

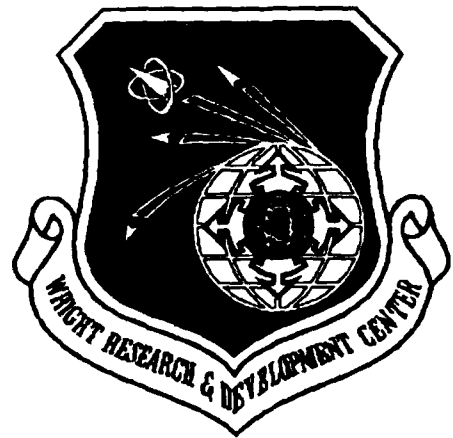
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WRDC-TR-89-2112

**AUTOMATED CODE DEMONSTRATION FOR  
COMPRESSOR BLADE STRESSES AND  
NATURAL FREQUENCIES**



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Final Report for Period November 1988 to April 1989

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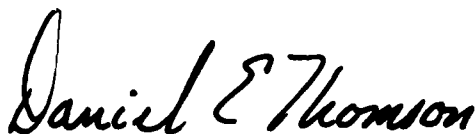


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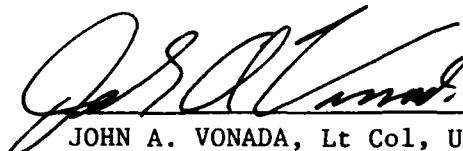
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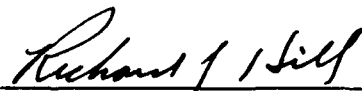


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## 1. INTRODUCTION

Accurate predictions of the stresses, natural frequencies and mode shapes of rotating blades for compressors are essential to the performance of aircraft engines. The first objective of this project is to demonstrate that a preprocessor for the modeling of compressor blades with a wedge type root attachment is feasible. A user-friendly program was devised to be used with the minimum possible input that describes the geometry of different components.

The second objective was to use this program in conjunction with BLADE to perform steady stress and natural frequency analyses for a selected example. The BLADE program, which is developed by STI, is a special purpose computer code for steam turbine blade stress and fatigue analysis. This program is sponsored by the Electric Power Research Institute (EPRI). A brief description of BLADE is included in the Appendix.

The results of these analyses were evaluated by their comparison to the ANSYS finite element program results. These results were validated using experimental data provided by the Wright Research and Development Center at WPAFB.

## 2. FINITE ELEMENT MODEL

A finite element model for the blade and disk attachment is constructed with minimum input information on the part of the user. In this section, the user input is described. An example which describes how the finite element model was generated is given.

### 2.1 User Input

The user should provide the program with some information about the following:

- ° the vane sections geometry
- ° the root features
- ° the disk geometry
- ° the blade materials
- ° forcing

The input file is a free format ASCII file. It consists of several records. Each record starts with a keyword which describes the input section for which the next set of data belongs to. The first record of the input file is an exception: it contains the title of the analysis, without being preceded by a keyword. In the following, the input file records are described:

Record 1 : **TITLE**

Record 2 : **COVER** - Keyword  
**IC** - Material reference number  
**R1** - Radius of blade tip trailing edge  
**R2** - Radius of blade tip leading edge

Record 3 : **AIRFOIL** - Keyword (Figure 2.1.1)  
**IA** - Material reference number  
**R** - Radius of airfoil section  
**YLE, ZLE** - Coordinates of leading edge  
**YTE, ZTE** - Coordinates of trailing edge

**NAP** - Number of defined airfoil points on the pressure side

**Y(1), Z(1)**  
:  
:  
**Y(2), Z(2).** } Pressure side coordinates

**Y(NAP), Z(NAP)**

**NAS** - Number of defined airfoil points on the suction side

**Y(1), Z(1)**  
:  
:  
**Y(2), Z(2)**  
**Y(NAS), Z(NAS)** } Suction side coordinates

Note: NAP and NAS do not have to be equal.

Record 3 may be repeated for different airfoil sections up to a maximum of 10 sections.

Record 4 : **PLATFORM** - Keyword (Figure 2.1.2)

**IP** - Material reference number

**RØ** - Radius to the underside of platform

**HD** - Height of platform at the downstream end

**HU** - Height of platform at the upstream end

**AL** - Axial length of platform

**CLE** - Distance from root centerline to leading edge of platform

**CTE** - Distance from root centerline to trailing edge of platform

**DC** - Distance from platform leading edge to the coordinate axis in the axial direction

**OPRET** - Offset of root centerline from the blade's global coordinate system

**RF** - Fillet radius of curvature between the airfoil and platform

**ANG** - Slant angle

**ZERO** - The number zero (this variable would be needed for curved roots)

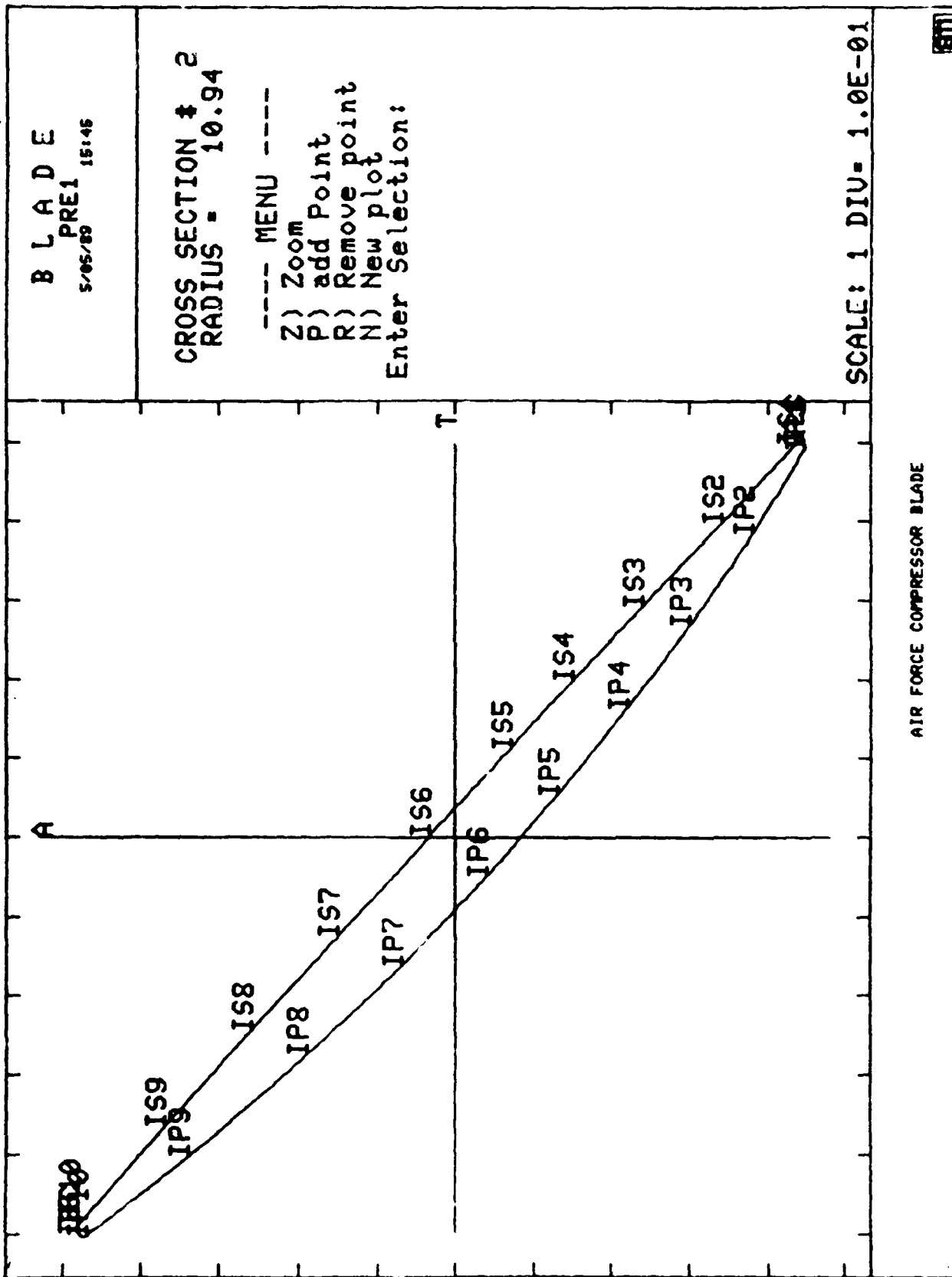


Figure 2.1.1: Typical Airfoil Section Input

Record 5 : **ROOT** - Keyword (Figure 2.1.2 and 2.1.3)

**ITYPE** - Root type number (set to 61)

**IR** - Material reference number

**X1, Y1, ...** } - Coordinates of input points

**...** }

**X8, Y8** }

**R1, R2,** - Input radii

**R3, R4**

**VS** - Stop width

**NP** - Number of points defining root end sections

**X(1), Z(1)** } - Coordinates of end section

**...** }

**X(NP), Z(NP)** }

Record 6 : **DISC** - Keyword (Figure 2.1.4)

**ID** - Material reference number

**NPD** - Number of pair of points describing the disk

**X1 (1), Z1 (1), X2 (1), Z2 (1)** } - Coordinates of disk points

**...** }

**X1(NPD), Z1(NPD), X2(NPD), Z2(NPD)** }

Record 7 : **MATERIAL** - Keyword

**I1** - Material user reference number

**IM** - Reference number from material library

**TM** - Temperature at which material is defined

Record 7 may be repeated for different material types up to a maximum of 10 material types.

Record 8 : **STAGE PARAMETERS** - Keyword

**NBLADES** - Total number of blades in the stage

**NBG** - Number of blades per group

**NBP** - Number of groups to be analyzed

**NZ** - Number of upstream nozzles

**DMPR** - Damping ratio

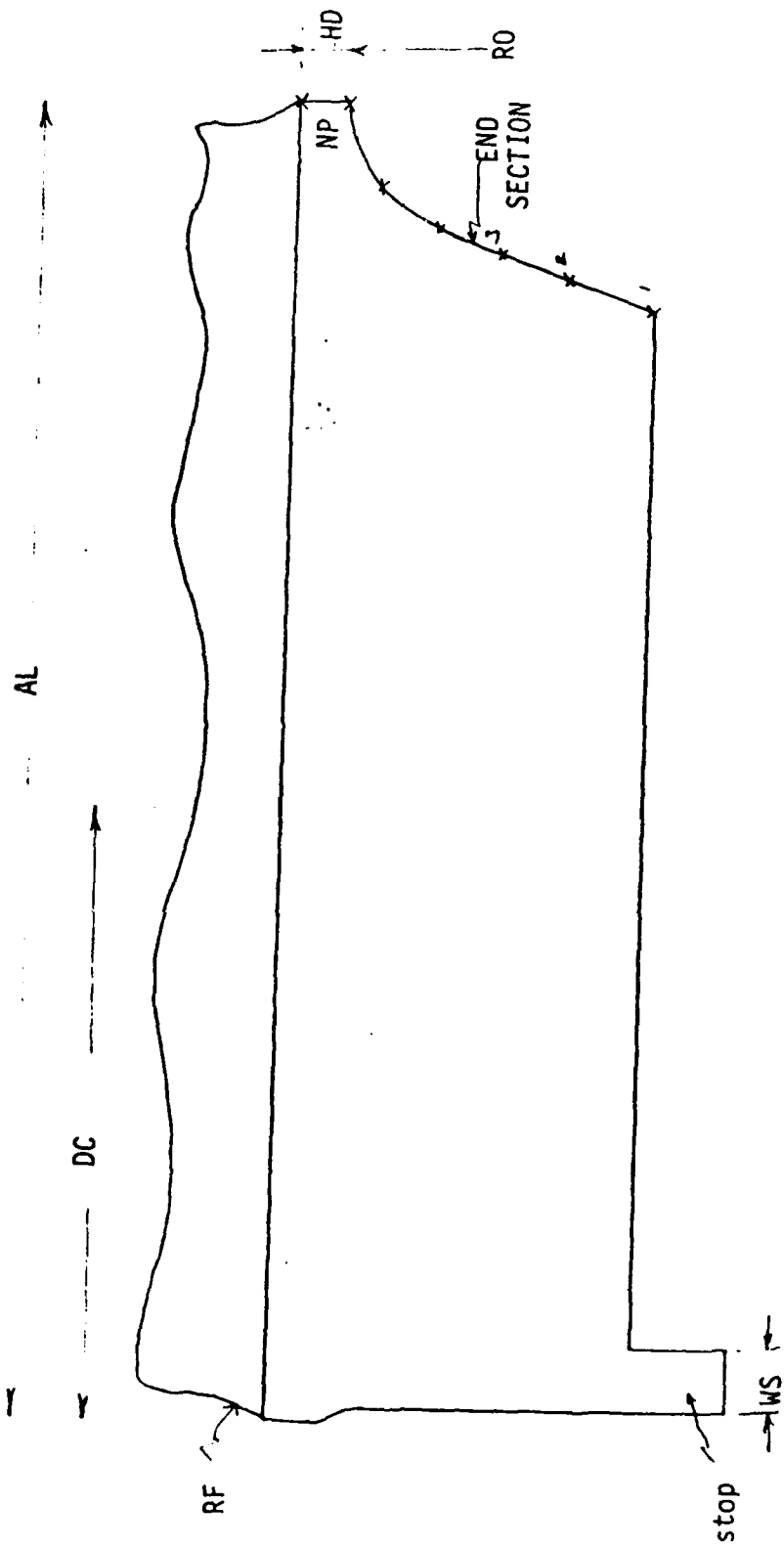


Figure 2.1.1.2: Root Side View

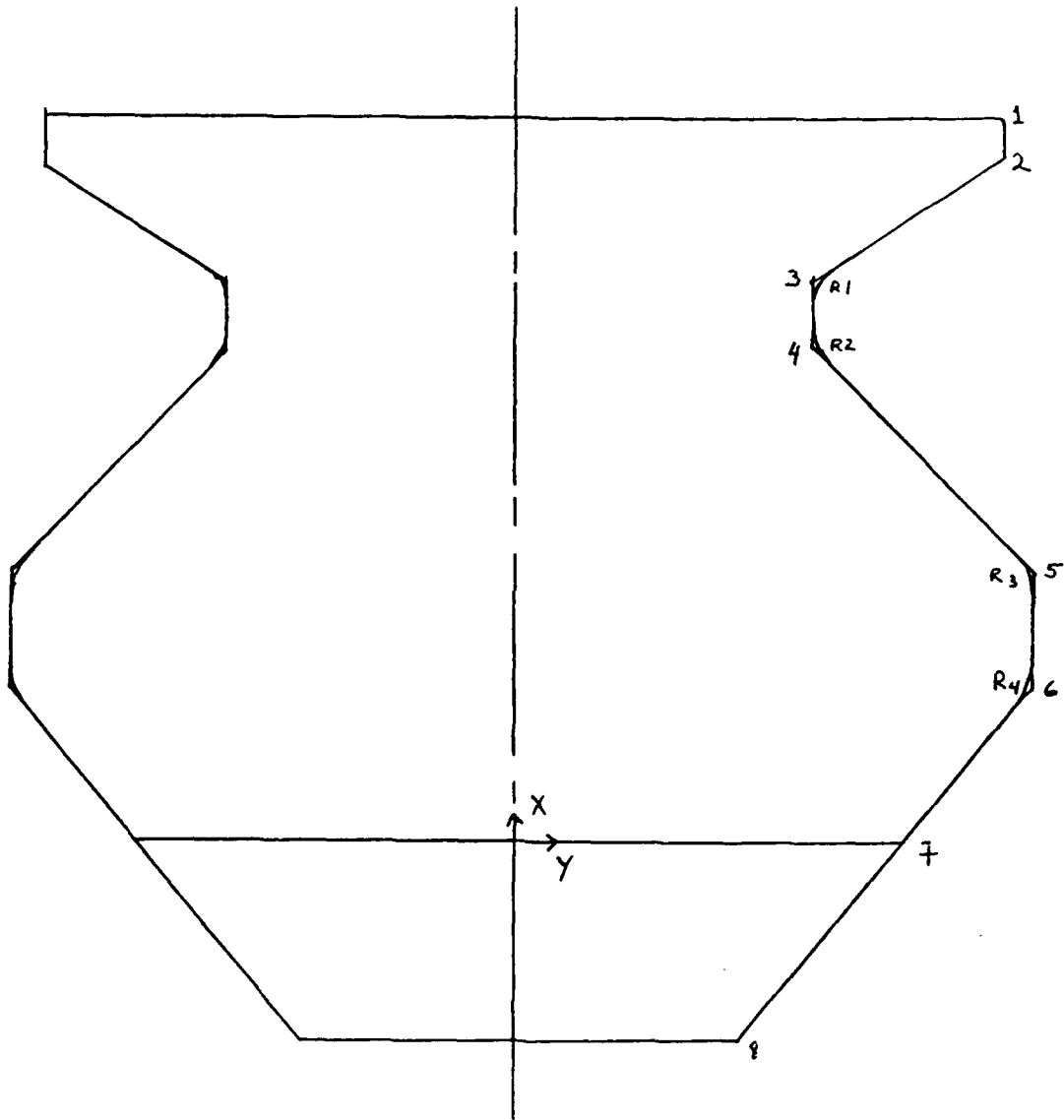


Figure 2.1.3: Root Front View

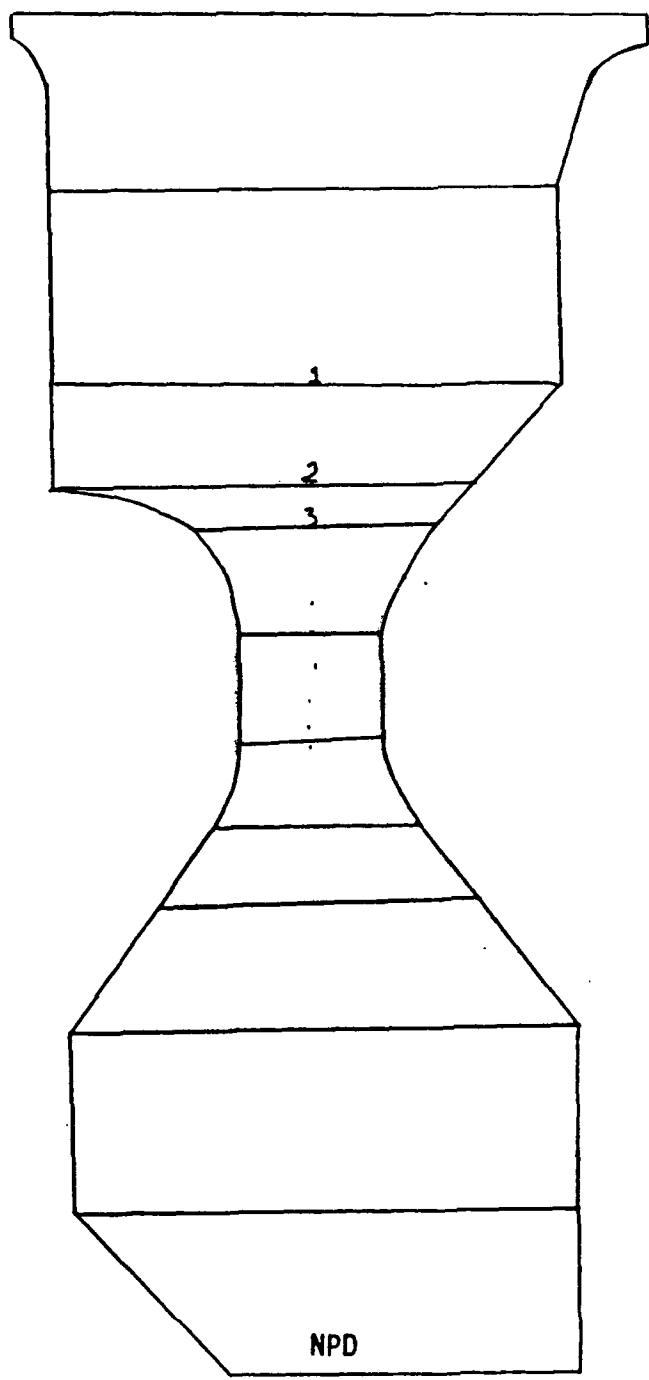


Figure 2.1.4: Disk Side View

- SPEED** - Rotor speed
- IB** - Blade of interest for back substitution
  
- Record 9 : **FORCING** - Keyword
  - C** -
  - A1** - If C = 0.0, tangential stream forces at hub and
  - A2** Tip, respectively: otherwise, power of the
  - stage and pressure drop, respectively
  - A3** - If C = 0.0, axial stream forces at hub and tip,
  - A4** respectively
  
- Record 10: **EXCITATION** - Keyword
  - NF** - Number of frequencies to be analyzed for dynamic stresses
  - FREQ (1)** } - First frequency and stimulus ratios in the
  - ST(1), SA(1)** } Tangential and axial direction
  
  - FREQ (NF)** } - Last frequency and stimulus ratios
  - ST (nf), SA (nf)** }
  
- Record 11: **END** - Keyword

In the case of natural frequency analysis, records 9 and 10 could be eliminated. They can also be eliminated in the case of steady stress analysis under the effect of centrifugal forces only.

## 2.2 Example

The blade to be analyzed is the 4th compressor stage of the F100 engine. A typical airfoil profile is given in Figure 2.1.1. Root section information is presented in Figures 2.2.1 and 2.2.2. Figure 2.2.3 shows the disk data and the input file is given in Table 2.2.1.

The finite element model was generated using the preprocessor written by STI for this specific purpose. The finite element meshes for the blade as

well as for the different components are given in Figures 2.2.4 through 2.2.10. Table 2.2.2 gives the number of eight-noded isoparametric finite elements used in modeling each component.

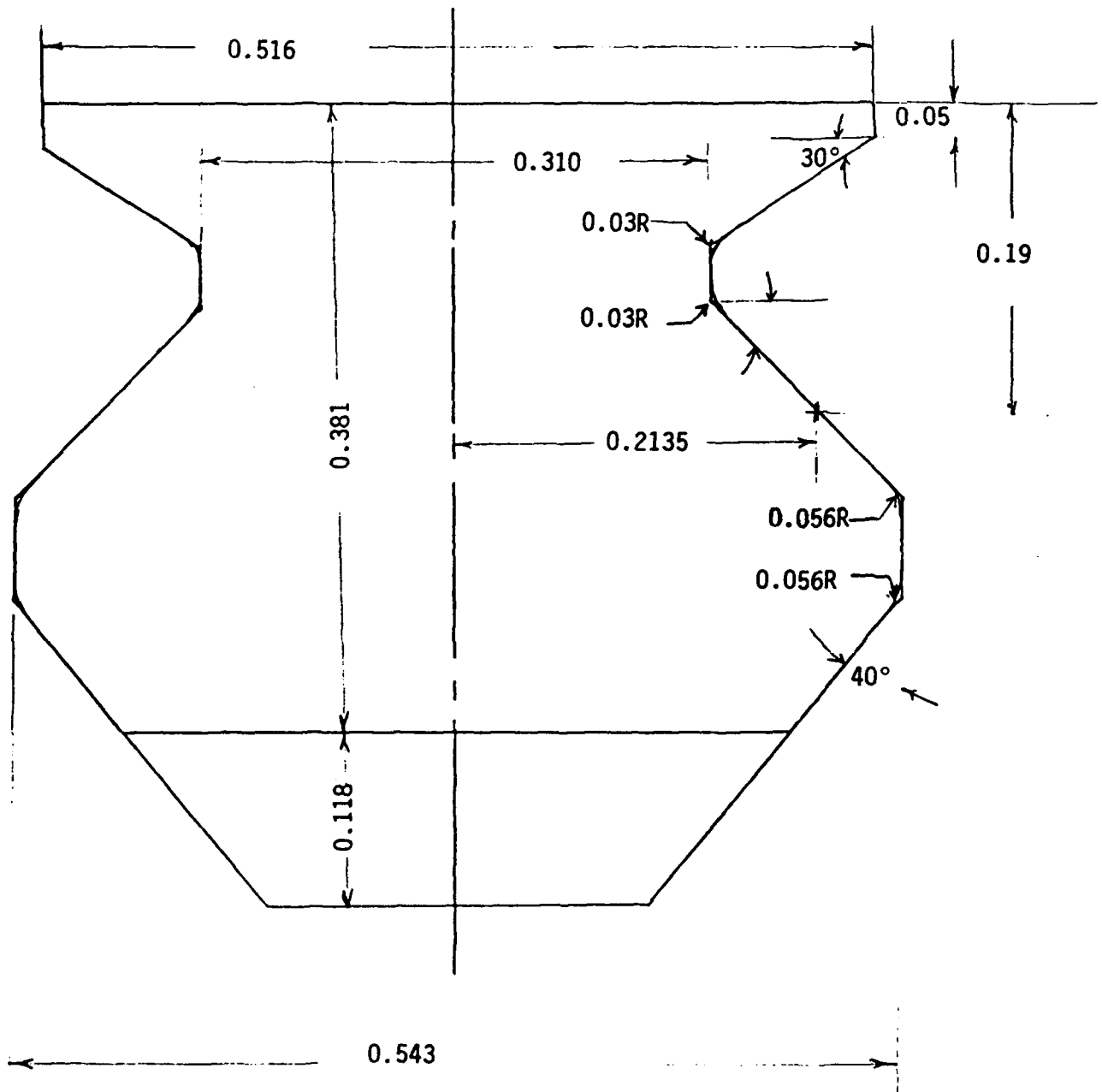


Figure 2.2.1: Root Section Front View

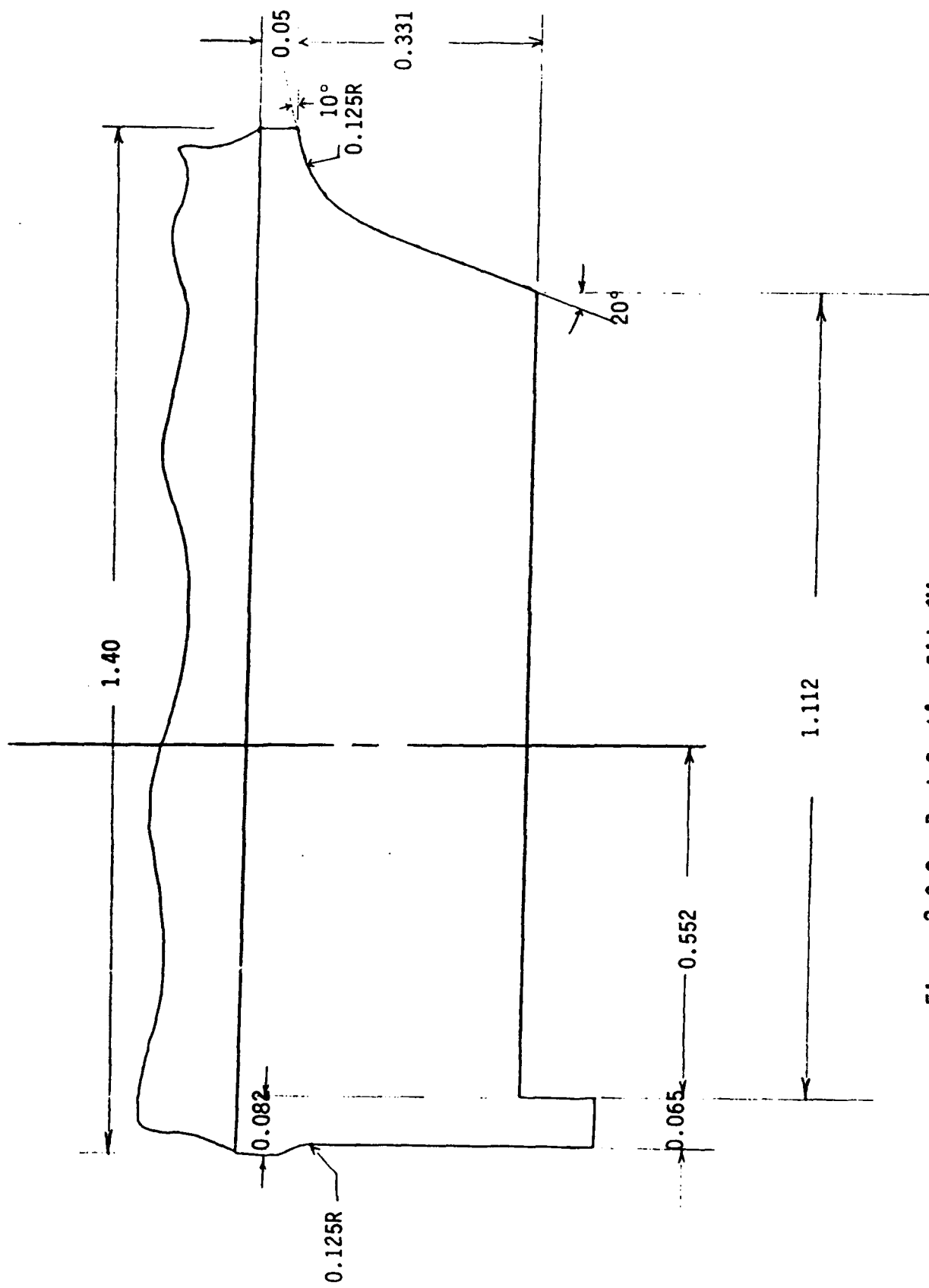


Figure 2.2.2: Root Section Side View

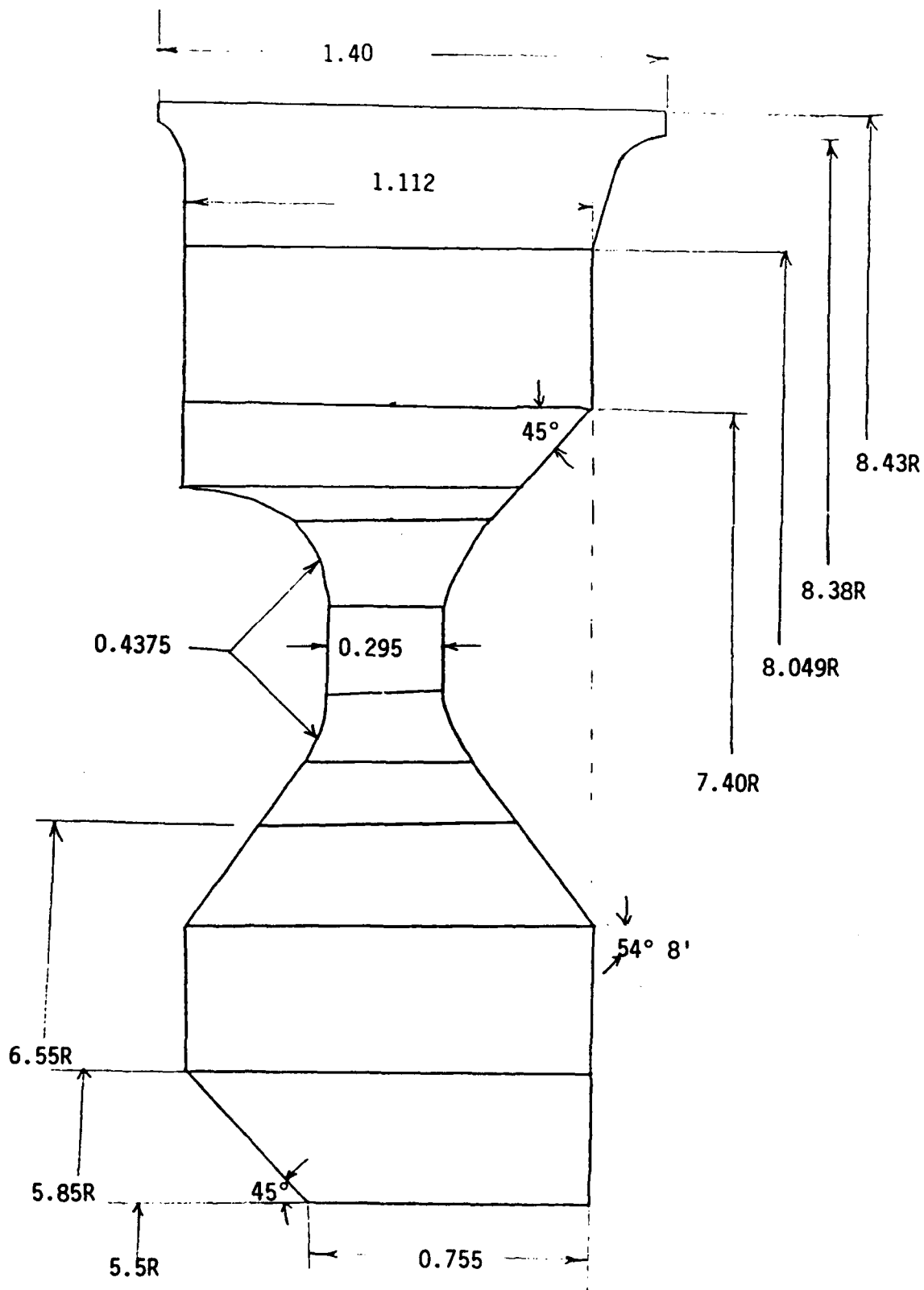


Figure 2.2.3: Disk Data

Table 2.2.1: Example Input File

AIR FORCE COMPRESSOR BLADE

COVER

0

11.639 11.729

AIRFOIL

1

11.292

-0.502 -0.394

+0.510 0.437

10

-0.500 -0.386 -0.389 -0.300 -0.286 -0.220 -0.172 -0.130

-0.072 -0.050 +0.025 0.030 +0.145 0.130 +0.264 0.230

+0.380 0.330 +0.501 0.437

10

-0.495 -0.397 -0.379 -0.330 -0.268 -0.260 -0.150 -0.180

-0.041 -0.100 +0.070 -0.010 +0.174 0.080 +0.290 0.190

+0.430 0.330 +0.509 0.430

AIRFOIL

1

10.757

-0.500 -0.445

+0.502 0.480

10

-0.499 -0.437 -0.397 -0.340 -0.292 -0.240 -0.198 -0.150

-0.112 -0.070 0.000 0.034 +0.125 0.150 +0.245 0.260

+0.366 0.370 +0.490 0.480

10

-0.491 -0.447 -0.383 -0.380 -0.268 -0.300 -0.162 -0.220

-0.052 -0.130 +0.050 -0.039 +0.163 0.070 +0.276 0.190

+0.404 0.340 +0.5025 0.469

AIRFOIL

1

10.383

-0.497 -0.479

+0.487 0.517

10

-0.496 -0.469 -0.430 -0.398 -0.300 -0.259 -0.205 -0.160

-0.118 -0.070 -0.020 0.030 +0.089 0.140 +0.210 0.260

+0.332 0.380 +0.473 0.516

10

-0.486 -0.480 -0.359 -0.400 -0.240 -0.310 -0.141 -0.230

-0.039 -0.140 +0.064 -0.040 +0.184 0.090 +0.292 0.220

+0.397 0.360 +0.488 0.500

Table 2.2.1: Example Input File (Con't)

AIRFOIL

1

9.848							
-0.492	-0.524						
+0.451	0.579						
10							
-0.490	-0.512	-0.430	-0.440	-0.330	-0.320	-0.230	-0.200
-0.130	-0.080	-0.026	0.044	+0.090	0.180	+0.220	0.330
+0.320	0.445	+0.435	0.577				
10							
-0.480	-0.521	-0.360	-0.449	-0.244	-0.360	-0.140	-0.270
-0.030	-0.163	+0.080	-0.041	+0.200	0.112	+0.310	0.280
+0.389	0.420	+0.456	0.563				

AIRFOIL

1

9.527							
-0.491	-0.546						
+0.423	0.612						
10							
-0.491	-0.533	-0.399	-0.420	-0.301	-0.300	-0.196	-0.170
-0.091	-0.040	+0.004	0.080	+0.106	0.210	+0.206	0.340
+0.325	0.500	+0.406	0.609				
10							
-0.480	-0.546	-0.360	-0.470	-0.233	-0.370	-0.120	-0.260
-0.000	-0.142	+0.107	-0.010	+0.211	0.140	+0.292	0.280
+0.370	0.440	+0.429	0.596				

AIRFOIL

1

9.046							
-0.484	-0.568						
+0.365	0.656						
10							
-0.484	-0.554	-0.370	-0.417	-0.291	-0.320	-0.188	-0.190
-0.093	-0.060	+0.004	0.080	+0.096	0.220	+0.183	0.360
+0.264	0.500	+0.345	0.650				
10							
-0.471	-0.569	-0.330	-0.480	-0.207	-0.380	-0.095	-0.270
+0.010	-0.150	+0.117	0.000	+0.208	0.160	+0.279	0.320
+0.330	0.470	+0.370	0.640				

Table 2.2.1: Example Input File (Con't)

AIRFOIL

```

1
  8.725
 -0.474 -0.588
 +0.291  0.695
10
 -0.473 -0.571 -0.377 -0.460 -0.285 -0.350 -0.190 -0.225
 -0.091 -0.080 -0.000  0.070 +0.081  0.220 +0.162  0.390
 +0.233  0.540 +0.276  0.686
10
 -0.460 -0.586 -0.335 -0.510 -0.210 -0.420 -0.105 -0.310
 +0.011 -0.170 +0.113 -0.010 +0.194  0.160 +0.257  0.350
 +0.291  0.530 +0.302  0.682
  
```

PLATFORM

```

1
  8.372  0.050  0.050  1.402  0.2764  0.2764  0.634  0  0.172
 21.0  0
  
```

ROOT

```

 61  1
 0.3925  0.2764  0.3425  0.2764  0.3030  0.1660  0.2625  0.1660
 0.1595  0.2908  0.0806  0.2908  0.0000  0.2287 -0.1065  0.1226
 0.0300  0.0300  0.0560  0.0560
 0.0820
 7
 0.0000  1.1935  0.2105  1.2701  0.2268  1.2760  0.2850  1.3105
 0.3000  1.3252  0.3425  1.4020  0.3925  1.4020
  
```

DISC

```

 1  10
 7.5950  0.5595  7.5950 -0.5520  7.4000  0.3574  7.4000 -0.5520
 7.3292  0.2796  7.3292 -0.2980  7.1200  0.1515  7.1200 -0.1435
 6.9516  0.1515  6.9516 -0.1435  6.6946  0.2351  6.6946 -0.2271
 6.5500  0.3534  6.5500 -0.3320  6.2457  0.5530  6.2457 -0.5520
 5.8500  0.5530  5.8500 -0.5520  5.5000  0.5530  5.5000 -0.2020
  
```

MATERIAL

```

1
 3707
 300
  
```

STAGE

```

 52  1  0  60
 0.002  6000  1
  
```

END

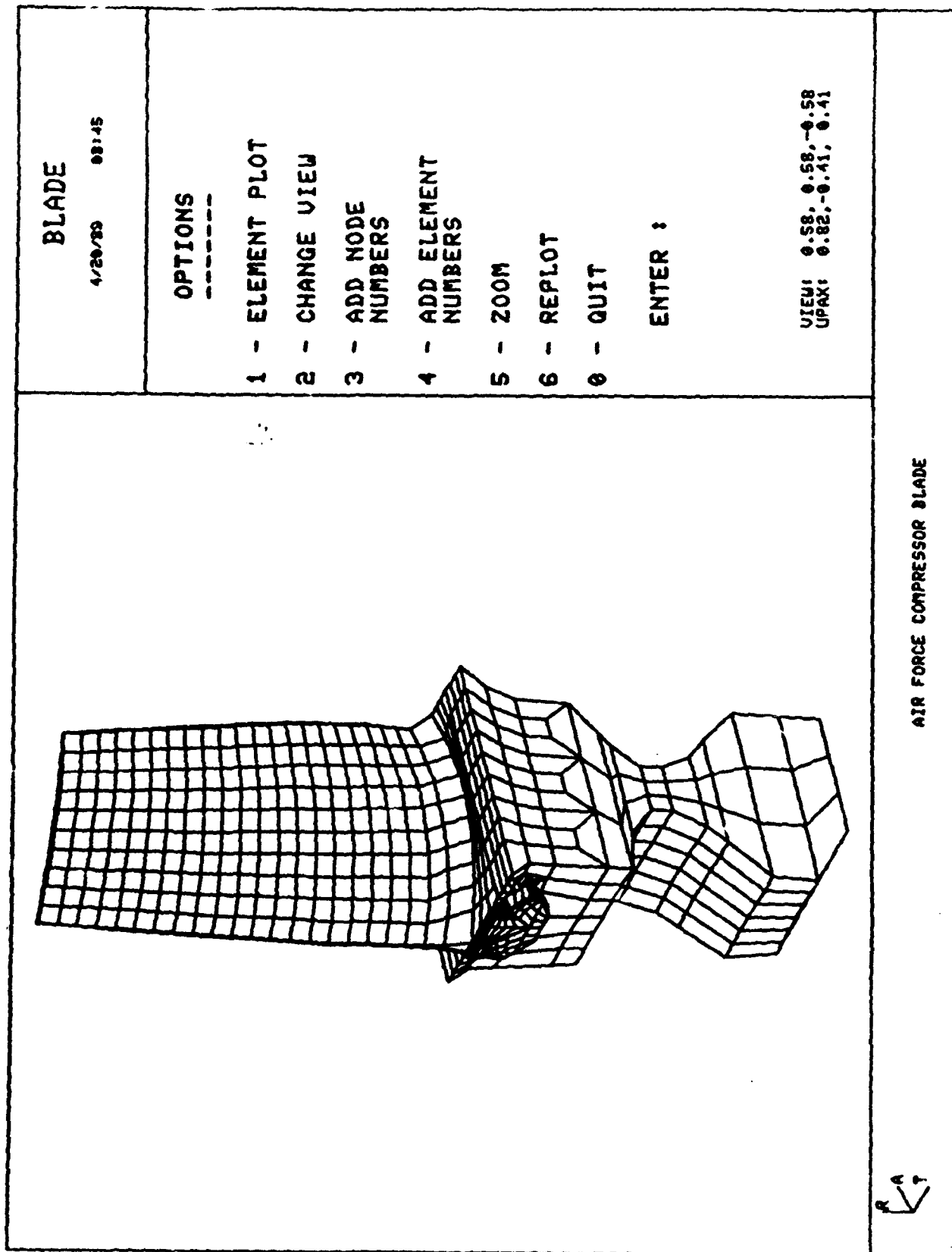


Figure 2.2.4: Example Finite Element Model

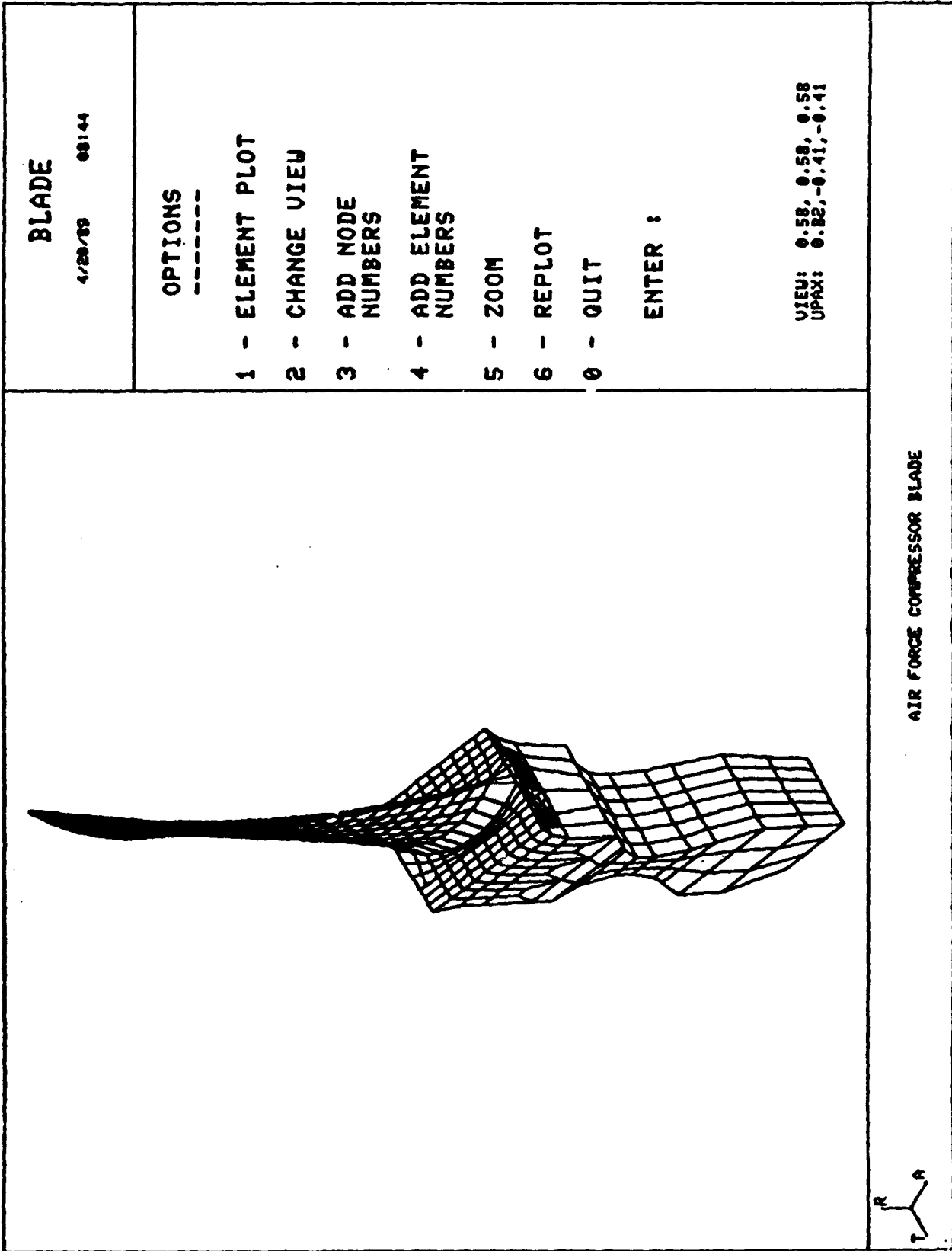


Figure 2.2.5: Finite Element Model for the Example

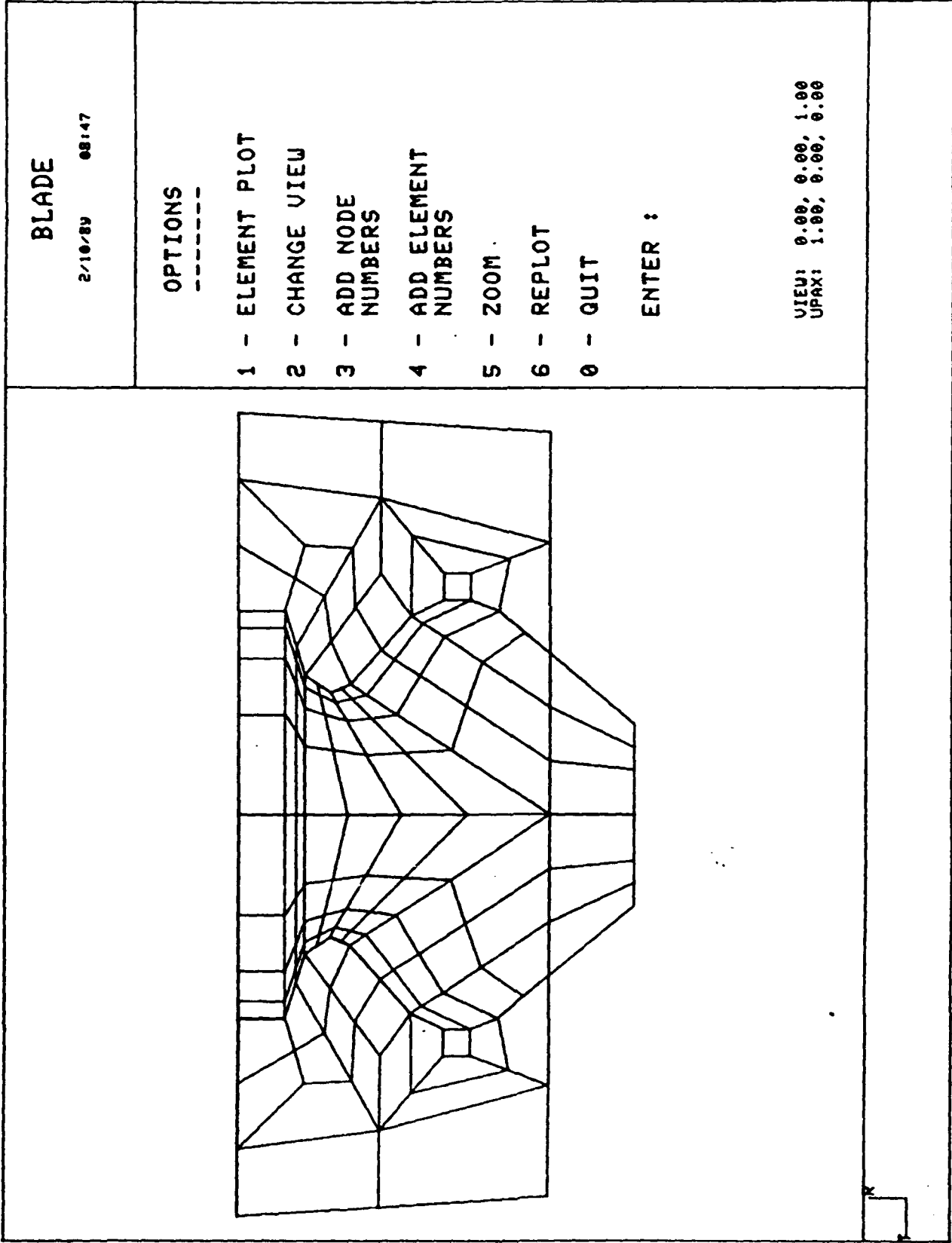


Figure 2.2.6: Finite Element Model of Root and Disk Section

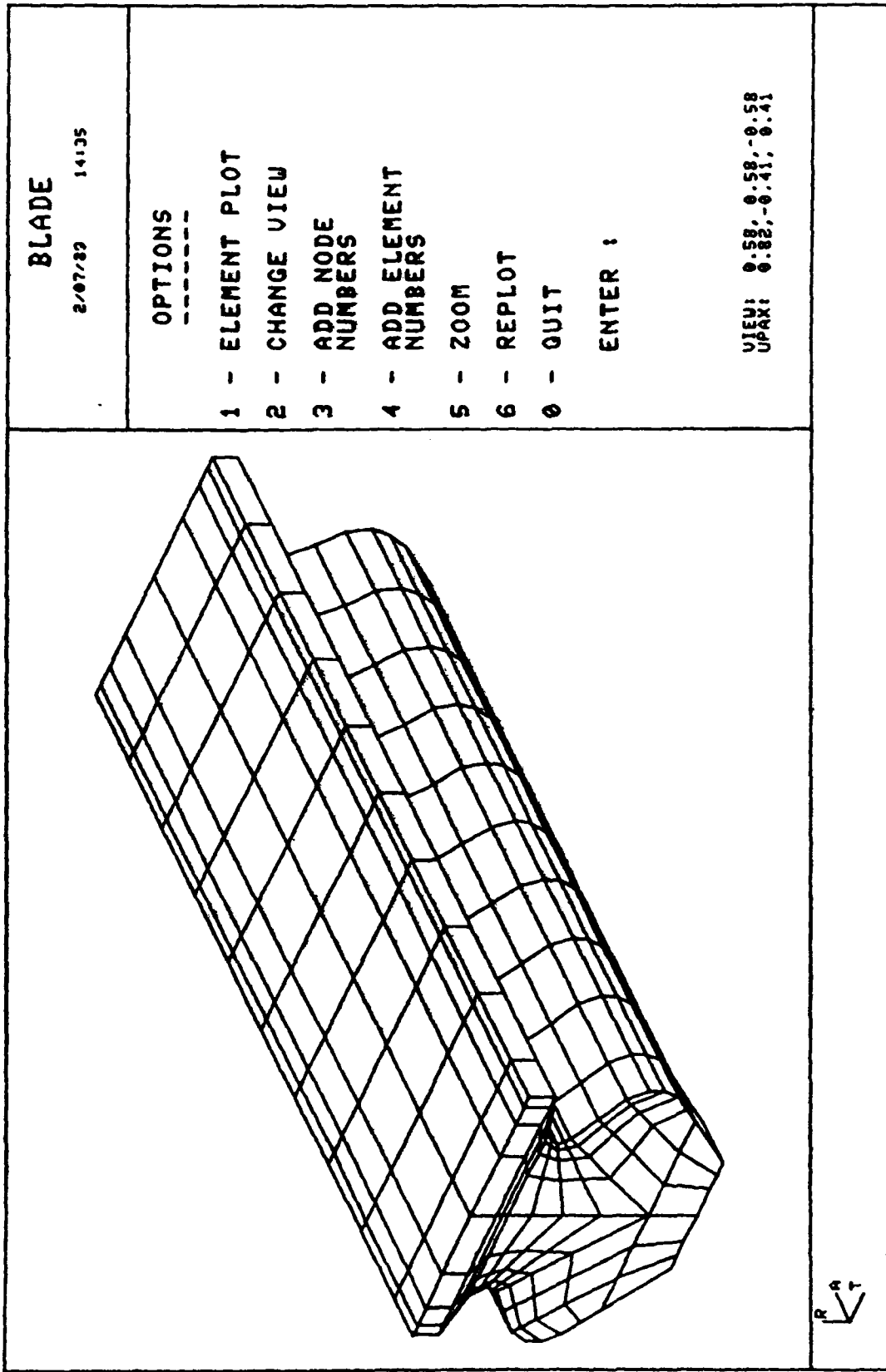


Figure 2.2.7: Finite Element Model for Root

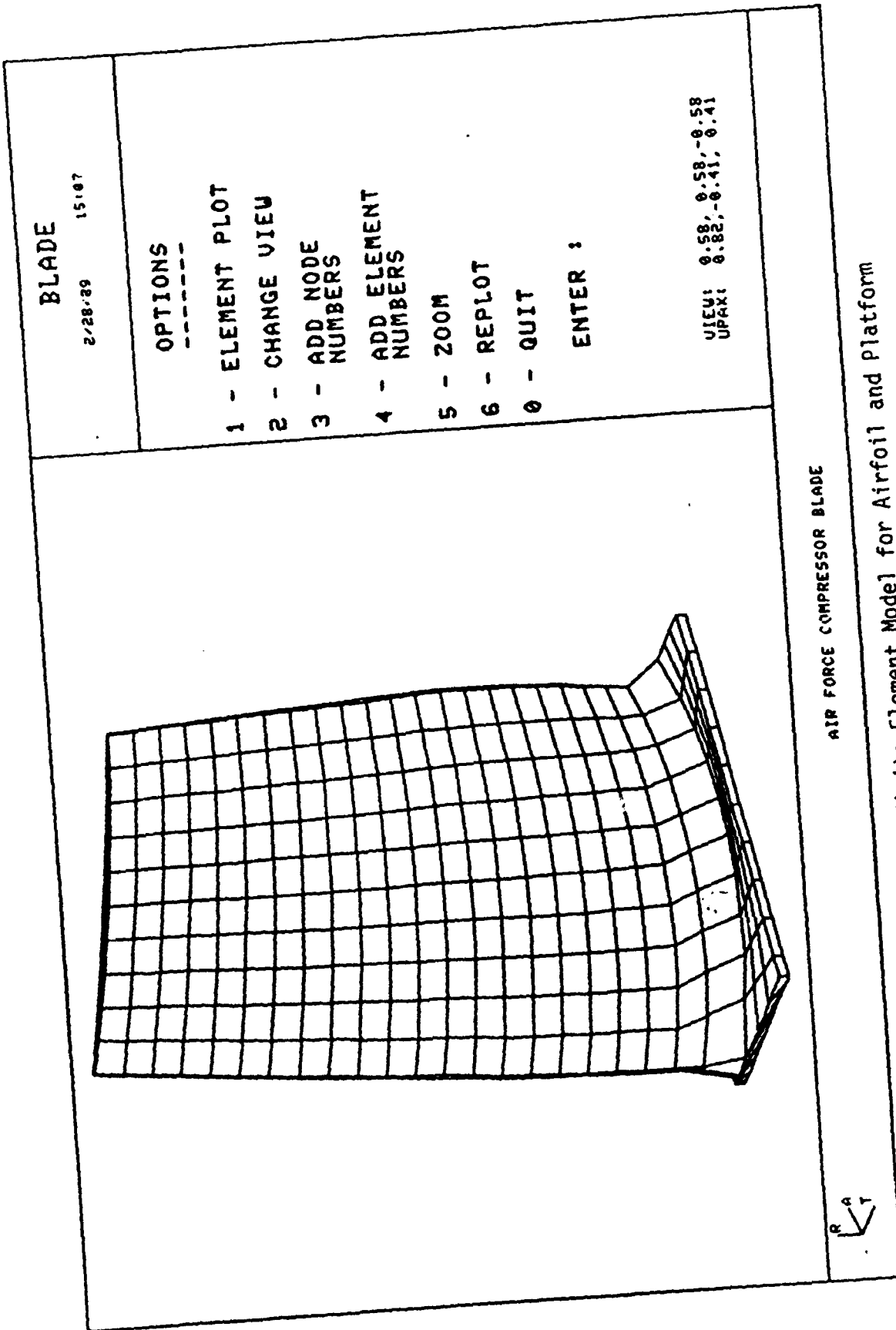


Figure 2.2.8: Finite Element Model for Airfoil and Platform

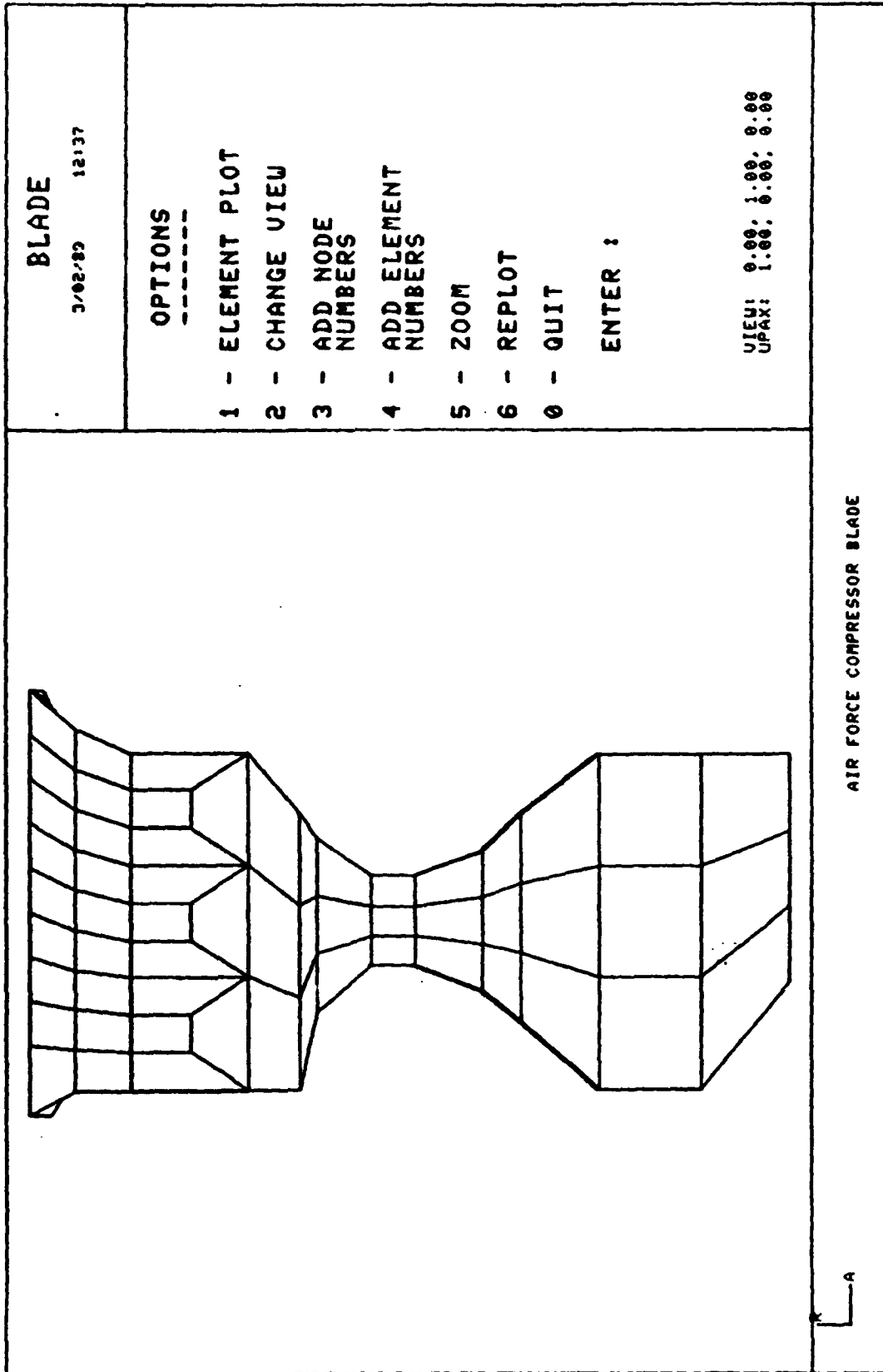


Figure 2.2.9: Finite Element Model for Disk Side View

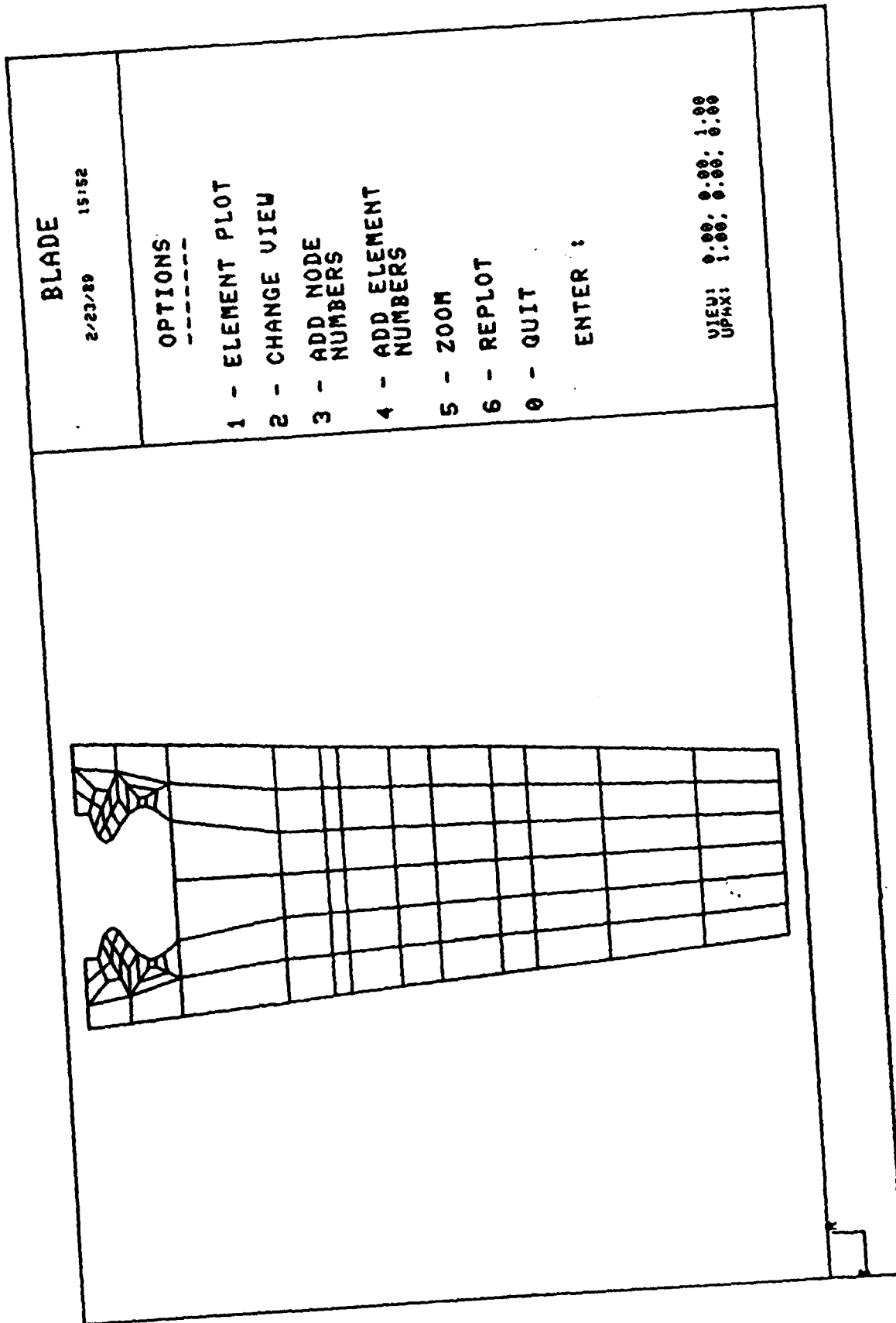


Figure 2.2.10: Finite Element Model for Disk Front View

**Table 2.2.2: Details of Compressor Blade Model**

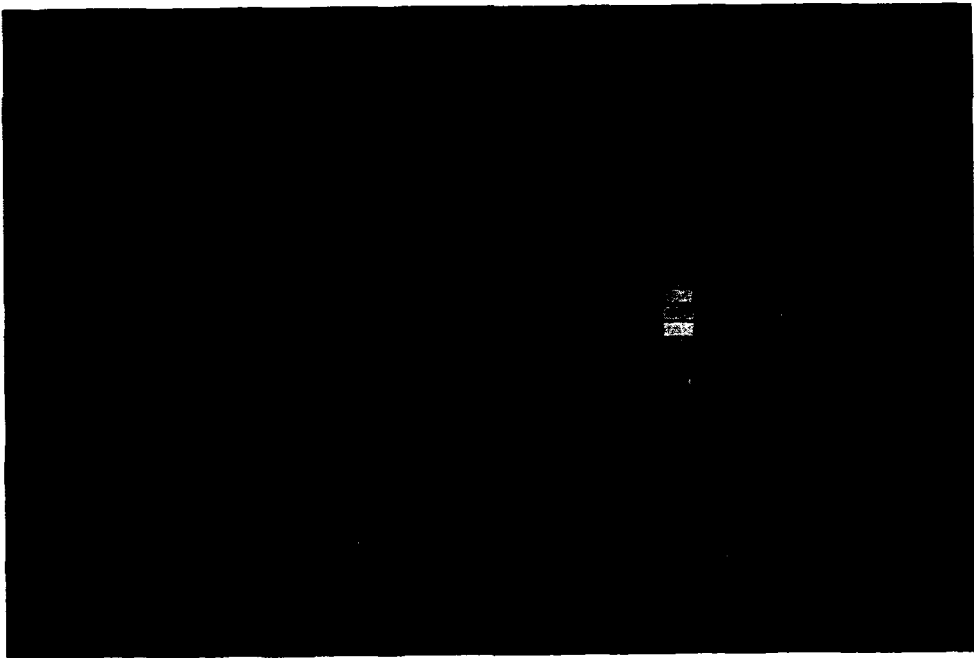
<b>Component</b>	<b>Number of elements</b>
<b>Airfoil</b>	<b>440</b>
<b>Platform</b>	<b>80</b>
<b>Root</b>	<b>726</b>
<b>Disk</b>	<b>594</b>
<b>Total</b>	<b>1840</b>

### **3. BLADE ANALYSIS**

The blade underwent two types of analysis, namely steady stress and natural frequencies analyses. Finite element programs ANSYS and BLADE were utilized to perform the computations. Both programs produced identical results which validate the BLADE program.

#### **3.1 Steady Stress Analysis**

The blade was analyzed statically under the effect of centrifugal forces. These forces result from the rotation of the rotor. A speed of 6000 rpm was used in the computation. Steady stress variations are shown in Figures 3.1.1 through 3.1.3. Maximum stress values are given in Table 3.1.1.



**Figure 3.1.1: Maximum Nodal Steady Stresses**

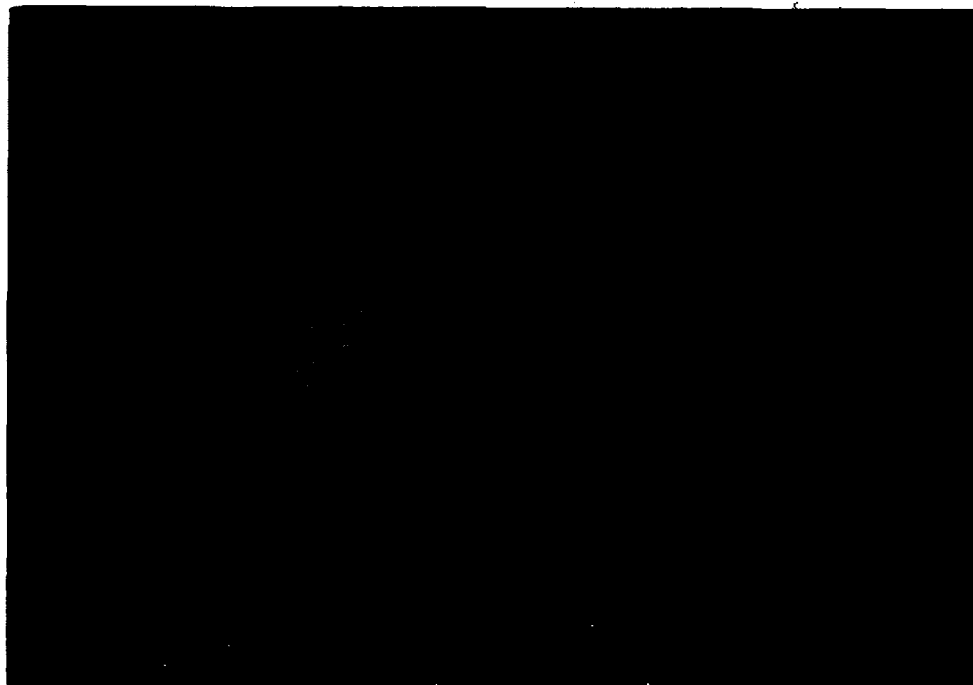


Figure 31.2: Maximum Nodal Stresses in Disk

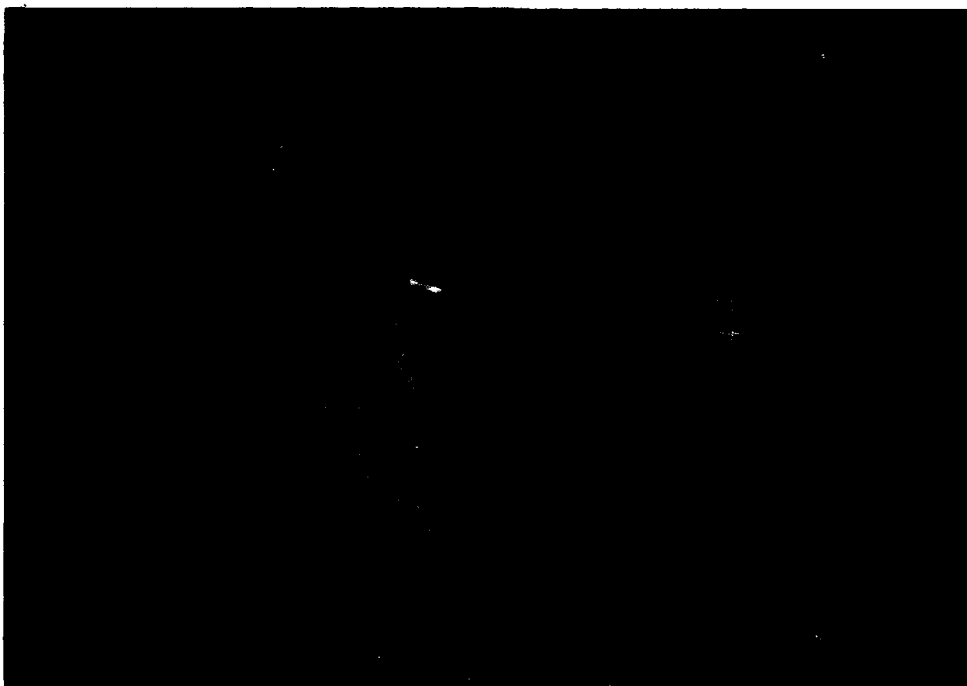


Figure 3.1.3: A Different View of Disk Nodal Stresses

Table 3.J.1 : Maximum Equivalent Stresses

<u>Element #</u>	<u>Location</u>	<u>Stresses, psi</u>	
		BLADE	ANSYS
1358	Disk	10085	10085
1359	Disk	8604.5	8604.5
1546	Disk	8460.0	8460.0
1361	Disk	8411.1	8411.1
1362	Disk	8375.8	8375.8
1360	Disk	8315.4	8315.4

Maximum equivalent stress in:

	<u>BLADE</u>	<u>ANSYS</u>
Airfoil	4364.9 psi	4364.9 psi
Platform	2530.9 psi	2530.9 psi
Root	7569.0 psi	7569.0 psi

### 3.2 Natural Frequency Analysis

Natural Frequency Analysis was performed on the blade model. The equation of motion for free vibration is of the eigenvalue type,

$$[M]\{\ddot{X}\} + [K]\{X\} = 0$$

where [M] and [K] are the mass and stiffness matrices and {X} is the displacement vector. By solving this eigenvalue problem, the eigenvectors are the mode shapes while the eigenvalues yield the natural frequencies. Due to the large number of degrees of freedom describing the model, Guyan reduction technique was implemented. At operation speed, centrifugal stresses stiffen the blade and therefore the natural frequencies at speed become higher than their corresponding values at zero rpm. The frequency values at speed and at zero rpm are given in Table 3.2.1. Mode shapes are presented in Figures 3.2.1 through 3.2.5. A Campbell diagram which shows the effect of speed on the values of the natural frequencies is given in Figure 3.2.6.

Table 3.2.1 : Natural Frequencies for a Single Blade

Mode #	Frequency (hz)		
	ANSYS at 0 rpm	BLADE at 0 rpm	at 6000 rpm
1	611.0	611.0	657.6
2	2173.8	2173.8	2212.8
3	2610.6	2610.6	2625.8
4	2854.3	2854.8	2866.6
5	4939.8	4939.8	4969.9
6	5466.2	5466.2	5502.3
7	8565.8	8565.8	8597.4
8	9313.4	9313.4	9347.1
9	10247.0	10247.0	10257.6
10	11118.0	11118.0	11141.9
11	12865.0	12865.0	12902.0

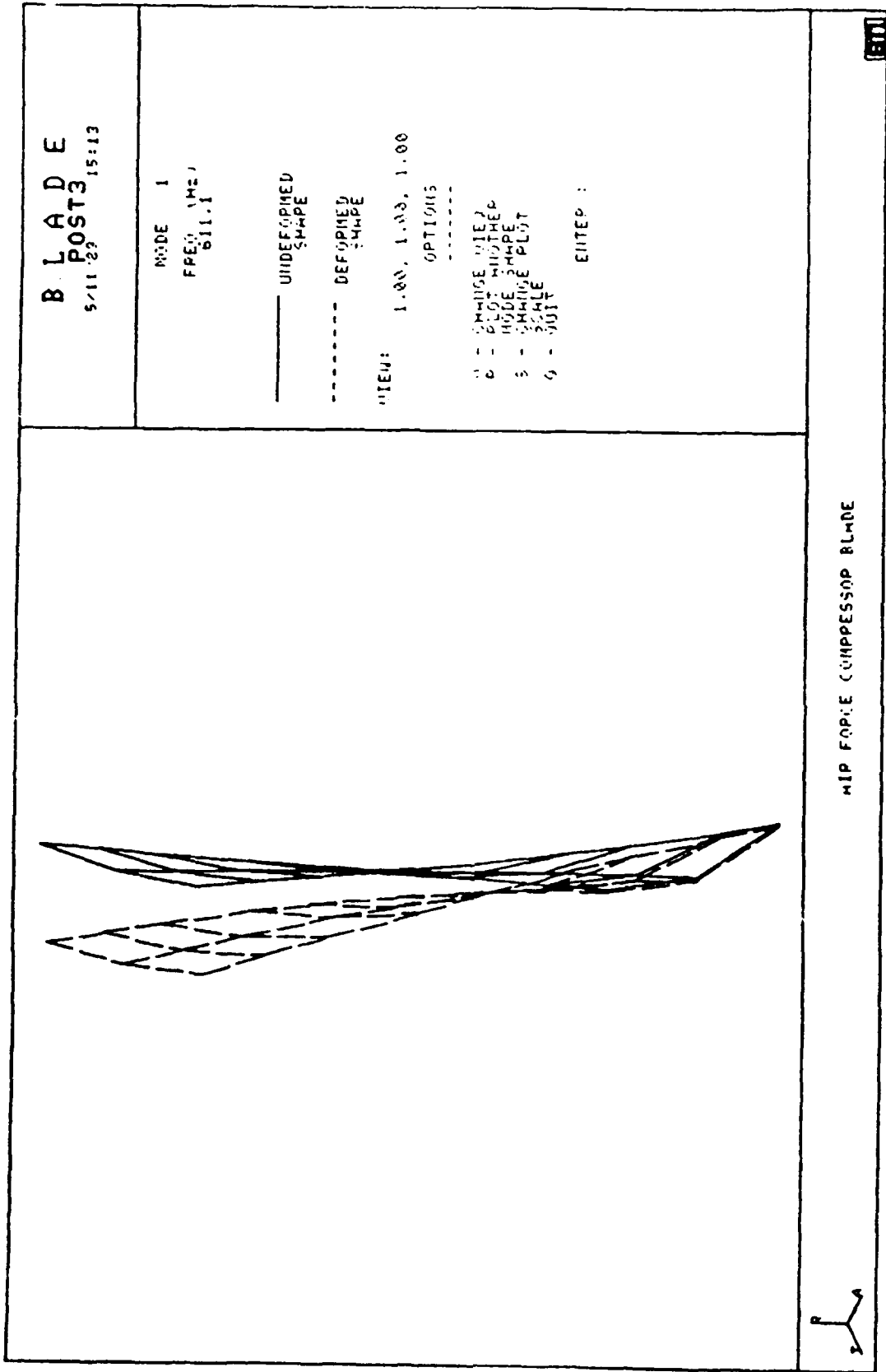


Figure 3.2.1: Mode Shape 1

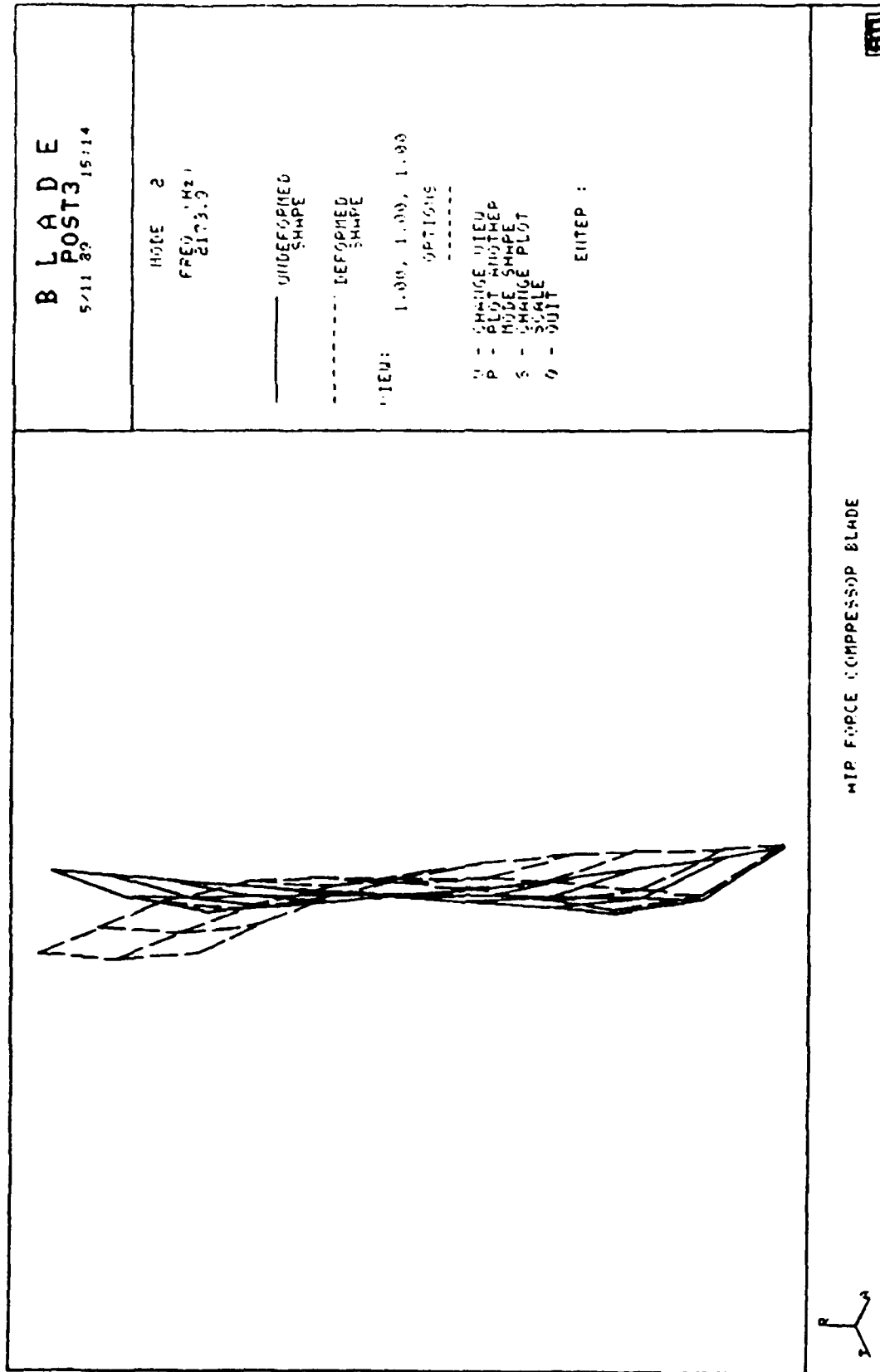


Figure 3.2.2: Mode Shape 2

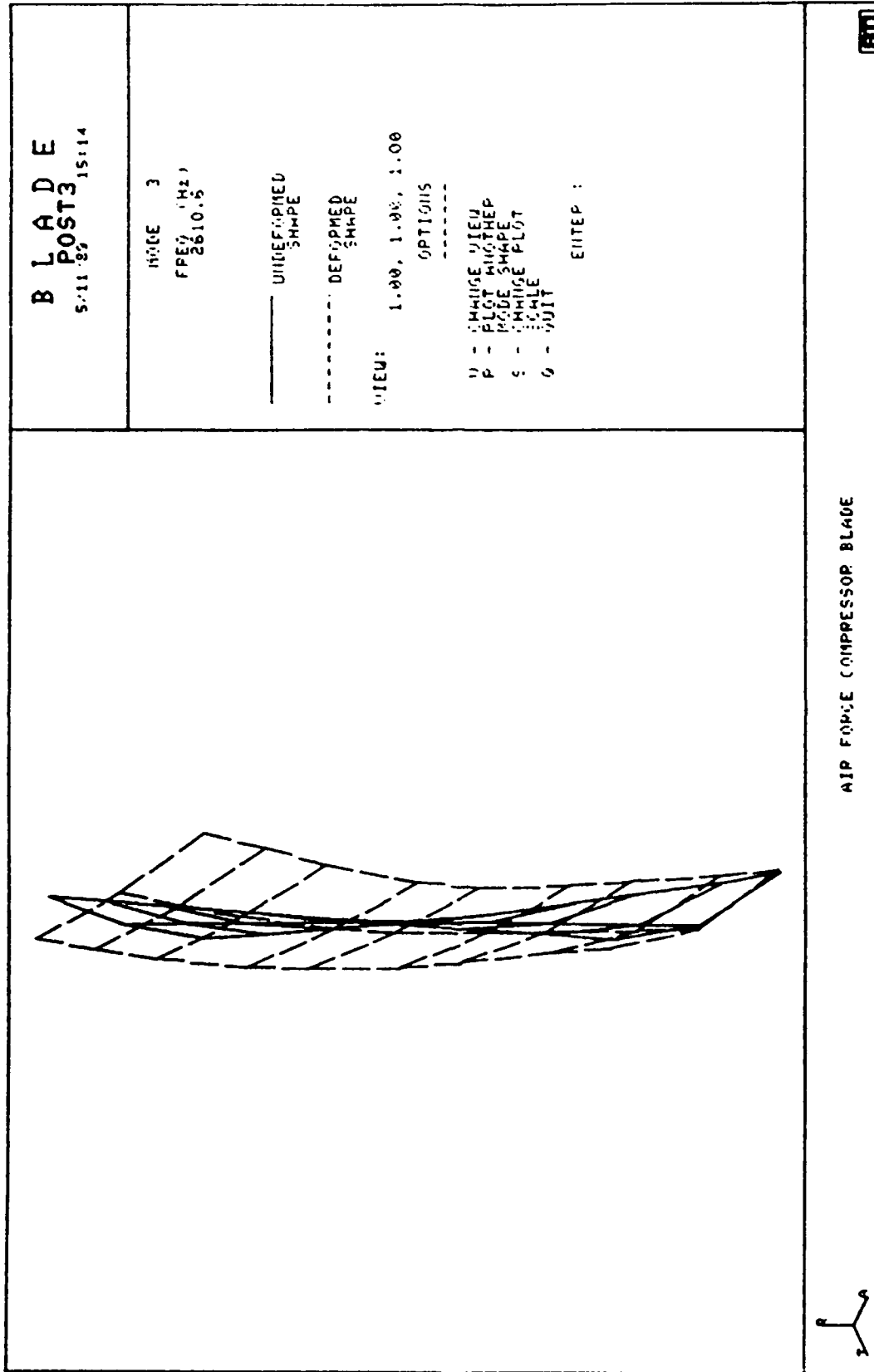


Figure 3.2.3: Mode Shape 3

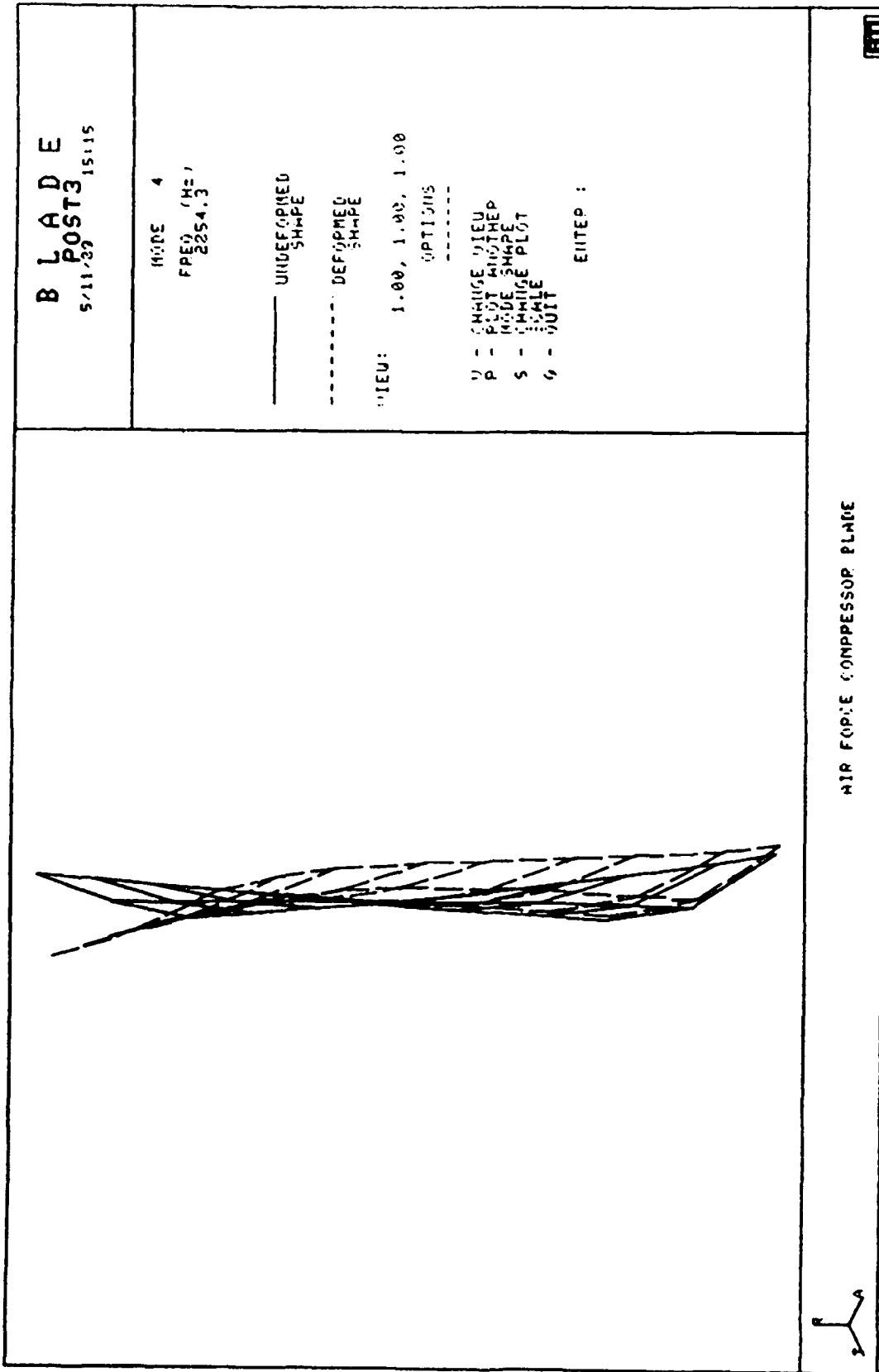


Figure 3.2.5: Mode Shape 4

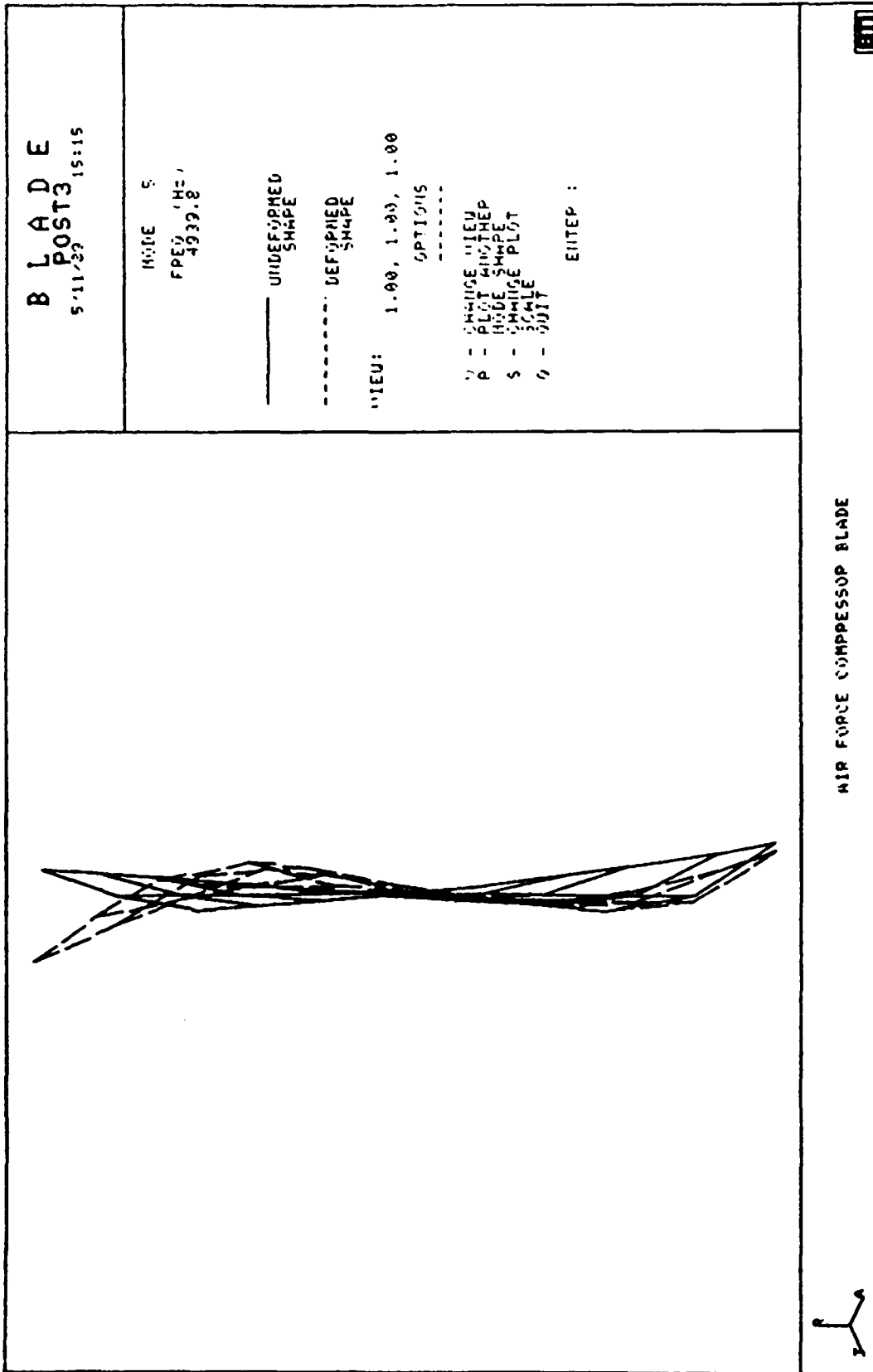


Figure 3.2.5: Mode Shape 5

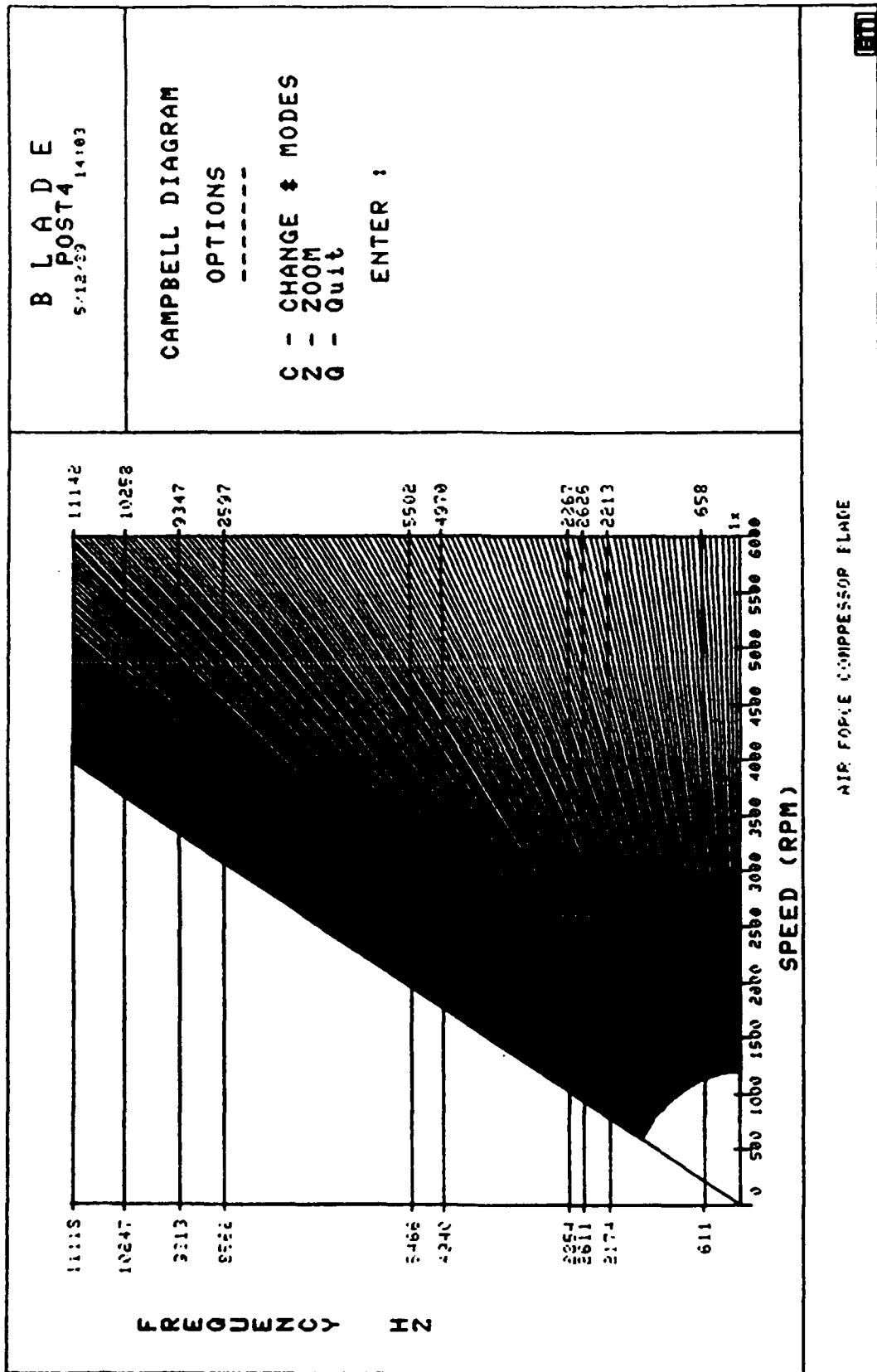


Figure 3.2.6: Campbell Diagram

### 3.3 Comparison with Test Results

Test results for natural frequencies test data were provided by AFWAL in the form of Holographic photos. The BLADE program was utilized to plot the displacement contours for the different mode shapes. Comparison of the calculated natural frequencies to the test data is given in Table 3.3.1. For Titanium material, it is known that the elastic modulus is strongly dependent on the thermal-mechanical processing. For bar stock, elastic modulus is about  $16 \times 10^6$  psi. For forged blade vane, the modulus of elasticity could become  $18.0 \times 10^6$  psi. As a result, the blade natural frequency can vary as much as 5 percent. As a matter of fact, the modulus for Titanium changes for one blade from one component to the next as shown in Table 3.3.2 obtained from tests conducted by Westinghouse. The analysis was carried out using a modulus of  $17.0 \times 10^6$  psi as shown in Table 3.3.1 in column (2). They compare favorably with the test data as shown by the percentage deviation in column (3). The table also shows that mode 4, 7, and 9 were missing from the experimental data. The Holograms are next compared to the calculated displacement contours in Figures 3.3.1 through Figures 3.3.8.

**Table 3.3.1: Comparison of Calculated Frequencies  
and Test Data**

<b>Test Frequency hz</b>	<b>Calculated Frequency hz</b>	<b>% Deviation</b>
602	611.0	+1.5
2138	2173.8	+1.67
2701	2610.6	-3.35
missed	2854.3	---
5200	4939.8	-5.0
5536	5466.2	-1.26
missed	8565.8	---
9480	9313.4	-1.76
missed	10247.0	---
10800	11118.0	+2.90
12620	12865.0	+1.94

Table 3.3.2: Dynamic Modulus of Prototype  
TI-6AL-4V Blades

<u>Row Number</u>	<u>Location</u>	<u>Direction</u>	<u>Average Dynamic Tensile Modulus (ksi)</u>
1	Airfoil	long	17.61
1	Root	long	17.33
1	Root	trans.	18.10
2	Airfoil	long	17.38
2	Airfoil	trans.	17.71
2	Root	trans.	17.97

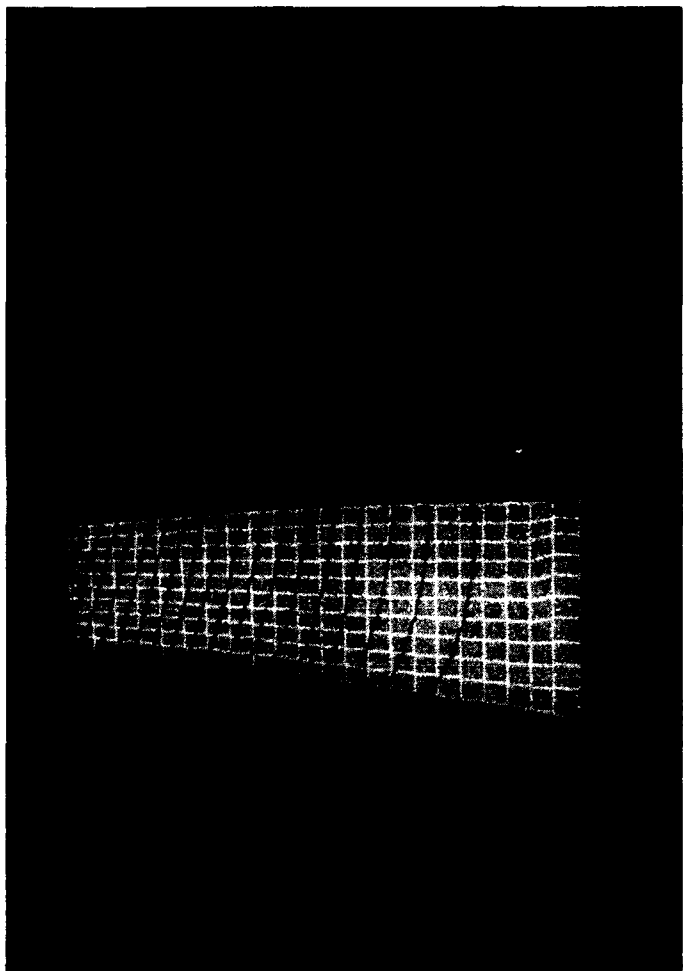


Figure 3.3.1: Comparison of Test Holograms and Analytical Results for Mode 1

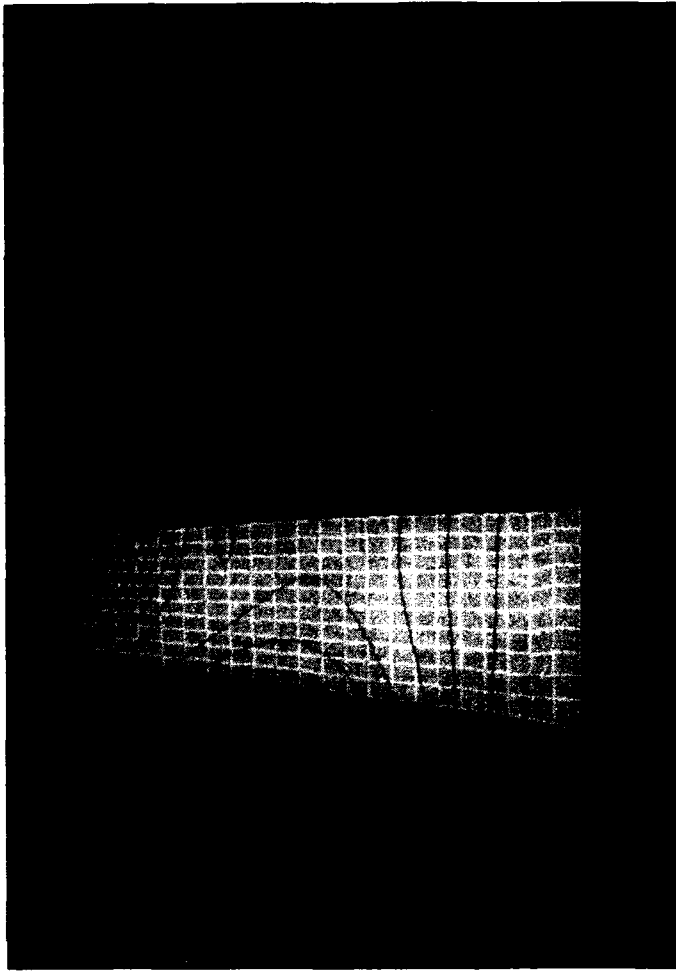
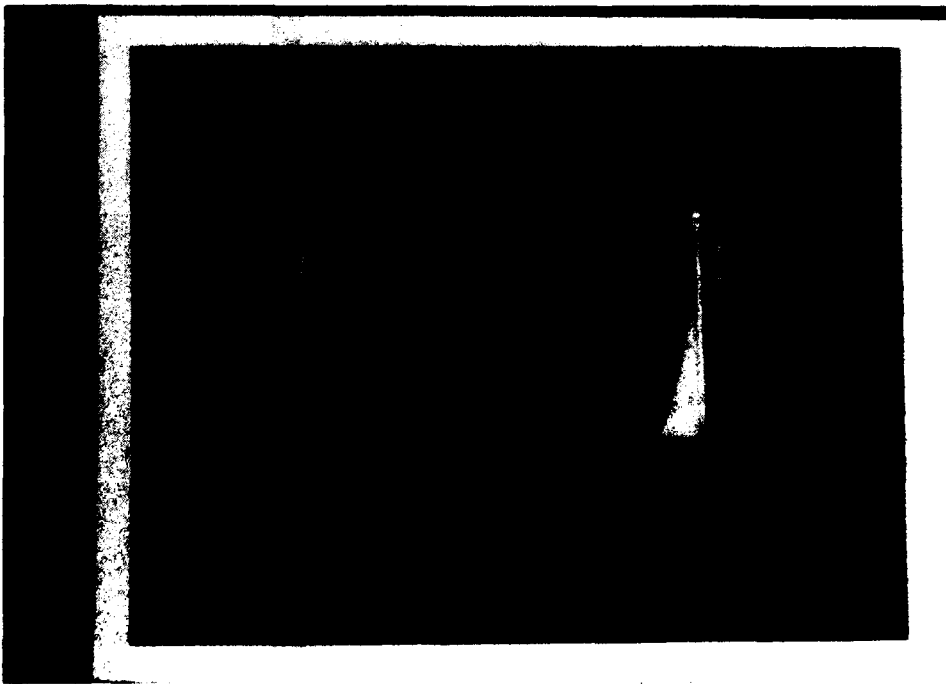


Figure 3.3.2: Comparison of Test Holograms and Analytical Results for Mode 2

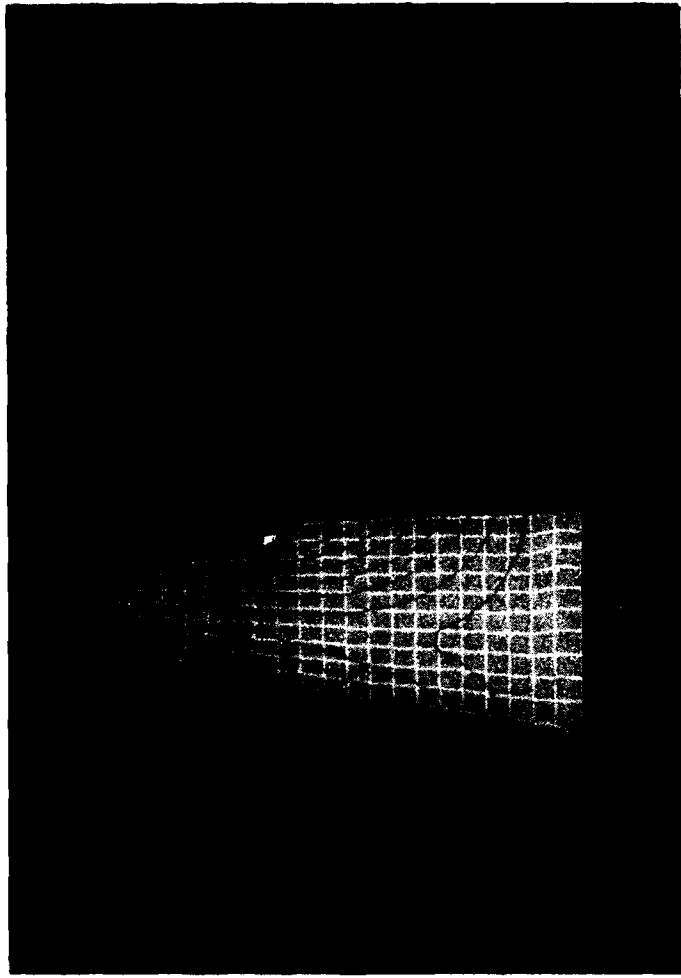
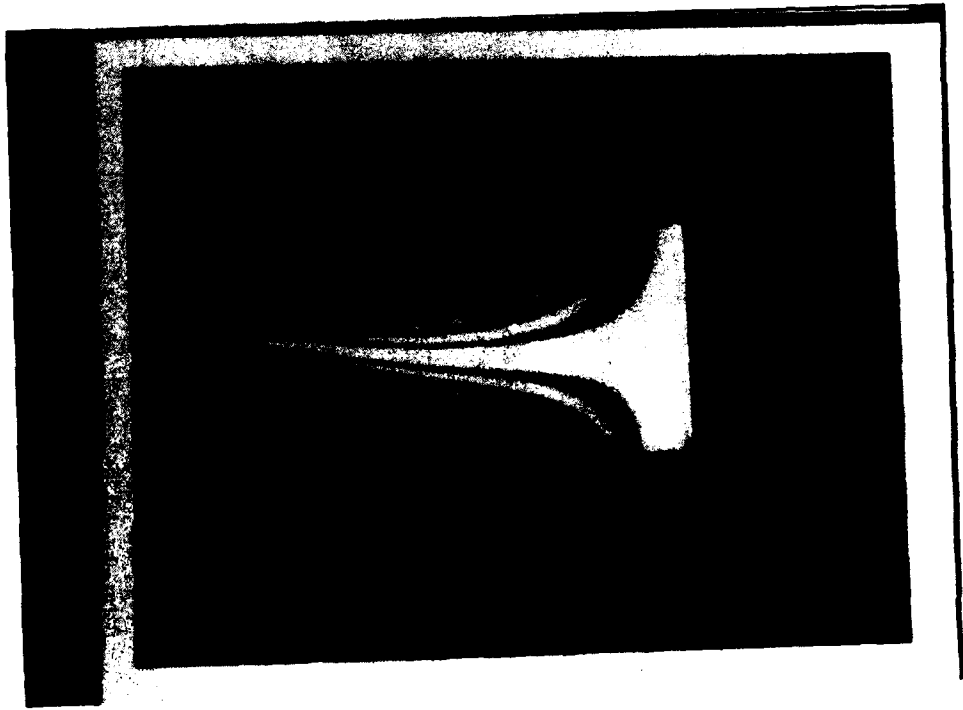


Figure 3.3.3.: Comparison of Test Holograms and Analytical Results for Mode 3

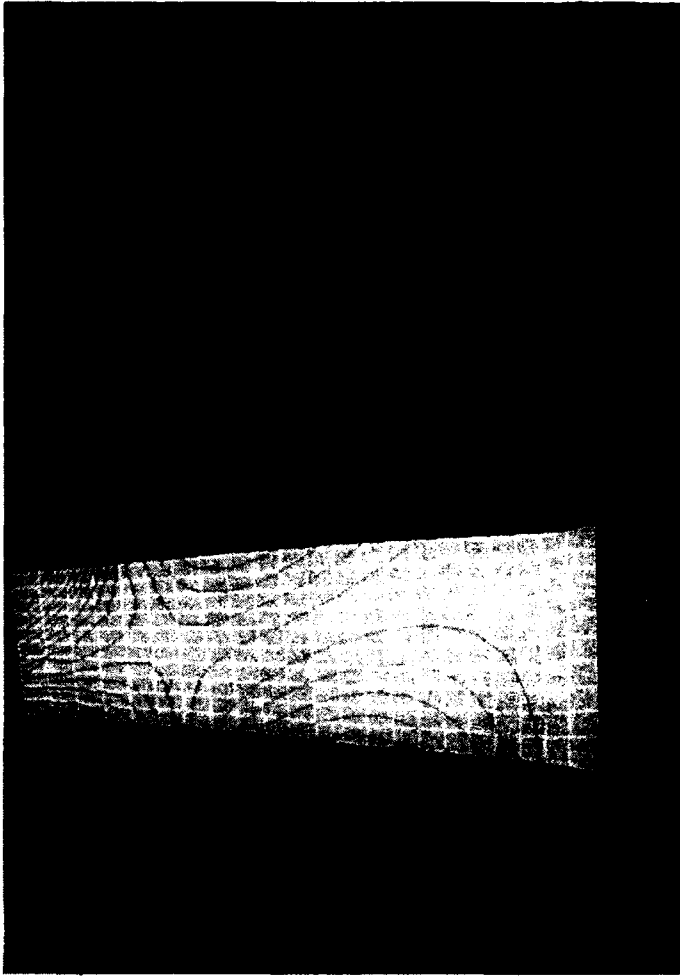
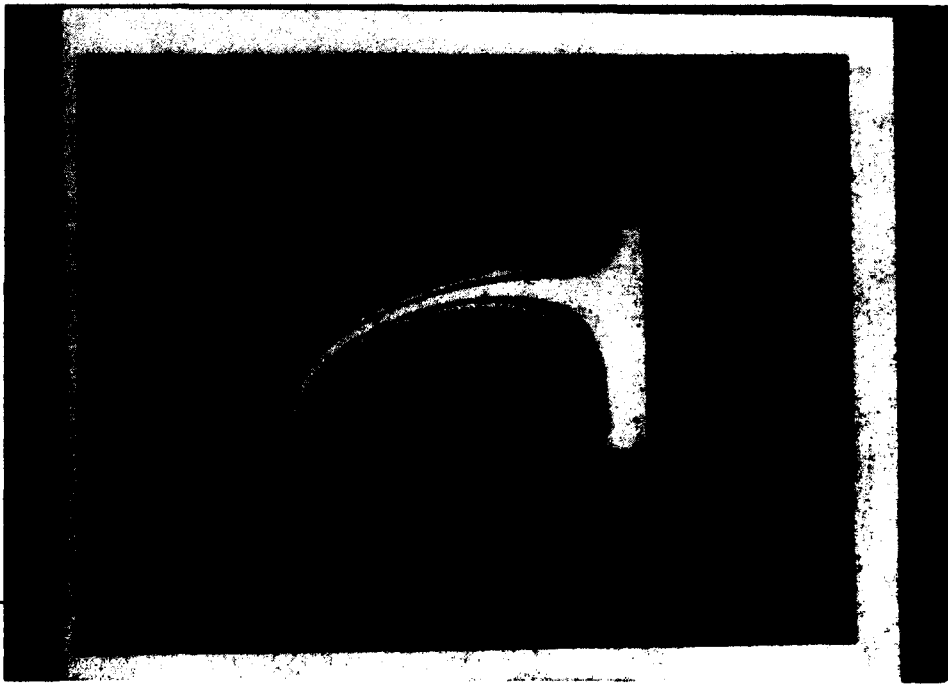


Figure 3.3.4: Comparison of Test Holograms and Analytical Results for Mode 5

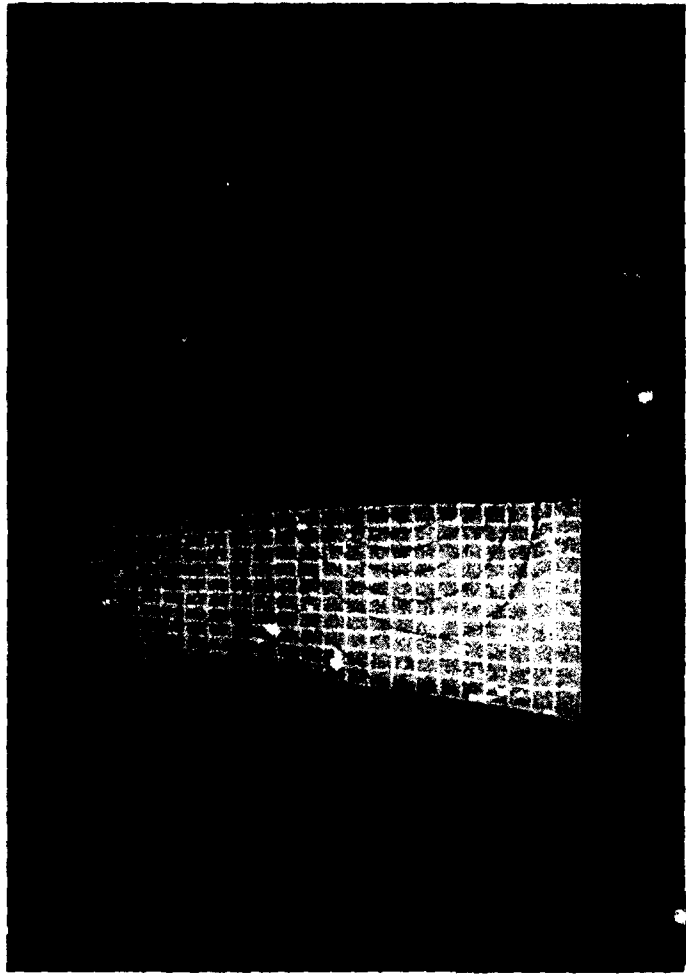


Figure 3.3.5: Comparison of Test Holograms and Analytical Results for Mode 6

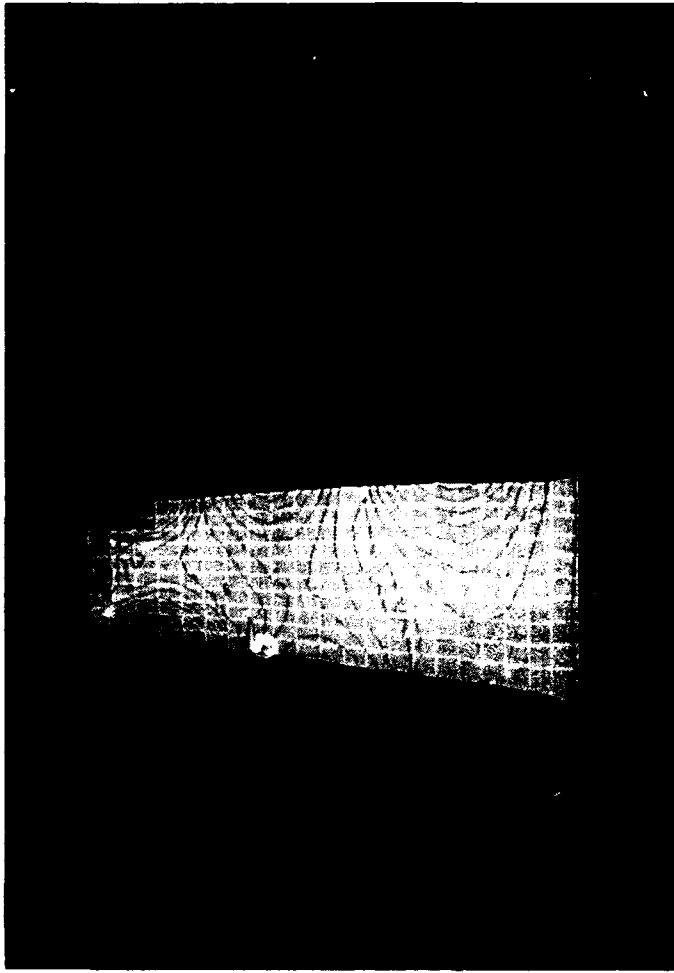


Figure 3.3.6: Comparison of Test Holograms and Analytical Results for Mode 8

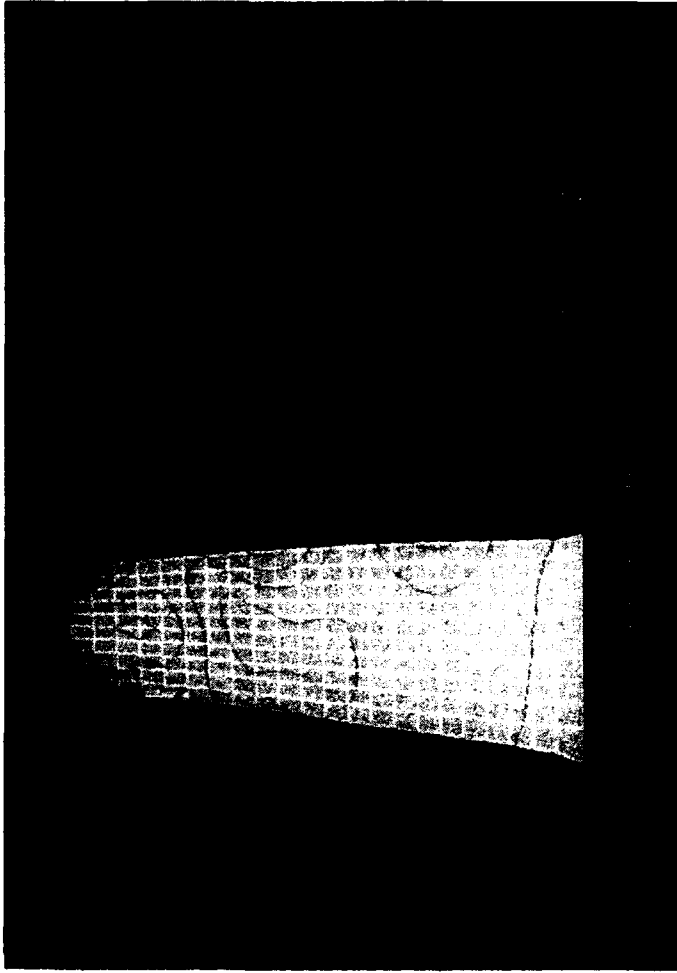
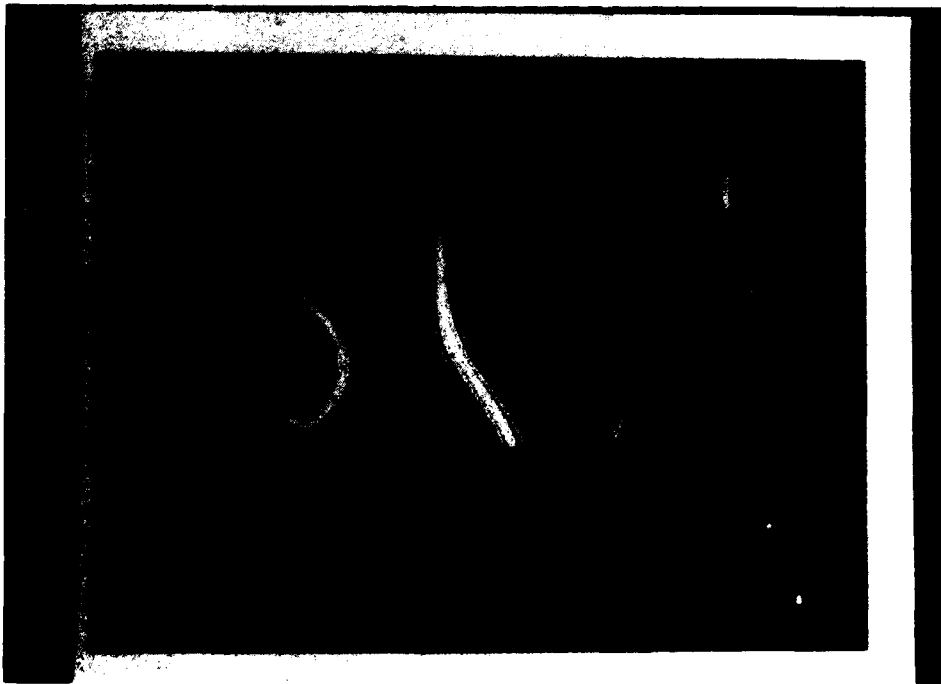


Figure 3.2.7: Comparison of Test Holograms and Analytical Results for Mode 10