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**THE APPLIED COMPUTATIONAL ELECTROMAGNETICS
SOCIETY JOURNAL**

**SPECIAL ISSUE
ON
BIOELECTROMAGNETIC COMPUTATIONS**

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THE APPLIED COMPUTATIONAL ELECTROMAGNETICS SOCIETY

SPECIAL ISSUE OF THE JOURNAL ON BIOELECTROMAGNETIC COMPUTATIONS

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INTRODUCTION TO THE SPECIAL ISSUE

ON

BIOELECTROMAGNETIC COMPUTATIONS

A.H.J Fleming, Guest Editor
K.H. Joyner, Guest Editor
Telecom Australia Research Laboratories

Bioelectromagnetics, the interaction of electromagnetic (EM) radiation with biological tissues and systems, is at present driven largely by two complementary disciplines: namely biomedical engineering and bio-hazards analysis. Their intersection with electromagnetics covers a wide variety of applications of critical concern to modern man.

Pertinent to this Special Issue, and because of the complexity of the anatomy or physiology involved, bioelectromagnetic computations are often required for a sufficient understanding of effects. Used in conjunction with analytic and experimental methods, EM numerical methods form an integral part of the tools at the disposal of the investigator. Even canonical shapes such as spheres, ellipsoids and cylinders, often used to provide a conceptual starting point, require considerable computational effort.

This being the first Special Issue of the ACES Journal devoted to computational bioelectromagnetics, the Guest-editors have attempted to provide the reader with a cross-section of current activity in the field. The first three papers present different methodologies for solving whole-body problems. The next four papers are applied to hyperthermia and concern levels of deposition in tissue. There are two papers that discuss methods for obtaining thermometric data during hyperthermia; one of these addresses the active imaging of this data. Following these is a paper that examines the imaging of tissues. Next discussed is a method for calculating magnetic fields at points near power line configurations. The final two papers examine methods of stimulation of neurons and tissues.

A synopsis of the methodologies employed by the authors of the selected papers is as follows.

The paper by K.D. Paulsen, X. Jia and D.R. Lynch presents a finite element method that employs an "extended weak form" using the magnetic field as the mesh variable. After discussing solution residuals, and testing against a homogeneous sphere problem, they demonstrate their method with whole-body models obtained from serial computer tomography (CT) data.

The paper by A.P.M. Zwamborn, P.M. van den Berg, J. Moolbroek and F.T.C. Koenis discusses a weak formulation of the domain-integral equation method for modeling full-vectorial, three dimensional EM problems that conjugate-gradient iterative technique with a Fast Fourier Transform technique. Error testing and convergence of the method is performed using Mie series solutions for an inhomogeneous sphere. Using a mesh generated from CT data, absorbed power densities inside an inhomogeneous block model are presented.

N. Kuster discusses the multiple multipole method, and applies it to a bio-hazards assessment of UHF-VHF transmitters often used for on-site, live radio and television broadcasts. Line multipoles are used to model the transmitters, and due to the frequency of transmission and the worst-case nature of the assessment, a simple inhomogeneous model is used for the whole body.

The paper by V. Sathiaselan, A. Taflov, M.J. Picket-May, C. Reuter and B.B. Mittal provides a review of numerical modeling as applied to hyperthermia, including a discussion of validity experimental studies using phantoms. Next, tissue deposition patterns from two-dimensional models of waveguide applicators and an inhomogeneous thigh are obtained using the finite difference time domain method (FDTD) code.

J.R. James and G. Andrasic apply the FDTD method to the analysis of a conformal microstrip hyperthermia applicator. To investigate its potential for delivery of deep focal resolution, necessary for effective abdominal and pelvic treatments, they use leaky-wave analysis and an experimental phantom, along with the FDTD method recast in curvilinear coordinates.

The FDTD method is again used by H.C. Taylor and R.W.M. Lau, who present an examination of clinical hyperthermia deposition. Two-dimensional models of an annular phased array applicator and patient abdomen, and the associated computed power deposition patterns, are utilized in case studies to determine the likely success of treatment.

C.M. Rappaport discusses the optimization of planar source distributions for the delivery of focused radio-frequency power deposition at depth within biological tissues. The method uses a surface field integral which results in a non-linear differential equation in terms of the phase of the source. An iterative method is used to determine the optimal depth of delivery.

F. Bardati, V.J. Brown and P. Tognolatti discuss the technique of multi-frequency microwave radiometry for monitoring tissue temperature during hyperthermia treatment. Using a layered cylindrical geometry appropriate for limbs, a two-dimensional inverse problem whose solution space is spanned by thermal functions, is solved via the FDTD method as two direct problems: an electromagnetic scattering problem and a heat balance one.

The paper by J.J. Mallorquí, A. Broquetas, L. Jofre and A. Cardama, discusses active thermometric imaging using a 64 element microwave tomographic scanner, reconstructing the image from the difference between the fields scattered by the reference and heated dielectric bodies. Data obtained using numerical simulations of the imaging algorithm are compared with those obtained using experimental phantoms.

An algorithm for electrical impedance tomography of three-dimensional volumes is discussed by J. Goble, M. Cheney and D. Isaacson. The algorithm is based upon the method of least squares, using one step of Newton's method to estimate conductivities inside the volume from electrical measurements on the surface. Images from an implementation of the algorithm are compared with experimentally obtained voltage data.

S. Tofani, G. d'Amore, G. Bonazzola and G. Flandino present a numerical method for evaluating the magnetic fields in the vicinity of power line configurations. The influences of spatial and electrical power lines parameters are considered. Calculated and measured data are presented.

K.P. Esselle and M.A. Stuchly discuss the stimulation of peripheral nerves and motor neurons in the cerebral cortex via current coils that induce pulsed magnetic fields in tissue. The coil dimensions and shape, are optimized, depending upon the depth of the nerve in tissue.

The final paper by G. D'inzeo, C. Giacomozzi and S. Pisa concerns the stimulation of nerve fibres surrounded by inhomogeneous, anisotropic, and dispersive tissue. A finite difference method is used to evaluate the thresholds for electric and magnetic forms of stimulation of the nerve in three-dimensional models of the forearm.

The Guest Editors would like to thank the contributors and reviewers for their wholehearted support during the preparation of this issue. Thanks also to David Stein for his support and encouragement during the initial stages and in the preparation of this Special Issue.

The Guest Editors would like to acknowledge the considerable efforts of Mr. Enn Vinnal for handling the day-to-day administrative detail of finalizing the Special Issue. The permission of the Director of Research, Telecom Australia Research Laboratories, for the Guest Editors to undertake this work is hereby acknowledged.

Tony Fleming and Ken Joyner work in the Electromagnetic Compatibility Section of the Telecom Australia Research Laboratories and their primary research interests are in the area of biological electromagnetic hazards.

3D Bioelectromagnetic Computation on Finite Elements

Keith D. Paulsen, Xilin Jia, and Daniel R. Lynch

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Abstract

Finite element computation of electric and magnetic fields induced in the body by noninvasive electromagnetic sources is discussed. Attention is focussed on three-dimensional calculations for full-scale body models with significant levels of internal anatomical structure. The finite element solution strategy including the sparse matrix approach which allows computation of over 100K degrees-of-freedom on standard reduced instruction set computer (RISC) workstation platforms is outlined. The finite element mesh generation problem is also described. Representative examples of the level of meshing detail and the type of 3D bioelectromagnetic solutions that can be achieved using finite elements in the workstation computing environment are shown.

I. Introduction

The need to quantify induced electromagnetic (EM) field distributions in biological bodies arises in numerous medical and health-related contexts, for example, in microwave imaging for diagnosis [Larson and Jacobi, 1986; Jofre *et al.*, 1990], and in hyperthermia cancer therapy [Hand, 1990; *Int. J. Hyperthermia*, Special Issue, 1988], among others. Of primary concern has typically been the nature of the detailed EM field distributions created inside the body, knowledge of which is required to quantify dose delivery or characterize tissue. The large electrical contrast between many tissues in the frequency ranges of interest [Stuchly and Stuchly, 1980] and the complex geometry of the human anatomy has made determination of internal field distributions a challenging problem requiring state-of-the-art computational methods.

While a variety of approaches are proving successful in the bioelectromagnetics context (e.g. see [Gandhi, 1990; Paulsen, 1990]), the finite element method (FEM) is a natural choice because of its inherent ability to treat highly heterogeneous, irregularly-shaped computational domains. For example, in hyperthermia treatment planning for cancer therapy, 2D FEM modeling has proven to be quite informative [Strohbehn *et al.* 1986]. Detailed EM field distributions and concomitant specific absorption rate (SAR) profiles have been calculated in 2D anatomical models derived from planar CT-scans of actual cancer patients. Comparisons have been made between various competing therapy devices [Paulsen *et al.*, 1985] as well as with actual clinical data [Strohbehn *et al.*, 1986; Paulsen and Ross, 1990] and the general trends predicted by these analyses have been borne out in clinical trials [Sapozink *et al.*, 1985]. The success of this type of 2D modeling has provided a strong rationale for extending the computational capabilities to 3D for potential use in planning the treatments of individual patients [Sullivan, 1990; Picket-May *et al.*, 1991]. Likewise, in other biomedical contexts the induced EM field distributions are essentially three-dimensional [Chen and Gandhi, 1989; Caorsi *et al.* 1989]; thus, the need for 3D computational methods and algorithms is critical to the understanding of the interactions of EM fields with biological bodies and the advancement of biomedical applications of EM energy.

In this paper, we report on our progress in computing 3D EM field distributions in full-body models with the finite element method. While the specific examples we present are of primary interest in hyperthermic treatment of cancer, the methods and approaches we utilize are relevant to bioelectromagnetic computing in general. We begin by reviewing our basic FEM formulation for calculating EM fields in biological tissue; we discuss our iterative solution strategy, our sparse matrix storage structure and the problem sizes/run-times that we can achieve on a typical reduced instruction set computer (RISC) workstation. We also discuss the grid generation techniques we

employ and show examples of the type of meshes we can create. We conclude by showing some EM field calculations in full-body models with noninvasive current excitations which are of importance in hyperthermia treatment of cancer.

II. FEM Formulation

In this section we briefly recount our finite element approach [Paulsen and Lynch, 1991; Lynch and Paulsen, 1991; Boyse *et. al.*, 1992]. We use an "extended weak form" that produces a special Helmholtz structure on homogeneous subregions with component coupling occurring only at media interfaces and boundaries where boundary integrals arise in terms of naturally specifiable quantities. The advantages of this approach are (1) the ability to use conventional FEM scalar bases as expansion functions while simultaneously eliminating the parasitic behavior that can occur in the discrete form of the double-curl operator [Lynch and Paulsen, 1991; Boyse *et. al.* 1992] and (2) the enhanced sparsity of the system matrix as a by-product of the decoupling of interior field components [Paulsen and Lynch, 1991].

In contrast to most of our earlier work where we solve directly for the electric field $\mathbf{E}$, herein, we compute the magnetic field $\mathbf{H}$ and subsequently determine $\mathbf{E}$ from the $\nabla \times \mathbf{H}$ if and when it is needed. For the problems of interest in many biomedical applications, the magnetic permeability of tissue, μ , can be considered constant whereas the permittivity, ϵ , and conductivity, σ , both vary with tissue type and frequency. Under these assumptions, all components of $\mathbf{H}$ are continuous in source-free regions (i.e. inside the body) and the strong enforcement of EM jump conditions [Yuan *et. al.*, 1991] can be delayed until the subsequent calculation of $\mathbf{E}$.

The "extended weak form" for $\mathbf{H}$ can be obtained from the weighted residual statement

$$\langle \left(\nabla \times \frac{1}{i\omega\epsilon^*} \nabla \times \mathbf{H} - \nabla \left(\frac{1}{i\omega\epsilon^*\mu} \nabla \cdot \mu\mathbf{H} \right) + i\omega\mu\mathbf{H} \right) \phi_i \rangle = 0 \quad (1)$$

where $\epsilon^* = \epsilon + i\sigma/\omega$ is the complex permittivity, $\langle \rangle$ indicates volume integration over the problem domain, ϕ_i is the locally-defined linearly-varying weighting function associated with each finite element nodal position, and an $e^{-i\omega t}$ time-variation is implied. Use of standard vector identities and integral theorems allows one to remove the second order differentiation in favor of relevant boundary integrals

$$\begin{aligned} \left\langle \frac{1}{i\omega\epsilon^*} \nabla \times \mathbf{H} \times \nabla \phi_i \right\rangle + \left\langle \frac{1}{i\omega\epsilon^*\mu} (\nabla \cdot \mu\mathbf{H}) \nabla \phi_i \right\rangle + \langle i\omega\mu\mathbf{H}\phi_i \rangle = \\ - \oint \hat{\mathbf{n}} \times \left(\frac{1}{i\omega\epsilon^*} \nabla \times \mathbf{H} \right) \phi_i ds + \oint \hat{\mathbf{n}} \frac{1}{i\omega\epsilon^*\mu} (\nabla \cdot \mu\mathbf{H}) \phi_i ds \end{aligned} \quad (2)$$

A symmetric linear algebraic system of equations $\mathbf{Ax} = \mathbf{b}$ is generated by expanding $\mathbf{H} = \sum_{j=1}^N \mathbf{H}_j \phi_j$ in the usual Galerkin fashion which leads to $\mathbf{A}$ being composed of the complex submatrices

$$\mathbf{A}_{ij} = \begin{pmatrix} \left\langle \frac{1}{i\omega\epsilon^*} \left(\frac{\partial\phi_j}{\partial y} \frac{\partial\phi_i}{\partial y} + \frac{\partial\phi_j}{\partial z} \frac{\partial\phi_i}{\partial z} \right) + \frac{\partial\phi_j}{\partial x} \frac{\partial\phi_i}{\partial x} \right\rangle + \langle i\omega\mu\phi_i\phi_j \rangle & \left\langle \frac{1}{i\omega\epsilon^*} \left(-\frac{\partial\phi_j}{\partial x} \frac{\partial\phi_i}{\partial y} + \frac{\partial\phi_j}{\partial y} \frac{\partial\phi_i}{\partial x} \right) \right\rangle & \left\langle \frac{1}{i\omega\epsilon^*} \left(-\frac{\partial\phi_j}{\partial x} \frac{\partial\phi_i}{\partial z} + \frac{\partial\phi_j}{\partial z} \frac{\partial\phi_i}{\partial x} \right) \right\rangle \\ \left\langle \frac{1}{i\omega\epsilon^*} \left(-\frac{\partial\phi_j}{\partial y} \frac{\partial\phi_i}{\partial x} + \frac{\partial\phi_j}{\partial x} \frac{\partial\phi_i}{\partial y} \right) \right\rangle & \left\langle \frac{1}{i\omega\epsilon^*} \left(\frac{\partial\phi_j}{\partial x} \frac{\partial\phi_i}{\partial x} + \frac{\partial\phi_j}{\partial z} \frac{\partial\phi_i}{\partial z} \right) + \frac{\partial\phi_j}{\partial y} \frac{\partial\phi_i}{\partial y} \right\rangle + \langle i\omega\mu\phi_i\phi_j \rangle & \left\langle \frac{1}{i\omega\epsilon^*} \left(-\frac{\partial\phi_j}{\partial y} \frac{\partial\phi_i}{\partial z} + \frac{\partial\phi_j}{\partial z} \frac{\partial\phi_i}{\partial y} \right) \right\rangle \\ \left\langle \frac{1}{i\omega\epsilon^*} \left(-\frac{\partial\phi_j}{\partial z} \frac{\partial\phi_i}{\partial x} + \frac{\partial\phi_j}{\partial x} \frac{\partial\phi_i}{\partial z} \right) \right\rangle & \left\langle \frac{1}{i\omega\epsilon^*} \left(-\frac{\partial\phi_j}{\partial z} \frac{\partial\phi_i}{\partial y} + \frac{\partial\phi_j}{\partial y} \frac{\partial\phi_i}{\partial z} \right) \right\rangle & \left\langle \frac{1}{i\omega\epsilon^*} \left(\frac{\partial\phi_j}{\partial x} \frac{\partial\phi_i}{\partial x} + \frac{\partial\phi_j}{\partial y} \frac{\partial\phi_i}{\partial y} + \frac{\partial\phi_j}{\partial z} \frac{\partial\phi_i}{\partial z} \right) \right\rangle + \langle i\omega\mu\phi_i\phi_j \rangle \end{pmatrix} \quad (3)$$

Boundary conditions are handled through the surface integrals appearing on the right-side of (2). When tangential $\mathbf{E}$ is specified, the first boundary integral is computed as the driving term for the tangential components of (2) while the normal component of (2) is discarded in favor of the known value of H_n (i.e. $\hat{\mathbf{n}} \cdot \mathbf{H} = \frac{-1}{i\omega\mu} \hat{\mathbf{n}} \cdot \nabla \times \mathbf{E}$). For boundaries where tangential $\mathbf{H}$ is specified, the tangential components of (2) are removed in favor of the known information whereas the normal component of (2) is enforced with homogeneous conditions (i.e. the second boundary integral in (2) vanishes). Finally, we implement current sources, if present, through strong enforcement of a jump condition on the tangential components of $\mathbf{H}$ at a current-carrying boundary:

$$\hat{\mathbf{n}} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J} \quad (4)$$

At such nodal locations, $\mathbf{H}$ has two computed values whose appropriate components are made to be discontinuous by the known amount of $\mathbf{J}$.

III. Sparse Matrix Solution

We solve the sparse set of algebraic equations generated by our FEM procedure with a symmetric, preconditioned form[§] of the Biconjugate Gradient Method (BCG)[Freund 1992]. Whenever i or j in (3) are not on a boundary or interface we save only 3 memory locations for the diagonal elements (all others vanishing on the elements we employ [Paulsen and Lynch, 1991]). For (i,j) combinations where both are on the boundary/interface, space is reserved for the full 3×3 submatrix in (3). With this memory economization, we have been able to compute solutions on meshes with more than 100K degrees-of-freedom using an IBM RS6000 series platform. Run-times have been on the order of 3-8 hours depending on the number of iterations consumed during a given solution. We monitor the equation residual, $\|\mathbf{r}_i\|_2 = \|\mathbf{b} - \mathbf{A}\mathbf{x}_i\|_2$, for each iteration $i = 1, 2, \dots$, and use this information as a measure of convergence. Typically, we normalize the initial equation residual (using an initial guess equal to the right-hand-side forcing vector) to unity and iterate until the initial equation residual has been reduced 3-4 orders of magnitude. In some of our longer runs (i.e. 8 hrs) we have used the stopping criterion of 2500 total iterations rather than terminating the solution when the equation residual has reached a prescribed value.

Because the equation residual for the BCG is not monotonic in its decent [Freund, 1992], choosing a definitive stopping rule has not been obvious. For problems with analytic solutions we have tracked both the equation residual and the solution residual (i.e. $\|\mathbf{r}_i\|_2 = \|\mathbf{x}_i - \mathbf{x}_{\text{analytic}}\|_2$) and found a strong correspondence between the two. Figures 1a-b show the equation and solution residuals as functions of iteration for small homogeneous sphere problems (≈ 9 -28K degrees-of-freedom) where we have imposed boundary conditions on tangential $\mathbf{H}$ at the outer sphere surface, namely

[§] We use right ILU preconditioning [Langtangen, 1989] with matrix reordering [George and Liu, 1981]

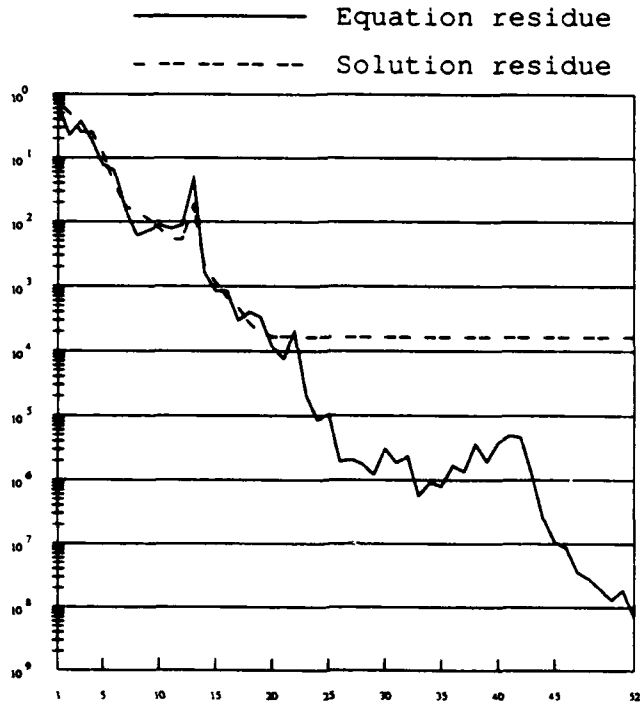
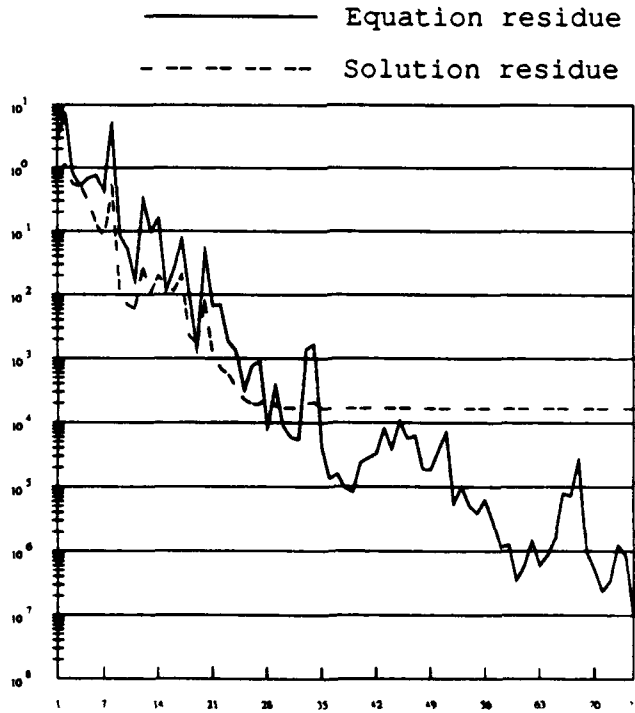


Figure 1a: Normalized solution and equation residuals as functions of iteration for a small sphere problem of 3144 nodes where an analytic solution exists.



$\mathbf{H} = \sin(\theta)\hat{\phi}$. The radius of the sphere used in these figures was 0.2 meters and its electrical properties were $\epsilon_r = 20$, $\sigma = 0.01 \Omega^{-1}m^{-1}$. The frequency of excitation was 70MHz. Note that the normalized solution residual does not decrease below 10^{-4} because only 4 significant figures

from the analytic solution were used in the comparisons. The behavior displayed in these plots is characteristic of the numerous sphere calculations we have made of this type. We find the solution accuracy is excellent for an equation residual which has been reduced 3-4 orders of magnitude from its initial value. Figure 1c shows that the computed H is essentially indistinguishable from the analytic solution for this level of convergence. Further, in nonanalytic cases (i.e. full-body models of the type we present in Section V) we have used a "converged" iterative solution† as the "analytic" solution needed to compute the solution residual and have studied the behavior of the equation residual relative to this solution residual as a function of iteration count.

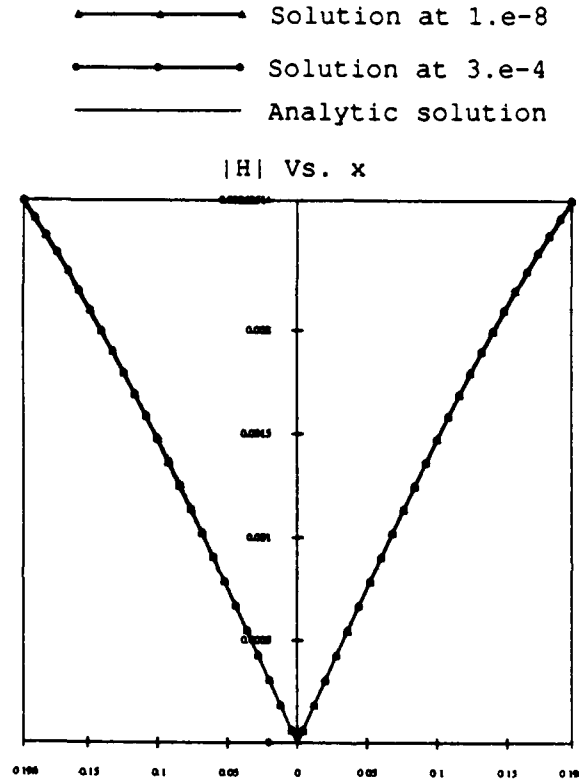


Figure 1c: Iterative H calculations with normalized equation residuals reduced to 10^{-4} and 10^{-8} compared to the analytic solution. Curves show magnitude as a function of radius for a homogeneous sphere with $\epsilon_r = 20$ and $\sigma = 0.1 \Omega^{-1}m^{-1}$.

A sample result is displayed in Figure 1d. Again, we found a strong correspondence between the two with the caveat that some early dips (within the first 100 iterations) in the equation residual often do not relate to decreases in the solution residual. Based on our experiences we believe a 3-4 orders of magnitude decrease in the initial equation residual is a reliable stopping criterion; however, as a conservative measure the solutions we report below have been iterated a fixed number (2500) of times and have equation residuals near 10^{-5} .

IV. Mesh Generation

One of the more challenging problems in using finite element analysis in 3D bioelectromagnetic applications is the generation of the FEM mesh. This is especially true if one wishes to take advantage of the real power of the method by using variable element sizes and faithful representations of the anatomical geometry of interest. Since in many biomedical contexts anatomical structure

† One with a normalized equation residual reduced to 10^{-5}

is available through serial CT-scans (this is certainly the case in hyperthermia cancer therapy), we begin the process of mesh generation by identifying the organs and tissues of interest on each CT-scan. The individual contours on each CT-slice are then linearly interpolated in the third dimension which produces a boundary element description of the anatomical model. Figure 2 shows a sample boundary description of a typical full-body anatomy which is to be meshed. We do not require that the actual nodes and surface elements shown in Figure 2 be part of the final FEM mesh, but only that the boundaries of the final FEM mesh conform to those displayed in the figure.

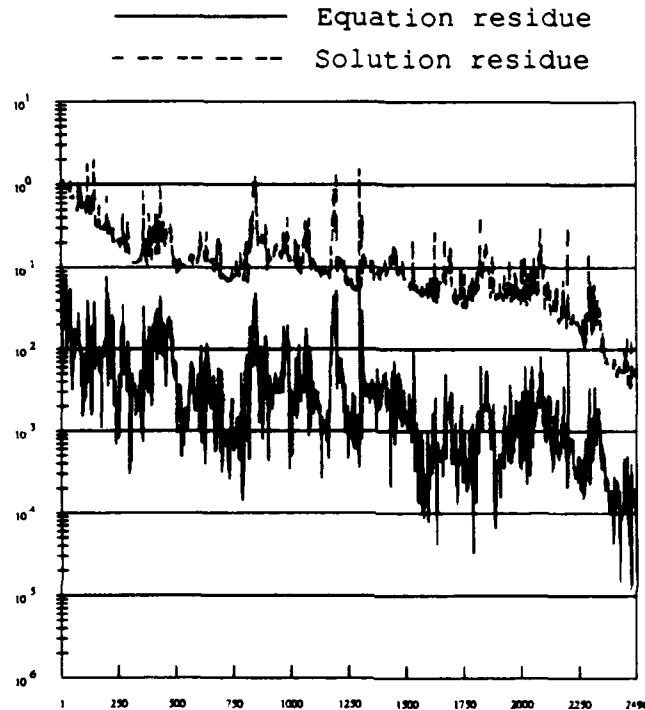


Figure 1d: Normalized solution and equation residuals as functions of iteration for a large full-body model ($\approx 135K$ degrees-of-freedom).

To generate the tetrahedral elements which are to fill the volume specified while simultaneously conforming to the boundaries delineated we use the nodal-pulling algorithm of Sullivan and Charron (1991). This is essentially the 3D version of a 2D mesh generator we have used successfully in the past [Lynch *et. al.*, 1987]. The strategy is to initially deploy the nodes and elements on a uniform lattice and to distort this lattice near the boundaries by pulling the nodes nearest to the predefined boundaries onto those boundaries. Localized refinements are possible over selected portions of the domain and result in a reduction in the characteristic nodal spacing by a factor of two for each refinement. Hence, the user specifies the largest mesh spacing that is to exist within the mesh as a starting gauge and subsequently refines the mesh in the appropriate areas until the desired resolution is achieved. During the entire process the nodal placement is continually altered near the boundaries such that the boundaries of the final mesh conform to those of the initially defined geometry. Figure 3 shows a sample mesh generated in this fashion. For ease of viewing and comparison with the initial boundary description (Figure 2) only the boundaries of the final mesh are displayed. It is clear from Figure 3 that several refinements have taken place in the areas of the most significant geometric detail. The final volume mesh for Figure 3 has 45K nodes (135K degrees-of-freedom) and 251K tetrahedral elements. To compute a solution on this mesh using a RISC workstation, our sparse matrix method (including auxiliary arrays and the preconditioner

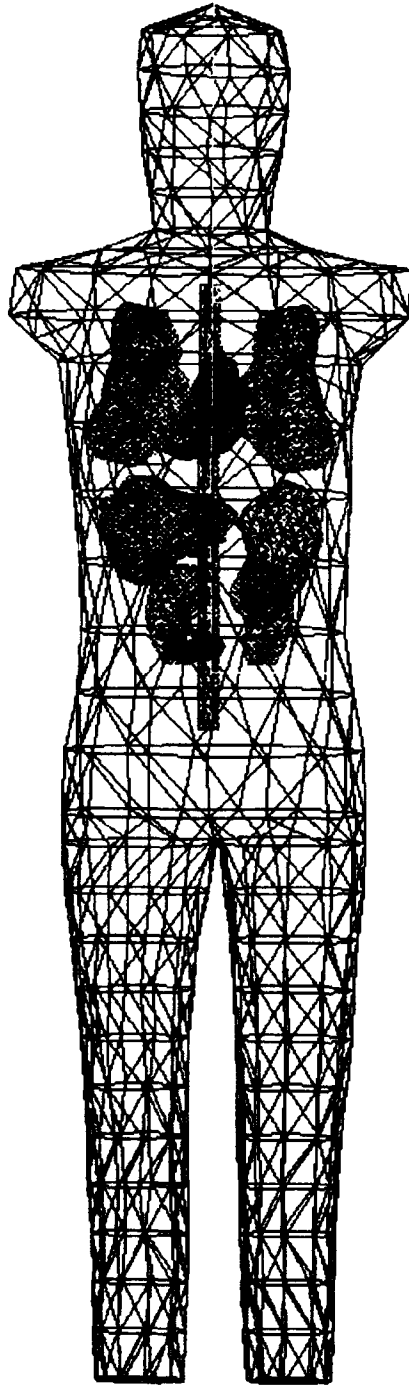


Figure 2: Boundary description of the anatomy to be meshed including internal organs and tissues which are electrically distinct. Geometry is derived from serial CT-scan data.

matrix) consumes 150 Mbytes of memory and run-times[†] increase by 11 Sec per BCG iteration. It is important to note that the anatomical model in Figs 2-3 does not yet contain a realistic representation of the body's skeleton. Reliable meshing of detailed skeletal structures has proved

[†] Processing the input data and performing the matrix and preconditioner assembly for a mesh of this size requires 30 minutes of CPU time

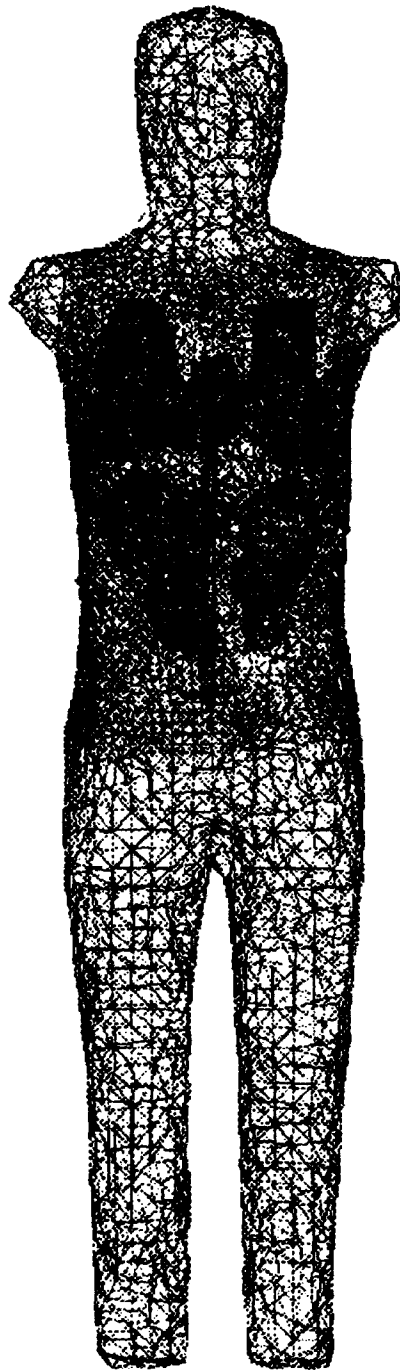


Figure 3: Actual boundary element mesh derived from the final volume element mesh generated to represent the anatomy specified in Fig 2. The boundaries produced conform to those of the original description given in Fig 2. Note the increased resolution of the body surface and internal organs relative to the original discretization in Fig 2.

to be beyond our present grid generation capabilities. Nonetheless, bony structures are important in many bioelectromagnetic applications because of their high electrical contrast relative to other soft tissues. Hence, it is imperative that we develop the ability to include relevant skeletal features

within our models in the future. Efforts along this dimension are currently underway.

V. Results

In this section we show some sample results of computed $\mathbf{H}$ and $\mathbf{E}$ distributions induced in the body model of Figure 3 by various noninvasive current source excitations. For convenience we enclose the body in a fictitious cylindrical boundary as shown in Figure 4 and enforce known current distributions over selected portions of the cylindrical surface which represent idealizations of physical devices used in hyperthermia treatment delivery. We handle the radiation fields away from the body by enclosing the model in $3 \times 3 \times 4$ meter region of equivalent dimensions to the shielded room in which most treatments of this type take place. On the outer boundary we require tangential $\mathbf{E}$ and normal $\mathbf{H}$ to vanish consistent with the nearly perfectly conducting nature of the walls of the treatment room. These conditions are implemented through the boundary integrals appearing in equation (2).

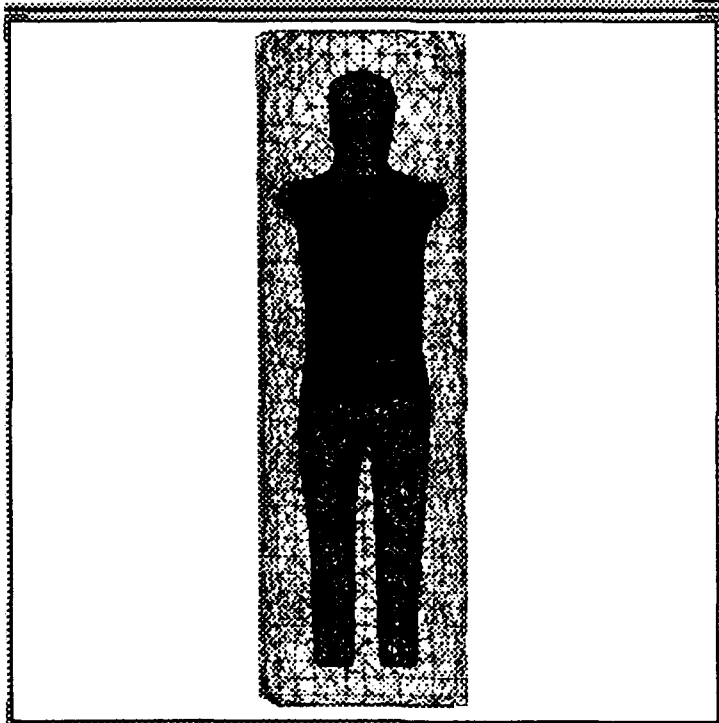


Figure 4: Body-model enclosed in a cylindrical boundary portions of which are selected to represent a noninvasive current source radiating EM fields into the body section of interest.

Specifically, we express (2) in a local normal/tangential coordinate system at nodes located on the walls. The first boundary integral in (2) vanishes identically since $\hat{\mathbf{n}} \times \mathbf{E}$ is assumed zero. The normal component of (2) is then discarded and the condition $H_n = 0$, consistent with $\hat{\mathbf{n}} \times \mathbf{E} = 0$, is enforced in its place as a Dirichlet condition on the primary field variable, $\mathbf{H}$. The finite element grid is generated by meshing the exterior region and body simultaneously so that the tetrahedral elements which fill both the body and the space between the body and the treatment room walls conform to their common surface. In meshing the exterior region and body as a composite object, we begin with a 0.18m mesh spacing (≈ 20 nodes per wavelength in air) and refine 4 times such that the internal organs have a 0.01125m characteristic nodal spacing. The electrical properties we use to simulate the various tissues and organs in our model are listed in Table 1.

In the first simulation example, we apply a uniform circumferential current which has a sinusoidal

dependence along a 100 cm length of the excitation surface described mathematically as

$$\mathbf{J} = \frac{1}{2} \left[1 + \cos \left(\frac{\pi(2z - z_1 - z_2)}{z_2 - z_1} \right) \right] \hat{\phi} \quad (5)$$

where z_2 and z_1 are the top and bottom heights of the excitation source surface. This excitation is representative of magnetic induction hyperthermia [Sapozink *et. al.*, 1985, 1990] where the body section to be heated is placed inside a current-carrying concentric sheath. The frequency of excitation is 70MHz.

TABLE I: Tissue Electrical Properties

	ϵ_r	σ ($\Omega^{-1}m^{-1}$)
Heart	72	0.89
Kidney	72	0.89
Liver	72	0.89
Lung	40	0.35
Muscle	66	0.593
Spine	10	0.20
Stomach	84	0.80
Tumor	72	0.89

Figure 5 shows the computed magnetic field distribution where the expected circulation of $\mathbf{H}$ around the current source is evident. In this figure the vector lengths as well as their gray level are proportional to their strength. Figure 6 displays the electric field derived by taking the curl of the numerical $\mathbf{H}$. We achieve nodal values of $\mathbf{E}$ by performing a Galerkin treatment of $\mathbf{E} = \frac{1}{i\omega\epsilon^*} \nabla \times \mathbf{H}$ where we use nodal quadrature to avoid the inversion of an additional matrix as described elsewhere [Lynch *et. al.*, 1986]. The $\mathbf{E}$ solution circulates as anticipated and is largely confined to 2D cross-sections suggesting that simpler 2D analysis of this type of excitation may be sufficient for hyperthermia purposes.

The second simulation example shows computed $\mathbf{H}$ and $\mathbf{E}$ solutions for a current excitation polarized along the long axis of the body. Specifically, we select $\mathbf{J}$ to have the same sinusoidal dependence along the 100 cm length of the excitation surface as before, but also vary the circumferential magnitude sinusoidally such that

$$\mathbf{J} = \frac{1}{4} \left[1 + \cos \left(\frac{\pi(2z - z_1 - z_2)}{z_2 - z_1} \right) \right] \left(1 + \cos \frac{\phi}{8} \right) \hat{z} \quad (6)$$

This idealization is representative of annular array applicators operating in the 70MHz range which are presently under investigation for noninvasive EM hyperthermia delivery [Sapozink *et. al.*, 1985, 1990]. The magnetic field is displayed in Figure 7 and its corresponding electric field in Figure 8. In this case the orientations of $\mathbf{H}$ and $\mathbf{E}$ are essentially reversed relative to the former simulation with $\mathbf{H}$ now circulating largely in cross-sectional planes and $\mathbf{E}$ oriented perpendicular to these cross-sections, but parallel to the long dimension of the body. The penetration of $\mathbf{E}$ within the body is greatly enhanced with this current source relative to the previous case. In Figure 9 we

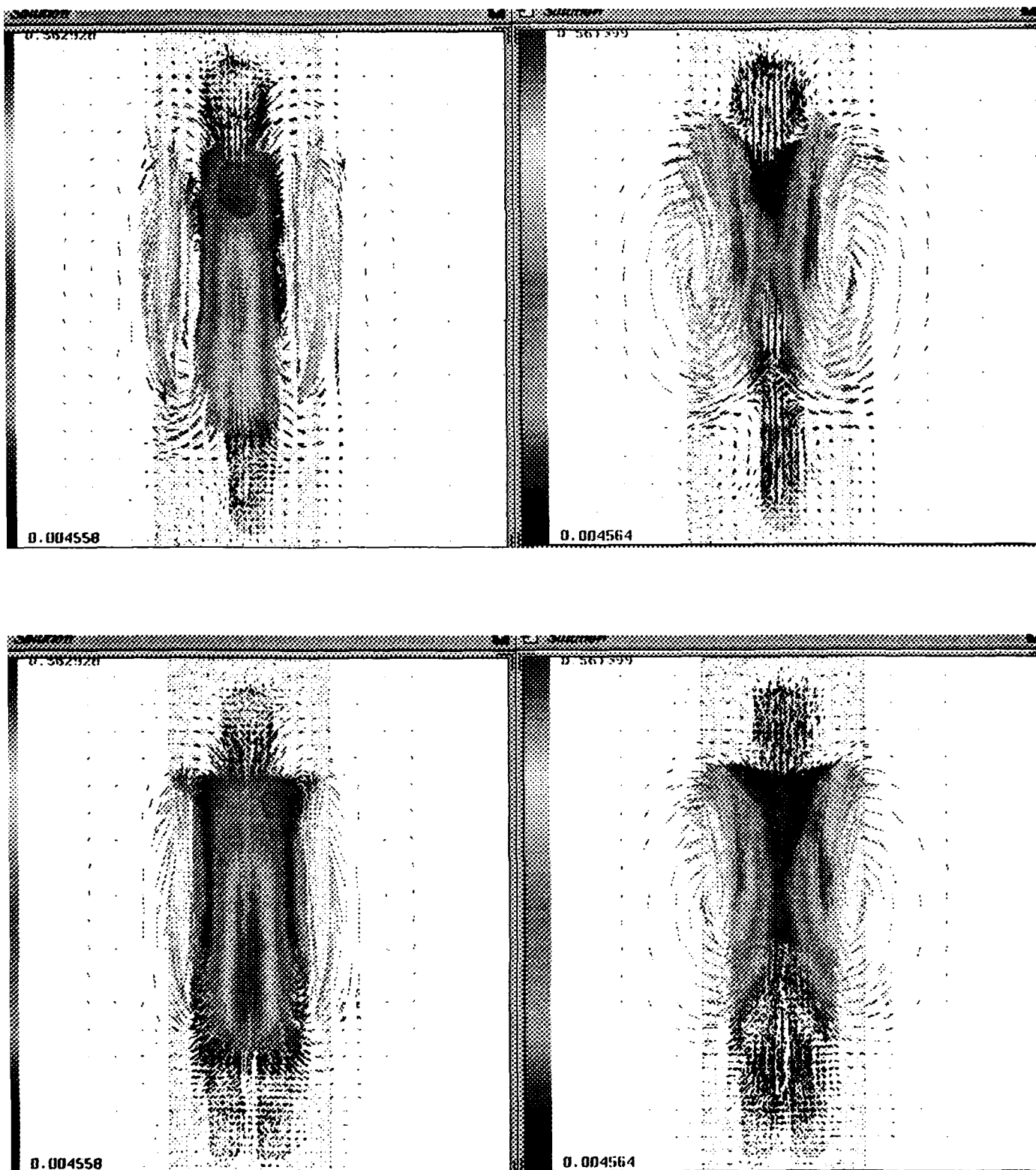


Figure 5. Computed $\mathbf{H}$ for the excitation given by equation (5) in the body model of Fig 4. Left: $Re(\mathbf{H})$, Right: $Im(\mathbf{H})$, Top: $x = 0$ plane, Bottom: $y = 0$ plane. Vector length and grey level are proportional to strength.

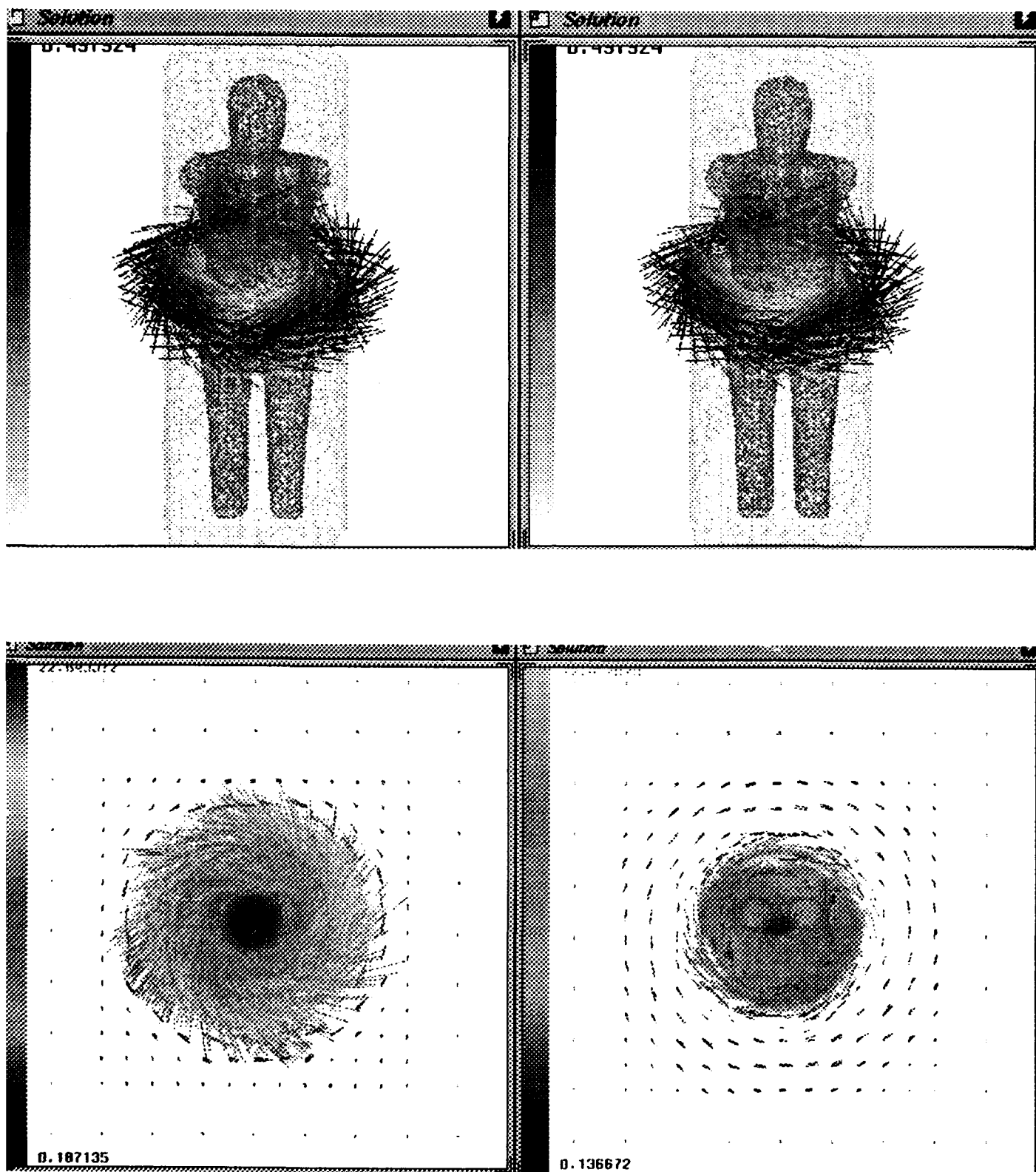


Figure 6: E solution computed from H in Fig 5

display only a subset of the overall vector solution in an attempt to highlight various features of

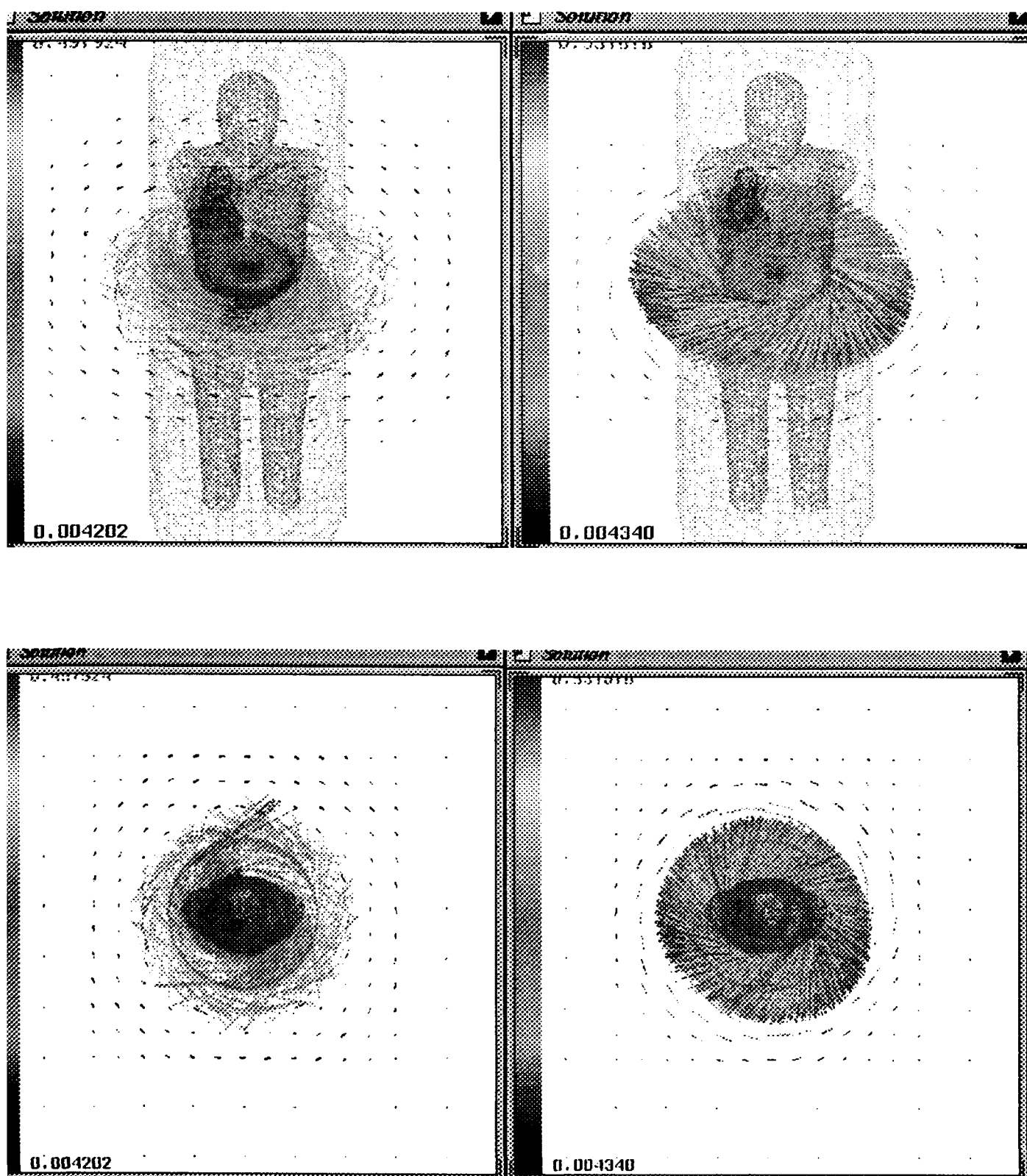


Figure 7. Computed H in the midplane of the current excitation given by equation (6) in the body model of Fig 4. Left: $Re(H)$, Right: $Im(H)$, Top: Front view, Bottom: End view. Vector length and grey level are proportional to strength.

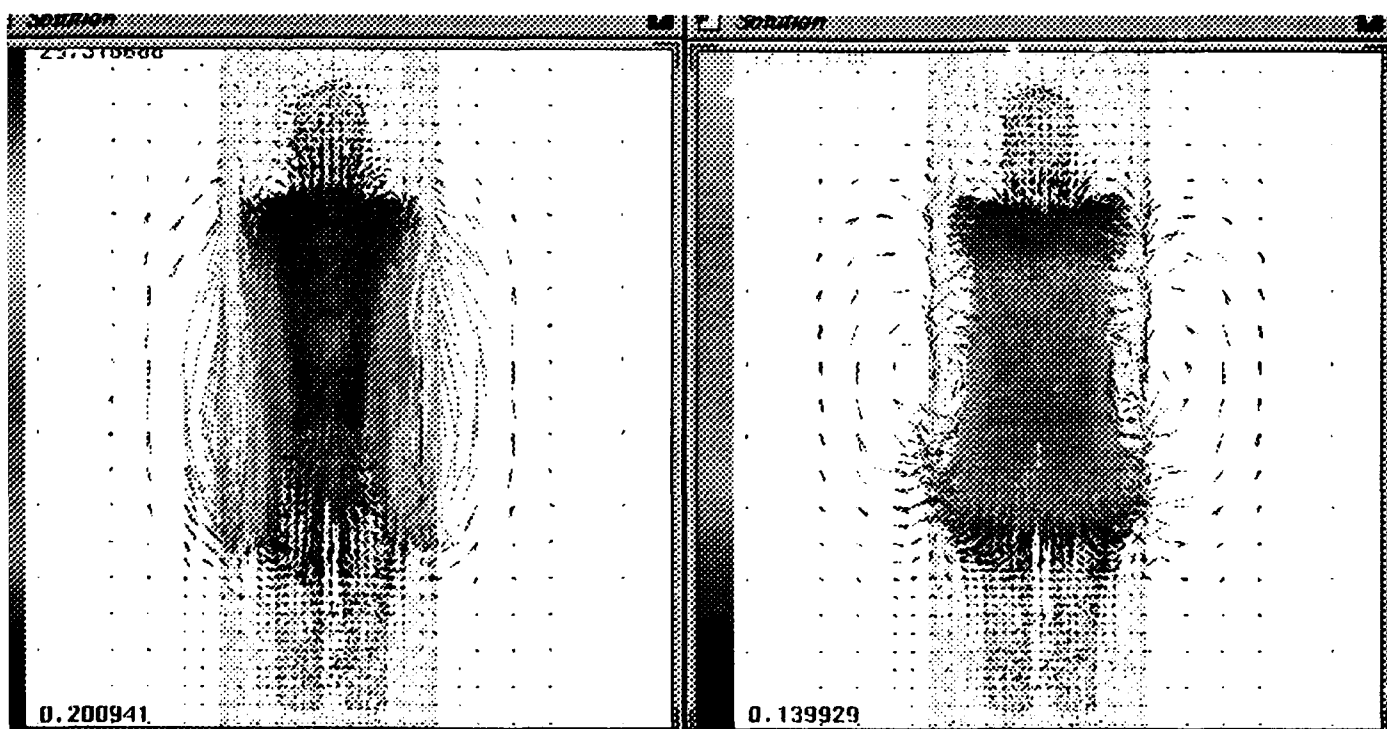
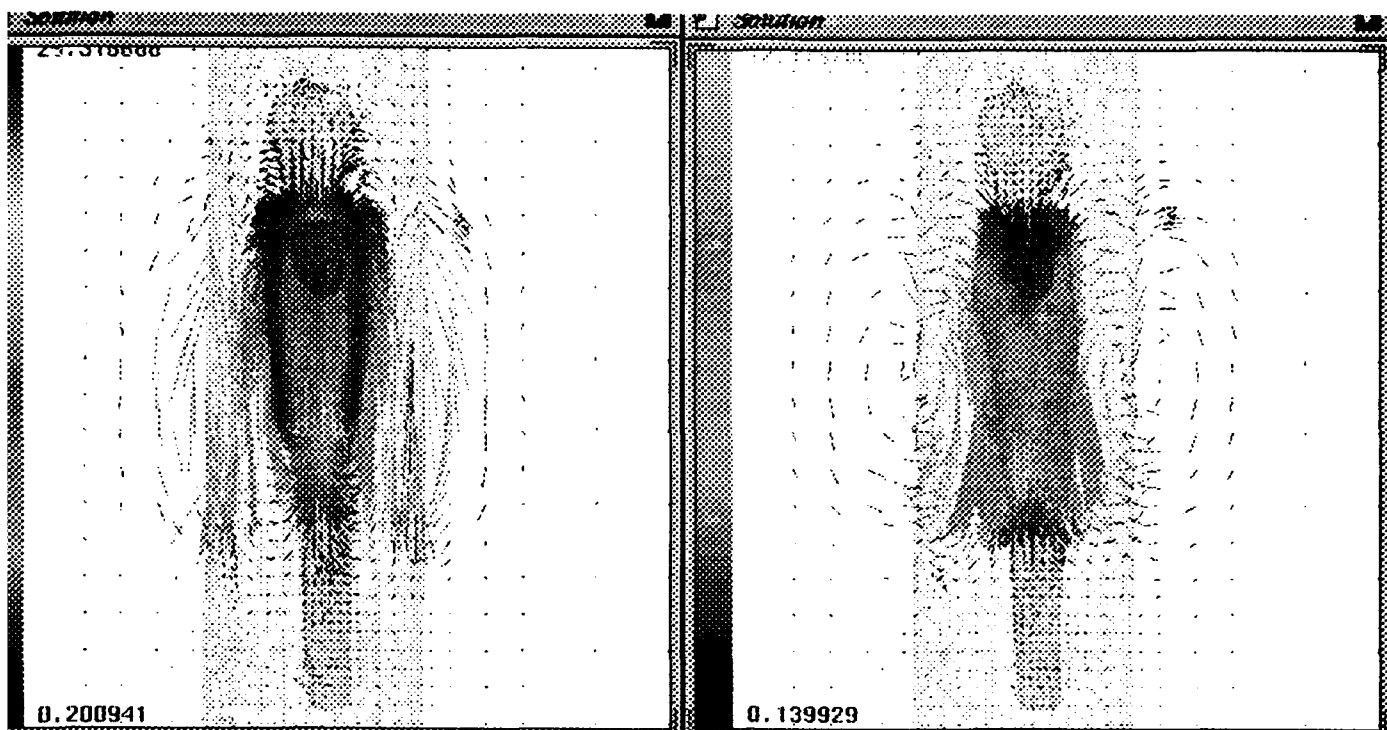


Figure 8. E solution computed from H in Fig 7. Top: $x = 0$ plane, Bottom: $y = 0$ plane.

the solution that one might be interested in. For example, it is interesting to note the phase change

in the E solution between the internal organs and the surrounding muscle tissue.

VI. Conclusions

We have demonstrated bioelectromagnetic computations on full-scale finite element man-models with significant levels of anatomical structure. Meshing is performed semi-automatically by developing a 3D boundary geometry of the region of interest from serial CT-scan data which is used as a template for guiding the creation of the final FEM grid. Irregular geometries and variable element sizes are possible such that the strength of the FEM is utilized in terms of providing flexibility in representing large computational domains. Sparse iterative matrix solution techniques allow problem sizes in excess of 100K degrees-of-freedom to be solved with run-times on the order of hours in the standard workstation computing environment.

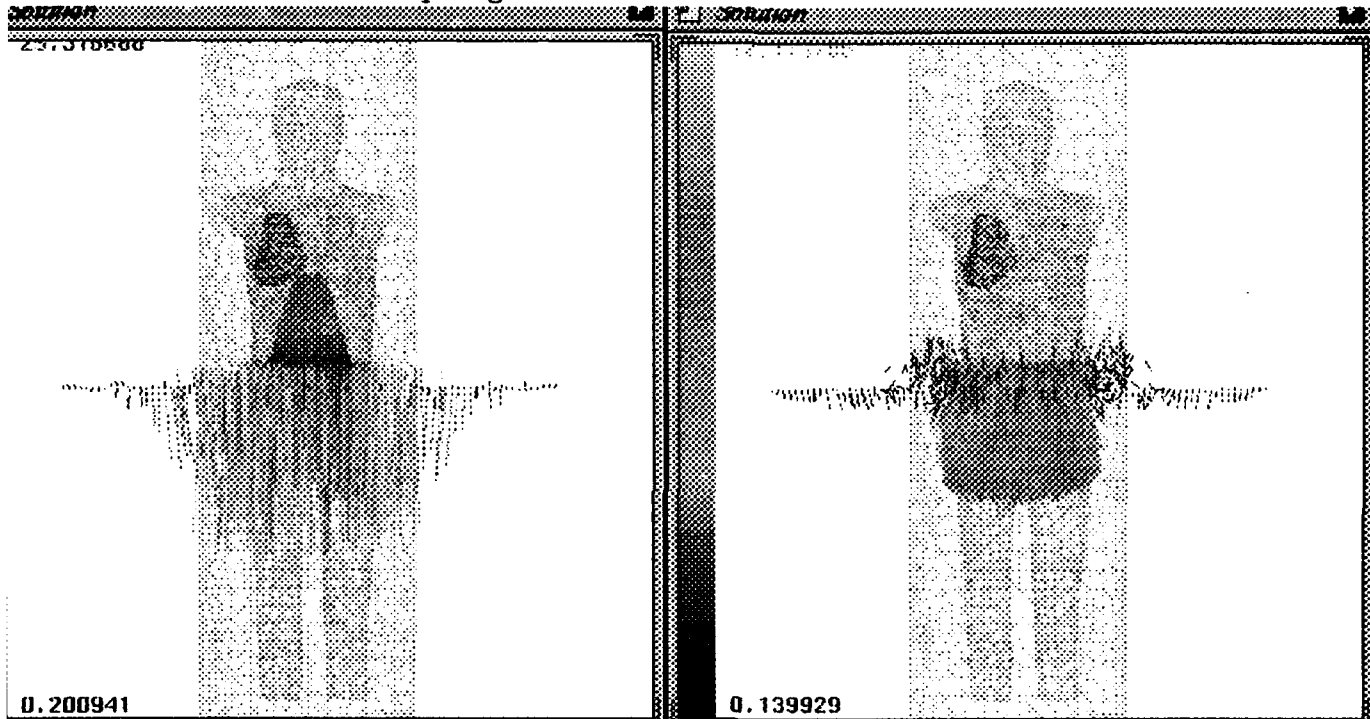


Figure 9. E solution in the midplane of the current excitation surface. The phase shift between the internal organs and the surrounding muscle tissue is evident.

Sample calculations in the hyperthermia cancer therapy context show that detailed H and E solutions are possible which highlight the salient features of EM interactions with electrically and geometrically distinct tissues and organs. Display of 3D vector data in an intelligible format is nontrivial and requires post-processing graphical routines which can plot solutions on subsets of the overall model (i.e. on arbitrarily-angled cutting planes through the geometry) or subsets of the overall collection of computed vectors (i.e. vectors within a prespecified magnitude range). In hyperthermia treatment planning, the ability to visualize specific anatomical structures and examine field phase and magnitude variations aids in the selection of source excitations which hopefully will lead to improved clinical hyperthermia delivery.

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COMPUTATION OF THREE-DIMENSIONAL ELECTROMAGNETIC-FIELD DISTRIBUTIONS IN A HUMAN BODY USING THE WEAK FORM OF THE CGFFT METHOD

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Abstract

The problem of the computation of electromagnetic-field distributions in a strongly inhomogeneous human body is formulated in terms of an integral equation over the body. A weak form of the integral equation is discussed, in which the spatial derivatives occurring in this equation are integrated analytically. The resulting equation can then be solved very efficiently using the advantageous combination of a conjugate-gradient iterative method and a fast Fourier technique (CGFFT). Numerical calculations have been carried out for a strongly inhomogeneous, lossy radially layered sphere. A comparison with the Mie-series solution shows that the present weak form of the CGFFT method yields accurate results. The absorbed power density inside a CAT-scan generated model of the body of one of the authors is computed. It demonstrates that the present method can be considered as a comparatively simple and efficient tool for solving electromagnetic wave-field problems in strongly inhomogeneous bodies.

1. Introduction

During the past several years considerable effort has been put into the development of efficient techniques for computation of electromagnetic wave fields in a strongly inhomogeneous object. One of the extensively utilized methods is the domain-integral-equation technique. It takes into account that the irradiated object is present in free space and that it manifests itself through the presence of secondary sources of contrast currents. The first method for solving the electric-field integral equation over the domain of a dielectric object was developed by Richmond for the two-dimensional TM case [Richmond, 1965], and for the two-dimensional TE case [Richmond, 1966]. Here the method of moments has been used with pulse expansion functions and point matching. The use of pulse expansion functions in the TE-case leads to numerical errors [Massoudi et al., 1984, Hagmann, 1985]. The method of moments requires the inversion of a (large) matrix, limiting the application of this

method. This problem has been circumvented by using a conjugate-gradient iterative technique [Van den Berg, 1984]. Bojarski [1982] has introduced the k -space method, obtaining an iterative approach that reduces the storage and the computation time by using a fast Fourier transform algorithm for the computation of the spatial convolution that occurs in the integral equation. A comprehensive review of Bojarski's work, together with the appropriate references to his k -space frequency domain method, can be found in his 1982 k -space time-domain paper.

In the numerical solution of two-dimensional TE and three-dimensional scattering problems, the applicability of the conjugate-gradient FFT method using pulse expansion functions casts some serious doubts [Borup et al., 1987, Joachimowicz and Pichot, 1990]. The operator involved consists of a grad-div operator that acts on a vector potential. The vector potential is an integral of the product of a Green's function and the electric contrast-current density inside the object. The vector potential is a spatial convolution. In the spectral Fourier domain this convolution is algebraic: a simple product. Recently, the weak formulation of the conjugate-gradient FFT method has proved to be an efficient and accurate scheme for solving two-dimensional TE scattering by strongly inhomogeneous lossy dielectric objects [Zwamborn and Van den Berg, 1991a]. Therefore, in this present paper, we present a weak formulation of the domain-integral equation for the modeling of *full vectorial, three-dimensional*, electromagnetic wave-field problems. The domain-integral equation that is obtained in its strong form is weakened by testing it with appropriate testing functions. This weak form is the operator equation to be solved by a CGFFT method. The advantages of this procedure are, firstly, that the grad-div operator acting on the vector potential is integrated analytically over the domain of the dielectric object only and, secondly, that we have maintained the simple scalar form of the convolution structure of the vector potential (in fact three scalar convolutions). The integral equation is formulated in terms of the unknown electric-flux density rather than in terms of the electric-field strength. The continuity of the normal component of the electric-flux density yields a correct implementation of the boundary condition of the normal component of the electric field at the interfaces of (strong) discontinuity. In contrast to the weak formulation of the two-dimensional TE-case [Zwamborn and Van den Berg, 1991a], the three-dimensional formulation is presented for different mesh sizes in the three Cartesian coordinates.

We present some numerical results for three-dimensional problems. Numerical computations have been carried out for a strongly inhomogeneous, lossy radially layered sphere. These numerical results are compared with existing analytical solutions (Mie series) and it is directly observed that the weak form of the conjugate-gradient FFT method yields accurate results. Subsequently, the absorbed power density inside a realistic model of the body of one of the authors, the Jaap phantom, is computed. These two test cases demonstrate that the present weak formulation of the conjugate-gradient FFT method can be considered to be a comparatively simple and efficient tool for solving scattering problems pertaining to (strongly) inhomogeneous lossy dielectric objects.

2. The domain-integral equation

The vectorial position in the three-dimensional space is denoted by $\mathbf{x} = (x_1, x_2, x_3)$. The unit vectors in the x_1 -, x_2 - and x_3 -directions are given by $\mathbf{i}_1$, $\mathbf{i}_2$ and $\mathbf{i}_3$. The time factor $\exp(-i\omega t)$ has been used for the field quantities in the frequency domain. We consider the problem of scattering by a lossy inhomogeneous dielectric object with complex permittivity

$$\varepsilon(\mathbf{x}) = \varepsilon_r(\mathbf{x})\varepsilon_0 + i\frac{\sigma(\mathbf{x})}{\omega}, \quad (1)$$

where ε_r denotes the relative permittivity of the object with respect to the lossless and homogeneous embedding with permittivity ε_0 , and where σ denotes the electric conductivity of the object. The incident electric field is denoted as $\mathbf{E}^i = (E_1^i, E_2^i, E_3^i)$. In this paper, we formulate the scattering problem as a domain-integral equation for the unknown electric-flux density $\mathbf{D} = (D_1, D_2, D_3)$ over

the object domain $\mathbf{D}^S$ as

$$\mathbf{E}^i(\mathbf{x}) = \frac{\mathbf{D}(\mathbf{x})}{\varepsilon(\mathbf{x})} - (k_0^2 + \text{grad div})\mathbf{A}(\mathbf{x}), \quad \mathbf{x} \in \mathbf{D}^S. \quad (2)$$

where $k_0 = \omega(\varepsilon_0\mu_0)^{\frac{1}{2}}$ and the vector potential $\mathbf{A} = (A_1, A_2, A_3)$ is given by

$$\mathbf{A}(\mathbf{x}) = \frac{1}{\varepsilon_0} \int_{\mathbf{x}' \in \mathbf{D}^S} G(\mathbf{x} - \mathbf{x}') \chi(\mathbf{x}') \mathbf{D}(\mathbf{x}') d\mathbf{x}', \quad (3)$$

in which the normalized contrast function χ is defined as

$$\chi(\mathbf{x}) = \frac{\varepsilon(\mathbf{x}) - \varepsilon_0}{\varepsilon(\mathbf{x})}. \quad (4)$$

Further, the three-dimensional Green's function G is given by

$$G(\mathbf{x}) = \frac{\exp(ik_0|\mathbf{x}|)}{4\pi|\mathbf{x}|}, \quad \mathbf{x} \in \mathbf{R}^3. \quad (5)$$

Eqs. (2) - (5) are equations in a strong form. In the next section we shall present a weak form.

3. The weak form

We first introduce a discretization in the spatial domain $\mathbf{x} = (x_1, x_2, x_3)$. We use a uniform mesh with grid widths of Δx_1 , Δx_2 and Δx_3 in the x_1 , x_2 and x_3 directions, respectively. For our convenience the discrete values of $\mathbf{x}$ are given by

$$\mathbf{x}_{M,N,P} = \left\{ (M - \frac{1}{2})\Delta x_1, (N - \frac{1}{2})\Delta x_2, (P - \frac{1}{2})\Delta x_3 \right\}, \quad (6)$$

denoting the centerpoints of the volumetric subdomains. The boundary of the discretized object now consists of surfaces parallel to the x_1 -, x_2 - or x_3 -axis. We assume that the discretized boundary $\partial\mathbf{D}^S$ of the object domain $\mathbf{D}^S$ lies completely in the embedding where $\chi = 0$. This is always possible, since we can extend the definition of the object domain $\mathbf{D}^S$ by extending it with a zero contrast function χ . In each volumetric subdomain with center $\mathbf{x}_{M,N,P}$ and dimension $\Delta x_1 \times \Delta x_2 \times \Delta x_3$, we assume the complex permittivity to be constant with values $\varepsilon_{M,N,P}$. Note that jumps in the (complex) permittivity function may occur at $x_1 = M\Delta x_1$, $x_2 = N\Delta x_2$ and $x_3 = P\Delta x_3$.

In order to cope with the grad-div operator in Eq. (2), we test the strong form of Eq. (2) by multiplying both the sides of the equality sign by a vectorial testing function $\psi_{M,N,P}^{(p)}(\mathbf{x})$, $p = 1, 2, 3$, and integrate the result over the domain $\mathbf{x} \in \mathbf{D}^S$ upon using Gauss' theorem on each subdomain where $\partial_p \psi_{M,N,P}^{(p)}(\mathbf{x}) \text{div } \mathbf{A}(\mathbf{x})$ is continuously differentiable and by using the continuity of the normal component of this function through the interfaces between these subdomains. Subsequently, we expand the generalized electric-flux density, the electric-contrast vector potential and the incident electric field in a sequence of vectorial expansion functions $\psi_{I,J,K}^{(q)}(\mathbf{x}) = \psi_{I,J,K}^{(q)}(\mathbf{x}) \mathbf{i}_q$, $q = 1, 2, 3$ and obtain

$$D_q(\mathbf{x}) = \varepsilon_0 \sum_{I,J,K} d_{I,J,K}^{(q)} \psi_{I,J,K}^{(q)}(\mathbf{x}) \quad \text{for } \mathbf{x} \in \mathbf{D}^S, \quad (7)$$

$$A_q(\mathbf{x}) = \sum_{I,J,K} A_{I,J,K}^{(q)} \psi_{I,J,K}^{(q)}(\mathbf{x}) \quad \text{for } \mathbf{x} \in \mathbf{D}^S, \quad (8)$$

$$E_q^i(\mathbf{x}) = \sum_{I,J,K} E_{I,J,K}^{i,(q)} \psi_{I,J,K}^{(q)}(\mathbf{x}) \quad \text{for } \mathbf{x} \in \mathbf{D}^S. \quad (9)$$

In view of the partial derivatives acting on the testing and expansion functions, the volumetric rooftop functions [Catedra et al., 1989] are chosen as basis in the test and expansion functions.

Using these functions we obtain the following weak formulation of the domain-integral equation, see [Zwamborn, 1991b, Zwamborn and Van den Berg, 1992],

$$e_{M,N,P}^{i,(1)} = \sum_{I=1}^3 \left[b_I^{(1)} d_{M+I-2,N,P}^{(1)} + c_I^{(1)} A_{M+I-2,N,P}^{(1)} \right] + \sum_{I=1}^2 \sum_{J=1}^2 t_{I,J}^{(3)} A_{M+I-2,N+J-1,P}^{(2)} + \sum_{I=1}^2 \sum_{K=1}^2 t_{I,K}^{(2)} A_{M+I-2,N,P+K-1}^{(3)}, \quad (10)$$

$$e_{M,N,P}^{i,(2)} = \sum_{I=1}^2 \sum_{J=1}^2 t_{J,I}^{(3)} A_{M+I-1,N+J-2,P}^{(1)} + \sum_{J=1}^3 \left[b_J^{(2)} d_{M,N+J-2,P}^{(2)} + c_J^{(2)} A_{M,N+J-2,P}^{(2)} \right] + \sum_{J=1}^2 \sum_{K=1}^2 t_{J,K}^{(1)} A_{M,N+J-2,P+K-1}^{(3)}, \quad (11)$$

$$e_{M,N,P}^{i,(3)} = \sum_{I=1}^2 \sum_{K=1}^2 t_{K,I}^{(2)} A_{M+I-1,N,P+K-2}^{(1)} + \sum_{J=1}^2 \sum_{K=1}^2 t_{K,J}^{(1)} A_{M,N+J-1,P+K-2}^{(2)} + \sum_{K=1}^3 \left[b_K^{(3)} d_{M,N,P+K-2}^{(3)} + c_K^{(3)} A_{M,N,P+K-2}^{(3)} \right], \quad (12)$$

in which the coefficients of the vectors $\mathbf{b}^{(p)}$ and $\mathbf{c}^{(p)}$ are obtained as

$$\mathbf{b}^{(1)} = \frac{\Delta x_1 \Delta x_2 \Delta x_3}{6} \begin{pmatrix} \frac{\epsilon_0}{\epsilon_{M-1,N,P}} \\ \frac{2\epsilon_0}{\epsilon_{M-1,N,P}} + \frac{2\epsilon_0}{\epsilon_{M,N,P}} \\ \frac{\epsilon_0}{\epsilon_{M,N,P}} \end{pmatrix}, \quad (13)$$

$$\mathbf{b}^{(2)} = \frac{\Delta x_1 \Delta x_2 \Delta x_3}{6} \begin{pmatrix} \frac{\epsilon_0}{\epsilon_{M,N-1,P}} \\ \frac{2\epsilon_0}{\epsilon_{M,N-1,P}} + \frac{2\epsilon_0}{\epsilon_{M,N,P}} \\ \frac{\epsilon_0}{\epsilon_{M,N,P}} \end{pmatrix}, \quad (14)$$

$$\mathbf{b}^{(3)} = \frac{\Delta x_1 \Delta x_2 \Delta x_3}{6} \begin{pmatrix} \frac{\epsilon_0}{\epsilon_{M,N,P-1}} \\ \frac{2\epsilon_0}{\epsilon_{M,N,P-1}} + \frac{2\epsilon_0}{\epsilon_{M,N,P}} \\ \frac{\epsilon_0}{\epsilon_{M,N,P}} \end{pmatrix}, \quad (15)$$

$$\mathbf{c}^{(p)} = \Delta \mathbf{x}_1 \Delta \mathbf{x}_2 \Delta \mathbf{x}_3 \left[-\frac{k_0^2}{6} \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} + (\Delta \mathbf{x}_p)^{-2} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} \right], \quad (16)$$

while the coefficients of the matrix $\mathbf{t}^{(p)}$ follow from

$$\mathbf{t}^{(p)} = \Delta \mathbf{x}_p \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}. \quad (17)$$

The values of $e_{M,N,P}^{i,(p)}$ follow from $E_{M,N,P}^{i,(p)}$ as

$$e_{M,N,P}^{i,(1)} = \frac{\Delta \mathbf{x}_1 \Delta \mathbf{x}_2 \Delta \mathbf{x}_3}{6} [E_{M+1,N,P}^{i,(1)} + 4E_{M,N,P}^{i,(1)} + E_{M-1,N,P}^{i,(1)}], \quad (18)$$

$$e_{M,N,P}^{i,(2)} = \frac{\Delta \mathbf{x}_1 \Delta \mathbf{x}_2 \Delta \mathbf{x}_3}{6} [E_{M,N+1,P}^{i,(2)} + 4E_{M,N,P}^{i,(2)} + E_{M,N-1,P}^{i,(2)}], \quad (19)$$

$$e_{M,N,P}^{i,(3)} = \frac{\Delta \mathbf{x}_1 \Delta \mathbf{x}_2 \Delta \mathbf{x}_3}{6} [E_{M,N,P+1}^{i,(3)} + 4E_{M,N,P}^{i,(3)} + E_{M,N,P-1}^{i,(3)}]. \quad (20)$$

With our particular choice of expansion functions, the quantities $d_{M,N,P}^{(p)}$, $A_{M,N,P}^{(p)}$ and $e_{M,N,P}^{i,(p)}$ follow from

$$d_{M,N,P}^{(p)} = \frac{D_p(\mathbf{x}_{M,N,P} - \frac{1}{2}\Delta \mathbf{x}_p \mathbf{i}_p)}{\epsilon_0}, \quad p=1,2,3, \quad (21)$$

$$A_{M,N,P}^{(p)} = A_p(\mathbf{x}_{M,N,P} - \frac{1}{2}\Delta \mathbf{x}_p \mathbf{i}_p), \quad p=1,2,3, \quad (22)$$

$$E_{M,N,P}^{i,(p)} = E_p^i(\mathbf{x}_{M,N,P} - \frac{1}{2}\Delta \mathbf{x}_p \mathbf{i}_p), \quad p=1,2,3. \quad (23)$$

The electric-contrast vector potential A_m is related to the electric-flux density $\mathbf{D}$ via Eq. (3). Note that with this procedure we have enforced the equality sign of Eqs. (21) - (23) exactly in a single point. Again, this is a strong form and we will weaken this form by taking the spherical mean. The computation of the electric contrast-vector potential is discussed in [Zwamborn, 1991b, Zwamborn and Van den Berg, 1992] and we will only summarize the results.

Replacing the continuous convolution integral of Eq. (3) and using the convolution theorem of the discrete Fourier transform (DFT), the discrete convolution is evaluated numerically by

$$A_{M,N,P}^{(p)} = \Delta \mathbf{x}_1 \Delta \mathbf{x}_2 \Delta \mathbf{x}_3 \text{DFT}^{-1} \left\{ \text{DFT} \{G_{M',N',P'}\} \text{DFT} \{ \chi_{M,N,P}^{(p)} d_{M,N,P}^{(p)} \} \right\}, \quad (24)$$

in which $p = 1, 2, 3$; the discrete values of the normalized contrast function follow from

$$\chi_{M,N,P}^{(1)} = \frac{\chi(\mathbf{x}_{M-1,N,P}) + \chi(\mathbf{x}_{M,N,P})}{2}, \quad (25)$$

$$\chi_{M,N,P}^{(2)} = \frac{\chi(\mathbf{x}_{M,N-1,P}) + \chi(\mathbf{x}_{M,N,P})}{2}, \quad (26)$$

$$\chi_{M,N,P}^{(3)} = \frac{\chi(\mathbf{x}_{M,N,P-1}) + \chi(\mathbf{x}_{M,N,P})}{2}. \quad (27)$$

The discrete values of the weakened Green's function are given by

$$G_{M,N,P} = [G](M\Delta \mathbf{x}_1, N\Delta \mathbf{x}_2, P\Delta \mathbf{x}_3), \quad (28)$$

where

$$[G](\mathbf{x}) = \begin{cases} \frac{(1 - \frac{1}{2}ik_0\Delta x)\exp(\frac{1}{2}ik_0\Delta x) - 1}{\frac{1}{6}\pi k_0^2(\Delta x)^3} & \text{if } |\mathbf{x}| = 0, \\ \frac{\exp(ik_0|\mathbf{x}|) \left[\frac{\sinh(\frac{1}{2}ik_0\Delta x)}{\frac{1}{2}ik_0\Delta x} - \cosh(\frac{1}{2}ik_0\Delta x) \right]}{\frac{1}{3}\pi(k_0\Delta x)^2|\mathbf{x}|} & \text{if } |\mathbf{x}| > \frac{1}{2}\Delta x. \end{cases} \quad (29)$$

Note that, for the limiting case $\Delta x \rightarrow 0$, the weak form of the Green's function $[G](\mathbf{x})$, $|\mathbf{x}| > \frac{1}{2}\Delta x$, tends to the strong form of the Green's function $G(\mathbf{x})$. The subscripts M' , N' and P' of $G_{M',N',P'}$ in Eq. (24) are dictated by Eq. (6) and the spatial periodicity of the discrete Fourier transform.

Let us assume that the domain $\mathbf{D}^S$ of the object lies completely inside a block with $M_{\mathbf{D}^S}$ meshpoints in the x_1 -direction, $N_{\mathbf{D}^S}$ meshpoints in the x_2 -direction and $P_{\mathbf{D}^S}$ meshpoints in the x_3 -direction. It is easily shown [Zwamborn, 1991b, Zwamborn and Van den Berg, 1992] that the numerical evaluation of Eq. (3) using a trapezoidal integration rule is equivalent with Eq. (24) inside the object domain $\mathbf{D}^S$ if the relevant DFT's are defined inside a block with M_{DFT} meshpoints in the x_1 -direction, N_{DFT} meshpoints in the x_2 -direction and P_{DFT} meshpoints in the x_3 -direction, such that

$$M_{\text{DFT}} \geq 2(M_{\mathbf{D}^S} + 1), \quad N_{\text{DFT}} \geq 2(N_{\mathbf{D}^S} + 1), \quad P_{\text{DFT}} \geq 2(P_{\mathbf{D}^S} + 1). \quad (30)$$

Finally, the quantity $E_{M,N,P}^{i,(p)}$ is given in case the incident field is taken to be a uniform plane wave. Then, $\mathbf{E}^i$ follows from

$$\mathbf{E}^i(\mathbf{x}) = \mathcal{E} \exp(ik_0\boldsymbol{\theta} \cdot \mathbf{x}), \quad (31)$$

in which $\mathcal{E}$ denotes the amplitude of the plane wave and $\boldsymbol{\theta}$ denotes the unit vector of the direction of propagation. The spherical mean (weak form) of this incident field is given by, see [Zwamborn, 1991b],

$$[\mathbf{E}^i](\mathbf{x}) = \mathcal{E} \exp(ik_0\boldsymbol{\theta} \cdot \mathbf{x}) \frac{12}{(k_0\Delta x)^2} \left[\frac{\sinh(\frac{1}{2}ik_0\Delta x)}{\frac{1}{2}ik_0\Delta x} - \cosh(\frac{1}{2}ik_0\Delta x) \right]. \quad (32)$$

The latter weak form is used in the representations for the quantity $E_{M,N,P}^{i,(p)}$ as

$$E_{M,N,P}^{i,(p)} = [\mathbf{E}_p^i](\mathbf{x}_{M,N,P} - \frac{1}{2}\Delta x_p \mathbf{i}_p). \quad (33)$$

Note that, for the limiting case $\Delta x \rightarrow 0$, the strong form of the incident uniform plane wave is obtained (cf. Eq. (31)).

Collecting all the results, the weak form of the domain-integral equation is given by Eqs. (10) - (20), (24) and (33). This domain integral equation is symbolically written as

$$\mathbf{e}^i = Ld. \quad (34)$$

The latter operator equation is solved numerically by applying a conjugate-gradient iterative scheme, where the DFT's are computed efficiently using fast Fourier transform (FFT) algorithms.

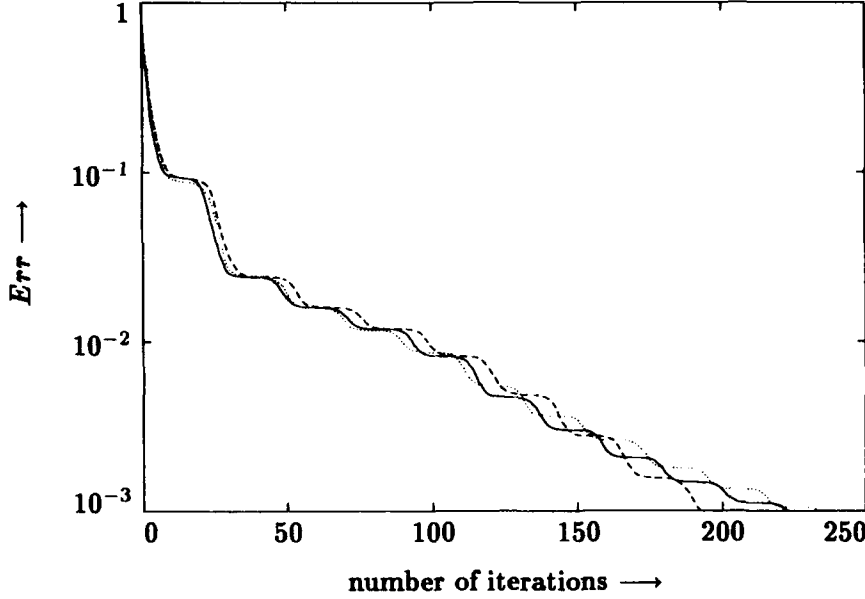


Figure 1: The numerical convergence obtained for the scattering by a inhomogeneous sphere. The numerical results pertaining to a mesh size of $15 \times 15 \times 15$ and $29 \times 29 \times 29$ (Fig. 2) are presented by the dotted line and the solid line, respectively, while the results for the refined mesh (Fig. 3) are presented by the dashed line.

4. Numerical results

The numerical convergence is measured by the normalized root-mean-square error Err

$$Err = \frac{\|\mathbf{r}^{(n)}\|}{\|\mathbf{r}^{(0)}\|}, \quad (35)$$

in which $\|\mathbf{r}^{(n)}\|$ denotes the norm of the residual error in the satisfaction of the operator equation of Eq. (34) over the domain $\mathcal{D}^S$ of the object in the n^{th} iteration. In all cases we have taken a zero initial estimate. All computations were carried out on a VAX 3100/76 workstation in double precision arithmetic. The DFT's are efficiently computed using fast Fourier transform (FFT) algorithms in single precision arithmetic only. The incident field is taken to be a uniform plane wave with (cf. Eq. (32))

$$\mathcal{E}_1 = 1 \text{ V/m}, \quad \mathcal{E}_2 = 0, \quad \mathcal{E}_3 = 0, \quad (36)$$

$$\theta_1 = 0, \quad \theta_2 = 0, \quad \theta_3 = -1, \quad (37)$$

while the frequency of operation is 100 MHz.

We firstly consider a radially layered lossy dielectric spherical object to be present with its origin at $\mathbf{x} = \{a, a, a\}$, where a denotes the outer radius of the sphere. It is noted that for this special test case, analytical results are obtained with the Mie series [Kong, 1986]. The relative permittivities

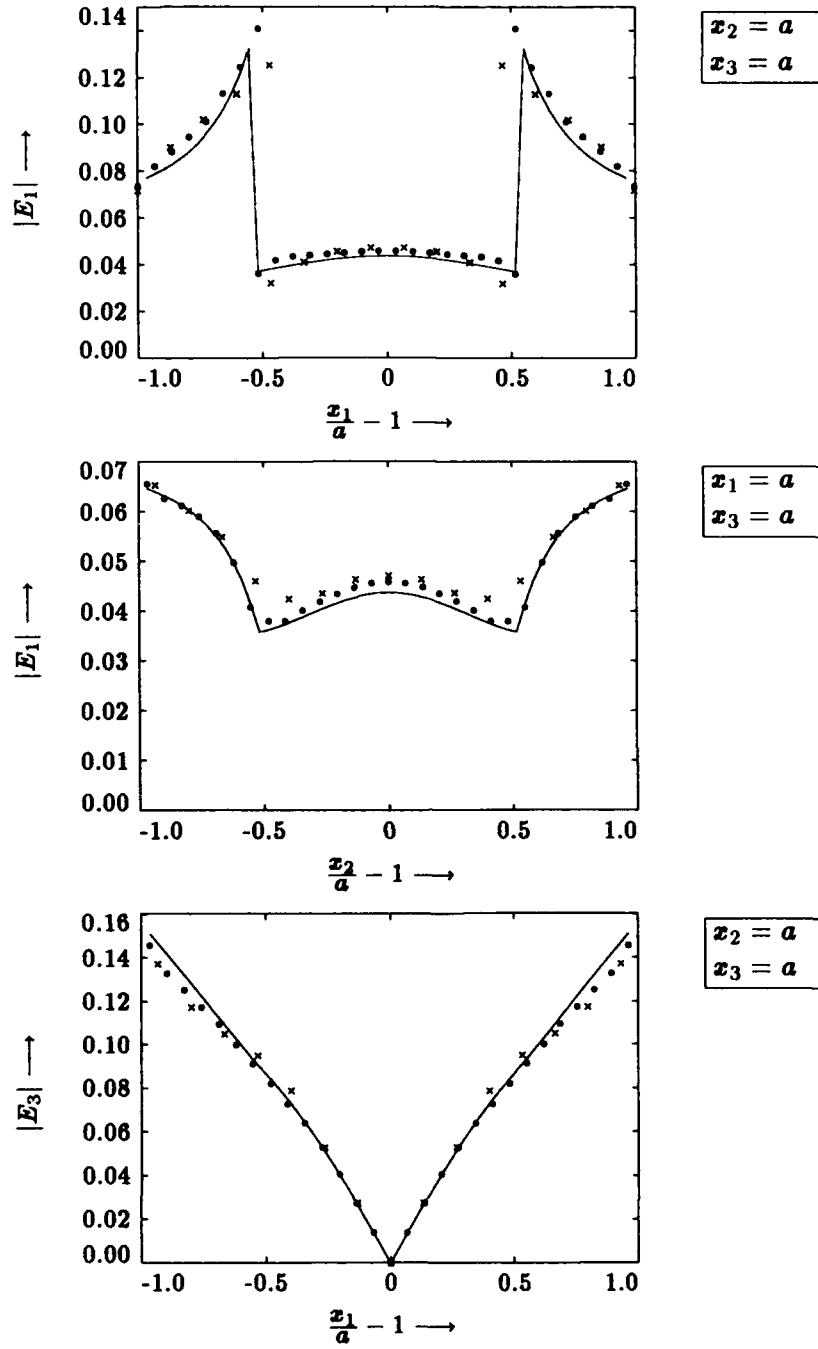


Figure 2: The magnitude of the components of the total electric field inside a lossy inhomogeneous sphere. The numerical results pertaining to a mesh size of $15 \times 15 \times 15$ are presented by the symbols $\times$ and the numerical results pertaining to a mesh size of $29 \times 29 \times 29$ are presented by the symbols $*$. The analytical solution of the inhomogeneous sphere is presented by the solid line.

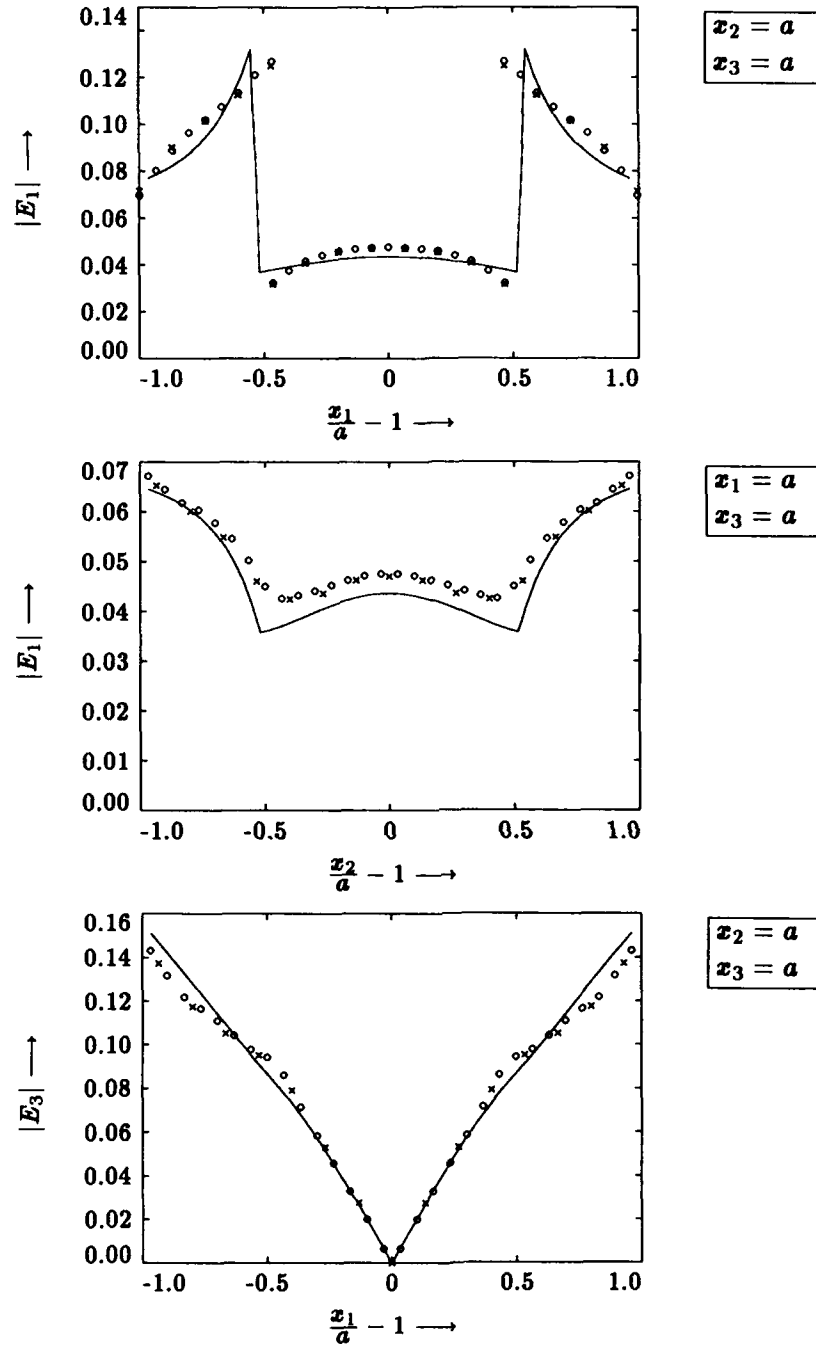


Figure 3: The magnitude of the components of the total electric field inside a $15 \times 15 \times 15$ discretized lossy inhomogeneous sphere. The numerical results pertaining to a mesh size of $15 \times 15 \times 15$ are presented by the symbols $\times$. The discretized object is refined with a mesh size of $30 \times 30 \times 30$ and the numerical results are presented by the symbols $\circ$. The analytical solution of the inhomogeneous sphere is presented by the solid line.

and conductivities are $\epsilon_{r,1} = 71.5$, $\sigma_1 = 0.83$ S/m, and $\epsilon_{r,2} = 15$, $\sigma_2 = 0.22$ S/m, respectively. The dimensions are given by $k_0 a_1 = 0.163$ and $k_0 a_2 = 0.314$. It is noted that a_1 denotes the radius of the inner sphere and $a_2 = a$ denotes the radius of the outer sphere. The computations are performed for different mesh sizes of $M_{\mathbf{D}^s} = N_{\mathbf{D}^s} = P_{\mathbf{D}^s} = 15$ ($M_{\text{DFT}} = N_{\text{DFT}} = P_{\text{DFT}} = 32$) and $M_{\mathbf{D}^s} = N_{\mathbf{D}^s} = P_{\mathbf{D}^s} = 29$ ($M_{\text{DFT}} = N_{\text{DFT}} = P_{\text{DFT}} = 64$), respectively. The numerical convergence rate of the iterative scheme is presented in Fig. 1, while the magnitudes of the components of the total electric field are presented in Fig. 2. In order to investigate the discrepancies of the numerical results and the analytical results, we have taken the discretized sphere of the case $M_{\mathbf{D}^s} = N_{\mathbf{D}^s} = P_{\mathbf{D}^s} = 15$ as new object. As next step, this new object has been subdivided with $M_{\mathbf{D}^s} = N_{\mathbf{D}^s} = P_{\mathbf{D}^s} = 30$. The number of iterations to obtain an error less than 0.1 percent is 194. From Fig. 3 it is observed that refining the mesh in the interior of the object yields hardly no improvement. The same discrepancies between the numerical results and the analytical results are observed. The latter reveals that the differences between the analytical and numerical results are caused by the block approximation of the spherical boundary. In order to obtain a better approximation of the spherical boundaries, the discretization of the sphere has to be improved.

As second test case we consider the absorbed power density in a CAT-scan generated model of a human body. In order to arrive at a standard discretized three-dimensional model of a human body, an X-ray computer tomographic scan of one of the present authors, Jaap Mooibroek, has been carried out. A model of $128 \times 128 \times 512$ subdomains with a mesh width of 4 mm in each direction is the result. This so-called "Jaap phantom" consists of muscle with permittivity $\epsilon_r = 71.5$ and conductivity $\sigma = 0.83$ S/m and fat with permittivity $\epsilon_r = 15$ and conductivity $\sigma = 0.22$ S/m. In this Jaap phantom we have increased the mesh size with a factor of $4 \times 4 \times 8$ and determined the smallest rectangular domain containing this Jaap phantom. This leads to the reduced Jaap phantom, subdivided with $M_{\mathbf{D}^s} = 21$, $N_{\mathbf{D}^s} = 31$ and $P_{\mathbf{D}^s} = 53$ and mesh sizes $\Delta x_1 = 1.33751$ cm, $\Delta x_2 = 1.6041$ cm and $\Delta x_3 = 3.1479$ cm. In order to obtain a global impression of the discretized model at hand, we present in Fig. 4 the projection of the Jaap phantom on the (x_1, x_3) -plane and (x_2, x_3) -plane, respectively. The incident field, see Eqs. (32), (36) and (37), is a plane wave propagating from head to feet. The number of iterations to obtain an error less than 1 percent amounts to 601 iterations. In Figs. 5 - 7, we present the absorbed power density,

$$\dot{w}_h = \frac{1}{2} \sigma |\mathbf{E}|^2, \quad (38)$$

together with the tissue composition of the Jaap phantom in some cross-sections. Note that we do not use the Specific Absorption Rate (SAR), since using SAR we also need the values of the volume density of mass. The latter quantity is not significant for solving the electromagnetic field problem. The absorbed power density is normalized to its maximum value in the Jaap phantom. From the latter figures, it is observed that the maximum absorbed power density can be found at the area above the lung. Although the incident plane wave travels from the head to the feet, the absorbed power density has a much smaller value in the area around the brains. This enforces the necessity of having an adequate computer code for the prediction of the electromagnetic field distribution in a strongly inhomogeneous body.

The numerical convergence of the iterative scheme is presented in Fig. 8. In Table 1, the computation time needed to evaluate one iteration on the VAX 3100/76 workstation and the number of unknowns in the pertaining field problems have been presented. It is noted that the VAX Fortran computer code pertaining to these values is, however, not optimized. Examining this table reveals that the computation time of each iteration is proportional to $(M_{\text{DFT}} \times N_{\text{DFT}} \times P_{\text{DFT}}) [1 + {}^2\log(M_{\text{DFT}} \times N_{\text{DFT}} \times P_{\text{DFT}})]$.

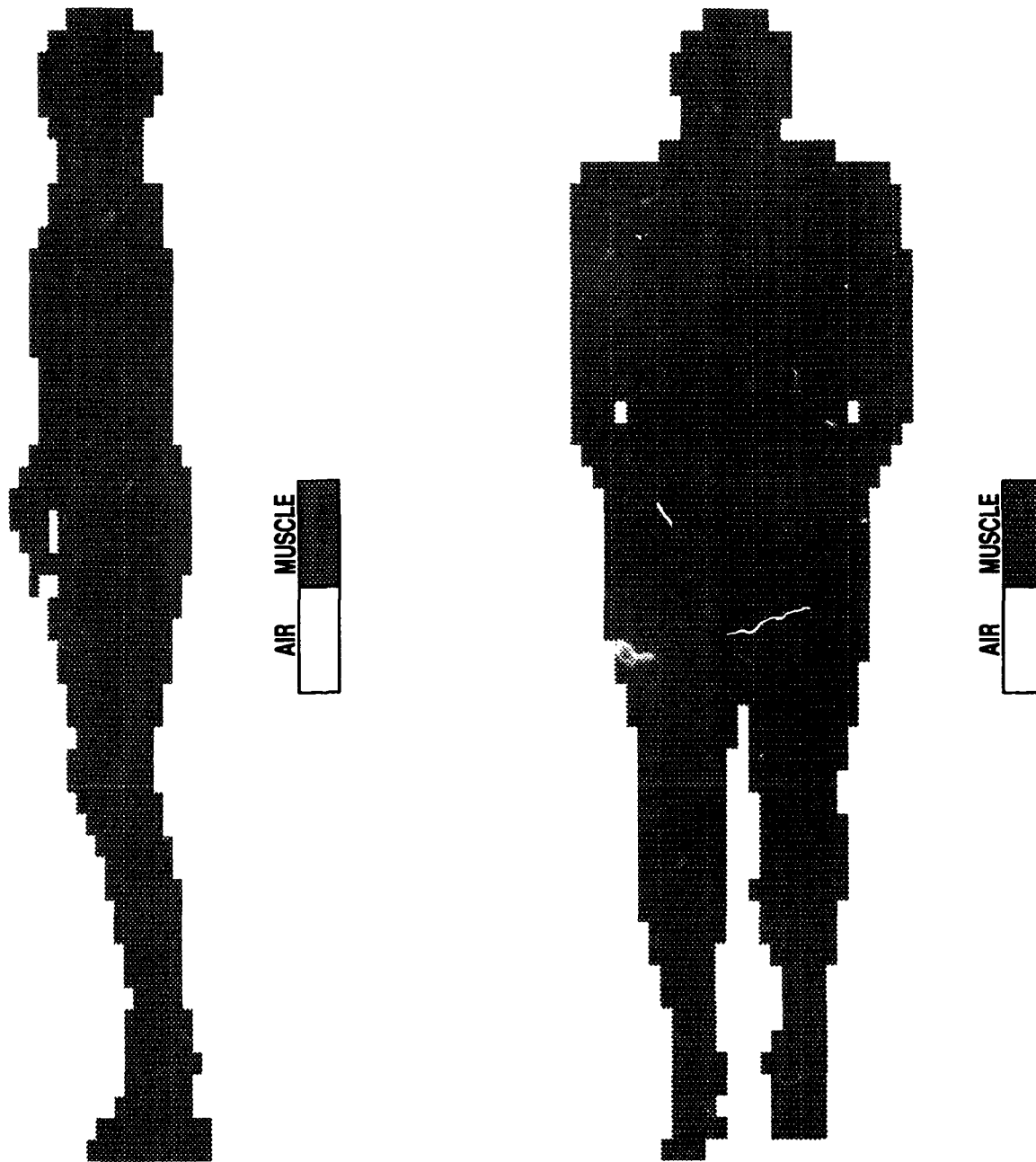


Figure 4: The projection of the Jaap phantom on the plane $x_2 = 0$ (left) and the plane $x_1 = 0$ (right).

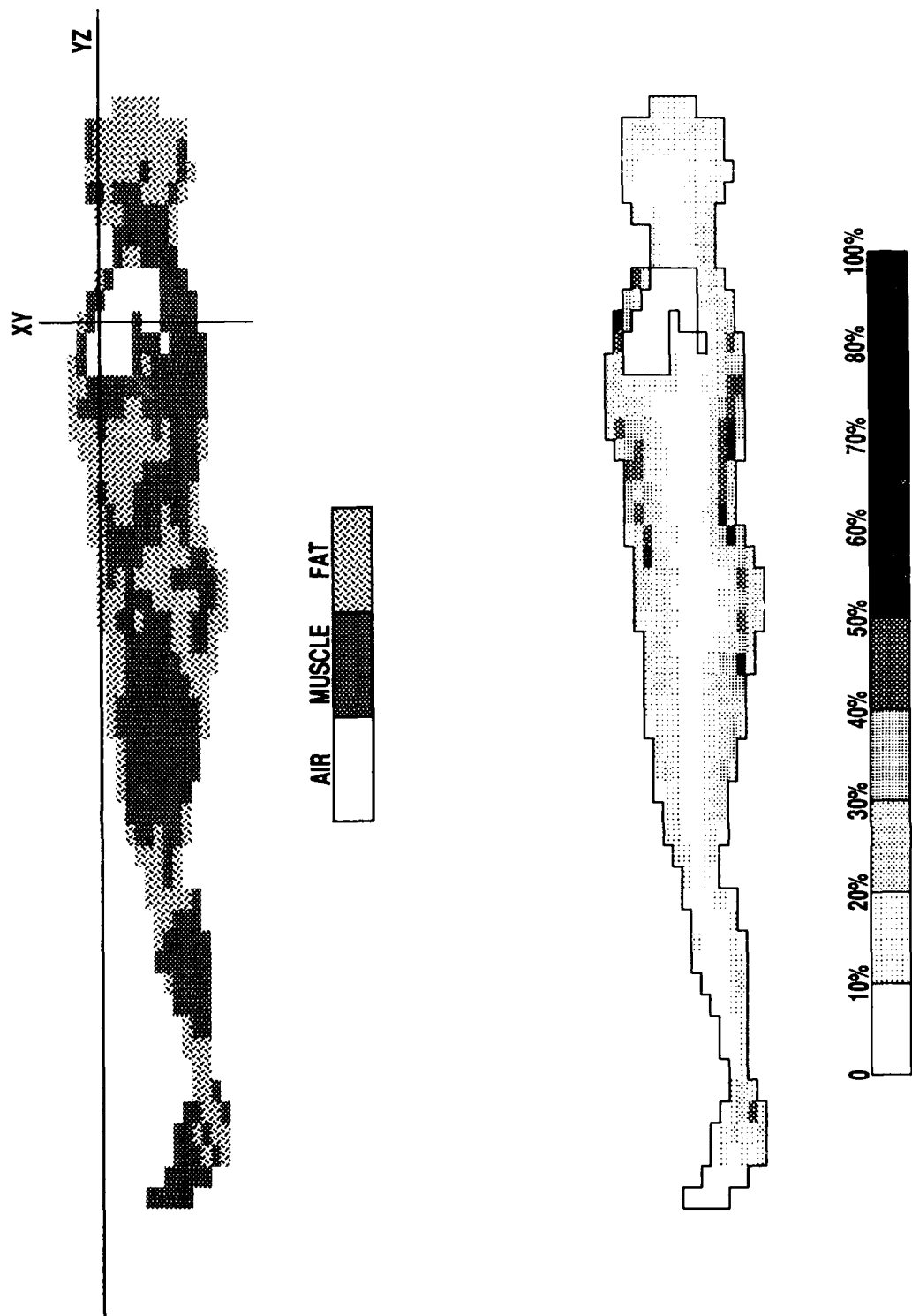


Figure 5: A cross-section of the Jaap phantom (left) and the absorbed power distribution (right) in a sagittal plane XZ.

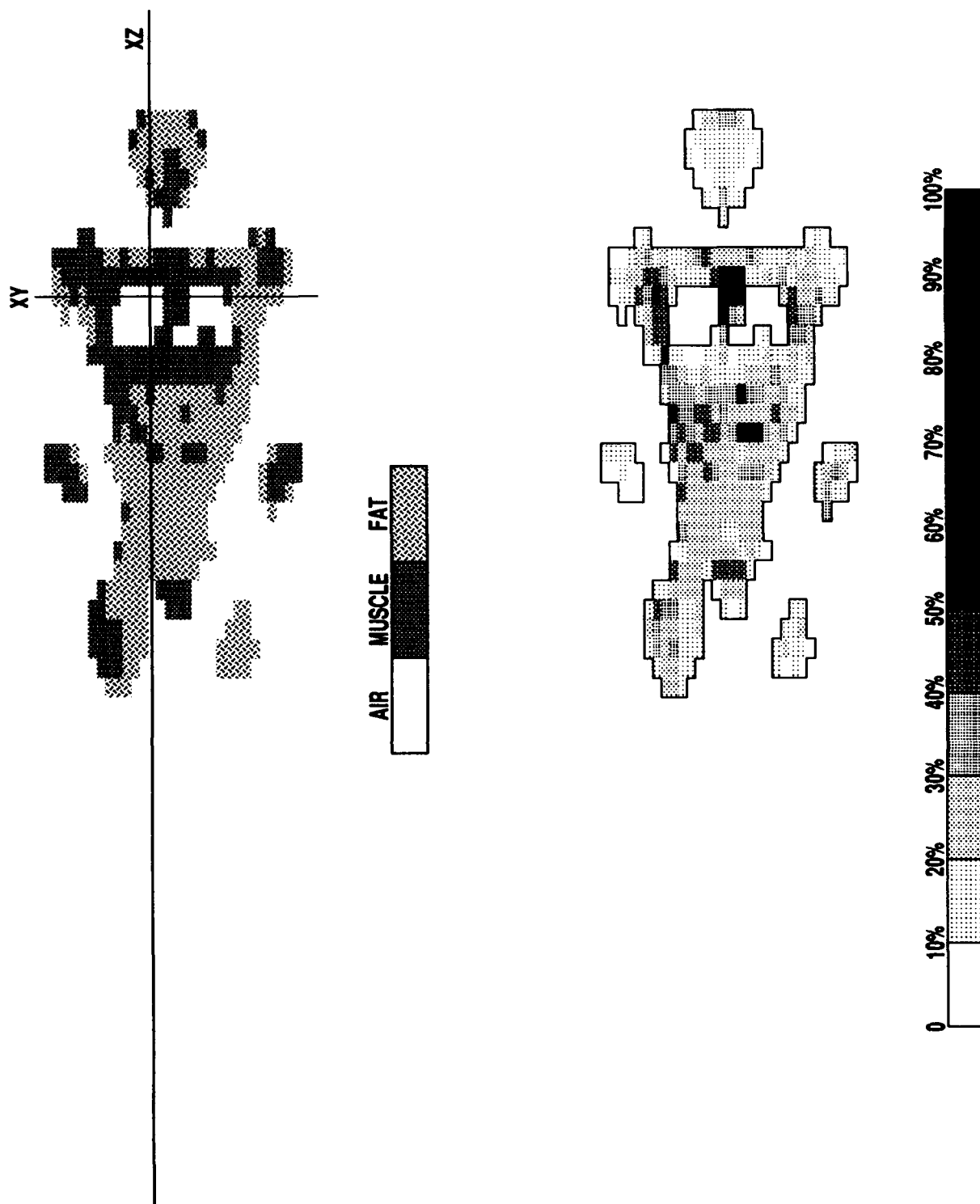


Figure 6: A cross-section of the Jaap phantom (left) and the absorbed power distribution (right) in a coronal plane YZ.

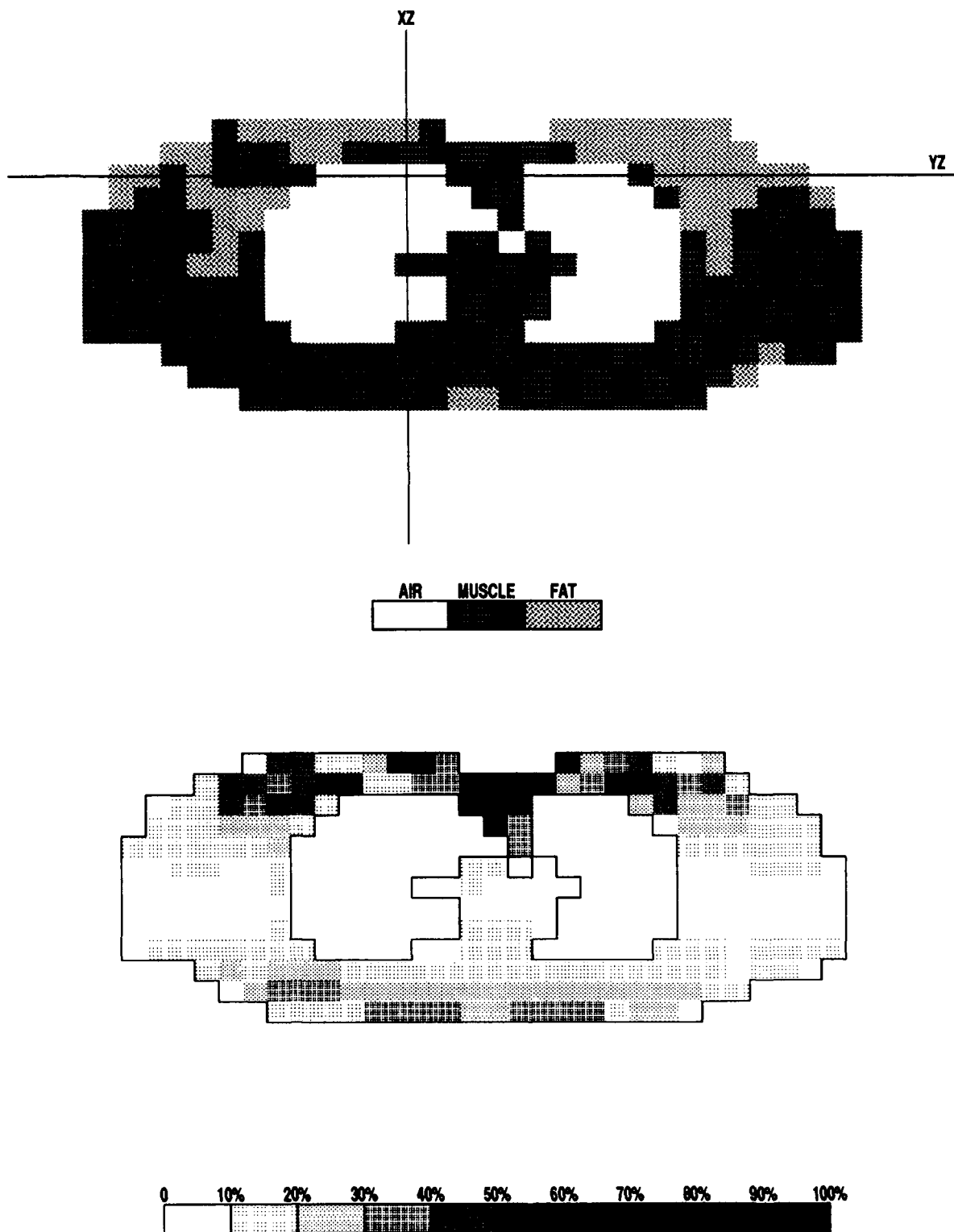


Figure 7: A cross-section of the Jaap phantom (top) and the absorbed power distribution (bottom) in a transverse plane XY.

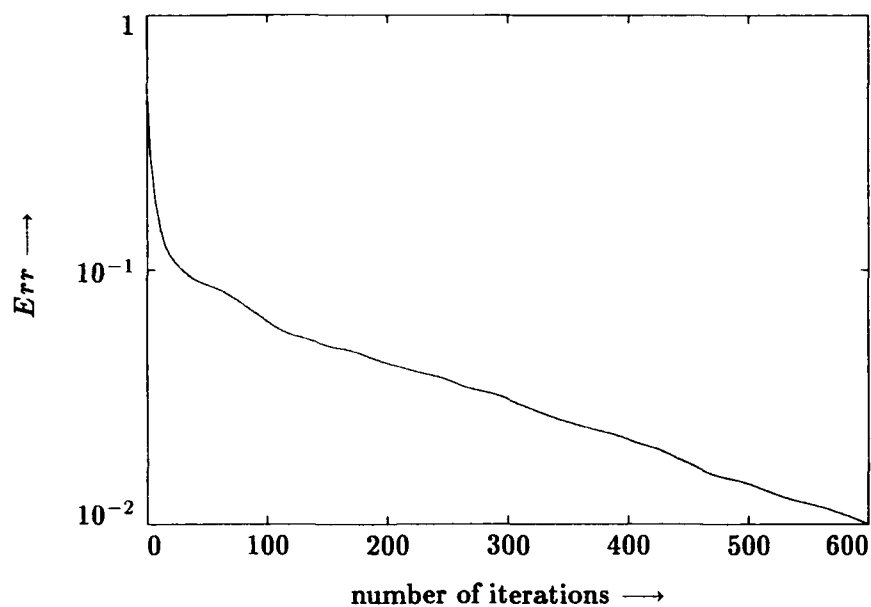


Figure 8: The numerical convergence obtained for the CAT-scan generated model of the human body, the Jaap phantom.

Table 1: Computation time and storage of VAX 3100/76 workstation.

$M_{\mathbf{D}^s} \times N_{\mathbf{D}^s} \times P_{\mathbf{D}^s}$	$M_{\mathbf{DFT}} \times N_{\mathbf{DFT}} \times P_{\mathbf{DFT}}$	number of unknowns	CPU-time of one iteration	computer storage
$15 \times 15 \times 15$	$32 \times 32 \times 32$ (sphere)	10800	0.55 min	5 Mb
$31 \times 31 \times 31$	$64 \times 64 \times 64$ (sphere)	92256	5 min	18 Mb
$21 \times 31 \times 53$	$64 \times 64 \times 128$ (Jaap phantom)	106916	9 min	25 Mb

5. Conclusions

In this paper we have presented a three-dimensional weak formulation of the conjugate-gradient FFT method for strongly inhomogeneous bodies. It is observed that the present weak formulation yields excellent agreement with the analytical results for the radially layered lossy dielectric sphere. Modeling the curved boundaries using a cubical mesh seems to be feasible and the discretization errors tend to vanish for increasingly finer discretizations.

Since we have maintained the simple scalar convolution structure of the vector potential, the computation time of our present weak form is even less than the computation time of the standard conjugate-gradient FFT methods discussed in the Introduction. A realistic model of the human body can now be handled by workstations, the exclusive need of a supercomputer for three-dimensional modeling is not necessary. It is noted that in contrast with the weak formulation of the two-dimensional TE scattering problems presented in [Zwamborn and Van den Berg, 1991a], the present formulation allows the use of different mesh sizes in each Cartesian coordinate. The latter enhances the applicability of the weak formulation to complex, strongly inhomogeneous structures. Finally, it is mentioned that the extension of the present formulation to anisotropic objects is rather straightforward [Zwamborn, 1991b].

6. Acknowledgement

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Multiple Multipole Method Applied to an Exposure Safety Study

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Abstract - A summary of the three-dimensional implementation of the multiple multipole method (3D MMP) is given, followed by discussions on the method's advantages and limitations on the basis of an example study. The advantages are seen in its quantitative validation capability, in its efficiency for smoothly shaped bodies and its achievable accuracy, in particular, near surfaces. However, MMP's greatest strength appears to be with problems with EM sources in the closest vicinity of lossy bodies. This is shown by a condensed presentation of a study investigating the RF exposure safety of UHF and VHF transmitters commonly used by radio and TV crews for onsite live reports. In addition to gaining knowledge about the local SAR distribution, the goal of this study was to assess necessary safety distances for given threshold values. The presented study was performed with the 3D MMP software package on a 80386 based PC laptop extended by a 80860 board with 32 MB RAM.

1 Introduction

In the 1970's and early 1980's, various analytical and numerical approaches were suggested and applied to study the absorption of biological bodies under far field exposures. Far field conditions were studied because of their relevance to setting exposure safety guidelines. The other reason is that the computers and numerical techniques available at that time were hardly suited for near field computations. Exposures to strong far fields are rare, even within occupational groups (i.e., radar workers), while extensive near field exposures are much more frequent (i.e., welders, antenna service workers, etc.). With the recent growth in popularity of mobile communications, more and more of the general population is exposed daily to small transmitters operating in the closest vicinity of the user's head.

Early attempts to study these dosimetric problems were of limited success because of shortcomings in the methodology of the applied numerical techniques, such as IEBCM [Iskander et. al., 1987] and MoM [Stuckly et. al., 1986] for near field exposures. More appropriate are two methods developed and utilized for such applications within the last few year, namely, FDTD [Chen and Gandhi, 1989] and 3D MMP [Kuster and Ballisti, 1989; Kuster and Balzano, 1992]. The latter is the subject of this paper. A comprehensive summary of MMP is given followed by a discussion on the basis of a dosimetric assessment study. More detailed overviews on methodology, code and simulation technique are described in [Hafner, 1990], [Bomholt, 1990], [Kuster, 1993].

2 MMP Method

The most fundamental ideas of MMP were first proposed by Hafner [Hafner, 1980]. However, similar techniques have been developed independently around the world during the last decade. [Ludwig, 1989] summarizes these techniques and adopts the name generalized multipole technique (GMT).

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2.1 Basics of GMT

Generalized multipole technique is the generic term for methods approximating the unknown (scattered) field F in each domain by several (P) sets of functions F_i with different origins located inside or outside the domain but not on the boundaries. Each set consists of a limited number N_i of basis functions f_{ij} , which are analytical solutions of Maxwell's equations for linear and homogeneous materials.

$$F = \sum_{i=1}^P F_i = \sum_{i=1}^P \sum_{j=1}^{N_i} c_{ij} f_{ij} \quad (1)$$

As the name GMT implies, the primary expansion functions are finite series of multipole solutions of the Helmholtz equations (in the following simply referred to as multipoles). The unknown parameters c_{ij} are numerically evaluated by enforcing the boundary conditions on a discrete set of points on the boundary.

Hence, the methods based on GMT are applicable to static or time-harmonic electromagnetic boundary value and eigenvalue problems within piecewise linear and homogeneous domains. The different methods distinguish themselves in the sets of basis functions used and the matching technique applied. Most use several multipoles with different origins for the same boundary adapted to its geometry in order to increase the flexibility and convergence properties of the technique. This approach was not considered initially because of a dependence between analytical, infinite expansions in the same locations. However, finite multipole functions have a very local behavior which can be exploited to achieve sufficient independence between different multipoles. This idea is analogous to the charge simulation method for electrostatics and is further illustrated by the Huyghens principle.

In the following, only the sets of basis functions and the matching technique implemented in the 3D MMP software package are discussed.

2.2 Basis Functions

The most flexible set of basis functions implemented in the 3D MMP software package are *multipoles*, which are obtained from the Helmholtz equation by separation of variables in spherical coordinates

$$\tilde{f}_{ij} = h_n^{(1)}(kr_i) P_n^m(\cos \vartheta_i) \frac{\cos}{\sin}(m\varphi_i) \quad (2)$$

where the $h_n^{(1)}$ are spherical Hankel functions of the first kind ($e^{-i\omega t}$ time dependence) and P_n^m are associated Legendre functions. The vectorial f_{ij} are deduced by simple derivations from the scalar $\tilde{f}_{ij}$ [Bomholt, 1990]. The functions f_{ij} are singular in their origins ($r_i = 0$) and must be placed outside the domain in which the field is expanded. The use of several multipoles of different orders along the same boundary led to the method's name "multiple multipole". Simple geometrical rules have been developed to obtain a sufficient base and while avoiding numerical dependence.

Multipoles are most efficient if a large area of the boundary can be covered by one multipole. For long and thin structures, a large number of multipoles must be set up because the largest possible distance to the boundary is small compared to the length of the structure.

This approach is especially poor for thin wire structures, which are also often involved in bioelectromagnetic applications (e.g., sources such as antennas or implants such as a pace maker). To avoid this rather significant shortcoming, *line multipoles* are implemented [Leuchtmann, 1991]. The electrical zero order term corresponds to the *thin wire approximation* with the segment currents

$$I(z') = I_a \cos kz' + I_b \sin kz' \quad (3)$$

along the z' -direction (k is the complex propagation constant of the surrounding medium). The corresponding field values can be evaluated in simple analytical terms. The fields of higher order multipoles are not axisymmetrical but vary with $\frac{\cos}{\sin}(m\varphi)$. This implementation might also be seen as a combination of NEC with GMT.

In some cases, so called *normal expansions* are very convenient and efficient to use. They are formally equal to (2), but the Hankel functions are replaced by Bessel functions. There is no singularity at the origin of the normal expansions, and they usually do not have a local behavior. Inside lossy media, the values of these functions even grow with distance. Physically they describe a superposition of incoming and outgoing waves. Only one normal expansion should be used for a domain, preferably placed inside the domain. They are especially suited for spherical or rather spherical domains and are often more efficient than multipoles, especially if symmetries are exploited. Furthermore the validation of the result is simplified.

Another important function, called *connection*, is a linear combination of basis functions, where each function has a *known* parameter c_k :

$$f_i = \sum_{k=1}^N c_k f_k \quad (4)$$

Connections may also be nested, i.e., f_k may itself be a connection. This is a very powerful macro feature and the basis of the iterative simulation technique, which considerably extends the capability of the code. In this context, the solution of one step is packed into a connection and used as excitation for the next step, i.e., the basis functions f_{ij} become f_k and the solved parameters c_{ij} become c_k .

Other expansion functions for special purposes may be introduced without any difficulty, e.g., *plane waves*, *waveguide modes* and many others. Typically they are used as excitation.

2.3 Matching Technique and Solution of the Matrix Equation

Besides using more general basis functions, MMP distinguishes itself from the other GMT based methods through its applied matching technique. Instead of the simple point matching, a *generalized point matching* technique (GPM) with many more matching points (and equations) than necessary is used. Moreover, all 6 boundary conditions for a general surface can be enforced instead of only 4 linearly independent ones. The resulting overdetermined system of equations is solved in the least squares (LS) sense. In 2D, a 2 to 3 times overdetermined system is considered to be sufficient [Hafner, 1990], whereas in 3D, the use of 4 to 10 more equations is recommended because of the directionality of the spherical expansions [Kuster, 1993]. With the selected weighting functions – important for LS – GPM becomes numerically equivalent to an error minimization technique or a moment method with Galerkin test functions [Hafner, 1990]. As a consequence, the error is much more smoothly distributed over the surface and the solution is much less sensitive to the location of single matching points. An additional and important advantage is that the discretization is now largely independent of the choice of basis functions, which dramatically simplifies the modeling process.

The price for these advantages is an increased numerical effort because the obtained system of equations

$$A c = b \quad (5)$$

is strongly rectangular. Although strong dependence is avoided by using empirical rules for pole-setting, weighting, etc., the matrix A tends to be ill-conditioned for more complex problems. QR decomposition methods are therefore preferred being less sensitive to the condition number $\kappa_2(A)$

[Gentleman, 1975], [Golup and Loan, 1989] than the more efficient method of normal equation. Among the QR decomposition methods, Givens transformations is selected. Although it requires more operations than comparable algorithms (e.g., Fast Givens or Householder), it has the important advantage that the matrix A can be processed row by row, as it is formed. Therefore, only an upper triangular matrix R (Cholesky factor of A^*A) with $n(n+1)/2$ elements needs to be stored.

The matrix A has additional numerically interesting properties that depend on the problem but are worth being exploited. For most applications, A can be arranged on the basis of physical considerations such that a square block matrix with strongly diagonal dominance is obtained. This matrix is solved through a block iterative technique that is equivalent to Gauss-Seidel block iteration. This technique has important advantages regarding modeling which even surpass the important numerical benefits. A detailed discussion of this technique is given in [Kuster and Bomholt, 1993].

2.4 Symmetries

The geometry of boundary value problems is often symmetric, i.e., invariant to certain symmetry transformations in space. As a result, the boundary value problem can be split up into a number of smaller, symmetry adapted problems which can be solved separately at lesser overall expense.

$$Ac = b \quad \rightarrow \quad A_l c_l = b_l \quad (l = 1, \dots, L) \quad (6)$$

In the 3D MMP code, reflective symmetries on the orthogonal planes $X = 0$, $Y = 0$ and $Z = 0$ can be fully exploited [Bomholt, 1990], i.e., $L_{max} = 8$. Additionally, some of these symmetry parts may be left out *a priori* if the corresponding symmetry part of the excitation disappears, which further reduces expense.

Connections are fully compatible with symmetries, i.e., connections of higher symmetries may be introduced into problems with lower symmetry and connections with lower symmetry into problems with higher symmetries.

2.5 Software and Hardware

The kernel software of the 2D and 3D MMP implementations are written in a very portable FORTRAN and published in [Hafner, 1990b] and [Bomholt, 1990], respectively. These codes run on a wide range of computers such as personal computers from the XT-compatible to the 8048 powered machines, floating point boards based on the 80860 processor, workstations and different mainframes. Additionally, the code was parallelized for transputer (T800) networks or SUN networks.

A software package with which the input files for general problems can be efficiently generated and the solution can be visualized and validated is at least as important as the method and its implementation. For the 3D MMP code, two sophisticated graphic input and output interfaces have been developed for PC's and SUN workstations [Regli, 1989]. The input interface allows for easy surface discretization and helps the user in the choice of multipoles. With the output interface a large variety of plots can be drawn and animated.

All of the simulations described in the following have been performed on a 386 based laptop computer extended by an 80860 board with 32 MByte memory.

3 Example Study

The more sincere way to illustrate and discuss the applicability and limitation of a technique and its implementation is on the basis of a performed study than with any method-matched examples.

Thus, some parts of a dosimetric assessment performed for public broadcasting organizations are presented and discussed in the following. The objective of the study was to clarify the safety questions of the mobile transmitters commonly used for on-site live reports by radio and TV crews. The broadcasters specifically were interested in information regarding the actual local distributions of the specific absorption rate (SAR) and for the evaluation of safety distances, which ensure the compliance with current and proposed SAR safety limits, in particular with the newest revision of the German standard [DIN VDE 0848 Part 2, 1991]. As the discussions on the most appropriate averaging weights still continue, different averaging weights are analyzed.

3.1 Transmitters

The investigated reportage transmitters were conceived for on site reports for radio and television crews. A high quality broadband transmitter with a matched antenna is used to submit reports, while a simple narrow-band receiver using the same antenna is sufficient to receive the producer's instructions. The devices are offered in various configurations which also use different frequency bands.

UHF-Types

The newer devices transmit in the UHF range between 540 and 790 MHz. Thirty two channels are distributed upon a total bandwidth of 185 MHz, which requires a tunable antenna (cf. Figure 1). The thicker part of the antenna corresponds to a center fed $\lambda/2$ dipole (monopole with a mode suppression filter or $\lambda/4$ balun) whose length may be adjusted for the given frequency with the help of an SWR-meter. The VHF signal is received by the same antenna, which is lengthened by the capacitive-coupled extension for this purpose. The antennas can be mounted directly on the device or on a special shoulder belt (cf. Figure 1c). The latter improves the quality of the transmission. However, in this case the antenna is in the closest vicinity of the head and, therefore, to those organs which may be the most sensitive to EMF (the eyes and brain).

First the output impedance of the HF amplifier and the feedpoint antenna depending on its position to the body is measured as both determine the feedpoint current and therefore the SAR values [Kuster and Balzano, 1992]. The equivalent output impedance of the HF amplifier is about $Z = (50 - i15)\Omega$ for both power levels. The structure of the symmetrical UHF-Antenna is relatively complex and difficult to measure. Extensive measurements for various well defined positions are unnecessary because reliable worst case SAR can be assessed with a good estimate on the minimum antenna impedance. The more qualitative measurements were performed with a network analyzer HP8783A calibrated at the location of the coaxial connector of the antenna. This calibration was then corrected by the electrical length between driving point and connector, which was approximately determined in a previous measurement. Using this set-up, measurements of the feed point impedance are performed at different antenna distances from the head of a test person. The lowest feedpoint impedance measured was larger than 40Ω . This finding indicates that the antenna current does not largely depend on the distance between antenna axis and surface of the body.

VHF-Types

Some types of the same device operates in the VHF-range between 220 and 240 MHz with no more than 16 channels of maximum 5 MHz bandwidth and a nominal power of 10 W (switchable to 5 watt). The ($\lambda/4$) monopole antenna, illustrated in Figure 1b, has a fixed length. Its impedance is matched to the respective transmitter. It can be mounted either on the device or just above the shoulder on a special belt. To obtain a better defined counterpoise for the shoulder mounted

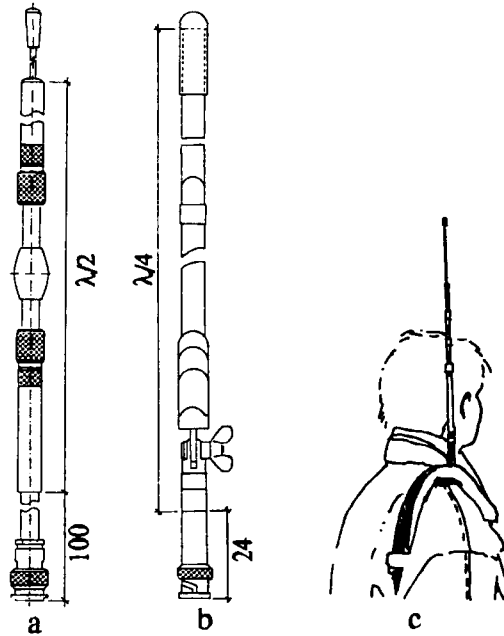


Figure 1: a) UHF dipole antenna; b) VHF monopole antenna; c) the UHF dipole and the upper band VHF monopole antennas may be worn on a special shoulder belt, as illustrated here. Measure in mm.

monopole antenna, a $\lambda/4$ choke for sheath current suppression is integrated into the cable, which feeds the antenna and is mounted on the shoulder belt. Therefore, the choke is almost attached to the surface of the body and results in a strong coupling to the body. In other words, the choke and the body of the user are an integral part of the radiation mechanism, providing the return current to the $\lambda/4$ monopole. This fact must be discussed in greater detail below.

Many different types of the former device family have been produced, which operate between 70 and 230 MHz. Generally, they all have a nominal power of 10 W which can be switched down to 1 W. The antenna is also a monopole antenna with constant length, which is mounted only on the device (cf. Figure 1b).

3.2 Numerical Models

Obviously, a detailed modeling of the human body with its immense complexity is far beyond any scope. In addition, the variations between people are significant on a coarse scale. Detailed simulations of the transmitters that cover all important parameters regarding the absorption are also not without any problems. Therefore, and particular with in respect to safety questions, the simulation of "worst case" situations is the primary goal. The basic idea behind safety standards is to define the limits by factors below the known threshold values (safety factor). Therefore, artificial "worst case" situations are not considered in the following. Instead realistic, extreme situations, which might occur in practice over a longer time period (> 6 min.) are studied.

The following numerical models are as simple as the "worst case" assessment allows. It is known from the interaction mechanism in the near field of dipole antenna [Kuster and Balzano, 1992] that the most important parameters are the frequency and feedpoint current and the distance of the antenna from the tissue. The inhomogeneties of the body are less important than previously anticipated.

Transmitters

The transmitting part of the UHF antenna is a symmetrical, centrally fed antenna with a well defined driving point impedance. A well suited model for this type of antenna is a thick dipole with the appropriate dimensions.

Not as well defined is the impedance of the monopole type of antenna for the VHF device, because the electrical counterpoise consists of either the enclosure of the device or the $\lambda/4$ choke together with the body of the user. A detailed numerical simulation of the transmitter would go beyond the scope of this project. The transmitting antenna was easily simulated with an asymmetric dipole with maximum current at the feeding point. Possible deviations in the absorption rate caused by these simplifications will be discussed and estimated at the end of this paper.

Using line multipoles, the modeling of the transceivers essentially corresponds to that of codes based on the thin wire approximation, i.e., NEC. More accurate models would be possible, but are not required for this assessment.

Human Model

The previously used models of the human body [Hafner and Kuster, 1991] are improved in order to obtain more realistic geometrical conditions for the simulation of the device-mounted or body-mounted antennas. The model is illustrated in Figure 2 and its simulation is described in detail in [Kuster, 1993]. Although the model is divided into several layers for computational reasons, it is homogeneously characterized using the electrical parameters of human muscle tissue. This is necessary as muscle tissue is the most highly conductive tissue and therefore most appropriate for assessing worst case SAR values. The parameters used for the head correspond to either brain-like or muscle-like tissues. Such significant simplifications of the human body are permitted because the body's inhomogeneities do not significantly modify absorption in this frequency range [Kuster and Balzano, 1992]. In order to underscore this argument, two further simulations are performed using the same human shape but additionally modeling either the skull or the kidneys (Figure 3). The values for the electrical parameters are obtained by interpolating the values given by [Pethig, 1984]. The mass density is in the following discussion assumed as 1 g/cm^3 for reason of simplicity.

3.3 Validation of the Solutions

Special care must be taken regarding the validation of the solutions because the quality of the assessment entirely depends on the quantitative accuracy. Two checks are performed.

First, the results are validated by applying the procedure for solution validation [Kuster, 1993], which was developed for MMP and the tools of which are integrated into the 3D MMP software package. These internal check routines have proven to be highly reliable, at least for simply shaped bodies, such that experimental control measurements can be avoided for many applications.

Second, the physical plausibility of all solutions are checked with the energy absorption mechanism extracted on the basis of experimental measurements and numerical computations [Kuster and Balzano, 1992]. The spatial local peak SAR values are compared with those obtained with the approximation formula derived in the same paper.

3.4 Results

The results of the simulations are illustrated in the following Figures 4 - 10 and summarized in Tables 1 - 4. It is important to note that all values are normalized to a feeding point current of 141 mA (1 W on 50 Ω) and 447 mA (10 W on 50 Ω), respectively. The figures show the SAR

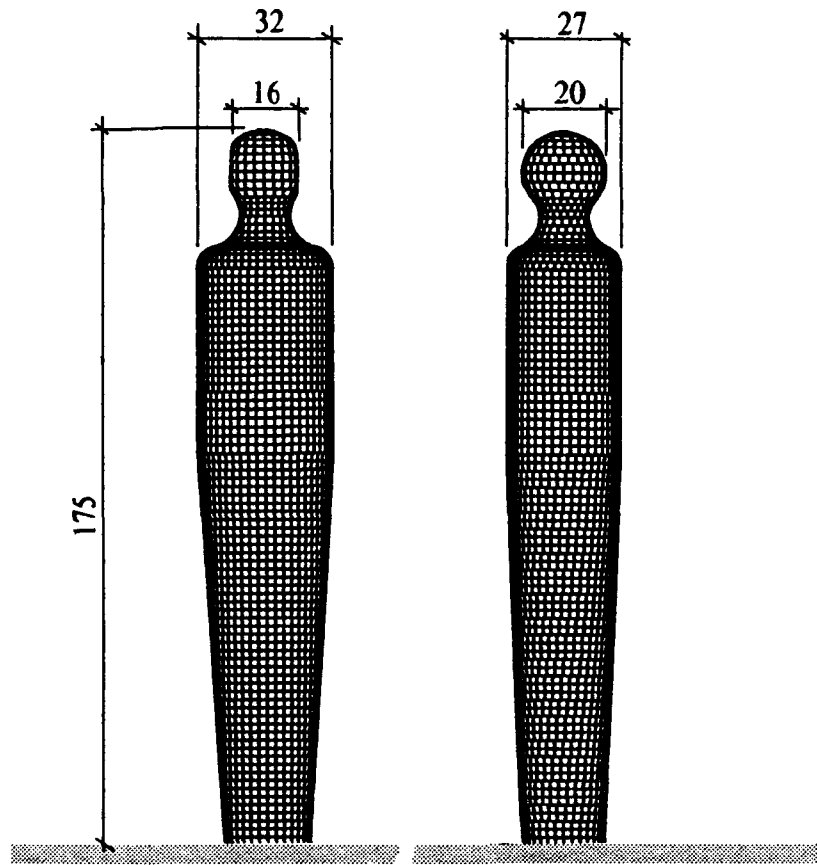


Figure 2: Longitudinal and cross section of the human model used (measured in cm).

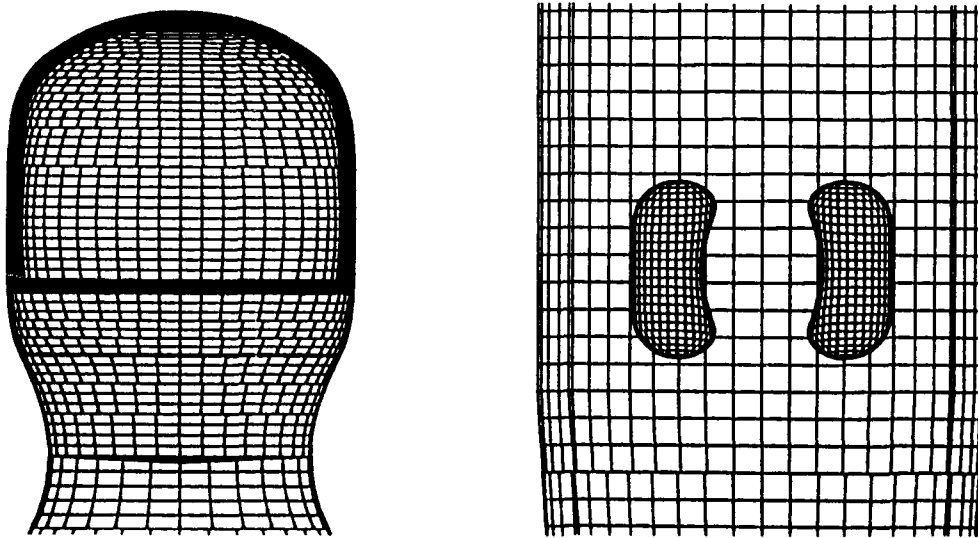


Figure 3: Refined human model (left) with skull of 5 mm thickness or (right) with kidneys ($h = 10$ cm).

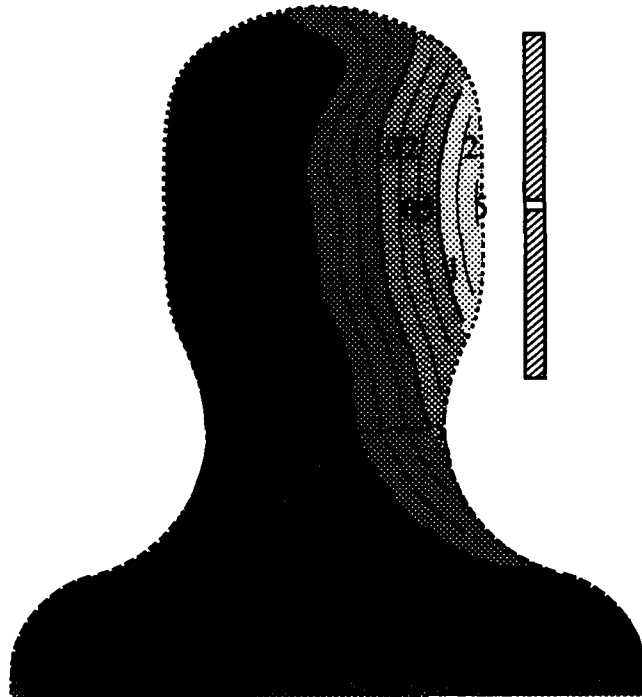


Figure 4: SAR [mW/cm^3] distribution in the longitudinal section. Simulation of the body mounted UHF antenna ($f = 750 \text{ MHz}$) at the distance of 2.5 cm from the surface of the head. The parameters of the simulation are: $I_{fp} = 141 \text{ mA}$; head and body: $\epsilon_r = 55$, $\sigma = 1.4 \text{ mho/m}$ (which correspond to muscle tissue).

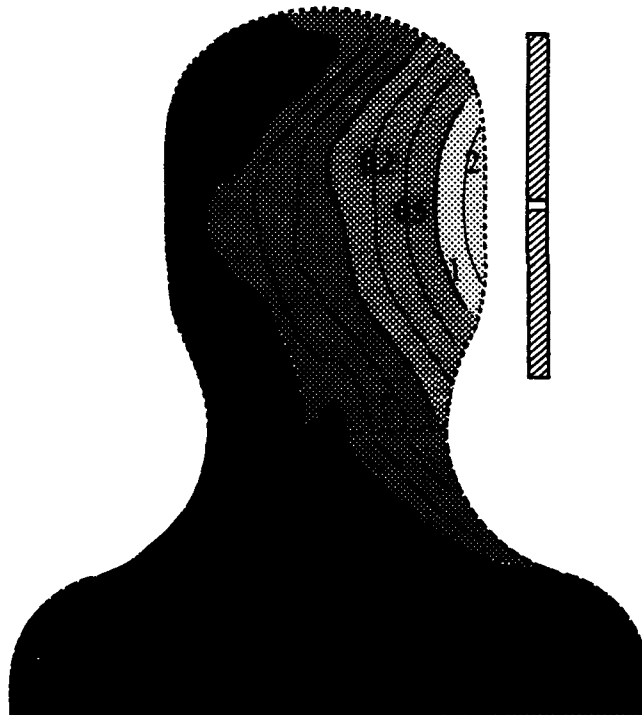


Figure 5: SAR [mW/cm^3] distribution in the longitudinal section. Simulation of the body mounted UHF antenna ($f = 750 \text{ MHz}$) at the distance of 2.5 cm from the surface of the head. The parameters of the simulation are: $I_{fp} = 141 \text{ mA}$; head: $\epsilon_r = 45$, $\sigma = 0.75 \text{ mho/m}$ (which correspond to brain tissue); body: $\epsilon_r = 55$, $\sigma = 1.4 \text{ mho/m}$ (muscle tissue).

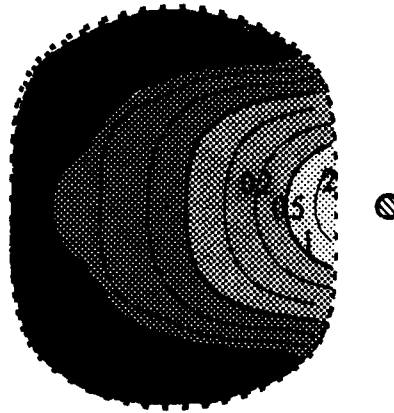


Figure 6: SAR [mW/cm^3] distribution in the cross section at the level of the antenna feedpoint. Simulation of the body mounted UHF antenna ($f = 750 \text{ MHz}$) at a distance of 2.5 cm from the surface of the head. The parameters of the simulation are: $I_{fp} = 141 \text{ mA}$; head: $\epsilon_r = 45$, $\sigma = 0.75 \text{ mho/m}$ (brain tissue); body: $\epsilon_r = 55$, $\sigma = 1.4 \text{ mho/m}$ (muscle tissue).

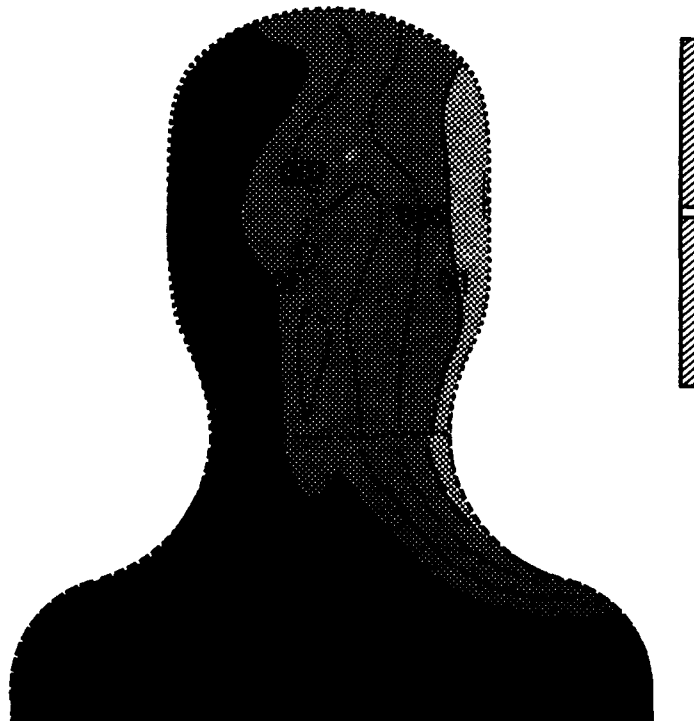


Figure 7: SAR [mW/cm^3] distribution in the longitudinal section. Simulation of the body mounted UHF antenna ($f = 750 \text{ MHz}$) at a distance of 10 cm from the surface of the head. The parameters of the simulation are: $I_{fp} = 141 \text{ mA}$; head: $\epsilon_r = 45$, $\sigma = 0.75 \text{ mho/m}$ (brain tissue); body: $\epsilon_r = 55$, $\sigma = 1.4 \text{ mho/m}$ (muscle tissue).

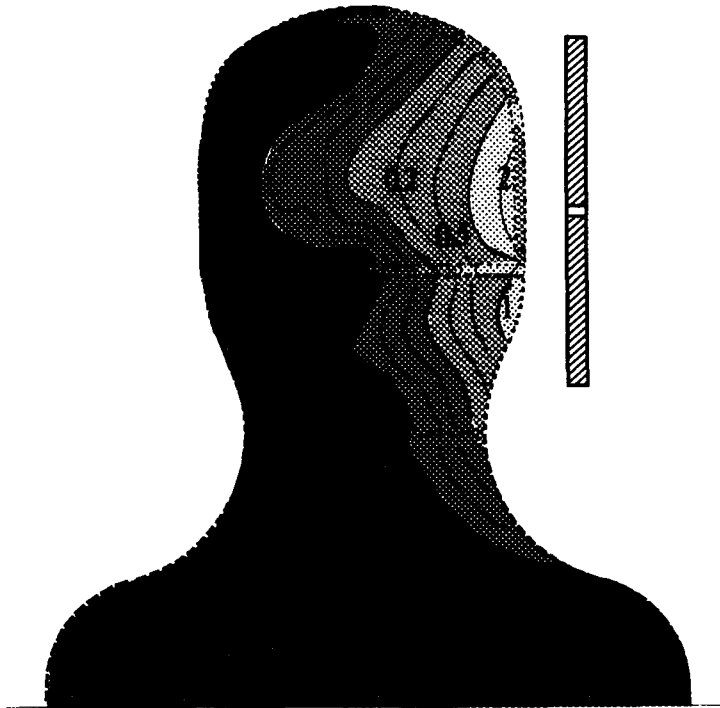


Figure 8: SAR [mW/cm^3] distribution in the longitudinal section of the refined model including skull bones. Simulation of the body mounted UHF antenna ($f = 750 \text{ MHz}$) at a distance of 2.5 cm from the surface of the head. The parameters of the simulation are: $I_{fp} = 141 \text{ mA}$; brain: $\epsilon_r = 45$, $\sigma = 0.75 \text{ mho/m}$ (brain tissue); skull: $\epsilon_r = 5$, $\sigma = 0.15 \text{ mho/m}$ (correspond to bone tissue); neck and rest of the body: $\epsilon_r = 55$, $\sigma = 1.4 \text{ mho/m}$ (muscle tissue).

distribution for different human models and transmitter types. As this does not greatly change with the distance d between the longitudinal axis of the antenna and the body surface, only a few representative examples are shown.

The solutions verify the expected relation [Kuster and Balzano, 1992] between the spatial distribution of the absorbed power and the spatial magnetic field distribution or the current distribution on the antenna surface, respectively. When comparing the solution of the refined model (Figure 8) with that of the homogeneous one (Figure 5), it is, at first glance, surprising that higher SAR values are found in the lower conducting skull base than in the nearby brain or muscle tissue. This is explained by the fact that the main component of the induced electric field is normal to the surface of the brain, bone and muscle tissue, which have strongly different dielectric properties. The peak local SAR values ($2.2 \text{ mW}/\text{cm}^3$, and $15 \text{ mW}/10 \text{ cm}^3$, respectively) are lower than that of the homogeneous head (Figure 5) not because of any screening effect of the skull but because of the greater distance ($d + \text{thickness of the skull}$) between the more conductive and absorbent brain tissue and the antenna axis. Figure 8 and Figure 10 in particular clearly show that inhomogeneity modifies the absorption distribution only locally and does not lead to significant changes. From this, it follows that models of higher complexity should only be used to study specific effects in detail, e.g., the absorption in the vitreous body of the eye or in its crystalline lens. Otherwise the validation of the solution becomes extremely tedious or even impossible.

The calculation of the numerical value of the peak local SAR averaged over a given volume is carried out as follows. First the point of highest absorption is determined by a scanning procedure. The averaged SAR values around this point are then computed in cubes of volume at 1 cm^3 , 10 cm^3 and 100 cm^3 by numerical integrations. Highest resolution is given because the fields inside the

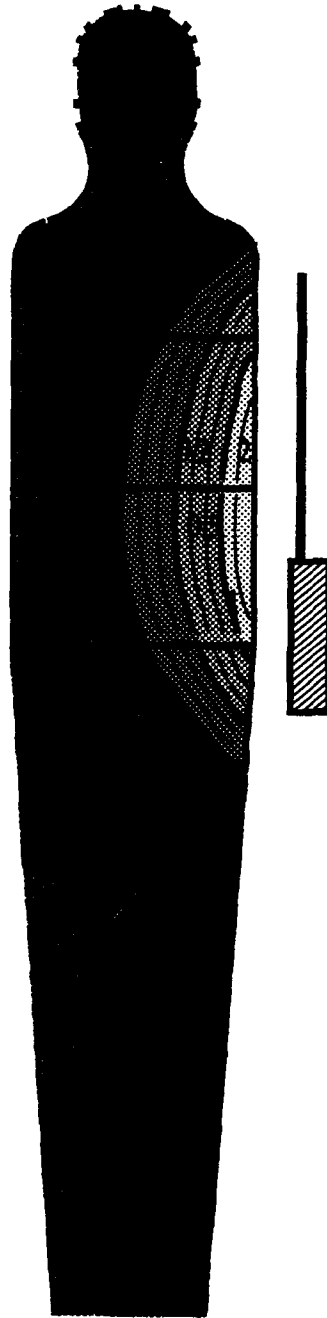


Figure 9: SAR [mW/cm^3] distribution in the longitudinal section. Simulation of VHF version of the newer devices ($f = 200 \text{ MHz}$) with a monopole antenna mounted on the device at a distance of 5 cm from the surface of the body. The parameters of the simulation are: $I_{fp} = 447 \text{ mA}$; head: $\epsilon_r = 70$, $\sigma = 0.45 \text{ mho/m}$ (which correspond to brain tissue); body: $\epsilon_r = 75$, $\sigma = 0.95 \text{ mho/m}$ (muscle tissue).

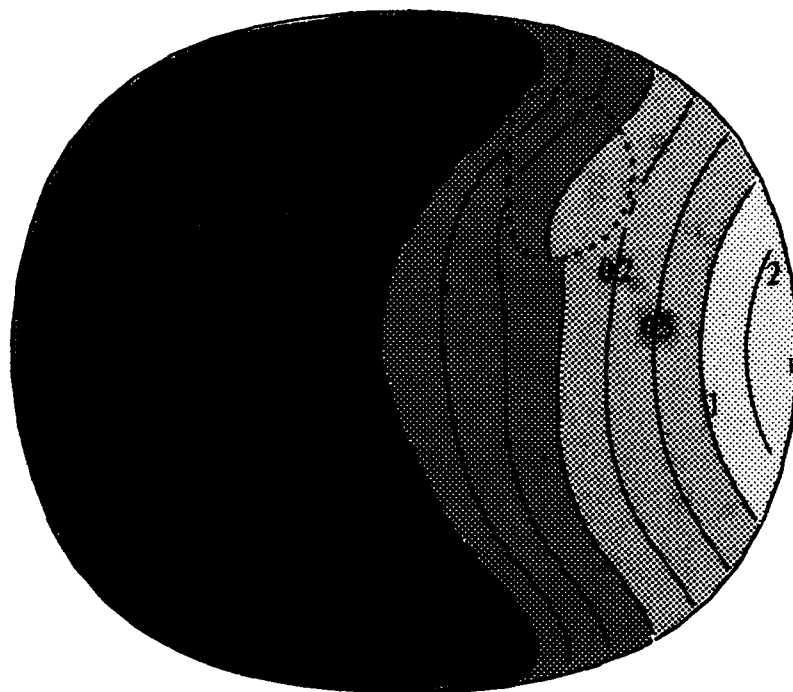


Figure 10: SAR [mW/cm^3] distribution in the cross section at the level of the antenna feedpoint. Simulation of VHF version of the newer devices ($f = 200 \text{ MHz}$) with a monopole antenna mounted on the device at a distance of 5 cm from the surface of the body. The parameters of the simulation are: $I_{fp} = 447 \text{ mA}$; head: $\epsilon_r = 120$, $\sigma = 0.4 \text{ mho/m}$ (which correspond to brain tissue); body: $\epsilon_r = 100$, $\sigma = 0.8 \text{ mho/m}$ (muscle tissue); kidneys: $\epsilon_r = 180$, $\sigma = 1.0 \text{ mho/m}$ (kidney tissue).

tissue are Maxwell fields, i.e., the volumes do not need to be discretized into a large number of cells in which the fields are approximated by linear functions.

The values in Table 1 and 2 are calculated by a series of simulations varying the distance d with the given model (Figure 2). For the computation of the local SARs for Table 3 and 4, an even more simplified model can be applied. In this case, the solution validation process, which is performed according to [Kuster, 1993], shows that the spatial local peak SAR values varies only within 1dB if a cylindrical model (length = 175 cm, diameter = 12 cm) and a symmetrical dipole are used instead of the initial models (Figures 9 and 10). This significantly reduced the required computer time without altering the reliability of the tables. It is important to note that the degree of E-field coupling depends on the resonance properties of the body in the case of greater distances. Therefore, the absorption strongly depends on the body, especially for larger distances. In order to obtain worst case estimates, the length of the cylinder used is close to resonance.

Only the values for the shoulder-mounted VHF monopole antenna must be estimated (Table 3). This is due to the fact that, although the radiating monopole is a few cm from the body surface, the electrical counterpoise combined with feeding line ($\lambda/4$ choke) is practically attached to the shoulder. A simulation of this situation would require a completely different modeling. In addition, the factors which govern the direct coupling to the body would have to be determined. All this would be extremely time consuming and is not performed because reliable estimates can be obtained with the given models. The values are extrapolated using the results when $d = 1.25 \text{ cm}$ and $d = 0.65 \text{ cm}$ and when the antenna is actually inside the muscle tissue. The exact SAR distribution cannot be given. However, it is obvious that the maximum values lie in the shoulder region.

If the values of Tables 1 to 4 are compared with the values obtained by the approximation

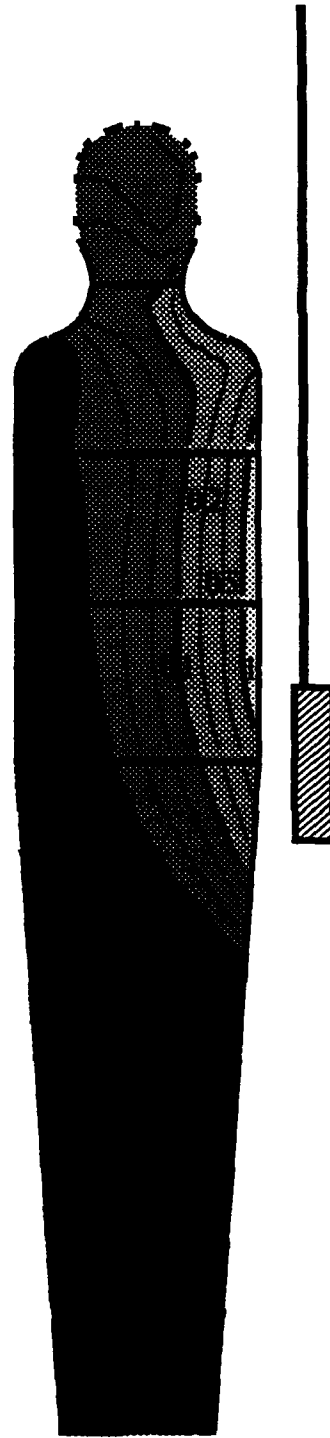


Figure 11: SAR [mW/cm^3] distribution in the longitudinal section. Simulation of an older devices ($f = 80 \text{ MHz}$) with a monopole antenna mounted on the device at 5 cm distance from the body surface. The parameters of the simulation are: $I_{fp} = 447 \text{ mA}$; head: $\epsilon_r = 120$, $\sigma = 0.4 \text{ mho/m}$ (which correspond to brain tissue); body: $\epsilon_r = 100$, $\sigma = 0.8 \text{ mho/m}$ (muscle tissue).

f [MHz]	d [cm]	SAR _p [mW/cm ³]	SAR _{av1cm³} [mW/cm ³]	SAR _{av10cm³} [mW/cm ³]	SAR _{av100cm³} [mW/cm ³]
750	2.5	6.0	4.0	2.5	1.0
750	10.0	0.36	0.26	0.17	0.09
550	2.5	4.3	2.9	1.9	0.90
550	10.0	0.27	0.20	0.14	0.08

Table 1: Spatial local peak SAR averaged over different volumes with the body mounted UHF antenna (shoulder-belt) ($I_{fp} = 141$ mA). Both body and head are simulated with the electrical properties of muscle tissue. These values can be not only considered as a worst case for the head, but also as a good approximation for the case if the antenna is mounted on the device enclosure. This is because the different curvature between head and body do not significantly modify the peak SAR values for this wavelength.

f [MHz]	d [cm]	SAR _p [mW/cm ³]	SAR _{av1cm³} [mW/cm ³]	SAR _{av10cm³} [mW/cm ³]	SAR _{av100cm³} [mW/cm ³]
750	2.5	4.1	3.0	2.1	1.1
750	5.0	1.1	0.85	0.64	0.37
750	10.0	0.25	0.20	0.15	0.10
550	2.5	2.9	2.2	1.6	0.85
550	10.0	0.19	0.15	0.12	0.09

Table 2: Spatial local peak SAR values averaged over different volumes for the UHF transmitters with the body mounted antenna ($I_{fp} = 141$ mA). Both body and head are simulated with the electrical properties of muscle and brain tissue, respectively.

formula (assuming an attenuation given by the skin effect), then the approximation values are lower than the computed ones for the VHF devices. The difference increases with larger distances d due to the resonance coupling mentioned above.

3.5 Assessment

Safety distances can be assessed using the summarized solutions above and the approximation given in [Kuster and Balzano, 1992]. The new draft of the DIN VDE norm [DIN VDE 0848 Part 2, 1991] considers 10 W/kg (Environment 1) as the basic limiting value for spatial local peak SAR values, whereby the average must be taken for each 10 g of muscle tissue and at each 6 minute intervals. For the extremities, such as the hand, wrist, foot and ankle, 20 W/kg are tolerated. Within Environment 2, the values are reduced by a factor of 5, i.e., 2 W/kg and 4 W/kg, respectively. The

f [MHz]	d [cm]	SAR _p [mW/cm ³]	SAR _{av1cm³} [mW/cm ³]	SAR _{av10cm³} [mW/cm ³]	SAR _{av100cm³} [mW/cm ³]
200	body mounted	7*	5*	3*	1.5*
200	2.5	1.5	1.2	0.95	0.56
200	5.0	0.70	0.58	0.47	0.30
200	10.0	0.27	0.23	0.19	0.13
200	20.0	0.08	0.07	0.06	0.04

Table 3: Spatial local peak SAR values averaged over different volumes for the VHF version of the newer devices with the monopole antenna mounted on the device enclosure ($I_{fp} = 141$ mA). The values for the body mounted antenna (shoulder belt) were approximated from different simulations (*).

f [MHz]	d [cm]	SAR _p [mW/cm ³]	SAR _{av1cm³} [mW/cm ³]	SAR _{av10cm³} [mW/cm ³]	SAR _{av100cm³} [mW/cm ³]
80	2.5	0.41	0.35	0.29	0.19
80	5.0	0.23	0.20	0.17	0.12
80	10.0	0.11	0.10	0.09	0.07
80	20.0	0.05	0.05	0.04	0.03

Table 4: Spatial local peak SAR values averaged over different volumes for the older VHF transmitters with the monopole antenna mounted on the device enclosure ($I_{fp} = 141$ mA).

major differences in the safety limits defined by IEEE [IEEE, 1992] and the IRPA [IRPA, 1988] are in the tissue mass upon which the average is based (1 g and 100 g, respectively). As discussions on the most appropriate tissue mass have not yet come to an end, the safety distances for the three volumes 1 cm³, 10 cm³, and 100 cm³ are calculated.

These distances are summarized in Tables 5 and 6 for both Environments 1 and 2 (10 W/kg and 2 W/kg). By applying the DIN VDE norm, the limit of Environment 1 must be applied for the reportage transmitter since the exposure is brief (less than 6 hours a day). According to this, a distance of only 2.5 cm for all devices would be sufficient, assuming a 50 Ω feedpoint impedance. This distance must be increased to 3.5 cm if the limits are based on 1 cm³. If we require that the values for 2 W/kg (Environment 2) must be respected, the necessary distance becomes 10 cm. For the body-mounted (shoulder belt) VHF antenna, a safety distance cannot be given because the safety limits are already violated independently of the distance to the head by the strong direct coupling to the shoulder.

Finally, possible deviations from the values given in this report to those for real devices and human beings must be discussed.

The used dipole model is highly suited to simulate the UHF transmitting antenna. For this reason, it is believed that no coarse violation of the limits should occur for distances greater than the safety distances given here.

The situation is less defined for the VHF transmitter. In these cases, because the enclosure represents the electrical counterpoise for the monopole antenna, the impedance, as well as the HF current distribution on the enclosure in function of the distance, is not accurately known. However, reliable information would be important because the induced power is proportional to the square of the magnetic field at the body surface [Kuster and Balzano, 1992]. These uncertainties can be taken into account by a safetyfactor of 2, i.e., by 1.4 times larger distances, making the limits now highly reliable. The only exception is the VHF antenna, as it must be assumed, that the limits are violated in the shoulder region in any case.

In conclusion, the investigated transmitters are in compliance with the limits for Environment 2 of DIN VDE 0848 Part 2, if a minimum distance of 4 cm from the body is respected. Only the use of the body-mounted VHF monopole antenna cannot be recommended.

4 Conclusion

The MMP codes are applicable to static or time-harmonic electromagnetic boundary value and eigenvalue problems with piecewise linear and homogeneous domains. Consequently, inhomogeneities can only be simulated by several homogeneous subdomains for each of which appropriated basis functions must be set up. At first sight, this appears to be a serious drawback to this approach as compared to FDTD. However, in most cases the inhomogeneities of biological bodies modify the

f [MHz]	P [W]	ant-mount method	d(1cm ³ av) [cm]	d(10cm ³ av) [cm]	d(100cm ³ av) [cm]
550-750	1	B,D	1.5	1.5	1
550-750	3	B,D	3	2	1.5
200	10	D	3.5	2.5	2
80	10	D	1.5	1.5	1

Table 5: Safety distances up to those of the limits for 10 W/kg is reached, which are only valid for the methods of antenna mounting (ant-mount) given in the Table (B = body mounted: the antenna is mounted on the shoulder belt; D = mounted on the device enclosure). These values assume a feedpoint impedance of 50 Ω .

f [MHz]	P [W]	ant-mount method	d(1cm ³ av) [cm]	d(10cm ³ av) [cm]	d(100cm ³ av) [cm]
550-750	1	B,D	4	3.5	2
550-750	3	B,D	7	6	3.5
200	10	D	11	10	7
80	10	D	4	3.5	3

Table 6: Safety distances according to Table 5 for the safety limits of 2 W/kg.

field distribution only locally beyond the uncertainties of the models. In these cases, the modeling of only the local inhomogeneities or disturbances, which are of interest, might be preferable because the simulations are more easily controlled and validated.

On the other hand, MMP has important advantages: 1) the modeling effort is reduced by one dimension because the basis functions are analytical solutions of the Maxwell equations and only the boundary conditions are numerically approximated, i.e., only the boundary must be discretized (also in the case of lossy materials). This allows fast and convenient creation and modification of the geometry with a graphical interface. 2) The modeling of wire objects is done as easily as with MoM codes such as NEC. 3) Multipoles are excellent expansion functions to match local disturbances, i.e., locations where high gradient fields occur. 4) The validation effort is also reduced by one dimension due to the same reasons as the modeling. This empowers reliable validation of the quantitative results on a numerical basis. 5) Any fields and integrals are easily computed because the solution corresponds to a Maxwell field in any point.

Because of the method's flexibility and efficiency in modeling and solving geometrically simple problems, the MMP codes are successfully employed in EMF education. However, they are also best suited for a wide range of scientific and commercial applications, in particular, for biomedical computations.

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APPLICATION OF NUMERICAL MODELING TECHNIQUES IN ELECTROMAGNETIC HYPERTHERMIA

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Abstract

Electromagnetic hyperthermia has been demonstrated to be a safe and useful adjuvant to ionizing radiation in the treatment of malignant tumors. However, applicators and systems for delivering the optimum treatment prescribed by the physicians are far from being available at present. Computer modeling can play a significant role in the design of better heating equipment and in improving the quality of the hyperthermia treatments currently being administered. There is an active ongoing research to develop suitable calculational models using a variety of numerical techniques. But several gaps exist in the current knowledge regarding the validity of these numerical simulations in the clinical context. The development of treatment planning systems similar to those used for radiation therapy requires resolution of these issues. Of the different numerical modeling approaches currently being developed, the finite-difference time-domain (FD-TD) technique has been extensively applied to calculate specific absorption rate (SAR) patterns in complex 3-D heterogeneous biological objects primarily because it is accurate and has a small computer burden relative to frequency-domain integral equation and finite-element techniques. Following a brief review of the historical development of numerical modeling of electromagnetic interaction with biological structures in the hyperthermia context, examples of recent calculations using FD-TD technique in realistic situations in electromagnetic hyperthermia are provided. It has been observed from 2-D calculations, that the water bolus, routinely used in the clinic to provide energy coupling and surface(skin) temperature control, and the inhomogeneous tissue structures significantly modify the SAR patterns compared to patterns computed in planar and homogeneous structures. In conclusion, future areas of work are identified and discussed.

Introduction

Computation of electromagnetic (EM) field interaction with biological objects have been extensively carried out over the past three decades primarily to quantify specific absorption rates (SAR) from a health hazard point of view [Durney, 1980; Spiegel, 1984]. In recent years the application of these numerical techniques in electromagnetic hyperthermia has become an active area of research [Strohbehn and Roemer, 1984]. Based on a number of clinical studies it has been established that hyperthermia, i.e. heating of tumors to temperatures greater than 42°C, can produce increased cell killing and act as an efficient adjuvant to ionizing radiation therapy in the treatment of localized superficial malignancies [Arcangeli et al, 1988; Overgaard, 1989].

At present, heating techniques using EM energy are commonly employed in the clinic to produce therapeutic temperatures in the tumor [Hand and James (eds), 1986]. The ideal objective is to heat all tumor tissues to a specified temperature without overheating surrounding normal tissues. In virtually all the clinical situations encountered, it is extremely difficult to know that adequate coverage has been achieved. A number of factors influence the EM power deposition patterns from commonly used applicators. Significant among these are the effect of relative applicator positioning with respect to the defined treatment volume, and the means of coupling the energy from the commonly used flat or planar aperture applicators to the irregularly contoured patient surface. Thin-walled plastic bags containing stationary or circulating distilled water are routinely

used in the clinic as bolus between the applicator and the treatment surface for improving the coupling efficiency and to provide surface cooling where required. In most instances, due to the curved geometry of the body contour, the bolus bag tends to be distorted resulting in asymmetrical loading of the applicator. Also, the integral type bolus used with some commercially available applicators can produce a collimation effect that alters the power deposition patterns. It is virtually impossible to intuitively visualize or measure the effects of these factors on the power deposition patterns. To obtain experimental 3-D power deposition patterns for all the situations encountered in the clinic is a time-consuming and difficult task.

Alternatively, computer modeling of hyperthermia treatments to predict SAR distributions would be extremely useful for pretreatment evaluation of suitable applicator set-up that will provide adequate coverage of the treatment volume. Such computerized simulation of hyperthermia treatments would be especially valuable in deep tumor treatments where the absorbed power patterns are further modified by complicated internal tissue structures [Wust et al, 1991]. For example, Sathiaselan et al [1986] used a 2-D numerical method of moments (MoM) model to investigate the effect of phase steering for an annular phased array (APA) applicator system. The results of this numerical study were then used in the clinic to devise phase steering techniques for the APA to change the deposited power patterns to improve the quality of treatments and reduce toxicity [Howard et al, 1986]. Also, numerical modeling studies would enable extensive parametric studies to be performed quickly and inexpensively so that sensitive and insensitive parameters can be identified. These studies will also help to evaluate the multi-applicator array systems that are being developed to provide better coverage. Another useful area is in the design of applicators. Numerical modeling can be used as a computer-aided design tool in testing different designs quickly thus saving time [Shaw et al, 1991; Iskander and Tumei, 1989]. However, there is a considerable need for research in this area and only recently software tools have become available for determining three-dimensional power deposition and temperature patterns for realistic patient anatomies. The current state of research in this area have been excellently reviewed and discussed in a recent publication on *Thermal Dosimetry and Treatment planning* [Gautherie (ed), 1990].

Detailed EM power deposition calculations have been performed so far using 2-D models and attempts are currently being made to develop detailed 3-D heterogeneous models. Although 2-D models have been useful, they imply "cylindrical type" structures when extrapolated to the real 3-D situation, and thus do not accurately model the biological system. However, the experimentally demonstrated effect of variations in anatomical configurations [Turner, 1984] suggests that 3-D numerical methods which take into account the overall interaction of the body with EM fields are required to accurately predict EM power deposition. Following a brief review of the historical development of numerical modeling of electromagnetic interaction with biological structures, examples of recent calculations using FD-TD technique in realistic clinical situations will be provided. Future area of work will be discussed in conclusion.

EM Modeling Background

In the 1970's and early 1980's, detailed predictive models of EM wave absorption by biological tissue structures were based largely on Harrington's frequency-domain method of moments (MoM) [Harrington, 1968]. Here, an integral or integro-differential equation is derived from Maxwell's equations either by enforcing the continuity of tangential electric/magnetic fields across dielectric media boundaries [Poggio and Miller, 1979], or by applying the concept of polarization currents to specify wave fields within the interior of dielectric objects [Richmond, 1965]. 3-D volumetric formulations of MoM using space-filling cubic [Livesay and Chen, 1974] and tetrahedral [Schaubert et al, 1984] elements have been formulated primarily for application to the biological tissue interaction problem. Yet, because MoM leads to systems of linear equations having dense, full, complex-valued coefficient matrices, the required computer storage has an N^2

dependence, where N is the number of field unknowns; and the required computer execution time to solve the MoM system has an N^2 to N^3 dependence. With spatial resolution requirements in the order of 0.1 to 0.2 wavelength to avoid aliasing of vital near-field magnitude and phase data, this implies that MoM modeling of arbitrary 3-D structures spanning more than a very few wavelengths would tax or exhaust even the most capable existing computers.

The literature indicates that the basic MoM treatment of whole-body human tissue structures culminated in models having in the order of several hundred discretization cells, each of multi-centimeter scale [DeFord et al, 1983]. This is clearly inadequate to provide levels of detail of internal tissue structure needed for hyperthermia treatment planning. Theoretical efforts therefore shifted to examine alternative formulations of MoM which promise a dimensional reduction of computer resources. One such formulation [Borup and Gandhi, 1984] exploits the convolutional nature of the volume integral equation based on polarization currents (when defined on a uniform grid or lattice) to permit use of the fast Fourier transform (FFT) in the solution algorithm. Although, in principle, extending the horizon of MoM modeling to structures having thousands of discretization cells, the literature indicates that this approach may provide substantial errors in the calculations of the internal fields in biological tissues for EM excitations having transverse electric (TE) field components [Borup et al, 1987]. This would greatly impede application of FFT approaches to the important 3-D tissue case.

An alternative to frequency-domain MoM formulations of EM wave penetration into biological tissues was the FD-TD method introduced by Taflove in 1975 [Taflove and Brodwin, 1975], based in part on an algorithm published by Yee [Yee, 1966]. FD-TD is a direct finite-difference solution of Maxwell's time-dependent curl equations which implements a sampled-data reduction of the continuous EM field in a volume of space over a period of time. Overall, FD-TD is a marching-in-time procedure which simulates the continuous actual waves by sampled-data numerical analogs. At each time step, the system of equations to update the field components is fully explicit, so that there is no need to set up or solve a set of linear equations. This feature permits the required computational resources to vary approximately as N , the number of field unknowns in the model, which is dimensionally low compared to that of the original MoM formulation and the finite element method. Further, this feature permits highly efficient processing of the FD-TD algorithm by both vectorizing and concurrent-processing supercomputers such as the Cray Y-MP/8 and the Intel Delta, respectively. Studies are beginning to emerge where detailed energy deposition patterns are being calculated inside inhomogeneous three-dimensional models of the body [Sullivan et al, 1988; Sullivan, 1990; Wang and Gandhi, 1989].

Unlike the FFT/MoM approaches, which also have dimensionally-low computer burdens, FD-TD has been shown to be robust, providing highly accurate modeling predictions for a wide variety of EM wave interaction problems in two and three dimensions [Taflove, 1988; Taflove and Umashankar, 1989]. A subset of these includes biological tissue interactions: detailed (order 1-cm resolution) whole-human-body dosimetry under plane-wave illumination [Sullivan et al, 1988], and detailed partial- or whole-body hyperthermia modeling [Lau et al 1986a; Sullivan et al, 1987; Sullivan et al, 1988; Sullivan, 1990; Wang and Gandhi, 1989]. The increasing availability of Cray-type vectorizing supercomputers to the engineering electromagnetics community has in fact permitted application of FD-TD to model EM wave interactions with arbitrary 3-D structures substantially larger in electrical size than MoM (approximately 10 times larger in span and 1000 times larger in volume). As of July 1992, the largest reported FD-TD models are the set of digital interconnect structures modeled by Picket-May & Taflove [1992]. These 3-D models, implemented on the Cray Y-MP/8, solve for up to 60-million unknown vector field components.

Largely due to the demonstrated success of FD-TD grid-based computational modeling of EM wave interactions with complex structures and the clear evolutionary trends in supercomputer capabilities, interest in other "partial differential equation (PDE)" direct solvers for Maxwell's

equations has greatly expanded since the mid-1980's. These include the class of finite-element (FE) and finite-volume (FV) numerical approaches. Rockwell International Corp., in finite-volume time-domain (FV-TD) methods [Shankar et al, 1989], and Boeing Aerospace Corp., in finite-element frequency-domain (FE-FD) methods [Bussoletti et al, 1988], have developed the most advanced alternative PDE Maxwell's solvers aimed primarily at radar cross section modeling. Both the Rockwell and the Boeing efforts to find alternatives to FD-TD were motivated by the need to model target surfaces in a conformal manner. This is vital in the RCS modeling area, since the original stepped-surface or staircase approximation of smooth surfaces by FD-TD generates computational noise in the fields scattered outward to free space which markedly degrades the accuracy of the RCS calculation. There appears to be no sparse matrix algorithm available that can resolve the dimensionally-large computer burdens involved in treating the large, sparse, non-banded matrices that arise in all FE-FD approaches in 3-D. In effect, the FE-FD user generating sparse matrices must resort to matrix inversion, LU decomposition, or iterative methods historically employed by MoM users generating dense matrices. As discussed above, these matrix problems stymied the further development of MoM, and it appears likely that the further development of pure FE-FD will be likewise stymied.

Since the mid 1980's, as well, there have been efforts at FE-FD solutions of Maxwell's equations in the area of EM wave interactions with biological tissue structures [Lynch et al, 1988; Paulsen et al, 1988a; Paulsen et al, 1988b]. Similar to the radar cross section work, the goals here involve nearly conformal modeling of tissue structures using well-characterized geometry generation software. In 2-D, qualitative as well as quantitative retrospective comparisons with limited clinical data suggested that FE-FD numerical models accurately predicted the dominant capabilities and limitations of the annular array applicator and magnetic induction coil studied [Paulsen et al, 1985; Roemer et al, 1984a; Roemer et al 1984b; Strohbehn et al 1986]. In 3-D, the formulation of FE-FD for hyperthermia problems has been numerically implemented. However, because of the same sparse matrix problems experienced by Boeing in the radar cross section area, the application of the FE-FD method to 3-D hyperthermia problems has been very limited. Paulsen et al [1992] report elsewhere in this special issue, recent progress in FE-FD by computing 3-D EM field distributions in a full-body model with limited internal organs but without any skeletal structures. To avoid some of the drawbacks of the pure FE-FD method in 3-D, an alternative frequency-domain hybrid and boundary element formulation has been developed and implemented [Paulsen et al, 1988b]. To date only 3-D calculations on simple homogeneous and heterogeneous cylindrical geometries have been attempted with this approach because dimensionally unacceptable amounts of computer memory are still required for large-scale problems. This has resulted in the migration of the finite-element technique to the time domain (where matrices can be avoided, as in FD-TD) with the development of finite-element time-domain (FE-TD) methods [Lynch et al, 1988]. These should possess dimensional computer resource burdens similar to FD-TD. However, work in this area is ongoing and details concerning the accuracy and computer requirements of 3-D FE-TD models of hyperthermia relative to FD-TD have not yet appeared.

Unlike the RCS area, the rationale for conformal FD-TD modeling of biological structures relative to EM wave interactions is not justified. Detailed studies [Borup et al, 1987; Sullivan et al, 1987] have shown that simple FD-TD surface staircasing of cylindrical, layered cylindrical, and spherical tissue structures (of typical permittivity and loss) is sufficient to permit calculation of the penetrating internal fields with a high degree of accuracy compared to the exact summed eigenfunction solutions. It is observed that the EM fields penetrating a lossy dielectric structure are much less sensitive to the nature of the surface approximation of the structure than the fields scattered away from the structure into free space. This observation permits effective use of the earlier staircasing FD-TD models for computing penetrating fields in tissue structures. These have substantially reduced geometry generation requirements relative to the latest fully conformal FD-TD models based upon local Faraday's Law and Ampere's Law contour paths [Jurgens et al, 1992], and will sacrifice little accuracy.

Experimental Validation Studies

The major thrust in this field to date has been in developing the computational algorithms and checking their validity using analytical solutions or other numerical techniques for simple structures. Rigorous testing of how well the theoretical predictions compare to measured EM absorption in realistic heterogeneous human phantoms intended to simulate clinical situations has not been carried out so far. Experimental validations in simple muscle equivalent phantoms irradiated using waveguide applicators of the types employed in superficial hyperthermia have been reported for 2-D predictions [Rine et al, 1990] and 3-D predictions [Lau et al, 1986b, Sullivan, 1990], as well as for interstitial antennas embedded in homogeneous phantoms [Wong et al, 1986]. Validations have also been carried out for radiative type regional heating systems. Paulsen and Ross compared the predicted and experimental measurements at a single frequency of 70 MHz for the older APA [Paulsen and Ross, 1990] using simple 2-D models and phantoms. They obtained quantitative agreement in phantoms containing moderate complexities. Sullivan has compared the predicted and measured results along the major and minor axes of the CDRH elliptical phantom [Sullivan, 1990, Sullivan, 1991] filled with homogeneous muscle phantom tissue in an earlier work and has recently published further results using a modified CDRH phantom with inhomogeneities introduced [Sullivan et al, 1992]. Good agreement was observed between the FD-TD predicted results and the measurement results. While the existing validations are qualitatively encouraging, they have not been rigorous. In effect, they have not precisely established the uncertainty bounds in the basic FD-TD near-field physics model for the applicators in question. More research using both theory and experiment will have to be carried out to quantitatively determine these uncertainty bounds for the classes of aperture and dipole/array applicators, as well as for the class of interstitial applicators.

FD-TD Modeling for EM Hyperthermia with Realistic Geometries

The theoretical formulation of the FD-TD method and its implementation to solve directly the time-dependent Maxwell's equations has been described extensively in recent literature [Lau et al, 1986a; Sullivan, 1987; Taflov, 1988] and will not be reviewed here. Only examples of the application of the FD-TD technique in modeling EM hyperthermia with realistic geometries will be presented. The goals in these studies were to first develop the computational model starting with simple geometrical structures excited by simple radiators, and then extend the analysis to more complex structures excited by more complex radiators. By this approach it was possible to rigorously validate the computational model at different stages. As an example of a simple set up, a model of the human thigh irradiated with a waveguide applicator has been studied. 2-D and 3-D models using the FD-TD technique have been developed to run on different computing machines. 2-D models have been run on Compaq 386/25/Weitek laboratory computer, MicroVax II computer and Cray Y-MP supercomputer. The 3-D models have been mainly run on Cray Y-MP supercomputer.

The results of our 2-D and 3-D theoretical validation studies of the code have been published previously [Katz et al, 1991; Piket-May et al, 1992]. Following the good agreement obtained in these extensive validation studies, a 3-D model of the human thigh was reconstructed from a collection of 29 serial computerized tomographic (CT) images. The SAR distributions when irradiated with a waveguide applicator were then calculated. Different patterns were obtained for the 2-D and 3-D models [Piket-May et al, 1992]. Further calculations investigating the effect of the water bolus (routinely used in the clinic to provide energy coupling and skin temperature control) and the inhomogeneous tissue structures have been carried out in 2-D. These results are presented in this paper. In these studies a high-resolution (5-mm uniform mesh size) 128 x 100 cell FD-TD grid was analyzed. All data were obtained after time stepping 15 periods of the incident wave at 915 MHz.

Waveguide applicator excitation of a planar phantom in 2-D

Muscle equivalent planar phantoms are routinely used in the clinic to characterize the performance of applicators [Chou et al, 1986]. The measured SAR patterns are then used for treatment planning and quality assurance purposes. To test the accuracy of the FD-TD model and also to study the effect of the water bolus on the SAR distributions, planar phantom /applicator configuration with and without the water bolus was studied in 2-D at 915 MHz.

The aperture size of the applicator modeled was 10 x 10 cm. It was assumed to be filled with a low-loss dielectric material having a relative permittivity, ϵ_r , of 6 and conductivity, σ , of 0 S/m. The thickness of the water bolus used was 2.5 cm, and the water was assumed to have $\epsilon_r = 80$ and $\sigma = 0$ S/m. The dielectric properties used for muscle were $\epsilon_r = 50.5$ and $\sigma = 1.307$ S/m.

The calculated normalized SAR distributions are shown in Figures 1a and 1b. The patterns look qualitatively very similar to measured patterns. The introduction of the water bolus did not make any significant changes in the SAR distributions.

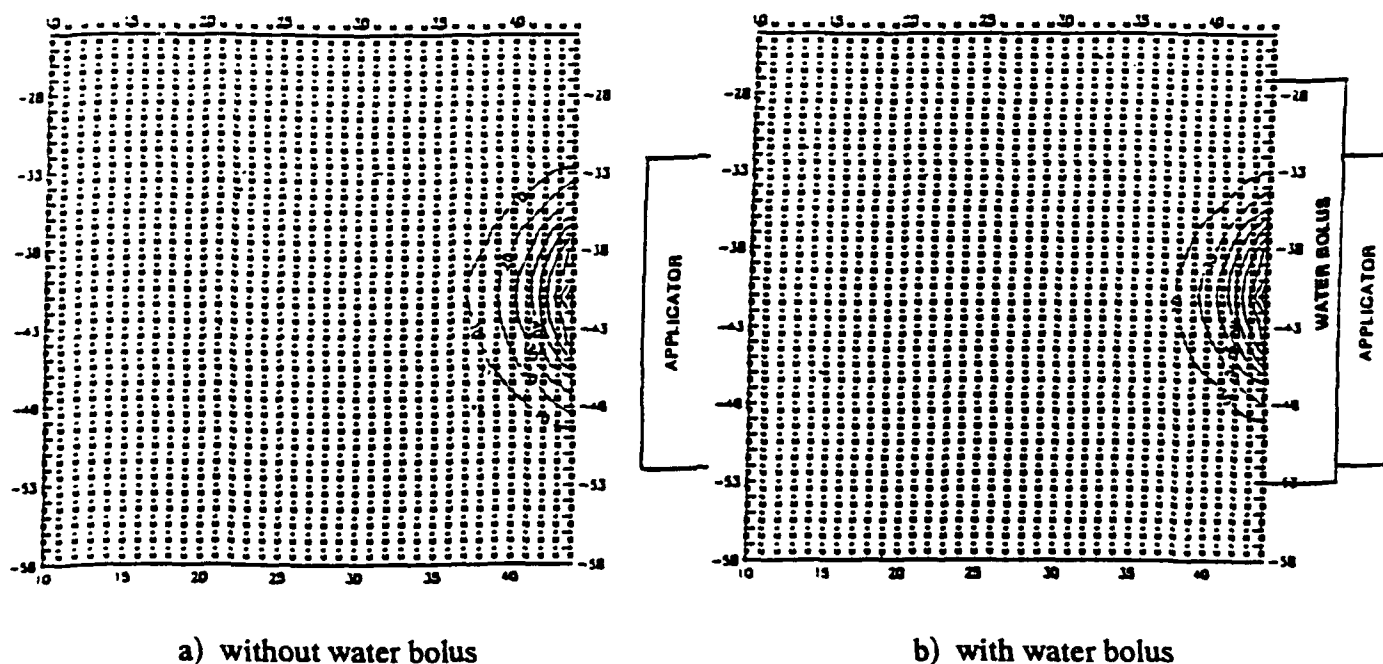


Figure 1. Normalized percent SAR distributions in a planar phantom

Waveguide applicator excitation of homogeneous and inhomogeneous thigh model in 2-D.

Using actual CT-scan data of a human thigh, the tissue structure (fat, muscle, bone, and bone marrow) was semi-automatically deduced as described in [Piket-May et al, 1992]. The thigh was assumed to be made up of homogeneous muscle in the first calculations. In Figures 2a and 2b normalized SAR distributions obtained without and with the water bolus are shown. Figures 3a

and 3b show the corresponding patterns when the thigh is assumed to be inhomogeneous. Some interesting results can be observed. With the water bolus, a hot spot can be observed in the inhomogeneous thigh case near the top edge of the applicator. This is not present in the case of the homogeneous thigh. Deepening and broadening of the SAR pattern is observed with the bolus compared to the case without the water bolus. Also in the inhomogeneous case, there are number of smaller hot and cold spots present largely due to tissue inhomogeneities.

These results indicate that the SAR patterns measured in planar phantoms can not give a very accurate picture of the SAR patterns in clinical situations where the inhomogeneous tissue structure and shape of the dielectric scatterer markedly influences the patterns. Further, these 2-D results do not take in to account the effect of 3-D variations which may further modify the SAR distributions and work is in progress to investigate these 3-D effects.

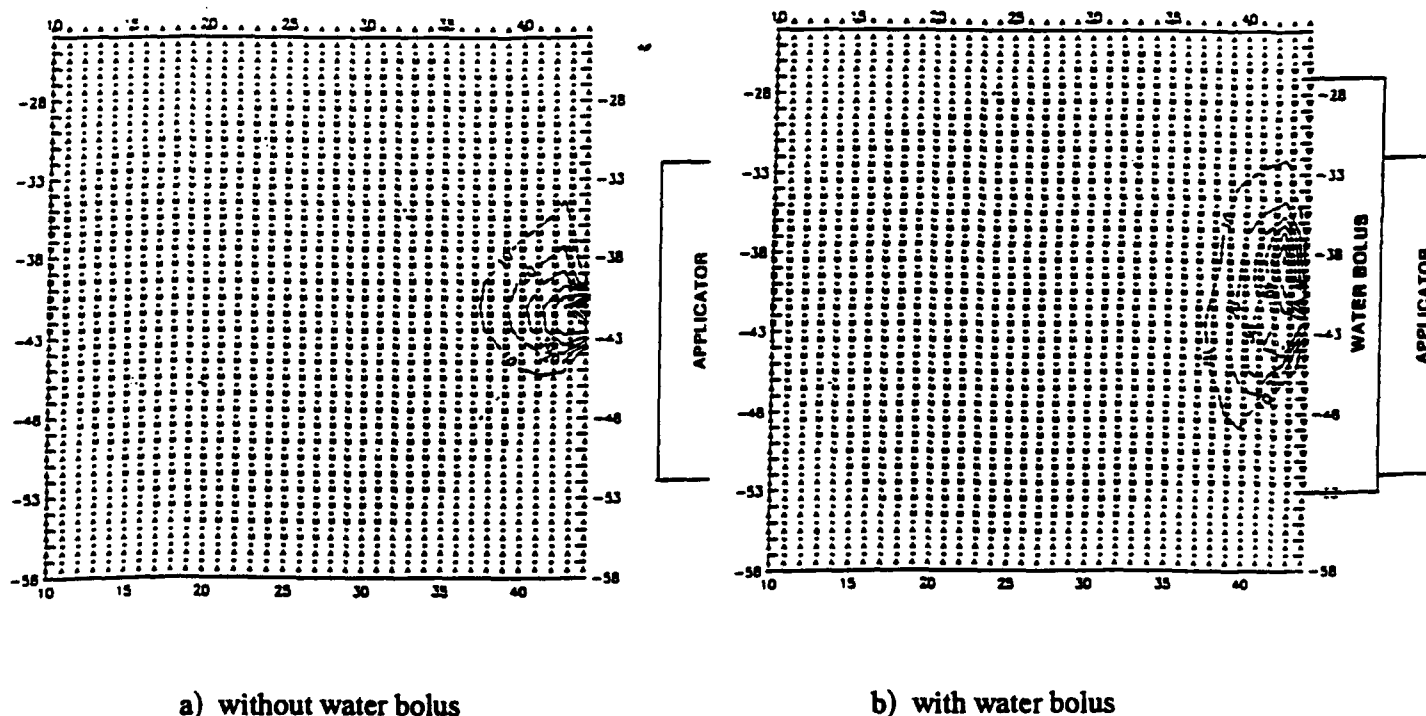


Figure 2. Normalized percent SAR distributions in a homogeneous thigh (M-Muscle, W-Water, A-Air)

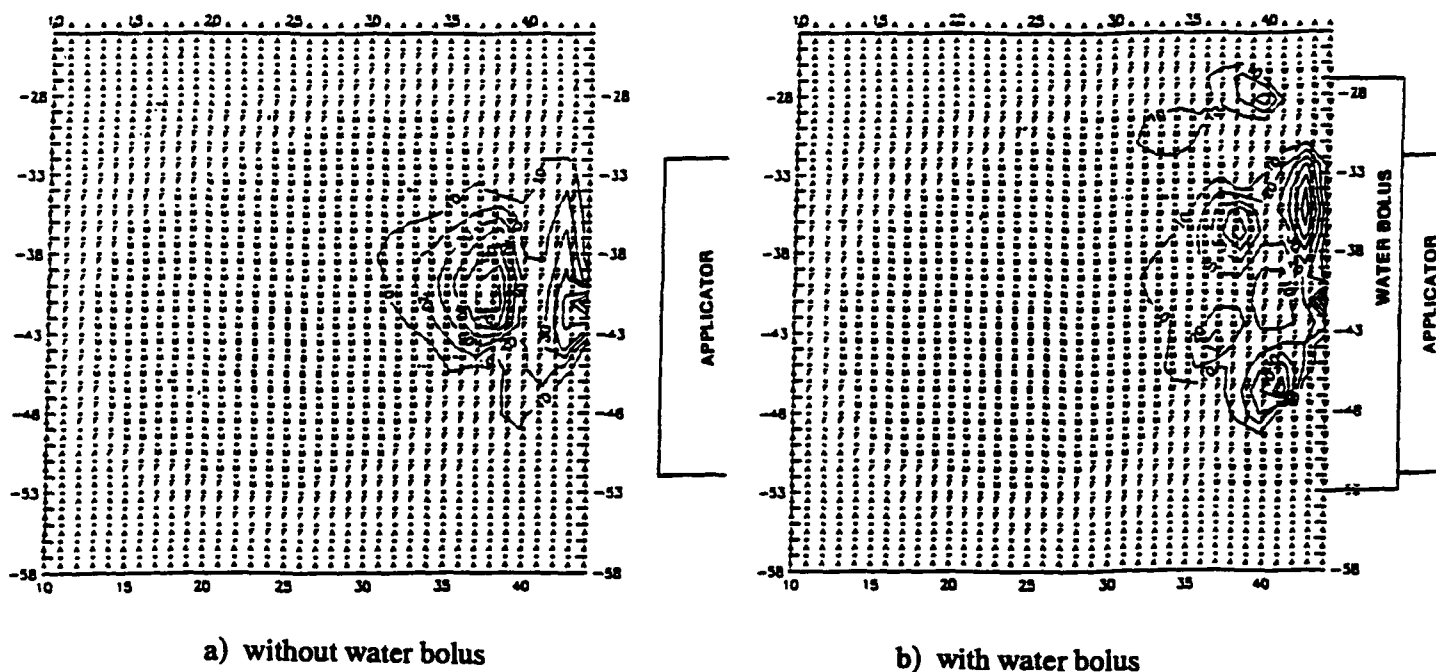


Figure 3. Normalized percent SAR distributions in an inhomogeneous thigh (M-Muscle, F-Fat, B-Bone, R-Bone marrow, W-Water, A-Air)

Future Needs in the EM Modeling Area

The computed SAR distributions for realistic geometries presented above show some interesting features which may not readily be appreciated from planar phantom measurements. Two important issues have to be addressed. First is the question of whether the observed distributions are real or are due to some numerical artifacts. The extensive validations carried out by comparing with MoM results give some confidence in the calculations. Yet it is important to carry out further experimental validations for selected cases before full confidence can be placed on these numerical models. Validation studies will have to consider both applicator free-space operation as well as loading by media having the electrical characteristics of biological tissue.

The second issue is whether the observed SAR hot spots will result in corresponding temperature hot spots. Two distinct modeling problems are involved in the computer simulation of hyperthermia treatments: computation of the absorbed EM power distribution in tissue and the prediction of resulting temperature distributions using a suitable thermal model. These are two complex problems requiring different theoretical basis and tissue property and physiological data. Development of suitable thermal models to predict the resulting temperature distributions is also an active area of research. However, the limited bloodflow data available and the heterogeneous blood flow distribution in tumors may limit the applicability of this approach. There is active research ongoing to obtain bloodflow data using positron emission tomography (PET) systems. Overall, accurate numerical models to calculate power deposition and temperature distributions would be valuable tools to improve the delivery of hyperthermia and thus significantly enhance its role in cancer treatment.

Computer modeling can play a significant role in the design of better heating equipment and in improving the quality of the hyperthermia treatments currently being administered. However, there are several gaps in the state of the existing knowledge regarding the validity of numerical

simulation in the clinical context. There are also a number of important EM field physics issues unique to hyperthermia: 2-D vs. 3-D modeling quantification; analytical and experimental validations; and understanding realistic antenna/patient configurations; that need to be addressed.

It is concluded that FD-TD computational modeling, with stepped-surface approximation of tissue structures, will remain for the near future as the most efficient means of obtaining predictive data for EM field penetration into biological tissue structures in 3-D. Research into time-domain finite-elements, however, may ultimately provide an alternative means of such modeling since the time-domain formulation permits in principle explicit or nearly-explicit time-marching of the fields. This is the key to dimensionally-low computer resources and processing by supercomputers.

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ANALYSIS AND COMPUTATION OF LEAKY-WAVE HYPERTHERMIA APPLICATOR

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ABSTRACT

The use of electromagnetic waves to induce hyperthermia in cancer therapy has been the subject of intensive research but obtaining good resolution at depth in the abdomen and pelvic regions remains a fundamental problem. This paper investigates the prospects for generating leaky-waves in tissue to improve field penetration and focussing at depth. An approximate analysis based on a planar structure illustrates the feasibility of obtaining good resolution at depth but leaky-wave action in a strict sense is not possible. Measurements on an applicator conformal with the phantom tissue simulated by a saline solution, demonstrate good resolution and penetration but hotspot regions are detected around the launcher region. A three dimensional finite difference time domain (FDTD) computation is performed in Cartesian, circular cylindrical and elliptic cylindrical coordinates to model all the effects for various geometries of the phantom region and different launching symmetries. The results illustrate the need for symmetric launching of the strip, computational convergence and mode diagnostic data, the shape of focal region and the useful property of being able to shift the focal region by changing frequency. It is concluded that this new quasi-leaky-wave applicator concept is potentially capable of giving improved focal resolution at depth with some positional control and only one generator is required. Optimising the launching of the quasi-leaky-waves on the applicator and preventing hotspot regions from creating unwanted tissue heating are remaining practical problems to address.

1. INTRODUCTION

The concept of exposing tumours to electromagnetic radiation in cancer therapy has been intensively investigated in recent years [Field and Hand, 1990]. Obtaining spatial resolution of the fields at deep locations in the tissue is a fundamental problem due to the high level of loss present in the media. The heterogeneous nature of the human body, and particularly its layered nature [Hansen 1989] compound the difficulties. Direct contact applicators typically give penetration depths of a few centimetres while annular arrays enable large volumes of tissue to be heated [Turner, 1984]. The natural transverse resonance of the abdomen and pelvic regions contributes to the behaviour of the annular array and recent computer modelling [Picket-May, Taflove and Sathiaselan, 1991] suggests that whispering gallery type modes may be an effect causing unwanted hotspots. Mutual coupling between the array apertures is also an effect to consider [Leybovich, Myerson, Emami and Straube, 1991]. The use of a large reflector antenna has been computer simulated [Chang and Mei, 1989] to demonstrate the improved spatial resolution, but no practical results are given; practical results for a microwave array focussing into the thorax region have been reported however [Melek, Anderson, Brown and Conway, 1982]. Some mention has been made [Taylor, 1983, Anderson, 1986] of the possible use of inhomogeneous waves to achieve greater penetration into lossy tissue and some initial work on leaky-waves was reported by Rappaport, 1987.

The aim of this present paper is to investigate the feasibility of generating leaky-waves into lossy tissue to achieve improved spatial resolution at depths compatible with the requirements for treating tumours in the pelvic and abdomen regions. It was considered that such a leaky-wave device would offer the prospects of electronic beam scanning simply by changing the applied radio frequency. Also the fact that only one radio frequency generator is required is a distinct advantage over the annular array concept and it should also be possible to further control the focal region by electronically varying the impedance of the leaky-wave transmission line.

Several questions were however apparent at the beginning, concerning the nature of leaky-waves in a lossy medium, the launching of such waves in practice and the intensity of near fields of the launcher. It was decided that full three-dimensional computations embracing all these aspects were required and the finite

difference time domain (FDTD) method was chosen. This method is capable of accommodating heterogeneous media as presented by the human body but the scope of the present work is confined to a homogeneous phantom simulating muscle. To gain insight into the preservation of leaky-wave action in the presence of lossy media, a simplified mathematical model is derived and some experimental work has been carried out on the practical problems associated with launching leaky-waves. These latter aspects will be reported first.

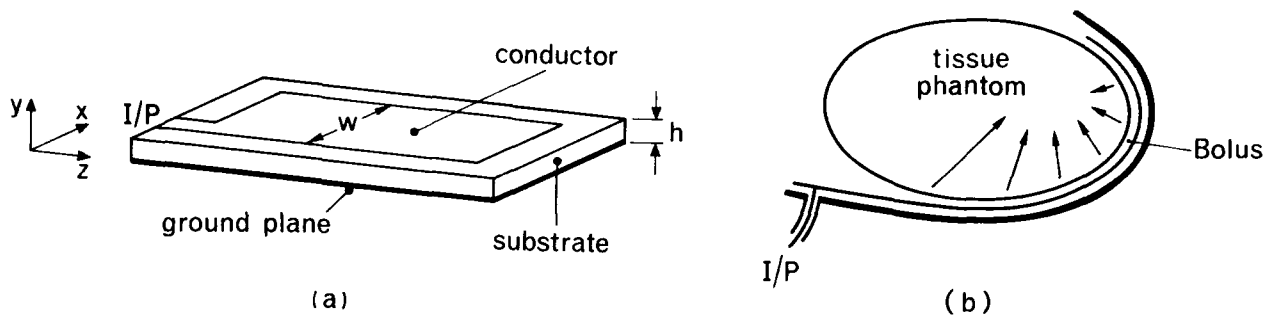


Fig 1 (a) Conventional leaky-wave microstrip antenna (b) Sectional view of microstrip structure conformal with cross-section of abdomen region and bolus.

2. APPROXIMATE ANALYSIS OF APPLICATOR

The clinical requirements suggest that a convenient form for the applicator would be a thin planar assembly in close contact with the abdomen. A suitable structure to investigate is the planar microstrip leaky-wave antenna sketched in Fig 1a, showing a conducting strip of width W on a dielectric substrate of thickness h backed by a conducting ground plane. This very wide microstrip line radiates leaky-waves with a narrow beamwidth when asymmetrically excited and this offers improved spatial resolution for an applicator. It was conjectured that the hyperthermic applicator would be similar in structure but bent conformally with the cross-section of the lossy abdomen into which the leaky-waves are to propagate. The spatial resolution would be further enhanced by the curvature as sketched in Fig 1b. The extent to which leaky-wave action gives improved spatial resolution depends on the degree of loss in the lossy media. For patient comfort and to avoid the burning of the abdomen skin, cooled water will be circulated in a bolus region between the applicator and the body. The length of the applicator is determined by the extent to which the leaky-wave discharges its power as it progresses along the conducting strip.

The following approximate analysis for electrically thin substrates adopts the Wiener-Hopf analysis of Chang and Kuester, 1981, in a similar way to Oliner, 1987, with the exception that the space above the conducting strip is occupied by lossy media with a complex relative permittivity $\epsilon_{mc} = \epsilon_m - j\sigma_m/(\omega\epsilon_0)$ where $j = \sqrt{-1}$ and ϵ_0 is the free space permittivity. We use the representation in terms of a relative dielectric constant ϵ_m and conductivity σ_m . Transverse electromagnetic (TEM) waves are launched in the parallel plate substrate region between the conducting strip of width W and the ground plane. The substrate of thickness h has a complex relative permittivity $\epsilon_{rc} = \epsilon_r - j\sigma_r/(\omega\epsilon_0)$. For a narrow conducting strip, the TEM waves reflect from the strip edges to create the quasi-TEM microstrip wave used in circuits defined by a longitudinal wavenumber

$$\begin{aligned} k_x &= \beta - j\alpha \\ k_x^2 &= k_0^2 \epsilon_{rc} - k_{ze}^2 \end{aligned} \quad (1)$$

where α and β are the attenuation and phase constants respectively, k_0 is the free space wavenumber and k_{ze} is the leaky-wave transverse wavenumber in the substrate in the x direction; the coordinate system is given in Fig 1a. For a time dependence of $\exp(j\omega t)$, the transcendental equation relating W and mode number n is

$$\exp j(\chi - k_{ze} W) = -1 = \exp(\pm j n \pi), \quad n \text{ odd} \quad (2)$$

or

$$\chi - k_{xe} W = \pm n \pi, n \text{ odd}$$

where

$$\chi = 2 \arctan \left[\frac{k_z}{k_{xe}} \tanh \Delta \right] - f_c \left[\frac{-k_{xe}}{k_0} \right]$$

where $k_0 h \sqrt{\epsilon_r} \ll 1$ and Δ and $f_c \left[\frac{-k_{xe}}{k_0} \right]$ are formulas given by Oliner, 1986 containing a transverse wavenumber k_z for the airspace region above the strip. $n = 0$ yields the dominant quasi-TEM microstrip mode while $n = 1$ is the first leaky-wave mode and the one we wish to generate. This mode has an electric field tangential to the strip in the airspace. For odd modes a vertical electrical wall exists mid-way beneath the strip in the y direction while the even modes have a magnetic wall.

To convert the analysis to the present case where the airspace is replaced by lossy tissue we assert that

$$k_x^2 = k_0^2 \epsilon_{mc} - k_z^2 \quad (3)$$

In the absence of lossy media above the strip leaky-waves exist for $\alpha > 0$ and $\beta < k_0$ which is associated with electrically wide strips. The presence of losses will rapidly attenuate the leaky-waves and we have previously referred to this action as a quasi-leaky-wave (Andrasic, James and Hand, 1990, 1991 a and b) and shown that improved spatial resolution is still possible and is superior to that produced by a conventional type of applicator. The choice of frequency is illustrated for planar applicators in Fig 2 for the $n = 1$ mode. We seek the condition where the phase shift along the z direction is small and α is neither too large to leak the power prematurely nor too small to enable the power to be transferred to the tissue. The planar structure in Fig 2 is similar to that in Fig 1a except that the space above the strip is occupied by lossy tissue with constants ϵ_m and σ_m . The substrate is low loss water. A change of frequency will cause the focal field point to move in position with some defocussing

To calculate the electrical field $E(P)$ at a point P within the elliptical shaped homogeneous phantom of complex relative permittivity ϵ_{mc} it is assumed that the modal conditions on the planar structure are largely maintained when it is bent conformal with the phantom.

$$E(P) = -\Delta \times F$$

$$F = \frac{\epsilon_{mc}}{2\pi} \int_{\phi=\phi_1}^{\phi_2} J_M \left[\frac{\exp(-jkR_1)}{R_1} + \frac{\exp(-jkR_2)}{R_2} \right] ds(\phi) \quad (4)$$

where J_M is the equivalent magnetic current around the edges of the conducting strip and $k = k_0 \sqrt{\epsilon_{mc}}$

$$J_M = \exp[-j(\beta - j\alpha) s(\phi)] \quad (5)$$

R_1 and R_2 are the distances from the current elements $ds(\phi)$ at each strip edge to point P and the angle ϕ rotates about the central axis of an elliptical phantom as J_M is integrated along the extent of the strip. The specific absorption rate (SAR) defines the hyperthermic heating effect within tissue and for the homogeneous phantom region is proportional to $|E(P)|^2$. The computed SAR result in Fig 3a substantiates the expectations showing a good spatial resolution at a depth of some 8cms into the phantom and its positional variability with frequency. The phantom conductivity is for the saline solution used and is lower than that in Fig 2 caption.

3. EXPERIMENTAL RESULTS

The main purpose here is to investigate how to connect the coaxial cable from the generator to the applicator to ensure that the $n = 1$ mode is generated with little $n = 0$ contribution and the minimum of unwanted hotspots around the launcher region. Saline solution (3gms/litre) with $\epsilon_{mc} = 76 - j150$, ($\sigma_m = 6 \times 10^{-3}$ S/cm)

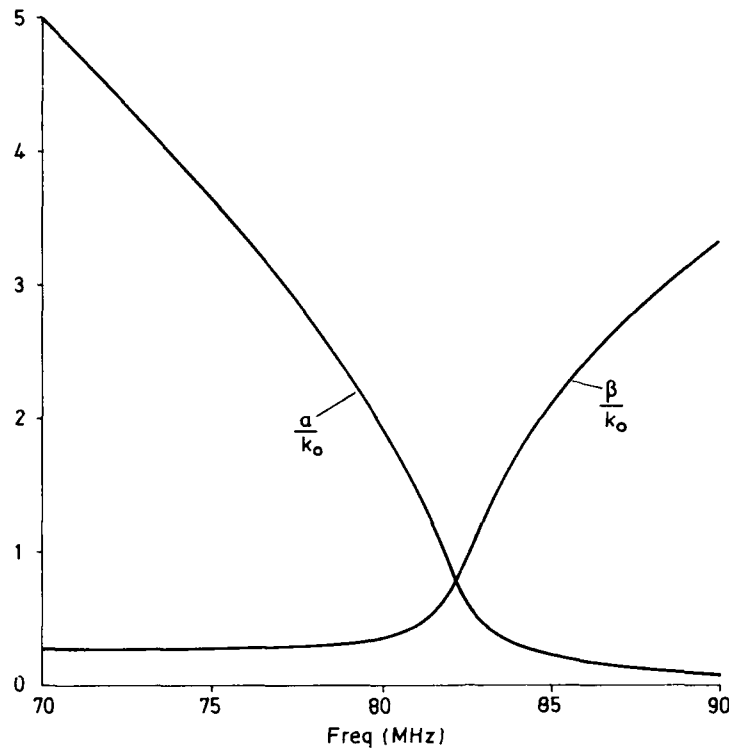


Fig 2 α and β values for planar structure for $n = 1$ mode eqns(1-3);
 $W = 20$ cms, $h = 1$ cm, $\epsilon_r = 76$, $\sigma_r = 6 \times 10^{-5}$ S/cm, $\epsilon_m = 76$, $\sigma_m = 8 \times 10^{-3}$ S/cm

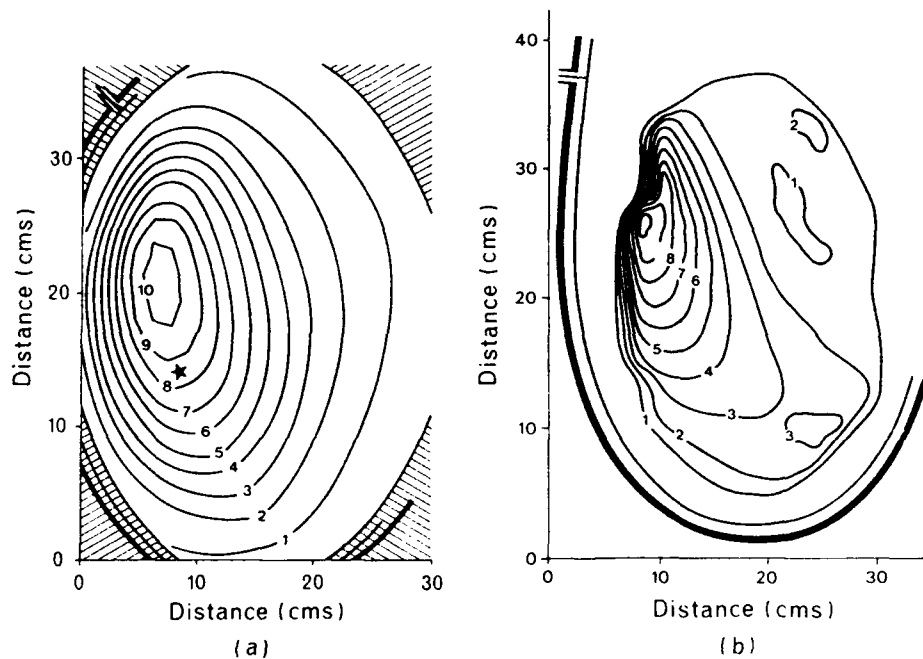


Fig 3 Relative Specific Absorption Rate (SAR) contours with linear increments, within the elliptical phantom with major and minor axis of 39.5cm and 31.5cm respectively, at 72MHz, in central transverse plane. Electrical constants as in Fig 2 caption, except that $\sigma_m = 6 \times 10^{-3}$ S/cm.

(a) Computed eqns (1) to (5). At 82MHz focal region shifts to starred location.

(b) Probe measurement of SAR corresponding to dominant electric field in direction of ellipse axis.

was used as the phantom held in a polythene container; Fig 4a illustrates the bolus region and surrounding water which will assist patient comfort. The field scanning monopole probe, the phantom container and the applicator ground plane can be seen in the corresponding photograph, Fig 4b. The coaxial cable was attached to one corner of the conducting strip of width W chosen to reject modes for $n > 1$ in the planar version and a typical measurement, Fig 3b, is in reasonable agreement with the approximate analytical result. The input field region is bent away from the phantom as in Fig 1b to facilitate probing of the fields. When the input field region was probed, field hotspots were detected around the launcher and furthermore the focal region showed little shift with frequency. Attempts at constructing a balanced launcher using two coaxial cable inputs of opposite polarity had some effect on the shape of the focal region, but the launcher hotspots persisted. The field perturbation created by the monopole probe and the corrosive effects of the saline solution, are some of the limitations that prevented an accurate quantification of these effects.

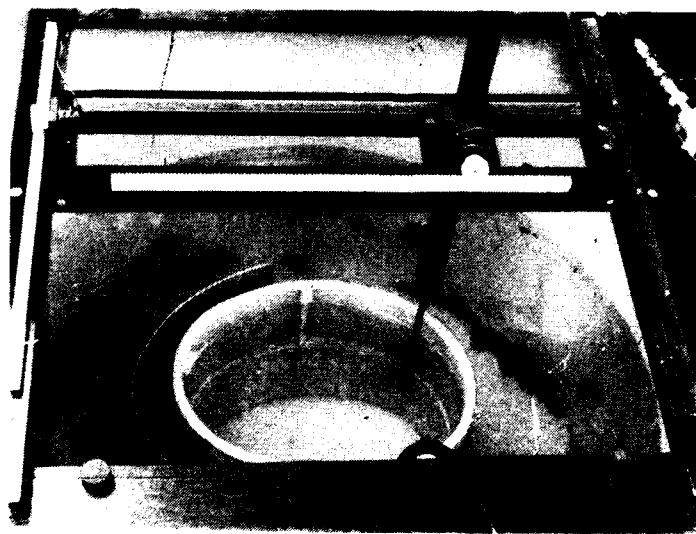
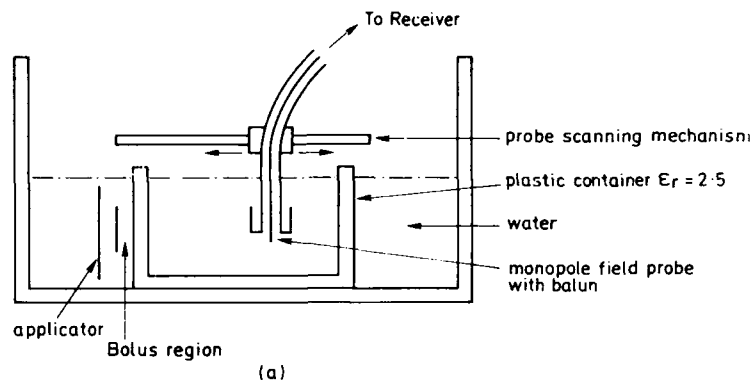


Fig 4 Experimental rig.

(a) Sketch of cross-section of elevation view showing phantom and the surrounding water.

(b) Photograph showing part of applicator pulled away from the container of phantom for the purposes of viewing.

4. FINITE DIFFERENCE TIME DOMAIN (FDTD) COMPUTATION

4.1 Planar Geometry

To investigate the likely resolution benefits described in section 2 we consider a planar version of the applicator as sketched inset in Fig 6. In the FDTD programming on a VAX 8800, the basic method of Yee, 1966 is adopted throughout. The planar structure shown inset in Fig.6 is resolved into cube cells of 1cm sides, having assigned values of permittivity, permeability and boundary conditions. The region between the conducting strip and ground plane has $h = 1$ cm and thus are one cell deep. In all, 63000 cells were allocated to a rectangular lattice of $30 \times 30 \times 70$ which embraced the applicator, and the lossy tissue of complex permittivity ϵ_{mc} above the conducting strip. In addition buffer regions each occupying $10 \times 30 \times 70 = 21000$ cells were placed on both sides of the lattice, thus there were 105000 cells in the computational domain. Absorbent boundary conditions based on the equations of Mur 1981, were placed at the extremes of the computational domain. A transient sinusoid was applied to one corner of the conducting strip for asymmetric excitation and antiphase transient sinusoids at each corner for symmetric excitation. The formulation of Maxwell's equation in differential form in terms of spatial increments Δx , Δy , Δz and the time step Δt together with the leap-frogging process and numerical stability condition follows the conventional methods [Yee, 1966, Sheen, Ali, Abouzahra, and Kong, 1990].

An example of the FDTD computation in Fig. 5 shows the steady state achieved at 81 MHz after 700 time steps. With asymmetric launching there will be some $n = 0$ TEM mode present and the decomposition in Fig. 5 quantifies this. Examination of the SAR in the phantom region Fig. 6, illustrates the undesirable standing wave patterns due to the presence of both $n = 0$ and $n = 1$ modes when asymmetrically excited. The symmetric excitation case is readily computed by inserting a central electric wall as previously mentioned. The SAR corresponding to the dominant E_x component is plotted in Fig. 7 and a well resolved focal region is achieved at a height of approximately 7cm above the conducting strip. The result in Fig 7 shows the ideal situation where only the $n = 1$ mode exists but in practice the launching of the latter will create field concentrations and other hotspot regions. Clearly improved resolution depends on the success in launching the $n = 1$ mode with the minimum launcher disturbance. The presence of a lossless bolus layer between the conducting strip and phantom has insignificant effect and is also omitted in these computations.

4.2 Curvilinear Geometry

There is evidence [Harms, Lee and Mittra, 1992] that the body fitting of cells in the FDTD method has an influence on convergence and accuracy. For this reason we have recast the FDTD formulation in curvilinear coordinates having greater geometrical compatibility with the phantom shape, to achieve improved resolution of the hotspots particularly in the launcher region. Both circular and elliptic cylindrical coordinate lattices have been developed as shown in Fig 8a and b respectively. For circular cylindrical geometry the Maxwell's equations in differential form in ρ, ϕ, z coordinates are given as,

$$\begin{aligned}\frac{\partial H_z}{\partial t} &= -\frac{1}{\mu\rho} \left[\frac{\partial(\rho E_\phi)}{\partial \rho} - \frac{\partial E_\rho}{\partial \phi} \right] \\ \frac{\partial E_z}{\partial t} &= \frac{1}{\epsilon} \left[\frac{1}{\rho} \left\{ \frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi} \right\} - \sigma E_z \right]\end{aligned}$$

which may be represented in difference form as,

$$\begin{aligned}H_z^{n+1/2}(i+1/2, j+1/2, k) &= H_z^{n-1/2}(i+1/2, j+1/2, k) \\ &+ \frac{\Delta t}{\mu\rho} \{ [E_\rho^n(i+1/2, j+1, k) - E_\rho^n(i+1/2, j, k)]/\Delta\phi + \\ &[\rho_i E_\phi^n(i, j+1/2, k) - \rho_{i+1} E_\phi^n(i+1, j+1/2, k)]/\Delta\rho \}\end{aligned}$$

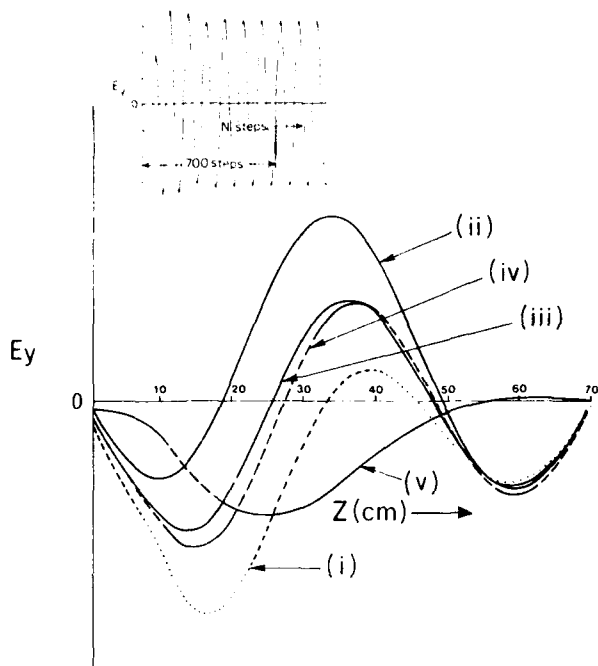


Fig 5 FDTD computation of E_y field at 81MHz and decomposition into symmetric and asymmetric modes. Electrical constant and lattice extent are as in Fig 6. E_y field reaching steady-state after 700 time steps is shown inset.

- (i) and (ii) Fields of edges of strip
- (iii) TEM field calculated from $((i) + (ii))/2$
- (iv) TEM field at centre of strip
- (v) $n = 1$ mode at one edge calculated from $((i) - (ii))/2$

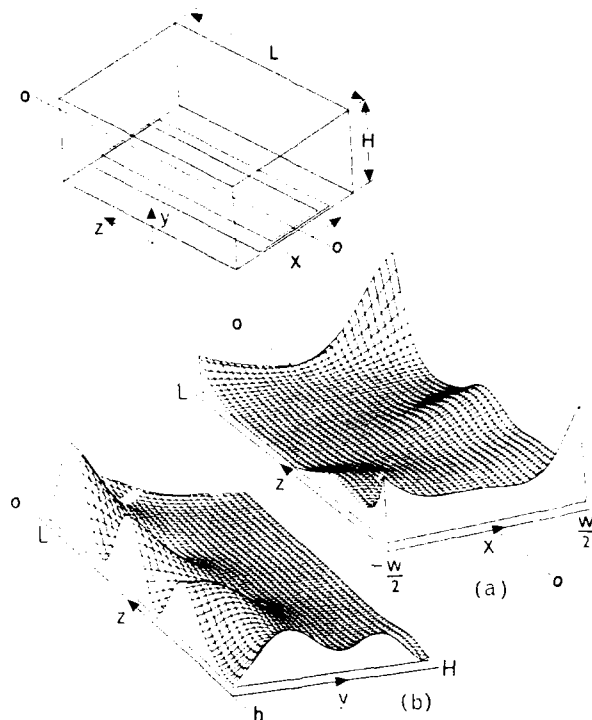
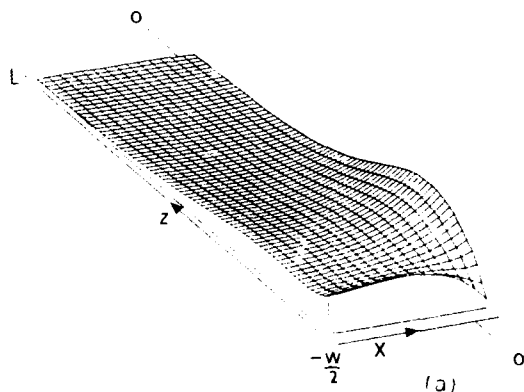


Fig 6 Computed plots of relative Specific Absorption Rate (SAR) levels at 81MHz for asymmetrically excited planar strip with $W = 20$ cm, ground plane width = 30 cm, $h = 1$ cm, $H = 30$ cm, $L = 70$ cm, $\epsilon_r = 76$, $\epsilon_m = 76$, $\sigma_r = 6 \times 10^{-5}$ S/cm, $\sigma_m = 6 \times 10^{-3}$ S/cm. Sketch of applicator is shown inset.

- (a) $|E_x|^2$ at height $y = 10$ cm
- (b) $|E_x|^2$ in yz plane at $x = 0$

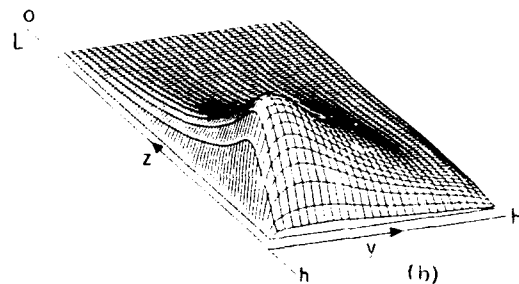


Fig 7 Computed plots of relative Specific Absorption Rate (SAR) levels as in Fig 6, but symmetrical excitation enforced by central electric wall.

- (a) $|E_x|^2$ at $y = 10$ cm
- (b) $|E_x|^2$ in yz plane at $x = 0$

$$\begin{aligned}
E_z^{n+1}(i, j, k + 1/2) = & \left(\frac{1 - S}{1 + S} \right) E_z^n(i, j, k + 1/2) \\
& + \frac{\Delta t}{\epsilon(1 + S)} \{ [H_\rho^{n+1/2}(i, j - 1/2, k + 1/2) - H_\rho^{n+1/2}(i, j + 1/2, k + 1/2)] / \Delta \phi + \\
& [\rho_{i+1/2} H_\phi^{n+1/2}(i + 1/2, j, k + 1/2) - \rho_{i-1/2} H_\phi^{n+1/2}(i - 1/2, j, k + 1/2)] / \Delta \rho \}
\end{aligned}$$

where $S = \sigma \Delta t / (2\epsilon)$ and $\rho_i = i \times \Delta \rho$

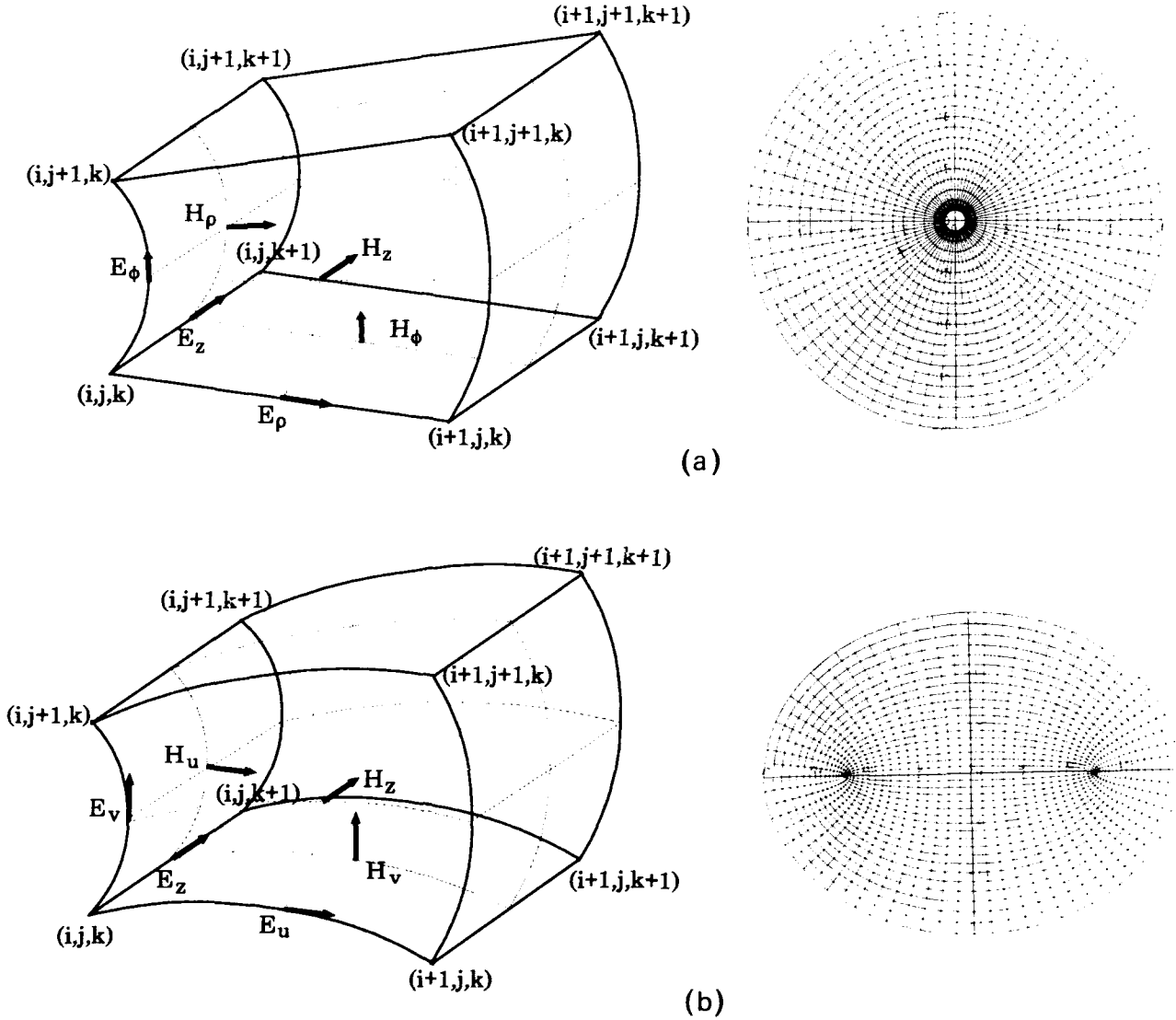


Fig 8 Geometry of leapfrogging unit cell and corresponding lattice for curvilinear coordinates
(a) circular cylinder coordinates
(b) elliptic cylinder coordinates

Mur's Absorbing boundary condition is applied in the plane boundaries at $z = 0$ and z_L which are the extreme planes of the computational domain. A perfectly conducting boundary surface is assumed at the circular boundary corresponding to an extended ground plane.

A similar set of equations were obtained for the elliptic cylinder case, where the coordinates u, v , and z were used and the corresponding difference equations are given by

$$H_z^{n+1/2}(i+1/2, j+1/2, k) = H_z^{n-1/2}(i+1/2, j+1/2, k) + \frac{\Delta t}{\mu h^2} \{ [h_{j+1} E_u^n(i+1/2, j+1, k) - E_u^n(i+1/2, j, k)] / \Delta v + [h_i E_v^n(i, j+1/2, k) - h_{i+1} E_v^n(i+1, j+1/2, k)] / \Delta u \}$$

$$E_z^{n+1}(i, j, k+1/2) = \left(\frac{1-S}{1+S} \right) E_z^n(i, j, k+1/2) + \frac{\Delta t}{\epsilon(1+S)h^2} \{ [H_u^{n+1/2}(i, j-1/2, k+1/2) - H_u^{n+1/2}(i, j+1/2, k+1/2)] / \Delta v + [h_{i+1/2} H_v^{n+1/2}(i+1/2, j, k+1/2) - h_{i-1/2} H_v^{n+1/2}(i-1/2, j, k+1/2)] / \Delta u \}$$

where $S = \sigma \Delta t (2\epsilon)$ and $h = d \sqrt{\cosh u^2 - \cos v^2}$, $h_i = d \sqrt{\cosh (i\Delta u)^2 - \cos v^2}$, d being the distance between the foci of the ellipse.

For the circular case the cell size is defined by $\Delta r = 1$ cm, $\Delta \phi = 5^\circ$ and $\Delta z = 1$ cm and in the r, ϕ and z directions these are respectively $20 \times 72 \times 30 = 43200$ cells encompassing the circular shaped applicator and phantom. The latter is sandwiched between buffer regions comprised of phantom material and each buffer region has, in the r, ϕ and z directions respectively $20 \times 72 \times 10 = 14400$ cells. The total number of cells in the computational domain is thus 72000 cells where $z_L - z_0 = 50\Delta z$. In elliptic geometry the cell structure in u, v and z coordinates is similarly organised with 43200 cells comprised of a central lattice containing the applicator and 28800 cells for the two adjacent buffer regions.

The six SAR results in Fig 9a to f compare FDTD computations with the approximate analysis eqn(4), illustrating the effect of circular and elliptic phantom geometry and the shift of the focal region with frequency in the range 75 to 82 MHz. The applicator is conformal with the phantom geometry and the circular phantom has a smaller cross-sectional area than the elliptic shaped phantom. Launcher hotspots in this central transverse plane of the phantom lie just outside the contour regions shown. The FDTD results Fig 9b at 75MHz shows a similar size focal region as the approximate analysis Fig 9a some 7cms deep. Altering the frequency to 82MHz certainly gives a worthwhile shift in the focal region's position Fig 9d and again the analytical results Fig 9c has a similar focal resolution but is more elongated. This is likely to be due to the various assumptions made in the analysis, but is otherwise seen as encouraging agreement in predicting the fundamental behaviour. The elliptical cases Figs 9e and f shows similar characteristics and, as may be anticipated, the focal region is more elongated. The distortion in the FDTD contours Fig 9f was found to be due to the excessive cell length near the elliptic foci.

It was found that the presence of hotspots around the launcher region, and to a lesser extent at the terminal end of the conducting strip, were greatly influenced by the phantom boundary diametrically opposite to the applicator. An example is given in Fig 10a for a circular phantom at 75 MHz where the hotspot regions near the launcher and terminal end of applicator are more pronounced than the desired focal region. The region outside the circular phantom is occupied by water and when the conductivity is increased to that of the phantom the hotspots are less prominent Fig 10b and one inference is that reflections are occurring in the configuration of Fig 10a at the phantom/water interface which disturb the launching process. In Fig 10c the water exterior to the phantom is made more conducting than the phantom and the hotspots become negligible suggesting that there are no reflections from the phantom boundaries. Finally FDTD computations in other than the central transverse plane indicate that the field markedly decreases but remains large near the edge of the conducting strip.

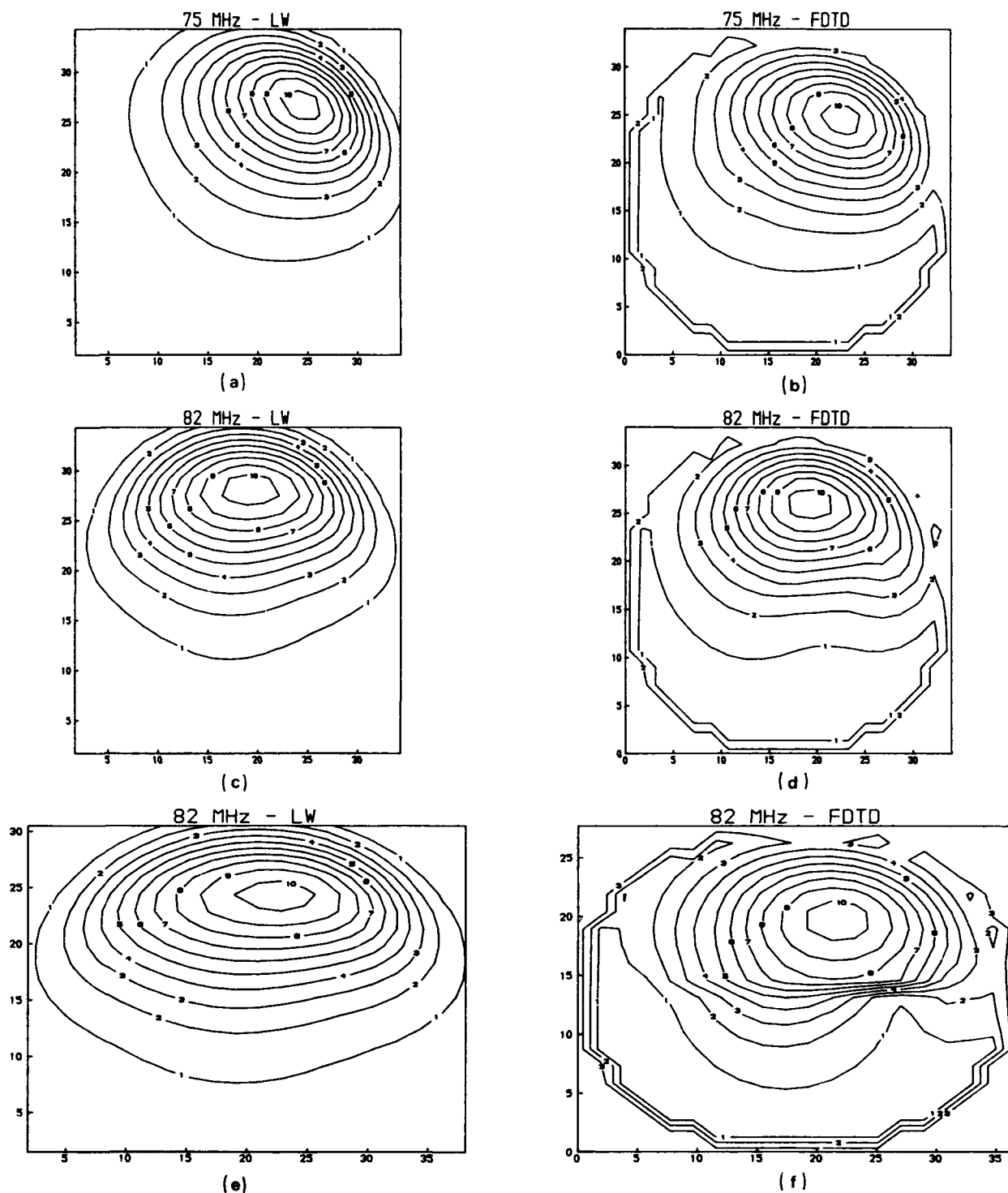


Fig 9 Computed relative Specific Absorption Rate (SAR) levels using approximate leaky-wave (LW) analysis eqns (1) - (5) and FDTD method. No bolus incorporated and $\epsilon_m = 76$, $\sigma_m = 6 \times 10^{-3}$ S/cm. Contour relative levels are on a linear scale with maximum of 10.

- | | |
|-------------------|--|
| (a) LW at 75MHz | } circular phantom diameter = 38 cm |
| (b) FDTD at 75MHz | |
| (c) LW at 82MHz | |
| (d) FDTD at 82MHz | } elliptical phantom as defined in Fig 3 caption |
| (e) LW at 82MHz | |
| (f) FDTD at 82MHz | |

At the phantom boundary, curves fragment due to interpolation routine used.

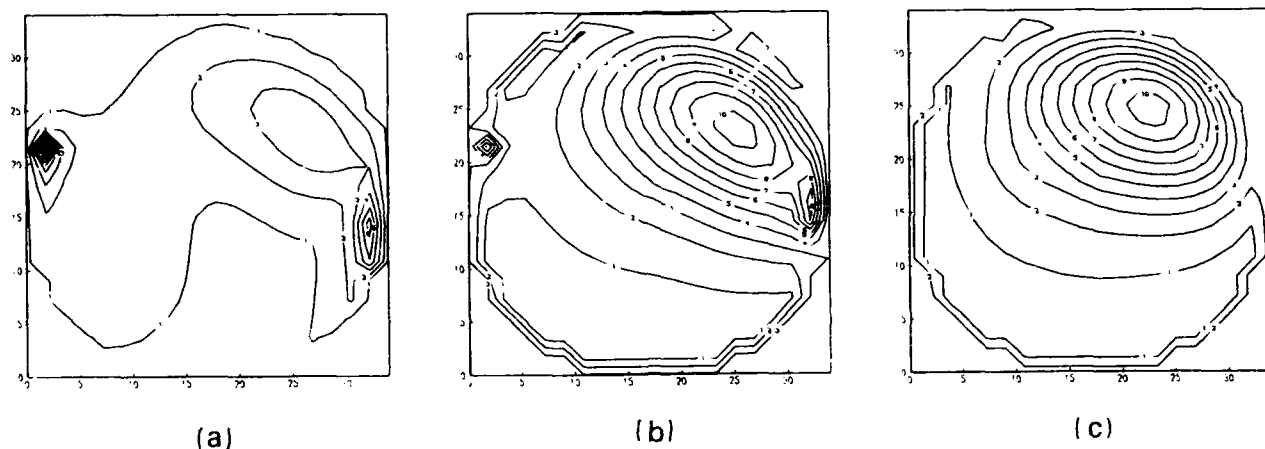


Fig 10 Examination of hotspots in computed 75MHz relative Specific Absorption Rate (SAR) levels using FDTD method. Contour relative levels are on a linear scale with maximum of 10 and electrical data for circular phantom is given in Fig 9 caption. The circular phantom is bounded by the square frame. Water at the phantom/water interface has the following loss.

- (a) 6×10^{-5} S/cm
- (b) 6×10^{-3} S/cm
- (c) 6×10^{-1} S/cm

5. DISCUSSION AND CONCLUSION

The combination of FDTD computations and independent approximate analysis together with some limited experimental results, confirm that a quasi-leaky-wave can be excited in a homogeneous tissue phantom using a wide microstrip-type applicator conformal with the phantom. The approximate analysis usefully highlights the need for a means of launching only the quasi-leaky-wave mode, since a TEM presence will not release the power to the phantom with the same focal resolution and standing waves will occur. These effects and the need for symmetric excitation of the strip is confirmed by FDTD computation of a planar version of the applicator. Unwanted hotspot regions around the launcher and the terminal end of the applicator have been examined by recasting the FDTD computations in curvilinear form and examples for both circular and elliptic cylindrical phantoms are given. It is considered that with greater computer capacity and speed it would have been sufficient to work with cube Cartesian lattices throughout, but the experience with the curvilinear version has proved of interest and in the elliptical case has posed difficulties. The presence of a lossless bolus region appears not to disturb the overall behaviour. The FDTD method is ideally suited to layered and other heterogeneous media as presented by realistic tissue and this has been outside of the present study.

The results of this study suggest that the quasi-leaky-wave applicator concept is potentially useful, offering good focal resolution at a depth that could be particularly useful in the abdominal and pelvic regions. The use of one generator and a frequency scanned beam are added advantages, but the development of such a device would demand attention to the hotspot behaviour, design of a symmetric launching system, the possible sensitivity of the focal region to tissue characteristics and a means of accommodating a variety of patient dimensions. Mode filters might be usefully incorporated in the conducting strip to assist the launching process. It is also possible that the launcher region could be offset from the tissue as sketched in Fig 1b and additional cooling applied to hotspot regions. The hotspot behaviour also appears to be sensitive to absorbent loading at the tissue/water interface and this needs further investigation.

In conclusion the analytical, computational and experimental work has confirmed the potentially useful properties of quasi-leaky-wave generation in hyperthermia cancer treatment and issues requiring further investigation have been identified.

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Evaluation of Clinical Hyperthermia Treatment Using Time Domain Finite Difference Modelling Technique

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Abstract

The use of numerical modelling techniques, especially the time domain finite difference method, has improved the understanding of power deposition in human bodies undergoing cancer treatment with microwave or radio-frequency waves. In this paper, clinical hyperthermia treatments are modelled with the time domain finite difference method. Quantitative assessment criteria are defined for the evaluation of computed power deposition patterns and are found to be useful in determining whether a particular treatment is likely to succeed.

1. Introduction

Microwave or radio-frequency wave induced hyperthermia is a method of treating cancer by raising the temperature of malignant tissues to 42-43°C for an extended period of time. One major concern in electromagnetic hyperthermia is to be able to deposit sufficient amount of power in the cancer tumour sites without overheating other healthy tissues. The human body is a complicated heterogeneous dielectric body and the physics of electromagnetic interactions dictates that the irradiation of such a body will result in highly non-uniform field distribution and heating pattern.

In the past decade, computational electromagnetics has been applied to the design and evaluation of antennae (applicators) used in electromagnetic hyperthermia [Iskander *et al.* 1982; Lau *et al.* 1986; Shaw *et al.* 1991] and the calculation of whole body power absorption of un-intended electromagnetic radiation [Spiegel *et al.* 1989]. Computational electromagnetics has been particularly useful in hyperthermia because of the immense difficulty in monitoring either power or temperature within the patient.

The time domain finite difference (TDFD) method [Taflove and Umashankar, 1989] of solving Maxwell's equations has become widely used in computational electromagnetics in general. This can be observed from the ever increasing number of papers published and presented [Miller, 1991] on this method. It has long been recognised that the TDFD method is well suited for the hyperthermia problem [Lau and Sheppard, 1986] due to its efficiency in the usage of computing resources.

With the potential ability to achieve better localization of power and deeper penetration into the body, the annular phased array (APA) hyperthermia system has been used in many hyperthermia centres over the world. The system is based on the principle of phased antenna array. It is hoped that by varying the phase and amplitude of individual applicators, field pattern can be steered to allow the tumour site to be selectively heated up.

The APA hyperthermia system has been modelled extensively with the TDFD method [Wang and Gandhi, 1989; Sullivan, 1990; Lau and Sheppard, 1990]. Computed data have also been validated by experiments performed on homogeneous tissue-equivalent phantoms [Sullivan, 1990; Sullivan, 1991]. Techniques based on TDFD simulations have been proposed for planning hyperthermia treatment [Sullivan, 1991]. Although simulations have been carried out on patient-specific models (i.e. those constructed from tomographic scans), the computed data have yet to be validated due to experimental difficulties. So far such studies yield valuable information on the performance of the APA system under various applicator placement, phase and amplitude settings. However, little work has been reported on if and how data from computer models can be related directly to real clinical treatments. Such correlation study must be pursued to build up the confidence of clinicians in the use of simulation data, in parallel with other effort such as building better and faster models.

This paper aims to demonstrate how TDFD models can be correlated, albeit in a qualitative manner by necessity, to clinical observations. Quantitative parameters are defined to assess the effectiveness of treatment using an APA hyperthermia system.

2. Method

As the TDFD method has been well documented in many publications [Taflove and Umashankar, 1989], only the briefest account of the algorithm will be given here. Maxwell's two time dependent curl equations are approximated by an explicit finite difference scheme in cartesian coordinates. The E and the H field components are positioned on a lattice first proposed by Yee [Yee, 1966]. Given an initial field excitation, explicit time-marching is performed alternately on E and H field components until steady state is reached. Dielectric and metallic objects are represented by collections of cubic lattice cells which have identical permittivity and permeability values assigned. Details of the code used for this study can be found in [Lau and Sheppard, 1986].

Two dimensional models based on the BSD-1000 hyperthermia system were used in the current study. Figure 1 illustrates a typical model consisting of eight waveguide applicators surrounding the patient. The gap between the apertures of the applicators and the patient was filled with de-ionized water. The patient area of the models was constructed from computer tomographic (CT) images obtained in a plane containing the maximum area of a tumour in the pelvic region. Closed boundaries of all the organs on the CT slice were first digitized manually and the TDFD code filled in each organ with the appropriate dielectric properties automatically. The dielectric parameters used for the models are shown in table 1.

Three field components, namely E_z , H_x and H_y were solved in the models. The waveguide applicators were excited with a sinusoidally varying E_z source across the width of each applicator. The amplitude of the source was modulated by the term $\sin(\pi x/a)$ where a is the width of the waveguide aperture and $0 \leq x \leq a$. The width of each waveguide was 0.2m and the length was 0.15m. The frequency modelled was 60 MHz. The spatial resolution of the uniform finite difference grid used was 0.004m and all models consisted of 200 by 200 lattice cells. The temporal resolution used for the time marching was 0.004m divided by twice the speed light in free space. Steady state $|E|$ field value at each lattice cell of the model was extracted after six cycles of the incident wave had elapsed, the equivalent number of time steps needed for each run was 14988. The power deposition in the patient was also computed at each cell by the specific absorption rate (SAR), defined as: $\sigma|E|^2/\rho$ where σ was the electrical conductivity and ρ was the density of the tissue. The unit for SAR is W/kg.

The computer platform used for the study was a VAXstation 3100 with 24MB main memory running VMS operating system. The TDFD code was written in standard Fortran 77 [Lau and Sheppard, 1986] and the processor time required for each run was on average three hours. The actual memory required for the simulation was about 1.25MB. Graphics pre- and post-processing offered by this TDFD code was based on either the UNIRAS package or the built in window software of the computer.

3. Case Studies

In this section, simulations and clinical studies on three patients are reported. In all the TDFD data computed, two parameters are used to assess the quality or the effectiveness of a treatment. The maximum tumour SAR is simply the maximum value of SAR (in W/kg) found in the tumour. The treatment ratio is defined as ratio of the average SAR in the tumour to that of the nearby sensitive organ. In clinical hyperthermia treatments, it is usual to monitor the tumour and a nearby organ which can easily be damaged by excessive heating. When the temperature in the nearby organ rises above the tumour being treated, the treatment is aborted. Hence the evaluation of treatment ratio as defined parallels the clinical monitoring procedure.

The size of each of the patients was measured in terms of the major and minor axes of the CT image of the patient as illustrated in figure 1. Table 2 shows the dimensions of the three patients reported in this study.

3.1 Patient I

This patient suffered from a rectal tumour and the nearby sensitive organ was chosen to be bladder. When the patient was treated with the BSD-1000 system with all applicators in phase and with equal power, the temperature of the bladder was observed to rise above that of the tumour. The session was aborted and a phase lag of 65° was introduced to half of the applicators (1,2,7 & 8) before treatment resumed. The bladder temperature was seen to drop below the tumour temperature which increased and the treatment was allowed to continue for a further period of time.

Computer simulations were performed on this patient with both the in-phase and the 65° phase lag settings. Figure 2 shows the field distribution in the entire system for the in-phase treatment. The corresponding SAR distribution in the patient is shown in figure 3. The SAR distribution of the 65° phase lag treatment is presented in figure 4.

The maximum tumour SAR's and treatment ratios were calculated for both treatments and are shown in table 3. It can be seen that the improvement in the treatment is reflected by the increased values of both parameters in the 65° phase lag case, especially in the substantial increase in the treatment ratio.

3.2 Patient II

It has been observed clinically that in patients with central pelvic tumours (e.g. bladder tumour), localisation of heating in the tumour is more difficult to achieve than other off-centre tumours such as those presented in Patient I. Generally, the whole body heats up fairly uniformly and treatment is then aborted due to excessive blood pressure and heart rate, both physiological responses to an overall temperature rise in the body.

In order to see if such observation can be made from a computed SAR pattern, a patient with bladder tumour was simulated with all applicators irradiating in-phase. The resulting SAR pattern is shown in figure 5. The nearby sensitive organ for this patient was the rectum. The maximum tumour SAR and the treatment ratio are shown in table 4, they indicate that power absorption in the tumour is low and selective heating is extremely poor. It should also be noted that the maximum tumour SAR in this patient is much lower than that of Patient I, indicating that it would be more difficult to raise the bladder tumour to the desired hyperthermic temperature range compared with patients with rectal tumours.

3.3 Patient III

This patient suffered from a bladder tumour as Patient II. However, the size of this patient was larger than Patient II. The nearby sensitive organ was also rectum.

TDFD simulation was carried out for this patient. The SAR pattern is shown in figure 6 and the assessment parameters for this patient can be found in table 4. The treatment ratio is very similar to that of Patient II but the maximum tumour SAR is lower. These values suggest poor localisation of heating inside the tumour and that it would be more difficult to raise the tumour temperature of a large patient than in a smaller patient. Such trends are again often observed in clinical hyperthermia treatments.

4. Conclusions and Discussions

In this study, it has been shown that by applying quantitative analysis to data obtained from TDFD simulations, clinically relevant information regarding the effectiveness of a treatment can be obtained. Specifically, it has been demonstrated that features obtained from computed data corresponded well to clinical observations in :

- (1) the improvement of treatment of rectal tumour when the APA system is operating with a phase shift between groups of applicators;
- (2) the difference between treatment of central and non-central tumours; and
- (3) the difference between treatment of patient with different sizes.

The three cases presented in this paper are samples from a large number of patients which have been similarly analyzed. Through systematic evaluation and organization of such data, a data base has been constructed. It can provide clinical practitioners of hyperthermia with useful guidelines which they can understand and apply with confidence to the treatment planning and have some idea as to the degree of success they can expect.

The simulation and analysis presented in this work cannot replace rigorous experimental validation through field and temperature measurements in patients or realistic tissue-equivalent phantom. Nevertheless, it provides a practical way to allow data from computer models to be related to clinical experiences.

This work was performed mostly on two dimensional models due to the fact that computing resources required to evaluate a large number of cases in full three dimensions are still prohibitively large. However, a small number of three dimensional TDFD models have been built and data from such simulations so far are consistent with the findings from the two dimensional work.

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Table 1 : Dielectric Properties Used for TDFD Models at 60 MHz

Dielectric	Relative Dielectric Constant	Conductivity (Sm^{-1})
Water at 15°C	82.24	0.001
Bone	7.35	0.053
Fat	15.05	0.215
Muscle	83.85	0.791
Skin	77.27	0.540
Bladder/Tumour/Rectum	87.38	0.750

Table 2 : Dimensions of Patients in Case Studies

	Major Axis (cm)	Minor Axis (cm)
Patient I (female)	32.2	22.5
Patient II (male)	33.1	18.8
Patient III (male)	35.0	24.1

Table 3 : Treatment Assessment for Patient I

	Maximum Tumour SAR (W/kg)	Treatment Ratio
All applicators in phase	11.7	2.81
App. 1,2,7,8 at 65° phase lag	14.5	5.21

**Table 4 : Treatment Assessment Comparison for Patients I,II and III
(treatment with all applicators in-phase)**

Patient	Maximum Tumour SAR (W/kg)	Treatment Ratio
I (rectal tumour)	11.7	2.81
II (bladder tumour)	4.6	1.31
III (bladder tumour)	2.1	1.29

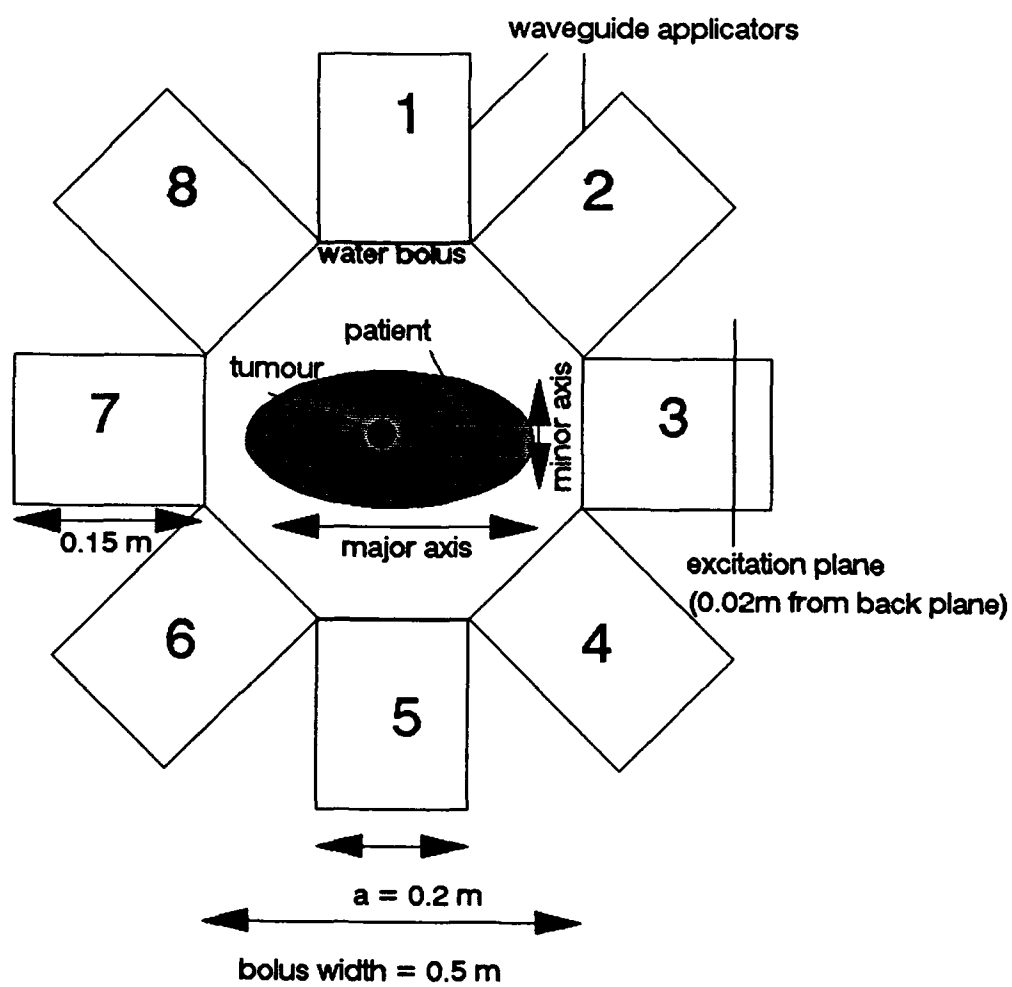


Figure 1 : TDFD Model of APA Hyperthermia System

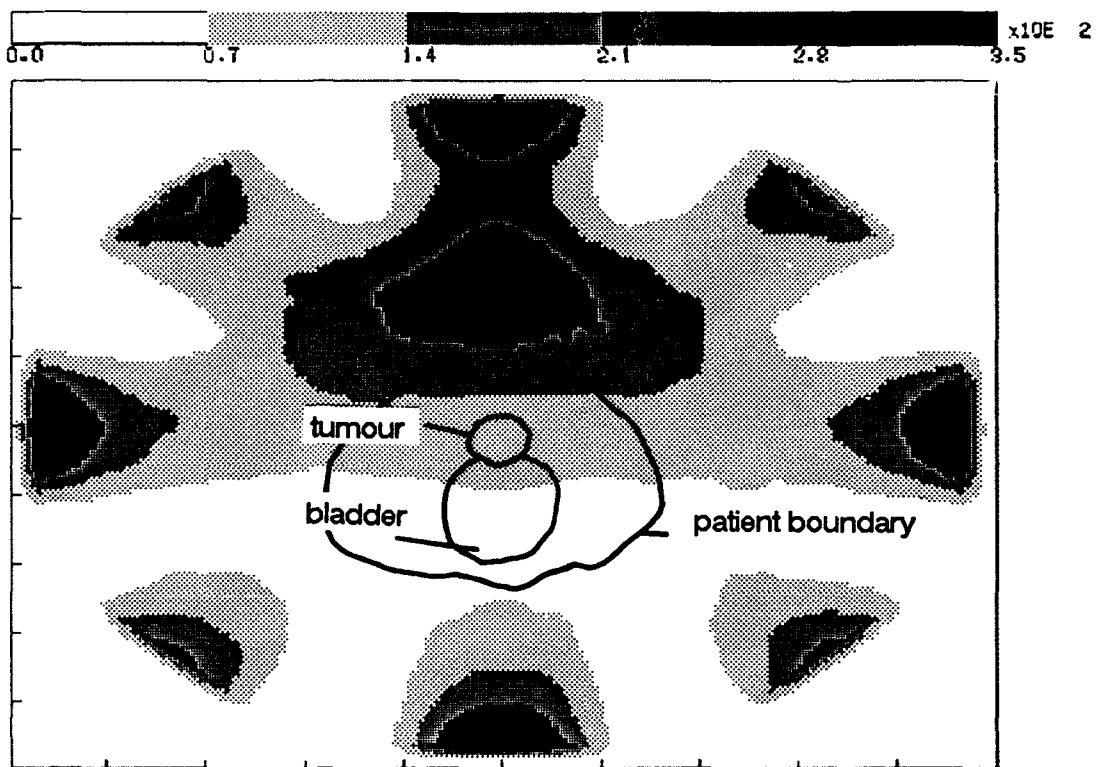


Figure 2 : $|E|$ field plot of model when Patient I is treated with all applicators in-phase.
Range of relative field values plotted is 0 - 350 V/m.

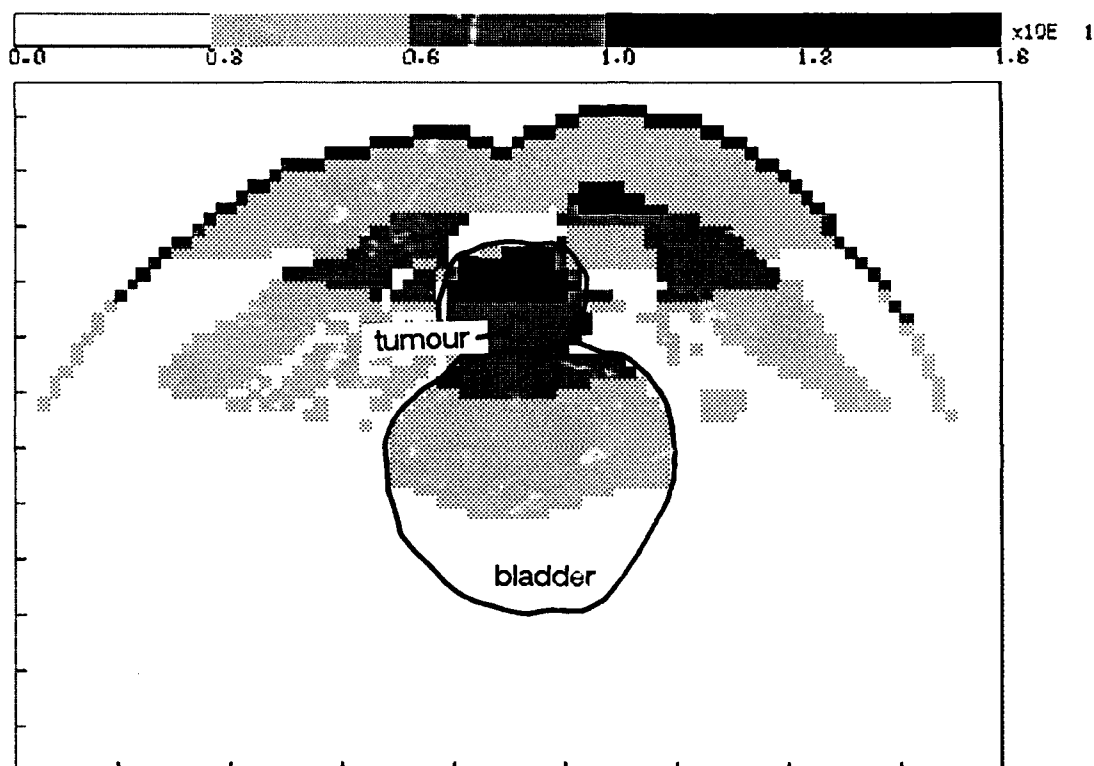


Figure 3 : SAR plot of Patient I when treated with all applicators in-phase.
 Range of grey scale displayed corresponds to 0.0 - 16.2 W/kg.
 Maximum tumour SAR is 11.7 W/kg.
 Physical dimensions plotted is 0.356 m (width) by 0.252 m (height).

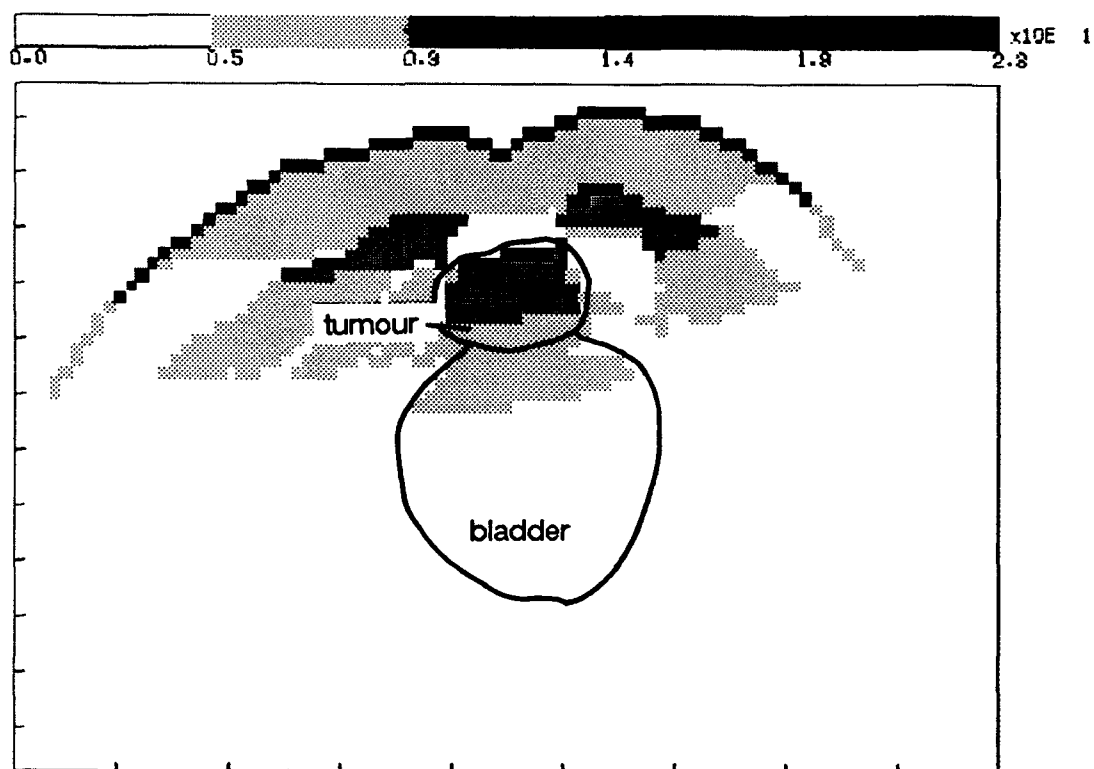


Figure 4 : SAR plot of Patient I when the phase of applicators (1,2,7 & 8) lags the rest of the applicators by 65° .
 Range of grey scale displayed corresponds to 0.0 - 23.0 W/kg.
 Maximum tumour SAR is 14.5 W/kg.
 Physical dimensions plotted is 0.356 m (width) by 0.252 m (height).

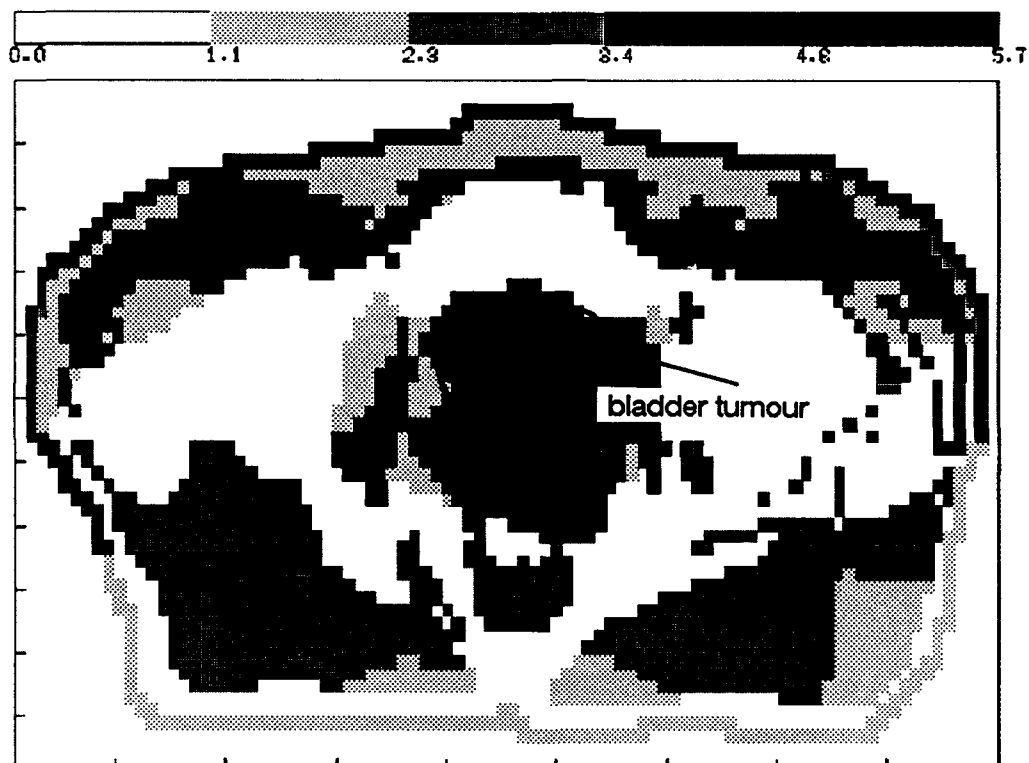


Figure 5 : SAR plot of Patient II when treated with all applicators in-phase.
 Range of grey scale displayed corresponds to 0.0 - 5.7 W/kg.
 Maximum tumour SAR is 4.6 W/kg.
 Physical dimensions plotted is 0.360 m (width) by 0.220 m (height).

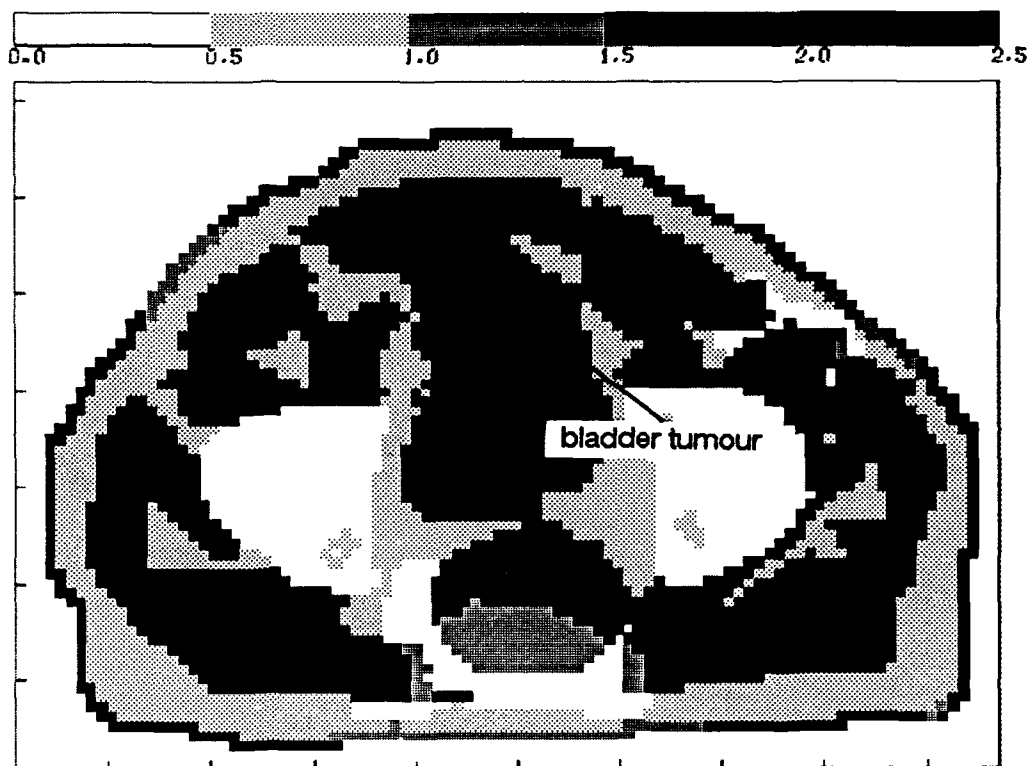


Figure 6 : SAR plot of Patient III when treated with all applicators in-phase.
Range of grey scale displayed corresponds to 0.0 - 2.5 W/kg.
Maximum tumour SAR is 2.1 W/kg.
Physical dimensions plotted is 0.388 m (width) by 0.288 m (height).

Optimizing Planar Source Distributions for Deep Power Deposition in Biological Tissue

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Abstract

In non-invasive hyperthermia, penetration depth in high water content biological tissue can be increased up to 3 times using a focused instead of uniform surface electric field distribution. The focusing involves maximizing the field integral at a focal point by solving for the surface phase function which makes the integrand real and positive for all surface points. The resulting non-linear differential equation is solved in using a series approximation. A focused power deposition pattern is presented using this ideal planar distribution which is the theoretical optimum for high resolution hyperthermia cancer treatment.

Introduction: EM Cancer Treatment

Favorable responses to electromagnetic heating of tumors to 42° - 45° C has generated interest in developing antenna applicators to assist in treating cancer in humans [Overgaard, 1972, and Hahn, 1982]. Electromagnetic radiation has advantages over other current forms of treatment since it can be administered non-invasively, without anesthesia, and causes minimal side effects. Hyperthermia also shows promise when used in conjunction with standard treatment modalities, by increasing the absorption and response of chemotherapeutic agents or by attacking radiation resistant cells [Strohbehn, 1984]. It is essential in designing these antennas to find field distributions which maximize the deposited power at the tumor, while keeping healthy intervening tissue from overheating. The "best" focused field pattern is one that has at least as much power at the focal target as on the surface or at any locations of secondary maxima.

The current analysis assumes a simple uniform geometric model of tissue volume with a localized tumor at its center, exposed to radiation from a surrounding antenna aperture. Previous work has optimized the source distributions for spherical and cylindrical geometries [Rappaport, 1987, Pereira, 1991, Jouvie, 1986]. This study will examine maximizing the power at the mid-plane of a planar slab, optimally illuminated from both sides. As

such, it provides a benchmark for possible microwave heating of relatively flat portions of the body: primarily chest and abdomen.

Electrical characteristics of human tissue vary non-linearly with frequency: with conductivity increasing and dielectric constant decreasing as frequency increases. To determine the best power dissipation pattern in a volume of tissue, the frequency must be selected to balance the competing effects of exponential wave decay and geometric focusing. A higher frequency wave has a smaller wavelength (and thus decays more rapidly), but can be focused better than a lower frequency wave. The present synthesis effort concerns power deposited in high water content tissue (HWC): muscle, organs, and blood, which has higher conductivity and dielectric constant, and so is more challenging to focus waves at depth in than low water content tissue (LWC): fat and bone.

It has been determined that for a deep, well-formed focal maximum in uniform HWC tissue, 915 MHz is the best standard frequency of electromagnetic radiation [Rappaport, 1988]. Above 915 MHz, the wavelength is too short to take advantage of lower loss per wavelength and increased constructive interference of HWC tissue; while below 915 MHz, the greater tissue conductivity eliminates a high-resolution, well-defined focal maximum. For uniform heating, lower frequencies are better, but to target a localized tumor and minimize heating of the surrounding healthy tissue, frequencies above 500 MHz are necessary. The dielectric constant and conductivity of HWC tissue at 915 MHz are about $\epsilon' = 51$ and $\sigma = 1.28$ S/m [Johnson, 1972].

Optimization Procedure

The method used for deriving the best power pattern in a planar slab involves integrating the surface field distribution. Although the electric field and deposited power can be increased at a focal target in lossy tissue by specifying a conjugate phase distribution on a single planar source irradiating a half-space, it is not possible to raise the power levels at depth to that of the surface [Loane, 1986, Gee, 1984]. To deposit as much heat in the center of a slab model of HWC tissue as on its surface requires balanced excitation from both sides. The maximum thickness of the tissue slab is determined by this focal point global maximum condition, and is a function of frequency and source distribution. It is the point of this study to find this maximum slab thickness.

The standard focusing practice of specifying a conjugate (or time-reversed) source phase distribution maximizes the focal field in free space or lossless media. However, in dissipative media, an additional correction phase function is required to ensure all points of the aperture constructively contribute to the field at the focus. It should also be noted that if the medium is lossy, an infinite, uniform amplitude aperture does not produce infinite field intensity at the focal point.

By first assuming the source fields on opposing source planes have uniform amplitude, the synthesis problem reduces to finding the best phase distribution. The phase function is determined so as to make the integrand purely real for the observation field-point at the focus. This ensures that every point of both source planes adds constructively at the focal target. For other points—both in the longitudinal and transverse directions—there will be some measure of destructive interference, which will enhance the relative power at the focus.

The opposing planar sources, shown in Figure 1, have circular symmetry about a focal axis. The observer point is identified by vector $\bar{r}$. When $\bar{r}$ lies along the axis of symmetry at the point $(0, 0, z)$, $d/2 + z$ is the distance to the source at $z = -d/2$, and $d/2 - z$ is the distance to the source at $z = d/2$. The tangential electric field on the source plane is assumed to be y -polarized with unit amplitude, and have both tangential (radial) and longitudinal (axial) real phase and amplitude dependencies expanded about the axial coordinate at each source:

$$\bar{E}_t(\bar{r}) = \hat{y} e^{j\Phi(\rho) + j\Gamma(\rho)(\frac{d}{2} \pm z) + j\Upsilon(\rho)(\frac{d}{2} \pm z)^2} e^{\Xi(\rho)(\frac{d}{2} \pm z)} e^{\Omega(\rho)O((\frac{d}{2} \pm z)^3)} \quad (1)$$

where $\Phi(\rho)$ is the surface phase radial dependence at $z = \mp d/2$, and the choice of sign preceding z is the positive for the source at $z = -d/2$ with wave propagation to the right, and negative for source at $z = d/2$ propagating to the left. Note that for each source at $z = \pm d/2$ there is no amplitude variation as a function of ρ , but the derivative of $\bar{E}_t$ with respect to z at the surface has non-zero amplitude with radial dependence $\Xi(\rho)$. It will be assumed that the amplitude varies exponentially (with linear exponent rather than as e^{az^2} , or higher order). Similarly, the Γ and Υ phase functions are necessary since the normal derivatives have non-zero phase. All phase dependence higher than second order on z are lumped into the last factor of Equation (1).

Inserting this assumed electric field into the circular cylindrical version of the Helmholtz Equation,

$$\left(\frac{\partial^2}{\partial z^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + k^2 \right) \bar{E} = 0, \quad (2)$$

yields one real and one imaginary equation:

$$\Xi^2 - \Gamma^2 - \Phi'^2 + \left(\frac{d}{2} \pm z \right) [\dots] = \alpha^2 - \beta^2 \quad (3)$$

and

$$-2\Gamma\Xi + \Phi'' + \frac{\Phi'}{\rho} + 2\Upsilon + \left(\frac{d}{2} \pm z \right) [\dots] = 2\alpha\beta \quad (4)$$

where Ω and the terms involving derivatives of Φ , Γ , Ξ , and Υ multiplying $(\frac{d}{2} \pm z)$ have been suppressed. The wave number is $k = \beta - j\alpha = (\omega/c) \sqrt{\epsilon' - \sigma/\omega\epsilon_0}$ for radian frequency ω and speed of light c .

At each source, separately, $z = \mp d/2$, Equations (3) and (4) reduce to:

$$\beta^2 - \alpha^2 - (\beta_z^2 - \alpha_z^2) - \Phi'^2 = 0 \quad (5)$$

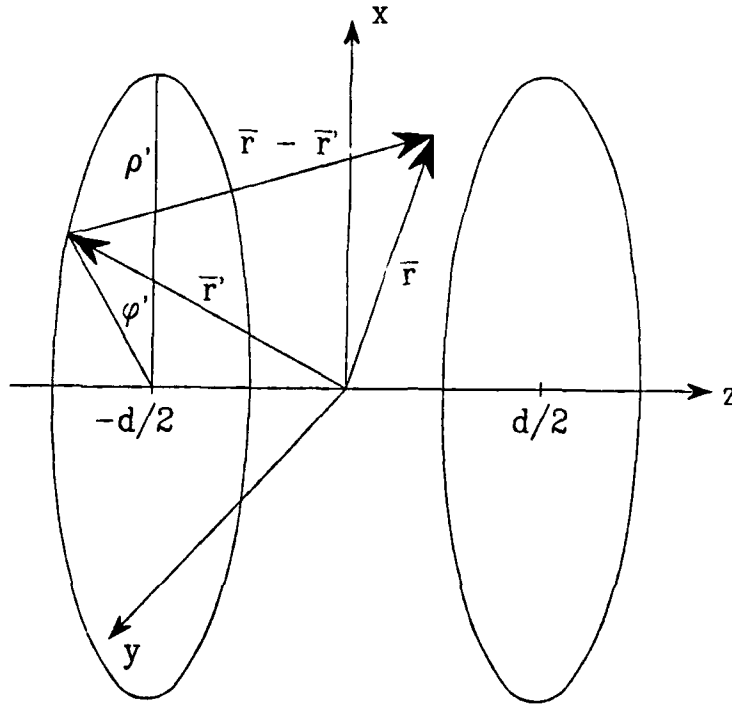


Figure 1: Opposing planar surface source excitation of tissue slab geometry with desired focal target at the origin.

and

$$-2\alpha\beta + 2\alpha_z\beta_z + \Phi'' + \frac{\Phi'}{\rho} + 2\Upsilon = 0 \quad (6)$$

where the natural substitutions, $\Xi(\rho) = -\alpha_z(\rho)$, $\Gamma(\rho) = \beta_z(\rho)$ have been made in analogy with inwardly propagating inclined plane waves.

The derivative of the source phase with respect to radial coordinate, $\Phi'(\rho)$, is thus given in terms of these unknown auxiliary functions representing the normal component of the propagation constant $k_z = \beta_z - j\alpha_z$.

One additional equation is necessary to uniquely specify the phase. This final equation is the focusing condition. The Kirchoff integral equation for electric field caused by unity amplitude surface electric field can be written in terms of the parameters and geometry discussed above as [Kong, 1986],

$$\begin{aligned} \bar{E}(\bar{r}) = \hat{y} \int_0^{2\pi} d\phi' \int_0^\infty \rho' d\rho' \left[j \left(\frac{d/2 \pm z}{R} \beta + \beta_z \right) \right. \\ \left. + \frac{d/2 \pm z}{R} \left(\alpha + \frac{1}{R} \right) + \alpha_z \right] \frac{e^{-j[kR - \Phi(\rho')]} }{4\pi R} \end{aligned} \quad (7)$$

where the sources are on the planes at $z = \pm d/2$, and $R = \sqrt{\rho'^2 + (\frac{d}{2} \pm z)^2}$ is the distance from the source point $\bar{r}' = (\rho', \phi', \frac{d}{2})$ to the observation point along the focal axis $\bar{r} =$

$(0,0,z)$. The terms α_z and β_z result from the normal derivatives of the surface electric field. They correspond to the surface magnetic field.

The task is to determine $\Phi(\rho')$ so that the integral of Equation (7) is maximized at the observer focal point, $(\rho, z) = (0,0)$. This condition occurs when the integrand is entirely real and positive for all values of integration arguments ϕ' and ρ' . Writing the entire integrand in magnitude/phase form and setting the phase to zero at the focus, $z = 0$, yields the nonlinear differential equation for Φ :

$$\Phi(\rho') = \beta R_0 - \tan^{-1} \frac{\beta d/2 + \beta_z R_0}{d/2 R_0 + \alpha d/2 + \alpha_z R_0} \quad (8)$$

where $R_0 = \sqrt{\rho'^2 + (d/2)^2}$.

Differentiating Equation (8) with respect to ρ' results in:

$$\Phi'(\rho') = \beta \frac{\rho'}{R_0} + \delta(\rho') \quad (9)$$

where the correction phase function $\delta(\rho')$ represents the derivative of the arctangent function. In the subsequent analysis, the primes on the source radius will be suppressed.

In order to solve for $\Phi(\rho)$ using Equations (5), (6) and (9), the values of $\alpha_z(\rho)$ and $\beta_z(\rho)$ must be determined.

If Equation (6) remains finite at $\rho = 0$, the lowest order term of δ must be linear (i.e. $\delta(\rho) = \delta_1 \rho + O(\rho^2)$), and thus $\delta(0) = 0$. Further, since $\delta(\rho)$ is proportional to the derivative of the argument of the arctangent function of Equation (8), which in turn has terms composed of either ρ/R_0 , $\partial\alpha_z/\partial\rho$, or $\partial\beta_z/\partial\rho$, then at $\rho = 0$, a possible constraint is:

$$\begin{aligned} \left. \frac{\partial\alpha_z}{\partial\rho} \right|_{\rho=0} &= 0 \\ \left. \frac{\partial\beta_z}{\partial\rho} \right|_{\rho=0} &= 0 \end{aligned} \quad (10)$$

Appealing to Equation (5) and the notion that at $\rho = 0$ the incident wave is entirely normal, it can be safely concluded that $\alpha_z = \alpha + O(\rho^2)$ and $\beta_z = \beta + O(\rho^2)$. Choosing the precise form of α_z and β_z is somewhat arbitrary. As their values increase, the electric field at the focus, given by Equation (7) at $z = 0$ increases, even though the surface electric field stays constant at unity. This is analogous to coupling to a resonator at a point near an electric field null. Since these parameters represent the magnetic field sources, they cannot be selected to be too large at any point on the source surface, since they produce large electric fields one-quarter of a wavelength away. As long as α_z and β_z are bounded to values close to α and β , there will be no non-physical fields resulting from the normal electric field derivatives. Also, since the contributions to the focused field from surface

source points more than several wavelengths away from the focal axis becomes negligible, only the limiting behavior of α_z and β_z near $\rho = 0$ matters.

The phase term $\Upsilon(\rho)$ of Equation (1) represents the second-order normal dependence of the field. As $\rho \rightarrow \infty$, $\beta_z \rightarrow 0$ (because the wave would be propagating entirely in the radial direction), and the entire z -dependent phase is proportional to $\Upsilon(\rho)z^2$ (similar to the expansion of the radius dependence of the spherical Green's function for small z).

For $\rho \rightarrow \infty$ the argument of the arctangent of Equation (8) approaches a constant, while $R_0 \rightarrow \rho$. Therefore, $\Phi'(\rho) \rightarrow \beta$, and $\Phi''(\rho) \rightarrow 0$, and thus $\delta(\rho) \rightarrow 0$. Equations (5) and (6) are solved in this limit to give:

$$\begin{aligned}\beta_z|_{\rho \rightarrow \infty} &= 0 \\ \alpha_z|_{\rho \rightarrow \infty} &= \alpha\end{aligned}$$

Armed with these physical arguments for limiting behavior, the approximation form of α_z and β_z are selected to be:

$$\alpha_z \approx \alpha \quad (11a)$$

$$\beta_z \approx \frac{\beta}{\sqrt{1 + (\frac{\rho}{B_1})^2 + (\frac{\rho}{B_2})^4 + (\frac{\rho}{B_3})^6}} \quad (11b)$$

with the coefficients B_1 , B_2 , and B_3 to be found by numerically solving the non-linear differential Equations (5) and (8). The assumption of Equation (11a) simplifies these equations considerably. This assumption is reasonable, since $\alpha_z(0) = \alpha$, $\alpha_z(\infty) = \alpha$, and all other radial variations in this problem are monotonic. Also, since the amplitude is specified to be uniform on the surface for all radii, it makes sense for the media loss to be solely and entirely dependent on the axial coordinate.

Equation (6) need not be solved to determine the ideal phase function. However, since $\Upsilon(\rho)$ is unconstrained, except for its value at $\rho = 0$, it can assume any functional dependence required to solve Equation (6).

To find the coefficients of Equation (11b), Equation (9) is squared and substituted into Equation (5), yielding:

$$1 - \frac{1}{1 + (\frac{\rho}{B_1})^2 + (\frac{\rho}{B_2})^4 + (\frac{\rho}{B_3})^6} \approx \left(\frac{\rho}{R_0} + \frac{\delta(\rho)}{\beta} \right)^2 \quad (12)$$

which is solved for each ρ^{2n} term. Note that since the argument of the arctangent function of Equation (8) is a function of ρ^2 , $\delta(\rho)$ is an odd function of ρ , and the right hand side of Equation (12) is an even function of ρ . The series form of β_z approximates the focusing phase derivative well, while maintaining the correct initial and final values. To solve exactly Equation (12) would require an infinite series of even powers of ρ in the approximation of β_z , but as long as the error between the left and right hand sides of Equation (12) is small—especially for small radii, where the source field most strongly affects the focused

field—only three terms are necessary. This is explicitly shown to be true in the numerical example in a following section.

Although the selection of a_z seems unsubstantiated, and Equation (6) is used only to establish limiting behavior, as long as the solution for the source phase distribution $\Phi(\rho')$ specifies electric field which is self-consistent with the Helmholtz Equation and also gives an entirely positive integrand in Equation (7), it represents the optimal solution. The computation leading to this result is approximate to the extent that the amplitude is assumed to vary no more rapidly than exponentially in the normal direction (which, in fact, is observed in the power pattern results section), and that the phase correction is approximated by a three-term series (with extremely small error).

Secondary Maximum Suppression

Solving for the phase in the preceding section yields the best uniform amplitude electric field source distribution for maximizing power at the center of the planar tissue slab. The thickest slab which can safely be heated would be that for which the focal power is the *same* as the surface power and higher than any secondary volume field maximum. Higher surface power than the focal power risks overheating tissue close to the sources; while lower focal than surface power would allow a thicker slab to be safely heated.

Using the source distribution derived in the preceding section at 915 MHz unfortunately generates a symmetric pair of secondary electric field maxima along the z axis. These maxima must be reduced to below the focal field value to avoid overheating healthy intervening tissue. By slightly adjusting the source distribution, it is possible to correct the pattern, lowering the secondary maxima fields, without significantly affecting the focal field.

These secondary maxima, each of which is closer to one source plane or the other than the focal maximum, are more sensitive to the surface source regions close to the z -axis. Introducing a minor constant phase shift for source points in the region $\rho < a$ will have a strong destructive effect on the secondary maximum, since the phase of each source contribution varies considerably with radius. However, the focal maximum will not change as much, because the entire in-phase source region $\rho < a$ is phase shifted by the same amount. Of course, the maximum slab thickness will have to be reduced slightly, since the focal field will no longer be the ideal integral of purely positive contributions. Instead, the field at the origin will be the vector sum of two phasors separated by a small angle θ . The vector sum must be unity, so the sum of the magnitudes of the vectors (which in the previous ideal case simply equaled the focal field) must be a slightly larger than one. The equation for determining θ follows from the Law of Cosines,

$$\theta = \cos^{-1} \frac{I_1^2 + I_2^2 - 1}{2I_1 I_2} \quad (13)$$

where

$$I_1 = \int_0^a \rho d\rho' \int_0^{2\pi} d\phi' \mathcal{I}$$

$$I_2 = \int_a^\infty \rho d\rho' \int_0^{2\pi} d\phi' \mathcal{I}$$

and $\mathcal{I}$ is the integrand of Equation (7).

The field throughout the volume is thus given by:

$$\bar{E}(\bar{r}) = I_1 e^{j\theta} + I_2 \quad (14)$$

with the same values of the functions α_z , β_z , and Φ as in Equation (7), and the source-to-observer distance $R = \sqrt{(d/2 \pm z)^2 + \rho'^2 + \rho^2 - 2\rho'\rho \cos(\phi' - \phi)}$.

It is not important what the "best" value of radius a might be, as long as the angle θ is small, and the secondary fields levels are lowered to below the focal and surface field levels.

Maximum Slab Thickness Determination

An iterative procedure is used to find the thickest HWC tissue slab model with a central global maximum at 915 MHz. First, an arbitrary slab thickness d is selected, say, one wavelength. The required focused phase distribution, $\Phi(\rho)$, is obtained as described above, and the power pattern is calculated using Equation (7) along with the values of β_z and $\alpha_z (= \alpha)$ as stated in Equation (11). If the focal target power at $z = 0$ is greater than the surface or other secondary peak power, the slab thickness d is increased.

When the maximum thickness is reached, a radius a is selected for the constant phase shift to lower the secondary maxima of the power pattern. Once the greatest focal power peak is found, d is again increased, and the ideal focusing phase is recalculated.

The material characteristics of HWC tissue at 915 MHz are $\lambda = 4.46$ cm, $\beta = 1.40$ cm⁻¹ and $\alpha = 0.322$ cm⁻¹. The result of the iteration gives a maximum thickness $d = 11.3$ cm. The best coefficients for the approximation of Equation (12) for $d = 11.3$ cm are: $B_1 = 5.615$, $B_2 = 13.74$, and $B_3 = 19.3$ cm. Note that several significant digits are necessary to accurately specify these coefficients since the phase correction function $\delta(\rho)$ is of order unity radians/cm, its approximation should be accurate to at least two places, and the series representing the approximation should be correct within two additional places. The series approximation error for $\delta(\rho')$ phase is shown in Figure 2. This phase *derivative* error is presented rather than phase error since the derivative is being approximated in Equation (12), and the numerical integration needed to obtain the phase may introduce further error. It is seen in Figure 2 that the worst error occurs at about 35 cm, or 8 wavelengths from the focal axis, with a value of one-half of one percent of a radian (0.3°)

per centimeter. Inside a two wavelength source radius, the error is less than 0.0003 radians per cm.—clearly acceptable within the scope of this biological problem, where the measured electrical parameters are accurate to at most 1 %.

The resulting focused phase is given in Figure 3, along with the simple conjugate phase function with zero offset. Figure 4 shows a detail of the correction to the phase as a function of radius. Mathematically, this is a plot of the arctangent function of Equation (8). The correction is greatest at the focal axis, lowering the conjugate phase value by almost 1.28 radians. It falls off with increasing radius to 0.92 radians at $\rho = 25$ cm or 5.5 wavelengths. This amounts to about 20° variation in a radius of 5.5λ . This correction function does not vary much compared to the conjugate phase distribution; however, to determine the optimum power deposition pattern, all contributing factors must be considered. The difference between the electric field at the focal point resulting from the conjugate phase and from the ideal phase distributions amounts to about 0.1%, so the conjugate phase distribution is sufficient for focusing.

Power Pattern Results

Figure 5 shows three power patterns for electromagnetic radiation at 915 MHz in an infinitely wide slab of muscle tissue with thickness $d = 2.54\lambda = 11.3$ cm, as a function of distance from the focal target at the mid-plane. Each pattern is normalized to the power at the slab surface. The pattern with the least power at the focus $z = 0$ represents dissipated power for uniform phase and unity amplitude opposing sources located at $z = \pm \frac{d}{2}$. Its functional dependence is $P = |\cos(\beta - j\alpha)z|^2 / |\cos(\beta - j\alpha)\frac{d}{2}|^2$ [Rappaport, 1986]. For this slab thickness, all tissue more than 1.76 cm, or 0.4λ away from the mid-plane is overheated.

With phase focusing, including the lossy media correction function, the power at the focus increases by almost an order of magnitude. The slab thickness for which the surface power equals the focal power is $d = 11.4$ cm. Slight overheating occurs at the secondary maximum near 3.0 cm out from the mid-plane. Of course, this secondary peak can be reduced with a thinner slab by placing the sources closer to the focal target.

Phase shifting the source within the region $\rho < a = 3$ cm and using Equation (13) to find the shift $\theta = .40081$ changes the absolute power at the focus very slightly from the uniform amplitude case, while reducing the secondary peak to the level of the focal power. The limiting thickness, $d = 11.3$ is determined by this pattern. The sacrifice from introducing the constant phase shift decreases the maximum thickness 0.1 cm, less than 1%. The maximum thickness has increased over the uniform phase illumination by a factor of 3.2. Figure 5c represents the maximum amount of power delivered to the deepest point in a planar slab without risking overheating any other regions. Thus 5.65 cm is the deepest possible safe heating depth from plane microwave sources.

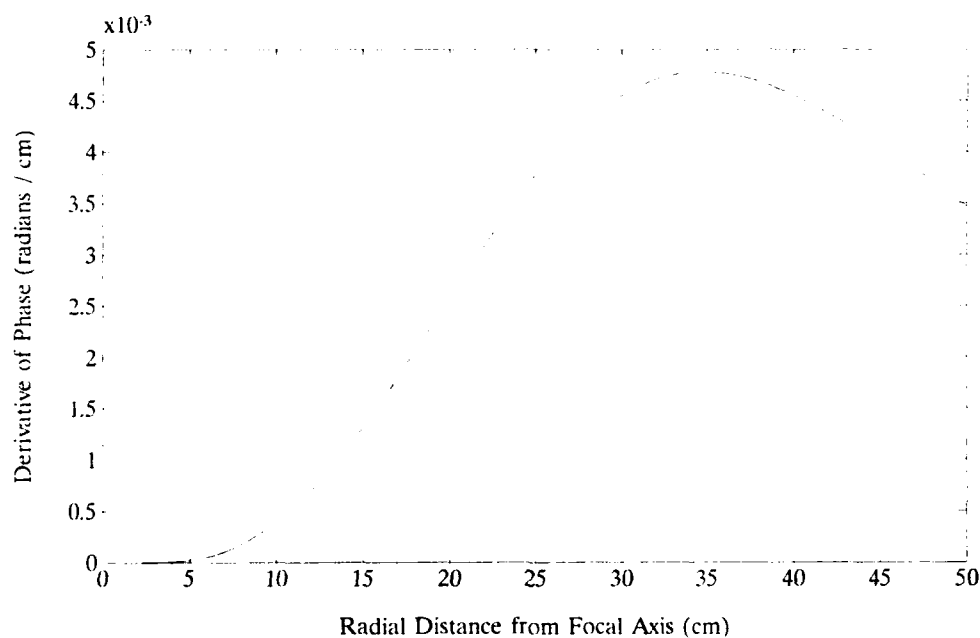


Figure 2: Error between the derivatives of focusing phase and the series approximation of $\sqrt{\beta^2 - \beta_z^2}$ for $d = 11.3$ cm.

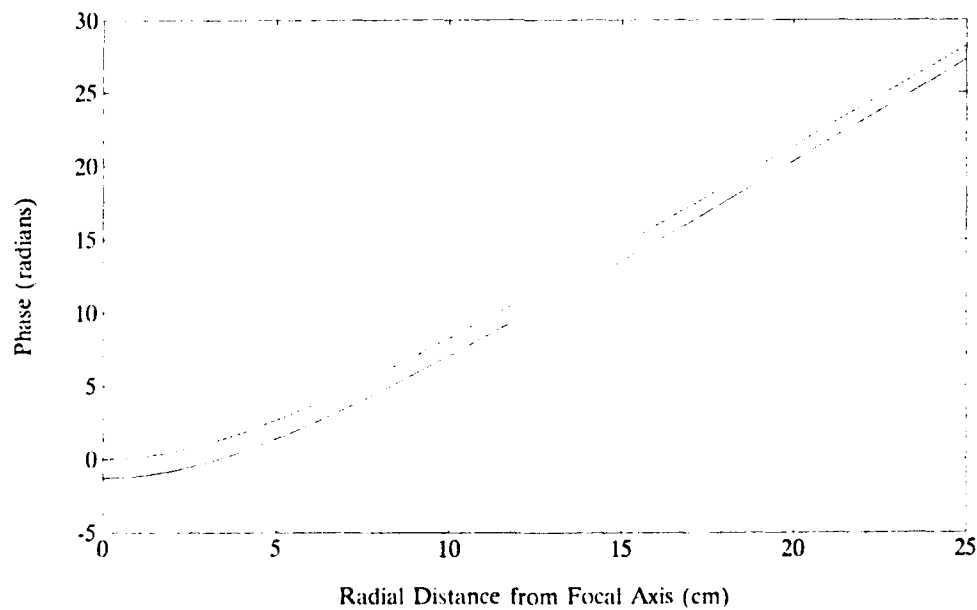


Figure 3: Conjugate (dashed) and ideal (solid) phase functions for focusing at a point on the mid-plane of a $d = 11.3$ cm thick slab of high water content biological tissue from the source plane at 915 MHz.

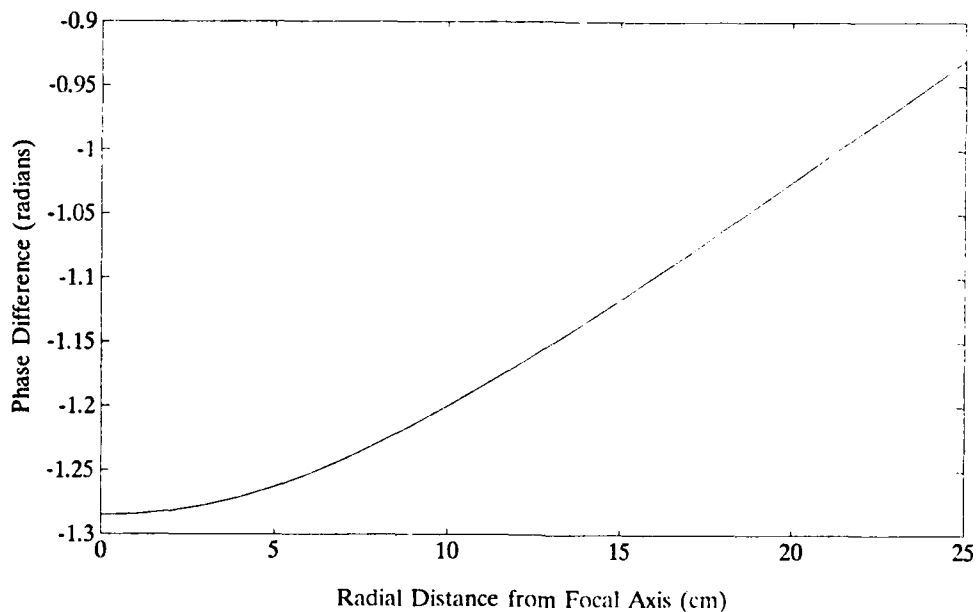


Figure 4: Correction function for complex integrand for focusing 915 MHz radiation at $d/2 = 5.65$ cm depth.

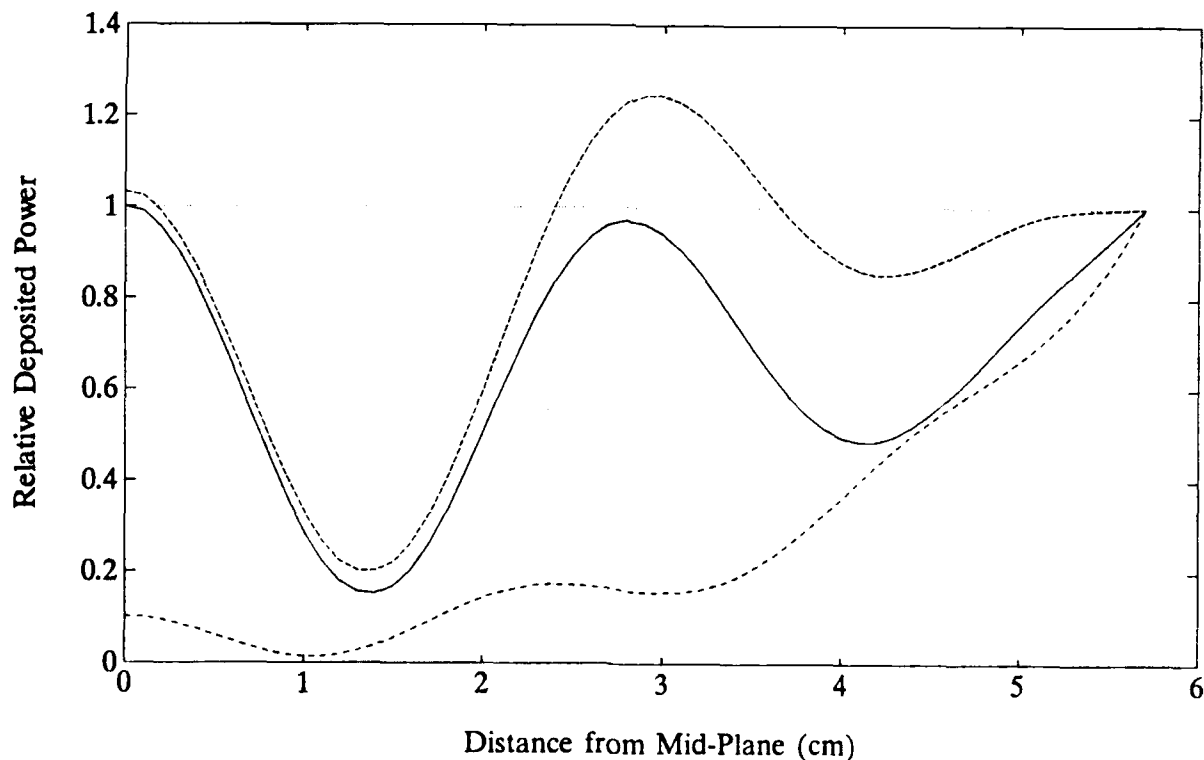


Figure 5: Power patterns for 915 MHz opposing planar sources: a) uniform source phase (dashed-dotted), b) phase focused unit amplitude (dashed), and c) phase focused with phase shift of .40081 for $\rho < 3$ cm (solid).

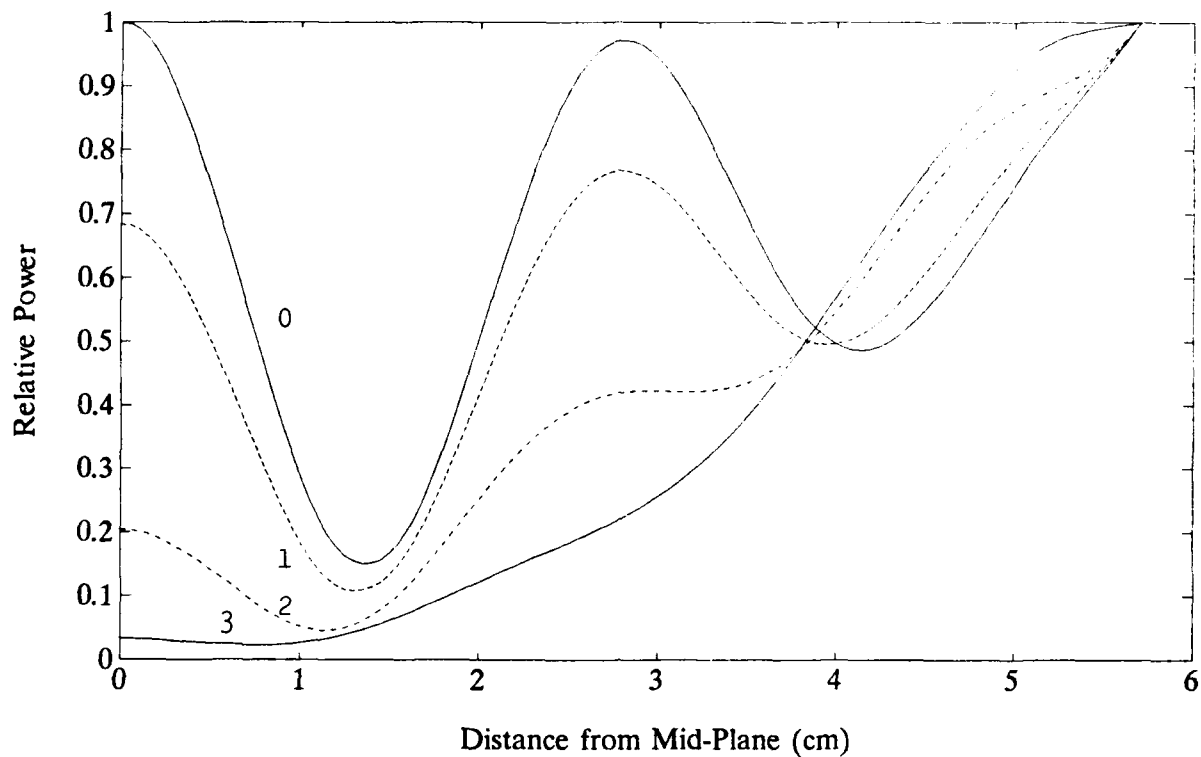


Figure 6: Optimum phase focused axial power deposition patterns for radial positions $\rho = 0, 1.0, 2.0,$ and 3.0 cm.

As a check, the axial power patterns for four radial positions: $\rho = 0, 1.0, 2.0,$ and 3.0 cm for the optimized source distribution of Figure 5c is shown in Figure 6. The focal power at $z = 0$ falls off to about two-thirds at $\rho = 1.0$ cm, and is almost negligible for greater radii.

Conclusions

The source excitation phase distribution for safely and efficiently heating the center of a planar slab of high water content tissue have been optimized. Corrections to the lossless conjugate phase focusing distribution produce only minimal increases in focal power.

The best planar source distribution can deposit a global power maximum in the center of a tissue volume more than three times thicker than for a uniformly excited slab. This power pattern is the best theoretical pattern. It may not be possible to duplicate the desired source phases and amplitudes with real hyperthermia applicators, but this pattern provides an ideal benchmark for determining feasibility of treatment.

It is conceivable that a different choice of constant phase shifting of part of the distribution might increase the ratio of focal point power to secondary maximum power without significantly suppressing the former below the surface power. However, this improvement is insignificant, especially in view of the large variation of measured tissue electrical characteristics.

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TWO-DIMENSIONAL TEMPERATURE RETRIEVAL IN BIOLOGICAL STRUCTURES BY MULTIFREQUENCY MICROWAVE RADIOMETRY: A SOBOLEV-SPACE SOLUTION

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Abstract

The problem of retrieving a two-dimensional temperature distribution from radiometric data measured at various frequencies and for different positions of the sensing antenna around the body has been considered. The retrieval has been modelled as an inverse problem whose solution is investigated in a suitably defined functional space which takes regularity properties of temperature functions into account. The retrieval of hot spots in a cylinder at uniform temperature has been numerically analyzed, the examples being relevant in the hyperthermia treatment of malignancies.

1. Introduction

Anti-tumoral hyperthermia is the application of heat to malignant cells as an adjuvant of radiotherapy and chemotherapy in cancer treatment. The monitoring and control of the temperature distribution, in an anatomic area heated during a hyperthermia session, are important aspects of treatments. Subcutaneous tissue temperatures are currently measured by inserting thermocouples or optical sensors. Drawbacks of these types of sensors exist and motivate an exploration of non-invasive methods of measurement. Among non-invasive methods different approaches have been studied. Multispectral microwave radiometry has been considered [Miyakawa, 1981; Bardati and Solimini, 1983; Chivé *et al.*, 1984; Mizushima *et al.*, 1986; Bocquet *et al.*, 1986] as being the only technique which measures the temperature directly, while other non-invasive techniques measure variations with temperature of physical quantities other than temperature.

Due to the strong attenuation of microwaves in high-water-content tissues, a microwave radiometer supplies data proportional to the average temperature in a tissue region up to a depth of 3-4 cm from the body surface. The average is weighted by a "weighting function" which takes into account (a) the morphology and dielectric properties of the tissue surrounding the antenna, (b) the frequency band of microwave receiver, and (c) the response of the contacting antenna used to receive the radiation. Properly, the antenna behaves as a transducer from fluctuating temperature-dependent electromagnetic radiation inside tissue to fluctuating electrical currents in the radiometer input device. Since the weighting function depends on frequency, the fine structure of the temperature can be obtained, in principle, by elaborating the data of radiometric measurements performed at different frequencies. The theoretical problem of the temperature reconstruction inside bodies of simple shapes (homogeneous and layered half-spaces) has received attention in the past. The retrieval has been modeled as the solution of a Fredholm integral equation of the first kind whose kernel is the weighting function. This inverse problem is an ill-posed one: the solution is not stable with respect to small variations of the data (noise). Therefore, only a finite number of components of the solution with respect to a suitable basis can be accurately determined. The appropriate basis is provided by the singular system of the integral operator [Bardati *et al.*, 1987; Bardati and Brown, 1991 a].

In a recent paper [Bardati *et al.*, 1991 b] it has been shown that the retrieval of temperature can considerably benefit from some *a priori* information on regularity properties of the function to be retrieved. The function space of physical temperatures has been assumed to be the space of continuous *thermal* functions. The latter have been defined as solutions of a steady-state heat-transfer equation together with an homogeneous Dirichlet condition on the boundary of the observed region. The computations, which have been carried out for the case of radiometric

observation of a homogeneous circular cylinder, show an improvement of retrievals with respect to previous reconstructions. However, closed-form expressions of thermal functions are readily obtained only for very simple structures. In this paper we consider the extension of the method to include cylinders whose electromagnetic and thermal behaviour is characterized by nonuniform parameters, therefore requiring a numerical approach. The data are simulated by letting the radiometric antenna sense the surface of a circular cylinder along a circle on a plane normal to the axis. In this way a two-dimensional inverse problem is generated, that is, the temperature is retrieved as a function of depth (i.e., in the direction from surface inwards) and in the transversal direction (along the scanning line).

2. Two-dimensional radiometer equation

Let us denote by Ω the cross-section of the cylindrical body, normal to the cylinder axis (z-axis). The antenna is modeled as a truncated parallel-plate waveguide Γ , width w , whose aperture lies on a plane α which is secant to the boundary, $\partial\Omega$, of Ω at points P_α , $P_{\alpha'}$ (Fig.1). Let P be intermediate between P_α and $P_{\alpha'}$. In the following $P(\phi)$ will identify the measurement point around the body, where ϕ is the polar angle in the fixed frame x,y . It is worth noting that a living tissue body undergoes a small deformation caused by the pressure of the contacting antenna, which is filled by a solid dielectric material in most practical situations. For the sake of simplicity, this deformation is accounted for in our model by cutting away $\Gamma \cap \Omega$.

The dependence of the radiometric datum, T_a , on the physical temperature $T(P')$ at a point $P' \in \Omega$ is given by the radiometric equation

$$T_a(P, \nu) = \int_{\Omega} W(P, P', \nu) T(P') dP' \quad (1)$$

where ν is frequency and W is a non-negative weighting function normalised according to

$$\int_{\Omega} W(P, P', \nu) dP' = 1 \quad (2)$$

T_a is called the available radiometric temperature of the source, when the latter is observed through the antenna at position P .

Equation (1) defines a mapping from an object space of functions of $P' \in \Omega$ into an image space of functions of ϕ and ν . In practice, only a discrete set of data is given, corresponding to radiometric data measured at M positions,

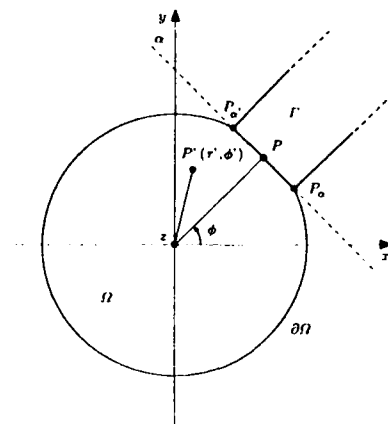


Fig.1 Geometry of the structure.

ϕ_m , and at N frequencies, ν_n , in a given frequency band, $[\nu_a, \nu_b]$. The radiometer equation for the case of discrete data is

$$p_{nm} = \int_{\Omega} W_{nm}(P') T(P') dP' , \quad (3)$$

where p_{nm} is an element of $\mathbf{p}$, a vector of $\mathbb{R}^{M \times N}$, and $W_{nm} = W(\phi_m, P', \nu_n)$. We are interested in a solution to (3) which is continuous in Ω and vanishes on $\partial\Omega$.

A suitable inner product between elements $\mathbf{p}$ and $\mathbf{q}$ of the image space of $\mathbb{R}^{M \times N}$ can be defined by means of the equation

$$(\mathbf{p}, \mathbf{q}) = \mu \sum_{n=1}^N \sum_{m=1}^M p_{nm} q_{nm} \quad (4)$$

where $\mu = (N-1)^{-1}(M-1)^{-1}$. It has been shown [Bardati *et al*, 1991 a; Bardati *et al*, 1991 b] that a suitable scalar product in the object space X_{Ψ} of physical temperature is the following

$$(u, v)_{X_{\Psi}} = \int_{\Omega} K(P') \nabla u(P') \cdot \nabla v(P') dP' \quad (5)$$

where K has the meaning of a thermal conductivity, while u and v are two elements of X_{Ψ} . To obtain a solution for T from equation (3) in the space equipped by the scalar product (5), we must consider thermal functions $\Psi_{nm}(P')$ such that, for any $T(P')$:

$$(\Psi_{nm}, T)_{X_{\Psi}} = \int_{\Omega} W_{nm}(P') T(P') dP' \quad (6)$$

Equation (6) enforces T to be the weak solution of

$$-\nabla \cdot (K \nabla \Psi_{nm}) = W_{nm}(P') \quad (7)$$

which satisfies a homogeneous Dirichlet condition on $\partial\Omega$ [Brezis, 1983]. Therefore the thermal functions are immediately recognized from (7) to be solutions of the classical stationary heat equation in Ω vanishing on the boundary.

Finally, solutions for T in the Sobolev subspace spanned by the above defined thermal functions are obtained by enforcing

$$(\Psi_{nm}, T)_{X_{\Psi}} = p_{nm} , \quad (8)$$

for $n=1, \dots, N$ and $m=1, \dots, M$. It is of interest, and will be done in the following, to compare solutions to (3) in X_{Ψ} to those which can be obtained in the object space equipped by the familiar $\mathbb{L}_2$ scalar product:

$$(u, v)_{X_w} = A^{-1} \int_{\Omega} u(P') v(P') dP' , \quad (9)$$

where $A = \text{mis}(\Omega)$. The integral equation obtained by using (9) instead of (5) is similar to equation (8) where, however, we find AW_{nm} instead of Ψ_{nm} .

We look, now, for the minimal-norm solution of equation (8) which can be written in terms of the singular system [Bertero *et al*, 1985] of the integral operator $\mathfrak{I}$, which is defined as

$$(\mathfrak{I}u)_{nm} = (\Psi_{nm}, u)_{X_\Psi}, \quad (10)$$

where the symbol $(\mathfrak{I}u)_{nm}$ denotes the nm -th component of vector $\mathfrak{I}u$. The adjoint of $\mathfrak{I}$ from (4) is

$$(\mathfrak{I}^*q)(P') = \mu \sum_{n=1}^N \sum_{m=1}^M \Psi_{nm}(P') q_{nm}. \quad (11)$$

The singular system is obtained from the solutions to the following shifted eigenvalue problem

$$(\mathfrak{I}u_k)_{nm} = \lambda_k (q_k)_{nm}, \quad n=1, \dots, N; \quad m=1, \dots, M; \quad (12)$$

$$(\mathfrak{I}^*q_k)(P') = \lambda_k u_k(P'), \quad P' \in \Omega,$$

where λ_k , $u_k(P')$ and $q_k \in \mathbb{R}^{M \times N}$ are called singular values, singular functions and singular vectors, respectively. By using (12) it is easy to show that the singular values are the square roots of the eigenvalues of the symmetrical Gram matrix:

$$[G]_{nm, n'm'} = \mu (\Psi_{nm}, \Psi_{n'm'})_{X_\Psi}. \quad (13)$$

By means of (7), after a short manipulation we obtain

$$[G]_{nm, n'm'} = \mu \int_{\Omega} \Psi_{nm}(P') W_{n'm'}(P') dP'. \quad (14)$$

The singular values are ordered according to $\lambda_k > \lambda_{k+1}$. The singular vector q_k is the eigenvector of $[G]$ corresponding to λ_k which is normalized according to the norm induced by (4). Finally, the singular functions are obtained from the second equation of (12). The singular functions form an orthonormal basis of X_Ψ , while the vectors q_k are an orthonormal basis of the image space. It is worth noting that orthonormality must be referred to the inner products (4) and (5).

The temperature is basically retrieved by means of the minimal norm solution of the equation (8), which is

$$\hat{T}(P') = \sum_{k=1}^{N \times M} \frac{1}{\lambda_k} (p, q_k) u_k(P'), \quad (15)$$

Minimal norm solutions are exact solutions of the radiometric equation in the object space in the absence of noise. When noise is considered, regularized solutions must be used to counteract the lack of continuous dependence of the normal solutions on the data. In this paper we shall use a rectangular filter, that consists of limiting the summation

in equation (15) to an integer $\Lambda \leq (M \times N)$. Such a regularized solution will be denoted by $\hat{T}^{(\Lambda)}$.

The norm $[(p,p)]^{\frac{1}{2}}$ is called *signal* and denoted by $\bar{S}$ in the following. The *channel* noise, δp_{nm} , is commonly defined as the rms value (averaged over the radiometer integration time) of the fluctuation superimposed on the datum measured at frequency ν_n and position ϕ_m , i.e. p_{nm} . The norm $[(\delta p, \delta p)]^{\frac{1}{2}}$ is called *noise* and is denoted by $\bar{N}$. A common definition of degrees-of-freedom of a measurement is the number of independent pieces of information which can be extracted from data in the presence of noise. It can be shown that a reasonable estimate of the number of degrees-of-freedom [Pike *et al*, 1984] of a temperature retrieval by means of (15) is the largest integer Λ' for which the quotient $\lambda_{\Lambda'}/\lambda_1$ is less than the signal-to-noise ratio $\bar{S}/\bar{N}$.

3. Numerical analysis

In this paper the antenna has been modelled as a parallel-plate waveguide, filled by a dielectric material of uniform relative permittivity $\epsilon_r = 30$. To a first approximation cylindrical segments of human body such as limbs and thighs can be described by two concentric cylinders. Therefore the body has been assumed to be composed of two materials, a bone for $r < r_B$ and a muscle for $r_B < r < r_M$. The complex permittivity $\epsilon(P, \nu)$ takes different values in bone and muscle. They will be denoted by ϵ_{Bn} and ϵ_{Mn} in the respective regions, at frequency ν_n . A microwave radiometer equipped by $N=4$ channels (1.1, 2.5, 4.5 and 5.5 GHz) has been considered in the numerical examples.

It is known from reciprocity in antenna theory that the weighting function is equal to the power deposition within Ω when the antenna is used in the active mode to radiate unitary power into the body. Therefore, at frequency ν_n

$$W_{nm}(P') = -C_{nm} \pi \nu_n \operatorname{Im} [\epsilon_n] |E_{nm}(P')|^2 \quad (16)$$

where C_{nm} is found by imposing condition (2) and $E_{nm}(P')$ is the electric field vector generated by the microwave incident radiation from the antenna at ϕ_m . The minus sign in (16) is due to the fact that the imaginary part of ϵ_n is negative for a time-dependence $\exp(j2\pi\nu_n t)$ of the electromagnetic field, where j is the imaginary unit. Note that E_{nm} and ϵ_n are the only functions defined in Ω .

The computation of E_{nm} has been performed numerically by a finite-difference time-domain technique [Yee, 1966].

The incident electric field has been assumed as $\hat{z} \cos(\pi\xi/w)$, $|\xi| \leq w/2$, ($\hat{z}$ is the unitary vector along the z -axis, ξ is the transversal coordinate on the xy -plane of a cartesian frame ξ, η, ζ solidal with the waveguide, and w is the width of the waveguide) at a depth inside the waveguide, where high-order modes, which originate on the aperture, sufficiently vanish. Contour level plots of the weighting functions at radiometer frequencies are shown in Figs. 2-5. It is worth mentioning that, due to the rotational symmetry of the assumed permittivities, the weighting functions depend on the angular difference ϑ between the source point and the angle-of-view ϕ_m : $\vartheta = \phi' - \phi_m$. Moreover, since the tangential electric field is continuous across two media of different permittivities, as a consequence of (16) the weighting functions are also discontinuous.

The thermal functions have been numerically calculated by means of equation (7), where K takes different constant values, K_B and K_M , in bone and muscle. Also for this computation a finite-difference time-domain method has been used, i.e., the term $\partial T/\partial t$ has been added to the left-hand side of (7) and the resulting time-dependent heat-equation has been numerically solved. The steady-state solution is coincident with the solution to (7). Diagrams of the thermal functions are recorded in Figs. 6-9.

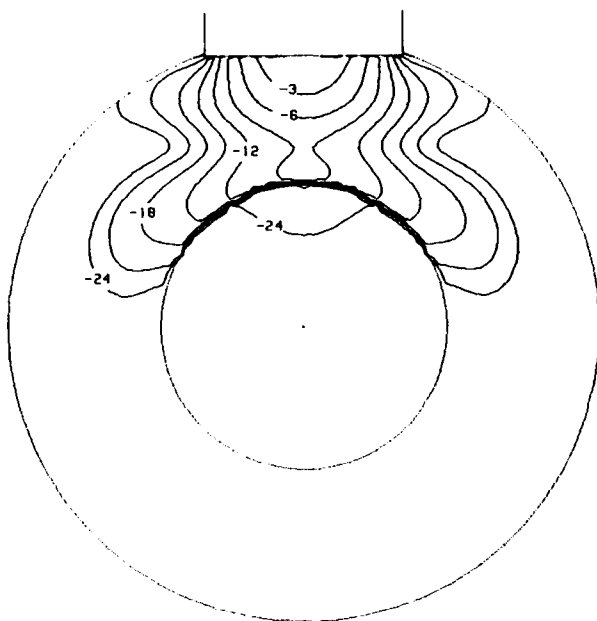


Fig.2 Contour plot of the weighting function at 1.1 GHz. Levels (dB) are referred to a maximum value on the aperture. The size of the grid used for finite-difference computation is 0.1cm. The numerical values used are: $r_B=2.8\text{cm}$, $r_M=6.0\text{cm}$, $\epsilon_B=6.4-j1.6$, $\epsilon_M=50.5-j24.3$.

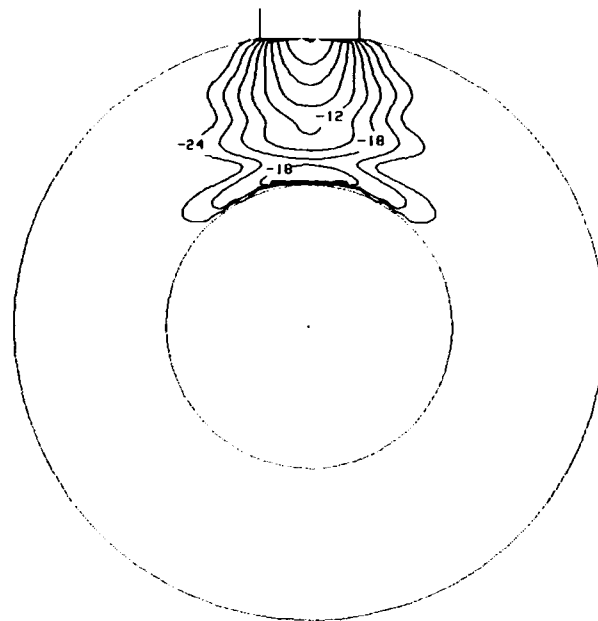


Fig.3 Contour plot of the weighting function at 2.5 GHz.

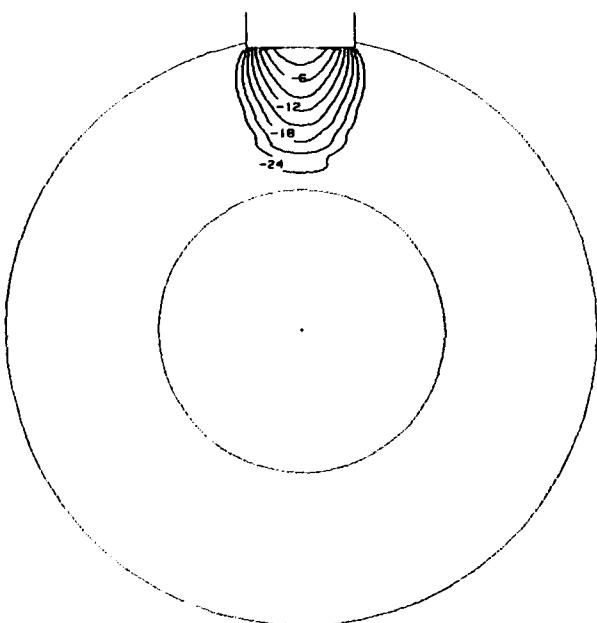


Fig.4 Contour plot of the weighting function at 4.5 GHz.

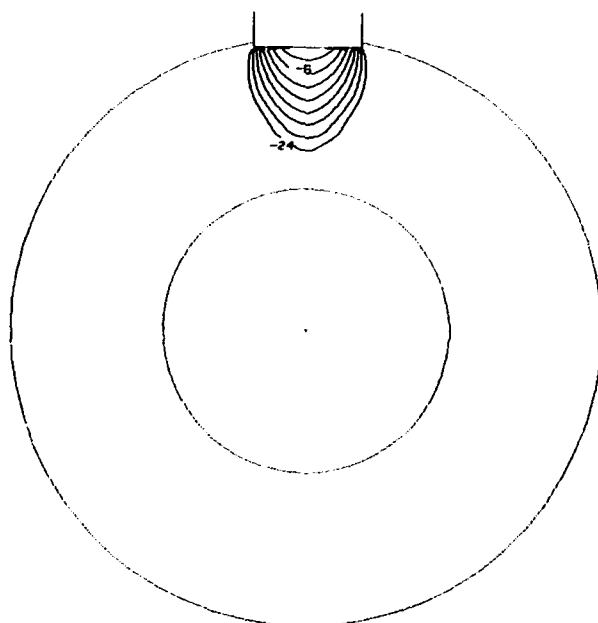


Fig.5 Contour plot of the weighting function at 5.5 GHz.

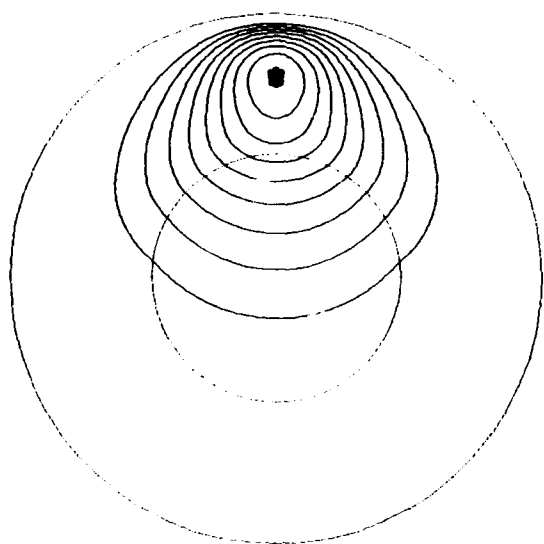


Fig.6 Thermal function Ψ_1 . The contour levels are at intervals of 10% of the maximum value. The ratio $K_B/K_M=0.74$ has been used.

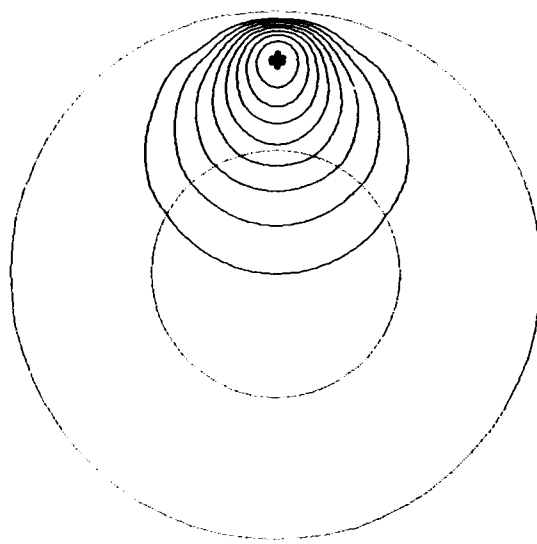


Fig.7 Thermal function Ψ_2 .

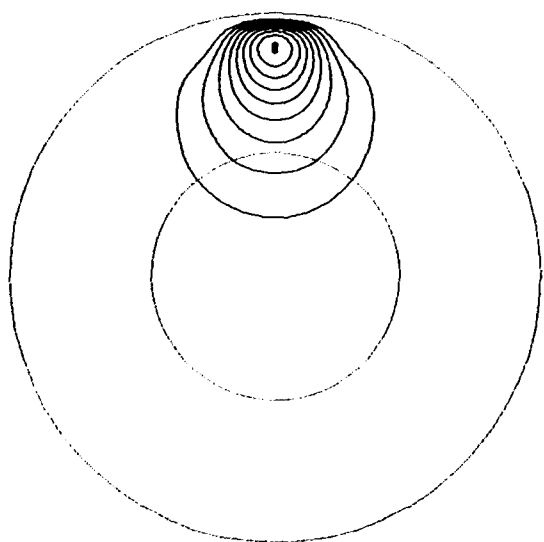


Fig.8 Thermal function Ψ_3 .

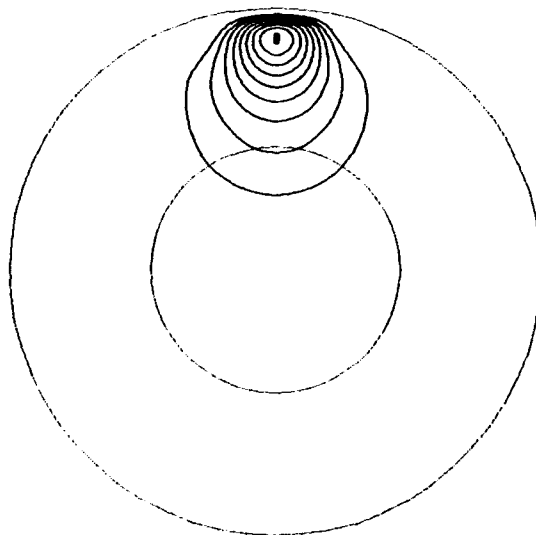


Fig.9 Thermal function Ψ_4 .

Let $\mathfrak{C}(e, r_c)$ be a circle fully contained in the annular muscle region $r_B < r < r_M$, of eccentricity e and radius r_c (Fig.10). We have numerically investigated retrievals of temperature functions defined as:

$$T(P') = 1, P' \in \mathfrak{C}; \quad T(P') = 0, P' \notin \mathfrak{C}. \quad (17)$$

The above defined functions have been considered for modelling a localized hot spot in a uniform temperature field as may be the case during hyperthermia treatments. Synthetic data have been generated by substituting $T(P')$ from (17) into equation (3) and by letting the antennas assume $M=32$ equiangular positions around the cylinder. Examples of reconstructions, i.e. contour level plots of $\hat{T}^{(\Lambda)}$, are shown in Figs. 11-16, for various values of e and two values of Λ .

A common feature of retrievals is distortion, in the sense that the sharp original function (17) is retrieved as a main lobe, which corresponds to the maximum of $\hat{T}^{(\Lambda)}$, while secondary lobes can occur (Fig. 11). Moreover a shift of the main lobe maximum with respect to the original function is present. A larger value of Λ , i.e. of the number of terms retained in the minimal norm solution (15), corresponds to an assumed larger ratio $\bar{S}/N$. As can be seen from the diagrams, increasing Λ improves the retrievals by reducing the shift and increasing the main lobe sharpness.

A retrieval in the L_2 subspace, X_W , spanned by the weighting functions is shown in Fig. 17 for a comparison with the retrieval in X_Ψ for the same signal-to-noise ratio. The retrieval in X_W exhibits discontinuities as a consequence of the discontinuities of the weighting functions at the boundary between two materials of different permittivities. Moreover a spurious lobe occurs close to the boundary $\partial\Omega$ where temperature is sensed by antennas. This lobe is due to the fact that the weighting functions reach the largest values in the vicinity of the antennas.

4. Conclusions

The problem of the retrieval of a 2D temperature distribution in a circular cylinder observed through a parallel-plane waveguide whose walls are parallel to the cylinder axis has been studied. The retrieval of temperature has been modelled as an inverse problem, whose solution has been investigated in a space spanned by thermal functions.

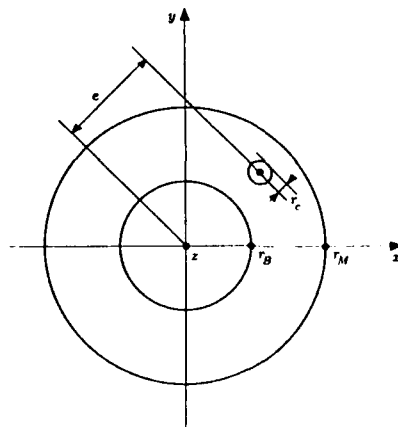


Fig.10 Thermal structure to be radiometrically retrieved for various values of the eccentricity e . The radius of the circular hot spot is $r_c=0.5\text{cm}$.

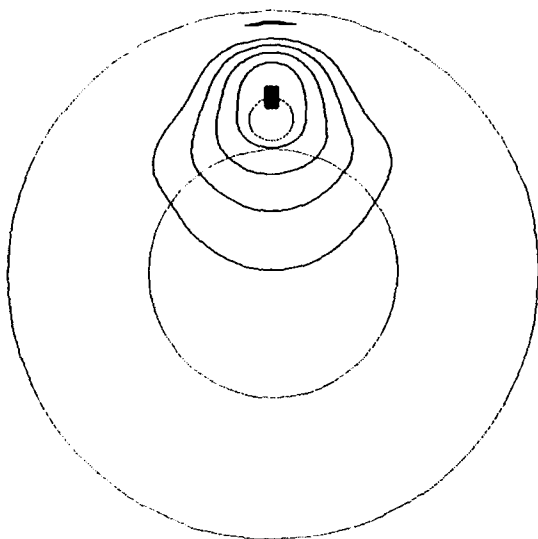


Fig.11 Temperature retrieval, $\hat{T}^{(\Lambda)}$, for $e=3.5\text{cm}$ and $\Lambda=27$. The value of Λ corresponds to $S/N=30$. The contour levels are at intervals of 20% of the maximum positive value. Black shaded areas correspond to regions where the retrieved temperature is not less than 98% of the maximum value.

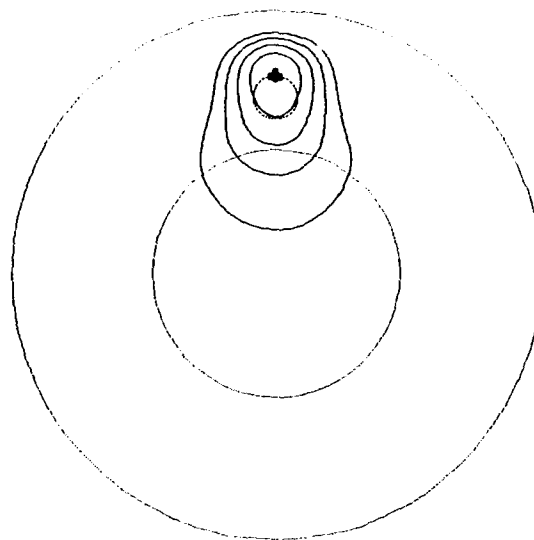


Fig.12 Temperature retrieval, $\hat{T}^{(\Lambda)}$, for $e=4.0\text{cm}$ and the same Λ as in Fig.11.

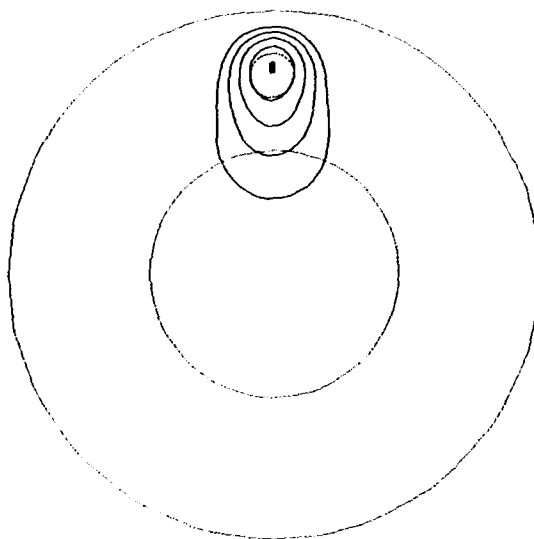


Fig.13 Temperature retrieval, $\hat{T}^{(\Lambda)}$, for $e=4.5\text{cm}$ and the same Λ as in Fig.11.

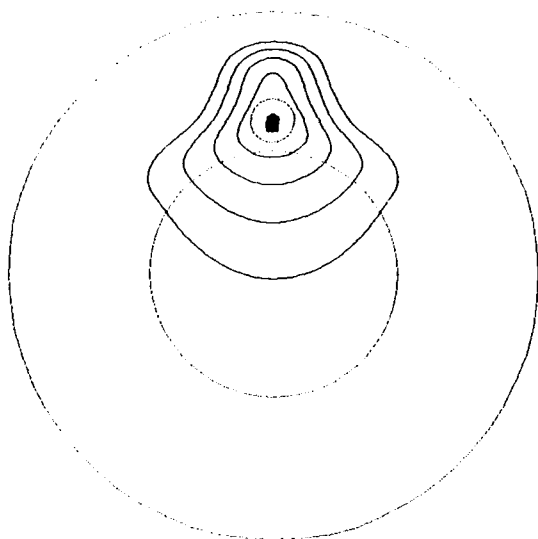


Fig.14 Temperature retrieval, $\hat{T}^{(\Lambda)}$, for $e=3.5\text{cm}$ and $S/N=100$ ($\Lambda=45$).

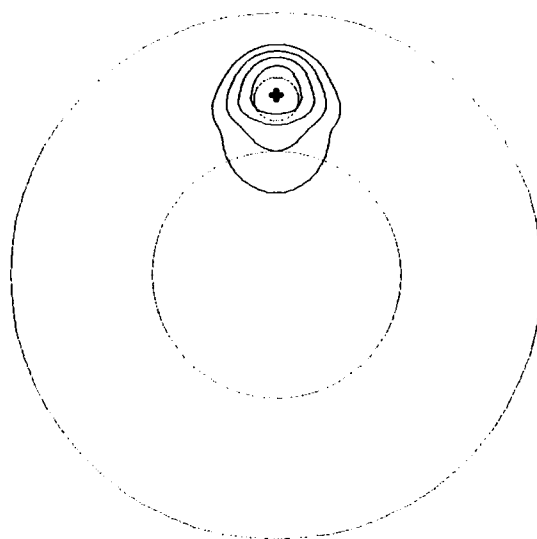


Fig.15 Temperature retrieval, $\hat{T}^{(\Lambda)}$, for $e=4.0\text{cm}$ and the same Λ as in Fig.14.

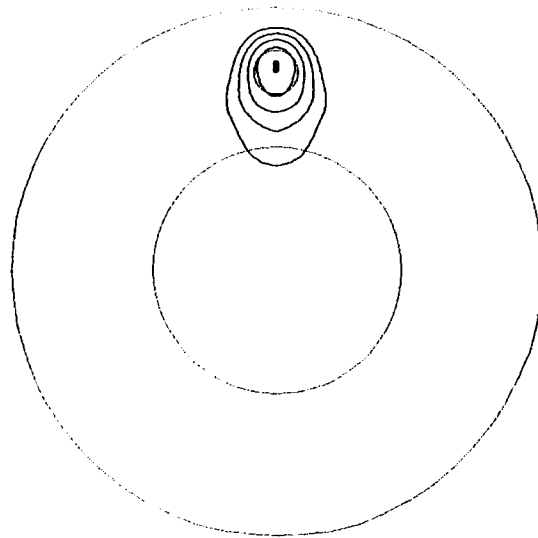


Fig.16 Temperature retrieval, $\hat{T}^{(\Lambda)}$, for $e=4.5\text{cm}$ and the same Λ as in Fig.14.

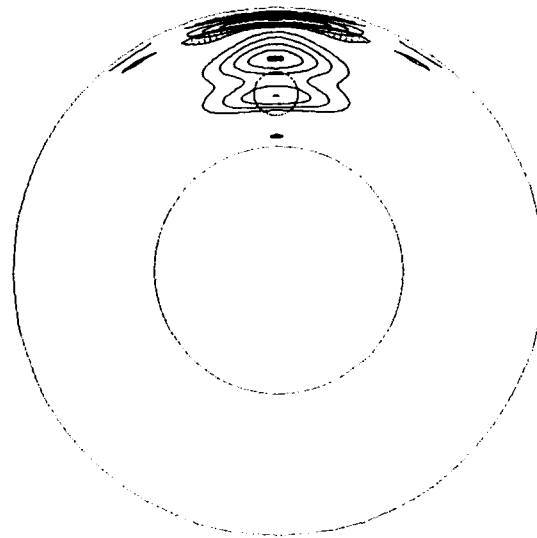


Fig.17 Temperature retrieval, $\hat{T}^{(\Lambda)}$, for $e=4.0\text{cm}$ and $\Lambda=44$, in the L_2 subspace, X_W , spanned by the weighting functions. The value of Λ corresponds to $S/N=30$. The same values of e and of S/N have been used for the retrieval (in X_Ψ) shown in Fig. 12. The negative portion of diagrams are indicated by gray shading.

The solution to the inverse problem needs the solutions to two direct problems, an electromagnetic scattering problem and a heat-balance one. In this paper both problems have been numerically solved by finite-difference time-domain schemes of computation. It is worth noticing that also the computation of the temperature distributions due to electromagnetic heating in hyperthermia is commonly performed by numerical procedures which first solve an electromagnetic boundary value problem and then a heat transfer equation. The finite-difference time-domain technique is widely used. Therefore the retrieval of temperature from radiometric data can benefit from numerical techniques which are developed for planning hyperthermia treatments.

Finally the results show that the use of a Sobolev space greatly improves the retrievals.

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NON-INVASIVE ACTIVE THERMOMETRY WITH A MICROWAVE TOMOGRAPHIC SCANNER IN HYPERTHERMIA TREATMENTS.

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ABSTRACT

In this work the active microwave tomographic imaging of tissue temperature changes induced in deep hyperthermia treatments is studied. The thermal images would allow to monitor and optimize the treatment, increasing its efficacy and avoiding the heating of healthy tissues. As some hyperthermia systems are based on a cylindrical geometry, it is possible to integrate a microwave imaging array in the same structure, providing a non-invasive tool for the monitoring and control. The application of this technique to deep hyperthermia treatments is investigated by numerical simulations and experimentally on two phantoms of thorax and pelvis, using a prototype for microwave tomography recently developed [Jofre et al, 1990].

INTRODUCTION

Forming images of the human body using ionizing radiation has been a part of the diagnostic process in medicine for many years. More recently, other probing radiations, notably ultrasound, have provided successful diagnostic images. Tomography is now in widespread use in the clinic with ionizing radiations (X-ray, isotopes) and with nuclear magnetic resonance imaging. In the past few years, other probing radiations have been considered for tomography, including ultrasound [Mueller et al 1979][Duchene and Tabara, 1985], very low-frequency electromagnetic fields in electrical impedance tomography [Barber et al., 1983] and microwave.

Hyperthermia has shown considerable promise for the adjuvant treatment of cancer, but it is essential to heat the tumor region with a high degree of precision. Ideally the temperature throughout the volume of both the tumor and surrounding normal tissue would be known so one could determine the degree of uniformity of treatment to the tumor and also estimate the extent of possible damage to the normal tissue. When radio-

frequency energy is used to induce hyperthermia, much of the heating may occur outside the region intended for treatment. Aberrant heating is especially significant when frequencies below 1 GHz are used to treat deep-seated tumors. Such undesired heating is likely to go undetected during a treatment session since there are practical limits on the number of temperature probes that may be used. In practice temperature probes are used for measurements at only a few intracavity or interstitial locations [Conway and Anderson, 1986]. Thus it will be useful to develop a non-invasive method to obtain a thermal map of a body under hyperthermia treatment, that allows improve hyperthermia treatments.

Microwave active imaging is a promising non-invasive method for thermography imaging, thus biological tissues have a high dependence of dielectric constant with temperature changes. Active microwave imaging consists of illuminating the body to be imaged with a low-power coherent microwave field and measuring the field scattered by the body. The measured data can be processed using specialized diffraction inverse algorithms to give information on the complex dielectric permittivity of the scattering body.

SYSTEM DESCRIPTION

A cylindrical tomographic microwave system working at 2.45 GHz has been recently developed. The scanner consists of a 64 element circular array with a useful diameter of 20 cm. Although the array is in fact circular, the name cylindrical is used because each antenna is assumed to produce an almost cylindrical wave on the illuminated region. Antennas are electronically scanned, alternately as sources and receivers, to give a complete scan of the object with no mechanical movement. The incident power is less than 0.1 mW/cm^2 at the object and require a coupling medium (in this case water) between object and the source/receiver [Broquetas et al, 1991].

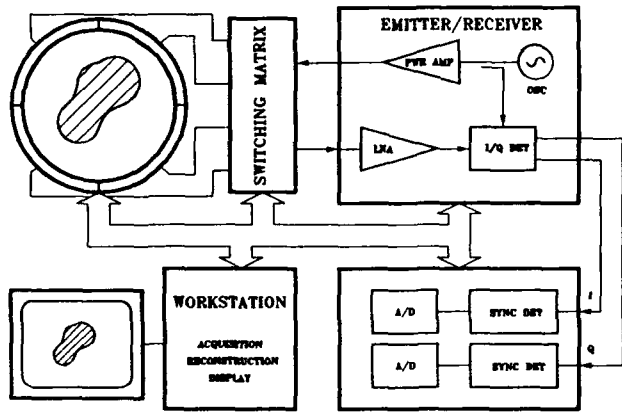


Fig. 1 Block diagram of the prototype system.

IMAGING ALGORITHM

Unlike X rays, microwave radiation has a wavelength comparable to the size of object; thus refraction and diffraction cannot be neglected and diffraction imaging algorithms must be used. In a cylindrical array the field produced by each element will be approximately a cylindrical wave emerging from the antenna if the field energy is assumed to be confined to the thickness of the array. For each emitter the scattered field will be measured on other elements of the array, yielding a matrix $E_s(\varphi_r, \varphi_e)$, where φ_r is the angular position of the receiver element and φ_e the angular position of the emitter, as shown in Fig. 2.

Using an electric field polarized axially and neglecting depolarization effects in the body, we can use an approximate scalar representation of currents and electric field [Slaney and Kak, 1985]. Conventional algorithms are based on linear geometries of measurement and make use of first-order approximations, which means that the body is assumed to perturbate slightly the incident field. Illuminating the body with a plane wave, the plane wave angular spectrum of the scattered field contains the spectrum of the dielectric contrast of the object along circles of radius k_0 (the wavenumber of the field in the embedding reference medium). The dielectric contrast $C(\vec{r})$ is the relative difference between the complex permittivity of each point of the body $\epsilon(\vec{r})$ and the reference permittivity ϵ_0 corresponding to the embedding

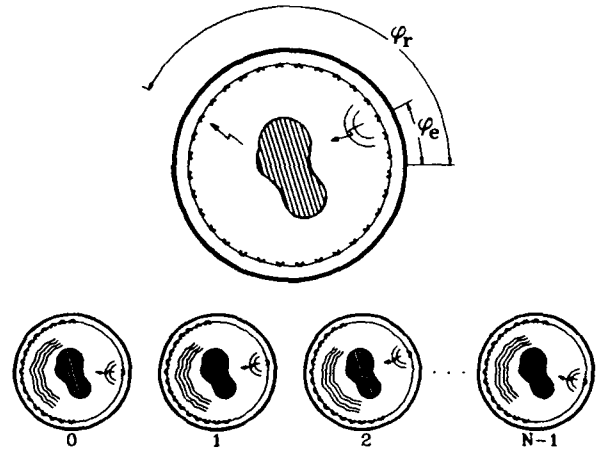


Fig. 2 Measurement procedure in a cylindrical system; with the angular coordinates of the emitter and receiver antennas in operation. The object is illuminated sequentially with the elements of the array (views); for each view the scattered fields are measured.

medium:

$$C(\vec{r}) = 1 - \epsilon(\vec{r})/\epsilon_0 \quad (1)$$

In our case the geometry of measurement is cylindrical and a similar algorithm can be obtained based on Hankel transforms; however this algorithm is difficult to implement on a digital computer and very inefficient because of the large number of Bessel functions involved.

A much more efficient algorithm can be obtained by using a synthetic approach to form plane waves as a combination of cylindrical waves. Due the linearity of the scattering and using the reciprocity theorem the Fourier bidimensional spectrum of $C(\vec{r})$

$$\tilde{C}(\vec{\eta}) = \mathcal{F}\{C(\vec{r})\} = \int_{-\infty}^{\infty} C(\vec{r}) e^{-j\vec{\eta} \cdot \vec{r}} d\vec{r} \quad (2)$$

can be related to the scattered fields [Rius et al., 1987]. The spectral domain is obtained on circular arcs of radius k_0 with coordinates $\vec{\eta} = k_0(\hat{\theta} - \hat{\theta}_0)$, where $\hat{\theta}_0$ and $\hat{\theta}$ are unit vectors in the directions of illuminating and

scattered plane waves:

$$\tilde{C}[k_0(\hat{\theta} - \hat{\theta}_0)] = \frac{j}{\omega \epsilon_0} \int_0^{2\pi} \int_0^{2\pi} E_s(\varphi_r, \varphi_s) J_a(\varphi_s; -\hat{\theta}) \cdot J_a(\varphi_s; \hat{\theta}_0) R^2 d\varphi_r d\varphi_s \quad (3)$$

where R is the radius of the array. Equation (3) is formally a double convolution of the measured scattered fields E_s with the currents on the array J_a that would produce a plane wave illumination. The convolution can be efficiently calculated using FFT techniques as a double product in the spectral domain between the Fourier series of E_s and J as described in [Rius et al., 1987]. The image obtained by inverse bidimensional Fourier transform once $\tilde{C}[k_0(\hat{\theta} - \hat{\theta}_0)]$ has been mapped on a rectangular grid by linear interpolation in a way similar to that in linear algorithms. The reconstruction time is 30 s on a HP9000-320 workstation [Rius et al., 1987].

Unfortunately, most biological bodies causes the breakdown of the Born approximation due to their high contrast characteristics and large size in terms of wavelengths. It has been found both numerically and experimentally that some biological bodies with sections of a few wavelengths can be reconstructed, revealing qualitatively their internal structures. When the size is larger, it is possible to get useful differential reconstructions showing dielectric changes.

In thermography, the image is reconstructed from the difference between the fields scattered by the reference and the heated body. Ideally the image should represent the dielectric changes induced by heating. Unfortunately there are terms of error and the image is qualitative and does not reconstruct the original permittivity values. However, in this case the image still supplies information on the situation of thermal changes induced. In preliminary experiments to estimate the temperature sensitivity of the system a rubber tube 0.05 mm thick and 3 cm internal diameter was placed off center in the water tank; heated water was pumped through the tube and a temperature resolution of the system of 0.5 °C was found [Jofre et al., 1990]. In the present work the thermal sensitivity of high contrast large biological bodies is investigated.

All images in the paper represent the object profile, defined as $C(\vec{r}) = 1 - \epsilon(\vec{r})/\epsilon_0$.

NUMERICAL SIMULATION

The frequency is a compromise between spatial resolution, with a theoretical limit of one half of the wavelength, and attenuation. The spatial resolution improves with frequency but, due to the high water content of biological tissues, the operating frequency is limited by the maximum attenuation that can be tolerated in the field measurements. The behaviour of the imaging algorithm depends on the electrical size of the object, this is an additional factor that must be considered in the choice of the optimum frequency. The useful imaging diameter depends on the number of antennas and the wavelength of the external medium.

$$D = \frac{2N\lambda_0}{4\pi} \quad (4)$$

The frequencies to study are 434 MHz, 700 MHz and 915 MHz allowing a reasonable resolution and level of attenuation. In all cases 64 antennas are enough to reconstruct a human trunk, with an average width of 38 cm. A simulated tomographic scanner based on a 60 cm diameter array with 64 antennas has been adopted.

Numerical models of sections of thorax and pelvis have been defined with a discretization step of 0.1 λ for each frequency. Fig. 3 shows the numerical model and phantoms of anatomic sections [Karitké and Sick, 1988]. Table 1 shows the permittivity of tissues taken from [Stuchly, 1980][Iskander, 1978].

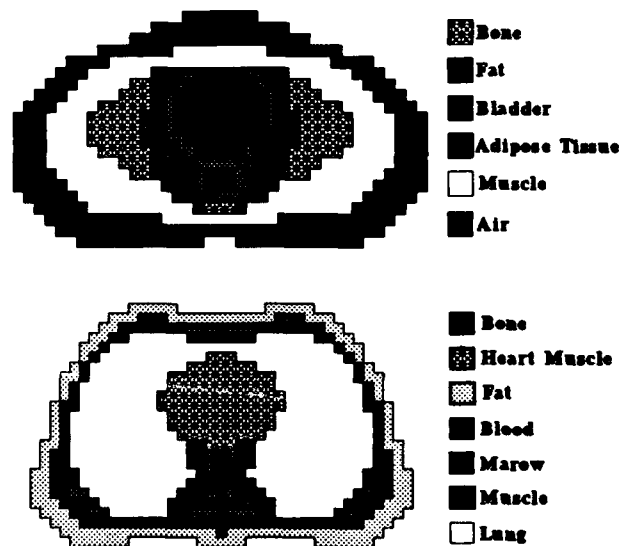


Fig. 3 Numerical and phantom models of thorax and pelvis.

TISSUE PERMITTIVITY			
Tissue	434 MHz	700 MHz	915 MHz
Fat	5.5-j3.3	5.5-j2.0	4.6-j1.1
Muscle	53-j41.4	53-j30.8	54-j23.6
Heart Muscle	56-j38.5	55-j28.2	55-j21.6
Bone	8.5-j1.3	8.5-j0.8	8-j1
Bladder	50-j16.5	50-j10.2	-
Lung	36-j25.2	34-j19.5	35-j14.3
Marrow	5.5-j1.2	5.5-j0.8	5.5-j0.6
Adipose Tissue	20-j9	20-j8.1	-
Blood	63-j49.7	62-j30.2	63-j24.5
Thermal Sensitivity	$100(\Delta \epsilon/\epsilon)^{\circ}C$		$100(\Delta \epsilon/\epsilon)^{\circ}C$
Fat	1.2		4.6
Muscle	-0.2		1.3
Lung	-0.2		1.5

Table 1. Permittivity and thermal sensitivity of human tissues

Using the Conjugate Gradient Method (CGM-MM) to calculate the scattered fields by the reference and heated body in the same way as with a tomographic system, the differential images have been obtained by processing the change of the scattered field produced by the induced temperature. Image artifacts are caused by the dielectric inhomogeneities of the materials explored. Therefore it is convenient to use an embedding medium with a dielectric constant similar to the body average. Several hypothetical lossy mediums have been evaluated.

With the pelvis model at 434 MHz, the best external medium was 50-j8.5. Fig. 4 shows the distortion induced by the model over the differential image. The algorithm reconstructs correctly the heated zones although with some artifacts caused for the model inhomogeneities, for example the high dielectric constant of bladder contents.

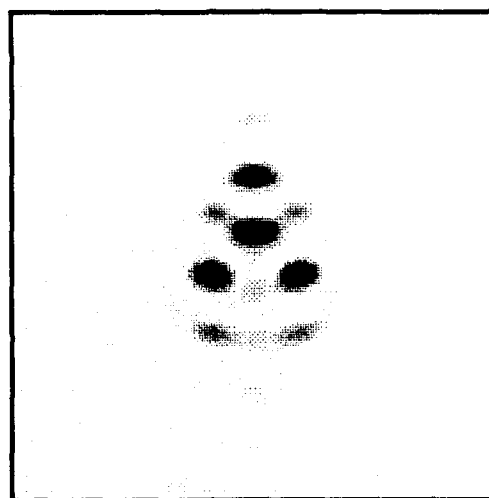


Fig. 4 Pelvis model at 434 MHz with 2 heated zones (5°C) in internal adipose tissue.

Fig. 5 shows 4 heated zones in the pelvis model at 434 MHz, the effect of mismatching between losses of body and external medium causes a radial variation in the image, resulting in lower thermal sensitivity at internal zones than external.

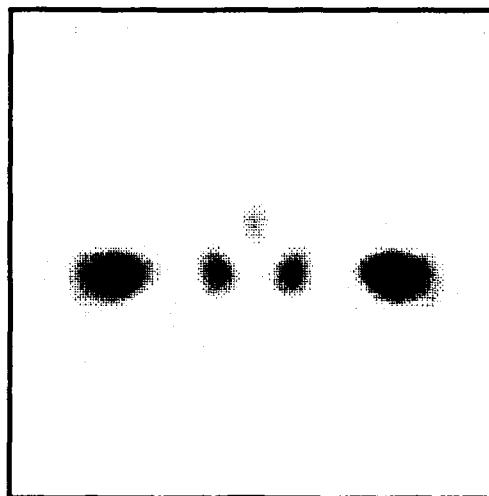


Fig. 5 Pelvis model at 434 MHz with 4 heated zones (5°C), 2 internal and 2 external, and a bladder contents equal to external media.

In the higher frequencies, the distortion introduced by the body on the incident field increases resulting in more artifacts. Thus, we can conclude that the optimal frequency for pelvis monitoring with a first order

algorithm is 434 MHz.

A similar study has been performed on a thorax model. The higher homogeneity of this body allows to reconstruct thermal changes up to 900 MHz with low artifact levels. Fig. 6 shows 6 heated zones with temperatures increases of 5 °C, 3 °C and 1 °C; the first one is perfectly visible, the second one have a minor level and the third one is visible, but it may be confused with other erroneous spots.

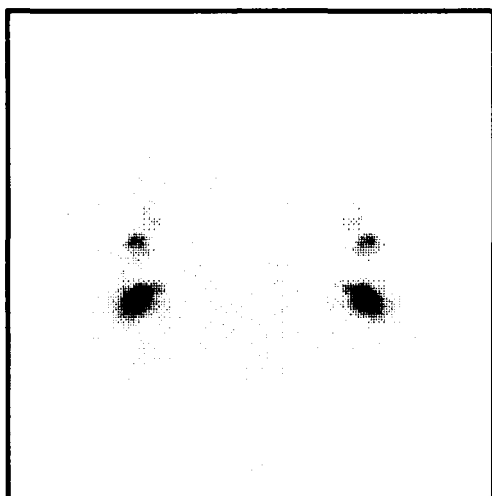


Fig. 6 Thorax model at 700 MHz with six heated zones.

EXPERIMENTAL STUDY

From the results obtained by numerical simulations, phantoms have been constructed in order to simulate at study frequencies the electrical properties of thorax and pelvis. The experimental prototype works at 2.45 GHz using water as outer medium. In order to reproduce the electrical size and contrast of the numerical simulations, the complex permittivity and dimensions of the phantoms have been calculated using the electromagnetic scaling expressions [Sinclair, 1948] at Table 2.

The phantoms are simplified versions of the numerical models consisting of a plaster outer shell to simulate skin and external fat, including gels simulating muscle, bladder and lung [Kato et al., 1986] and plaster simulating bone. Several thin plastic tubes were added to simulate internal heating by chemical dielectric alterations.

Due to the prototype sensitivity limitations and the high attenuation of biological tissues, only large dielectric

changes were reconstructed. Fig. 7 shows a pelvis phantom at 434 MHz with a focalized change in bladder of $14 + j0.2$ in permittivity. This was the minor change detected in this model.



Fig. 7 Pelvis phantom at 434 MHz with a focalized change in bladder.

In thorax phantoms better results have been obtained. Fig. 8 shows a phantom at 700 MHz with a increment in permittivity of $5 + j15$. Also this case was the minor change detected in the model.

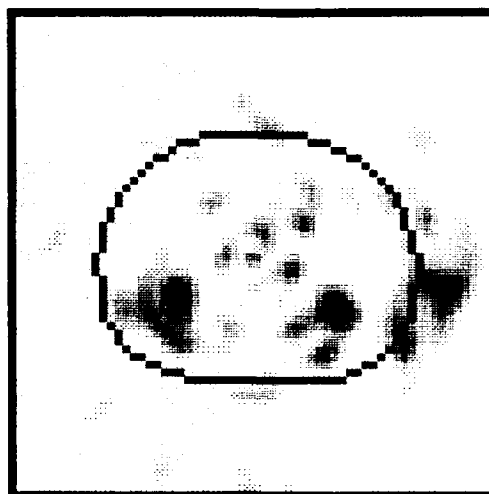


Fig. 8 Thorax phantom at 700 MHz with 2 heated zones.

Real System A	Simulated System A'
$\vec{r} = p\vec{r}'$	
$t = \gamma t'$	$\omega = \frac{1}{\gamma} \omega'$
$\vec{E}(\vec{r}, t) = \alpha \vec{E}'(\vec{r}', t')$	
$\vec{H}(\vec{r}, t) = \beta \vec{H}'(\vec{r}', t')$	
$\sigma(\vec{r}) = \frac{\beta}{\alpha p} \sigma'(\vec{r}')$	
$\epsilon(\vec{r}) = \frac{\gamma \beta}{\alpha p} \epsilon'(\vec{r}')$	
$\mu(\vec{r}) = \frac{\alpha \gamma}{\beta p} \mu'(\vec{r}')$	
If magnetic materials not scaled ... $\mu(\vec{r}) = \mu'(\vec{r}')$	
$t = \gamma t'$	$\vec{r} = \frac{\alpha \gamma}{\beta} \vec{r}'$
$\sigma(\vec{r}) = \frac{\gamma}{p^2} \sigma'(\vec{r}')$	$\epsilon(\vec{r}) = \frac{\beta^2}{\alpha^2} \epsilon'(\vec{r}')$

Table 2. Relations obtained from Maxwell's Equations.

DISCUSSION AND CONCLUSION

The results presented show the potential capability of the microwave tomography system to reconstruct induced temperature changes in large biological bodies. From numerical simulations the optimum frequencies and external media have been estimated in this application.

For a medical prototype, in order to measure the weak changes in the fields caused by small thermal changes, a system with a high repeatability and dynamic range is required (100 dB).

The algorithm is capable to reconstruct thermal changes in homogeneous bodies, but in the case of large inhomogeneities, erroneous spots appear in the final image. This is due to the limitations caused by the Born approximation algorithm. New algorithms based in iterative techniques are in study; and first results

obtained are considerably better than using Born approximation. The problem with this new type of algorithms is the large computational time and large amount of memory necessary to obtain good results. The big advantage of presented imaging algorithm is the computational speed, for example in an HP-Apollo 700/RX station measured data could be processed in real time.

Research will continue to improve reconstruction algorithms, increase sensitivity (dynamic range) and system repeatability.

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Electrical Impedance Tomography in Three Dimensions

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Abstract

An algorithm is developed for electrical impedance tomography (EIT) of three-dimensional volumes using multi-planar electrode arrays. This algorithm is based upon the method of least squares, and uses one step of Newton's method to estimate the conductivity distribution inside the volume using electrical measurements made on the boundary.

An implementation of the algorithm for right cylindrical volumes is described. This computer code, called *N3D*, permits reconstructions with up to 2016 degrees of freedom. The code uses an initial guess consisting of a uniform conductivity, allowing many of the computations to be done analytically.

Although the code does not reconstruct the conductivity distribution accurately (unless it differs very little from a constant,) it does yield useful images at reasonable computational cost. The algorithm is demonstrated using three-dimensional resistivity distributions reconstructed from experimental data.

Introduction

The inverse problem in Electrical Impedance Tomography (EIT) considers the reconstruction of the electrical characteristics inside the body from measurements made on the skin.

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Material	Resistivity (ρ) ohm-cm
Urine	30
Plasma	63
Cerebrospinal fluid	65
Blood	150
Skeletal muscle	300
Cardiac muscle	750
Lung	1275
Fat	2500
Copper	1.724×10^{-6}

Table 1: Typical Tissue Resistivity Values

Although superficially similar, this task is fundamentally different from that encountered in X-ray Computed Tomography(CT) or Positron Emission Tomography(PET), where the photon paths through tissue are essentially straight lines. In contrast, the current paths in EIT are functions of the unknown conductivity distribution, and the reconstruction problem is highly non-linear. We focus here on biomedical applications of EIT, but note that the technique has been used in geophysical exploration and may have important uses in non-destructive testing, and in the monitoring of industrial processes such as solidification and multi-phase flow.

Biological Basis of the EIT Signal

Biological tissues contain free charge carriers that permit them to act as (relatively poor) electrical conductors. This ability to conduct varies substantially among various types of tissue: some typical values of resistivity for tissues of interest have been tabulated [Baker, 1989] and are presented as Table 1. For reference, the resistivity of pure copper is included. The goal of EIT is to compute and display the spatial distribution of the resistivity inside the body.

Also present in tissue are bound charges. In the presence of an electromagnetic field, these charges give rise to displacement currents that are manifested as phase shifts in the measured voltages relative to the applied currents. Phase shifts that vary from about 7° for lung to nearly 20° for cardiac muscle at an excitation frequency of 10 kHz have been reported [Schwan and Kay, 1957]. This ability of a material to store charge is referred to as its permittivity. Because the permittivity is related to the tissue type and is spatially variable, we can also consider producing an image of the permittivity distribution inside the body.

Some work suggests that these electrical characteristics are substantially altered in a variety

of pathological conditions. For instance, some investigators have demonstrated permittivity changes in excised malignant breast tumor specimens compared to normal tissue surrounding the tumor [Surowiec, *et al*, 1988]. Similarly, the non-invasive diagnosis and monitoring of pulmonary edema represents an attractive goal, since differences in the bulk resistivity properties of the lung may be detectable even with crude reconstruction algorithms [Isaacson, *et al*, 1990].

Biological Application of EIT

One of the first instruments designed to image the body's resistivity distribution was the "impedance camera" [Henderson and Webster, 1978]. This device consisted of one large electrode, positioned on the back, and an array of 144 electrodes, in a 12 by 12 grid, positioned on the chest, through which currents could be applied. In practice, measurements were made using 100 electrodes in an inner 10 by 10 grid, and currents were established on the outer "guard" electrodes in an attempt to "linearize" the current paths.

The authors monitored the current flow through each of the 100 measurement electrodes, and displayed iso-admittance curves on the body surface. They then proposed that reconstruction algorithms for the purpose of imaging the internal impedance should be developed.

Following this paper, many authors published descriptions of systems designed to reconstruct the impedance inside the body from electrical measurements made on the skin. Researchers began to apply mathematical tools developed for X-ray CT scanning to produce images of the body's internal impedance.

Investigators at the University of Sheffield [Barber, *et al*, 1983, Brown, *et al*, 1983] described a back-projection technique that was based on pre-computed equipotential lines in a homogeneous medium. In this technique, current was injected on two adjacent electrodes, and the resulting voltages were measured differentially on the remaining electrode pairs. The ratio of measured to predicted voltage was used to update the resistivity estimate in a strip between the isopotential lines ending on the measuring electrodes. This process was continued, with each electrode pair serving as the current injection electrodes.

More recently, these investigators have presented modifications to their original back-projection algorithm that uses a weighting system and logarithmic projection of the voltage ratios [Brown, *et al*, 1985]. Although strictly two dimensional, this algorithm is the basis for much of the *in-vivo* research in EIT.

In a 3-D body, current flow is not restricted to the plane of electrode placement, and the resistivity of tissues outside this plane contributes to errors in the Barber-Brown reconstruction scheme. Even in a 2-D body, current flow may be substantially perturbed by objects of varying resistivity.

In an attempt to address the problem of 3-D equipotential line distortion, a technique

that offers some improvement over the strictly 2-D algorithm has recently been described [IDER and Gencer, 1989, Ider, *et al*, 1990]. In this scheme, the authors compute an "equivalent translationally uniform" object that has the same equipotential lines as the three dimensional resistivity distribution. They used finite element techniques to compute modified 2-D equipotential lines by assuming that the resistivities obtained in the plane of electrode placement were translationally uniform. Other authors report experimental measurements of the sensitivity of back-projection reconstructors in 3-D domains [Guardo, *et al*, 1991].

Searching for more efficient algorithms, investigators have extended these techniques to the case of a cylinder with radial variations in conductivity [Andersen and Berntsen, 1988] and more recently, to a general formulation for the problem applicable to two and three-dimensional geometries [Berntsen, Andersen and Gross, 1991].

Other reconstruction techniques, including current path projection methods, "double constraint" techniques [Wexler, *et al*, 1985], and perturbation methods [Kim, *et al*, 1983] and [Tarassenko and Rolfe, 1984] have been applied to the EIT problem.

Iterative Newton's Method Techniques

Another approach to the solution of the inverse problem (and the one taken in this work) is the use of iterative linearization of the underlying nonlinear equations. This technique was suggested as early as 1978 [Lytle and Dines, 1978].

Later, these authors suggested an improved algorithm, using Newton's method to solve for each resistor in a 7x7 2-D array. This algorithm performed well on simulated data, even with the addition of 10 % additive noise [Dines and Lytle, 1981]. A patent [Bai, *et al*, 1984] appears to use a similar iterative technique.

Yorkey implemented an iterative algorithm using Newton's method. In his thesis, he compared the performance and computational costs associated with a variety of reconstruction algorithms of many reconstruction schemes, including the perturbation techniques [Kim, *et al*, 1983, Tarassenko and Rolfe, 1984], equi-potential back-projection techniques reminiscent of CT algorithms [Barber, *et al*, 1983, Barber and Brown, 1984] and double-constraint techniques [Wexler, *et al*, 1985]. He concluded that his "modified Newton--Raphson" was always the most accurate, at least on certain simulated data with and without added noise [Yorkey, 1986].

Simske implemented a two dimensional algorithm based on the Levenberg-Marquardt method using an analytic forward solver [Simske, 1987, Cheney, *et al*, 1990]. This method forms the basis for the extension of these concepts to three dimensions.

Mathematical Formulation of the Problem

Mathematically, the EIT problem can be formulated as follows:

Let $u(p)$ denote the electric potential at an arbitrary point p in Ω , a bounded domain in $\mathbb{R}^3$, and let $\sigma(p) = \frac{1}{\rho(p)}$ denote the conductivity at p . Then we assume that u satisfies

$$\nabla \cdot \sigma(p) \nabla u(p) = 0 \text{ for } p \text{ in } \Omega. \quad (1)$$

We choose to solve the Neumann problem, so at the boundary, we have

$$\sigma(p) \frac{\partial u(p)}{\partial \nu} = j(p) \text{ on } \partial\Omega \quad (2)$$

where ν denotes the unit outward normal and j denotes the current density applied on $\partial\Omega$. From conservation of charge, we require that

$$\int_{\partial\Omega} j(p) = 0. \quad (3)$$

To select a unique solution, we choose our reference potential so that

$$\int_{\partial\Omega} V(p) = 0, \quad (4)$$

where $V(p) = u(p)$ for p on $\partial\Omega$.

Notation for the 3-D Problem

Consider a right cylindrical volume with a three-dimensional electrode array on the surface, and with boundary conditions as in Figure 1. Let M denote the number of electrode planes and Λ denote the number of electrodes on any ring. Consequently, the total number of electrodes is $L = M\Lambda$. We denote an individual electrode by e_l , $l = 1, 2, 3, \dots, L$.

Using external electronics, steady state patterns of current are established on the electrode array and measurements resulting from $L - 1$ orthogonal current patterns over the L electrodes are used to reconstruct the resistivity distribution within the object. Denote the k^{th} applied current pattern by the L -vector I^k and the corresponding voltage measurement by another L -vector V^k , so that

$$I^k = \begin{bmatrix} I_1^k \\ I_2^k \\ I_3^k \\ \vdots \\ I_L^k \end{bmatrix}; \quad V^k = \begin{bmatrix} V_1^k \\ V_2^k \\ V_3^k \\ \vdots \\ V_L^k \end{bmatrix}. \quad (5)$$

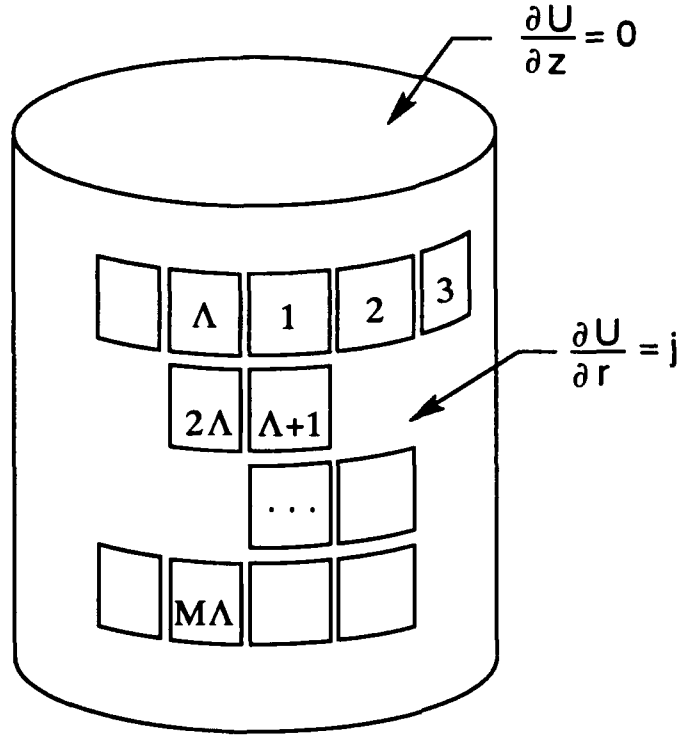


Figure 1: Nomenclature for the 3-D Problem

We will require a “reference” electrode, in order to enforce equation 4 . We may designate one electrode in the existing array, and simply measure all our voltages with respect to this electrode. Alternatively, we may supply an additional electrode which is tied directly to ground potential. We constrain each of the applied current patterns so that $\sum_{i=1}^L I_i = 0$, so that the current flowing through this ground electrode is negligible. To preserve this symmetry, we normalize the measured voltages so that $\sum_{i=1}^L V_i^k = 0$.

We define an operator $\mathbf{R}(\rho)$ that maps the applied current vector into the vector of measured voltages. We define another L-vector U^k

$$U^k = U(\rho) = \begin{bmatrix} U_1^k \\ U_2^k \\ U_3^k \\ \vdots \\ U_L^k \end{bmatrix} = \mathbf{R}(\rho) I^k \quad (6)$$

which represents the predicted voltages on the electrodes due to some resistivity estimate ρ . The **forward problem** consists of predicting the voltages on L electrodes from knowledge (or an assumption) about the resistivity distribution inside the tank. In the case of a homogeneous cylinder, we can analytically solve for the predicted voltages U^k using a variety of models for the boundary conditions [Cheng, *et al*, 1990]. The more general case requires

use of finite difference or finite element techniques to solve the forward problem for an arbitrary resistivity distribution.

The **inverse problem** consists of predicting $\rho(p)$ throughout the body from knowledge of the applied currents and measured voltages. Because the operator $\mathbf{R}(\rho)$ is self-adjoint, we have at most

$$N_{max} = \frac{L(L-1)}{2} \quad (7)$$

degrees of freedom. Consequently, we subdivide the volume of interest into $N \leq N_{max}$ voxels, or volume elements, on which the resistivity is assumed constant. Hence we will compute a resistivity vector of length N such that

$$\rho(p) = \sum_{n=1}^N \rho_n \chi_n(p) \quad (8)$$

where

$$\chi_n(p) = \begin{cases} 1 & \text{if } p \text{ in voxel } n \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

We use ρ to denote the resistivity solution vector

$$\rho = \begin{bmatrix} \rho_1(p) \\ \rho_2(p) \\ \rho_3(p) \\ \vdots \\ \rho_N(p) \end{bmatrix}.$$

Figure 2 depicts the geometry of a typical voxel as used in our 3-D reconstructor.

Instrumentation Considerations

In this work, all experimental data was collected using **ACT IIB**, a 64 electrode system whose general design features have been described by Gisser *et al.* in 1987. This system is an improvement of the the original **ACT I** design to incorporate better voltmeter precision and phase stability [Fuks, 1989]. This instrument, which has subsequently been extended from 32 to 64 channel operation, can be used for 2-D or 3-D operation, depending on the software.

In operation, **ACT II** uses a Wein bridge one op-amp oscillator operating at about 15 kHz. Its amplitude is stabilized to better than 0.1 % by a DC feedback loop, and serves as a system-wide reference for amplitude and phase.

This primary frequency is fed simultaneously to 64 12 bit CMOS digital-to-analog converters, which act as digitally controlled attenuators for the master oscillator. This voltage

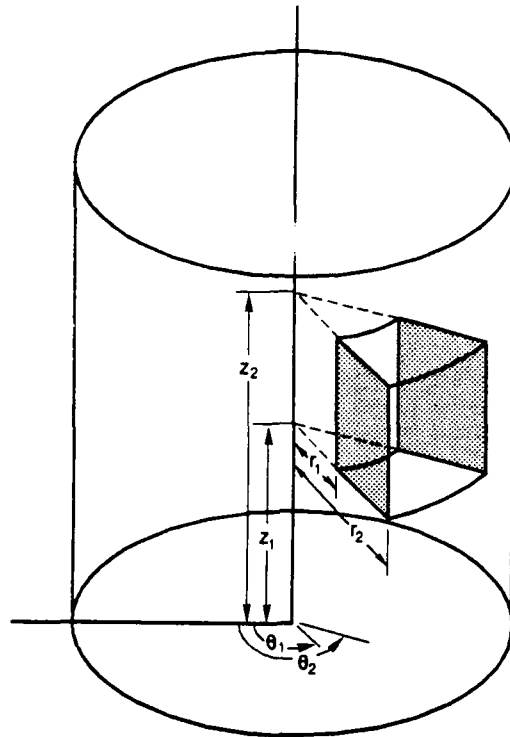


Figure 2: Geometry for the 3-D Resistivity Voxel

reference for each electrode is applied to op-amp voltage-to-current converters, and the resulting current is applied on the skin through shielded cables to the electrode.

The voltage required to maintain this fixed current flow will vary, depending on the resistivity of the object to be imaged. The voltage at the output of each current generator is fed through a 64:1 multiplexer into two demodulators synchronized to the master oscillator.

The first of these devices is synchronized by a signal that is in phase with the applied currents, and after rectification and low pass filtering, produces a DC signal proportional to the in-phase voltages on the electrodes. The second demodulator is synchronized with a signal that is 90° out of phase, producing a quadrature voltage related to the permittivity of the object. The resulting DC voltages are fed to a commercially available 16 bit analog-to-digital converter, a DT2801/5716¹. Table 2 is adapted from [Fuks, 1989].

A new instrument known as **ACT III** [Saulnier, *et al*, 1991] will incorporate a design philosophy similar to previous instruments but will exhibit substantially improved performance. Each electrode channel contains a digital signal processor (DSP), a current source, a voltmeter and associated amplifiers and calibration circuitry. These improvements will permit acquisition of all data, both in-phase and quadrature, with 16 bits of accuracy in 140 mSec.

¹Data Translation, Inc., Marlborough, MA.

Characteristic	Specification
Oscillator	
Frequency	15.0 kHz
Frequency Stability	< 2 Hz
Amplitude Stability	< 0.1 %
Harmonic Distortion	< -60 dB
Waveform	Sinusoidal
Current Generators	
Number	64
Maximum Current	5 mA
Amplitude Precision	12 bits or 2.4 μ A
Phase Coherence	0.3°
Voltmeter	
Type	Synch. Demodulation
Precision	16 bits (nominal)
Accuracy	10 ppm
Settling Time	1 ms for 12 bits 3 ms for 16 bits
System Speed	
Write Time	
32 electrodes	0.55s
64 electrodes	0.95s
Read Time	
32 electrodes	0.38s
64 electrodes	0.80s
Total Image Time	
20 averages	390s

Table 2: ACT II Operational Specifications

A Least Squares 3-D Reconstruction Algorithm

The Plan

In our approach, the goal is to minimize the least square error functional $E(\rho)$,

$$E(\rho) = \sum_{k=1}^{L-1} \|V^k - U^k\|^2 = \sum_{k=1}^{L-1} \sum_{l=1}^L (V_l^k - U_l^k)^2. \quad (10)$$

Here, V^k represents the voltages experimentally measured for the k^{th} current pattern and U^k represents the predicted voltages for the same pattern. U^k is produced by the forward solver. It should be apparent that even if we have a perfect forward solver, and our initial estimate of the resistivity is uncannily correct, then our error function may not be equal to zero since the measured voltages V_l^k contain measurement errors. In fact, there may be no resistivity distribution $\rho(p)$ that produces the measured data!

We would like to improve our estimate ρ^0 . We use Newton's method to compute a new estimate ρ^1 . At a minimum,

$$0 = \frac{\partial E}{\partial \rho_n} = F_n(\rho) = -2 \sum_{k=1}^{L-1} \sum_{l=1}^L (V_l^k - U_l^k) \frac{\partial U_l^k}{\partial \rho_n}. \quad (11)$$

We then compute a new estimate,

$$\rho^{i+1} = \rho^i - [F'(\rho^i)]^{-1} F(\rho^i), \quad (12)$$

where

$$F'(\rho) = \begin{bmatrix} \frac{\partial F_1}{\partial \rho_1} & \cdots & \frac{\partial F_1}{\partial \rho_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_N}{\partial \rho_1} & \cdots & \frac{\partial F_N}{\partial \rho_N} \end{bmatrix} \quad (13)$$

and

$$F'_{n,m}(\rho) = \frac{\partial F_m}{\partial \rho_n} = \frac{\partial^2 E}{\partial \rho_n \partial \rho_m} \quad (14)$$

$$= 2 \sum_{k=1}^{L-1} \sum_{l=1}^L \frac{\partial U_l^k}{\partial \rho_n} \frac{\partial U_l^k}{\partial \rho_m} - 2 \sum_{k=1}^{L-1} \sum_{l=1}^L (V_l^k - U_l^k) \frac{\partial^2 U_l^k}{\partial \rho_n \partial \rho_m}. \quad (15)$$

Regularization of F'

It would at first seem that finding ρ^{i+1} in equation 12 involves only inverting the known matrix, F' , applying it to the vector F , and subtracting this term from the old resistivity estimate ρ^i . Unfortunately, F' is ill-conditioned [Simske, 1987].

Consider the first term of equation 13,

$$\mathbf{A}_{n,m} = 2 \sum_{k=1}^{L-1} \sum_{l=1}^L \frac{\partial U_l^k}{\partial \rho_n} \frac{\partial U_l^k}{\partial \rho_m}. \quad (16)$$

We can think about this term as follows: If we apply the k^{th} current pattern and calculate the voltage on the l^{th} electrode, then perturb the resistivity on the n^{th} voxel, then $\frac{\partial U_l^k}{\partial \rho_n}$ represents the voltage change. Similarly, $\frac{\partial U_l^k}{\partial \rho_m}$ represents the voltage change associated with a change in the resistivity on element m . If the m^{th} and n^{th} voxels are in some sense "close" to each other, then the voltage changes they produce will be similar. Consequently, the product $\frac{\partial U_l^k}{\partial \rho_n} \frac{\partial U_l^k}{\partial \rho_m}$ will be large and positive.

Following a similar line of reasoning, we might presume that if the m^{th} and n^{th} elements are quite remote, then the product $\frac{\partial U_l^k}{\partial \rho_n} \frac{\partial U_l^k}{\partial \rho_m}$ may likely be small. If the numbering of the elements is such that nearby elements have indices that are close together, then the matrix $\mathbf{A}$, whose elements are described by equation 16, will have its largest, positive elements on or near the diagonal.

The second term in equation 13,

$$\mathbf{B}_{n,m} = -2 \sum_{k=1}^{L-1} \sum_{l=1}^L (V_l^k - U_l^k) \frac{\partial^2 U_l^k}{\partial \rho_n \partial \rho_m} \quad (17)$$

has no simple interpretation. But notice that if our estimate ρ is close to the true ρ , then this entire term will be close to zero. In this algorithm, we improve the condition of F' by neglecting the term $\mathbf{B}$ entirely, replacing it with

$$\mathbf{B}_{n,m} \approx \gamma \mathbf{A}_{n,m} \delta_{n,m}$$

where γ is a regularization parameter and $\delta_{n,m}$ is the Kroenecker delta. So

$$F'_{n,m} \approx \mathbf{A}_{n,m} + \gamma \mathbf{A}_{n,m} \delta_{n,m}. \quad (18)$$

The regularization parameter γ is chosen empirically to produce a balance between numerical stability and image contrast and definition. This well known regularization is used in the Levenberg-Marquardt method in, for example, [Press, *et al*, 1986].

Calculation of ρ^0

We now consider the problem of computing an initial resistivity estimate ρ^0 that is needed in equation 12. If we choose a uniform resistivity estimate, then we have

$$\rho^0 = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \vdots \\ \rho_N \end{bmatrix} = c \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

where c is a scalar constant. We choose for the initial resistivity estimate ρ^0 the value that minimizes the error between the measured voltage values and the values predicted by the forward solver for a constant resistivity. Here, because the differential equation is linear, we can simplify

$$U_l^k(c\mathbf{1}) = cU_l^k(\mathbf{1}).$$

We would like to minimize

$$E(c\mathbf{1}) = \sum_{k=1}^{L-1} \sum_{l=1}^L [V_l^k - cU_l^k(\mathbf{1})]^2. \quad (19)$$

Again, setting the derivative equal to zero, we have

$$\frac{\partial E}{\partial c} = -2 \sum_{k=1}^{L-1} \sum_{l=1}^L [V_l^k - cU_l^k(\mathbf{1})] U_l^k(\mathbf{1}) = 0. \quad (20)$$

Now, we can solve for c to obtain

$$c = \frac{\sum_{k=1}^{L-1} \sum_{l=1}^L [V_l^k U_l^k(\mathbf{1})]}{\sum_{k=1}^{L-1} \sum_{l=1}^L [U_l^k(\mathbf{1})]^2}. \quad (21)$$

Thus we choose our initial guess so that $\rho^0 = c\mathbf{1}$, with c defined as in equation 21.

Computing $U_l^k(\rho^0)$

The next task is to predict the observed boundary voltages U_l^k for a homogeneous tank of resistivity ρ^0 . We can solve this problem analytically. We have

$$\nabla^2 u = 0 \quad (22)$$

in the interior. The Neumann boundary conditions are

$$\frac{\partial u}{\partial z}(r, \theta, 0) = \frac{\partial u}{\partial z}(r, \theta, h) = 0 \quad (23)$$

on the top and bottom of the tank. On the sides of the tank,

$$\sigma^0 \frac{\partial u}{\partial r}(r_0, \theta, z) = j(\theta, z). \quad (24)$$

We assume that the current on the l^{th} electrode is equal to the area of the electrode multiplied by $\sigma^0 \frac{\partial u}{\partial r}$ evaluated at the center of the electrode. Similarly, we assume that U_l^k , the voltage on the l^{th} electrode, is just the potential u at the center of the l^{th} electrode. These assumptions we refer to as the "continuum model" [Cheng, *et al*, 1990, Cheney, *et al*, 1990].

We can use separation of variables to solve

$$\nabla^2 u = u_{rr} + \frac{u_r}{r} + \frac{u_{\theta\theta}}{r^2} + u_{zz} = 0. \quad (25)$$

for the potentials on the boundary. We discover two sets of solutions. For $m = 0$ we find solutions that are constant in z :

$$U(r, \theta) = \frac{\rho^0 r_0}{n} \left(\frac{r}{r_0} \right)^n \cos(n\theta) \quad (26)$$

and

$$U(r, \theta) = \frac{\rho^0 r_0}{n} \left(\frac{r}{r_0} \right)^n \sin(n\theta) \quad (27)$$

For $m \neq 0$, we have solutions of the form:

$$U(r, \theta, z) = \frac{\rho^0 h}{m\pi} \frac{I_n\left(\frac{m\pi r}{h}\right)}{I'_n\left(\frac{m\pi r_0}{h}\right)} \cos(n\theta) \cos\left(\frac{m\pi z}{h}\right) \quad (28)$$

and

$$U(r, \theta, z) = \frac{\rho^0 h}{m\pi} \frac{I_n\left(\frac{m\pi r}{h}\right)}{I'_n\left(\frac{m\pi r_0}{h}\right)} \sin(n\theta) \cos\left(\frac{m\pi z}{h}\right). \quad (29)$$

where h is the height of the cylinder and $I_n(x)$ denotes the n^{th} "modified Bessel function."

From these results, we can write down characteristic current patterns for the 3-D problem in the test tank. These current patterns T^k satisfy $U_l^k = \rho_{m,n} T_l^k$, i.e., they are the eigenvectors of the operator $R(\rho^0)$ when ρ^0 is a constant.

These characteristic current patterns are the values at the electrode centers of

$$\begin{aligned} & C \cos(n\theta) \cos\left(\frac{m\pi z}{h}\right), \text{ for } m = 0, 1, \dots, M-1; n = 1, 2, \dots, \frac{\Lambda}{2} \\ & C \sin(n\theta) \cos\left(\frac{m\pi z}{h}\right), \text{ for } m = 0, 1, \dots, M-1; n = 1, 2, \dots, \frac{\Lambda}{2} - 1 \\ & C \cos\left(\frac{m\pi z}{h}\right), \text{ for } m = 1, 2, \dots, M-1; n = 0, \end{aligned} \quad (30)$$

where C is the maximum amplitude available from our current generator electronics. We denote these characteristic patterns by $T^k, k = 1, 2, \dots, \Lambda M - 1$. For instance, for a system with 4 rows of 16 electrodes, this defines a total of 63 orthogonal current patterns over the 64 electrodes.

Computing $\frac{\partial U^k(\rho^0)}{\partial \rho_n}$

From equation 12, we require the vector $\frac{\partial U^k(\rho^0)}{\partial \rho_n}$, and we can expand this vector in the orthogonal basis of equation 30:

$$\frac{\partial U^k(\rho^0)}{\partial \rho_n} = \sum_{s=1}^{L-1} \frac{\langle T^s, \frac{\partial U^k(\rho^0)}{\partial \rho_n} \rangle}{\langle T^s, T^s \rangle} T^s. \quad (31)$$

We show in [Cheney, *et al*, 1990] that

$$R_{s,k}^b = \langle T^s, \frac{\partial U^k(\rho^0)}{\partial \rho} \rangle \approx \frac{1}{(\rho^0)^2} \int_{\text{voxel}_n} \nabla u^s \cdot \nabla u^k, \quad (32)$$

where u^α satisfies equations 22–24 and with the j of equation 24 corresponding to T^α .

Computation of these elements $R_{s,k}^b$ requires most of the computer time for the reconstruction. From equation 32, we see that these quantities depend only on the mesh geometry and can be precomputed. We demonstrate the computation for a typical 3-D integral below.

Typical R Matrix Calculation

We demonstrate a typical calculation for the R matrix elements. In total, there will be 25 such cases, related to the each type of current pattern from equation 30 with each of the other types of current patterns. Fortunately, the calculations are highly degenerate. We use the continuum model for all the derivative calculations. Consider the case where I^α is the vector composed of

$$c^\alpha \cos(n\theta) \cos\left(\frac{m\pi z}{h}\right) \quad (33)$$

evaluated at the center of the electrodes, and I^k corresponds in a similar way to

$$c^k \cos(n\theta) \cos\left(\frac{m\pi z}{h}\right). \quad (34)$$

The scalar c has been chosen so that $\sum_{l=1}^L (I_l^\alpha)^2 = 1$.

Using the “continuum” electrode model,

$$U^\alpha(r, \theta, z) = \frac{c^\alpha \rho^0 h}{m\pi} \frac{I_n\left(\frac{m\pi r}{h}\right)}{I'_n\left(\frac{m\pi r_0}{h}\right)} \cos(n\theta) \cos\left(\frac{m\pi z}{h}\right), \quad (35)$$

and U^k is governed by a similar equation. From equation 32, we have

$$R_{\alpha,k}^b = \frac{1}{(\rho^0)^2} \int_{\text{voxel}_b} \nabla u^\alpha \cdot \nabla u^k dv. \quad (36)$$

Substituting our expression for the potential yields

$$R_{\alpha,k}^b = \frac{c^\alpha c^k h^2 (\rho^0)^2}{(\rho^0)^2 m^\alpha m^k \pi^2 I_{n^\alpha} \left(\frac{m^\alpha \pi r_0}{h} \right) I_{n^k} \left(\frac{m^k \pi r_0}{h} \right)} \int_{\text{voxel}_b} \nabla \left[I_{n^\alpha} \left(\frac{m^\alpha \pi r}{h} \right) \cos(n^\alpha \theta) \cos \left(\frac{m^\alpha \pi z}{h} \right) \right] \cdot \nabla \left[I_{n^k} \left(\frac{m^k \pi r}{h} \right) \cos(n^k \theta) \cos \left(\frac{m^k \pi z}{h} \right) \right] \quad (37)$$

after constants are factored and removed from beneath the integral. Now we use the fact that in cylindrical coordinates,

$$\iiint \nabla u \cdot \nabla v = \int \frac{u_\theta v_\theta}{r^2} + \int u_r v_r + \int u_z v_z. \quad (38)$$

to carry out the integration in equation 37. The integral involving only the θ derivatives is

$$\int_{\text{voxel}_b} \frac{I_{n^\alpha} \left(\frac{m^\alpha \pi r}{h} \right) I_{n^k} \left(\frac{m^k \pi r}{h} \right)}{r^2} \cos \left(\frac{m^\alpha \pi z}{h} \right) \cos \left(\frac{m^k \pi z}{h} \right) n^\alpha n^k \sin(n^\alpha \theta) \sin(n^k \theta) r dr d\theta dz, \quad (39)$$

which is

$$n^\alpha n^k \int_{r_1}^{r_2} \frac{I_{n^\alpha} \left(\frac{m^\alpha \pi r}{h} \right) I_{n^k} \left(\frac{m^k \pi r}{h} \right)}{r} dr \int_{\theta_1}^{\theta_2} \sin(n^\alpha \theta) \sin(n^k \theta) d\theta \int_{z_1}^{z_2} \cos \left(\frac{m^\alpha \pi z}{h} \right) \cos \left(\frac{m^k \pi z}{h} \right) dz. \quad (40)$$

Undaunted, we continue with the term involving only r derivatives of equation 38, finding

$$\frac{m^\alpha m^k \pi^2}{h^2} \int_{r_1}^{r_2} I'_{n^\alpha} \left(\frac{m^\alpha \pi r}{h} \right) I'_{n^k} \left(\frac{m^k \pi r}{h} \right) r dr \int_{\theta_1}^{\theta_2} \cos(n^\alpha \theta) \cos(n^k \theta) d\theta \int_{z_1}^{z_2} \cos \left(\frac{m^\alpha \pi z}{h} \right) \cos \left(\frac{m^k \pi z}{h} \right) dz. \quad (41)$$

Finally, we turn our attention to the term involving z derivatives:

$$\frac{m^\alpha m^k \pi^2}{h^2} \int_{r_1}^{r_2} I_{n^\alpha} \left(\frac{m^\alpha \pi r}{h} \right) I_{n^k} \left(\frac{m^k \pi r}{h} \right) r dr \int_{\theta_1}^{\theta_2} \cos(n^\alpha \theta) \cos(n^k \theta) d\theta \int_{z_1}^{z_2} \sin \left(\frac{m^\alpha \pi z}{h} \right) \sin \left(\frac{m^k \pi z}{h} \right) dz. \quad (42)$$

Computer	Operating System	CPU Utilization (seconds)
Sun 4/150	SunOS	285
IBM 3092 w/VF	AIX	16.2
Cray Y/MP	UNICOS	2.6

Table 3: Some Typical Run Times for *N3D*

We find that if *both* patterns are of the form of equation 33, then equations 40-42 provide an expression for $R_{\alpha,k}$ for voxel b . We (or at least *Mathematica*²) can analytically integrate the terms involving only sine and cosine, but terms containing the modified Bessel functions I and their derivatives I' must be evaluated numerically. Here, these integrations are performed using Simpson's rule.

Expressions similar to equations 40-42 can be worked out for other combinations of current patterns.

Images from *N3D*

N3D has been implemented in FORTRAN for a cylindrical domain. In this section, we demonstrate images from *N3D* using experimentally obtained voltage data. Typical execution times for several machines are displayed in Table 3 for a problem with 420 degrees of freedom.

Figure 3 demonstrates a test configuration consisting of a tank, 30 cm in diameter and 26 cm high. This tank was filled with saline adjusted to approximately $900 \Omega - cm$. The cylinder's wall was lined with 64 electrodes, arranged as 4 rows of 16 electrodes per row. The electrodes consisted of stainless steel about 5 cm square, shaped to conform to the tank inner surface. Also indicated in the figure is the discretization of the domain into 420 resistivity voxels.

Two objects were introduced into this volume. The first object consisted of a cylinder of copper, 3.3 cm in diameter and 5 cm high. It was placed at the level of the third electrode plane. The second target was a teflon cylinder, 3.3 cm in diameter and 13 cm high, placed so that it was confined to the topmost and second electrode planes.

Currents in several configurations were applied to the test tank. The first experiment was designed to simulate two-dimensional acquisition in the three-dimensional volume. Here, an orthogonal set of currents were applied on the third plane of electrodes, and the resulting

²Wolfram Research, Inc., Champaign, IL.

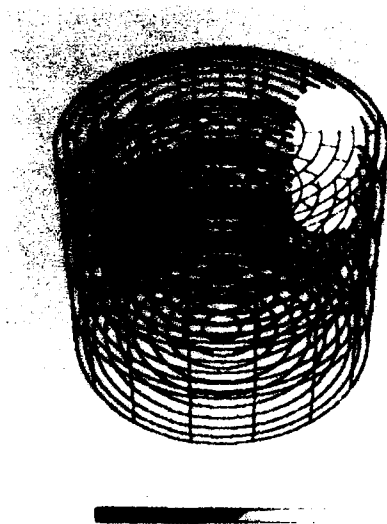


Figure 3: Test Tank Configuration

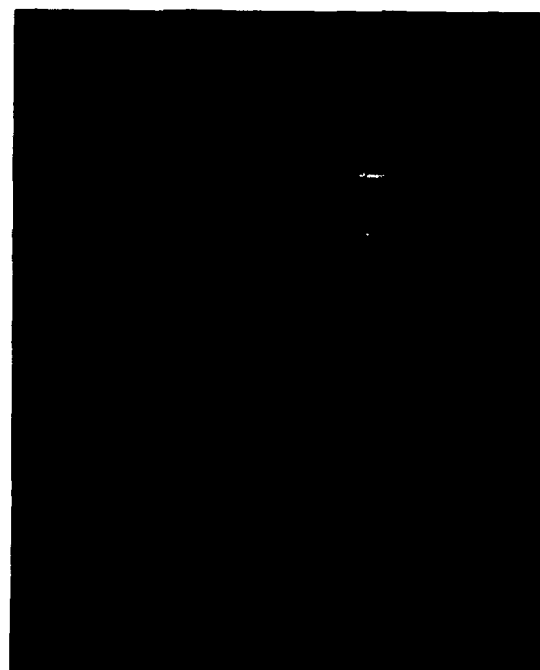
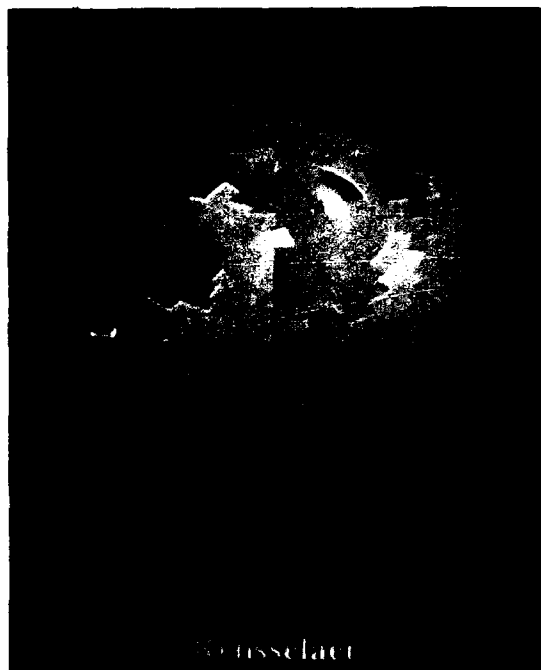


Figure 4: *NOSER* Reconstruction (l.) and *N3D* Reconstruction (r.)

voltages were used to reconstruct the resistivity distribution in the plane using a sixteen electrode variation of the 2-D *NOSER* code [Cheney, *et al*, 1990]. The targets were removed from the tank, an appropriate volume of saline added to replenish the displaced volume of the targets, and a second set of voltages were acquired. This "background" data was also reconstructed using the *NOSER* code. Resistivity reconstructions with and without the targets were subtracted, and the resulting net resistivity vector was processed using 3-D visualization software.

The resulting image, displayed on the left in Figure 4, clearly demonstrates the presence of the conductive target. However, an artifact of increased resistivity also appears in the image in an area that underlies the position of the resistive cylinder. The net resistivity of this reconstruction ranges from -4.2 to $+2.1 \Omega - cm$. The presence in the image of resistivity perturbations outside the plane of the electrodes demonstrates the importance of truly three-dimensional reconstruction algorithms.

To demonstrate the improved performance of *N3D*, the canonical current patterns T^k were scaled to 5 mA and established on the electrode array. Again, the resistivity vector resulting from the *N3D* reconstruction and the "background" vector were subtracted to produce a net resistivity image. This image, displayed as a three-dimensional cylinder cut away through the plane of the targets to reveal the resistivity reconstruction in the interior, demonstrates the ability of the algorithm to obtain information about the resistivity distribution along the longitudinal axis of the cylinder. The net resistivity image displayed on the right in Figure 4 ranges from -9.7 to $+14.3 \Omega - cm$.

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COMPUTATIONAL METHOD FOR POWER LINE MAGNETIC FIELD EVALUATION

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ABSTRACT - The paper reports a model for the power line magnetic field evaluation. The obtained results for three different power lines (380 kV double circuit, 380 kV single circuit, 220 kV double circuit) was validated with measurement data. Calculated and corresponding measured data were found in good agreement. The influence of geometrical and electrical power line parameters, such as the spatial configuration of the conductors and line phase sequences, is evaluated and discussed. Data on the influence of adjacent line span on the field are also reported.

The method can be used for prediction of human exposure in epidemiological studies.

INTRODUCTION

In the past years some papers [Wertheimer and Leeper, 79-82; Tomenius, 86; Savitz, 88] have proposed a possible correlation between exposure to low-level magnetic field generated by transmission lines and the increase of cancer and leukaemia risk.

A more accurate way of assessing magnetic field human exposure has been considered necessary by the International Agency for Research on Cancer in order to better understand the results of epidemiological studies [Ad Hoc Working Group, 90]. Computational methods allowing an accurate evaluation of the magnetic field pattern around transmission lines, can be of great help in the human exposure assessment. Other authors [Deno, 76; Hart and Marino, 77; CIGRE', 80; Olsen and Pankaskie, 83; Olsen and Jaffa, 84; Olsen and Rouseff, 85] have reported theoretical analyses of transmission line magnetic fields. These analyses are based on several assumptions to simplify the calculation procedure such as conductors being parallel to the ground, a flat earth and two dimensional space.

Our paper regards a calculation method to be used for the evaluation of magnetic field emitted by transmission lines. The method takes also in account for the real conductor configuration, geometrical characteristic of the line and earth orography.

Comparison between calculated and measured magnetic field values for different kinds of transmission lines is also reported.

CALCULATION METHOD

In a transmission line span the conductors describe a catenary curve Γ which is represented, in the conductor plane, by the following function:

$$Y(x) = \frac{Y_v}{2} (e^{x/Y_v} + e^{-x/Y_v}) = Y_v \cosh \frac{x}{Y_v} \quad (1)$$

where we have considered a reference system with y axis coincident with catenary axis and x axis at a

distance Y_v from the catenary vertex (see fig.1).

The magnetic field B generated by a generic line conductor which describes a Γ curve at a P point in space is given, using the Ampere-Laplace law, by:

$$\vec{B} = \mu_0 \int_{\Gamma} I \frac{d\vec{l} \times \vec{r}}{4\pi r^3} \quad (2)$$

where I is the current which flows in the line conductor element dl , r is the distance between the dl element and the P point and μ_0 is the vacuum permeability.

In order to obtain data of the three field orthogonal components, we have solved the integral of eq. (2) by using the numerical Simpson method [Gerald, 70] for the integral corresponding to the three orthogonal components of vector $dl \times r$.

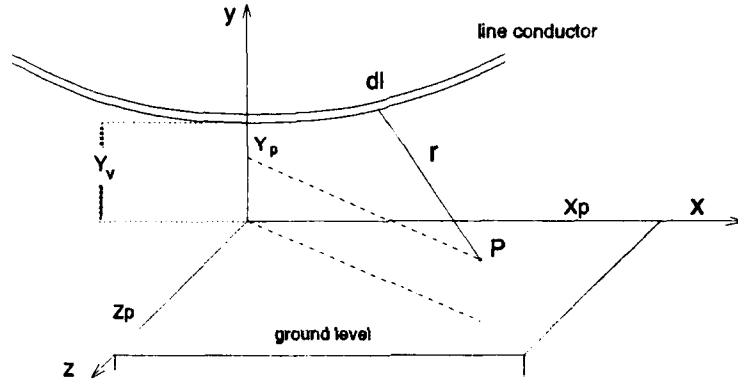


Fig.1 The adopted orthogonal reference system.

If we assume that the conductor plane is the y - x plane (see fig. 1), the z component of dl is zero. So, we can write the three components of differential contribution (dB) to B field as:

$$dB_x = \mu_0 I \frac{Z_p dy}{4\pi r^3} \quad (3a)$$

$$dB_y = -\mu_0 I \frac{Z_p dx}{4\pi r^3} \quad (3b)$$

$$dB_z = \mu_0 I \frac{[(y-Y_p) dx - (x-X_p) dy]}{4\pi r^3} \quad (3c)$$

where dy and dx are the dl components in the y and x direction respectively and Z_p , Y_p and X_p are the three orthogonal P coordinates in the reference system considered.

By differentiation of function reported in eq. (1) we obtain:

$$dy = \sinh \frac{x}{Y_v} dx \quad (4)$$

Substituting dy contained in equations (3a) and (3c) with expression obtained in eq. (4), the three components of differential magnetic field (dB) can be written as:

$$dB_x = \frac{\mu_o I Z_p}{4 \pi r^3} \sinh \left(\frac{x}{Y_v} \right) dx \quad (5a)$$

$$dB_y = \frac{\mu_o I Z_p}{4 \pi r^3} dx \quad (5b)$$

$$dB_z = \frac{\mu_o I}{4 \pi r^3} \left[(Y_v \cosh \left(\frac{x}{Y_v} \right) - Y_p) - (x - X_p) \sinh \left(\frac{x}{Y_v} \right) \right] dx \quad (5c)$$

By simple algebraic calculations and using the expression (1), we can express the distance r as a function of only x variable:

$$r = \sqrt{(x - X_p)^2 + Z_p^2 + (Y_v \cosh \left(\frac{x}{Y_v} \right) - Y_p)^2} \quad (6)$$

Once we have expressed r as a function of only x , the equations (5a), (5b), (5c) become:

$$dB_x = \frac{\mu_o I}{4 \pi} \frac{\sinh \left(\frac{x}{Y_v} \right)}{[(x - X_p)^2 + Z_p^2 + (Y_v \cosh \left(\frac{x}{Y_v} \right) - Y_p)^2]^{3/2}} dx \quad (7a)$$

$$dB_y = \frac{\mu_o I}{4 \pi} \frac{dx}{[(x - X_p)^2 + Z_p^2 + (Y_v \cosh \left(\frac{x}{Y_v} \right) - Y_p)^2]^{3/2}} \quad (7b)$$

$$dB_z = \frac{\mu_o I}{4 \pi} \frac{[(Y_v \cosh \left(\frac{x}{Y_v} \right) - Y_p) - (x - X_p) \sinh \left(\frac{x}{Y_v} \right)]}{[(x - X_p)^2 + Z_p^2 + (Y_v \cosh \left(\frac{x}{Y_v} \right) - Y_p)^2]^{3/2}} dx \quad (7c)$$

The result of the integration on the catenary is a magnetic field B whose j -th orthogonal component is:

$$B_j = A_j \cos(\omega t + \phi_j) \quad (8)$$

where A_j is the amplitude of the field and ϕ_j is its phase.

The total magnetic field generated by the transmission line has to be evaluated considering the contributions of the single conductors and the interferences resulted from the superposition of the fields generated by the several conductors which constitute the line.

The j -th orthogonal component of the total B field ($B_{\omega j}$) is given:

$$B_{\omega j} = A_{\omega j} \cos(\omega t + \phi_{\omega j}) \quad (9)$$

where:

$$\phi_{totj} = \arctan \left[\frac{\sum_{i=1}^3 A_{ji} \sin \phi_{ji}}{\sum_{i=1}^3 A_{ji} \cos \phi_{ji}} \right] \quad (10a)$$

$$A_{totj} = \sqrt{\sum_{i=1}^3 A_{ji}^2 - \frac{1}{2} \sum_i \sum_k A_{ji} A_{jk}} \quad , \quad i \neq k \quad (10b)$$

In the above reported expressions we have represented with A_{ji} and ϕ_{ji} the amplitude and the phase respectively of the j -th component of the field produced by the i -th conductor. In writing the equation (10b) we assume that the difference between the phases relative to the conductors is equal to 120° .

Equations (10a) and (10b) can be applied also to a double circuit transmission line, which is constituted (see fig. 2) by three couples of phased conductors (on the whole six conductors). For this type of line i index refers to each couple of phased conductors and so A_{ji} have to be equal to the sum of the B_j components produced by the single conductors of the i -th couple.

The earth influence on field pattern can be evaluated using the image method, which considers the contribution to B_{tot} due to image currents placed under the ground. These image currents must be chosen such that the following boundary conditions at air-ground interface are satisfied:

$$B_{1n} = B_{2n} \quad (11a)$$

$$\frac{B_{1t}}{\mu_0} = \frac{B_{2t}}{\mu_1} + J_s \quad (11b)$$

where B_{1n} and B_{1t} are respectively the normal and tangential B components in the air and B_{2n} and B_{2t} are respectively the normal and tangential B components in the earth, J_s is a surface current flowing upon the interface, and μ_0 and μ_1 are respectively the air and the ground permeability.

As the earth has a finite conductivity ($\sigma = 10^{-1} \div 10^{-3}$ mhos/m) J_s cannot exist on the interface and the boundary condition for tangential component of B field (eq. 11b) becomes:

$$\frac{B_{1t}}{\mu_0} = \frac{B_{2t}}{\mu_1} \quad (11c)$$

By using the above reported Ampere-Laplace law, see eq. (2), and considering the geometric parameters shown in the scheme of figure 3, the tangential and normal components of B field generated by a current with a J density can be given, in a P point on the ground (see fig. 3), by the following expressions:

$$B_{1n} = \frac{\mu_0}{4\pi} JA \int_{\Gamma} \frac{x_0 dl}{(x_0^2 + y_1^2)^{3/2}} ; \quad B_{1t} = \frac{\mu_0}{4\pi} JA \int_{\Gamma} \frac{x_0 dl}{(x_0^2 + y_1^2)^{3/2}} ; \quad (12)$$

where A is the area of conductor section, and x_0 and y_1 are the projection of position vector r respectively

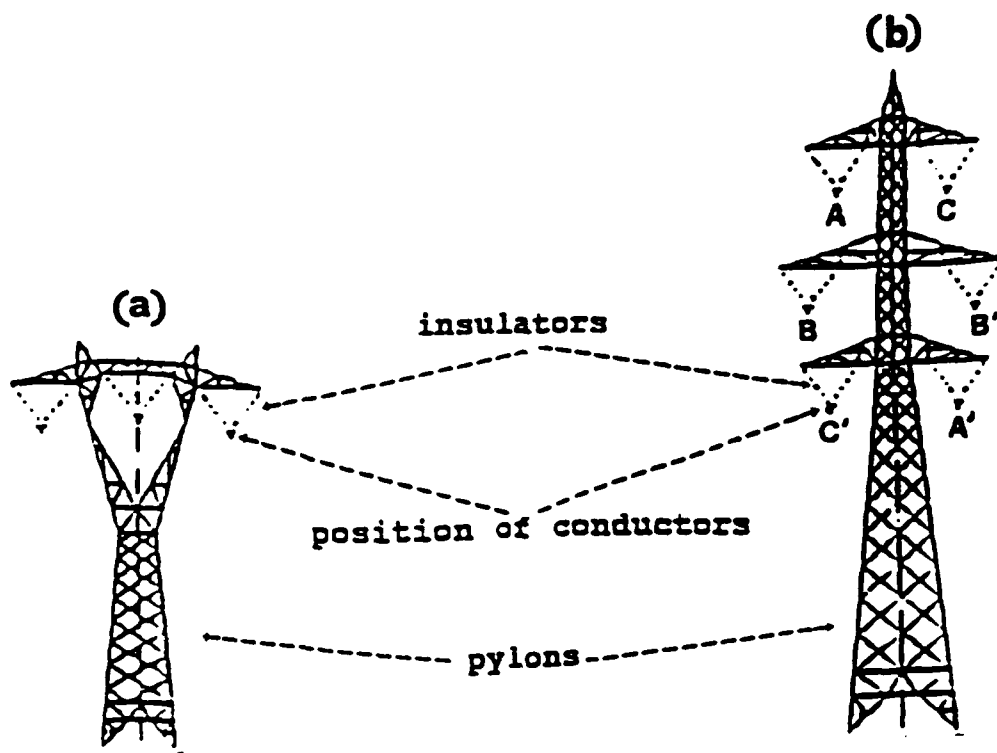


Fig.2 The basic structure of: (a) a single circuit 3-phase line; (b) a double circuit 3-phase line.
 In the figure (b), A-A', B-B' and C-C' letters indicate the three couples of conductors in which current with the same phase flows. In this figure is represented one of the possible configurations of phased conductors, which is named crossed phase configuration.

on x-z plane and y-z plane. We note that both x_0 and y_1 are dependent of l integration variable.

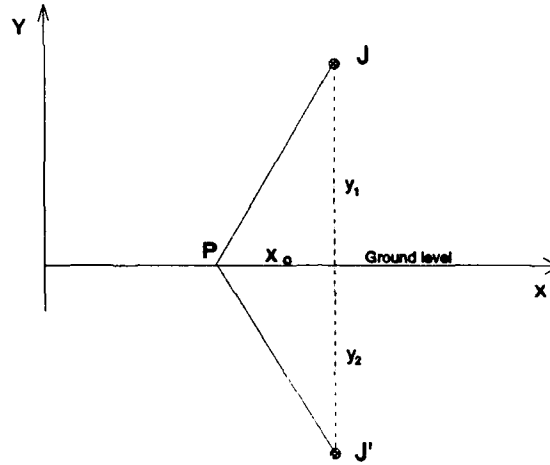


Fig.3 Cross section of real (J current density) and image (J' current density) conductors placed in opposite sides of the ground level.

If we express the B tangential and normal components by the equations (12), the boundary conditions (11a) and (11c) can be written as:

$$\frac{\mu_0}{4\pi} JA\alpha - \frac{\mu_0}{4\pi} J'A\beta = \frac{\mu_1}{4\pi} J''A\alpha \quad (13a)$$

$$\frac{JA}{4\pi} \gamma - \frac{J'A}{4\pi} \epsilon = \frac{J''A}{4\pi} \gamma \quad (13b)$$

where J' is the image current and J'' is a current density distributed in the space as J , generating the B field (B_2) under the ground; the coefficients α , β , γ and ϵ are respectively given by:

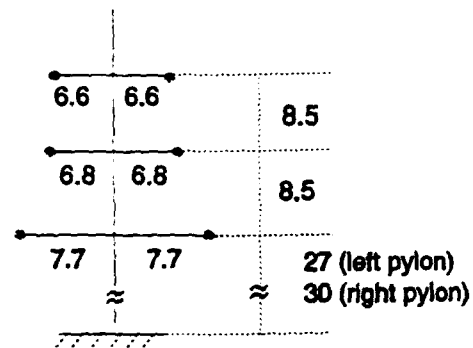
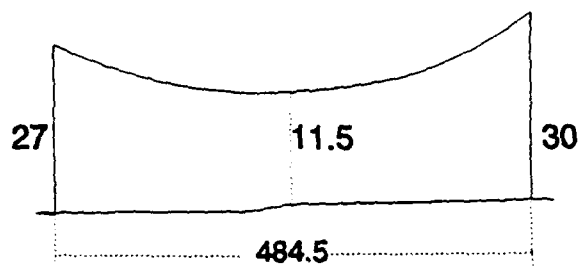
$$\alpha = \int_{\Gamma} \frac{x_0 dl}{(x_0^2 + y_1^2)^{3/2}}; \quad \beta = \int_{\Gamma} \frac{x_0 dl}{(x_0^2 + y_2^2)^{3/2}};$$

$$\gamma = \int_{\Gamma} \frac{y_1 dl}{(x_0^2 + y_1^2)^{3/2}}; \quad \epsilon = \int_{\Gamma} \frac{y_2 dl}{(x_0^2 + y_2^2)^{3/2}};$$

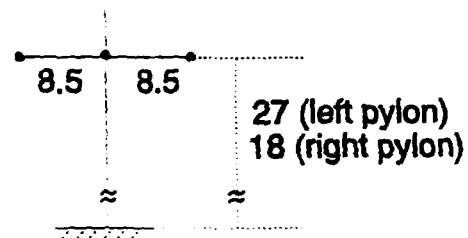
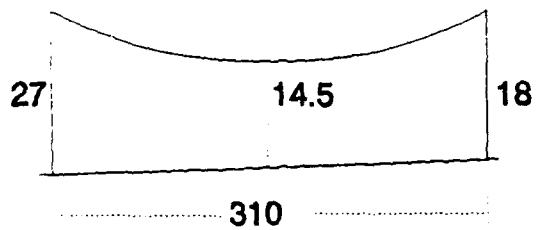
The simultaneous solution of equations 13a and 13b gives the following relation between image current density J' and real current density J :

$$J' = \frac{\mu_0 - \mu_1}{\mu_0 \frac{\beta}{\alpha} + \mu_1 \frac{\epsilon}{\gamma}} J \quad (14)$$

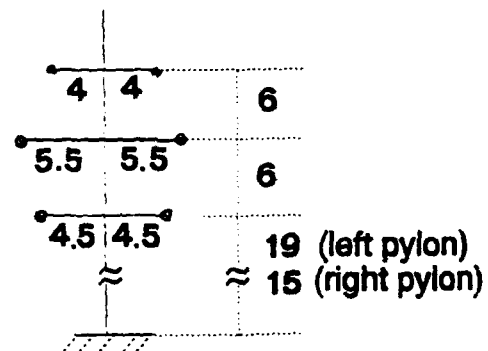
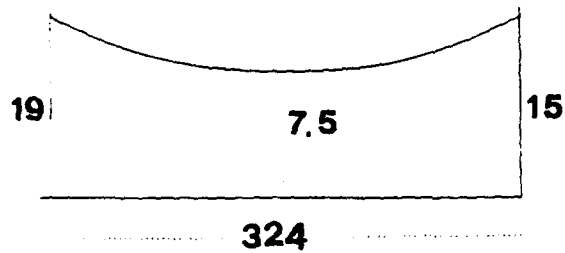
As immediate consequence of equation (14) it results that $J' = 0$ if $\mu_0 = \mu_1$. So if we assume that air permeability is approximately equal to ground permeability ($\mu_0 \approx \mu_1$) we can neglect image currents in the



380 Kv Double circuit Line



380 Kv Single circuit Line



220 Kv Double circuit Line

Fig.4 The geometric characteristics of line spans and pylons relative to the three types of transmission lines considered in this work. All the distances reported in this figure are expressed in meters.

power line magnetic fields calculation procedure.

The above reported computational procedure has been applied to three different transmission lines. The lines taken in consideration were: a) 380 kV double circuit line, b) 380 kV single circuit line, c) 220 kV double circuit line. In figure 4 we have reported the geometric characteristics of pylons and spans relative to the three type of line that we have considered in this paper.

Current flowing in the conductors was measured by National Electricity Generating Board (ENEL) at the station which receive the high voltage electricity and redistribute it at lower voltages. Calculation of the current variation due to the electric characteristics of the line was used, together with the measured values, to assess the current flowing in the considered line span.

Using the calculation procedure, the per cent current variation is found to be about 10% for a line segment length of 100 km.

The calculated B values have been compared with measurement data. The measurements were performed using broad-band and narrow-band measurement systems. The use of these systems and the obtained results are reported in detail in [Tofani and al.].

RESULTS

Comparison between magnetic field profiles calculated assuming rectilinear and catenary conductor distribution is reported in figure 5. We have used two different rectilinear conductors distributions. In the first one (distribution a) the conductors are approximated to straight lines crossing the two points in which each conductor is connected to the two pylons of a span, while in the second distribution (distribution b) the conductors are approximated by straight lines, parallel to the ground and crossing the point of intersection between the conductors itself and the test plane (perpendicular to the line). This last approximation is widely used [Deno, 1976], while the first one can be used when the pilons heights are the only data known with accuracy. Figure 5 shows that the approximation of catenary conductors with rectilinear conductors can lead to a sensible disagreement in the B evaluation.

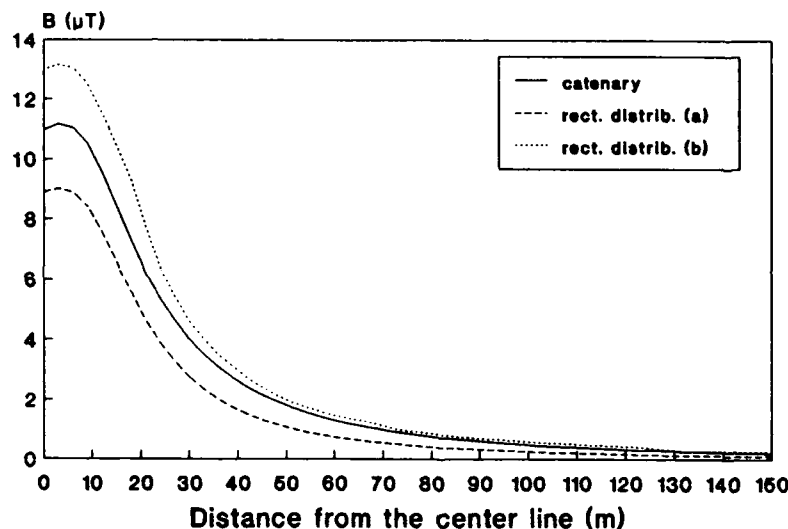


Fig.5 Magnetic field magnitudes calculated, 1.5 m above the ground, considering catenary conductors and rectilinear conductors (distributions (a) and (b)) with a load current of 1000 A. These profiles refer to a 380 kV, 50 Hz double circuit line. The minimum height of conductors is 13 m from the ground.

In the examined case of rectilinear distribution (distribution a, see fig.5) the deviation from the catenary distribution is on the order of -30% up to a distance of about 30 meters from the center line and becomes greater as the distance increases and B values decrease reaching about -50 % at a distance of 150 m, and

vanishing for a further increase of the distance; in the case of the second rectilinear distribution (distribution b, see fig.5) the deviation is on the order of 16% up to a distance of about 30 meters from the center line and vanish for a further increase of the distance. The reported differences due to modeling the line with linear conductors instead of catenary one can increase when adjacent spans taking angles different 180° are concerns.

The influence of the line phase arrangements in the magnetic field profiles for a double circuit power line can be seen in Figure 6.

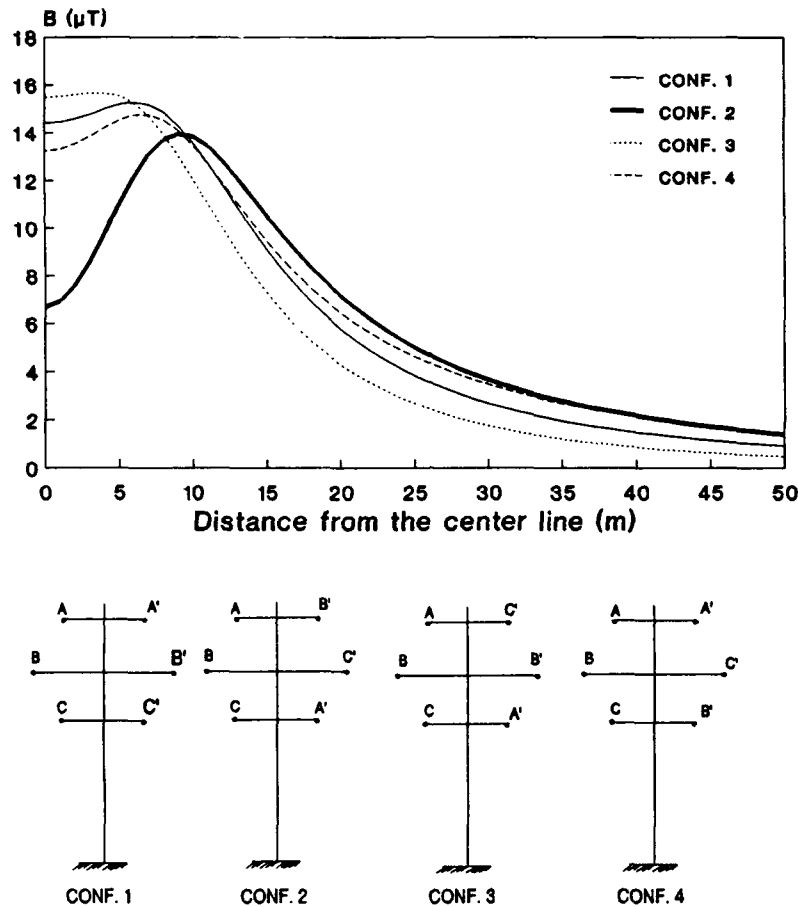


Fig.6 Profiles of magnetic field magnitudes calculated at a 1.5m above ground, under a 380 kV, 50 Hz double circuit line for four different phase arrangements, illustrated in the lower part of this figure (conf.1 ÷ 4). The minimum line height is 12 m. The line loading is 1000 A.

The Figure 6 reports B values versus distance for four different phase configurations. Configuration 3, also named configuration with crossed phases, gives the lowest B values further than about seven meters from the center line. The highest values are given by the configuration 1 further than about 10 meters from the center line.

Furthermore from figure 6 we can see that the per cent differences between B values corresponding to the configuration 3 and those of configuration 1 vary from 30 % at a distance of 15 m from the center line to 70 % at a distance of 50 m from the center line.

Under the line the differences are amplified and the configuration 1 gives the lowest field, while the configuration 3 gives the highest one.

Comparison between calculated and measured B values are reported in figure 7. The measurements were made at the middlespan level in a plane perpendicular to the line, where 380 kV single circuit line and 220

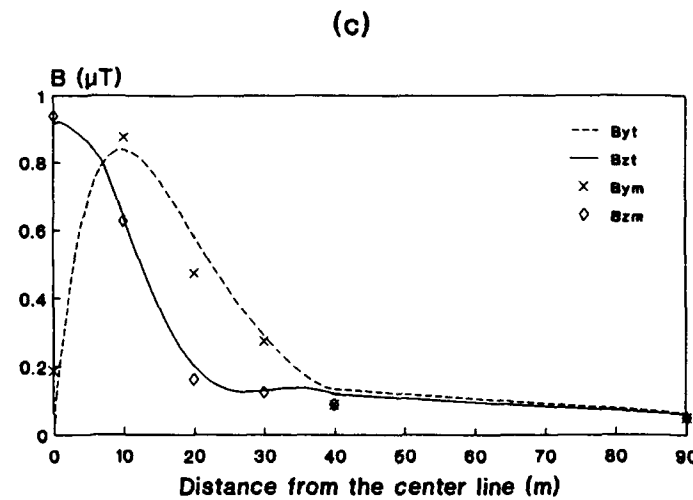
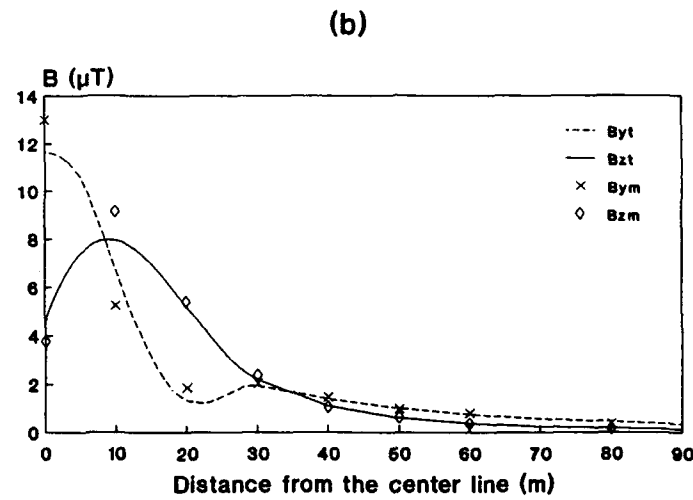
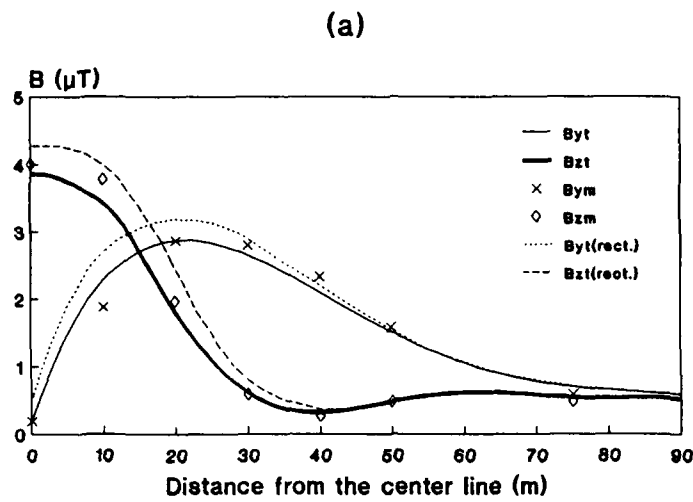


Fig.7 Theoretical (lines — and ---) and experimental (symbols $\diamond$ and $\times$) profiles of two magnetic field components (B_y and B_z) calculated at a 1.5 m above ground, under: (a) 380 kV double circuit line, (b) 380 kV single circuit line, (c) 220 kV double circuit line. B_{yt} and B_{zt} are the calculated B_y, B_z component, while B_{ym} and B_{zm} are the respectively correspondent experimental components. Load current is 850 A for (a) transmission line 1160 A for (b) transmission line and about 100 A for 220 kV line. The catenary vertex of the lowest wire was at 12 m ((a) transmission line), 13 m ((b) transmission line) and 12 m ((c) transmission line) from the line. The additional theoretical data reported in (a), i.e. $B_{yt}(\text{rect.})$ and $B_{zt}(\text{rect.})$, refer to the assumption of rectilinear line conductors parallel to the ground and tangential to the minimum of line catenary.

kV double circuit line were considered, and at a distance of 124 m from the middlespan, where 380 kV double circuit line was considered.

Figure 7 doesn't report data regarding the B_x component because the values of this component are found to be negligible in respect to the other two orthogonal component values (B_y , B_z).

We observe that the used model gives results in a good agreement with the experimental data. The mean value of the differences between all the corresponding theoretical and experimental data is resulted to be about 9.5 %. This value is very close to the accuracy (8%) evaluated for the used measurement systems [Tofani and al.].

The obtained agreement can be very satisfactory considering also that the used measurement systems don't take in account for the accuracy in the positioning of the meter.

If we compare theoretical data, obtained assuming rectilinear conductors ($B_{y,t(rect.)}$ and $B_{z,t(rect.)}$) with the experimental ones, we find a mean difference of 20%.

The decrement of the accuracy between experimental and theoretical data going from catenary assumption to the rectilinear conductors approximation justify the added effort in increasing the mathematical complexity needed for the catenary approach.

So far we have calculated the B field considering only one line span and neglecting the field generated by the adjacent spans. However, there are regions in space in which can be of interest to evaluate also the B field emitted by closer adjacent span, and to compare it with the B field calculated considering only one span.

For example we can examine the case illustrated in figure 8, where are represented two adjacent spans S_1 and S_2 which form an angle $\vartheta = 150^\circ$, with vertex in the common pylon P_c .

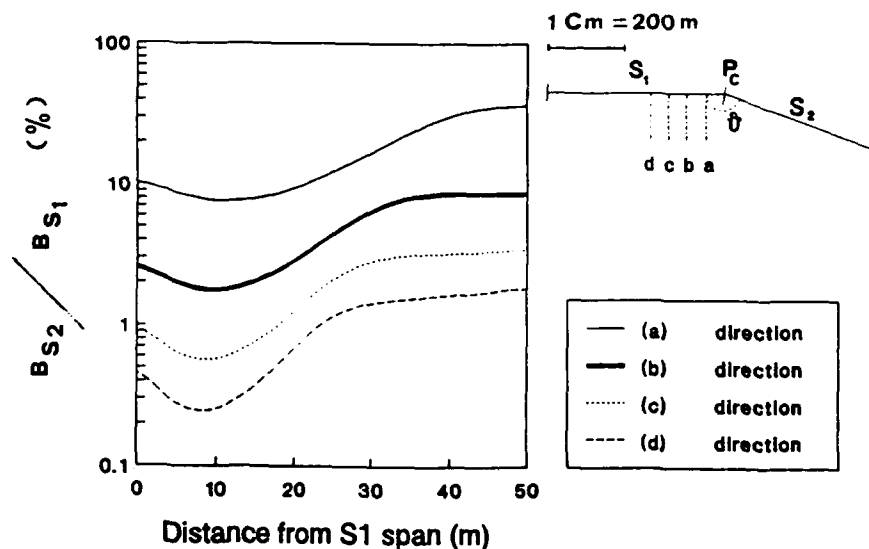


Fig.8 Percentage ratio between the B field produced by S_2 line span (B_{S2}) and that produced by S_1 line span (B_{S1}) versus distance from the center of S_1 span. a, b, c, and d are the four different perpendicular directions located respectively at distances of 50, 100, 150, 200 m from the common pylon P_c .

The "a", "b", "c" and "d" lines represent the directions, located respectively at distances of 50, 100, 150, 200 m from the common pylon P_c , along which we have calculated both the B field emitted by only S_1 span (B_{S1}) and the B field emitted by only S_2 span (B_{S2}).

Always in figure 8 is shown the comparison between B_{S1} and B_{S2} fields in the "a", "b", "c" and "d" directions up to a distance of 50 m from the S_1 line span.

We observe that in the considered case the magnetic field produced by S_2 span is negligible in respect to that produced by S_1 span ($B_{S2}/B_{S1} < 2\%$) along "c" and "d" directions. The B_{S2}/B_{S1} ratio is greater in "a" direction, where increases from 10% to 40 %, respectively at distances of 0 m and 50 m from the S_1 span. These results are strongly dependent from the angle (ϑ) formed by the two adjacent spans. When ϑ

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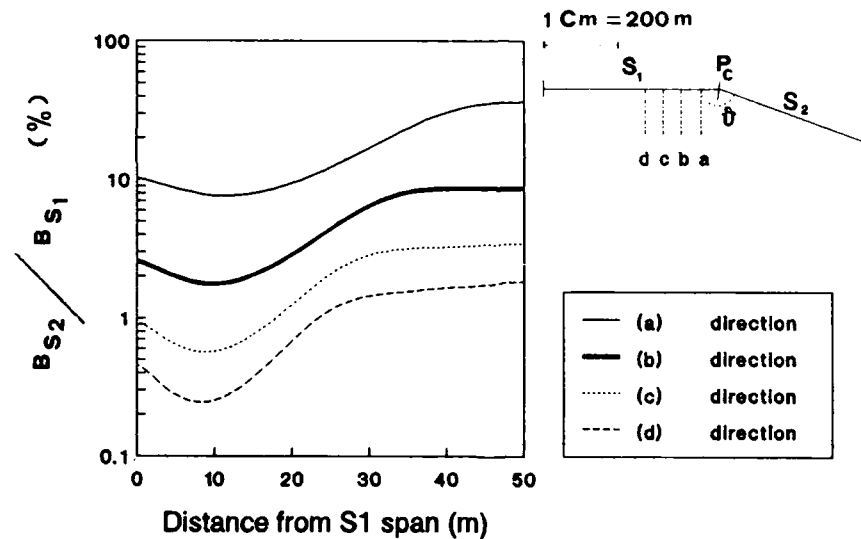


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increases B_{s2} values decrease in the considered directions.

So to determine the magnetic field along directions which are at a small distance (in the above examined case < 100 m) from the common pylon of two adjacent line spans, it is necessary to execute the above reported calculation procedure for both spans. Neglecting the contribution of one of the two spans to total B field, can give rise to large errors in the B amplitude estimation.

CONCLUSIONS

The reported transmission line model, elaborated considering also the actual spatial configuration of conductors (catenary) and the earth orography has given a good prediction of the magnetic field pattern under the line. Considering the measurement accuracy (not less than 8 %), the B calculated values are in good agreement with the measured ones (mean per cent difference equal to 9.5 %)

The more mathematical complexity needed for handling the method, in comparison with the previous methods, is justified by the accuracy improvement in the B estimation. In fact, in the examined lines, the differences between calculated and measured values decrease from 20 (using rectilinear conductors approximation) to 9.5 %.

The influence of geometrical and electrical line parameters, such as spatial configuration of the conductors and line phase sequences, in the B field has been analyzed.

Depending upon the type of the rectilinear conductors assumption used, the disagreement on the B estimate in comparison with the catenary data range, in the considered case, from 16 to 30 % up to a distance of 30 m from the center line, and can be of 50 % at 150 m.

At distances from the center line where the population exposure can be concerned (greater than 20 m from the center line), the influence of the phase sequences on B increases with the distance reaching 70 % at 50 m.

The model has been also used to evaluate the field contributions due to one of two adjacent line spans. In the considered case the contribution is significant only at distance closer than 100 m from the common pylon.

Considering the obtained results, the model can be of good help in the prediction of population exposure to power line magnetic field. This prediction can then be of interest for the interpretation of epidemiological results.

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SIMPLIFIED ANALYTICAL SOLUTIONS FOR MAGNETIC STIMULATION OF NEURONS

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ABSTRACT

Strong pulses of magnetic field are used to stimulate peripheral nerves and motor neurons in the cerebral cortex. Such stimulation is used in neurology for numerous diagnostic purposes. The electric field induced in tissue along the neuron and its spatial derivative are the parameters determining neural response. Another important parameter influencing the efficiency of stimulation is the inductance of a coil producing the magnetic field, as it defines the current time derivative for a given pulse generator. For arbitrarily located coils of arbitrary shapes, a semi-analytical solution is presented to calculate spatial distributions of the electric field and its spatial derivatives in a semi-infinite tissue model. Analytical solutions are given for coils composed of linear segments parallel or perpendicular to the air-tissue interface. Expressions for inductance of coils having suitable geometries for neural stimulation are derived. Coils can be optimized for stimulation of nerves at given orientation and distance from the air-tissue interface. In the optimization, coil dimensions and shape are considered as they affect both the induced field and inductance. A quadruple coil consisting of triangular sections appears to offer some advantages over other shapes for stimulation of shallow nerves. For deep nerves spaced quadruple square and three-dimensional coils are preferred. Analyses described are useful in evaluating various options, gaining an insight into the physical phenomena involved, and as the first step before undertaking a numerical analysis of models more closely representing the tissue electrical and geometrical complexities.

I. INTRODUCTION

Since the mid-1980s magnetic stimulation of neurons has gained rapid acceptance as a diagnostic and research tool in neurology [Geddes, 1991]. Various medical conditions associated with abnormal conduction in motor pathways can be diagnosed by stimulating an appropriate neuron and measuring the conduction velocity by recording motor action potentials. Diseases such as multiple sclerosis, cervical myelopathy, myelopathy, cerebral palsy, Bell's palsy and others can be diagnosed [Barker *et al.*, 1986, 1987, Chokroverty, 1990, Maccabee *et al.*, 1991, Murray, 1991]. Mapping of the motor cortex has been used to evaluate congenital mirror movements, amputations, spinal cord injury and effects of hemispherectomy [Cohen *et al.*, 1991].

Neural stimulation occurs when an externally applied stimulus has a sufficient amplitude and duration to cause current to flow through a cellular (neural) membrane and change its potential above a threshold value. In case of magnetic stimulation the stimulus is produced by a pulse of magnetic field from an external coil. The electric field induced in tissue is proportional to the time derivative of the coil current. For overdamped current pulses, which are preferred in neural stimulation, the current derivative is [Basser and Roth, 1991]:

$$\frac{dI(t)}{dt} = \frac{V}{L} e^{-\omega_1 t} \left[\cosh(\omega_0 t) - \frac{\omega_0}{\omega_1} \sinh(\omega_0 t) \right] \quad (1)$$

1) This work was performed at the Bureau of Radiation and Medical Devices, Health and Welfare Canada.

where V is the voltage, L is the coil inductance, $\omega_1 = R/2L$, R is the series resistance, and

$$\omega_0 = \left[\left(\frac{R}{2L} \right)^2 - \frac{1}{LC} \right]^{1/2}$$

where C is the capacitance of the stimulator capacitor charged to voltage V , which then is discharged through the coil. This relationship indicates that for a given generator (V and C), the stimulus (at $t=0$) is inversely proportional to the inductance.

For long nerves, the sub-threshold transmembrane potential can be derived from a passive cable equation, and for magnetic stimulation it takes the following form [Basser and Roth, 1991]:

$$\lambda_0^2 \frac{\partial^2 V_m}{\partial x^2} - \tau_0 \frac{\partial V_m}{\partial t} - (V_m - V_r) = \lambda_0^2 \frac{\partial E_x}{\partial x} \quad (2)$$

where V_m is the transmembrane potential, V_r is the resting potential, λ_0 is the neural space constant, τ_0 is the neural time constant, E_x is the electric field strength along x , i.e. the direction of the neuron.

The electric field spatial derivative $\partial E_x / \partial x$, also referred to as the activating function, therefore plays an important role in magnetic nerve stimulation. Depending on the sign of the activation function, the neuron is either depolarized (negative derivative) or hyperpolarized. All neurons that are depolarized above a threshold level (determined by nerve size, pulse width, etc.) are stimulated. Equation (2) and these considerations apply directly to long, straight axons; the situation is more complex in case of short neurons with dendrites and it is likely that both the electric field and its derivative will be important.

The importance of evaluation of the induced electric field and its derivatives is apparent. Furthermore, various coil geometries and orientations can produce vastly different spatial distributions of the fields. A simplified analysis of induced fields using an analytical method was previously performed, but the charge accumulation on the air-tissue interface was neglected [Grandori and Ravazzani, 1991]. Numerical techniques were used to analyze homogeneous and layered tissue models such as a semi-infinite tissue space [Roth *et al.*, 1990a], cylinder [Roth *et al.*, 1990a] and sphere [Branston and Tofts, 1991, Roth *et al.*, 1991a]. The numerical analyses performed so far require a considerable computing time, of the order of hours, and large memories, yet do not account for tissue heterogeneity. Another limitation of these methods is that physical insight is lost, hindering their use for optimization or improvement of the coil geometry.

We have previously derived analytical expressions for the induced electric field and its spatial derivatives in a semi-infinite tissue half-space produced by a very short element of current-carrying coil [Esselle and Stuchly, 1992]. Using this method it is possible to calculate within minutes using a microcomputer the induced fields from an arbitrarily shaped and oriented coil. In this contribution we further extend this method and develop closed-form expressions for coils consisting of linear segments of wire parallel or perpendicular to the air-tissue interface. Methods are developed to calculate the inductance of such coils. The coil size is optimized to obtain the strongest stimulus at a given depth. Three promising coil configurations: quadruple square coil (QS), quadruple triangular coil (QT) and three-dimensional (3D) coil are described, analyzed and compared. Performance of these coils is evaluated by considering the following criteria: the strength of the peak stimulus for a given voltage, the rate at which the peak stimulus decreases with depth, the peak hyperpolarization level as a fraction of the peak depolarization level, and the size of the stimulation region (spot).

II. CALCULATION OF INDUCED ELECTRIC FIELD

Infinitesimal Coil Element

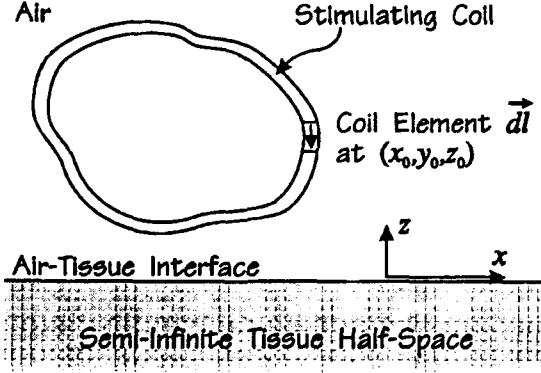


Figure 1. A stimulating coil of arbitrary shape, above a semi-infinite tissue block.

The computation of the induced electric field in the semi-infinite tissue half-space, shown in Fig. 1, from a time-varying current in an external, arbitrarily-shaped coil usually requires several numerical steps. However the electric field from an infinitesimal element of the coil can be evaluated analytically [Esselle and Stuchly, 1992], provided that the time-variation of current is slow enough to justify quasi-static approximations [Roth *et al.*, 1991b]. For a coordinate system with its origin on the interface and z-axis perpendicular to the interface, and for a tissue half-space homogeneous in the x- and y-directions, the quasi-static electric field components induced at point (x,y,z) in tissue by a coil-element $\vec{dl}$ located at (x_0, y_0, z_0) in free-space are as follows [Esselle and Stuchly, 1992]:

$$dE_x = -\frac{\mu_0(dI/dt)}{4\pi} \left\{ \frac{dl_x}{R} + \frac{dl_z(x-x_0)}{\rho^2} \left(1 + \frac{z-z_0}{R} \right) \right\} \quad (3)$$

$$dE_y = -\frac{\mu_0(dI/dt)}{4\pi} \left\{ \frac{dl_y}{R} + \frac{dl_z(y-y_0)}{\rho^2} \left(1 + \frac{z-z_0}{R} \right) \right\} \quad (4)$$

$$dE_z = 0 \quad (5)$$

where

$$\rho = \sqrt{(x-x_0)^2 + (y-y_0)^2}, \quad (6)$$

$$R = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}, \quad (7)$$

dl_x , dl_y and dl_z are the components of the vector $\vec{dl}$, μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ H/m) and dI/dt is the time rate of change of coil current. The induced current is given by $\vec{J} = \sigma \vec{E}$ where σ is tissue conductivity. The spatial derivatives of the induced electric field components are given by:

$$d \left(\frac{\partial E_x}{\partial x} \right) = \frac{\mu_0(dI/dt)}{4\pi} \left\{ \frac{(x-x_0)dl_x}{R^3} + \left[\frac{(x-x_0)^2 - (y-y_0)^2}{\rho^4} \right] dl_z + \left[\frac{(x-x_0)^2 - (y-y_0)^2}{R\rho^4} + \frac{(x-x_0)^2}{\rho^2 R^3} \right] (z-z_0) dl_z \right\} \quad (8)$$

$$d \left(\frac{\partial E_y}{\partial y} \right) = \frac{\mu_0(dI/dt)}{4\pi} \left\{ \frac{(y-y_0)dl_y}{R^3} + \left[\frac{(y-y_0)^2 - (x-x_0)^2}{\rho^4} \right] dl_z + \left[\frac{(y-y_0)^2 - (x-x_0)^2}{R\rho^4} + \frac{(y-y_0)^2}{\rho^2 R^3} \right] (z-z_0) dl_z \right\} \quad (9)$$

The total electric field and the spatial derivatives can be obtained by integrating numerically the corresponding

expression along the length of the coil. This method of field computation is more efficient than methods used previously, since the numerical computation used here is very simple and requires little time.

It is interesting to note that neither the electric field nor its spatial derivatives depend on tissue conductivity σ . This is a result of the following quasi-static assumptions made in the derivation: (1) tissue is a good conductor where the displacement current is negligible compared with the conduction current ($\sigma \gg 2\pi f\epsilon$), (2) tissue is not an excellent conductor, and therefore, the skin depth ($1/\sqrt{\mu\sigma\pi f}$) is much larger than the system dimensions. At sufficiently low frequencies (< 10 kHz), these conditions are valid for most biological tissues [1991a]. As long as the tissue behaves as a good-but-not-excellent conductor, the induced electric field is independent of its conductivity and permittivity.

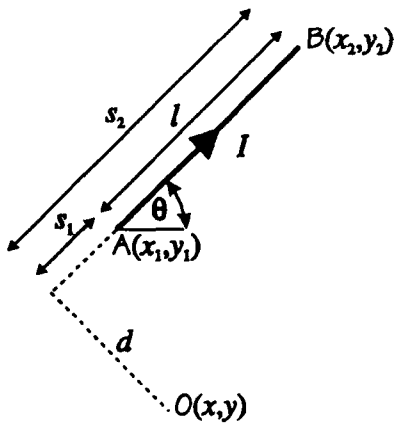
Linear Coil Segment Parallel to the Interface

Most of the best-performing stimulating coils, such as double and quadruple square coils, are composed of several linear segments [Esselle and Stuchly, 1992]. Others, such as circular coils, can be approximated by polygon-shaped coils composed of linear segments [Cohen *et al.*, 1990]. In these cases, the fields and their spatial derivatives produced by each linear segment can be evaluated analytically saving considerable amounts of computer time, and allowing for physical insight.

Let us consider a linear segment of a coil parallel to the interface and extending from (x_1, y_1, z_0) to (x_2, y_2, z_0) . The projection of the segment on the x - y plane is shown in Fig. 2, together with the projection of the point (x, y, z) where the field is evaluated. The electric field components produced by the segment can be obtained by integrating expressions (3) and (4) along the length of the segment. Analytical integrations result in the following expressions:

$$E_x = \frac{\mu_0(dI/dt)}{4\pi} [\sinh^{-1}(\cot \alpha_2) - \sinh^{-1}(\cot \alpha_1)] \cos \theta \quad (10)$$

$$E_y = \frac{\mu_0(dI/dt)}{4\pi} [\sinh^{-1}(\cot \alpha_2) - \sinh^{-1}(\cot \alpha_1)] \sin \theta \quad (11)$$



$$\text{where } \alpha_1 = \cos^{-1} \left[\frac{(x_1 - x)(x_1 - x_2) + (y_1 - y)(y_1 - y_2)}{R_1 l} \right] \quad (12)$$

$$\alpha_2 = \cos^{-1} \left[\frac{(x_2 - x)(x_1 - x_2) + (y_2 - y)(y_1 - y_2)}{R_2 l} \right] \quad (13)$$

$$R_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_0)^2} \quad (14)$$

$$R_2 = \sqrt{(x - x_2)^2 + (y - y_2)^2 + (z - z_0)^2} \quad (15)$$

$$l = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad (16)$$

$$\theta = \tan^{-1} \left(\frac{y_2 - y_1}{x_2 - x_1} \right) \quad (17)$$

Figure 2. The projection of a linear segment and the projection of the point where the fields are calculated is also shown (O).

The spatial derivatives of the electric field due to the linear

parallel segment can be obtained either by differentiating eqs. (10) and (11) with respect to x and y , respectively, or by integrating eqs. (8) and (9) along the length of the segment. Either method leads to the following results:

$$\frac{\partial E_x}{\partial x} = \frac{\mu_0(dl/dt)}{4\pi} \left\{ \frac{\cos\theta}{R_2} \left[\cos\theta + \frac{s_2 d \sin\theta}{d^2 + (z-z_0)^2} \right] - \frac{\cos\theta}{R_1} \left[\cos\theta + \frac{s_1 d \sin\theta}{d^2 + (z-z_0)^2} \right] \right\} \quad (18)$$

$$\frac{\partial E_y}{\partial y} = \frac{\mu_0(dl/dt)}{4\pi} \left\{ \frac{\sin\theta}{R_2} \left[\sin\theta - \frac{s_2 d \cos\theta}{d^2 + (z-z_0)^2} \right] - \frac{\sin\theta}{R_1} \left[\sin\theta - \frac{s_1 d \cos\theta}{d^2 + (z-z_0)^2} \right] \right\} \quad (19)$$

where

$$s_1 = (x_1 - x) \cos\theta + (y_1 - y) \sin\theta \quad (20)$$

$$s_2 = (x_2 - x) \cos\theta + (y_2 - y) \sin\theta \quad (21)$$

and

$$d = (x - x_1) \sin\theta - (y - y_1) \cos\theta. \quad (22)$$

The physical meanings of s_1 , s_2 , l , d and θ are indicated in Fig. 2.

It is interesting to consider two special cases: a segment parallel to the x -axis and a segment parallel to the y -axis. For the former, the following expressions are obtained by substituting $\theta = 0^\circ$ in eqs. (18) and (19):

$$\frac{\partial E_x}{\partial x} = \frac{\mu_0(dl/dt)}{4\pi} \left[\frac{1}{R_2} - \frac{1}{R_1} \right] \quad (23)$$

$$\frac{\partial E_y}{\partial y} = 0. \quad (24)$$

It should be noted that R_1 is the distance from the point where spatial derivatives are evaluated to the segment end where the current flow starts, and R_2 is the similar distance but to the other end of the segment, i.e. where the current flow ends. For a segment parallel to the y -axis, substitution of $\theta = 90^\circ$ in eqs. (18) and (19) leads to the following results:

$$\frac{\partial E_x}{\partial x} = 0 \quad (25)$$

$$\frac{\partial E_y}{\partial y} = \frac{\mu_0(dl/dt)}{4\pi} \left[\frac{1}{R_2} - \frac{1}{R_1} \right]. \quad (26)$$

Linear Coil Segment Perpendicular to the Interface

For a linear segment of the coil perpendicular to the interface and carrying a current from (x_0, y_0, z_1) to (x_0, y_0, z_2) , the electric field components are obtained by integrating expressions (3) and (4) along the segment. For $\rho \neq 0$:

$$E_x = \frac{\mu_0(dl/dt)(x-x_0)(z_1-z_2-R_1+R_2)}{4\pi\rho^2} \quad (27)$$

$$E_y = \frac{\mu_0(dl/dt)(y-y_0)(z_1-z_2-R_1+R_2)}{4\pi\rho^2} \quad (28)$$

where ρ is given by eq. (6), and

$$R_1 = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_1)^2} \quad (29)$$

$$R_2 = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_2)^2}. \quad (30)$$

When $\rho = 0$, i.e. the observation point is right below the coil-segment, both E_x and E_y vanish.

The electric-field spatial derivatives, obtained by integrating eqs. (8) and (9) along the segment, are equal to ($\rho \neq 0$)

$$\frac{\partial E_x}{\partial x} = \frac{\mu_0(dl/dt)}{4\pi\rho^2} \left\{ \frac{[(x-x_0)^2 - (y-y_0)^2](z_2-z_1-R_2+R_1)}{\rho^2} + (x-x_0)^2 \left[\frac{1}{R_2} - \frac{1}{R_1} \right] \right\} \quad (31)$$

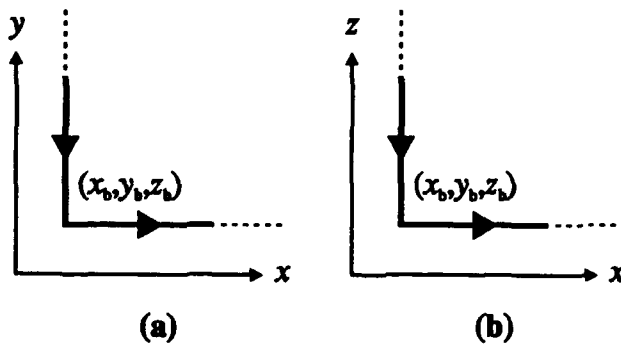
$$\frac{\partial E_y}{\partial y} = \frac{\mu_0(dl/dt)}{4\pi\rho^2} \left\{ \frac{[(y-y_0)^2 - (x-x_0)^2](z_2-z_1-R_2+R_1)}{\rho^2} + (y-y_0)^2 \left[\frac{1}{R_2} - \frac{1}{R_1} \right] \right\} \quad (32)$$

When $\rho = 0$, they are

$$\frac{\partial E_x}{\partial x} = \frac{\partial E_y}{\partial y} = \frac{\mu_0(dl/dt)}{8\pi} \left[\frac{1}{R_2} - \frac{1}{R_1} \right]. \quad (33)$$

Right-Angle (90°) Bends

The expressions for electric-field spatial derivatives produced by linear coil segments reveal that only the segment ends contribute to electric-field spatial derivatives. In other words, in a coil composed of several linear segments, only the bends (where current changes the direction) generate electric-field spatial derivatives.



A 90° bend with two semi-infinite segments, one parallel to the x-axis and the other parallel to the y-axis, is shown in Fig. 3(a). Such bends are found in many stimulating coils, e.g. square coils parallel to the interface. The electric-field spatial derivatives produced by this bend can be obtained by substituting $R_1 = R_b$, $R_2 \rightarrow \infty$ in eq. (23) and $R_1 \rightarrow \infty$, $R_2 = R_b$ in eq. (26):

$$\frac{\partial E_x}{\partial x} = -\frac{\partial E_y}{\partial y} = -\frac{\mu_0(dl/dt)}{4\pi R_b} \quad (34)$$

Figure 3. (a) A right-angle bend parallel to the air-tissue interface, (b) A right-angle bend perpendicular to the interface.

where R_b is the distance from the bend to the point of observation, and is equal

to:

$$R_b = \sqrt{(x-x_b)^2 + (y-y_b)^2 + (z-z_b)^2} \quad (35)$$

When $x = x_b$ and $y = y_b$, the spatial derivatives reach the following maximum or minimum values:

$$\frac{\partial E_x}{\partial x} = -\frac{\partial E_y}{\partial y} = -\frac{\mu_0(dI/dt)}{4\pi(z_b-z)} \quad (36)$$

Figure 3(b) shows a 90°-bend with one segment parallel to the x -axis and the other parallel to the z -axis. Such bends are present in square coils perpendicular to the interface. The electric field spatial derivatives produced by this bend are obtained by substituting $R_1 = R_b$, $R_2 \rightarrow \infty$ in eq. (23) and $x_0 = x_b$, $y_0 = y_b$, $z_1 \rightarrow \infty$, $R_1 \rightarrow \infty$, $R_1 - z_1 = -z$, $z_2 = z_b$, $R_2 = R_b$ in eqs. (31) and (32):

$$\frac{\partial E_x}{\partial x} = -\frac{\partial E_y}{\partial y} = \frac{\mu_0(dI/dt)[(x-x_b)^2 - (y-y_b)^2]}{4\pi\rho^2} \left[\frac{z_b - z - R_b}{\rho^2} + \frac{1}{R_b} \right] \quad (37)$$

where R_b is given by eq. (35) and

$$\rho = \sqrt{(x-x_b)^2 + (y-y_b)^2}.$$

For the special case of $\rho = 0$, the spatial derivatives become:

$$\frac{\partial E_x}{\partial x} = -\frac{\partial E_y}{\partial y} = -\frac{\mu_0(dI/dt)}{8\pi(z_b-z)}. \quad (38)$$

Comparing eqs. (36) and (38), it can be seen that the peak magnitude of the spatial derivatives produced by a parallel right-angle bend is twice as large as that produced by a perpendicular right-angle bend. This explains why a square coil stimulates nerves more efficiently when it is placed parallel to the interface [Esselle and Stuchly, 1992]. It also suggests that, the relatively strong hyperpolarization produced by some parallel bends in single and multiple square coils can be significantly reduced by replacing such parallel bends with perpendicular bends. This observation has led to the three-dimensional (3D) coil design described later.

Relationship between the Spatial Derivatives

Equations (34) and (37) show that the two orthogonal spatial derivatives of electric field, produced by a parallel or perpendicular 90° bend, are related in the following manner:

$$\frac{\partial E_y}{\partial y} = -\frac{\partial E_x}{\partial x}. \quad (39)$$

It can be shown that this relationship is true for the spatial derivatives produced by any coil of an arbitrary shape and orientation [Esselle and Stuchly, 1992].

Quadruple Coils

Several quadruple coils have been proposed for nerve stimulation [Roth *et al.*, 1990b, Esselle and Stuchly, 1991]. Although there are substantial differences between them, all the quadruple coils have several common advantages over single and double coils when used for stimulation of long nerves. They almost always produce only one stimulation spot (the primary depolarization peak) in a well-defined location (right below the coil centre). Their

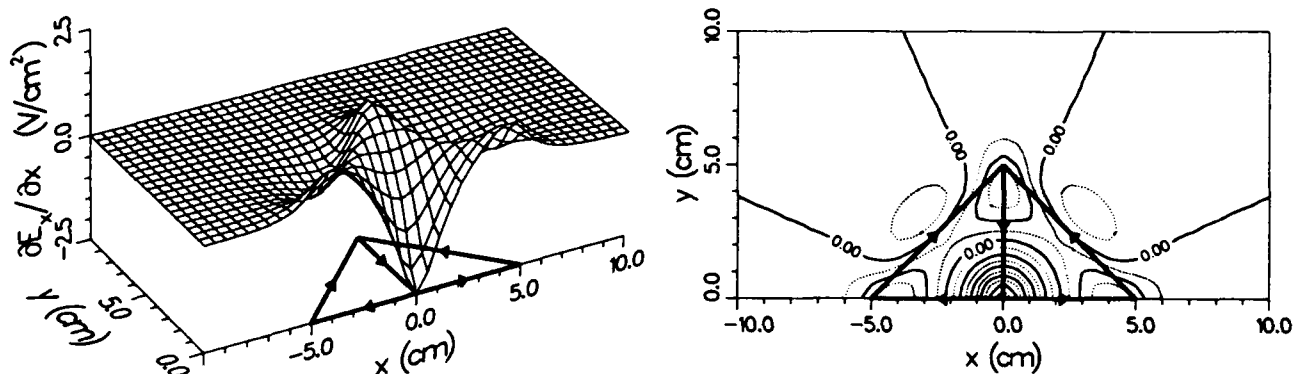


Figure 4. Plots of $\partial E_x/\partial x$ in a plane 1-cm below a QT coil. Each triangular section is 5-cm long, has 10 turns, and carries a current rising at 100 A/ μ s.

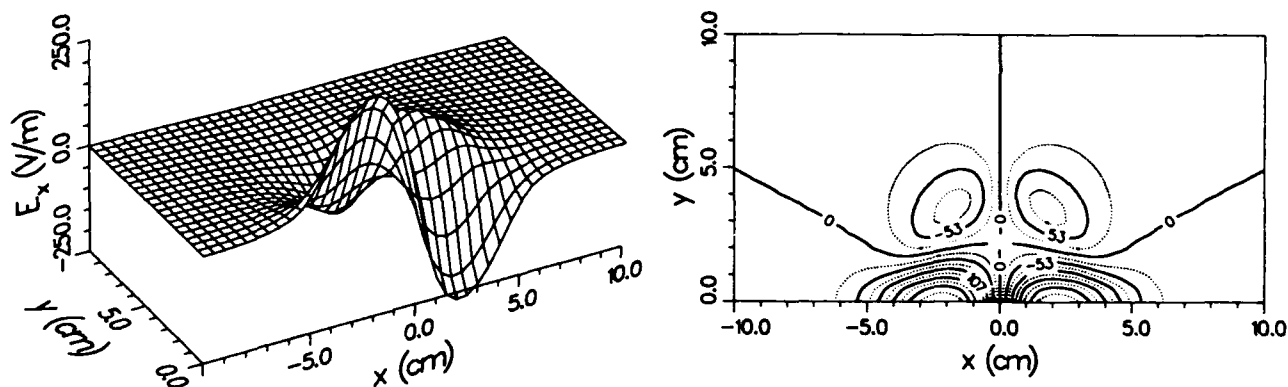


Figure 5. Plots of E_x in a plane 1-cm below a QT coil. Each triangular section is 5-cm long, has 10 turns, and carries a current rising at 100 A/ μ s.

secondary depolarization peaks and hyperpolarization peaks are relatively weak (<50% of primary depolarization peak). If the time rate of change of current and the total number of turns are constant, properly designed quadruple coils will create the same peak spatial derivative of the electric field as their single counterparts. However, the inductance of a quadruple coil is lower than that of a single coil with the same total number of turns, and therefore, the quadruple coil draws a larger current and produces a stronger peak spatial derivative of electric field when used with a given stimulator (for a fixed stimulator voltage).

The electric field and its spatial derivatives produced by quadruple Focalpoint™ coils [Roth *et al.*, 1990b] and quadruple square (QS) coils [Esselle and Stuchly, 1992] have been analyzed previously. Two other coil configurations are discussed here. Figure 4 shows the electric-field spatial derivative ($\partial E_x/\partial x$) produced by a

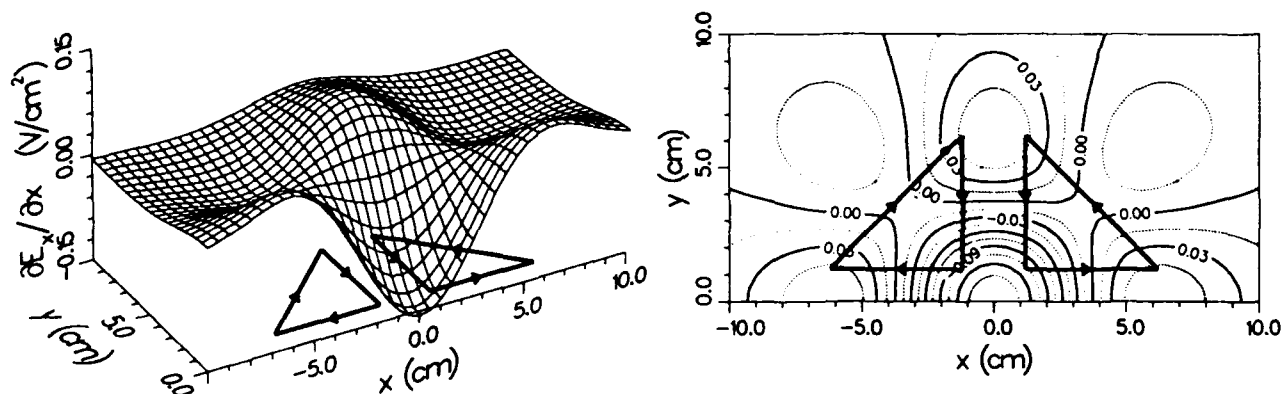


Figure 6. Plots of $\partial E_x/\partial x$ in a plane 4-cm below a QT coil. Each triangular section is 5-cm long, has 10 turns, and carries a current rising at 100 A/ μ s. Gap between sections (s) = 2.4 cm.

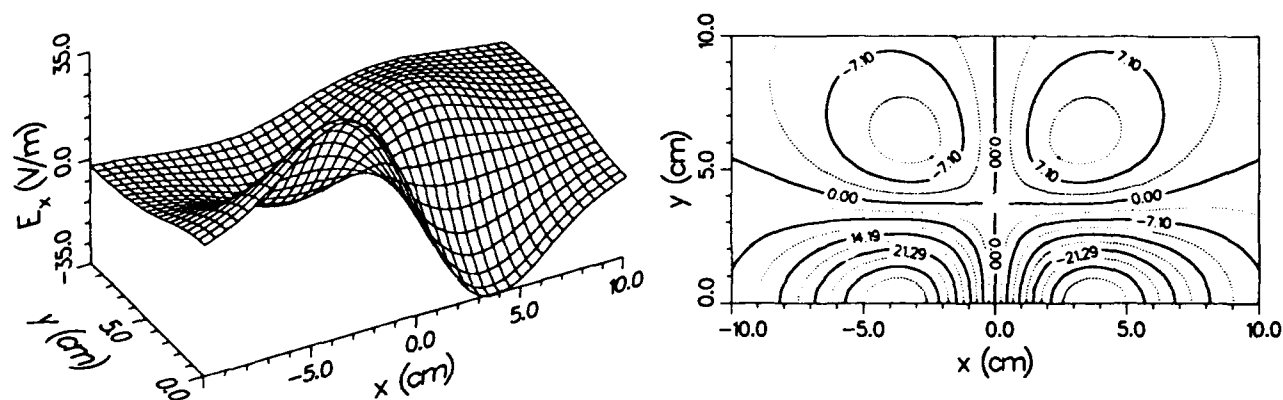


Figure 7. Plots of E_x in a plane 4-cm below a QT coil. Each triangular section is 5-cm long, has 10 turns, and carries a current rising at 100 A/ μ s. Gap between sections (s) = 2.4 cm.

quadruple triangular (QT) coil, in a relatively shallow plane inside tissue 1-cm below the coil. These and the following 3D and contour plots are symmetrical about the x -axis, and therefore only the $y \geq 0$ region of the plots is shown. In all contour plots, the contour interval is equal to one tenth the peak magnitude ($|\partial E_x/\partial x|_{\text{peak}}$ or $|E_x|_{\text{peak}}$). The x -component of the electrical field (E_x) produced by the QT coil is shown in Fig. 5. Figures 6 and 7, respectively, show $\partial E_x/\partial x$ and E_x produced by the same coil, but in a deeper plane 4-cm below the coil. The four triangular sections have been optimally spaced for maximum $|\partial E_x/\partial x|_{\text{peak}}$. The distance between turns in each triangle has been neglected in these calculations.

For a 10-cm x 10-cm QS coil, the relative peak hyperpolarization level (RPHL, defined as the ratio of the greatest positive value of $\partial E_x/\partial x$ and the greatest negative value of it) in a plane 1-cm below the coil is 47%

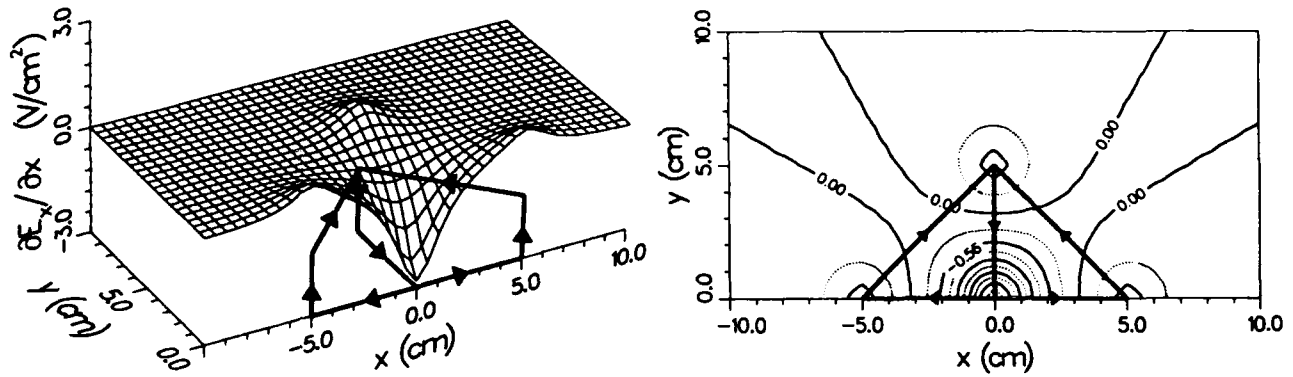


Figure 8. Plots of $\partial E_x/\partial x$ in a plane 1-cm below a 3D coil. Each section is 5-cm long and 2-cm high, has 10 turns, and carries a current rising at 100 A/ μ s.

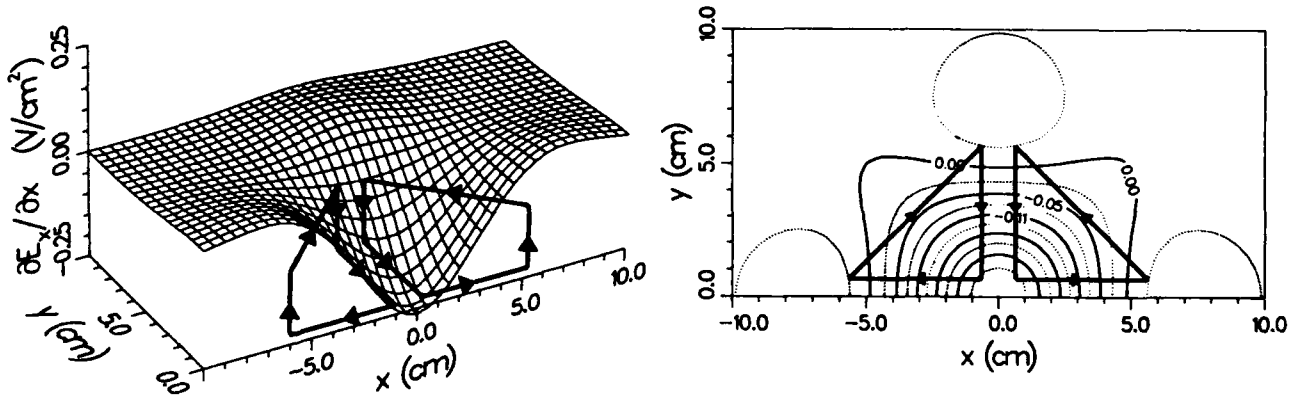


Figure 9. Plots of $\partial E_x/\partial x$ in a plane 4-cm below a 3D coil. Each section is 5-cm long and 2-cm high, has 10 turns, and carries a current rising at 100 A/ μ s. Gap between sections (s) = 1.3 cm.

[Esselle and Stuchly, 1992]; for a QT coil, it is 40% (Fig. 4). In both cases, the primary hyperpolarization spots as well as the secondary depolarization spots are created by additional bends in the coil that are parallel to the interface. Since these bends are not necessary for creation of the primary stimulation spot (at $x=0$, $y=0$), it is convenient to replace these parallel bends with perpendicular bends that produce smaller spatial derivatives. A three-dimensional (3D) coil obtained by modifying the QT coil, and its $\partial E_x/\partial x$ in a plane 1-cm below its bottom, are shown in Fig. 8. For these calculations, the coil height was assumed to be 2 cm, which results in a RPHL of 24%. This level can be further reduced by increasing the coil height, but at the expense of increased coil inductance and hence reduced coil current. Figure 9 shows the electric field spatial derivatives in a plane 4-cm below the coil. Again, coil sections are spaced for maximum $|\partial E_x/\partial x|_{\text{peak}}$.

III. COIL INDUCTANCE

In practical magnetic nerve stimulators, the time-rate of change of coil current strongly depends on the coil inductance, as expressed in eq. (1). Inductance can be calculated using a general equation for an arbitrary thin-wire coil given by Ramo *et al.* [1965], and applying it to coil segments as detailed in the Appendix. Analytical expressions derived therein have been used to calculate inductances of several square, QS, QT and 3D coils. Representative results are shown in Fig. 10 and Table I. In Fig. 10, the inter-turn distance (d) is also taken into account, assuming that the turns are stacked to form a single-layer tube, as shown in Fig. 11.

Table I. The inductance (L) and the peak spatial derivative of electric field per unit voltage (S). The square coil is 5-cm long and has 4 turns. All quadruple coils consist of 5-cm-long single-turn sections. Wire radius is 0.9 mm. Depth is measured from the bottom of the coil; d is the distance between turns; s is the gap between sections; b is the height of the 3D coil.

Coil type and parameters	Inductance L (μH)	S at 1-cm depth (m^{-2})	S at 4-cm depth (m^{-2})
Square ($d=0$)	2.03	14.7	1.3
QS ($s=0$)	0.63	47.1	4.0
QS ($s=1.6$ cm)	0.54	32.9	5.0
QT ($s=0$)	0.44	57.1	2.5
QT ($s=2.4$ cm)	0.37	28.5	4.3
3D ($s=0$, $b=2$ cm)	0.67	42.1	3.7
3D ($s=1.3$ cm, $b=2$ cm)	0.54	35.5	4.9

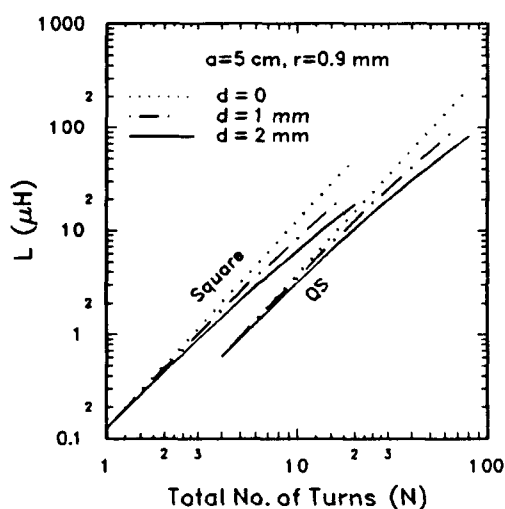


Figure 10. Inductance of square and QS coils versus number of turns. The inter-turn distance is d . Square coil is 5 cm x 5 cm; QS coil has four such square sections. Wire radius (r) = 0.9 mm, gap (s) = 0.

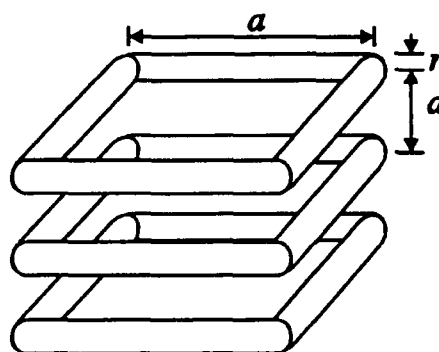


Figure 11. A square coil, or a section of a QS coil. Wire turns are stacked to form a single-layer tube.

With zero inter-turn distance, the inductance of any coil should increase according to the square of the number of turns. A finite inter-turn distance results in lower inductance. The inductance of a QS coil is 1.7 to 3.3 times less than that of a square coil with the same total number of turns. This is when the gap (s) between the four square sections in the coil is zero, as assumed in Fig. 10; otherwise, the inductance is even less. Table I shows the inductance of several 4-turn coils. In general, the QT coil has lower inductance than a QS coil, but the 3D coil has a higher inductance unless the coil height is very small.

IV. COIL OPTIMIZATION AND COMPARISON

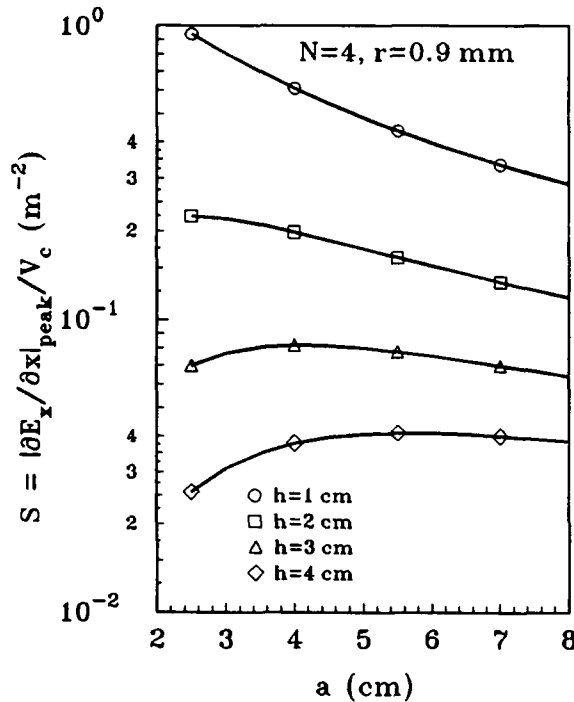


Figure 12. The peak electric-field spatial derivative per unit voltage (S) versus the length of one square section (a), for 4-turn QS coils. h is the depth of the nerve measured from the coil.

characteristic dimension (length of one section) of 5 cm. It can be seen that as far as the strength of the stimulus is concerned, unspaced QT coils are preferred for shallow nerves, and spaced QS or 3D coils are preferred for deep nerves.

In addition to the parameter S , there are other factors to be considered before choosing a coil for magnetic nerve application. The rate at which the peak value of $|\partial E_x/\partial x|$ decreases with depth is critical when stimulating deep-lying nerves. Figure 13 shows this drop for several quadruple coils. It can be seen that all spaced coils have approximately equal drop-rates. Unspaced coils have higher drop-rates, the QT having the highest. Figure 14 shows how the peak value of E_x decreases with depth. Again, the QT coil has the highest drop-rate.

The focality of a coil is determined by the size of the stimulation region. When the peak stimulus is 10% higher than the threshold stimulus, the stimulation region is marked by the 90% (of $-\partial E_x/\partial x|_{\text{peak}}$) contour. The width (ΔX) of the 90%-contour is plotted as a function of nerve depth (distance from the coil) in Fig. 15. In

First, we search for the optimum size of a coil assuming that its shape, wire radius, number of turns and the voltage applied across the coil terminals (V_c) are fixed. The peak electric field and the peak electric-field spatial derivative the coil produces at a given depth in tissue depend on the coil size. Using the methods described in Sections II and III, peak electric-field spatial derivative per unit voltage,

$$S = \frac{|\partial E_x/\partial x|_{\text{peak}}}{V_c} = \frac{|\partial E_x/\partial x|_{\text{peak}}}{L (dl/dt)} \quad (40)$$

can be calculated for different coil sizes. Results for QS coils are shown in Fig. 12. For a given depth of the nerve (h), there is an optimum QS coil size that gives maximum S . However, for shallow nerves, the optimum QS coil size is too small for practical realization [Cohen and Cuffin, 1991]. Not only are they difficult to wind, but also have low inductance, draw a heavy current and require large capacitor banks. On the other hand, optimum QS coils are practical and useful for deep-nerve stimulation.

Next, we alter the shape of, and the gap between, the four sections of a quadruple coil, keeping the size approximately constant. The last two columns in Table I give the value of S for several useful coils at 1-cm and 4-cm depths. All coils have the same

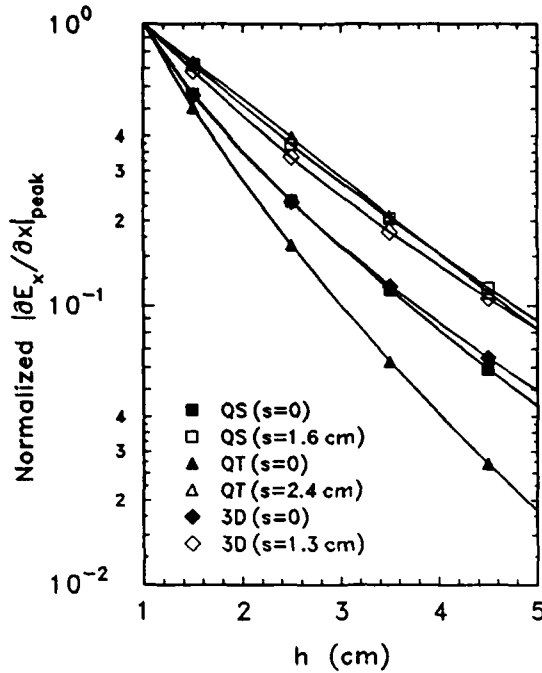


Figure 13. Peak electric-field spatial derivative normalized to value at 1-cm depth, versus depth (h) measured from the bottom of the coil. s is the gap between the four sections. $N = 40$, $d = 0$.

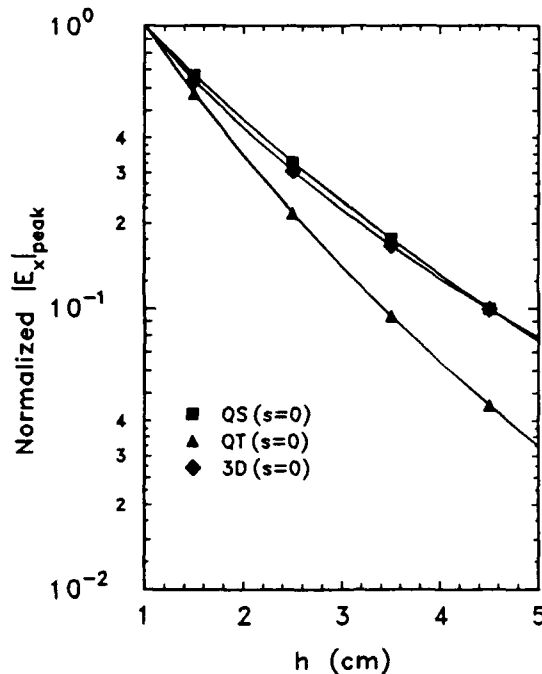


Figure 14. Peak electric field normalized to value at 1-cm depth, versus depth (h) measured from the bottom of the coil. There is no gap between the four sections. No. of turns = 40. Inter-turn distance is neglected.

general, QT coils are more focal than QS and 3D coils. Unspaced coils are usually more focal than spaced coils, but the difference is trivial in deep-nerve stimulation.

In practice, the coil is usually placed in such a position that the stimulation region (spot) coincides with the nerve to be stimulated. However, some other nerves that are at the secondary depolarization peaks of the coil may also be stimulated unintentionally if they have sufficiently lower stimulation thresholds. Also some other nerves may be hyperpolarized at the hyperpolarization peaks of the coil. Since the secondary depolarization peaks and hyperpolarization peaks of quadruple coils are equal in magnitude, the severity of both these potentially harmful effects can be expressed by a single parameter: the relative peak hyperpolarization level (RPHL) defined before (See sub-section: *Quadruple Coils*). Figure 16 shows the RPHL of quadruple coils; it is 100% for single and double coils. At all depths, 3D coils have significantly lower RPHL; for deep nerves it is less than half of the RPHL produced by other coils.

V. CONCLUSIONS

In stimulation of nerves with pulsed magnetic field several parameters of the stimulating coil have to be considered to produce effective stimulation of a desired nerve. For long, straight nerves the activation function is the spatial derivative of the electric field along the neuronal axis. This derivative depends on the coil design, shape and location with respect to the nerve. Ideally, the coil should produce a small region at a desired location of the nerve where the stimulus produces nerve depolarization. In practice, more than one region of depolarization as well as regions of hyperpolarization are produced. Furthermore, for a given stimulator, having a bank of capacitors that are charged to a given voltage, the stimulus is inversely proportional to the coil inductance.

Simplified analyses described here, that heavily rely on analytical solutions in addition to simple numerical computations, provide tools for coil optimization and comparison. This can lead to a selection of the best coil geometry and location for selective stimulation of a desired long nerve. Furthermore, the physical insight gained from this analysis should prove useful for more complex numerical analyses of tissue models better reflecting tissue geometry and heterogeneous electrical properties.

Quadruple coils of various shapes emerged from our evaluation as the optimal coils for stimulation of long

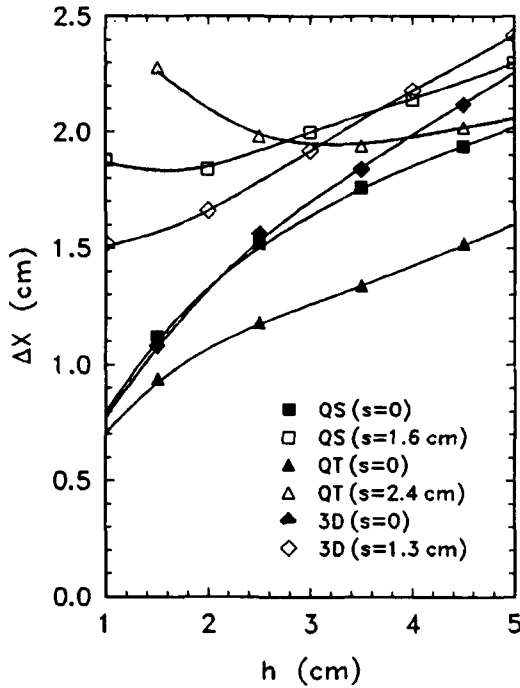


Figure 15. The size of the stimulation spot versus depth (h) measured from the bottom of the coil. The gap between the four coil sections is s .

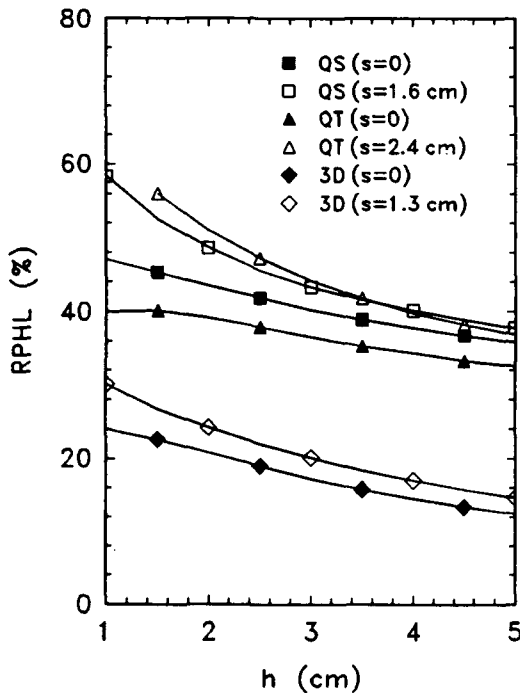


Figure 16. Relative peak hyperpolarization level versus depth (h) measured from the bottom of the coil. The gap between the four coil sections is s .

straight nerves.

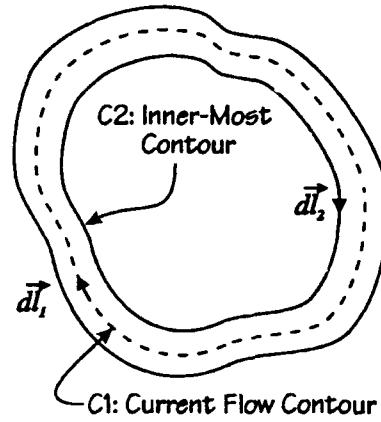


Figure A1. A thin-wire coil of arbitrary shape. Current is assumed to be concentrated along the centre-line.

VI. APPENDIX

For a general thin-wire coil shown in Fig. A1, inductance is given by [Ramo *et al.*, 1965]:

$$L = \frac{\mu_o}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\vec{l}_1 \cdot d\vec{l}_2}{d} \quad (\text{A1})$$

where C_1 is the contour along the centre-line of the wire, C_2 is the inner-most contour, $d\vec{l}_1$ is an elemental vector on C_1 , located at (x_1, y_1, z_1) , $d\vec{l}_2$ is an elemental vector on C_2 , located at (x_2, y_2, z_2) , and

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}.$$

It should be noted that eq. (A1) is valid only for coils composed of relatively thin wires where the current can be assumed to be concentrated along the centre-line.

For an arbitrarily-shaped coil, evaluation of eq. (A1) requires two numerical integrations. However, for coils composed of linear segments, at least one of the contour integrations can be done analytically. In this case, it is desirable to break the total inductance (L) into partial inductances due to each pair of linear segments. If the partial inductance due to i^{th} and j^{th} segment-pair as a result

of the current in the j^{th} segment is L_{ij} ,

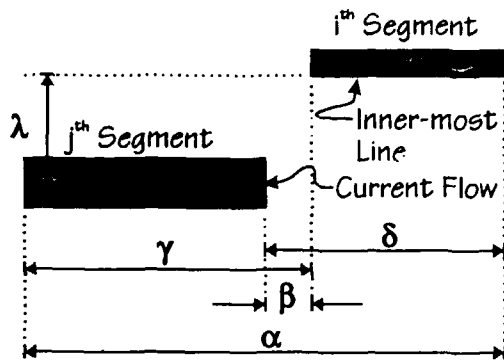


Figure A2. Two linear parallel segments of a coil.

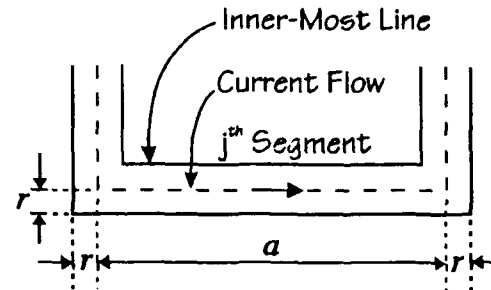


Figure A3. A segment of a square or QS coil.

$$L = \sum_{i=1}^{N_s} \sum_{j=1}^{N_s} L_{ij} \quad (\text{A2})$$

where N_s is the number of segments in the coil.

When i^{th} and j^{th} segments are perpendicular to each other, the dot product in eq. (A1) leads to $L_{ij} = 0$. When the centre-line of the j^{th} segment is parallel to the inner-most line of the i^{th} segment, as shown in Fig. A2, both contour integrations can be performed analytically, giving [Ramo *et al.*, 1965]:

$$L_{ij} = (\mu_o/4\pi) [\alpha \ln(\alpha + \sqrt{\alpha^2 + \lambda^2}) - \sqrt{\alpha^2 + \lambda^2} + \beta \ln(\beta + \sqrt{\beta^2 + \lambda^2}) - \sqrt{\beta^2 + \lambda^2} \\ - \gamma \ln(\gamma + \sqrt{\gamma^2 + \lambda^2}) + \sqrt{\gamma^2 + \lambda^2} - \delta \ln(\delta + \sqrt{\delta^2 + \lambda^2}) + \sqrt{\delta^2 + \lambda^2}] \quad (\text{A3})$$

where α , β , γ , δ and λ are the dimensions shown in Fig. A2. The above expression assumes that both segments carry current in the same direction; the sign of L_{ij} should be reversed otherwise. As a special case, eq. (A3) can be used to evaluate L_{jj} , the self-contribution from the j^{th} segment. For example, L_{jj} for the segment shown in Fig. A3 can be obtained by substituting $\alpha = a-r$, $\beta = -a+r$, $\gamma = r$, $\delta = -r$ and $\lambda = r$ in eq. (A3), which results in

$$L_{jj} = \frac{\mu_o}{4\pi} \left\{ (a-r) \ln \left[\frac{a-r + \sqrt{(a-r)^2 + r^2}}{r-a + \sqrt{(a-r)^2 + r^2}} \right] - 2\sqrt{(a-r)^2 + r^2} - r \ln \left(\frac{\sqrt{2}+1}{\sqrt{2}-1} \right) + 2\sqrt{2}r \right\}. \quad (\text{A4})$$

The calculation of L_{ij} due to two linear segments, which are parallel to the x - y plane but not parallel to each other (Fig. A4), can be also be done without numerical integration. The inner-most line of the i^{th} segment extends from $A \equiv (x_1^i, y_1^i, z_1^i)$ to $B \equiv (x_2^i, y_2^i, z_1^i)$, and the centre-line of the j^{th} segment extends from $C \equiv (x_1^j, y_1^j, z_1^j)$ to $D \equiv (x_2^j, y_2^j, z_1^j)$. Lengths of AB and CD, denoted by l^i and l^j , are given by

$$l^i = \sqrt{(x_2^i - x_1^i)^2 + (y_2^i - y_1^i)^2} \quad (\text{A5})$$

$$l^j = \sqrt{(x_2^j - x_1^j)^2 + (y_2^j - y_1^j)^2} \quad (\text{A6})$$

and their direction cosines are given by

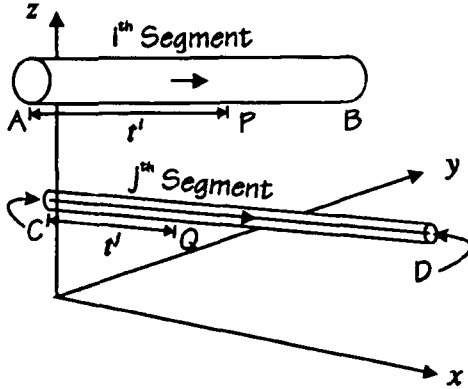


Figure A4. Two linear segments parallel to the x-y plane.

$$m^i = \frac{x_2^i - x_1^i}{l^i}, \quad m^j = \frac{x_2^j - x_1^j}{l^j} \quad (A7)$$

$$n^i = \frac{y_2^i - y_1^i}{l^i}, \quad n^j = \frac{y_2^j - y_1^j}{l^j} \quad (A8)$$

For two generic points: $P = (x^i, y^i, z^i)$ on AB, at a distance of t^i from A, and $Q = (x^j, y^j, z^j)$ on CD, at a distance of t^j from C, the following relationships apply:

$$x^i = x_1^i + m^i t^i, \quad x^j = x_1^j + m^j t^j \quad (A9)$$

$$y^i = y_1^i + n^i t^i, \quad y^j = y_1^j + n^j t^j \quad (A10)$$

$$z^i = z_1^i, \quad z^j = z_1^j \quad (A11)$$

According to eq. (A1), the partial inductance is given by

$$L_{ij} = \frac{\mu_0 (m^i m^j + n^i n^j)}{4\pi} \int_0^{t^j} \int_0^{t^i} \frac{dt^i dt^j}{\sqrt{(x^i - x^j)^2 + (y^i - y^j)^2 + (z^i - z^j)^2}}. \quad (A12)$$

The first integration in the above expression with respect to t^i is straightforward, but the resulting general analytical expression is fairly lengthy. This expression can be significantly simplified in some special cases, such as the segment-pairs in QT and 3D coils. The second integration, with respect to t^j , can also be done analytically, using the following integral:

$$\begin{aligned} \int \ln[p t^j + q + \sqrt{t^{j2} + r t^j + s}] dt^j = & -\frac{r}{2p^2} - t^j - \frac{2\Delta}{p^2 - 1} \tan^{-1} \left[\frac{q + r/2 + (1+p)(t^j + \Phi)}{\Delta} \right] \\ & - \frac{q - r/2}{p - 1} \ln \left(\frac{r}{2} + t^j + \Phi \right) + t^j \ln(q + p t^j + \Phi) \\ & + \frac{pq - r/2}{p^2 - 1} \ln \left\{ 2s - \frac{r^2}{2} + (r + 2t^j + 2\Phi) \left[q + \frac{r}{2} + (1+p)t^j \right] \right\} \end{aligned} \quad (A13)$$

where

$$\Delta = \sqrt{(p^2 - 1)(q^2 - s) - \left(pq - \frac{r}{2}\right)^2} \quad (A14)$$

and

$$\Phi = \sqrt{t^{j2} + r t^j + s}. \quad (A15)$$

The above integral as well as many other analytical expressions were obtained using the symbolic computation capability in the Mathematica[®] software package [Wolfram, 1991].

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ANALYSIS OF THE STIMULATION OF A NERVE FIBER SURROUNDED BY AN INHOMOGENEOUS, ANISOTROPIC, AND DISPERSIVE TISSUE

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ABSTRACT

A method for evaluating the threshold for electric and magnetic stimulation of nerves surrounded by an inhomogeneous, anisotropic, and dispersive tissue has been developed. The scalar potential distribution induced by a given electric or magnetic source is evaluated by using the Finite Difference Technique. Calculations are performed with a FORTRAN code, and for non dispersive tissues a spread-sheet is also used. Nerve fiber excitation is described by using the Frankenhaeuser-Huxley model in which the stimulating current density is obtained by extending the method proposed by Rattay. The analysis results predict that, with reference to the homogeneous case usually considered in literature, the larger differences in the current threshold are due to the tissue inhomogeneity, while the consideration of the dispersive properties of the tissues has less effect.

INTRODUCTION

Functional electrical stimulation (FES) is a technique for the rehabilitation of spinal-cord injured patients. It makes use of electrical stimulators to induce currents through the membrane of nerve fibers. These currents trigger action potentials (APs) that propagate along the nerve and induce muscular contraction by means of the synaptic mechanisms still active in paraplegic subjects.

Practical FES realizations concern both upper and lower limbs. For lower limbs Marsolais and Kobetic [1987] have developed a hand-operated stimulator (open loop) that applies trains of biphasic current pulses to the motor point of muscles by using percutaneous intramuscular wire electrodes. Graupe et al. [1988] have realized a closed loop stimulator in which the stimulating current was controlled by the electromyographic (EMG) signal of the upper trunk. Extracutaneous electrodes have been used to stimulate and to measure the response-EMG signal of the stimulated muscles. Paraplegic subjects using this system were able to walk for 300 meters and some of them could climb stairs.

With regard to upper limbs' applications, Buchett et al. [1988] have graded hand movement in quadriplegic subjects by using, as a controller, a shoulder position transducer. The transducer delivers its output to the stimulator module that, in turn, applies current pulses to the muscle by means of percutaneous electrodes. In the controller of their stimulator, Crago et al. [1976] have linearly combined the force and position signals of the hand grasping an object. In this manner and by varying the pulse width of the stimulating current, they could control the finger position, before object acquisition, and both grasp force and opening, after contact.

All the cited authors apply monophasic or biphasic current pulses to the limb to stimulate nerve fibers innervating muscles. For this reason many researchers have studied the threshold response of a nerve fiber to a current signal by introducing different electrical models for the excitation and propagation of the APs.

McNeal [1976] studied the response of a myelinated nerve fiber to a cathodic pulse applied with a spherical electrode. The electrical behavior of the fiber was simulated with a cascade of 11 dipoles (each representing a Ranvier's node) with the central one modelled by means of the Frankenhaeuser-Huxley (F&H) equations [1964]. To evoke an AP with the electrode 1 mm away from the fiber, 100 μ s and 226 μ A current pulses were necessary. Rattay [1986, 1988, 1989] extended the McNeal's study by considering both myelinated and unmyelinated fibers and electrodes with different geometries. Gorman and Mortimer [1983] analyzed the possibility of a selective excitation of the fibers inside the nerve by varying the electrical parameters of the stimulus. They obtained selectivity by using biphasic stimuli of appropriate widths. Veltink et al. [1989] studied the same problem as a function of the electrode position showing the possibility of selective excitation of the fibers using intraneuronal electrodes.

Nerve stimulation has been also performed by using time varying magnetic fields. This method seems painless compared to the conventional electrical stimulation and, for this reason, has been recently investigated. In particular, Polson et al. [1982], proved experimentally the feasibility of magnetic stimulation of nerve trunks. In their experiments the magnetic field was generated by the discharge through a coil of the charge accumulated on a

capacitor bank. Theoretical investigations have been performed, by using both the F&H [1964] and the Hodgkin-Huxley (H&H) models [1952], to evaluate the threshold of nerve stimulation induced by magnetic fields [Reilly, 1989; Roth and Basser, 1990; Basser and Roth, 1991].

All the quoted authors have studied the action on the nerve of currents flowing through a homogeneous, isotropic, and non dispersive medium. These conditions are never fulfilled practically. The purpose of this work is to develop a method for evaluating the nerve current threshold for electric and magnetic stimulation for the nerve fiber surrounded by an inhomogeneous, anisotropic, and dispersive tissue. The proposed method first evaluates the scalar potential distribution induced by a given electric or magnetic source by using the Finite Difference Technique (FDT) (in the magnetic case, the vector potential distribution is previously evaluated starting from the coil current and position), then it relates the scalar and vector potential distribution along the nerve to the current density crossing the nerve membrane, and finally it computes the nerve response to the resulting current density.

The emphasis in this paper is on the description of the numerical technique used to solve the EM problem, on the characterization of the dispersive properties of the tissue, and on the introduction of a modification of the activating function proposed by Rattay [1986].

METHODS

Finite Difference Technique

FDT has been widely used for studying the scalar potential distribution in a biological tissue due to known electrical [Sowinski and Van Den Berg, 1990; Heringa et al., 1982; Stock et al., 1988] and magnetic sources [Armitage et al., 1983; Gandhi et al., 1984; Polk, 1990; Polk and Song, 1990]. In this section, it will be shown that it is possible, starting from the Maxwell equations, to associate an electrical network with the region under study. The network consists of admittances, whose values are related to the electrical properties of the tissues forming the region, and generators representing the electric and magnetic sources.

To define the electrical network we divide the volume of interest into homogeneous parallelepipedic cells of sides Δx , Δy , Δz centered in (x,y,z) (Fig. 1):

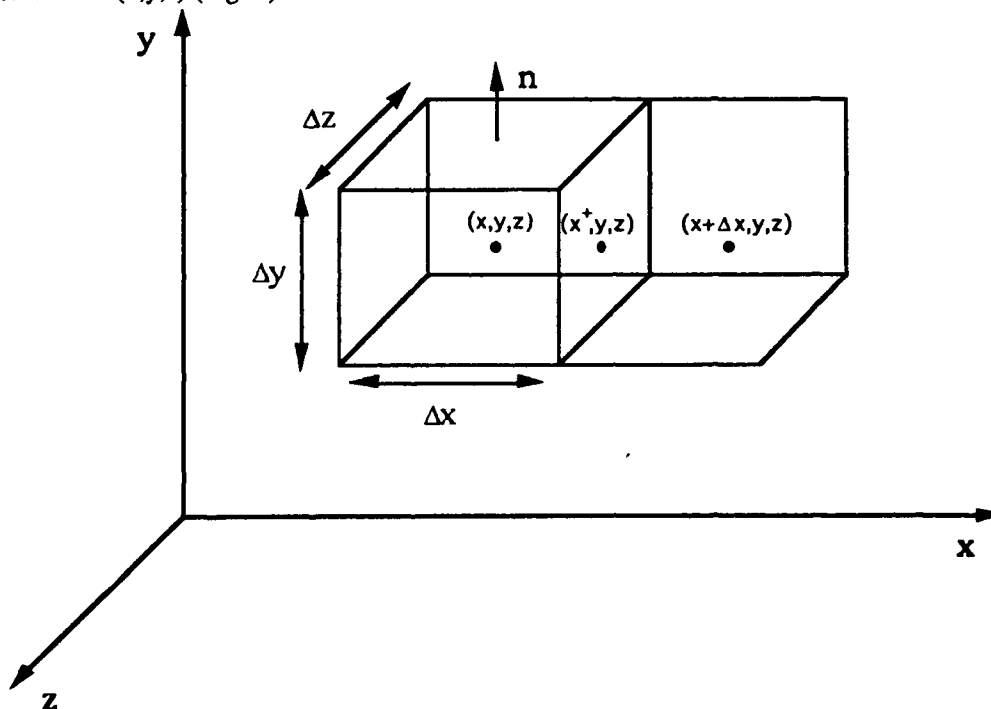


Fig. 1 Unit cell of the FDT lattice.

For each cell we consider the Maxwell equation (assuming harmonic fields):

$$\nabla \times \mathbf{H} = \mathbf{J}_c + \mathbf{J}_d + \mathbf{J}_{ie} = [\sigma] \cdot \mathbf{E} + j\omega[\epsilon] \cdot \mathbf{E} + \mathbf{J}_{ie} = [\sigma'] \cdot \mathbf{E} + \mathbf{J}_{ie} \quad (1)$$

where:

- J_c = conduction electric current density,
- J_d = displacement electric current density,
- J_{ie} = impressed electric current density,
- $[\sigma]$ = conductivity tensor,
- $[\epsilon]$ = permittivity tensor,
- $[\sigma^*] = [\sigma] + j\omega[\epsilon]$ = complex conductivity tensor.

In the Cartesian coordinates, the conductivity tensor takes the form (in the absence of any static magnetic field):

$$[\sigma^*] = \begin{bmatrix} \sigma_x^* & 0 & 0 \\ 0 & \sigma_y^* & 0 \\ 0 & 0 & \sigma_z^* \end{bmatrix} \quad (2)$$

In eq. (1), the electric field can be expressed in terms of scalar (V) and vector (A) complex potentials as:

$$E = -j\omega A - \nabla V \quad (3)$$

Upon taking the divergence of (1), volume integration and subsequent application of the divergence theorem, we obtain:

$$-\oint_S [\sigma^*] \cdot \nabla V \cdot \mathbf{n} dS - \oint_S [\sigma^*] \cdot j\omega \mathbf{A} \cdot \mathbf{n} dS + \oint_S \mathbf{j}_{ie} \cdot \mathbf{n} dS = 0 \quad (4)$$

The FDT consists in approximating the derivatives in (4) by the corresponding incremental ratios, thus obtaining:

$$\begin{aligned} & \sigma_x^* \frac{(V_{x,y,z} - V_{x-\Delta x,y,z})}{\Delta x} \Delta y \Delta z - \sigma_{x+}^* \frac{(V_{x+\Delta x,y,z} - V_{x,y,z})}{\Delta x} \Delta y \Delta z + \\ & + \sigma_y^* \frac{(V_{x,y,z} - V_{x,y-\Delta y,z})}{\Delta y} \Delta x \Delta z - \sigma_{y+}^* \frac{(V_{x,y+\Delta y,z} - V_{x,y,z})}{\Delta y} \Delta x \Delta z + \\ & + \sigma_z^* \frac{(V_{x,y,z} - V_{x,y,z-\Delta z})}{\Delta z} \Delta x \Delta y - \sigma_{z+}^* \frac{(V_{x,y,z+\Delta z} - V_{x,y,z})}{\Delta z} \Delta x \Delta y + \\ & + \sigma_x^* \Delta y \Delta z j\omega A_x - \sigma_{x+}^* \Delta y \Delta z j\omega A_{x+} + \sigma_y^* \Delta x \Delta z j\omega A_y - \sigma_{y+}^* \Delta x \Delta z j\omega A_{y+} + \\ & + \sigma_z^* \Delta x \Delta y j\omega A_z - \sigma_{z+}^* \Delta x \Delta y j\omega A_{z+} + I_{ie} = 0 \end{aligned} \quad (5)$$

where I_{ie} represents the impressed current at the node (x,y,z), positive when going out of the cell. The superscripts - and + in the expression mean that the quantity is evaluated on a specific face of the parallelepipedic cell. For example σ_{x+}^* represents the complex conductivity on the face between the cells centered in x and x+Δx (see Fig. 1). Continuity of the complex current at the x^+ surface requires that $\sigma_x^* E_x = \sigma_{x+\Delta x}^* E_{x+\Delta x} = \sigma_{x+}^* E_{x+}$, where E_{x+} is given by the negative of the potential gradient in (5) (in the absence of the vector potential and impressed current) and E_x and $E_{x+\Delta x}$ are the x-directed component of the electric fields evaluated, respectively, at x and x+Δx. To satisfy this condition we introduce the complex admittance:

$$Y_{x+} = (\sigma_{x+}^*) \frac{\Delta y \cdot \Delta z}{\Delta x} = \frac{2\sigma_x^* \sigma_{x+\Delta x}^* \Delta y \cdot \Delta z}{\sigma_x^* + \sigma_{x+\Delta x}^* \Delta x} \quad (6)$$

Similar admittances can be obtained for the other coordinates. With this substitution (5) becomes the Kirchhoff's current equation for an electrical circuit made by a three-dimensional star of admittances and current generators. Fig. 2 shows this circuit for the two-dimensional case.

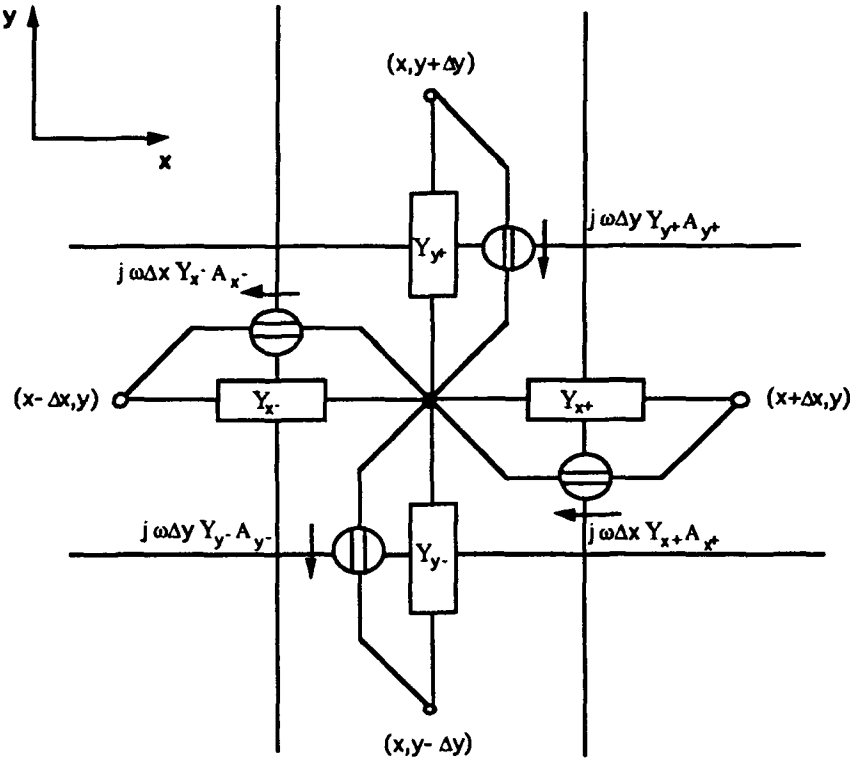


Fig. 2 Two-dimensional representation of the basic circuit.

Both for an electric current stimulation applied directly to nodes, and for a magnetic stimulation applied through an external coil, the potential distribution in the volume of interest can be evaluated by solving a set of simultaneous equations, namely, one per node:

$$\begin{aligned}
 V_{x,y,z} = \frac{1}{Y_x + Y_{x+} + Y_y + Y_{y+} + Y_z + Y_{z+}} [& Y_{x+} V_{x+\Delta x,y,z} + Y_{x-} V_{x-\Delta x,y,z} + \\
 & + Y_{y+} V_{x,y+\Delta y,z} + Y_{y-} V_{x,y-\Delta y,z} + Y_{z+} V_{x,y,z+\Delta z} + Y_{z-} V_{x,y,z-\Delta z} + \\
 & - Y_{x-} \Delta x j\omega A_{x-} + Y_{x+} \Delta x j\omega A_{x+} - Y_{y-} \Delta y j\omega A_{y-} + Y_{y+} \Delta y j\omega A_{y+} + \\
 & - Y_{z-} \Delta z j\omega A_{z-} + Y_{z+} \Delta z j\omega A_{z+} - I_{ie}]
 \end{aligned} \quad (7)$$

This set of equations is solved using an iterative technique with successive overrelaxation [Ralston and Rabinowitz, 1975]. This technique computes successive estimates of the potential by taking the previous estimate plus a correction term:

$$V^{n+1}(x,y,z) = V^n(x,y,z) + \alpha [V_{x,y,z} - V^n(x,y,z)] \quad (8)$$

where $V^{n+1}(x,y,z)$ and $V^n(x,y,z)$ are the potentials at the (x,y,z) node evaluated at steps $n+1$ and n respectively, $V_{x,y,z}$ is given by (7) and α is the relaxation constant ($1 < \alpha < 2$). The procedure is terminated when the following criterion is verified:

$$\frac{\sum_{i=1}^N |V_i^{n+1} - V_i^n|}{\sum_{i=1}^N |V_i^n|} < 0.003 \quad (9)$$

where N is the total number of cells [Veltink et al., 1989].

In the magnetic stimulation we firstly evaluate the vector potential on the cell surfaces, produced by the coil taking into account its position, geometry and current. As reported by Polk [1990], for the frequencies, conductivities, and distances considered in our work ($1 \div 10^6$ Hz; $0.013 \div 0.5$ S/m; $1 \div 100$ mm), we can apply the quasi-static approximation, and the vector potential can be expressed as:

$$\mathbf{A} = \frac{\mu}{4\pi} \iiint \frac{\mathbf{J}_i d\mathbf{v}}{R} \quad (10)$$

where $\mathbf{J}_i$ represents the volume current density that flows in the coil and produces the magnetic field. The vector potential at the center point of the cell surfaces is evaluated by approximating the coil with a 64-sided polygon and summing the contributions of each side [Roth and Bassar, 1990]. Starting from the value of the vector potential on the cell surfaces it is possible to evaluate the intensity of the current generators (Fig. 2). Finally eq. (7) is iterated up-to convergence.

Cross-Sectional Geometry and Electrical Properties of the Forearm

In our study we model the stimulation of the right forearm cutaneous nerve "antebrachii medianus" at the level of the third proximal. This nerve is not usually stimulated for rehabilitation purposes and it is chosen just to show the technique.

To identify the cell grid for the FDT, the cross-sectional anatomical picture of the right forearm at the level of the third proximal, reported in Sobotta and Becher [1983], is digitized into a computer by using a scanner (PC Scan 1000 DEST) with a resolution of 300 pixels per inch. The section is then subdivided into 30x25 square cells of 3mm side, each homogeneously filled with the prevailing tissue (Fig. 3).

In the three-dimensional study we consider parallelepipedic cells of 3x3x2mm sides. For each cell, the longitudinal dimension (2mm) is chosen to correspond to the internodal distance in myelinated fibers and it allows us to evaluate, by using the FDT, the scalar potential for Ranvier's nodes [McNeal, 1976]. The forearm structure is assumed to be cylindrical and it is obtained by superimposing a given number of parallelepipedic cell layers.

With each cell we associate a conductivity and a permittivity that, in some cases, are different in the transverse and longitudinal directions (anisotropic tissues). The conductivity values are taken from various experimental reports [Geddes and Baker, 1967; Stuchly and Stuchly, 1980; Polk and Postov, 1986]. For the permittivity we consider the frequency dependence, and use the analytical expression for the permittivity to interpolate the experimental data from Polk and Postow [1986] (see third and fifth column of Table I). The interpolating function gives a good fit of the experimental data for frequencies from dc to about 10MHz.

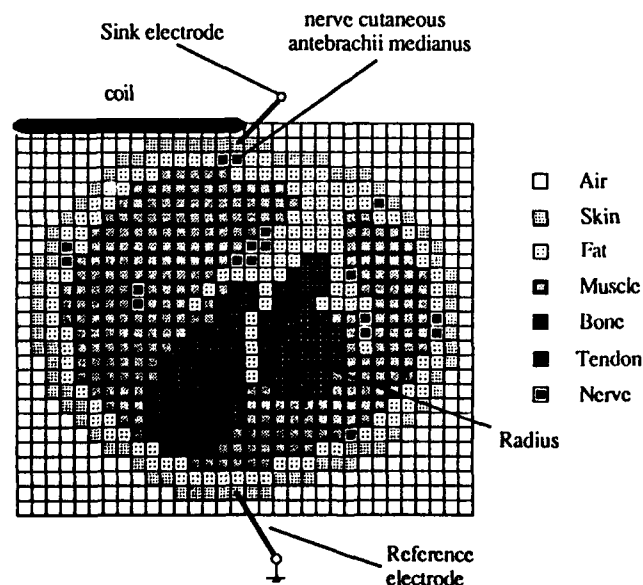


Fig. 3 Diagram of the forearm cross-section.

Biological Tissue		Transversal Conductivity σ_t (S/m)	Transversal Permittivity ϵ_{Rt}	Longitudinal Conductivity σ_z (S/m)	Longitudinal Permittivity ϵ_{Rz}
Air	0	0.0	1.0	0.0	1.0
Skin	1	0.5	$10^{6.5-0.5\log_{10}(f+1)}$	0.5	$10^{6.5-0.5\log_{10}(f+1)}$
Fat	2	0.05	$10^{6-0.5\log_{10}(f+1)}$	0.05	$10^{6-0.5\log_{10}(f+1)}$
Muscle	3	0.08	$10^{8-0.75\log_{10}(f+1)}$	0.5	$10^{6.5-0.5\log_{10}(f+1)}$
Bone	4	0.013	$10^{4-0.33\log_{10}(f+1)}$	0.013	$10^{4-0.33\log_{10}(f+1)}$
Tendon	5	0.013	$10^{4-0.33\log_{10}(f+1)}$	0.013	$10^{4-0.33\log_{10}(f+1)}$
Nerve	6	0.05	$10^{6-0.5\log_{10}(f+1)}$	0.5	$10^{6-0.5\log_{10}(f+1)}$

Table I

Modeling of Nerve Fiber Excitation

The nerve model used to evaluate the threshold response of the fiber is that of Frankenhaeuser and Huxley (F&H) [1964] (Fig. 4). The stimulating current density in this model is evaluated extending the method proposed by Rattay [1989]. Rattay showed that, in simulating the stimulation of a nerve fiber with external electrodes, the current density used in the F&H model is proportional to the longitudinal derivative, evaluated at the node of Ranvier, of the longitudinal component of the electric field.

Because of the dispersive properties of the tissues the scalar potential at each point inside the forearm varies with the frequency of the stimulating current. For this reason we introduce in the frequency domain a complex transfer function $T(f)$ defined, at each frequency, as the ratio between the current density stimulating the nerve ($J_S(f)$) and the applied electrode current (I_e). Extending the Rattay model in the frequency domain this ratio can be expressed as a function of the scalar and vector potential distribution along the nerve:

$$T(f) = \frac{J_S(f)}{I_e} = \frac{d\Delta z}{4\rho_i L I_e} \cdot \left\{ \frac{V_{z-\Delta z}(f) - 2V_z(f) + V_{z+\Delta z}(f)}{\Delta z^2} + j\omega \frac{A_{z'} - A_z}{\Delta z} \right\} \quad [m^{-2}] \quad (11)$$

To evaluate the time behavior $J_S(t)$ of the stimulating current density, first we apply the Fourier-transform to the electrode current, then the obtained frequency spectrum is multiplied by $T(f)$, and, finally, the inverse Fourier transform of the result gives the time behavior of the current density $J_S(t)$ used in the F&H model.

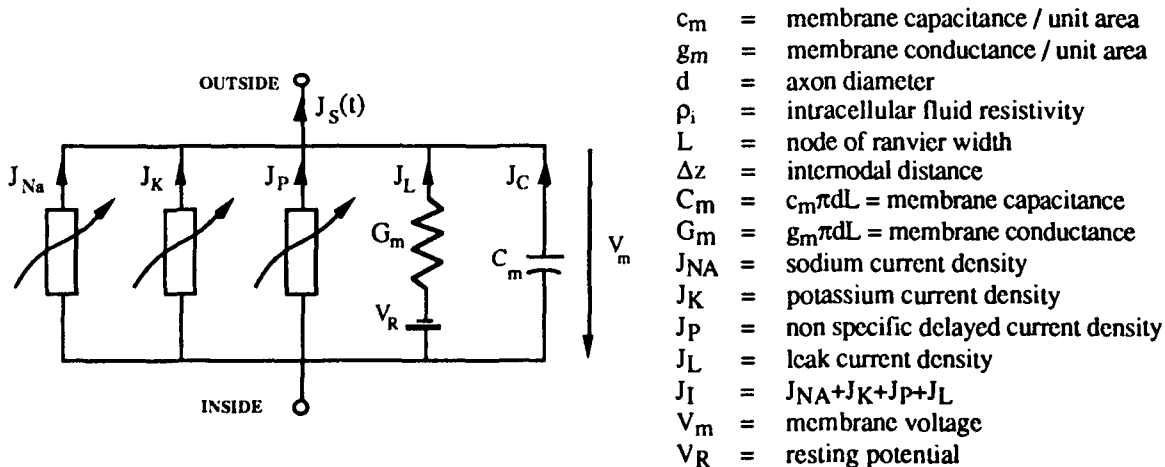


Fig. 4 Equivalent circuit representation of the F&H model and list of symbols used.

The time behavior of the membrane voltage V_m in response to the stimulating current density is evaluated by solving the differential equation:

$$\frac{dV_m}{dt} = \frac{1}{C_m} [-J_I(t) + J_S(t)] \quad (12)$$

Four additional differential equations with nonlinear coefficients are necessary to evaluate $J_I(t)$ [Frankenhaeuser and Huxley, 1964]. This set of five differential equations has been solved by using the Runge-Kutta numerical method [Ralston and Rabinowitz, 1975]. As an example Fig. 5 shows the time behavior of the membrane voltage in response to 120 μ s current density pulses with different amplitudes. The figure shows that excitation occurs for amplitudes above 775 μ A/cm² with the considered signal.

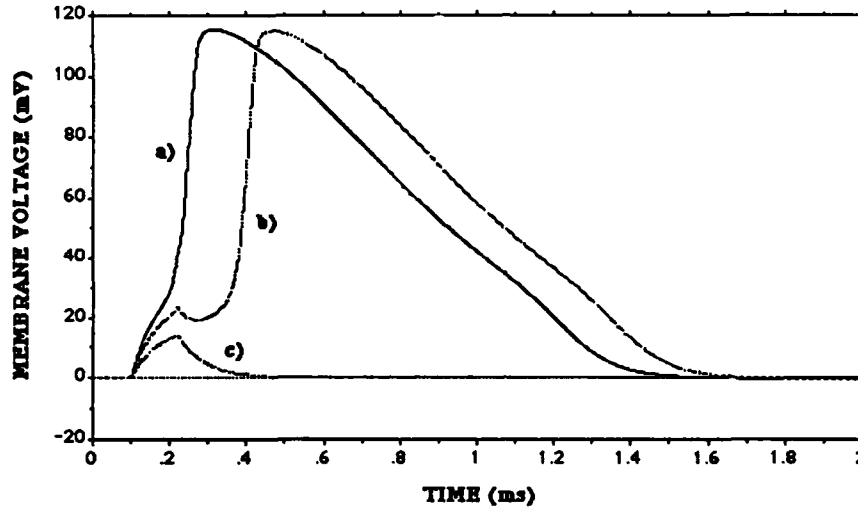


Fig. 5 Time behavior of the membrane voltage in response to 120 μ s current density pulses with different amplitudes; a) 1000 μ A/cm², above threshold response; b) 775 μ A/cm², threshold response; c) 500 μ A/cm², subthreshold response.

FDT Solution Methods

The electromagnetic problem is numerically solved using a FORTRAN code. For non dispersive tissues, both a FORTRAN code and a spread-sheet (WingzTM) are used. Spread-sheets have already been introduced in the past to get iterative solutions of finite difference problems [Hart, 1989a, 1989b, 1990]. In this work the spread-sheet WingzTM has been chosen for its capability of managing a large number of cells simultaneously, and because of its excellent graphic capabilities. In the application of the spread-sheet to solve the FDT problem, we first fix the number of transverse sections of the forearm to be considered, then, for each section, three regions of 30x25 cells are defined on the spread-sheet. The conductance values of the forearm cells are inserted in each cell of the first region; in the cells of the second section we place, in the electrical stimulation the sink and references current values, and in the magnetic stimulation, the sum of the currents of the generators dependent on the vector potential. Starting from the knowledge of conductivity and electrode matrices it is possible to implement Eq. (7) in a cell of the third region and to copy it into all the remaining cells. To find a solution of the problem, the iterative technique is applied by using the spread-sheet property of calculating a new value in a specific cell whenever a variation takes place in one of the adjacent cells. It must be noted that a further aid to the implementation of this procedure is provided by the possibility of programming the spread-sheet WingzTM. It uses a high-level programming language, and the calculations can be executed automatically.

As an example, let us consider a forearm model obtained by superimposition of five sections, that is 3750 grid points. By considering an electrical stimulation ($f = 0$) with a relaxation constant $\alpha=1.5$, the convergence criterion (9) is reached after 225 iterations. This number corresponds to an execution time of 70 minutes on a Macintosh SE/30. The same time is necessary to solve the problem with the FORTRAN program on the same computer.

RESULTS

Electrical Stimulation

The electrical stimulation is performed by placing the electrodes on a transverse section of the forearm. The sink electrode is placed on the skin, 3mm from the cutaneous nerve 'antebrachii medianus'; the reference electrode is placed on the other side of the forearm (Fig. 3). The stimulating signal with a pulse width of 120 μ s and a slew-rate of 1mA/ μ s is shown in Fig. 7a. The pulse amplitude (I_p) is evaluated as the threshold value (current threshold) necessary to obtain the excitation of the nerve membrane, that is a stimulating current density amplitude of 775 μ A/cm² (threshold current density).

In the following study, three cases will be evaluated:

- a) homogeneous and non dispersive,
- b) inhomogeneous and non dispersive,
- c) inhomogeneous and dispersive.

In a) the forearm tissue is assumed homogeneous with a conductivity of 0.08 S/m that corresponds to the transverse conductivity of the muscle (see Table I); in b) the inhomogeneities of the forearm are considered and for each tissue the conductivity values shown in Table I are used; finally in c) the permittivity of the forearm tissues is introduced and its frequency dependent behavior is considered as given by the analytical expressions in Table I. The c) situation is the real one, however, the a) and b) cases are analyzed to show the effect of the subsequent approximations on the current threshold.

a) homogeneous and non dispersive. In this situation the scalar potential inside the forearm does not vary with the frequency of the electrode current. The electromagnetic problem is solved in the $f=0$ case (no vector potential) and by considering five longitudinal layers of parallelepipedic cells.

The transfer function $T(f)$ is evaluated using Eq. (11). It can be noted that by using the FDT the potential is calculated apart from an additive term that can be ignored since, to compute the transfer function only differences of potential are considered. Since $T(f)$ is frequency independent, the stimulating current density is simply proportional to the electrode current. By applying this current density to the F&H model a current threshold value of 65 μ A is obtained. This value is close to that found by using, for the evaluation of the potential, the equation: $V_e = \rho_e I_e / 4\pi R$ where I_e is the electrode current, ρ_e the tissue resistivity, and R the distance between the electrode and the nerve [Rattay, 1986]. From this equation evaluated for $R=3$ mm, a current threshold of 100 μ A is found. In our simulation we obtain a lower current threshold since the current does not flow in all the directions but only in a confined region (Fig. 6a), so for the same electrode current the voltage gradient and the corresponding stimulating current density are higher.

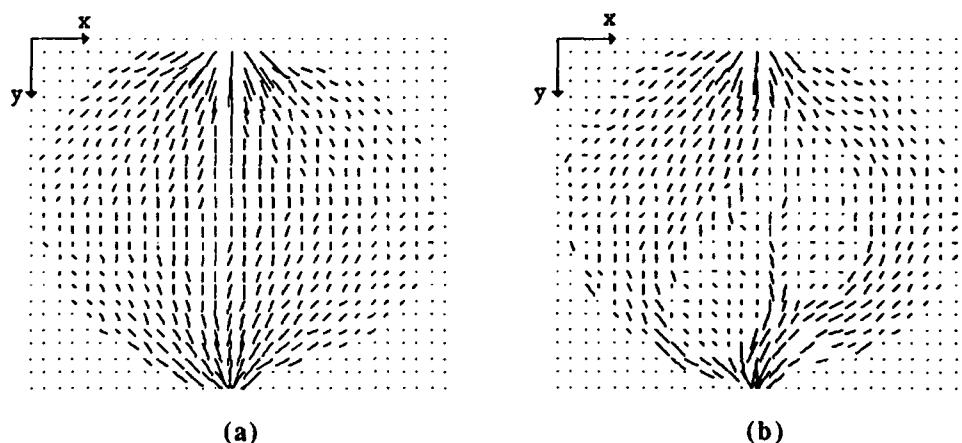


Fig. 6 Current density in the central layer of the forearm for the homogeneous and non dispersive tissue (a), and for the inhomogeneous and non dispersive tissue (b).

b) inhomogeneous and non dispersive. The effects of the tissue inhomogeneities on the current density distribution are evidenced in Fig. 6b. Low current values inside the low conductance tissues (bone and tendon in Fig. 3) can be seen. In this situation, the transfer function is still constant with frequency, but 20 times lower than in a). Higher pulse amplitudes are necessary to evoke an action potential ($I_p=1.22$ mA with 5 layers).

c) *inhomogeneous and dispersive*. In this case the potential and $T(f)$ vary with frequency. The stimulating current density is obtained by using the technique previously described for the dispersive case. By applying the electrode current shown in Fig. 7a we obtain the stimulating current density $J_S(t)$ shown in Fig. 7b. In spite of the difference in signal shape, the current threshold does not change very much (1.20 mA with 5 layers) with respect to the non dispersive case b). A time behavior similar to that reported in Fig. 7b has been experimentally obtained for upper limbs [McGill et al., 1982].

The performed analysis shows that, with reference to the homogeneous case, usually considered in literature, the most important differences are due to the tissue inhomogeneity, while the non dispersive tissue hypothesis does not cause significant errors. Since WingzTM can be used only for the nondispersive case, the obtained results suggest that the spread-sheet is a very useful tool for this kind of application.

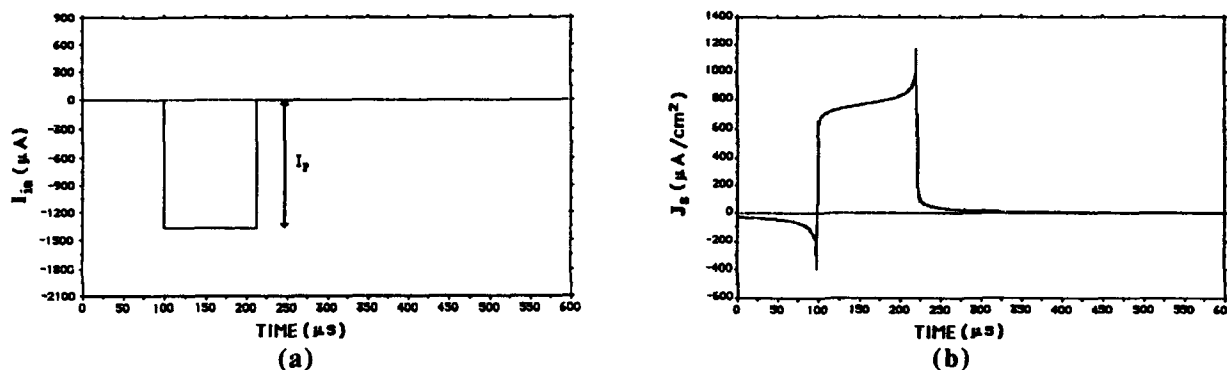


Fig. 7 Time behavior of the input electrode current (a) and of the stimulating current density (b) evaluated in the dispersive case.

Magnetic Stimulation

As in the electric situation, also in the magnetic one, stimulation of the nerve 'antebrachii medianus' will be modeled. The stimulation is obtained by a 30 turn external circular coil having a radius of 2.4 cm ($L=0.165\text{mH}$, $R=3\Omega$) placed 3mm from the forearm (Fig. 3) [Roth and Basser, 1990]. The coil current is generated by the discharge of a capacitance ($C=300\mu\text{F}$) initially at a voltage V_0 . The time behavior of the coil current is:

$$I_c(t) = \frac{V_0}{\sqrt{\left(\frac{R}{2}\right)^2 - \left(\frac{L}{C}\right)}} \exp\left(-\frac{R}{2L}t\right) \sinh\left(\sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}t\right) \quad (13)$$

Since in eq. (13) all the parameters are fixed except the voltage V_0 , we relate the stimulation of the nerve membrane to this parameter (voltage threshold).

In the magnetic stimulation we will analyze two situations for the forearm tissue:

- a) homogeneous and non dispersive,
- b) inhomogeneous and dispersive.

a) *homogeneous and non dispersive*. In this situation the vector potential in eq. (11) is evaluated by eq. (10). By solving this problem, we found that scalar potential values give a negligible contribution to the $T(f)$. The small scalar potential values depend on the homogeneous tissue assumption; in this case, in fact, there is no charge accumulation at the boundaries between different tissues. To further reduce the boundary effect due to the longitudinal truncation of the forearm, we have increased the number of cell layers to 49. Fig. 8a shows the transfer function ($f=50\text{ Hz}$) on a plane containing the nerve and parallel to the coil surface. The maximum of $T(f)$ corresponds to the point of the nerve that will be excited with the lowest voltage. By shifting the coil center with respect to the nerve by a distance equal to the coil radius the higher current density value is obtained at the nerve position. Fig. 8b shows $T(f)$ (obtained as in Fig. 8a) in a transverse plane 24 mm from the central layer. The figure shows that, as in the electrical case, the points with highest current density are those close to the skin and the current density decreases by increasing the distance from the skin. In the magnetic stimulation, however, the region with high current density is larger than for the electric case.

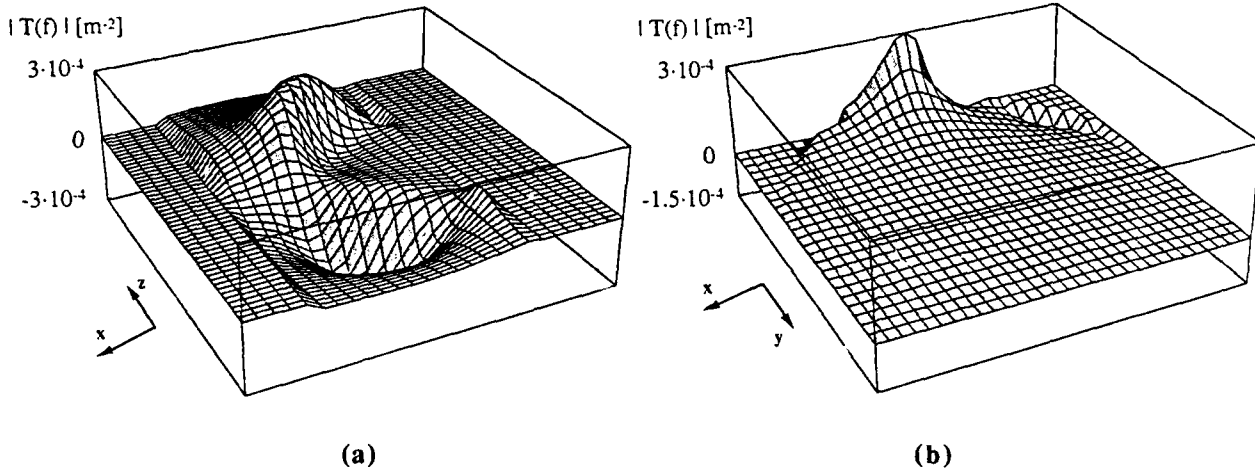


Fig. 8 Transfer function on a plane parallel (a) and transverse (b) to the coil surface.

Since the vector potential gives the main contribution to the transfer function $T(f)$, the stimulating current density is practically related only to the time derivative of the coil current I_c (a multiplication factor $j\omega$ in the frequency domain corresponds to a derivative in the time domain).

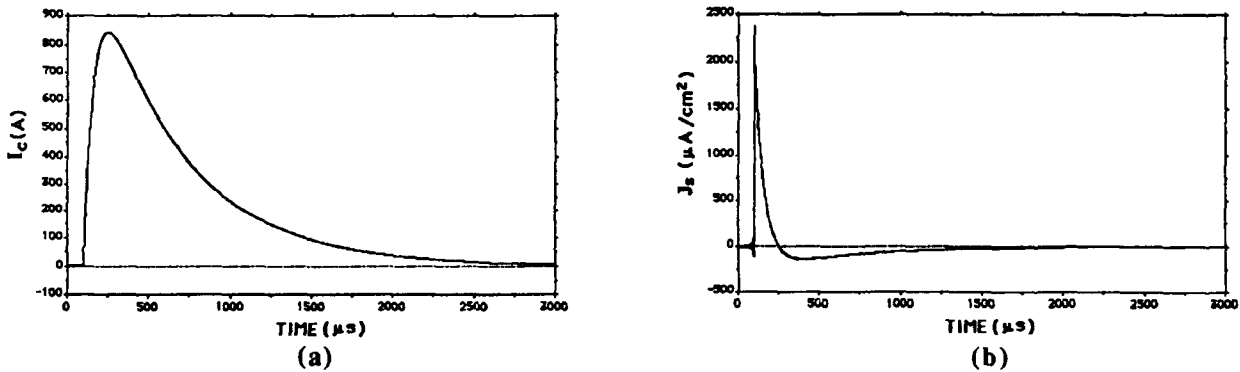


Fig. 9 The coil current (a) and the stimulating current density (b).

Fig. 9 shows the time behavior of the current in the coil (a) and of the stimulating current density (b). These figures are obtained by applying a voltage V_0 of 3250 Volts that represents the voltage threshold for this situation. By using a model of the mammalian myelinated axon and lightly different coil parameters, Bassar and Roth [1991] obtained voltage threshold values of the same order of magnitude.

b) inhomogeneous and dispersive: By considering the dispersive and inhomogeneous properties of the tissue of the forearm, the scalar voltage gives a contribution to the transfer function that is opposite to the vector potential one and it causes a reduction in the $T(f)$ values and consequently an increase in the coil current necessary to excite the nerve. In this situation we observe a 40% increase in the voltage on the capacitance necessary to evoke a spike on the nerve ($V_0 = 4700$ Volts). The obtained values for the voltage on the capacitance are similar to those found experimentally by Polson et al. [1982]. An exact comparison with the Polson data is not possible since the distance between the coil and the nerve is not reported.

CONCLUSION

The proposed technique allows to study the threshold for electric and magnetic nerve stimulation by assuming the nerve fiber surrounded by an inhomogeneous, anisotropic and dispersive tissue. The study shows the importance of considering the tissue inhomogeneities for evaluating the threshold, while, in the cases examined, the dispersive properties of the tissue are less important. The proposed method can be used both for the analysis and the synthesis of FES stimulators, as well as for the study of multi-electrode configurations. To improve the technique, a reduction of the cell dimensions should be considered. For the same anatomical section, this reduction causes an increase in the computation time and necessary computer memory. With modern computers, and in view of the small memory occupations by the FDT used in the present study, it should be possible to extend the technique. Another improvement can be obtained by considering a more realistic anatomical picture of the forearm.

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