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# THE ADVANTAGE OF CYCLIC SPECTRAL ANALYSIS (U)

by

Eric April

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# THE ADVANTAGE OF CYCLIC SPECTRAL ANALYSIS (U)

by

**Eric April**

*Communications Electronic Warfare Section  
Electronic Warfare Division*

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## ABSTRACT

This report shows that cyclic spectral analysis, as a general tool for signal spectral analysis, is much superior to more conventional spectral analysis. Three major advantages are pointed out: (1) its signal's discriminatory capability enables signal selectivity (even for highly corrupted environments); (2) it provides a richer domain for signal analysis; and (3) the theory of spectral correlation allows a much more complete mechanism for modeling communication signals in several applications. A conceptual view of the cyclic spectrum is presented along with a working example. A strong mathematical foundation for the theory of spectral correlation involved in cyclic spectral analysis is provided. Finally, investigations are made on some interesting applications where exploitation of the inherent redundancy associated with spectral correlation can be used advantageously.

## RÉSUMÉ

Il est démontré dans ce rapport qu'en tant qu'outil général d'analyse spectrale de signaux, l'analyse spectrale cyclique arbore une nette supériorité sur d'autres approches plus conventionnelles d'analyses spectrales. Trois de ses principaux avantages y sont soulignés: (1) la capacité de discrimination de signaux immanent de l'analyse spectrale cyclique permet la sélectivité de ceux-ci (même s'il s'agit d'environnements spectraux hautement corrompus par bruits de fond et interférences diverses), (2) l'analyse spectrale cyclique offre un domaine plus riche facilitant l'analyse de signaux, et (3) la théorie de corrélation spectrale jouit d'un mécanisme beaucoup plus complet en ce qui a trait à la modélisation de signaux utilisés en télécommunications pour de nombreuses applications. Une vue conceptuelle du spectre cyclique est présentée à l'aide d'un exemple. Ensuite, la théorie de corrélation spectrale nécessaire à l'analyse spectrale cyclique est formulée à l'intérieur d'une solide base mathématique. Finalement, on examine quelques intéressantes applications dans lesquelles l'inhérente redondance associée à la corrélation spectrale peut être avantageusement exploitée.

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## EXECUTIVE SUMMARY

Most signal processing methods deal with signals assumed to be statistically stationary. However, in communication systems, most manmade signals do not meet the stationarity assumption and are in fact better modelled as cyclostationary processes where the statistical parameters of the signals are assumed to be varying in time with single or multiple periodicities. This motivates the needs for cyclic spectral analysis, allowing identification of underlying periodicities.

In this report, the cyclic spectral analysis, as a general tool for signal spectral analysis, is shown to be much superior to more conventional spectral analysis. Three major advantages are pointed out:

1. its signal's discriminatory capability enables signal selectivity (even for highly corrupted environments);
2. it provides a richer domain for signal analysis and therefore more information about the signal; and
3. the theory of spectral correlation allows a much more complete mechanism for modelling communication signals in several applications.

A conceptual view of the cyclic spectrum is presented along with a working example which demonstrates most of these advantages. A strong mathematical foundation for the theory of spectral correlation involved in cyclic spectral analysis is provided.

The objective of all cyclic spectral analysis applications is to exploit signals spectral redundancy to improve the accuracy and reliability of information extracted from the measurements of corrupted signals. Some of the most promising applications are investigated: detection, classification, parameter estimation, TDOA estimation, direction finding, and frequency-shift filtering for signal extraction.

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## 1.0 INTRODUCTION

Most signal processing methods deal with signals assumed to be statistically stationary. Hence, the signal is presumed to have statistical parameters (i.e., mean, autocorrelation, etc) that do not vary with time. Unfortunately, this stationarity assumption is not applicable to most manmade signals encountered in a variety of areas such as the one of particular interest in this paper: communication systems. In communication systems, the statistical parameters of manmade signals usually vary with time and in many cases vary periodically with time. Examples of this include: sine carriers modulated in amplitude, phase, or frequency, periodic keying of the amplitude, phase, or frequency in digital modulation systems, repeating spreading codes, etc. In many cases, the performance of signal processors can be improved considerably by identifying and exploiting underlying periodicity. The approach discussed in this report is to model the signal as a cyclostationary process where the statistical parameters of the signal are assumed to be varying in time with single or multiple periodicities.

The first part of this paper, Section 2, presents a conceptual view of the *cyclic spectrum* through a working example. The following section, Section 3, investigates the numerous advantages of using the cyclic spectrum over the conventional spectrum. Section 4 is devoted to a mathematical description of the basis of the spectral correlation theory of cyclostationary signals and the cyclic spectrum. An analysis of the effects of some basic signal processing operations on the cyclic spectrum is discussed in Section 5. Finally, the last section, Section 6, gives a brief overview of some interesting applications where exploitation of the inherent redundancy associated with spectral correlation can be used advantageously. This includes signal detection, classification, time-difference-of-arrival (TDOA) estimation, spatial filtering, direction finding, and frequency-shift filtering for signal extraction.

## 2.0 A CONCEPTUAL VIEW OF THE CYCLIC SPECTRUM

Most books on signal processing when dealing with communications signals typically assume them as being stationary, i.e., signals whose statistical parameters, such as mean and variance, do not vary with time. The signal is then modeled as a one-dimensional autocorrelation function from which the power spectrum density (or PSD) is generated by computing the Fourier Transform (FT) of the autocorrelation function. However, it is known that most manmade signals encountered in radio communication systems are in fact cyclostationary [11], that is, statistical parameters are periodically or cyclically

Table 1: Signals Parameters

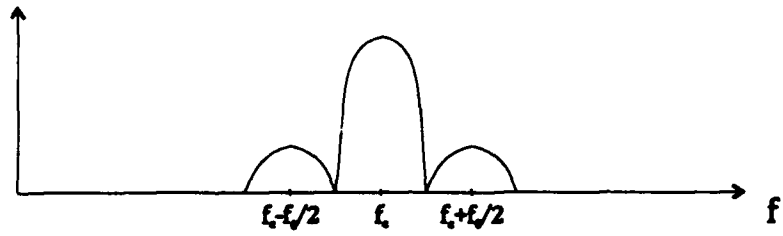
	BPSK-1	BPSK-2
# of samples	32768	32768
symbol rate	1/32	1/16
carrier frequency	3.3·1/32	4·1/32
power	0 dB	0 dB
pulse type	raised-cosine	raised-cosine
roll-off factor	1.0	1.0

stationary. Modeling the signal as cyclostationary results in a two-dimensional autocorrelation function where the extra dimension is denoted  $\alpha$ , i.e., the cycle frequency at which the one-dimensional autocorrelation function has been computed. For each  $\alpha$ , a F.T. of the cyclic autocorrelation function produces a cyclic-spectrum-cut for that particular frequency separation  $\alpha$ . More easily visualized in the spectral domain, Figure 1 shows all the steps involved in producing one cyclic-spectrum-cut for  $\alpha = f_0$ . In Figure 1(a), the magnitude of the F.T. with frequency resolution  $1/T$  (where  $T$  is the duration) of a signal  $x(t)$  at  $t = t_1$  is shown, i.e.,  $|X_T(t, f)|$ . Figure 1(b) and Figure 1(c) show two frequency-shifted versions of the original spectrum shifted by  $f_0/2$  and  $-f_0/2$  respectively. The spectral correlation measure is shown in Figure 1(d) and is simply the product of the two frequency-shifted versions. Since the statistics of the signal are not stationary, the above process is repeated for  $\{t = t_1, t_2, t_3, \dots, t_n \mid (t_1 < t_2 < t_3 < \dots < t_n)\}$  and the results averaged over time. The result is denoted  $S_{x_T}^{\alpha=f_0}(f)$ , which signifies a cyclic-spectrum-cut at  $\alpha = f_0$ . Repeating this process for other values of  $\alpha$  gives rise to a three-dimensional *cyclic spectrum* plot with the three axes being:  $f$ ,  $\alpha$ , and magnitude. Note that at  $\alpha = 0$ , the cyclic-spectrum-cut corresponds to the original spectrum, the conventional PSD, since the spectrum completely correlates with itself.

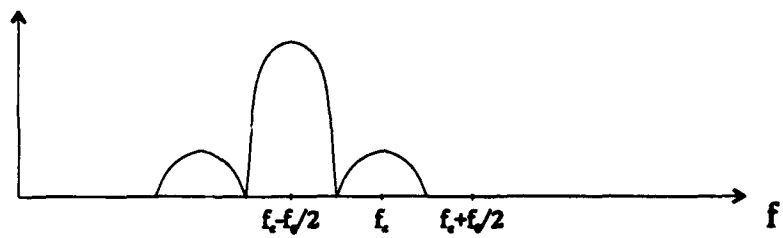
A working example will illustrate the concept of cyclic spectral analysis. Two Binary-Phase-Shift-Keying (BPSK) modulated signals having the parameters shown in Table 1 are used. The signal is corrupted by additive white Gaussian noise with a total power of 5 dB over the Nyquist band.

Applying conventional spectral analysis using frequency-smoothed estimation<sup>1</sup> (with  $\approx 3\%$  width smoothing window), the conventional PSD's of BPSK-1, BPSK-2, BPSK-1+BPSK-2, and BPSK-1+BPSK-2+noise are shown in Figure 2.

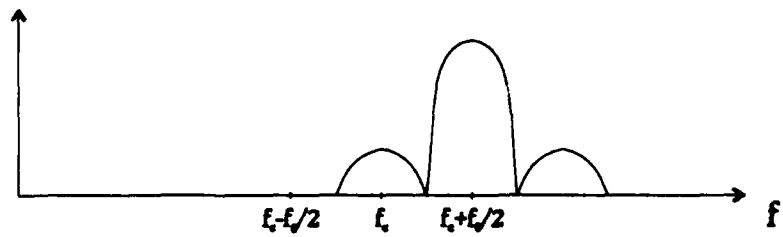
<sup>1</sup>Instead of using time-averaging as explained earlier, averaging in the spectral domain is used in this case. Note that the smoothing window represents the spectral width that is averaged (see [17], [35]).



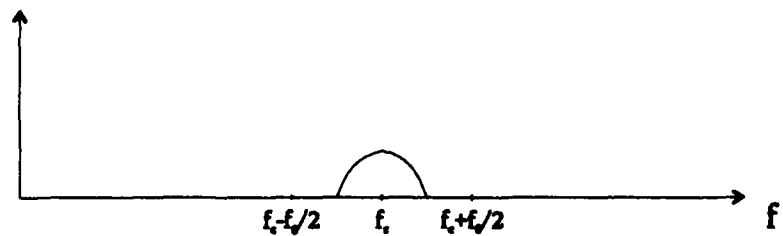
(a)  
Magnitude of the F.T. of the signal at instant  $t$ ,  $|X_T(t, f)|$



(b)  
Frequency-shifted versions by  $+f_0/2$  of (a),  $|X_T(t, f + f_0/2)|$

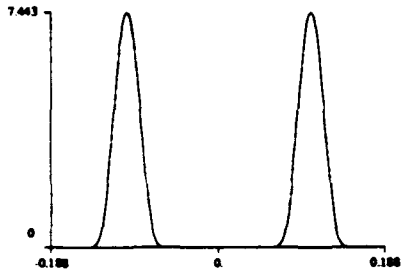


(c)  
Frequency-shifted versions by  $-f_0/2$  of (a),  $|X_T(t, f - f_0/2)|$

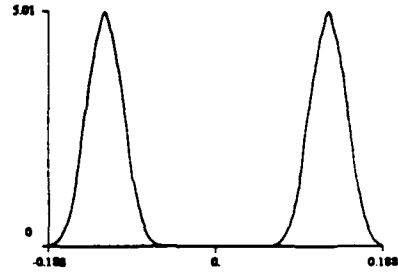


(d)  
Product of (b) and (c),  $|S_{x_T^{\alpha=f_0}}(t, f)| = |1/T X_T(t, f + f_0/2) X_T^*(t, f - f_0/2)|$

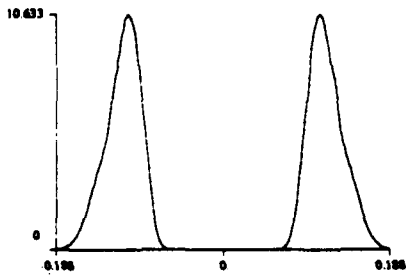
Figure 1: Steps involved for producing a cyclic-spectrum-cut at  $\alpha = f_0$



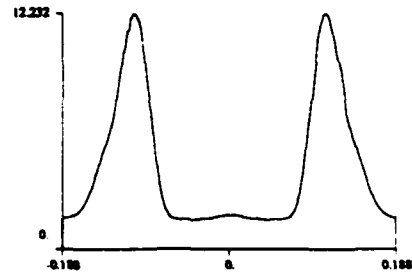
(a) BPSK-1



(b) BPSK-2



(c) BPSK-1+BPSK-2



(d) BPSK-1+BPSK-2+noise

Figure 2: Conventional PSD examples

It is important to note that in this and following examples the frequency has been normalized with respect to the sampling frequency so that the total frequency span is 1, with  $f$  ranging from -0.5 to 0.5. In Figure 2, for clarity, only a portion of this range is shown i.e.,  $f$  ranging from -0.188 to 0.188 (-6 times the symbol rate of BPSK-1 to +6 times the symbol rate). Additionally, all plots are shown with a linear scale.

Some comments can be made about the plots shown in Figure 2. Between Figure 2(a) and Figure 2(b), one can observe some differences: the peaks are not centered at the same carrier frequency; and the bandwidths are different. In order to find out the symbol rate, an approximation of the bandwidth of the signals can be made if and only if it has been assumed that only one signal is present at a time (in either (a) or (b)) and that one has an idea of the type of modulation one is looking at. Figure 2(c) is intended to demonstrate the spectral overlapping that may easily occur between signals. The PSD of the two added BPSK signals is somewhat unusual. It is not apparent from this plot that two BPSK signals are present at two different carrier frequencies and two different symbol rates. Rather, this looks like a single unusual signal instead of two BPSK signals. In other words, two BPSK signals can not be distinguished using the conventional spectral analysis. In Figure 2(d), the PSD of the same two BPSK signals with band limited (Gaussian noise added shows the raised noise floor that results compared to the preceding noise-free spectrum.

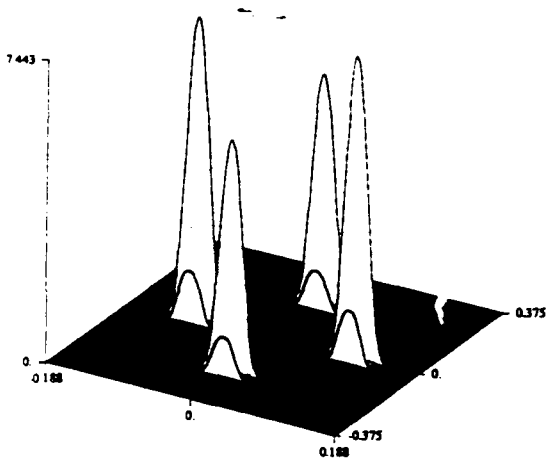
Applying the more powerful tool known as the cyclic spectral analysis, the four cyclic spectra analogous to PSD's of Figure 2 are shown in Figure 3. Again, the sampling frequency has been normalized as before. The front axis represents the ordinary frequency (the  $f$ -axis) and the side axis denotes the cycle frequency axis, also called  $\alpha$ -axis. Here again, only a portion of the complete spectrum is used in order to better see the features of interest: the  $f$ -axis ranges from -6 times the smallest symbol rate ( $1/32$ ) to +6 times  $1/32$  while the  $\alpha$ -axis<sup>2</sup> ranges from -12 times  $1/32$  to +12 times  $1/32$ . Note also that the cyclic spectrum has been computed for 120 discrete  $\alpha$ -values from -0.375 to 0.375. All plots are shown with a linear scale.

For all the plots in Figure 3, the conventional PSD is easily recognizable as the cyclic spectrum at the zero cycle frequency. The peaks in the cyclic spectrum reveal the cyclic features one would theoretically expect from a BPSK modulated signal having the parameters as described in Table 1. Denoting the symbol rate  $f_0$  and the carrier frequency  $f_c$ , then, for such a BPSK modulation, the signal peaks will be present when

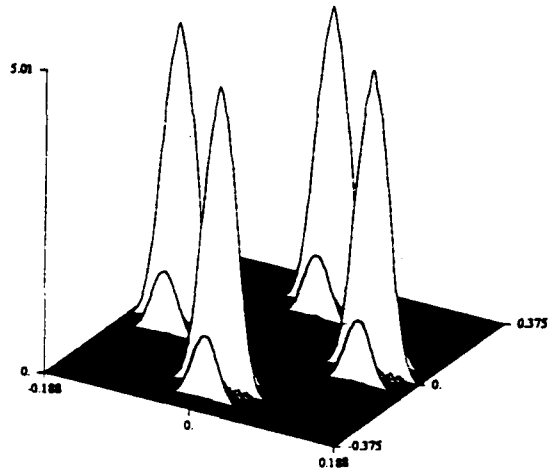
$$\alpha = f_0, \quad f = \pm f_c,$$

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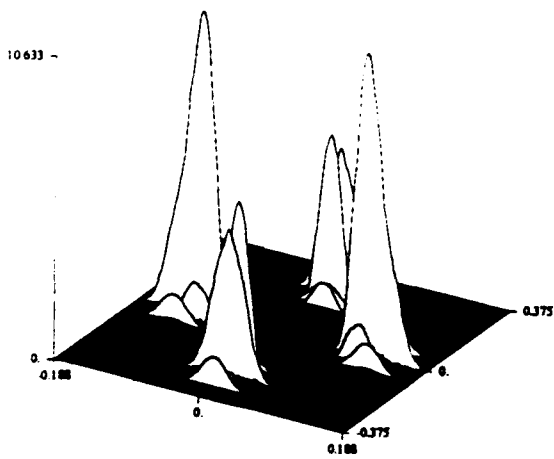
<sup>2</sup>The total span on the  $\alpha$ -axis is always twice the one of the  $f$ -axis (see [17]).



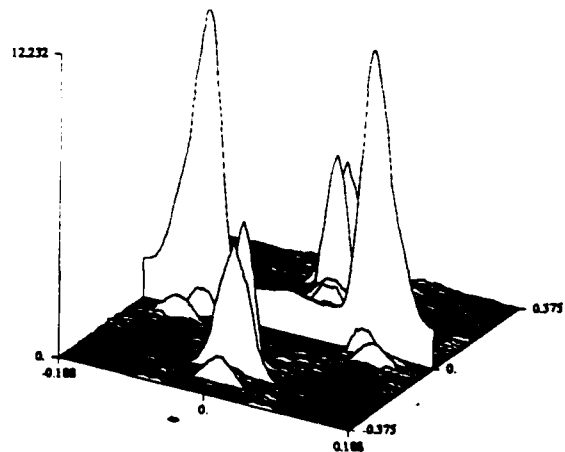
(a) BPSK-1



(b) BPSK-2



(c) BPSK-1+BPSK-2



(d) BPSK-1+BPSK-2+noise

Figure 3: Cyclic spectrum examples

$$\begin{aligned}\alpha &= 2f_c, & f &= 0, \\ \alpha &= 2f_c \pm f_0, & f &= 0.\end{aligned}$$

Note that since the signal is real, the cyclic spectrum is symmetric with respect to the  $\alpha$ -axis (as well as to the  $f$ -axis). Clearly, the cyclic spectrum provides a lot more information about the signal than the conventional PSD. Looking at Figure 3(a), the cyclic features of the BPSK-1 signal are easily determined ( $\alpha = 1/32, f = \pm 3.3 \times 1/32; \alpha = 2 \times 3.3 \times 1/32, f = 0; \alpha = 2 \times 3.3 \times 1/32 \pm 1/32, f = 0$ ) and similarly in Figure 3(b), the cyclic spectrum of BPSK-2 reveals its cyclic features ( $\alpha = 1/16, f = \pm 4 \times 1/32; \alpha = 2 \times 4 \times 1/32, f = 0; \alpha = 2 \times 4 \times 1/32 \pm 1/16, f = 0$ ).

Since the cyclic features are discretely distributed in the cyclic spectrum, one can recognize the two BPSK signals with two different symbol rates and two slightly different carrier frequencies by simply using appropriate pattern recognition techniques in Figure 3(c). The two BPSK signals appear very clearly in the cyclic spectrum and are therefore distinguishable. The effect of adding noise to the signals to the cyclic spectrum is shown in Figure 3(d). The most interesting result is the fact that since the background noise is not cyclostationary, it has no cyclic features and therefore only appears at the zero cycle frequency. Outside the zero cycle frequency, all the noisy components not present in Figure 3(c) are due to the measurement noise, that is the noise due to the fact that the cyclic spectrum is estimated from a finite number of samples. As can be seen however, the cyclic features still remain prominent even when 5 dB of noise has been added to the two 0 dB BPSK signals.

### 3.0 CYCLIC SPECTRUM ADVANTAGES OVER CONVENTIONAL SPECTRUM

This section analyses the various advantages of using the cyclic spectrum, as described in the previous section, over using the conventional spectrum in order to analyse and detect a number of features contained in a signal (cf [19]).

#### 3.1 DISCRIMINATORY CAPABILITY

One nice property of the cyclic spectrum is that signal features are discretely distributed in cycle frequency in the cyclic spectrum. Note that this is satisfied even in the case of a signal having continuous distribution in the power spectrum. It follows that signals with overlapping features in the power spectrum can have non-overlapping features in the cyclic spectrum. For instance, background noise such as Gaussian noise has no features at any cycle frequency other than at zero ( $\alpha \neq 0$ ). This important result means that

a cyclic spectral analysis at a non-zero cycle frequency of a signal of interest reveals its cyclic features without any component due to the noise. In fact, the only noise will be due to measurement noise or from other interference signals having the exact same cycle frequencies. The measurement noise is due to the way the cyclic spectrum is computed and tends asymptotically to zero as the collect time measurement increase. In the case of interferers, the signal of interest will generally have features at other cycle frequencies than those of the interferers. Thus, analysis of the cyclic spectrum at those particular cycle frequencies will show these features without any component due to the interference. In brief, the cyclic spectrum allows signal separation.

### **3.2 A RICHER DOMAIN FOR SIGNAL ANALYSIS**

The cyclic spectrum (magnitude and phase) reveals information about the signal that is not easily identifiable in the conventional spectrum. This information includes signal parameters like carrier and pulse-train frequency, bandwidth, phase and timing information, baud and frequency-hopping rate, etc. In short, it provides much useful information about the signal.

### **3.3 THEORY UNDERLYING THE TECHNIQUES**

Since most man-made signals are cyclostationary, a much more complete mechanism for modeling communication signals is provided through the theory of spectral correlation functions when compared to analysis of the power spectrum. Hence, a more powerful model is used and conventional spectrum analyzers and feature detection techniques can both be theoretically studied and analyzed in terms of the cyclic spectrum. Moreover, based on the cyclic spectral analysis, optimal parametric approaches as well as feature detection and nonparametric approaches used to detect and analyze signals can be devised.

## **4.0 SPECTRAL CORRELATION THEORY**

After having looked at the conceptual description of the cyclic spectrum, this section will state mathematical results related to spectral correlation theory (cf [14], [17], [22], and [5]).

### **4.1 CYCLIC AUTOCORRELATION FUNCTION**

As mentioned in the introduction, a signal modeled as cyclostationary will have probabilistic or statistical parameters varying periodically with time. The symmetric conventional

autocorrelation function of a time series  $x(t)$  is given by

$$R_x(t + \tau/2, t - \tau/2) = \langle x(t + \tau/2)x^*(t - \tau/2) \rangle, \quad (1)$$

where  $x^*(\cdot)$  denotes the complex conjugate of  $x(\cdot)$  and  $\langle \cdot \rangle$  is the time-averaging operation

$$\langle \cdot \rangle \triangleq \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} (\cdot) dt. \quad (2)$$

If  $x(t)$  contains more than one source of periodicity and the periods are all different (i.e., no fundamental period of which all periods are integral divisors), then the process is said to be *almost cyclostationary* because its parameters are almost periodic with time. Hence, the autocorrelation function can be expressed by using the Fourier series representation

$$R_x(t + \tau/2, t - \tau/2) = \sum_{\alpha} R_x^{\alpha}(\tau) e^{j2\pi\alpha t}, \quad (3)$$

where  $R_x^{\alpha}(\tau)$  represents the Fourier coefficients depending on the lag parameter  $\tau$  and is given by

$$R_x^{\alpha}(\tau) = \langle R_x(t + \tau/2, t - \tau/2) e^{-j2\pi\alpha t} \rangle. \quad (4)$$

Assuming  $x(t)$  is a cycloergodic process<sup>3</sup>, then (4) reduces to

$$R_x^{\alpha}(\tau) \triangleq \langle x(t + \tau/2)x^*(t - \tau/2) e^{-j2\pi\alpha t} \rangle, \quad (5)$$

where  $R_x^{\alpha}(\tau)$  is called the *cyclic autocorrelation function*.

When the cycle frequency  $\alpha = 0$ , then (5) becomes the conventional autocorrelation function, i.e.,  $R_x^0(\tau) = R_x(\tau)$  which is the autocorrelation function of a signal assumed to be wide sense stationary. It is observed that  $R_x^{\alpha}(\tau)$  for  $\alpha \neq 0$  is the ac component of the lag-product waveform  $x(t + \tau/2)x(t - \tau/2)$  for each value of  $\tau$  corresponding to sine-wave frequency  $\alpha$ .

## 4.2 CYCLIC SPECTRUM

From the *Wiener relation*, the conventional power spectrum density is obtained by

$$S_x(f) = \mathcal{F} \{ R_x(\tau) \} = \int_{-\infty}^{\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau. \quad (6)$$

---

<sup>3</sup>This means that ensemble averages equal time-averages (using synchronized averaging) of a cyclostationary time-series signal (see [17])

Similarly, it can be shown that

$$S_x^\alpha(f) = \mathcal{F} \{R_x^\alpha(\tau)\} = \int_{-\infty}^{\infty} R_x^\alpha(\tau) e^{-j2\pi f\tau} d\tau, \quad (7)$$

which is called the *cyclic Wiener relation*. It is easily seen that (6) is a particular case of (7), i.e., when  $\alpha = 0$ .  $S_x^\alpha(f)$  found in (7) is called the *cyclic spectral density function* or the *spectral correlation function*. For brevity, it will be referred to as *cyclic spectrum*. Let's define

$$u(t) = x(t)e^{-j\pi\alpha t}, \quad (8)$$

and

$$v(t) = x(t)e^{+j\pi\alpha t}. \quad (9)$$

Recall from the Fourier transforms properties that multiplying a signal by  $e^{\pm j\pi\alpha t}$  shifts its spectral content by  $\pm\alpha/2$ . We can therefore express  $R_x^\alpha(\tau)$  in terms of a conventional cross-correlation, i.e.,

$$R_{uv}(\tau) \triangleq \langle u(t + \tau/2)v^*(t - \tau/2) \rangle = R_x^\alpha(\tau). \quad (10)$$

From this definition, one can state that  $x(t)$  exhibits second-order periodicity if and only if frequency-shifted versions of  $x(t)$ , namely  $u(t)$  and  $v(t)$ , are correlated with each other. This will be the case if and only if  $R_x^\alpha(\tau)$  is not identically zero for some  $\alpha \neq 0$ . From (10),  $R_x^\alpha(\tau)$  can be normalized to give the *temporal correlation coefficient*  $\gamma_x^\alpha(\tau)$  which provides a measure of the strength of correlation between  $u(t)$  and  $v(t)$  and is defined as

$$\gamma_x^\alpha(\tau) \triangleq \frac{R_x^\alpha(\tau)}{R_x(0)}, \quad (11)$$

where  $R_x(0)$  is given by

$$R_x(0) = (|u(t)|^2 |v(t)|^2)^{\frac{1}{2}}. \quad (12)$$

Similarly, the *spectral autocorrelation coefficient* is given by

$$\rho_x^\alpha(f) \triangleq \frac{S_{uv}(f)}{[S_u(f)S_v(f)]^{\frac{1}{2}}}, \quad (13)$$

where  $S_{uv}(f) = S_x^\alpha(f)$ ,  $S_u(f) = S_x(f + \alpha/2)$ , and  $S_v(f) = S_x(f - \alpha/2)$ . Clearly,  $\rho_x^\alpha(f)$  can also be expressed as

$$\rho_x^\alpha(f) \triangleq \frac{S_x^\alpha(f)}{[S_x(f + \alpha/2)S_x(f - \alpha/2)]^{\frac{1}{2}}}. \quad (14)$$

From the above discussion, some conclusions can be drawn. A signal  $x(t)$  for which the autocorrelation function  $R_x(\tau)$  exists and is not identically zero is qualified as being *stationary* (more precisely wide-sense stationary). The terminology has to be refined in order to distinguish between stationary signals that exhibit cyclostationarity and those who do not. The case for which  $R_x^\alpha(\tau) = 0$  is said to be *purely stationary* (of second-order) whereas when  $R_x^\alpha(\tau) \neq 0$ , it is said to be *cyclostationary* (of second-order). Any  $\alpha$  for which  $R_x^\alpha(\tau) \neq 0$  is called a *cycle frequency* and the special condition  $\alpha = 0$  can be considered as a *degenerate cycle frequency* ( $e^{j2\pi\alpha t} = e^0 = 1 \rightarrow$  degenerate sinusoid). Thus, stationary signals can also be cyclostationary if it contains a cycle frequency other than zero. The discrete set of cycle frequencies is called the *cycle spectrum*.

### 4.3 DEGREE OF CYCLOSTATIONARITY

An important measure of the strength of cyclostationarity of a signal  $x(t)$  is denoted DCS (degree of cyclostationarity). Two measures of DCS, one which is *frequency-decomposed* and one which is *time-decomposed* are defined respectively as

$$DCS_f^\alpha \triangleq |\rho_x^\alpha(f)|^2, \quad (15)$$

and

$$DCS_\tau^\alpha \triangleq |\gamma_x^\alpha(\tau)|^2. \quad (16)$$

Also, a third measure of DCS known as *cycle-frequency-decomposed* measure of DCS is defined as

$$DCS^\alpha \triangleq \frac{\int_{-\infty}^{\infty} |S_x^\alpha(f)|^2 df}{\int_{-\infty}^{\infty} |S_x^0(f)|^2 df} = \frac{\int_{-\infty}^{\infty} |R_x^\alpha(\tau)|^2 d\tau}{\int_{-\infty}^{\infty} |R_x^0(\tau)|^2 d\tau}. \quad (17)$$

Moreover, this latter measure can also be expressed as

$$DCS^\alpha = \frac{\int_{-\infty}^{\infty} DCS_\tau^\alpha d\tau}{\int_{-\infty}^{\infty} |\gamma_x^0(\tau)|^2 d\tau} = \frac{\int_{-\infty}^{\infty} DCS_f^\alpha S_x^0(f + \alpha/2) S_x^0(f - \alpha/2) df}{\int_{-\infty}^{\infty} |S_x^0(f)|^2 df}. \quad (18)$$

Finally, from the cycle-frequency-decomposed measure of DCS, one can find the degree of cyclostationarity, i.e.,

$$DCS = \sum_{\alpha \neq 0} DCS^\alpha = \sum_{\alpha \neq 0} \left[ \frac{\int_{-\infty}^{\infty} |S_x^\alpha(f)|^2 df}{\int_{-\infty}^{\infty} |S_x^0(f)|^2 df} \right] = \sum_{\alpha \neq 0} \left[ \frac{\int_{-\infty}^{\infty} |R_x^\alpha(\tau)|^2 d\tau}{\int_{-\infty}^{\infty} |R_x^0(\tau)|^2 d\tau} \right], \quad (19)$$

or

$$DCS = \frac{\sum_{\alpha \neq 0} \left[ \int_{-\infty}^{\infty} |S_x^\alpha(f)|^2 df \right]}{\int_{-\infty}^{\infty} |S_x^0(f)|^2 df} = \frac{\sum_{\alpha \neq 0} \left[ \int_{-\infty}^{\infty} |R_x^\alpha(\tau)|^2 d\tau \right]}{\int_{-\infty}^{\infty} |R_x^0(\tau)|^2 d\tau}. \quad (20)$$

It is easily observed that if  $DCS = 0$ , then the signal is purely stationary but if the process exhibits cyclostationarity, then  $DCS > 0$ .

## 5.0 EFFECTS OF BASIC SIGNAL PROCESSING OPERATIONS ON THE CYCLIC SPECTRUM

Three different operations are considered, i.e., filtering, multiplication, and time sampling. The results come from [17], [5], and [10] and are reformulated in the following.

### 5.1 FILTERING

Let  $z(t)$  be defined as

$$z(t) = h(t) * x(t) \triangleq \int_{-\infty}^{\infty} h(u)x(t-u)du, \quad (21)$$

where  $x(t)$  is the incoming signal and  $h(t)$  is the impulse response of a linear time-invariant (LTI) filter. The well-known effect of filtering on the conventional spectrum of  $x(t)$  is expressed as

$$S_z(f) = |H(f)|^2 S_x(f). \quad (22)$$

The extension to the input-output cyclic spectrum relation for LTI filtering is given by

$$S_z^\alpha(f) = H(f + \alpha/2)H^*(f - \alpha/2)S_x^\alpha(f), \quad (23)$$

where  $H(f)$  is the Fourier transform of  $h(t)$ . Note that this latter expression includes (22) as a special case.

Now, let's assume that  $h(t)$  is replaced by  $h(t, u)$  to denote a periodic time-variant filter impulse response. The impulse response of the output is expressed as

$$z(t) = \int_{-\infty}^{\infty} h(t, u)x(u)du. \quad (24)$$

But since  $h(t, u)$  is a periodic impulse response function, then it can be expanded into the Fourier series

$$h(t + \tau, t) = \sum_{\beta \in A} g_\beta(\tau)e^{j2\pi\beta t}, \quad (25)$$

where  $g_\beta(\tau)$  is given by

$$g_\beta(\tau) = \langle h(t + \tau, t) e^{-j2\pi\beta t} \rangle \quad (26)$$

and  $A$  is the set of sinusoid frequencies associated with the product modulators in the system representation. Thus, the filter output  $z(t)$  is given by

$$z(t) = \sum_{\beta \in A} [x(t) e^{j2\pi\beta t}] * g_\beta(t), \quad (27)$$

and the input-output cyclic spectrum relation for a periodic time-variant filtering is found to be

$$S_z^\alpha(f) = \sum_{\beta, \gamma \in A} G_\beta(f + \alpha/2) G_\gamma^*(f - \alpha/2) S_x^{\alpha - \beta + \gamma} \left( f - \frac{\beta + \gamma}{2} \right). \quad (28)$$

## 5.2 MULTIPLICATION

Another common signal processing operation is the multiplication (or mixing) of two waveforms. Assuming two statistically independent time-series  $x(t)$  and  $s(t)$ , then

$$z(t) = x(t)s(t) \quad (29)$$

is the product of the two waveforms. It is found that the input-output cyclic spectrum relation for waveform multiplication is [22]

$$S_z^\alpha(f) = \int_{-\infty}^{\infty} \sum_{\beta} S_x^\beta(v) S_s^{\alpha - \beta}(f - v) dv. \quad (30)$$

## 5.3 TIME SAMPLING

Since digital processors are widely used, the signal  $x(t)$  is typically transformed into samples  $\{x(nT_s)\}$ . To calculate the cyclic autocorrelation function of  $x(t)$ , an asymmetric definition is required, i.e., [17]

$$\tilde{R}_x^\alpha(kT_s) \triangleq \langle x(nT_s + kT_s) x^*(nT_s) e^{-j2\pi\alpha nT_s} \rangle e^{-j\pi\alpha kT_s}, \quad (31)$$

where  $\langle \cdot \rangle$  is the discrete-time averaging over  $n$  and the correction factor  $e^{-j\pi\alpha kT_s}$  is used to make the asymmetric definition agreed with the symmetric one. From (31), the expression for the discrete cyclic spectrum is

$$\tilde{S}_x^\alpha(f) = \sum_{-\infty}^{\infty} \tilde{R}_x^\alpha(kT_s) e^{-j2\pi kT_s f}. \quad (32)$$

## 6.0 APPLICATIONS OF THE CYCLOSTATIONARY MODEL

When a signal  $x(t)$  is found to be cyclostationary, it reveals the existence of correlation between widely separated spectral components (with separation equal to  $\alpha$ ). This is called *spectral redundancy*.

The objective of all cyclic spectral analysis applications is to exploit the redundancy of a signal to improve the accuracy and reliability of information extracted from the measurements of corrupted signals. The degree of enhancement obtained in comparison to performance of more commonly chosen methods (which ignores this spectral redundancy) is dependant on the severity of the signal corruption as well as on the degree of cyclostationarity (DCS) exhibited by the signal  $x(t)$ . As discussed briefly in section 3.1, spectral redundancy is not usually observed in noise (noise is not cyclostationary) but does appear in most manmade signals. In addition, in cases where there are multiple signals of interest plus interferers (signals not of interest), overlap may occur in both time and frequency, but it is very likely that the spectral redundancy functions of those signals do not overlap providing they have distinct cycle frequencies  $\alpha$ . Thus, signals occupying the same spectral band but having different carrier frequencies and/or pulse rates or keying rates can be distinguished. In brief, this makes signal selectivity possible, a very basic feature of spectral redundancy.

In a practical situation, a signal can be modeled as being

$$x(t) = \sum_{i=1}^I s_i(t) + n(t), \quad (33)$$

where the set  $\{s_i(t), i = 1, \dots, I\}$  is formed by both signals of interest and interference (statistically independent of each other) and where  $n(t)$  is the background noise. The cyclic spectrum of  $x(t)$  is then given by

$$S_x^\alpha(f) = \sum_{i=1}^I S_{s_i}^\alpha(f) + S_n^\alpha(f). \quad (34)$$

However, if the only signal having the particular cycle frequency  $\alpha_k$  is  $s_k(t)$ , then

$$S_x^{\alpha_k} = S_{s_k}^{\alpha_k}(f) \quad (35)$$

assuming  $\Delta t \rightarrow \infty$  (measurement time).

In this section, some applications of the cyclostationary model will be analyzed.

## 6.1 DETECTION

One of the most difficult tasks in communications systems, particularly in tactical radio communications where the environment is highly corrupted by noise and interference, is to detect the presence of one or more signals of interest. The cyclic spectrum permits signal detection in this type of environment. Because of its signal selectivity capability, the spectral-correlation plane approach as a general approach to interception gives great flexibility and more importantly tolerates unknown and changing noise level and interference activity. As shown in (35), provided  $n(t)$  is not cyclostationary and all interferers do not exhibit cyclostationarity at  $\alpha_k$ , it is possible to detect the presence of a signal at that particular cycle frequency. At the same time, classifying the modulation type of this particular signal type will be a lot easier since all noise and interference will have been removed from the signal.

Assuming the incoming signal  $x(t)$  is modeled as in (33), a list of all cycle frequencies  $\alpha_k$  where  $S_x^{\alpha_k}(f) \neq 0$  (defined earlier as the cycle spectrum) along with a value *representative*<sup>4</sup> of  $S_x^{\alpha_k}(f)$  should be recorded. A threshold may be used in order to circumvent the problem of false detection due to measurement noise.

## 6.2 CLASSIFICATION

The problems of tolerance to noise and interference have long been encountered in modulation recognition systems. Some of these systems are commercialized and show good results for signal to noise ratios down to 20 dB but this is not sufficient in a typical environment where lower signal to noise ratios are often encountered and automatic modulation recognition is still required. After having detected the signals of interest (and cycle spectrum) as explained in section 5.1, classification according to modulation type becomes a matter of applying and/or developing cyclic spectrum modulation pattern recognition rules to almost *clean* signals. Also, the number of signals will be found at the same time. This new approach has not been well explored yet but looks promising.

To this date, a great amount of work has been carried out to find the theoretical cyclic spectrum expected for a number of different modulation types (see [17], [15], and [16]). The results are briefly stated here.

- *Amplitude Modulation (AM)*:

$$x(t) = a(t) \cos(2\pi f_0 t + \phi_0) \quad (36)$$

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<sup>4</sup>A value representative of  $S_x^{\alpha_k}(f)$  means either the maximum value or the integral of the cyclic spectrum at the particular cycle frequency  $\alpha_k$ .

$$S_x^\alpha(f) = \begin{cases} \frac{1}{4}S_a(f+f_0) + \frac{1}{4}S_a(f-f_0) & , \alpha = 0 \\ \frac{1}{4}S_a(f)e^{\pm j2\phi_0} & , \alpha = \pm 2f_0 \\ 0 & , \text{otherwise} \end{cases} \quad (37)$$

• *Pulse Amplitude Modulation (PAM):*

$$x(t) = \sum_{-\infty}^{\infty} a(nT_0)p(t - nT_0 + \epsilon) \quad (38)$$

$$S_x^\alpha(f) = \begin{cases} \frac{1}{T_0}P(f + \alpha/2)P^*(f - \alpha/2) \cdot \left[ \sum_{n,m=-\infty}^{\infty} S_a^{\alpha + \frac{m}{T_0}} \left( f - \frac{n}{T_0} - \frac{m}{2T_0} \right) \right] e^{j2\pi\alpha\epsilon} & , \alpha = \frac{k}{T_0} \\ 0 & , \text{otherwise} \end{cases} \quad (39)$$

where  $p(t)$  is a deterministic finite-energy pulse and  $\epsilon$  is a fixed pulse-timing phase parameter. Also note that  $\{a(nT_0)\}_n$  is a random sequence assumed to be purely stationary and  $k$  is an integer value.

• *Quadrature Amplitude Modulation (QAM):*

$$\begin{aligned} x(t) &= a(t) \cos(2\pi f_0 t + \phi(t)) \\ &= cs(t) \cos(2\pi f_0 t) + sn(t) \sin(2\pi f_0 t) \end{aligned} \quad (40)$$

$$S_x^\alpha(f) = \begin{cases} \frac{1}{4}[S_{cs}(f) - S_{sn}(f)] \pm \frac{1}{2}jS_{cs,sn}(f)_r & , \alpha = \pm 2f_0 \\ 0 & , \alpha \neq 0, \alpha \neq \pm 2f_0 \end{cases} \quad (41)$$

where  $S_{cs,sn}(f)_r$  is the real part of the cross-spectrum between  $cs(t)$  and  $sn(t)$ .

• *Phase Modulation (PM) and Frequency Modulation (FM):*

$$x(t) = \cos(2\pi f_0 t + \phi(t)) \quad (42)$$

$$R_x^\alpha(\tau) = \begin{cases} \frac{1}{2}\Re \{ \Psi_\tau(1, -1)e^{j2\pi f_0 \tau} \} & , \alpha = 0 \\ \frac{1}{4}\Psi_\tau(1, 1) & , \alpha = 2f_0 \\ \frac{1}{4}\Psi_\tau^*(1, 1) & , \alpha = -2f_0 \\ 0 & , \alpha \neq 2f_0, \alpha \neq 0 \end{cases} \quad (43)$$

where  $\Psi_\tau(\omega_1, \omega_2)$  is the joint characteristic function for  $\phi(t + \tau/2)$  and  $\phi(t - \tau/2)$  defined as

$$\Psi_\tau(\omega_1, \omega_2) \triangleq \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} e^{j[\phi(t+\tau/2)\omega_1 + \phi(t-\tau/2)\omega_2]} dt \quad (44)$$

Using the cyclic Wiener relation in (7), one can find the expression for  $S_x^\alpha(f)$ . Note that

$\phi(t)$  is assumed to contain no periodicity. The difference between PM and FM can be found when making assumptions about the type of incoming signal. For instance, if for a PM signal  $\phi(t) = y(t)$  (typical of speech, zero-mean purely stationary Gaussian waveform) where  $y(t)$  has the spectral density given by

$$S_y(f) = \begin{cases} S_0 & , 300 \leq |f| \leq 3300 \\ 0 & , \text{otherwise} \end{cases} \quad (45)$$

then for a FM signal,  $\phi(t)$  is given by

$$\phi(t) = \int_0^t e^{-\beta(t-u)} y(u) du, \quad 0 < \beta \ll 300. \quad (46)$$

• *Digital Pulse Modulation:*

$$x(t) = \sum_{n=-\infty}^{\infty} \sum_{m=1}^M \delta_m(n) p_m(t - nT_0) \quad (47)$$

Defining  $S_1$  and  $S_2$  as

$$\begin{aligned} S_1 &= \frac{1}{MT_0} \sum_{m=1}^M P_m(f + \alpha/2) P_m^*(f - \alpha/2) \\ S_2 &= \frac{1}{M^2 T_0} \left[ \sum_{m=1}^M P_m(f + \alpha/2) \right] \cdot \left[ \sum_{n=1}^M P_n^*(f - \alpha/2) \right] \\ &\quad \cdot \left[ 1 - \frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(f + \frac{\alpha}{2} - \frac{n}{T_0}\right) \right], \end{aligned}$$

then

$$S_x^\alpha = \begin{cases} S_1 - S_2 & , \alpha = \frac{k}{T_0} \\ 0 & , \alpha \neq 0 \text{ and } \alpha \neq \frac{k}{T_0}. \end{cases} \quad (48)$$

• *Amplitude Shift Keying (ASK):*

$$x(t) = a(t) \cos(2\pi f_0 t + \phi_0) \quad (49)$$

where

$$a(t) = \sum_{n=-\infty}^{\infty} a_n p(t - nT_0 - t_0) \quad (50)$$

and

$$\begin{aligned}
S_x^\alpha(f) = & \frac{1}{4T_0} \left\{ \left[ P\left(f + f_0 + \frac{\alpha}{2}\right)P^*\left(f + f_0 - \frac{\alpha}{2}\right)\tilde{S}_a^\alpha(f + f_0) \right. \right. \\
& + \left. P\left(f - f_0 + \frac{\alpha}{2}\right)P^*\left(f - f_0 - \frac{\alpha}{2}\right)\tilde{S}_a^\alpha(f - f_0) \right] e^{-j2\pi\alpha t_0} \\
& + P\left(f + \frac{\alpha}{2} + f_0\right)P^*\left(f - \frac{\alpha}{2} - f_0\right)\tilde{S}_a^{\alpha+2f_0}(f)e^{-j[2\pi(\alpha+2f_0)t_0+2\phi_0]} \\
& \left. + P\left(f + \frac{\alpha}{2} - f_0\right)P^*\left(f - \frac{\alpha}{2} + f_0\right)\tilde{S}_a^{\alpha-2f_0}(f)e^{-j[2\pi(\alpha-2f_0)t_0-2\phi_0]} \right\} \quad (51)
\end{aligned}$$

where  $\tilde{S}_a^\alpha(f)$  is the discrete-time correlation of  $a(t)$ .

Other cyclic spectrum expressions for modulation types such as *Phase Shift Keying* (PSK), *non-coherent* and *coherent Frequency Shift Keying* (FSK), *Frequency-Hopped FSK*, *Direct Sequence PSK*, and others can be found in [17] and [10]. More experiments are required to analyze the cyclic spectrum of other modulation types to gain a better knowledge of the best way to classify them. This will be something to investigate in the near future (using simulated modulated signals as well as real off air data).

### 6.3 PARAMETER ESTIMATION

The carrier frequencies and phases, keying rates and phases, or pulse rates and phases are found almost directly from the cyclic spectrum once the signals have been detected and classified. For instance, almost all modulation types having a carrier frequency  $f_c$  generate a spectral line at a cycle frequency  $\alpha = 2f_c$ . Also, the keying rate or pulse rate is found when the cyclic spectrum exhibits spectral lines at multiples of  $f_0$ , i.e., at  $\alpha = \frac{k}{T_0}$  where  $k$  is an integer. Other parameters such as bandwidth are quite easy to estimate once the cycle frequency of the signal and the modulation type are known and the signal spectrum has been extracted from background noise and interference. Finally, synchronization parameters such as frequencies and phases can be estimated as well.

### 6.4 TDOA ESTIMATION

The problem of resolving time-difference-of-arrival (TDOA) of overlapping signals is eliminated with the utilization of the cyclic spectrum, again because of its signal selectivity property in the  $\alpha$  domain. Denoting TDOA's as  $\{t_i\}$ , the incoming signal at one reception platform as  $x(t)$ , the incoming signal from the other platform as  $w(t)$ , then  $x(t)$  is

modeled as in (33) and  $w(t)$  is given by

$$w(t) = \sum_{i=1}^I s_i(t - t_i) + m(t) \quad (52)$$

where  $m(t)$  is the background noise received at the second platform. Provided  $s_i(t)$  is the only signal having the cycle frequency  $\alpha$ , then  $t_i$  is found to be an accurate estimate of the TDOA in the expression

$$S_{wx}^\alpha(f) = S_{s_i}(f) e^{-j2\pi(f+\alpha/2)t_i} \quad (53)$$

Here, the problem of resolving the TDOA's of overlapping signals has been circumvented (cf [9] and [10]).

## 6.5 SPATIAL FILTERING

An other important application of the cyclic spectral analysis concerns spatial filtering. It is found in [1] that it is possible to blindly and adaptively extract a signal of interest with only knowledge of the cycle frequencies  $\alpha$  of the signals.

With enough elements in an array to make the nulling possible, a linear combiner will restore spectral redundancy in its input at a particular cycle frequency  $\alpha$  and it will adapt to null out all other signals not having that cycle frequency. This will increase the degree of cyclostationarity at the output. Therefore, the system will null out all signals other than  $s_i(i)$  at the output.

An algorithm developed by Dr. Agee (see [1] and [2]) is called the *Spectral Self-Coherence Restoral (SCORE) algorithm*. The greatest advantage of this technique is that it accomplishes the nulling without any knowledge of the reception characteristics of the array (no calibration).

## 6.6 DIRECTION FINDING

For direction finding purposes, a narrowband model is considered, i.e.,

$$\mathbf{x}(t) = \sum_{i=1}^I \mathbf{a}(\theta_i) s_i(t) + \mathbf{n}(t) \quad (54)$$

where  $\mathbf{x}$  is the received data vector of dimension  $r$ , and  $\mathbf{a}(\theta_i)$  is the direction vector associated with the  $i^{\text{th}}$  received signal  $s_i(t)$  specified by the calibration data for the array.

The particular interest in using

$$R_x^\alpha(\tau) = R_{s_k}^\alpha(\tau) = \mathbf{a}(\theta_k) R_{s_k}^\alpha(\tau) \mathbf{a}^H(\theta_k), \quad (55)$$

which is the  $r \times r$  cyclic autocorrelation matrix, to solve  $\mathbf{a}(\theta_i)$  is that unlike the conventional autocorrelation matrix, advanced knowledge of the correlation properties of the noise  $R_n(0)$  and interference  $R_{s_i}(0)$  for  $i \neq k$  is not required. As a result, the constraint imposed by conventional methods that the number of elements in the array must exceed the total number of signals,  $I$ , impinging on the array is avoided. Also, better spatial resolution is achieved because the signals are resolved in  $\alpha$  instead of the direction of arrival. A number of approaches to signal selective DF have been derived so far ([40], [1], and [37]).

## 6.7 FREQUENCY-SHIFT FILTERING FOR SIGNAL EXTRACTION

A means for filtering a signal based on the cyclic spectral correlation function is called *Frequency-Shift Filtering* (FRESH filtering). This technique allows separation of  $I$  spectrally overlapping signals if they have the same keying rate but different phases or carrier frequencies. In fact, it is a time-variant filtering technique which can also be viewed as a combination of several frequency shifting and filtering operations. It is a technique well suited for adaptive co-channel interference removal and adaptive signal extraction (see [5], [17], and [7]).

A number of other applications have been considered so far [17] which are still under investigations, however, the applications described briefly above should give a general idea of the capability of the cyclostationarity model in communications systems. An important thing to recall is that a cyclic spectral analyzer provides more information than the conventional spectrum analyzer.

## 7.0 CONCLUSION

In this report, the cyclic spectral analysis as a general tool for signal spectral analysis was shown to be much superior to more conventional spectral analysis. Three major advantages have been pointed out. First, its discriminatory capability enables signal separation of spectrally overlapping signals not distinguishable in the conventional spectrum. Second, the cyclic spectrum provides more information about the signal and is therefore a richer domain for signal analysis. Third, the theory of spectral correlation is a much more complete mechanism for modeling communication signals in several applications. A

working example showing two BPSK signals with different parameters and one Gaussian noise signal has demonstrated most of these advantages.

The most important results on spectral correlation theory have also been provided but without detailed proofs. Mathematical definitions for cyclic autocorrelation function, cyclic spectrum, cycle frequency, spectral and temporal correlation coefficients, and degrees of cyclostationarity have been summarized. Subsequently, the results of filtering, multiplication, and time sampling operations on the cyclic spectrum have been formulated.

Exploitation of signals spectral redundancy to improve the accuracy and reliability of information extracted from the measurements of corrupted signals is the objective of all cyclic spectral analysis applications. Among others, detection, classification, parameter estimation, TDOA estimation, direction finding, and frequency-shift filtering for signal extraction have all been considered as very promising applications. An important topic not discussed in details in this paper concerns the problems related to measurements of spectral correlation. Clearly, computational complexity of cyclic spectral analysis far exceeds that of conventional spectral analysis. However, it is recognized in [34] and [35] that time-smoothing algorithms will result in more computationally efficient algorithms than those involving spectral smoothing especially when dealing with discrete-time signals. At least, one of these efficient algorithms called the Strip Spectral Correlation Algorithm (SSCA) will be implemented shortly and then compared to the basic spectral-smoothing algorithm given in [17] which was used to produce the figures of Section 2.

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