, at $r < r_o$.

The transit time was also given for potential distributions formed of the simple power–law well with a superimposed central anode potential whose shape varied linearly inversely with radius as

$$[E_o - e\phi_a(r)] = (E_o - e\phi_{ao})(r_o/r) \tag{2a}$$

or

$$e\phi_a(r) = E_o[1 - (r_o/r)] + e\phi_{ao}(r_o/r) \tag{2b}$$

The transit time was computed by integration of the electron speed taken as though all electrons were at the potential given by these expressions, from the integral equation

$$t_{trans} = 2\int_0^R \frac{dr}{v(r)} = 2\int_0^{r_o} \frac{dr}{v_c} + 2\int_{r_o}^R \frac{dr}{v(r)} \tag{3}$$

where the integral has been split into two parts, one within the convergence–limited core radius (r_o), where electron speed is taken as constant at v_c, and one outside this region, to the outer boundary of the system.

2. INVERSE–SQUARE CENTRAL VIRTUAL ANODES

The simple power–law wells are a good approximation only for cases of very low ion and electron core densities (i.e. for very low currents). At high particle densities, as for reactor level conditions, the potential wells all have a central virtual anode formed by the convergence of ions circulating through the device and compressing at least as fast as the inverse–square of their radial distance as they approach the center. As central ion density builds up, a virtual anode will be formed which will, in turn, slow down the ions and lead to still greater ion density rise with decreasing radius. In the previous note[1] the virtual anode potential height was modelled as decreasing inversely with distance from the origin. Further analytic work and EKXL computer code studies for relevant cases have shown that the central anode potential varies more rapidly than this, proceeding approximately as the inverse square of the distance from the well center, thus

$$[E_o - e\phi_a(r)] = (E_o - e\phi_{ao})(r_o/r)^2 \tag{4a}$$

or

$$e\phi_a(r) = E_o[1 - (r_o/r)^2] + e\phi_{ao}(r_o/r)^2 \tag{4b}$$

down to a radius r_w, at which the potential well reaches a minimum value ϕ_w, and rises with decreasing radius thereafter, as shown in Figure 1. Outside of r_w the electrons slow down according to the variation of the main potential (following the B field). Inside this minimum–well–depth position the electron radial speed is accelerated by the virtual anode "hill". The equation describing the total potential variation is

$$e\phi_t(r) = e\phi(r) - [E_o - e\phi_a(r)]$$

$$= E_o[1 - (r/R)^m] - E_o[(r/R)^2][1 - (e\phi_{ao}/E_o)] \quad (5)$$

where $e\phi(r)$ is the main system potential from eq. (1). Differentiation of eq. (5) allows solution for the radial position r_w at which the net potential reaches a (negative) maximum (or well minimum). This is found to be at

$$(r_w/R) = [2(r_o/R)^2(E_{ao}/E_o)/m]^{1/(m+2)} \quad (6a)$$

or

$$<r_w> = [2\eta<r_o>^2/m]^{1/(m+2)} \quad (6b)$$

where $E_{ao} = (E_o - e\phi_{ao})$ is the anode height above the absolute limiting well bottom E_o, and $\eta = (E_{ao}/E_o) = 1 - (e\phi_{ao}/E_o)$. The minimum energy of electrons at this point is

$$E_{ew} = E_o[<r_w>^m + <r_o>^2\eta/<r_w>^2] \quad (7)$$

For the case $m = 3$, $<r_o> = 1E{-}3$, $E_{ao} = 2E4$ eV and $E_o = 1.1E5$ eV, for example, eqs. (6) give $<r_w> = 0.041$; the EKXL code shows the well minimum to be essentially flat from 0.035 to 0.090 (mean ≈ 0.062). For a similar case, with the core at $<r_o> = 1E{-}2$, it is found that $<r_w> = 0.104$ and $E_o = 309$ eV from eq. (7).

The device can be operated so as to produce any desired central anode potential ϕ_{ao} by adjusting the available ion injection current for any given electron drive current. The EKXL code uses this central anode potential as a target control parameter in its calculational algorithm for adjustment of ion edge (input) density in successive time increments of computation. Real physical devices also can be controlled in this fashion.

A closed analytic form giving an approximate transit time t_{rtrn} for electrons moving only radially, with maximum injection speed v_{in}, across the system can be estimated by integration over three terms: one for the core region $r \leq r_o$, where the electron energy is assumed constant, $E_{eo} = \eta E_o + E_w/2$, at speed $v_a = (2E_{eo}/m_e)^{0.5}$; one for $r_o \leq r \leq r_w$, where $v(r) = [2E_{eo}(r_o/r)^2/m_e]^{0.5}$; and one for the region $r_w \leq r \leq R$, where $v(r) = [(2E_o + E_{ew})(r/R)^m/m_e]^{0.5}$. Note that this integration will yield an expression valid ONLY for maximum energy electrons with no transverse momentum, and strictly valid only for electron motion under the full wiffle–ball (WB) mode of operation. Systems operating at conditions below the full WB state will exhibit slightly longer transit times.

Carrying out these integrations yields a general expression for the ratio t_{rtrn}/t_{in} ($t_{in} = 2R/v_{in}$, the "free–run" transit time) in terms of the power law exponent (m), the electron energy E_o at injection and the (small) energy height E_{ew} of the well at the maximum–well–depth or "turning–point" radius ratio $\langle r_w \rangle$, the core convergence radius $\langle r_o \rangle$ and the anode height parameter (η). For the case of interest when m = 3, this is

$$\frac{t_{rtrn}}{t_{in}} = \left[\frac{\langle r_o \rangle + (r_w^2 - r_o^2)/2r_o}{[\eta + (E_{ew}/2E_o)]^{0.5}}\right] + \left[\frac{2}{[1+(E_{ew}/2E_o)]^{0.5}}\right]\left[\frac{1}{\langle r_w \rangle^{0.5}} - 1\right] \qquad (8)$$

The general behavior of this transit time ratio is shown in Figure 2, for several values of the anode height parameter (η) and for several convergence ratios $\langle r_o \rangle$. Note that this algorithm suggests that the transit time becomes long compared to the "free–run" transit time t_{in} at small values of r_o and η. This result is correct only for those electrons that move radially with maximum energy, at the top of the injection energy distribution. It is clearly non–physical in the real machine, where the electrons have a significant spread in energy in both radial and transverse directions.

3. AVERAGE TRANSIT TIME

To account for this it is necessary to consider the radial motion of electrons all across their radial and transverse energy distribution. For the square distribution used in the EKXL code, an approximate expression for the average radial speed v_{avg} across this distribution is found to be given by

$$v_{avg} \approx 2/[1+(dE_{perp}/E_o)] \tag{9}$$

This factor can account for the variation of transit time of all those electrons moving only radially, over the function that describes their radial energy distribution. However, the electrons also possess transverse energy whose "angular" momentum is conserved in their recirculating flow. Thus most electrons do not follow purely radial paths but, rather, move past the core with relatively high speed far above the well bottom, at distances considerably away from the system center.

Exact analysis of this system can not be done by analytical methods, and is quite beyond the capabilities of any 1–dimensional computer code such as EKXL. An approximate analysis has been done, using simple expansions of the velocity/potential functions for the integration process to find the average velocity of the average electron. The correction factor thus obtained for use in determing the average electron transit time is comparable to that in eq. (9), above, and is given by

$$f_{trn} = LN(t_{rtrn}/t_{in})/[(t_{rtrn}/t_{in})-1] \tag{10}$$

With this the average transit time of the average electron can be estimated approximately by use of eqs. (8, 10) according to

$$(t_{avg}/t_{in}) = f_{trn}(t_{rtrn}/t_{in}) \qquad (11)$$

This is shown in Figure 3 for a range of anode height parameter, η, over a range of core convergence radius, r_o. Note that the average transit time is about twice the "free–run" transit time over a wide range of parameters of interest for these systems. Thus although the existence of the potential well "slows down" the transit of electrons passing through it, the degree of this "slowing–down" is not great. This factor (eq. 11) is used in the EKXL code in the determination of electron transit time, for internal computations of electron lifetime and system population.

REFERENCES

[1] Robert W. Bussard and Katherine E. King, "Electron Transit Time in Simple Potential Wells", Energy/Matter Conversion Corporation (EMC2), technical report, EMC2–1090–02, Nov. 1990

LIST OF FIGURES

Figure 1. General potential distribution in well with central virtual anode, $E_{ao} = (E_o - e\Phi_{ao})$, with potential maximum (well minimum) at turning–point radius, r_w

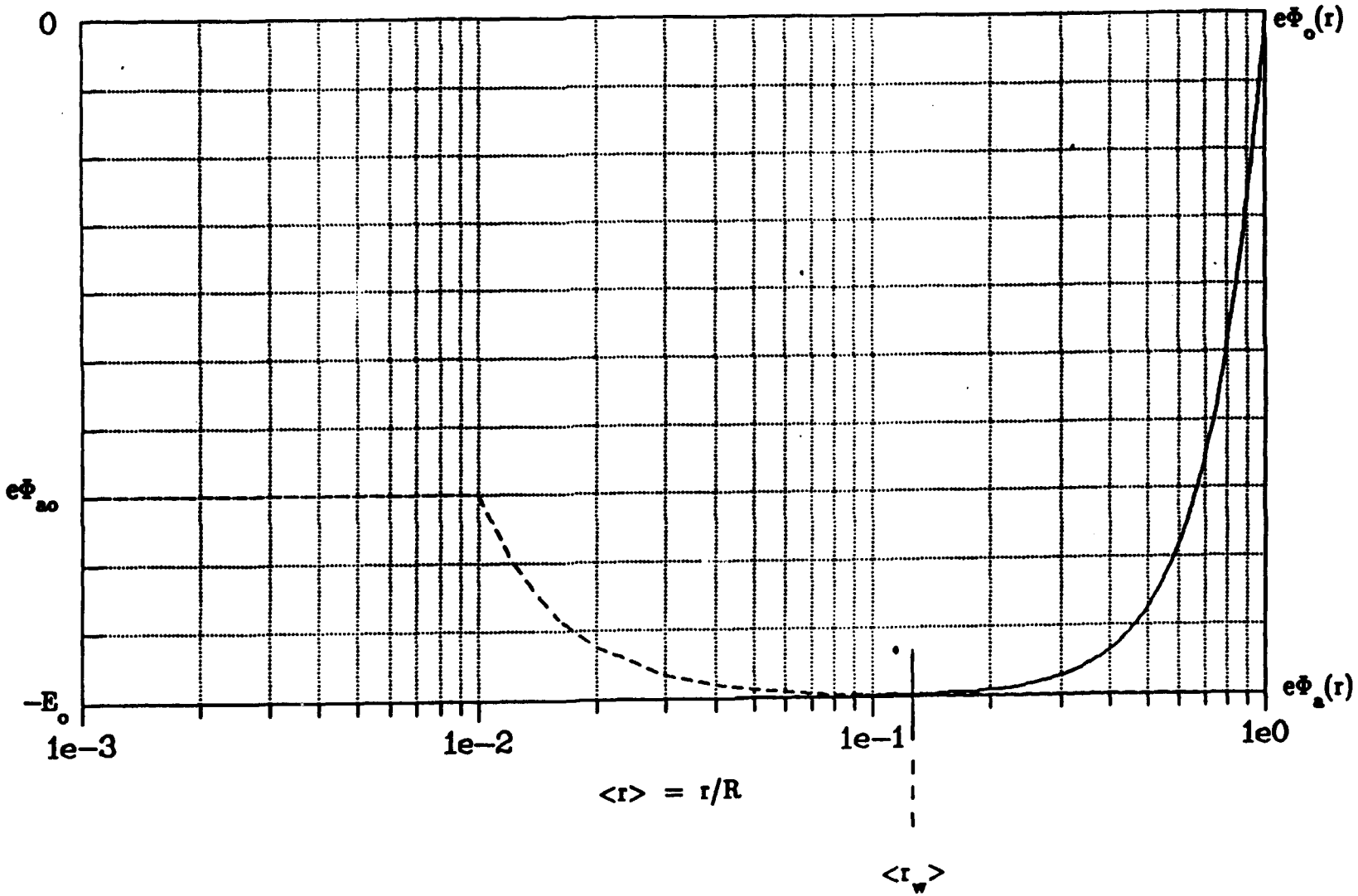

Figure 2. Transit time of most energetic radially–moving electrons across potential well with central virtual anode, as a function of anode height parameter, $\eta = E_{ao}/E_o$, and core convergence–limited radius, $\langle r_o \rangle = r_o/R$.

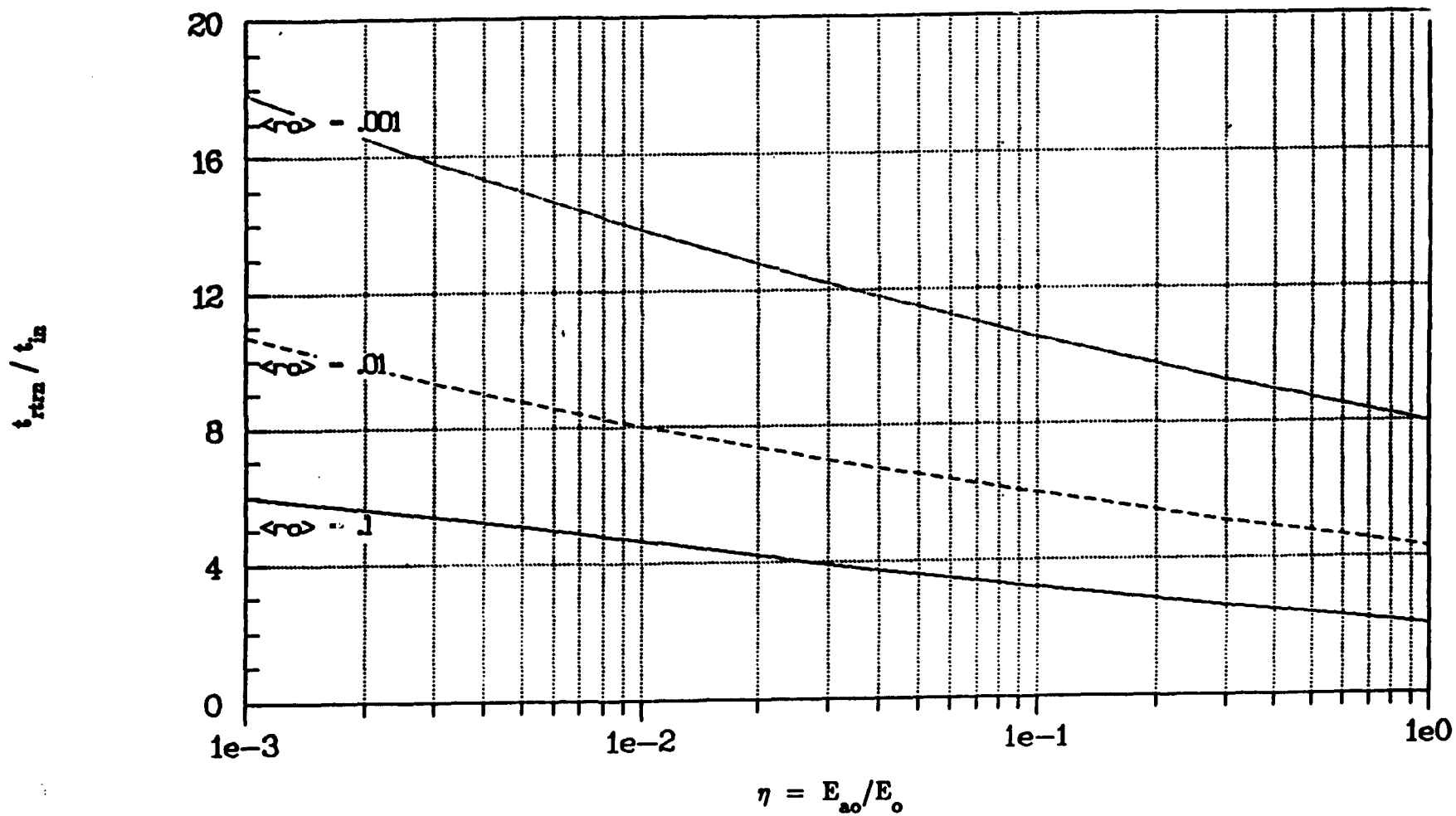

Figure 3. Approximate average transit time of electrons, averaged over their initial radial and transverse velocity distributions, through potential well with central virtual anode, as a function of η and $\langle r_o \rangle$.