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**Workshop on Trajectory
Optimization Methods
and Applications**

Presentations from the 1992 AIAA
Atmospheric Flight Mechanics Conference



Compiled by:

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FINAL REPORT FOR 10 AUGUST 1992

November 1992



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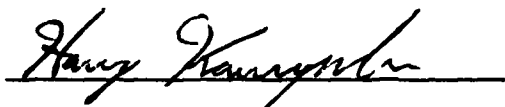


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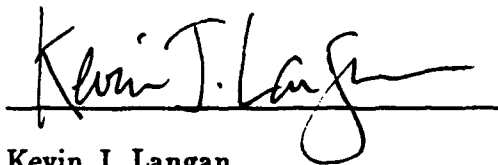
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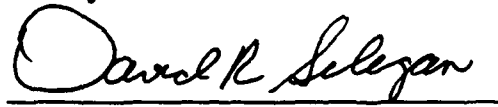
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Preface

This report is a compilation of presentations given at the "Workshop on Trajectory Optimization Methods and Applications", held at the 1992 AIAA Atmospheric Flight Mechanics Conference in Hilton Head, South Carolina. This workshop was co-chaired by Harry Karasopoulos and Kevin J. Langan, both of the former Flight Performance Group of the High Speed Aero Performance Branch in Wright Laboratory.

It is hoped that this document will help the attendees retain some of the ideas presented in the workshop, in addition to providing useful information to those who were unable to attend. Appreciation is expressed to the presenters and attendees. For the third year in a row, this workshop has proved to be a successful forum for highlighting current work in trajectory optimization. Thanks are also due to the American Institute of Aeronautics and Astronautics for making this workshop possible.

The following was the workshop schedule:

SCHEDULE

- **1:00 - Introduction - Harry Karasopoulos, Wright Laboratory.**
- **1:00 - OMAT: An Autonomous Optimal Solution to Rendezvous Problems with Operational Constraints - Don Jezewski, McDonnell Douglas Space Systems Company, Houston, TX.**
- **1:15 - MULIMP: Multi-Impulse Trajectory and Mass Optimization Program - Darla German, Science Applications International Corporation, Schaumburg, IL.**
- **1:30 - Phillips Laboratory Applications of POST - Jim Eckmann, SPARTA Inc., Edwards AFB, CA.**
- **1:45 - OTIS Advances at the Boeing Company - Steve Paris, The Boeing Company, Seattle, WA.**
- **2:00 - OTIS Activities at McDonnell Douglas Space Systems Company - Rocky Nelson, McDonnell Douglas Space Systems Company, Huntington Beach, CA.**

- **2:15 - Advances in Trajectory Optimization Using Collocation and Non-linear Programming** - Dr. Bruce A. Conway, University of Illinois, Urbana, IL.
- **2:45 - BREAK**
- **3:00 - Flight Path Optimization of Aerospace Vehicles Using OTIS** - Dr. Rajiv S. Chowdhry, Lockheed Engineering and Sciences Company, NASA LaRC, Hampton, VA.
- **3:15 - Trajectory Optimization of Launch Vehicles at LeRC: Present and Future** - Dr. Koorosh Mirfakhraie, ANALEX Corp., NASA Lewis Research Center, Cleveland, OH.
- **3:30 - Collocation Methods in Regular Perturbation Analysis of Optimal Control Problems** - Dr. Anthony J. Calise, Georgia Institute of Technology, Atlanta, GA.
- **3:45 - Automatic Solutions for Take-Off from Aircraft Carriers** - Lloyd H. Johnson, AIR-53012D, Naval Air Systems Command, Washington, DC.
- **4:00 - Current Pratt-Whitney OTIS Applications** - Russ Joyner¹, United Technologies, Pratt-Whitney, West Palm Beach, FL.
- **4:15 - Airbreathing Booster Performance Optimization Using Microcomputers** - Ron Oglevie, Irvine Aerospace Systems Co., Fullerton, CA.
- **4:30 - Scheduled Session End**

Unscheduled Speakers

- Dr. Klaus Well, University of Stuttgart.²
- Dr. Mark L. Psiaki, Cornell University.

¹Cancelled

²Presentation copy not available at the time of printing

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OMAT
An Autonomous Optimal Solution to
Rendezvous Problems with
Operational Constraints
by

D. J. Jezewski, J. P. Brazzel,
B. R. Haufler, and E. E. Prust

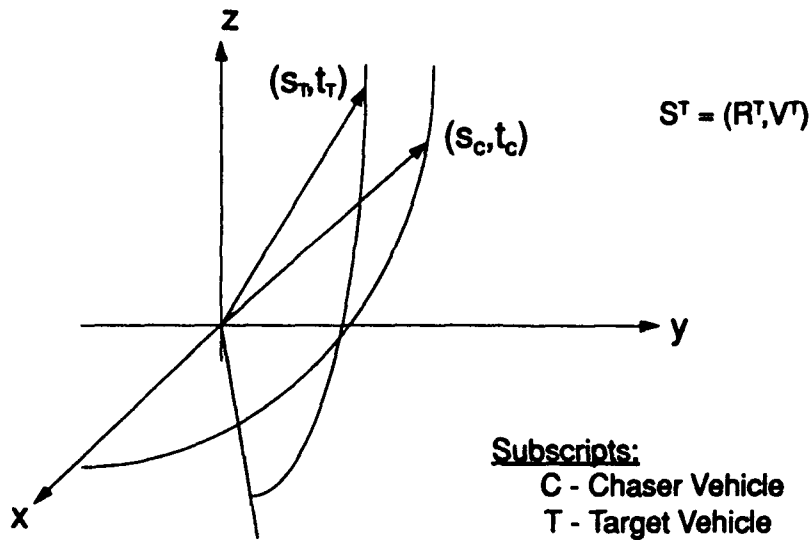
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Houston Division
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Houston, Texas 77062-6208

AIAA Atmospheric Flight Mechanics Conference
Hilton Head, South Carolina
August 10, 1992

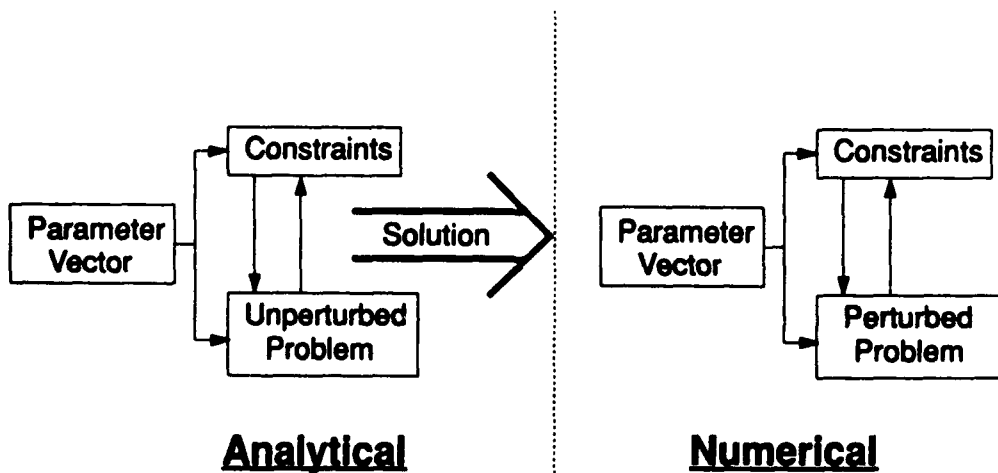
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Sketch of Rendezvous Problem



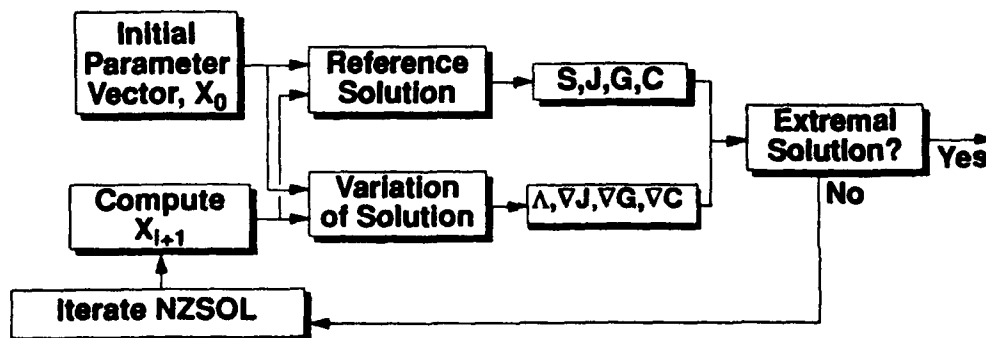
General Approach for Solving Optimal Rendezvous Problems



Definition of an Optimal Rendezvous Problem

- Given:
 - Chaser and Target States: (S_C, t_C) , (S_T, t_T)
 - Attracting Body
 - Objective Function, i.e., Delta-V, Fuel, Time, etc.
- Subject To:
 - Force Field
 - Perturbations
 - Terminal & Inflight Constraints and Limits
- Define:
 - Optimal Sequence of Maneuvers (Impulses, Finite Burns)
 - Number, Location, Magnitude or Duration, and Direction
- Such That:
 - Chaser & Target Vehicles Achieve a Relative Configuration at Some Time

A General Optimization Approach



X - Parameter Vector
S - State Vector
 Λ - Costate Vector
J - Objective Function
G - Active Constraint Vector
C - Solution Cost

Forms of Constraints

- ❑ Three Types of Constraints
 - Parameter Constraints
 - $a < X < b$
 - Linear Constraints (Constant Jacobian Matrix)
 - $L(S,t) \geq 0$
 - Non-Linear Constraints
 - $NL(S,t) \geq 0$
- ❑ Bounds
 - Constraints are Bounded by Upper and Lower Bounds (B_L, B_U)
 - Equality Defined by $B_U = B_L$
 - Unbounded Defined by $B_L = -\infty, B_U = +\infty$

Constraints are Defined by Five Integers

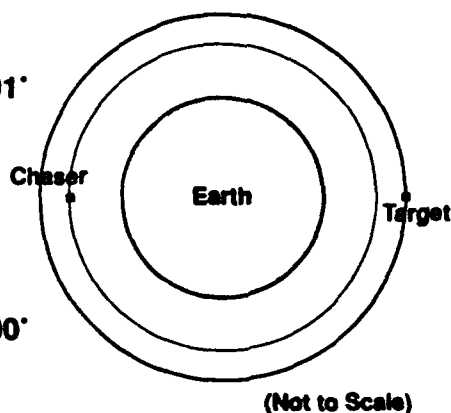
- ❑ I - Constraint Number
- ❑ J - What the the Constraint is Referenced to:
 - An Impulse
 - Another Constraint
- ❑ K - Reference Impulse or Constraint Number
- ❑ L - Condition that Triggers or Initiates Constraint
 - Time from GMT or Reference Event (Impulse or Constraint)
 - Phase Angle
 - Lighting Condition between Chaser and Target Vehicles
 - Delta-Angular Measurement in Chaser Orbit
 - Number of Revolutions
 - Chaser Vehicle's nth Periapsis, Apoapsis Crossing
 -
 -
 -

Constraints are Defined by Five Integers (concl'd)

- N - Type of Constraint
 - Periapsis, Apoapsis
 - Differential Height between Chaser Vehicle & Target Orbit
 - Phase Angle
 - Sleep Cycle or Quiet Time
 - Chaser Orbit Coelliptic with Target Orbit
 - Chaser Position Vector Relative to Target LVLH Frame
 - Chaser Velocity Vector Relative to Target LVLH Frame
 - Bounded Delta-V in Chaser LVLH Frame
 - Inertial Line-of-Sight Angular Rate
 - Wedge Angle
 -
 -
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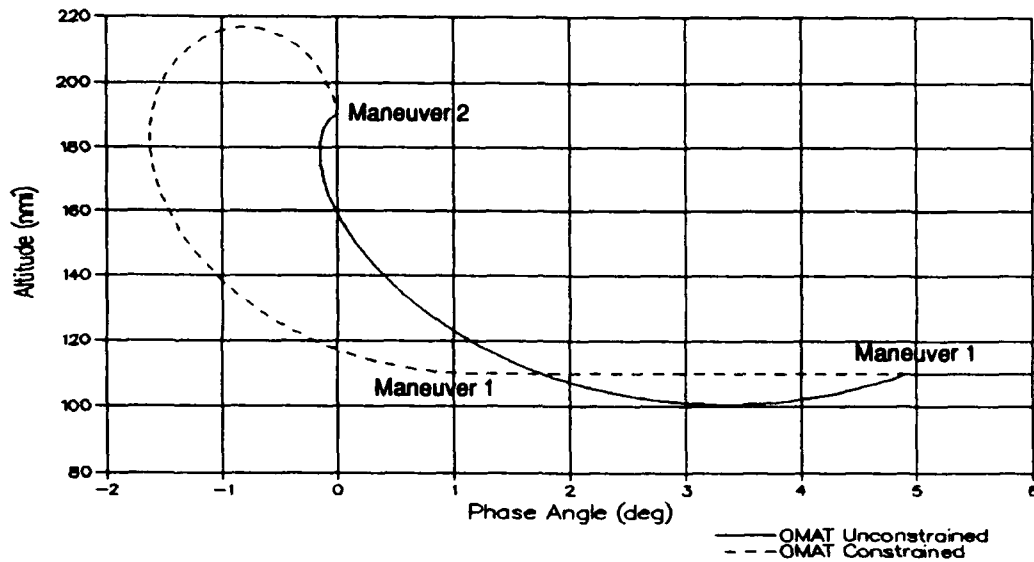
Demonstration 2: Typical Shuttle Rendezvous

- Shuttle (Chaser)
 - 110 circular altitude
 - Inclination = 28.5°
 - Longitude of ascending node = 101°
 - Argument of perigee = 0°
 - True anomaly = 180°
- SSF (Target)
 - 190 nmi circular altitude
 - Inclination = 28.5°
 - Longitude of ascending node = 100°
 - Argument of perigee = 0°
 - True anomaly = 0°
- Limited to 2 maneuvers



Demonstration 2 (Concl'd)

Demonstration 2: Shuttle Rendezvous with SSF (Unperturbed)
OMAT Unconstrained and Constrained Solution



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Demonstration 2 (Cont'd)

□ Results

- OMAT handles perigee constraint for unperturbed orbits
- Optimum unconstrained trajectory required 461 ft/s, constrained trajectory required 603 ft/s

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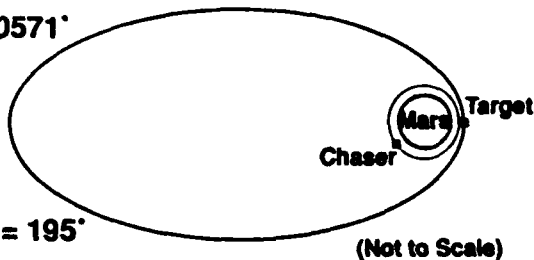
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Perturbations

- Primer Vector Theory (Presently) Requires Perturbations to have Form
 - $\mathcal{R}(R,t)$
- Largest Geopotential Perturbation for Earth & Mars Is J_2 , Has Form
$$\mathcal{R}_{J2} = j_r R + j_k K$$
 - Where K is a Unit Vector Normal to the Equator and j_r and j_k are Functions of the Position Vector.
- Presently Incorporating NxM Geopotential Model
- Need to Extend Theory to Functions $\mathcal{R}(R,V,t)$
- Need to Develop Theory for Third-Body Effects (Libration Point Rendezvous)

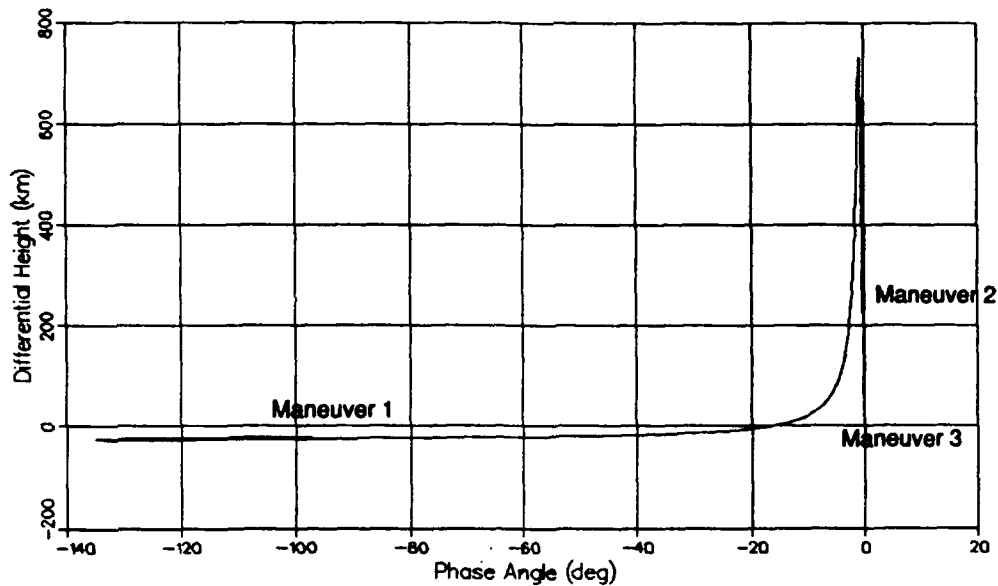
Demonstration 4: Mars MEV Post-Ascent Rendezvous with MTV

- MEV (Chaser)
 - 117 x 135 nmi altitude
 - Inclination = 164.264662°
 - Longitude of ascending node = 194.39079°
 - Argument of perigee = 159.820571°
 - True anomaly = 0°
- MTV (Target)
 - 135 x 18,294 nmi altitude
 - Inclination = 164.2°
 - Longitude of ascending node = 195°
 - Argument of perigee = 16°
 - True anomaly = 0°



Demonstration 4 (Concl'd)

Demonstration 4: Mars MEV Post-Ascent Rendezvous with MTV
OMAT J2 Perturbed Solution



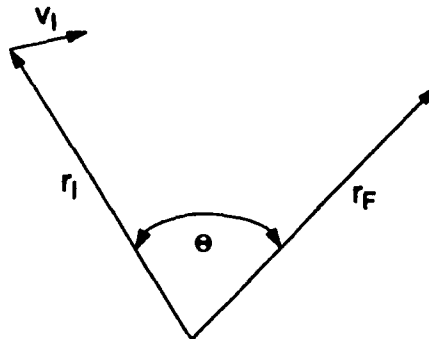
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Perturbed Lambert Problem

□ A Definition of Lambert's Problem:

- What is the Initial Velocity Vector, V_i , that Generates a Trajectory that Passes between Two Radii Separated by a Given Angle in a Specified Time Interval?



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Perturbed Lambert Problem (cont'd)

□ Standard Approach

- V_1 Obtained from Classical Lambert Problem
- δV_1 Obtained from Solution to Variational Equations, i.e.,

$$\delta V_1 = \phi_{12}^{-1} \delta R_F \quad \Phi(t_1, t_F) = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$$

▪ Difficulties with this Approach

- δR_F Must be "Small" (Linear Approximation)
- ϕ_{12} Must be Well Conditioned
- J_2 Frequently Must be Reduced (Sub-Problem)
- Excessive Number of Iterations and Integrations

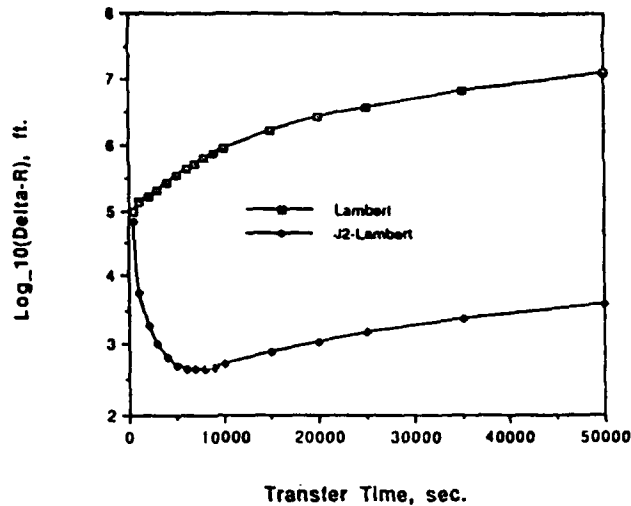
Perturbed Lambert Problem (concl'd)

▪ Improved Approach

- V_1 Obtained from First-Order Correction to the Inverse-Square Problem Resulting from the J_2 Perturbation
 - Analytic Solution
 - Expressed in "Ideal Reference Frame"
 - Regular, No Singularities
 - Solution Expressed in Terms of Elements & their Variations
- δV_1 Obtained from Solution to Variational Equations, i.e.,
$$\delta V_1 = \phi_{12}^{-1} \delta R_F$$
- $\delta R_F \approx O(10^{-3})$ Smaller than Classical Approach
- No Requirement for J_2 Reduction (Sub-Problems)
- Solution to TPBVP Requires Only a Few Iterations and Integrations

Comparison of Final Position Vector Errors for the Classical Lambert and Predictor/Corrector Solutions

- ☐ Earth Centered
- ☐ Transfer Angle = 170 degrees



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Outline of Optimal Solution Approach

- ☐ Unperturbed Problem
 - Force Field (Inverse-Square), i.e.,
$$\vec{V} = -\mu \left(\vec{R}/r^3 \right)$$
 - State and Variation in State Obtained from Solution of
 - Kepler's Problem (Goodyear, Analytic)
 - Boundary Value Problem Satisfied by Solution of
 - Lambert's Problem (Gooding, Analytic)
 - Constraints and Variation of Constraints Evaluated Using
 - Keplerian Elements
 - Non-Linear Programming Algorithm, NZSOL, Solves Constrained Optimal Problem
 - Solution Obtained in seconds on Sun Sparcstation 2

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Outline of Optimal Solution Approach (concl'd)

□ Perturbed Problem

- Force Field (Perturbed Inverse-Square)

$$\dot{V} = -\mu(R/r^3) + \mathfrak{R}$$

- State and Variation in State Obtained by Numerical Integration of Differential Equations
 - Variable-Step Runge-Kutta-Fehlberg 7/8 Algorithm
 - Maximum of 42 Linear & Non-Linear Differential Equations
- Boundary Value Problem Satisfied by Solving
 - Perturbed Lambert Problem
- Constraint Targeting Evaluated Using Non-Periodic Elements
- Constraints and Variation of Constraints Evaluated on Perturbed Trajectory
- NZSOL Solves Perturbed, Constrained Optimal Problem
- Solutions Obtained (Presently) in Minutes

What Have We Done, Where Are We Going?

□ Present Status of Development:

- Proof of Concept, Using Unperturbed Solution to Solve Perturbed Problem
- Verify Solution Approach for Handling Perturbations and Constraints and their Conditions
- Developed Solution Approach for Perturbed Lambert Problem
- Illustrate Initial Capability of the Algorithm, (OMAT), to Efficiently Solve Optimal Rendezvous Problems with Operational Constraints

□ Planned Future Development:

- Expand Perturbation and Constraint Models
- Develop Multi-Rev. Capability
- Develop Finite-Thrust Model
- Libration Point Rendezvous
- Solve Advanced Problems
-

Concluding Remarks

- ❑ Integration of Theoretical & Operational Aspects Of Optimal Rendezvous
- ❑ Development of an Autonomous Optimal Rendezvous Solution
- ❑ Primer Vector Theory Basis
- ❑ Approach Based on Solution to Lambert's Problem and it's Extention in a Perturbed Force Field
- ❑ Premise is Made that Perturbed Problem is an ϵ away from Unperturbed
- ❑ Constraints are Adjoined to Objective Function by Lagrange Multipliers
- ❑ General Mapping for Constraints from State to Parameter Space
- ❑ NZSOL Used to Solve Constrained Non-Linear Programming Problem
- ❑ Program is Fast, Reliable, Robust, Flexible, and Readily Extended
- ❑ Solution Time: Unperturbed (sec.), Perturbed (min.)

MULIMP

Multi-Impulse Trajectory and Mass Optimization Program

Darla German

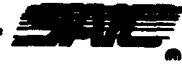
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MULIMP GENERAL DESCRIPTION

- DESIGNED TO COMPUTE A MULTI-TARGETED TRAJECTORY AS A SEQUENCE OF "TWO-BODY" SUBARCS IN A CENTRAL GRAVITATIONAL FIELD USING KEPLER AND LAMBERT ANALYTICAL SOLUTION ALGORITHMS
- BODIES MAY BE PLANETS (ORBITAL ELEMENTS STORED INTERNALLY), ASTEROIDS OR COMETS (ORBITAL ELEMENTS CONTAINED IN ASTCOM.ELM FILE), OR FICTITIOUS (ELEMENTS INPUT BY USER)
- CENTRAL BODY MAY BE THE SUN, ANY OF THE 9 PLANETS OR AN ARBITRARY BODY (GRAVITATIONAL CONSTANT INPUT BY USER)
- UP TO 19 SUBARCS MAY BE SPECIFIED



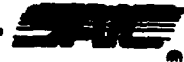
VARIABLES IN OPTIMIZATION SEARCH

- TIMES (DATES) OF THE NODAL POINTS CONNECTING TRAJECTORY SUBARCS
- POSITION COORDINATES OF MIDCOURSE ΔV POINTS NOT MADE AT AN EPHEMERIS BODY



DEPARTURE CONDITIONS

- RENDEZVOUS DEPARTURE IN WHICH CASE THE FIRST IMPULSE ΔV_1 , IS EQUAL TO THE HYPERBOLIC EXCESS SPEED V_{∞}
- PARKING ORBIT DEPARTURE IN WHICH CASE THE FIRST IMPULSE IS THAT NECESSARY TO ATTAIN THE HYPERBOLIC EXCESS SPEED FROM THE PARKING ORBIT (r_p, e) WITH THE MANEUVER ASSUMED TO BE COPLANAR
- A "FREE" DEPARTURE IN WHICH CASE THE FIRST ΔV IMPULSE IS EXCLUDED FROM THE PERFORMANCE INDEX
- A GRAVITY-ASSIST DEPARTURE IN WHICH CASE THE APPROACH HYPERBOLIC VELOCITY VECTOR MUST BE SPECIFIED BY INPUT



INTERMEDIATE TARGET CONDITIONS

ARRIVAL

- RENDEZVOUS
- ORBIT CAPTURE (ORBIT IS USER DEFINED)
- UNCONSTRAINED FLYBY SPEED
- CONSTRAINED FLYBY SPEED (HYPERBOLIC FLYBY) SPEED IS USER INPUT

DEPARTURE

- RENDEZVOUS DEPARTURE
- ORBIT DEPARTURE (ORBIT IS USER-DEFINED)

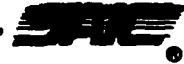
OTHER

- GRAVITY-ASSISTED SWINGBY



GRAVITY-ASSISTED SWINGBY

- **MODEL IS FORMULATED WITH POWERED MANEUVER AS THE GENERAL CASE**
- **ΔV WILL OFTEN ITERATE TO ZERO VALUE IF THE PROBLEM IS NOT OVERLY CONSTRAINED BY SWINGBY DATE AND DISTANCE**
- **USER OPTION TO SPECIFY POWERED MANEUVER LOCATION**
 - **INBOUND ASYMPTOTE**
 - **PERIAPSIS**
 - **OUTBOUND ASYMPTOTE**
 - **BEST CHOICE**



TERMINAL TARGET CONDITIONS

- **RENDEZVOUS**
- **TARGET-BODY ORBIT CAPTURE**
- **SATELLITE ORBIT CAPTURE**
- **UNCONSTRAINED FLYBY**
- **CONSTRAINED FLYBY**
- **SPECIFIED ORBIT ELEMENTS (a,e,i) RELATIVE TO CENTRAL BODY; FINAL TARGET MUST PROVIDE GRAVITY-ASSIST**



"FREE" MIDCOURSE ΔV POINTS

MIDCOURSE VELOCITY CHANGES MAY BE MADE AT INTERIOR IMPULSE POINTS NOT OCCURRING AT AN EPHEMERIS BODY. THESE MIDCOURSE ΔV POINTS MAY BE INCLUDED IN TWO WAYS:

- TIME AND POSITION COORDINATES MAY BE ESTIMATED AND INPUT; ON USER OPTION, THE TIME AND/OR POSITION COORDINATES WILL BE OPTIMIZED
- AUTOMATIC IMPULSE ADDITION MAY BE REQUESTED



MULTIPLE REVOLUTIONS AND RETROGRADE MOTION

- MULTIPLE REVOLUTIONS AND/OR RETROGRADE MOTION ARE SPECIFIED BY INPUT
- TWO OPTIONS ARE PROVIDED FOR HANDLING MULTIPLE REVOLUTION SOLUTIONS:
 - THE NUMBER OF COMPLETE REVOLUTIONS AND ENERGY CLASS MAY BE SPECIFIED
 - THE MAXIMUM NUMBER OF COMPLETE REVOLUTIONS TO CONSIDER MAY BE ENTERED IN WHICH CASE ALL INCLUSIVE SOLUTIONS WILL BE EXAMINED AND THE "BEST" ONE SELECTED ON THE BASIS OF A VELOCITY CHANGE CRITERIA



IR&D ENHANCEMENTS

- ADDITION OF THE UNPOWERED SPECIFICATION FOR PLANETARY SWINGBYS
- A NEW DEPARTURE OPTION OF SPACE STATION LAUNCHES
- A NEW TERMINAL ARRIVAL OPTION OF 3-IMPULSE PLANET ORBIT CAPTURE TO A FINAL ORBIT DETERMINED BY USER INPUT r_p , r_a , AND INCLINATION
- INCLUSION OF JPL SATELLITE EPHEMERIDES ROUTINES FOR MOST NATURAL SATELLITES
- CONVERSION OF THE WORKING COORDINATE SYSTEM FROM EMO50 TO J2000
- ABILITY TO CONSTRAIN TOTAL TRANSIT TIME


Science Applications International Corporation



RKSA - Applications Branch

Phillips Laboratory Applications of POST

*James B. Eckmann
SPARTA, Inc.
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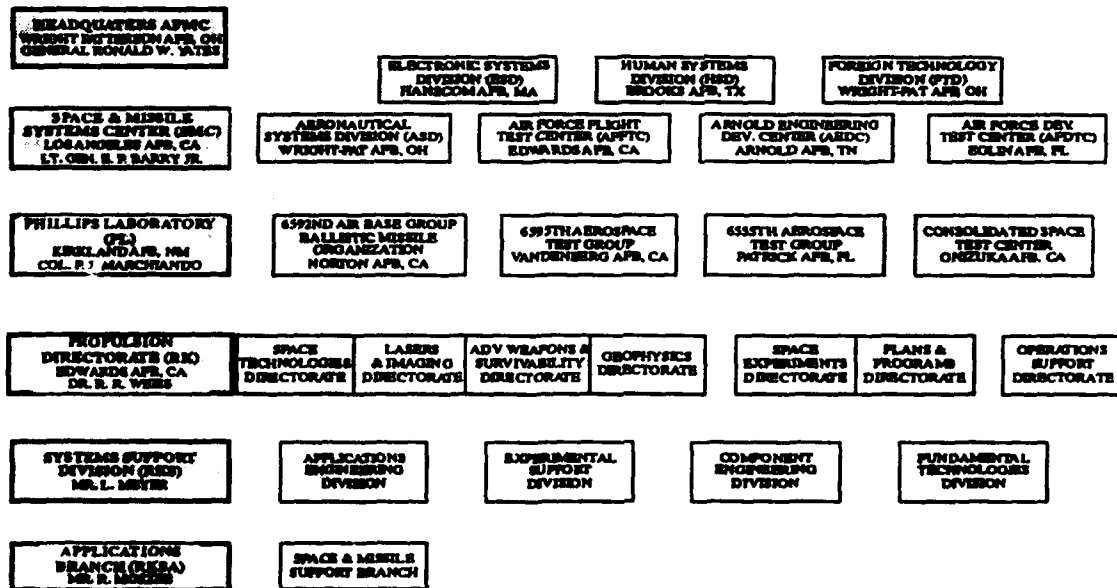
Presentation Overview

- Organization and Mission
- Simulation Work Environment
- Summary of POST Models
 - Applications and Some Results
- Future Plans



RKSA - Applications Branch

Organizational Hierarchy





RKSA - Applications Branch

Propulsion Directorate (RK) Mission

- Provide propulsion technology and expertise for U.S. space and missile systems.
- Be a center of excellence in propulsion research and development.
- Develop a broad, advanced technology base for future propulsion system designers.
- Demonstrate propulsion concepts for current systems designers.
- Assist in solving operational problems.



RKSA - Applications Branch

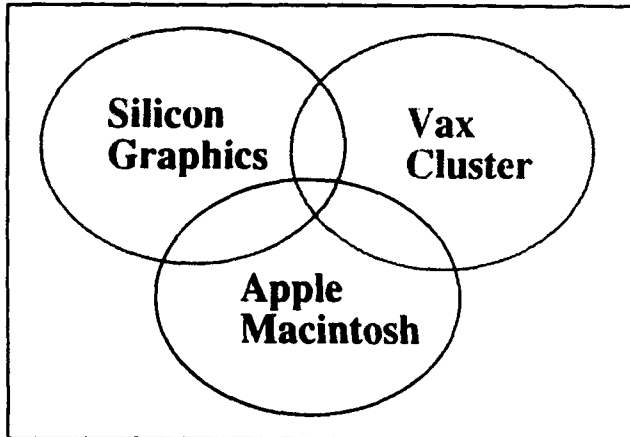
System Support Division, Applications Branch (RKSA)

- Mr. Raymond Moszee, Branch Chief
 - Capt. Tim Middendorf
 - 1 Lt. Paul Castro
 - 2 Lt. Naftali Dratman
 - Mr. Francis McDougall
 - Mr. Gerry Sayles
 - Ms. Pamela Tanck, SPARTA
 - Mr. James Eckmann, SPARTA
 - Maj. Leo Matuszak, AF Reservest



RKSA - Applications Branch

RKSA Simulation Environment



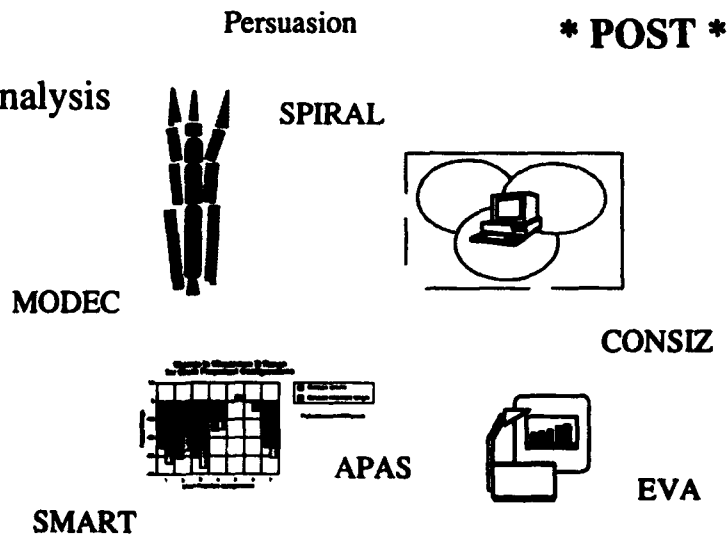
- Integrated Tools
- Ethernet Network (TCP/IP)
- Connectivity Software (NFS, Versaterm Pro)



RKSA - Applications Branch

Integrated Analysis

- Trajectories, Vehicle Analysis
- Sizing, Geometry
- Structures and Weights
- Presentation





Summary of POST Models

- Atlas II
- Delta
- Titan IV *
- Space Shuttle *
- Pegasus *
- Ground Based Interceptors
- Small ICBM
- Minuteman III
- National Launch System (NLS)
- Single Stage To Orbit (SSTO)
 - Delta Clipper - VTVL Concept *
 - RASV - HTHL Concept
 - RST - VTHL Concept *



Applications of POST Models

- Atlas II
 - Monopropellants; High Energy Density Mater (HEDM) propellants; Composite shroud
- Delta
- Titan IV
 - Soviet RD-170 strap-on LRB's to replace SRB's
- Space Shuttle
 - Clean propellant SRM's
- Pegasus
 - Advanced Liquid Axial Stage (ALAS) as 4th stage; Potential booster for NASP program flight test experiment
- Ground Based Interceptors
 - Single-stage, two-stage, three-stage, and dual-pulse motor boosters; Standard Missile and SRAM 2 boosters for LEAP tests
- SICBM
 - Advanced ICBM studies baseline



Minuteman III Model

- APPLICATION:

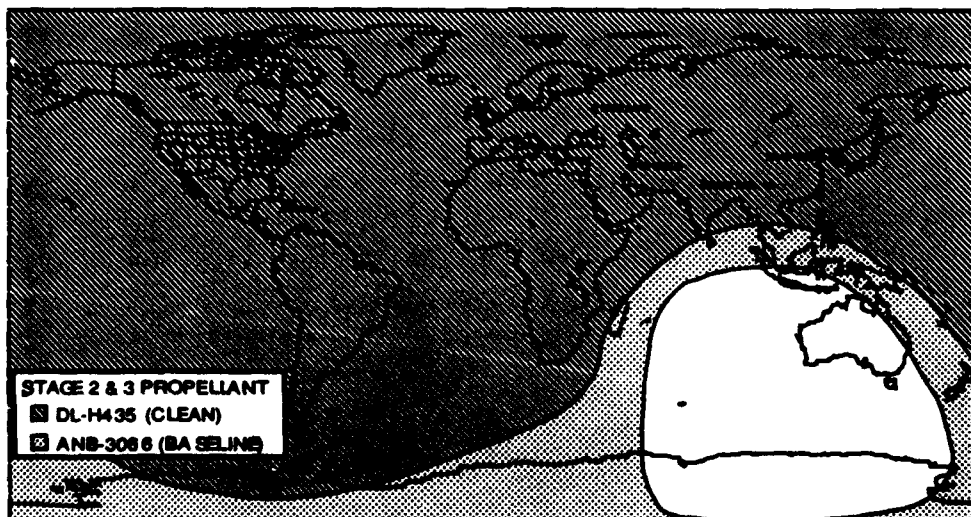
- The ICBM system of the future. Baseline for assessing advanced technology payoffs.
 - Clean Propellant Trade Studies
 - Impact of Reducing the Number of Warheads
 - Two-stage Missile Studies

- CONSTRUCTION:

- Objective Function: Maximum range for fixed payload or Maximum payload for fixed range
- Constraints (2): Maximum dynamic pressure, Minimum re-entry angle
- Control Variables (6): Pitch rate at motor ignition for each of 3 stages; Time at which inertial attitude is held constant for each of the 3 stages
- Phasing Events (16): 3 motor firings, 3 stage separations, initial pitch over, 3 constant attitude segments, 3 ballistic flight segments, payload shroud separation, atmospheric re-entry, ground impact.



Sample Minuteman III Results





RKSA - Applications Branch

National Launch System (NLS) Vehicles

Trade Studies Performed:

Castor 120's on NLS-3

Mixture Ratio

Tank Sizing

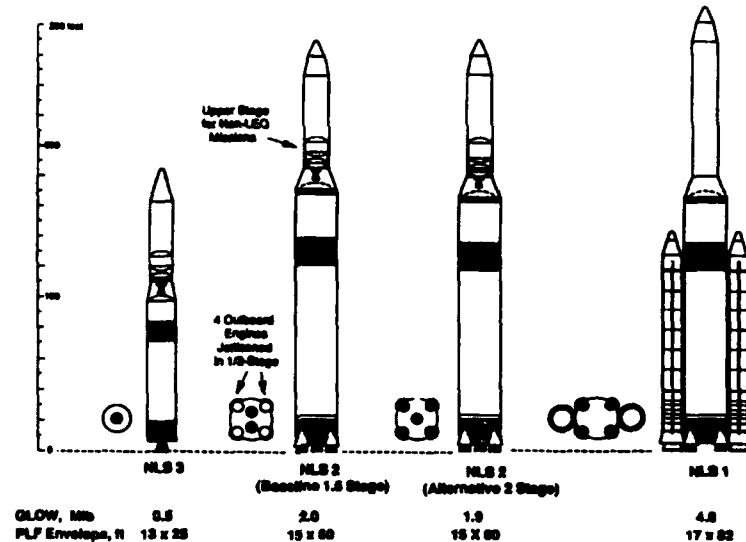
Thrust Level

Engine Out

Throttling Effects

Staging Algorithms

Upper Stages



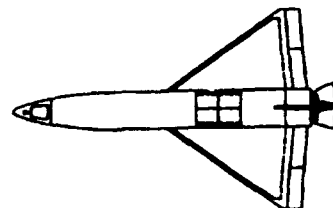
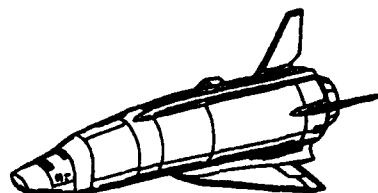
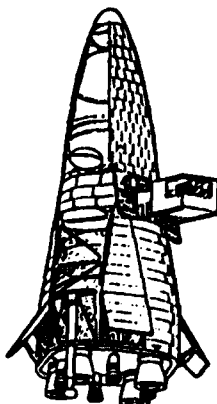
RKSA - Applications Branch

Single-Stage-To-Orbit (SSTO) Models

McDonnell Douglas Vertical Takeoff Vertical Landing (VTVL) Delta Clipper model developed and provided by NASA/Langley

Rockwell International Vertical Takeoff Horizontal Landing (VTHL) Reusable Space Transport (RST) model developed by Rockwell and provided by NASA/Langley

Boeing Horizontal Takeoff Horizontal Landing (HTHL) Reusable Aerodynamic Space Vehicle (RASV) model developed in-house





RKSA - Applications Branch

Future Plans

- **DEVELOPE A COMPLETE VEHICLE SIMULATION CAPABILITY**
 - Apply SMART and CONSIZE to current analysis tasks
 - Complete integration of Silicon Graphics machines
 - Develop a cost analysis capability
 - Continually evaluate new analysis tools

OTIS Advances at the Boeing Company

Steve Paris
Boeing Defense & Space Group

Optimal Trajectories by Implicit Simulation (OTIS) Development

Collocation based Optimal Control Methods

Chebytop ——— CTOP ———→

Indirect Trajectory Methods

Dickmanns ———→

TOP ———→

Direct Explicit Trajectory Schemes

AS2530 ——— NTOP / SPOT ———→

POST ———→



- State of the Art
- Optimal Control and Simulation of Aerospace Vehicles
- Flexible Operation
- Explicit Integration
- Collocation
- Non-linear Programming

OTIS Modes

Direct Trajectory Optimization
using Nonlinear Programming
and Collocation

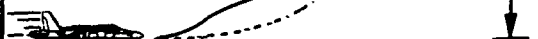
Mode 4 Trajectory Optimization
Optimal Control



Explicit Integration

- RK4
- Adams-Moulton
- Hermite
- Euler

Mode 3 Trajectory Optimization
Parameter



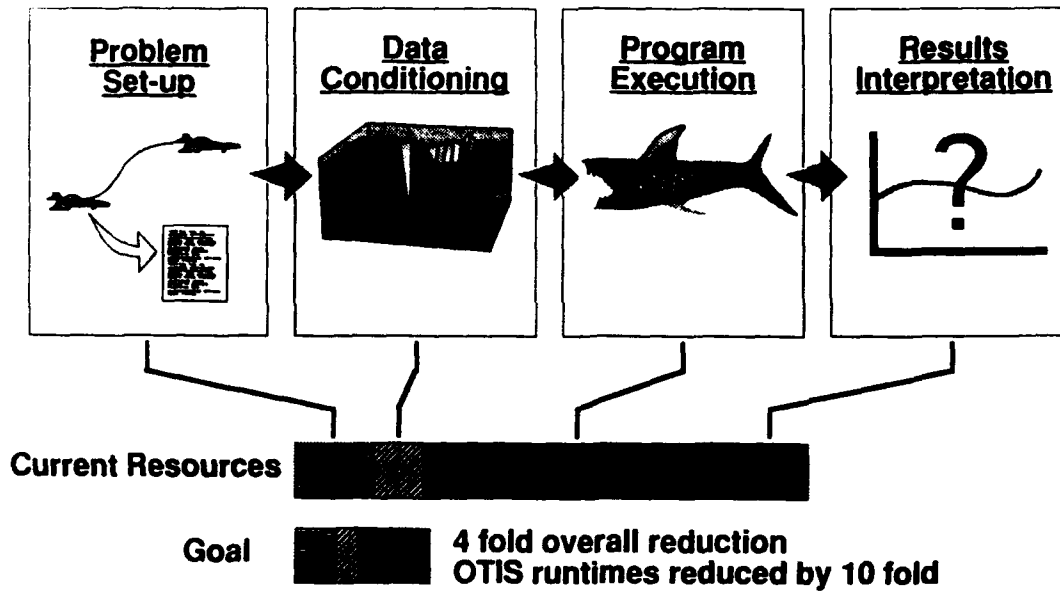
Mode 2 Targeting
Shooting



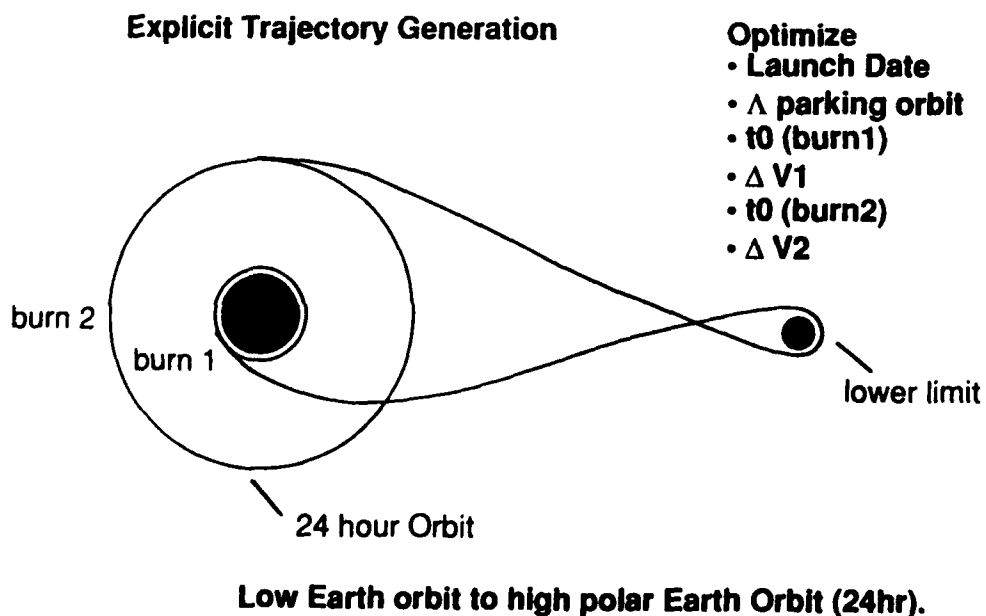
Mode 1 Trajectory Simulation
GIGO



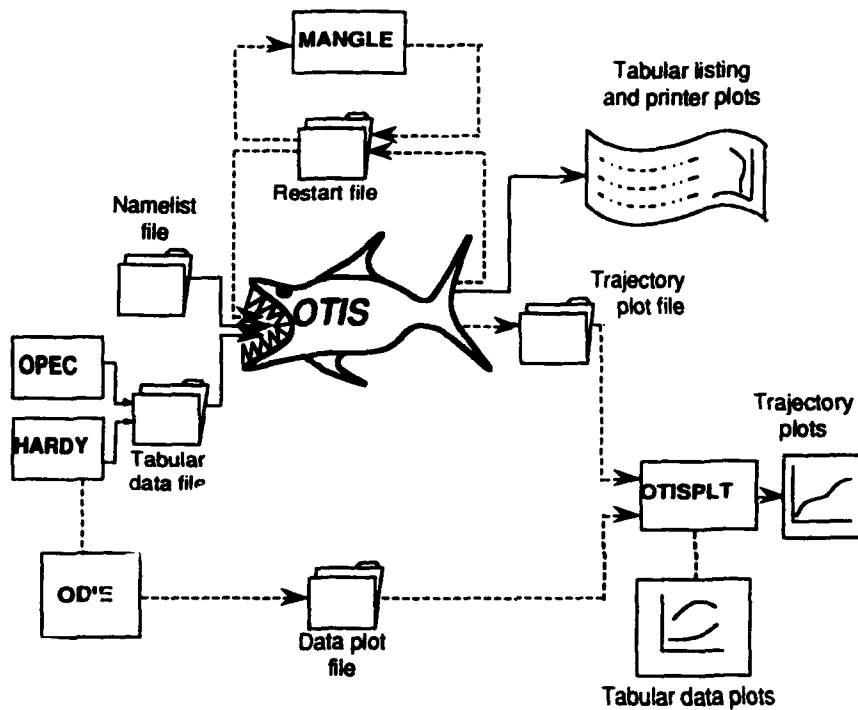
Next Wave of OTIS Advances



OTIS 3.0 Lunar Test Case

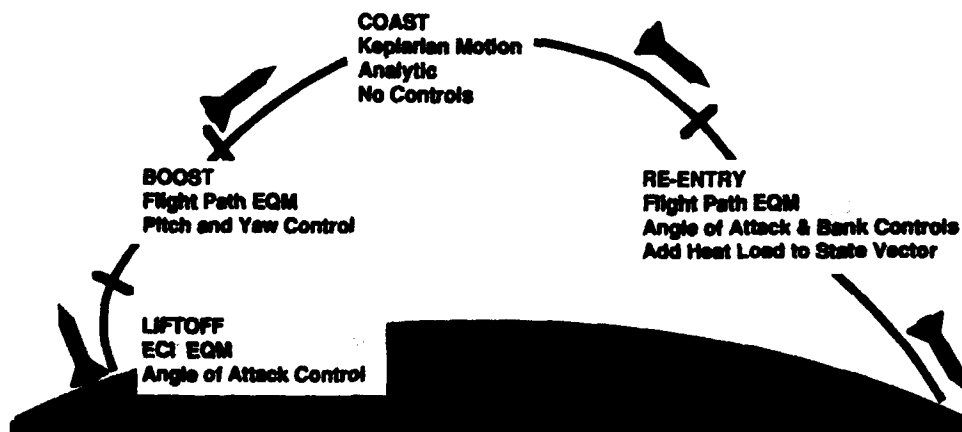


OTIS Elements

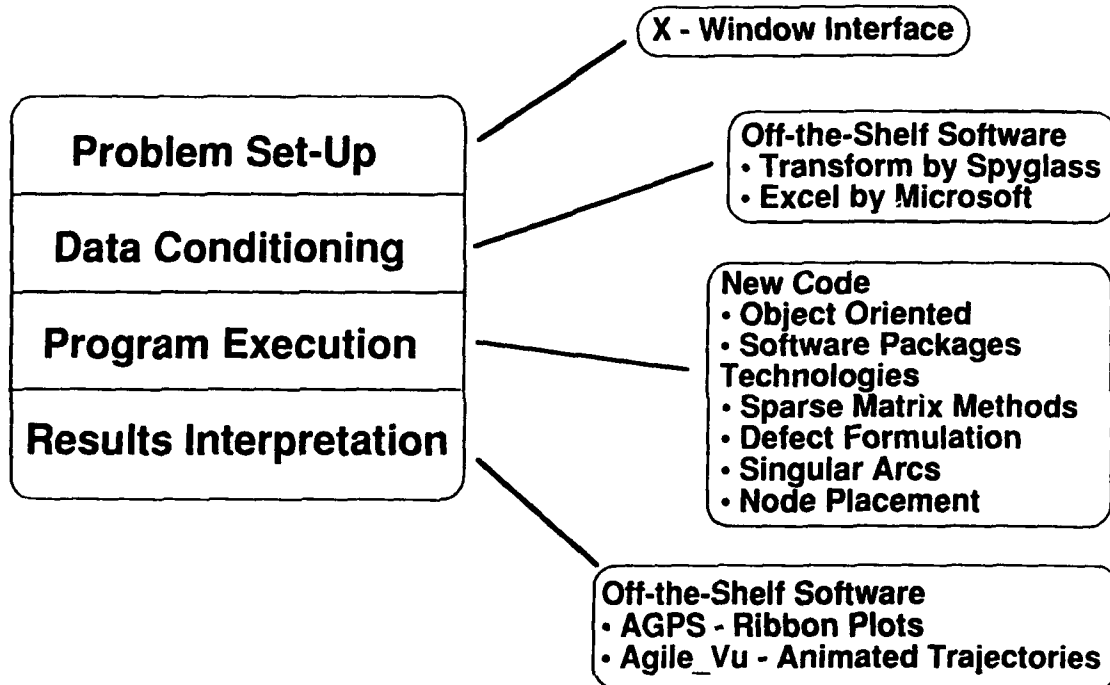


OTIS 3.0 Provides Extreme Flexibility

- Global Constraints
- Analytical Arcs
- Phase Dependent
 - Equations of Motion (EQM)
 - Control Variables
 - Quadrature Variables



Future Trends



Summary

- **Boeing Continues OTIS Development**
- **Focus on Speed & Usability**
- **Exploit Off-the-Shelf Software**
- **Goal is a "Better" OTIS**

— A F S —

**OTIS ACTIVITIES
AT
MCDONNELL DOUGLAS SPACE SYSTEMS COMPANY**

**R. L. NELSON
10 AUGUST 1992**

R. Nelson/2

— *McDonnell Douglas Space Systems Company* —

OTIS ACTIVITIES

— AFS —

- Applications
- OTIS Project Development (PD 1-301) at MDSSC-HB
- Launch Vehicle Sizing: ELVIS/OTIS
- OTIS upgrades for Wright Labs-AFB

— McDonnell Douglas Space Systems Company —

R. Nelson/2

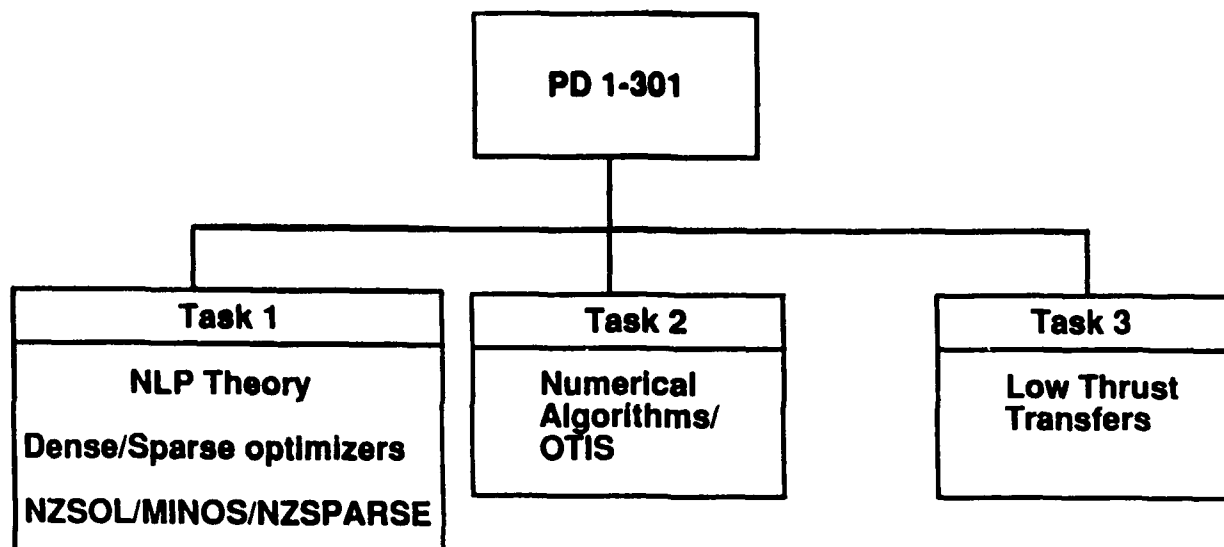
ADVANCED APPLICATIONS PD 1-301 OPTIMIZATION TECHNIQUES FOR ADVANCED SPACE MISSIONS

— MDSSC-HB —

- ☐ Theater High Altitude Area Defense (THAAD)
- ☐ ENDO/EXO Atmospheric Interceptor (E²I)
- ☐ HEDI
- ☐ DELTA
- ☐ National Aerospace Plane (NASP)
- ☐ SSRT
- ☐ Aerobrakes
- ☐ Hypersonic Advanced Weapon (HAW)
- ☐ Fighter Aircraft
 - Evasive maneuvers
 - Agility
- ☐ Military Space
- ☐ Space Transfer Vehicles

TASK FLOW
PD 1-301 OPTIMIZATION TECHNIQUES FOR
ADVANCED SPACE MISSIONS

MDSSC-HB



TASK 1-APPROACH (1992-1993)-NLP THEORY
PD 1-301 OPTIMIZATION TECHNIQUES FOR
ADVANCED SPACE MISSIONS

MDSSC-HB

- ☐ Develop strong robust globally convergent nonlinear optimizers
 - Dense and sparse optimizers
 - NZSOL, a dense optimizer
 - Initial feasibility algorithms
 - Min - Max optimizer
 - NZSPARSE, a sparse optimizer
 - Dr. Phillip E. Gill, Professor, University of California, San Diego
 - Dr. Michael Saunders, Research Professor, Stanford University

PD 1-301

TASK 1 - PROGRESS-NLP THEORY PD 1-301 OPTIMIZATION TECHNIQUES FOR ADVANCED SPACE MISSIONS

MDSSC-HB

- ☐ **Developed State of the Art optimizer, NZSOL (dense)**
 - **NPSOL 2.1**
 - **NPSOL 4.02**
 - **Continual testing**
 - **Tuned NZSOL for OTIS type problems**
- ☐ **BREAKTHROUGH ALGORITHM: NZSPARSE (sparse)**
 - **Theoretical formulation**
 - **Development and checkout**
 - **MINOS**
- ☐ **Modified OTIS structure to accept dense / sparse optimizers**

PD 1-301

TASK 2 - APPROACH (1992-1993)-NUMERICAL ALGORITHM: PD 1-301 OPTIMIZATION TECHNIQUES FOR ADVANCED SPACE MISSIONS

MDSSC-HB

- ☐ **Algorithms for OTIS**
 - **Automatic scaling**
 - **Automatic node placement (University of Illinois)**
 - **Automatic tabular data smoothing**
 - **Lagrange multiplier Interpretation (Continuous / discrete)**
 - **Minimum curvature cubic control splines**

TASK 2 - PROGRESS - ALGORITHMS

— AFS —

- **Tabular Data Smoothing**
- **Enhanced Velocity Loss Model for Launch Vehicles**
- **Generalized Stage - Phase Concept for Sizing**
- **Automatic Node Placement**

TASK 3 - APPROACH (1992-1993)-LOW THRUST PD 1-301 OPTIMIZATION TECHNIQUES FOR ADVANCED SPACE MISSIONS

— MDSSC-HB —

- ☐ **Develop restricted and general 3-body equations of motion**
- ☐ **Boundary conditions and coordinate systems**
- ☐ **Quantify the transition region for earth-moon low thrust / weight transfers**
- ☐ **SECKSPOT / NASA Code - COSMIC Library**
 - **Strong gravity field**
 - **Orbit averaging techniques**
- ☐ **QT2 Interplanetary code**
- ☐ **Dr. Richard Shi, MDSSC-HB**

PD 1-301

SSS/ROCKY2/RLN

TASK 3 - PROGRESS-LOW THRUST PD 1-301 OPTIMIZATION TECHNIQUES FOR ADVANCED SPACE MISSIONS

MDSSC-HB

- ☐ Key Idea to solve problem
 - The existence of the Jacobi (energy) integral for the restricted three-body problem will aid us in the general three-body problem.
 - Zero velocity or zero energy curves are the regions where the low thrust earth-moon transfers are possible
 - No integral available for the general 3-body problem
- ☐ Develop for OTIS
 - 3-body equations of motion
 - boundary conditions
 - coordinate systems

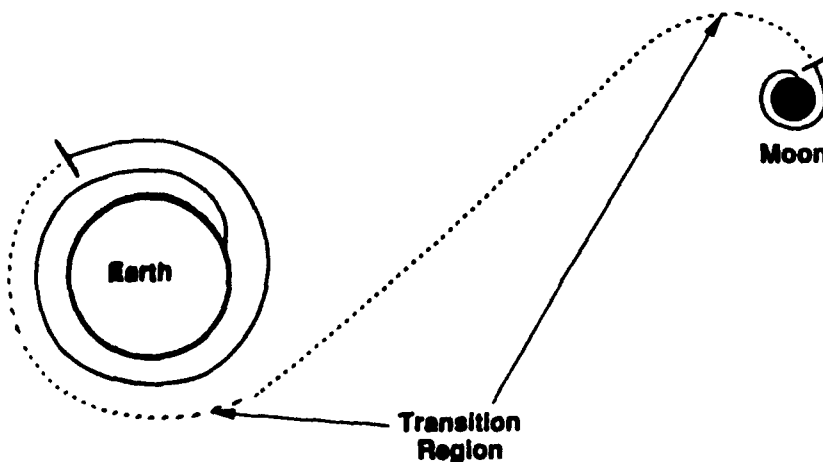
PD 1-301

553/ROCKY2/RLN/bm

PD 1-301 OPTIMIZATION TECHNIQUES FOR ADVANCED SPACE MISSIONS

VKB1563

MDSSC-SSD

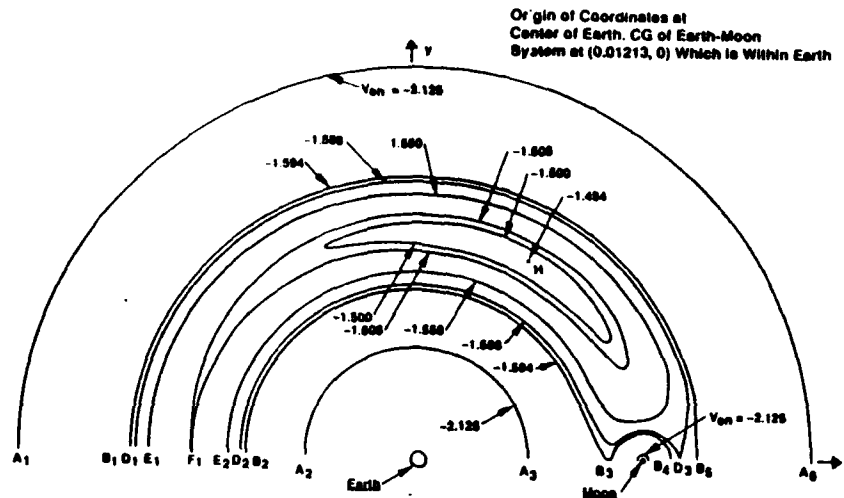


Earth-Moon Transfer

CONTOURS OF CONSTANT POTENTIAL ENERGY (REFERRED TO
ROTATING SYSTEM) IN PLANE $Z = 0$ WITH $\mu = 0.01213$

MDSSC-SSD

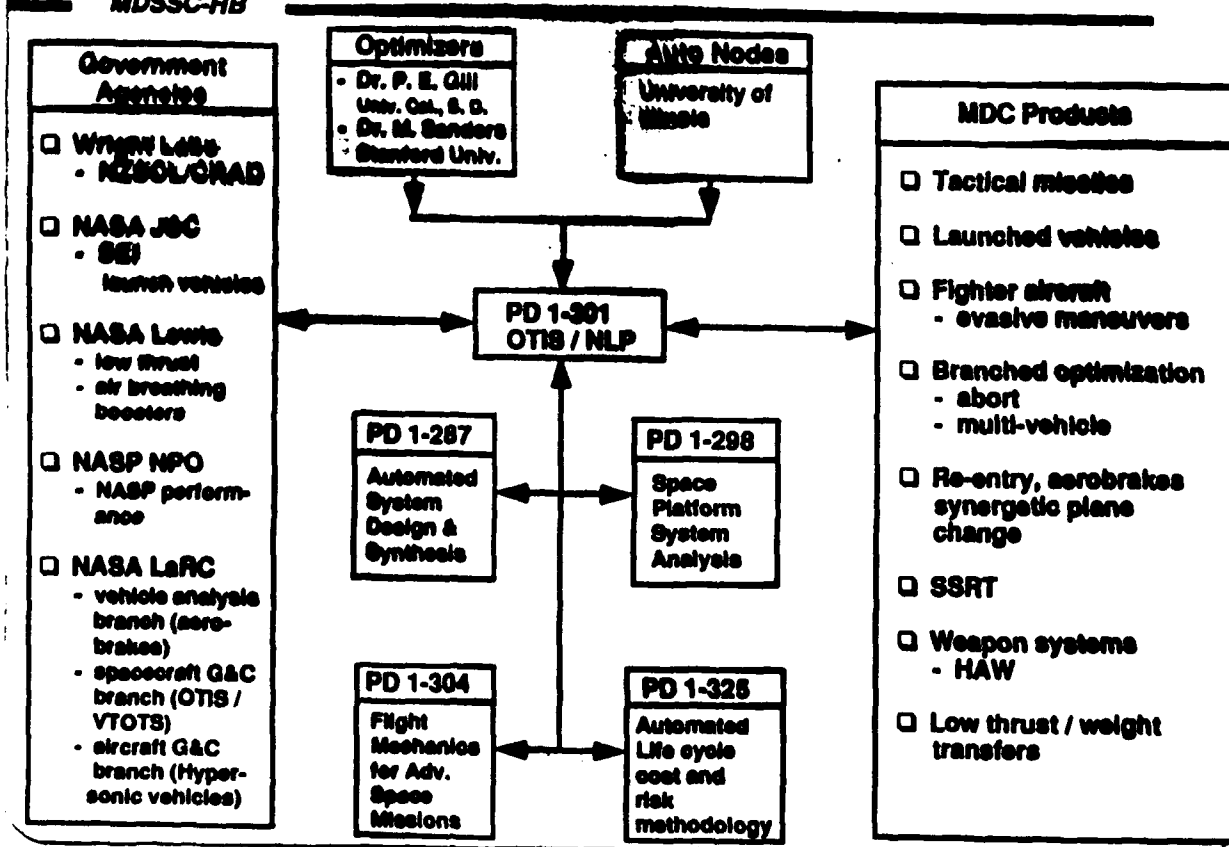
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PD 1-301

PROJECT INTERRELATIONSHIPS
PD 1-301 OPTIMIZATION TECHNIQUES FOR
ADVANCED SPACE MISSIONS

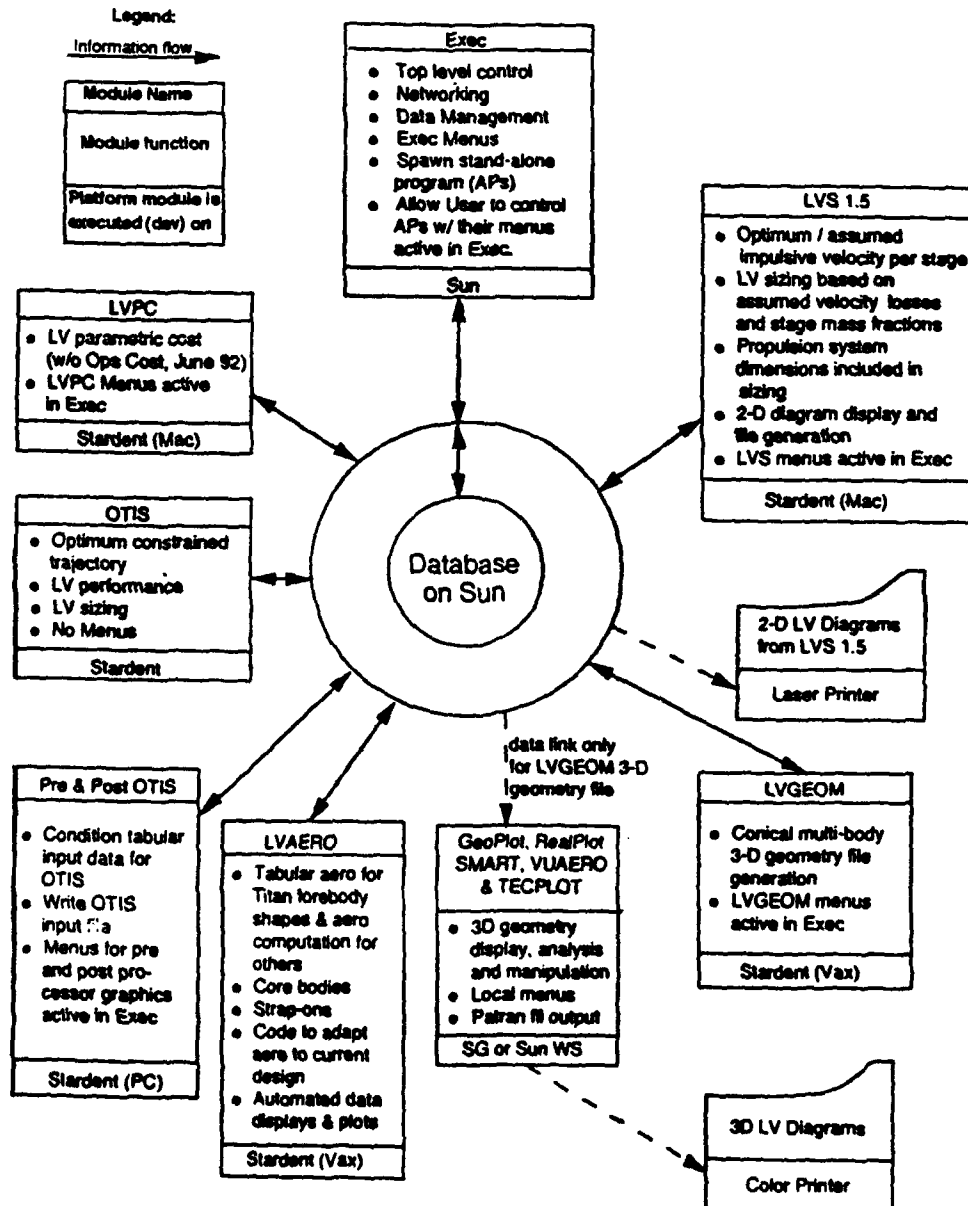
MDSSC-HB



ELVIS / OTIS ARCHITECTURE

R. Nelson/2

McDonnell Douglas Space Systems Company



OTIS UPGRADES FOR WRIGHT LABS

— AFS —

- TASK 1: NZSOL
- TASK 2: Automatic Variable Scaling
- TASK 3: Variable Names for NZSOL Output
- TASK 4: Minny Heating Model

McDonnell Douglas Space Systems Company

R. Nelson/2

**Advances in Trajectory
Optimization Using Collocation
and Nonlinear Programming**

**Bruce A. Conway
Dept. of Aeronautical & Astronautical
Engineering
University of Illinois
Urbana, IL**

August 1992

Outline

Introduction

Progress to Date - Theory

Progress to Date - Solved Problems

Continuing and Proposed Research - Problems

Progress - Theory

1. **Use of costates to improve an optimal trajectory.**

Lagrange multipliers for the discrete (NPSOL) solution are a representation of the Lagrange multipliers of the continuous case. (Enright & Conway, JGC&D 15, No. 4, 1992)

Knowledge of the Lagrange multipliers allows a posteriori determination of the optimality of the solution, e.g., can examine the switching function.

2. **Generalized defects**

Can be used when the differential equation for a state variable is integrable, e.g., on a coast arc.

May significantly reduce the number of NLP parameters and hence execution time.

3. **Coordinate transformation within the H-P structure**

Necessary for orbit transfer when changing sphere of influence

Keeps state variables near one order of magnitude, as NPSOL prefers

4. Method of parallel shooting

Replaces single Hermite-Simpson "integration step" with multiple Runge-Kutta steps allowing use of larger intervals.

Results in smaller NLP problems for a given accuracy.

5. Automatic node placement

Computer solves a succession of NLP problems in which additional nodes are inserted as needed to achieve a given accuracy.

More efficient than using a uniform distribution of nodes

6. Neighboring optimal feedback control

Determines gains for linear feedback controller to yield neighboring optimal controller

Unnecessary to solve NLP problem for small change in initial or terminal conditions

Feedback gain history easily loaded into small memory

Illustration of Generalized Defects

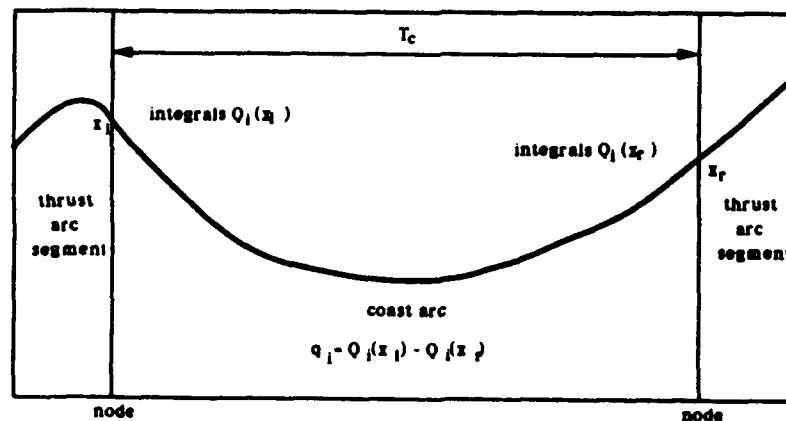
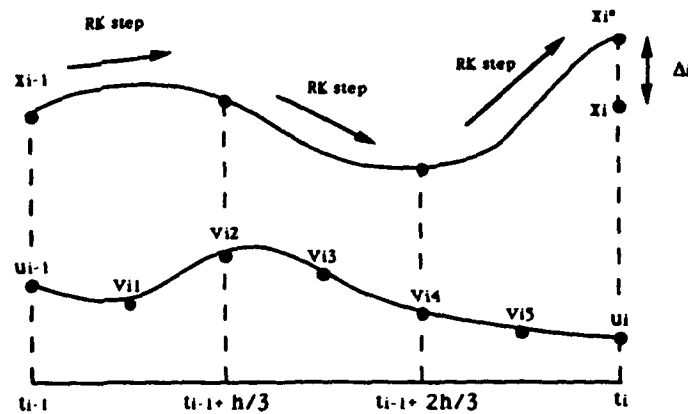


Illustration of Parallel Shooting

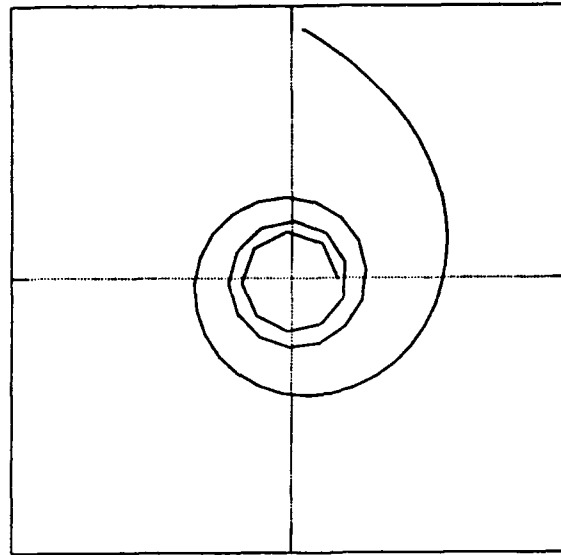


p - 3

Progress - Solved Problems

1. Optimal low-thrust escape trajectory (Enright Ph. D. thesis)
2. Optimal 2 and 3 burn circle-circle low-thrust rendezvous (Enright Ph. D. thesis)
3. Optimal low-thrust Earth-Moon transfer (Enright Ph. D. thesis)
4. Optimal spacecraft detumbling (A. Herman M.S. thesis)
5. Optimal low-thrust insertion- Mars Observer (Enright Ph. D. thesis)
6. Optimal 2D and 3D direct ascent time-bounded interception (J. Downey Ph. D. thesis)
7. Neighboring optimal feedback control for continuous-thrust ascent maximizing horizontal velocity (F. Chen Ph. D. research)

Low-Thrust Minimum Fuel Escape



$\mu = 1$
 $r_i = 1$
 $a_i = 0.0125$
 $t_{\text{final}} = 16\pi$
 All in canonical units

Method	Variables	CPU
Hermite/Simpson (60)	427	190 sec
Parallel shooting (34 x 3)	385	95 sec
Parallel shooting (5 x 20)	270	72 sec

Optimal 2 and 3 Burn Circle-Circle Rendezvous

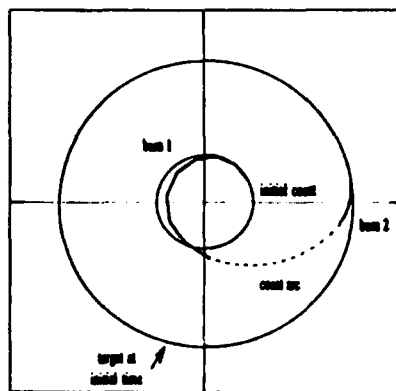


Fig. 3 Two-burn rendezvous trajectory.

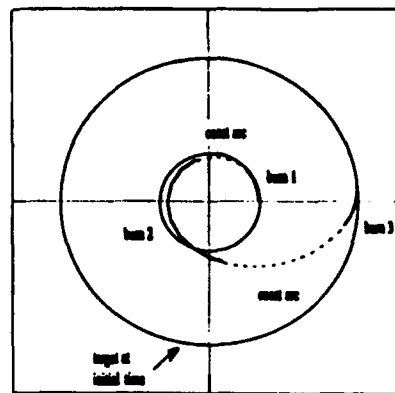


Fig. 4 Three-burn rendezvous trajectory.

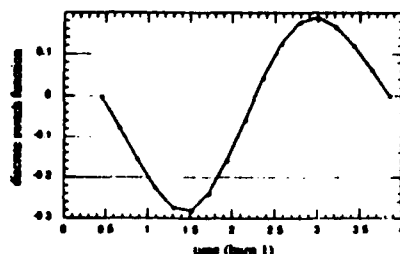


Fig. 4 Two-burn rendezvous discrete switch function for burn 1.

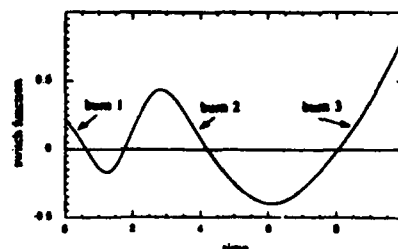


Fig. 5 Three-burn rendezvous switch function.

Optimal Low-Thrust Earth-Moon Transfer

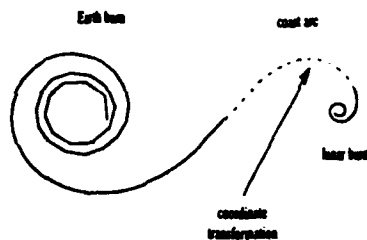
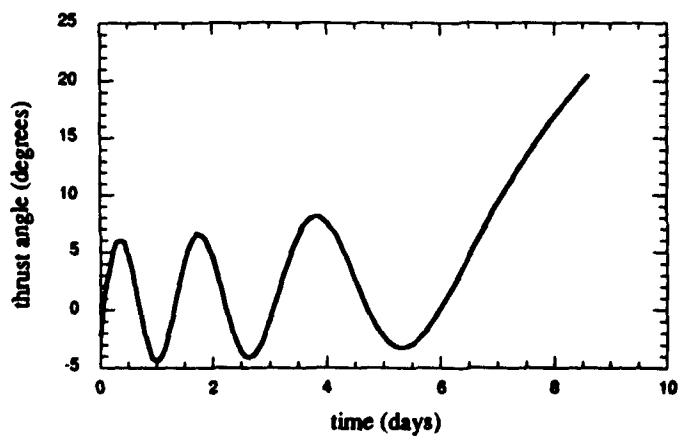
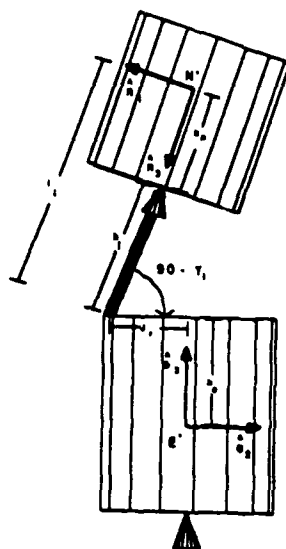


Fig. 7 Earth-moon transfer trajectory.

Optimal Low-Thrust Earth-Moon Transfer, cont'd



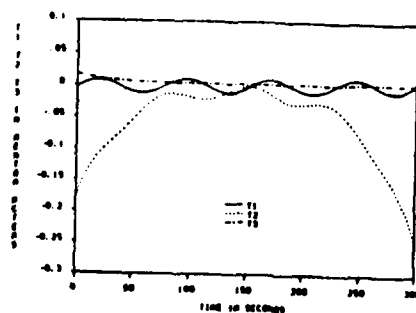
Optimal Spacecraft Passivation (Detumbling)



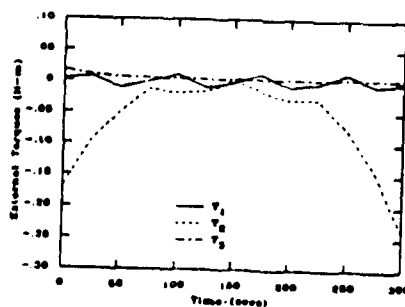
View of OMV / Disabled Satellite System

Spacecraft Passivation, cont'd.

Results from the
TPBVP solver



Results from the
NLP method



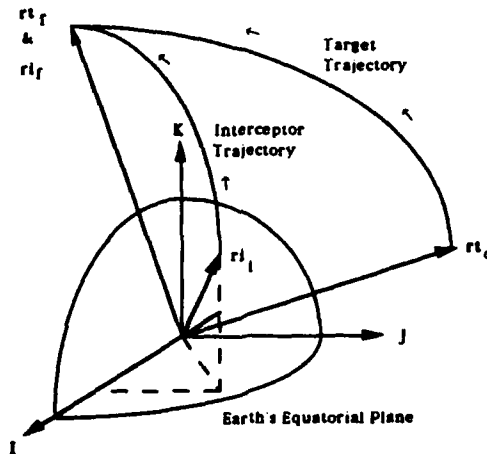
External Torque Histories

Optimal 2D & 3D Direct -Ascent Interception

- Target is assumed to be in a general Keplerian orbit with orbital elements

$$\mathbf{E}^T = [a, e, i, \Omega, \omega, f]$$

- Geometry of the problem



Continuing Research - Problems

1. Automatic node placement. (A. Herman)
2. Optimal very-low-thrust trajectories (W. Scheel)
3. Optimal Earth-Mars low-thrust transfer including escape and arrival spirals and coordinate transformations at sphere of influence of each planet. (S. Tang)
4. Neighboring optimal feedback control for complex problems
Automation of NOFC using symbolic programming (F. Chen)
5. Optimal trajectories for interception of Earth-crossing asteroids (B. Conway)

**FLIGHT PATH OPTIMIZATION OF
AEROSPACE VEHICLES USING
OTIS**

Rajiv S. Chowdhry

**Lockheed Engineering & Sciences Company
MS 489
Aircraft Guidance & Control Branch
NASA, LaRC, Hampton VA.**

Outline

- **Accuracy of OTIS solutions.**
- **Overview of OTIS applications at AGCB**

Accuracy of OTIS Solutions

OTIS : Optimal control solutions via direct transcription
Combination of collocation and nonlinear programming

Question :

How do OTIS solutions compare to the "exact" or TPBVP solutions ?

- **Analytical Approach**

estimate adjoint variables, examine discretized necessary conditions

- **Engineering Approach**

numerical comparison of collocation solution to exact solution for a representative problem

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numerical comparison of collocation solution to exact solution for a representative problem

Example : ALS Ascent to Orbit

Example Problem :

Steer a two stage launch vehicle from a given initial condition to a specified target orbit in minimum fuel.

- **Exact or TPBVP solution available in literature (Ref. Hans Seywald and E. M. Cliff)**
- **Care was taken to keep the vehicle/atmosphere/planet models same in OTIS**
- **Only solution methodologies were different**

Comparison of Optimal ALS Ascent with OTIS solutions

	OTIS Solutions				TPBVP solution
	12 nodes	22 nodes	32 nodes	40 nodes	
t_f sec	477.2	477.2	477.2	477.2	477.2
Mass (t_f) Kgs	149,881	149,877	149,895	149,891	149,900
Velocity (t_f) m/sec	7855	7855	7857	7857	7857
Altitude (t_f) km	148.68	148.74	147.80	147.96	148.2
Apogee (km)	274.58	274.11	279.52	278.77	277.81
Perigee (km)	148.53	148.66	147.8	147.9	148.16
CPU Time (secs)	74	377	935	1675	?

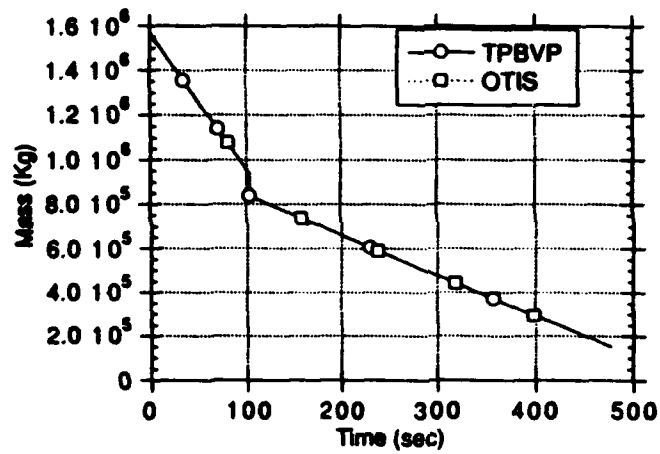


Figure [1]. Comparison of optimal ALS ascent with OTIS solution, mass (kg) vs. time.

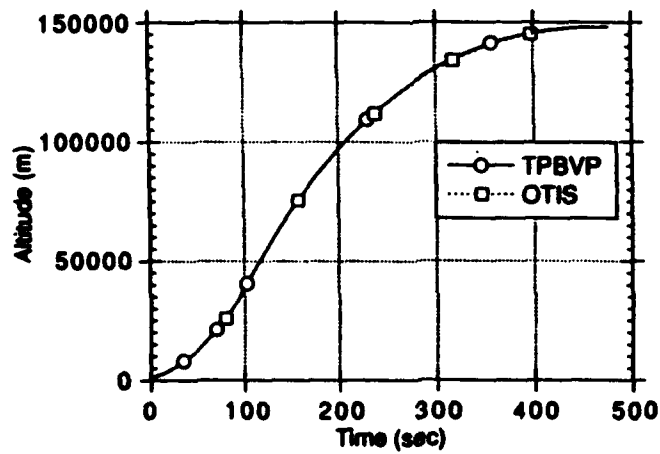


Figure [2]. Comparison of optimal ALS ascent with OTIS solution, altitude above spherical Earth vs. time.

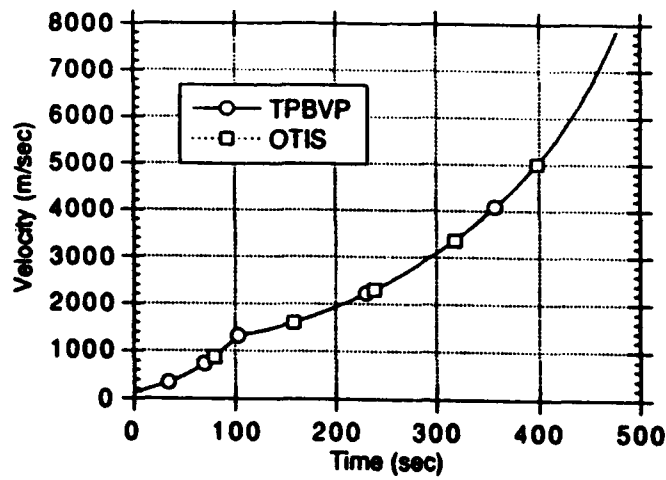


Figure [3]. Comparison of optimal ALS ascent with OTIS solution, Earth relative velocity (m/sec) vs. time.

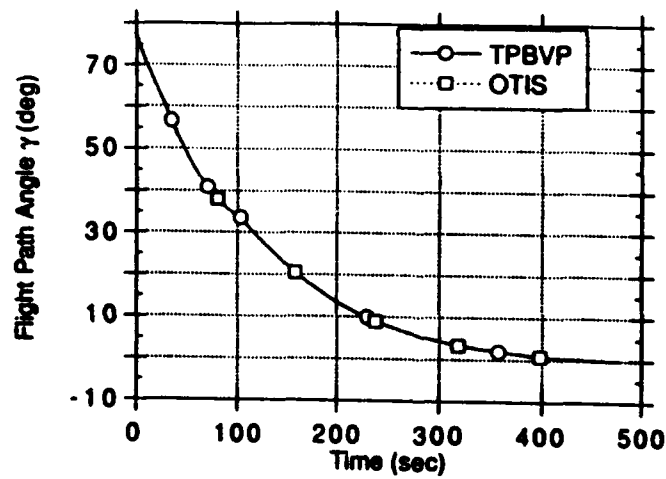


Figure [4]. Comparison of optimal ALS ascent with OTIS solution, local horizontal flight path angle (deg) vs. time.

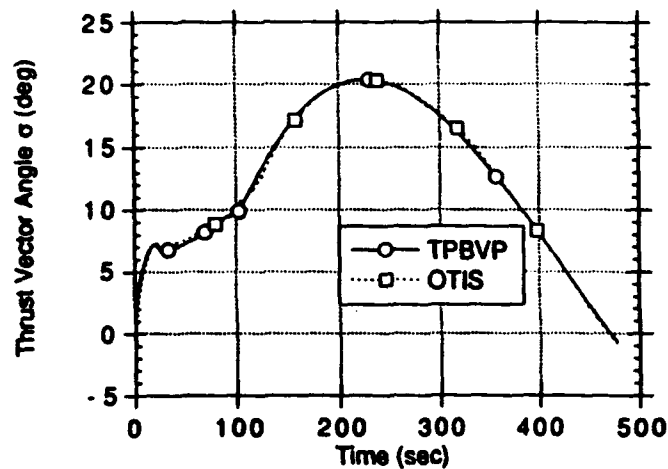


Figure [5]. Comparison of optimal ALS ascent with OTIS solution, thrust vector angle (deg) vs. time.

OTIS Applications at AGCB

- Fuel efficient ascent for SSTD airbreathing hypersonic vehicle.
- fuel optimal path definition for G&C studies

- HL-20 abort maneuvers : ELSA (Efficient Launch Site Abort)
- Parameter sensitivity studies to support design activities.
- Guidance algorithm development & real time validation.

- ALS ascent for OTIS calibration.

- Optimal maneuvers for a high performance fighter aircraft (HARV) in air combat situation.

Conclusions

For the ALS Ascent Problem :

- **Excellent match of the collocation solution to the TPBVP solution**
- **Relatively quick turnaround time for OTIS solutions**
- **Very robust to initial guesses**

Trajectory Optimization of Launch Vehicles at LeRC: Present and Future

**Presented by
Koorosh Mirfakhraie**

**at
Workshop on Trajectory Optimization Methods and Applications**

**Hilton Head, SC
August 10, 1992**

**ANALEX
CORPORATION**
KM 8/10/92

NASA Lewis Research Center
Advanced Space Analysis Office

Outline

- **Introduction**
- **Present method of solution and code**
- **Capabilities of the present code**
- **Motivation for replacing the code (and method)**
- **Examination of methods using collocation**

Introduction

Trajectory optimization* of ELV's at the Advanced Space Analysis Office at LeRC is performed for:

- **Mission design for approved programs**
- **Feasibility and planning studies**
- **Corroboration of contractors' data for NASA missions flown on Atlas and Titan**

*** Trajectory optimization: Maximizing the final payload subject to a set of intermediate and final constraints.**

Introduction (Cont'd)

Mission profiles for launch vehicle systems with booster and upper stages include:

- **Launches from ER and WR**
- **LEO, GTO, and GSO insertion**
- **Interplanetary escape trajectories**
- **Orbit transfers**

Present Method of Solution and Code

- **Calculus of Variations is used to formulate the problem. The resulting two point boundary value problem is solved using a Newton-Raphson algorithm.**
- **The computer program (DUKSUP) was written entirely at LeRC during 1960's and early 70's.**
- **DUKSUP is a 3-D.O.F. code written for performance analysis of multi-stage high-thrust launch vehicles.**

DUKSUP Features

- Detailed modeling (e.g., propulsion and aerodynamic) of a launch vehicle is possible.
- A variety of constraints can be imposed on the model. They include:
 - Instantaneous and total aerodynamic heating
 - Maximum dynamic pressure
 - Parking orbit parameters (e.g., radius of perigee, energy, velocity, etc.)
 - G-limit staging

DUKSUP Features (Cont'd)

- Several in-plane and out-of-plane final target conditions can be specified (e.g., energy, radius, true anomaly, inclination, declination of outgoing asymptote, etc.).
- Variables free for optimization include:
 - Upper stage burn and coast times
 - 'Kick angle'
 - Payload fairing jettison time
 - Thrust angle in the non-atmospheric flight

Motivation for Replacing DUKSUP

- **Sensitivity to initial guesses**
- **Difficulty in reformulating the C.O.V. problem when adding new features and constraints to the code**
- **Difficulty in modifying and expanding the code due to lack of documentation and outdated programming practices**

Examination of Methods Using Collocation

- **Two main features of collocation making it attractive are**
 - **Lack of sensitivity to initial guesses**
 - **Relative ease of formulation**
- **Concerns about using collocation for ELV optimization are**
 - **Ability to handle complex modeling requirements and constraints typical of ELV flight**
 - **Computer run time**
 - **Fidelity of the solution vis a vis C.O.V.**
- **Evaluation of collocation uses DUKSUP as the benchmark for comparison.**

Using Collocation (Cont'd)

- **Available collocation codes are used as testbeds with necessary modifications.**
- **A simple LV model is used first and moved progressively to a full DUKSUP model.**
- **Enright's orbit transfer program was used for the first simple model comparison. Results matched those of DUKSUP.**
- **OTIS is used for the more sophisticated comparisons.**
- **OTIS is currently used to model an Atlas II/Centaur to LEO.**

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NASA Lewis Research Center
Advanced Space Analysis Office

**Collocation Methods in Regular Perturbation Analysis
of Optimal Control Problems***

August 10, 1992

Prepared for

**Workshop on Trajectory Optimization Methods and Applications
AIAA Guidance, Navigation, and Control Conference
Hilton Head, SC**

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* See conference paper no. 92-4304. Research supported by NASA Langley under grant No. NAG-1-939

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Overview

Motivation

Regular Perturbation Analysis

The Method of Collocation

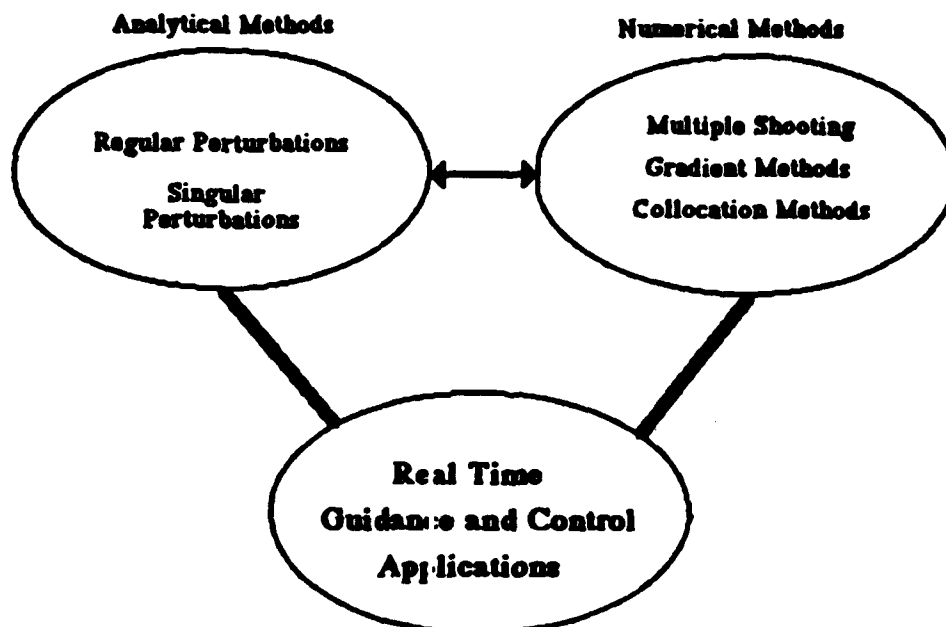
Hybrid Collocation / Regular Perturbation Analysis Approach

Examples

Duffing Equation

Launch Vehicle Guidance Application (presented at 1-GNC-1)

Motivation



Advantages / Disadvantages

Analytical Methods

Approximates solution by expansion in an asymptotic series in a small parameter

**Zero order problem is simpler to solve $\Rightarrow$ Insight
Higher order problems are linear**

Zero order problem must reasonably approximate the full order problem

For practical applications, zero order problem must be analytically tractable or reducible to a simple algebraic problem

Significant amount of analysis is required for each problem formulation of interest

Advantages / Disadvantages (continued)

Numerical (Collocation) Methods

Finite element method that enforces interpolatory constraints at specific points within each element

Simple to use for a wide variety of optimization problems

Large dimensional nonlinear programming problem

No general guarantee of convergence

Note: Advantages of analytical and numerical methods are in many respects complimentary in the sense that if the advantages can be combined in some way, then most of the important disadvantages for real-time applications can be removed.

Regular Perturbations in Optimal Control

Given:

$$\frac{dx}{dt} = f(x, u, t) + \varepsilon g(x, u, t); \quad x(t_0) = x_0$$

$$J = \phi(x, t) \big|_{t_f}$$

Find the control that minimizes J subject to the terminal time constraints:

$$\psi(x, t) \big|_{t_f} = 0$$

Optimality condition:

$$H_u = 0 \quad \text{assuming } H_{uu} > 0 \quad \Rightarrow \quad u = U(x, \lambda, t)$$

where:

$$\begin{aligned} H &= \lambda^T \{f + \varepsilon g\}; & H(t_f) &= -(\phi_t \big|_{t_f}); & \Phi &= \phi + \lambda^T \psi \\ d\lambda/dt &= -H_x; & \lambda(t_f) &= \Phi_x \big|_{t_f} \end{aligned}$$

Regular Perturbation Analysis

Based on a simplified model (when ε is set to zero)

- Treat neglected dynamics as perturbation
- Define a normalized independent variable, $\tau = (t - t_0) / T$
where $T = t_f - t_0$
- Compute zero order solution

Consider an asymptotic series in x , λ , and T

Evaluate high order corrections from sets of nonhomogeneous, time-varying linear O. D. E's.

$$\frac{d}{d\tau} \begin{bmatrix} x_k \\ \lambda_k \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_k \\ \lambda_k \end{bmatrix} + \frac{T_k}{T_0} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} P_{1k} \\ P_{2k} \end{bmatrix}$$

enforcing all boundary conditions to k th order

Compute feedback control at current time (t_0) using $x(t_0)$ and k th order approximation for $\lambda(t_0)$

Regular Perturbation Analysis (continued)

- A's and C's depend only on the zero order ($k = 0$) values.
- C's are the explicit correction term for free final time, T.
- P's are the forcing functions involving lower order ($k-1, \dots, 1, 0$) terms.

Solution:

$$\begin{bmatrix} \mathbf{x}_k(\hat{t}) \\ \lambda_k(\hat{t}) \end{bmatrix} = \Omega_A(\hat{t}, t_0) \begin{bmatrix} \mathbf{x}_k(t_0) \\ \lambda_k(t_0) \end{bmatrix} + T_k \frac{\hat{t} - t_0}{T_0} \begin{bmatrix} \dot{\mathbf{x}}_0(\hat{t}) \\ \dot{\lambda}_0(\hat{t}) \end{bmatrix} + \int_{t_0}^{\hat{t}} \Omega_A(\hat{t}, \tau) \begin{bmatrix} P_{1k}(\tau) \\ P_{2k}(\tau) \end{bmatrix} d\tau$$

Higher order correction involves simple operations of quadrature and solution of linear algebraic equations

Can be easily modified to account for discontinuous dynamics

Solution of Optimal Control Problems by Collocation

Methodology

- a finite element approach
- approximates the solution with interpolating functions
- consider first order polynomials

$$\mathbf{x}(\hat{t}) = \mathbf{x}_{j-1} + \mathbf{p}_j(\hat{t} - \hat{t}_{j-1}) \quad ; \quad \lambda(\hat{t}) = \lambda_{j-1} + \mathbf{q}_j(\hat{t} - \hat{t}_{j-1}) \quad ; \quad j = 1, 2, \dots, N$$

- enforce the derivative constraints at the mid point of each element

$$\mathbf{p}_j = \frac{\mathbf{x}_j - \mathbf{x}_{j-1}}{\hat{t}_j - \hat{t}_{j-1}} = \frac{\partial \mathbf{H}}{\partial \lambda} \Big|_{\hat{t}=(\hat{t}_j + \hat{t}_{j-1})/2; \mathbf{x}=(\mathbf{x}_j + \mathbf{x}_{j-1})/2; \lambda=(\lambda_j + \lambda_{j-1})/2}$$

$$\mathbf{q}_j = \frac{\lambda_j - \lambda_{j-1}}{\hat{t}_j - \hat{t}_{j-1}} = - \frac{\partial \mathbf{H}}{\partial \mathbf{x}} \Big|_{\hat{t}=(\hat{t}_j + \hat{t}_{j-1})/2; \mathbf{x}=(\mathbf{x}_j + \mathbf{x}_{j-1})/2; \lambda=(\lambda_j + \lambda_{j-1})/2}$$

- N is the number of elements, $\mathbf{x}_j$ and λ_j are nodal values
- control assumed to be eliminated using optimality condition

Hybrid Collocation / Regular Perturbation

A Regular Perturbation Formulation

- rewrite the actual dynamics as

$$\dot{x} = p_j + \varepsilon \left(\frac{\partial H}{\partial \lambda} - p_j \right) \quad ; \quad \dot{\lambda} = q_j + \varepsilon \left(-\frac{\partial H}{\partial x} - q_j \right)$$

- perturbation terms are zero at mid point of each element.
- for cases that control cannot be eliminated explicitly, use an analytic portion $\Pi(x, \lambda, u)$

$$0 = \Pi + \varepsilon \left(\frac{\partial H}{\partial u} - \Pi \right)$$

Carry out a Regular Perturbation Analysis

- expand about the zero order solution (derived from collocation)
- provides higher order corrections to collocation solution
- further exploitation of the analytically tractable portion of the dynamics will result in more intelligent interpolating functions (see simple example)

A Simple Example

Duffing's equation in first order form:

$$\dot{x} = v \quad ; \quad x(0) = x_0$$

$$\dot{v} = -x - ax^3 + u \quad ; \quad v(0) = v_0$$

$$J = S_x x^2(t_f) + S_v v^2(t_f) + \int_0^{t_f} (1 + u^2/2) dt$$

Notes:

- hardening effect is given by the nonlinear term, ax^3
- the optimal control problem is a fourth order example
- will demonstrate different levels of intelligent interpolating functions that enhance the approximation with fewer number of elements

Simple Example (continued)

Level 0 Formulation:

- degenerate case, uses only regular perturbation with a completely analytic zero order solution
- let $\epsilon = a = 0.4$, and treat the nonlinear terms as perturbations
- $S_x = S_v = 100$

$$\dot{x} = v \quad ; \quad x(0) = x_0$$

$$\dot{v} = -x + u - \epsilon x^3 \quad ; \quad v(0) = v_0$$

$$\dot{\lambda}_x = \lambda_v + \epsilon 3\lambda_v x^2 \quad ; \quad \lambda_x(t_f) = 2S_x x(t_f)$$

$$\dot{\lambda}_v = -\lambda_x \quad ; \quad \lambda_v(t_f) = 2S_v v(t_f)$$

$$H_u = u + \lambda_v = 0 \quad ; \quad \left\{ H = \lambda_v v + \lambda_x (-x + u - \epsilon x^3) + 1 + u^2 / 2 \right\} \Big|_{t_f} = 0$$

- zero order problem is linear and time-invariant
- compute up to second order corrected solutions (Fig's. 4.1 and 4.2)
- series not convergent, most accurate approximation is first order
- nonlinear term ax^3 is too large to be neglected in the zero order problem

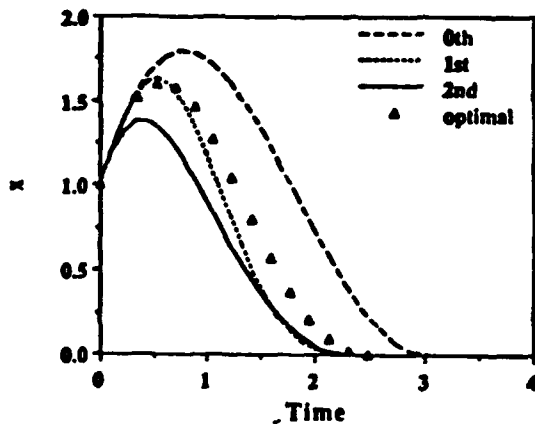


Figure 4.1. Level 0 Result in x .

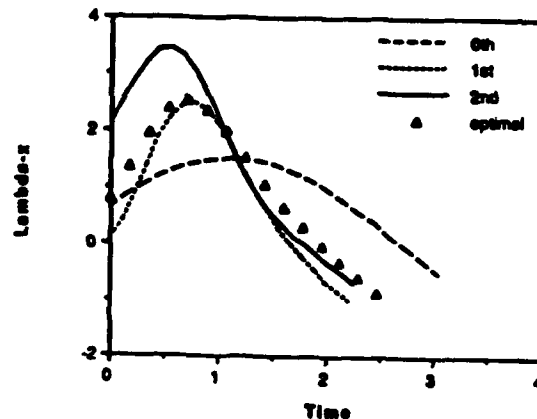


Figure 4.3. Level 0 Result in λ_x .

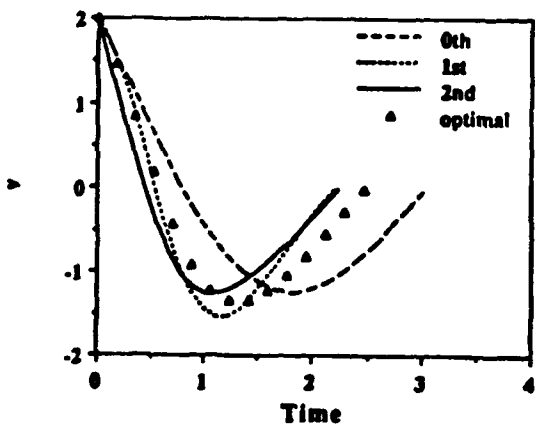


Figure 4.2. Level 0 Result in v .

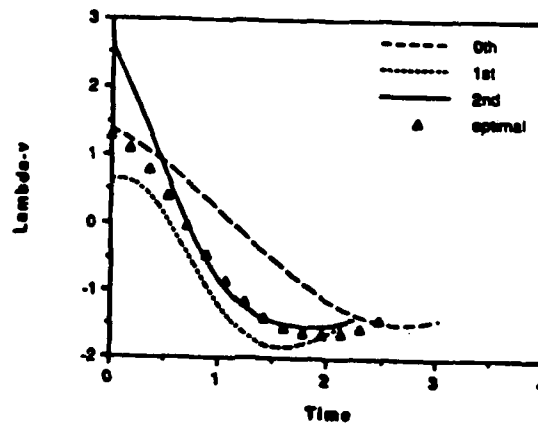


Figure 4.4. Level 0 Result in λ_v .

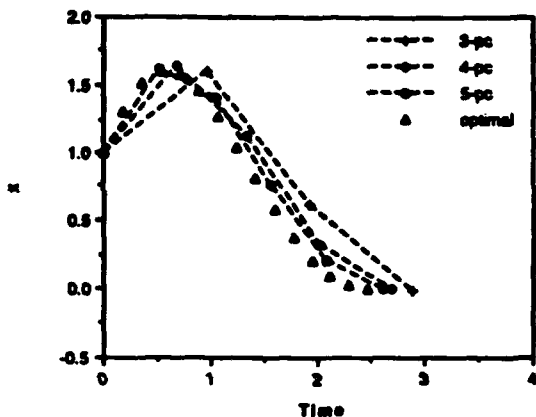


Figure 4.5. Level 1 Zero Order Results in x for Different N .

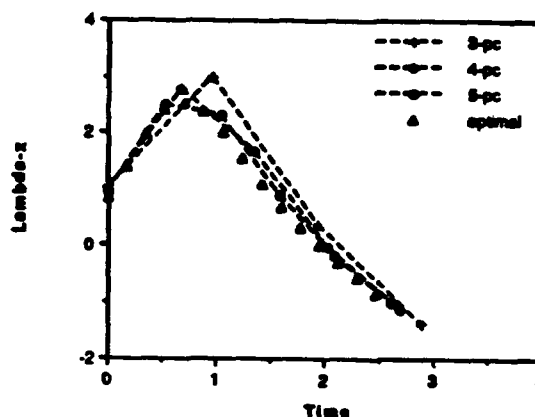


Figure 4.7. Level 1 Zero Order Results in λ_x for Different N .

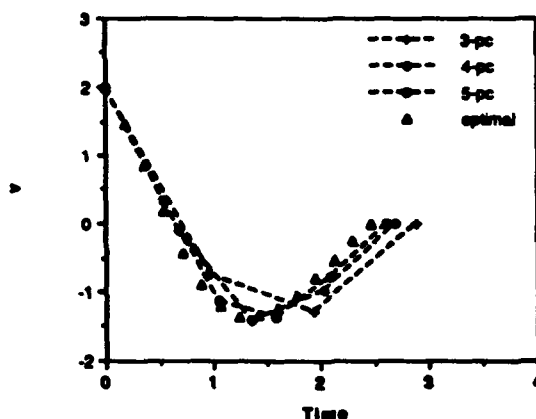


Figure 4.6. Level 1 Zero Order Results in v for Different N .

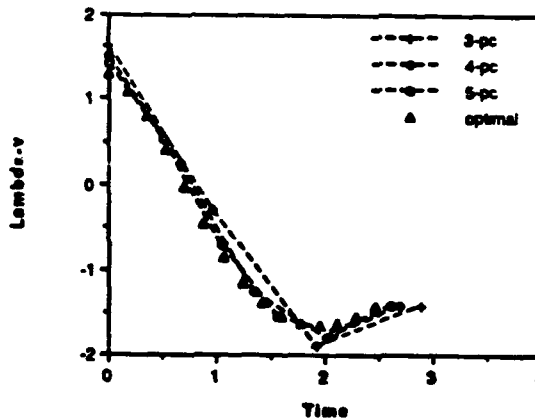


Figure 4.8. Level 1 Zero Order Results in λ_v for Different N .

Simple Example (continued)

Level 1 Formulation:

- use hybrid approach, approximate all state and costates as piecewise linear functions

$$x_0(\hat{t}) = x_{0j-1} + p_{xj}(\hat{t} - \hat{t}_{j-1}) \quad ; \quad v_0(\hat{t}) = v_{0j-1} + p_{vj}(\hat{t} - \hat{t}_{j-1}) \quad ; \quad j = 1, \dots, N$$

$$\lambda_{x0}(\hat{t}) = \lambda_{x0j-1} + q_{xj}(\hat{t} - \hat{t}_{j-1}) \quad ; \quad \lambda_{v0}(\hat{t}) = \lambda_{v0j-1} + q_{vj}(\hat{t} - \hat{t}_{j-1})$$

- number of unknowns is $4N + 5$
- 1st and 2nd order corrections are computed for $N = 3$ (Fig's. 4.9 - 4.12)
- discontinuity in slope is smoothed as order of correction increases
- correction by regular perturbation analysis allows use of crude number of element representation in the zero order collocation solution

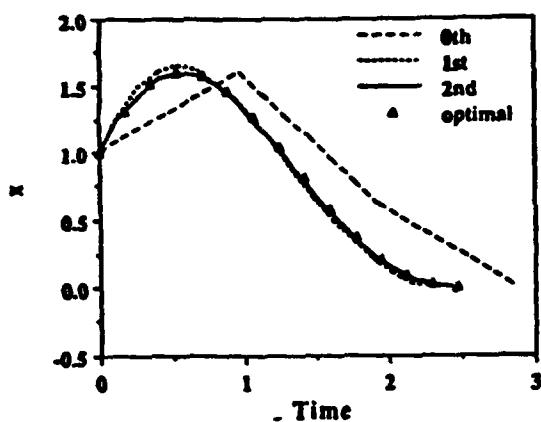


Figure 4.9. Level 1 Higher Order Results in x for $N = 3$.

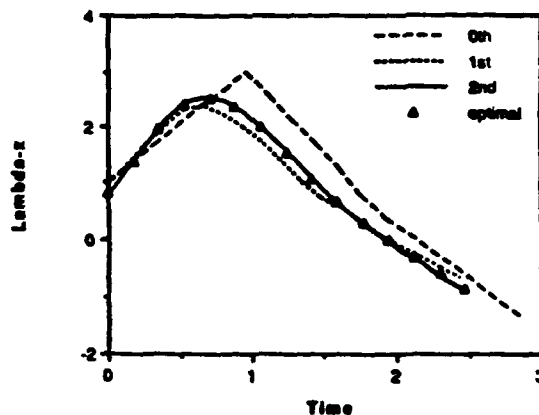


Figure 4.11. Level 1 Higher Order Results in λ_x for $N=3$.

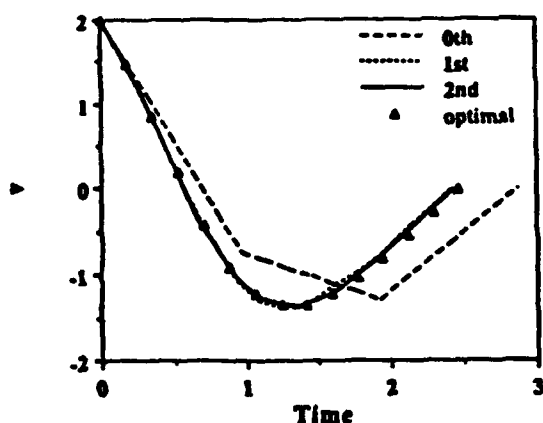


Figure 4.10. Level 1 Higher Order Results in v for $N = 3$.

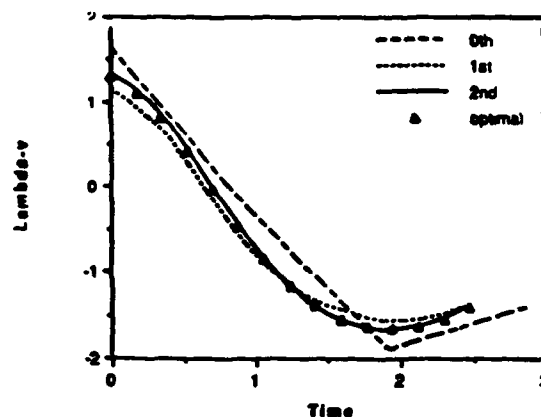


Figure 4.12. Level 1 Higher Order Results in λ_v for $N=3$.

Simple Example (continued)

Level 2 Formulation:

- enhanced level 1 formulation by interpolating only those variables that have nonlinear coupling
- decompose the dynamics as:

$$dx/dt = v$$

$$dv/dt = p_{vj} + \epsilon \{-x - \lambda_v - ax^3 - p_{vj}\} \quad j = 1, 2, \dots, N$$

$$d\lambda_x/dt = q_{xj} + \epsilon \{ \lambda_v(1 + 3ax^2) - q_{xj} \}$$

$$d\lambda_v/dt = -\lambda_x$$

- number of unknowns is $2N + 5$
- both zero and first order results for $N=2$ are superior than the $N=3$ results for the Level 1 formulation (Fig's. 4.13 - 4.16)

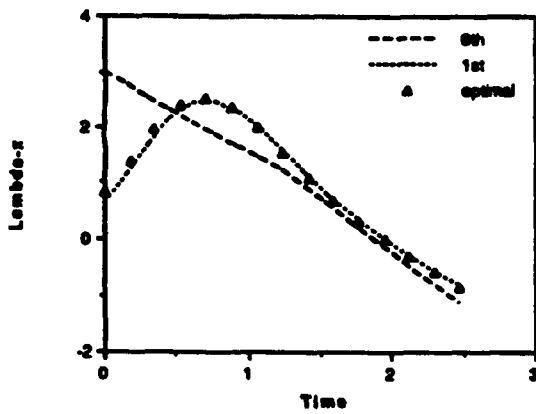


Figure 4.15. Level 2 Higher Order Results in λ_x for $N=2$.

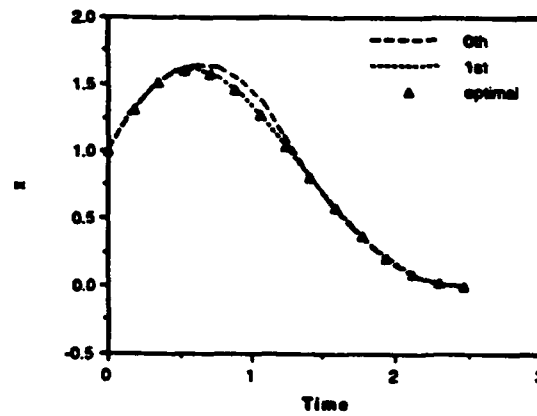


Figure 4.13. Level 2 Higher Order Results in x for $N=2$.

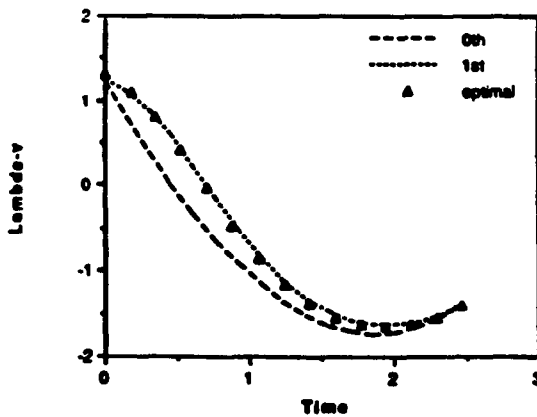


Figure 4.16. Level 2 Higher Order Results in λ_v for $N=2$.

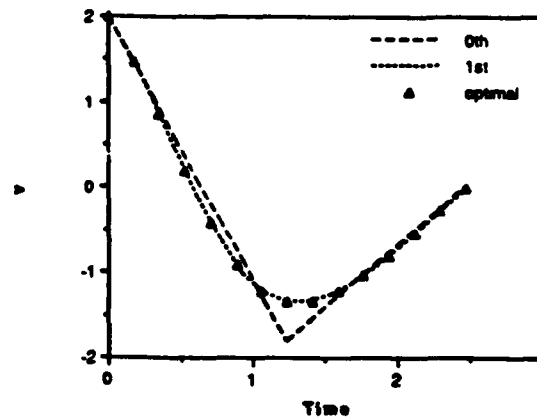


Figure 4.14. Level 2 Higher Order Results in v for $N=2$.

Simple Example (continued)

Level 3 Formulation:

- enhanced level 2 formulation by fully utilizing analytically tractable portion of the necessary conditions
- decompose the dynamics as:

$$\dot{x} = v$$

$$\dot{v} = -x - \lambda_v + p_{vj} + \varepsilon(-ax^3 - p_{vj}) \quad ; j = 1, 2, \dots, N$$

$$\dot{\lambda}_x = \lambda_v + q_{xj} + \varepsilon(3a\lambda_v x^2 - q_{xj})$$

$$\dot{\lambda}_v = -\lambda_x$$

- similar to Level 0 except for additional unknown constants p_{vj} , q_{xj}
- use piecewise constant terms to approximate the nonlinear parts
- both zero and first order results: for $N=1$ are superior than the Level 0 case (Fig's. 4.17 - 4.20)
- Level 2 and 3 cases demonstrate the use of intelligent interpolating functions

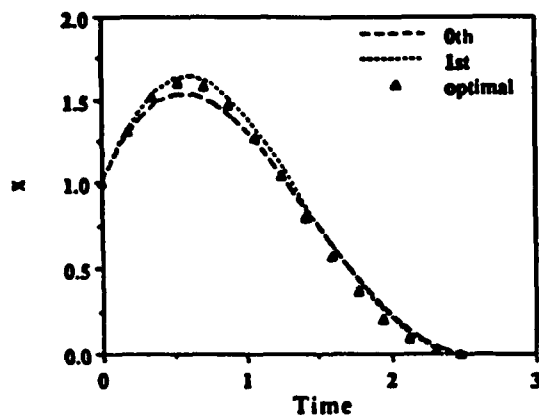


Figure 4.17. Level 3 Higher Order Results in x for $N = 1$.

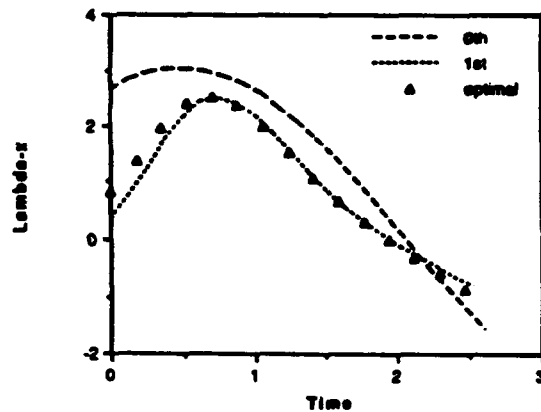


Figure 4.19. Level 3 Higher Order Results in λ_x for $N=1$.

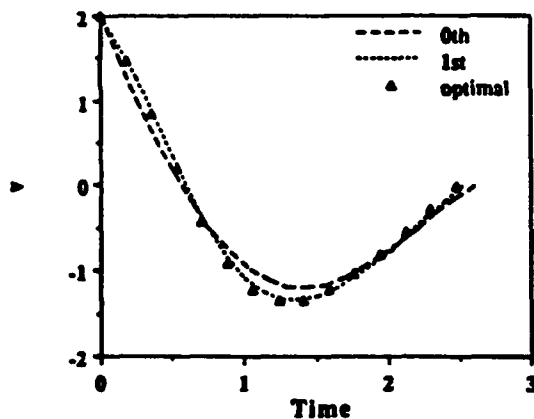


Figure 4.18. Level 3 Higher Order Results in v for $N = 1$.

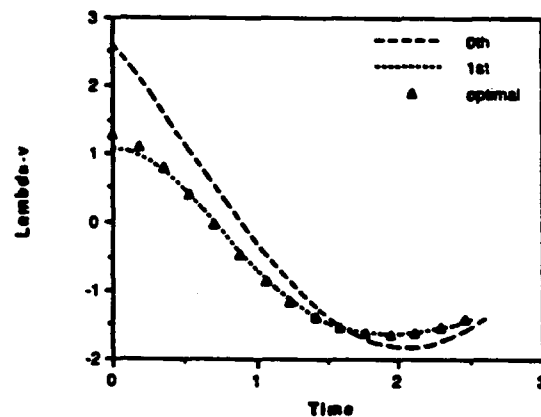


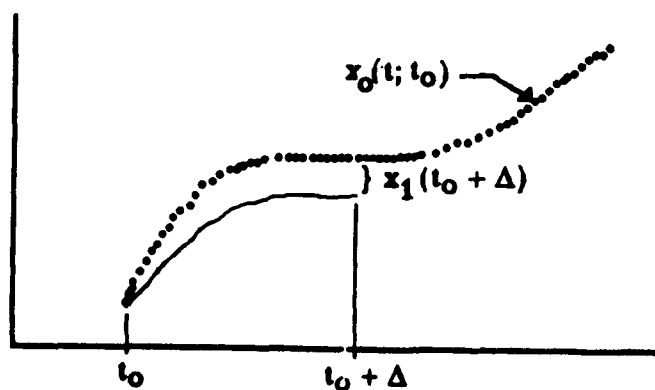
Figure 4.20. Level 3 Higher Order Results in λ_v for $N=1$.

Alternative Implementations

Repeat zero order solution and perform quadratures at each control update interval

Or

Compute zero order solution and quadratures off line, and store for in-flight use



Improves reliability and computational efficiency with some loss in accuracy

Summary

Benefits of Hybrid Approach:

Significantly improves a collocation solution

First and higher order corrections are obtained by quadrature

Intelligent interpolation functions obtained by retaining as much of the analytically tractable portion of the solution as possible

Possible to implement the control solution so that the zero order solution and quadratures are performed once off-line and stored

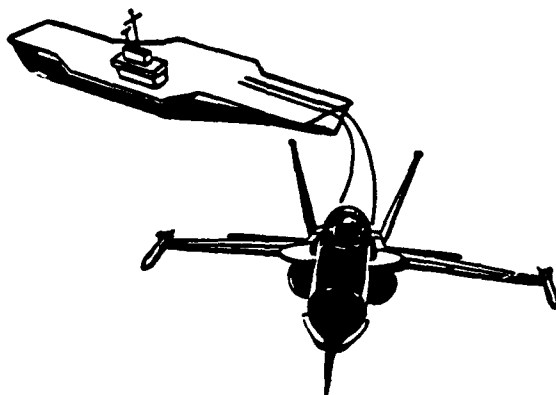
Significantly improve a regular perturbation solution

Retain more of the nonlinearities in the zero order problem by using finite elements and collocation to construct an improved zero order solution

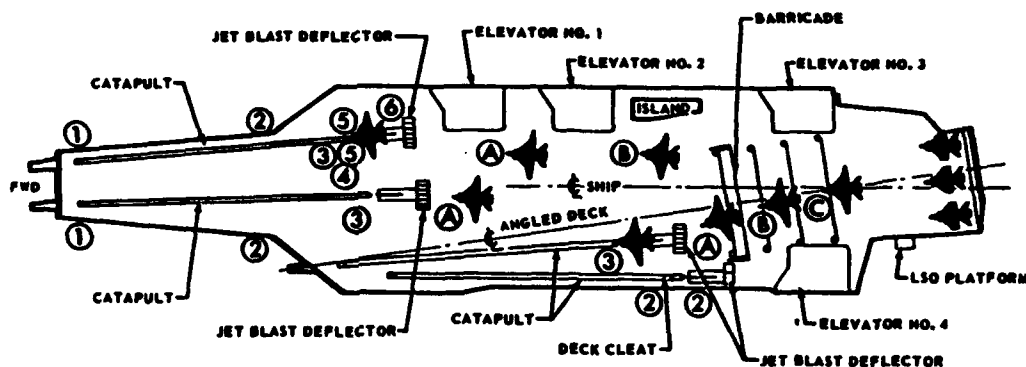
Important implications in real-time guidance applications

Computational efficiency and reliability

**AUTOMATIC SOLUTIONS FOR TAKE-OFF
FROM AIRCRAFT CARRIERS**



Lloyd H Johnson
AIR-53012D



- (A) Forward Zone Director
 (B) Amidships Director
 (C) Aft Zone Director
- (1) Bridle Arresting Crew Members
 (2) Catapult Deck Edge Operator
 (3) Catapult Director
 (4) Catapult Officer
 (5) Bridle Crew Members
 (6) Holdback Crew Members

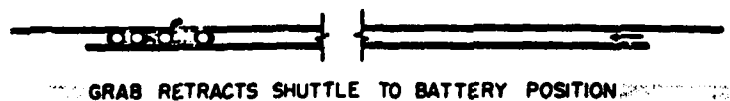
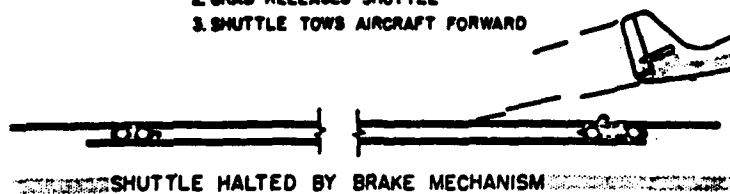
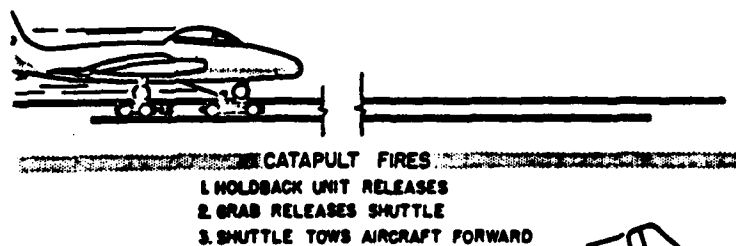
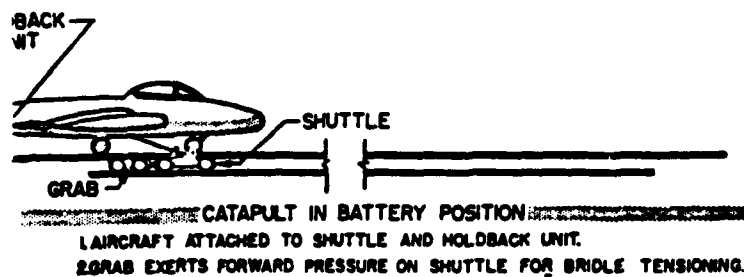
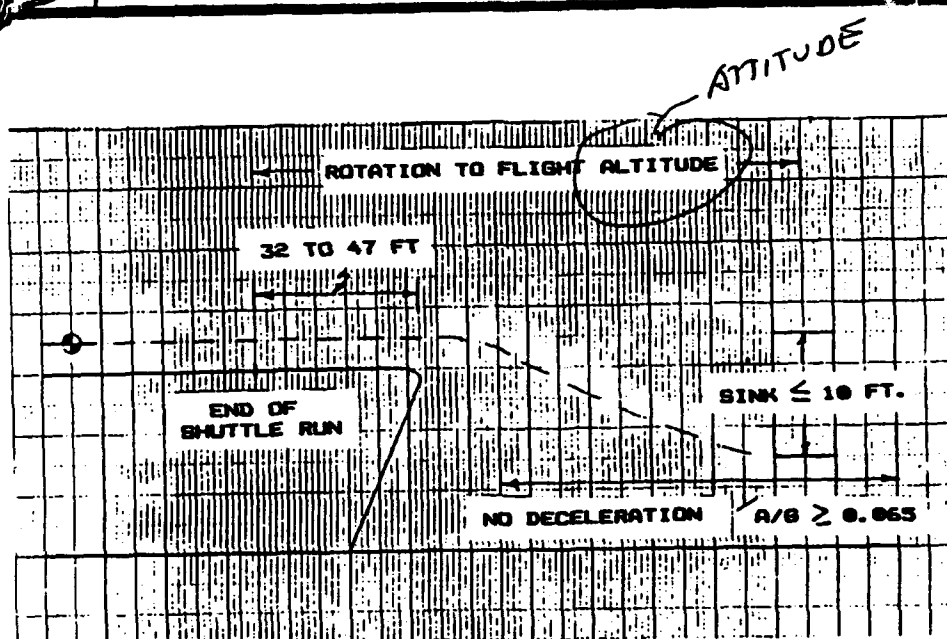
Figure 2-2. Location of Launching Personnel

NAEC 04136, REV B

U. S. NAVY AIRCRAFT CARRIER CATAPULT COMPARISON CHART					
HULL NO.	SHIP NAME	CLASS	CATAPULT MODEL	CATAPULT QUANTITY	CATAPULT NUMBER
AVT 16	USS LEXINGTON	16	C11-1	2	1 AND 2
CV 43	USS CORAL SEA	43	C11-1	3	1, 2 AND 3
CV 59	USS FORRESTAL	59	C11-1	2	3 AND 4
CV 60	USS SARATOGA	59	C11-1	1	3
CV 60	USS SARATOGA	59	C7	3	1, 2 AND 4
CV 61	USS RANGER	59	C7	2	3 AND 4
CV 62	USS INDEPENDENCE	59	C7	2	3 AND 4
CV 59	USS FORRESTAL	59	C7	2	1 AND 2
CV 61	USS RANGER	59	C7	2	1 AND 2
CV 62	USS INDEPENDENCE	59	C7	2	1 AND 2
CV 41	USS MIDWAY	41	C13	2	1 AND 2
CV 63	USS KITTY HAWK	63	C13	4	1, 2, 3 AND 4
CVN 65	USS ENTERPRISE	65	C13	4	1, 2, 3 AND 4
CV 64	USS CONSTELLATION	63	C13	4	1, 2, 3 AND 4
CV 66	USS AMERICA	63	C13*	3	1, 2 AND 4
CV 67	USS JOHN F. KENNEDY	63	C13*	3	1, 2 AND 4
CV 66	USS AMERICA	63	C13-1	1	3
CV 67	USS JOHN F. KENNEDY	63	C13-1	1	3
CVN 68	USS CHESTER W. NIMITZ	68	C13-1	4	1, 2, 3 AND 4
CVN 69	USS DWIGHT D. EISENHOWER	68	C13-1	4	1, 2, 3 AND 4
CVN 70	USS CARL VINSON	68	C13-1	4	1, 2, 3 AND 4

* REFERS TO OTHER CATAPULT MODELS INSTALLED WITHIN THE SAME DESIGNATED HULL NUMBER.

Figure 4-48. Shipboard Catapult Minimum Performance and Load Factors (Sheet 2 of 7)



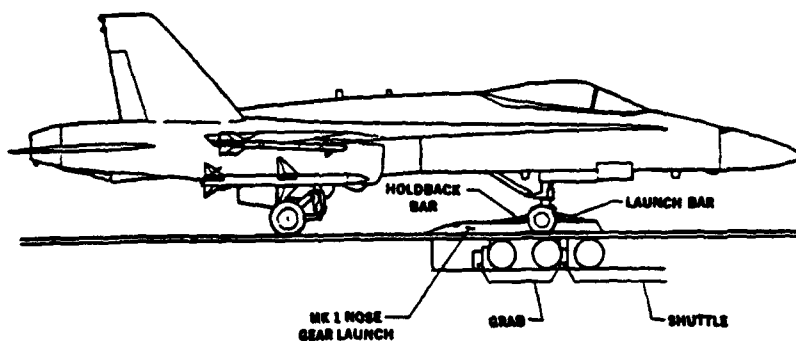


Figure 4-1. Nose Gear Launch Configuration

4.0.2 BRIDLE/PENDANT LAUNCH METHOD.

This method of launching aircraft is no longer a design option since the nose gear launch method became standard (see paragraph 4.0.1). With this method, the aircraft is coupled to the catapult tow fitting by means of a wire rope bridle or pendant. The bridle is "V" shaped and requires two tow fittings on the aircraft, whereas the pendant needs only one tow fitting on the aircraft.

The holdback device used with bridle/pendant launch is wire rope, chain, or metal links. The release element is either ring or a tension bar. The holdback assembly attachment point on the aircraft is well aft and the deck end attaches to the holdback deck cleat. Section VIII describes typical holdback and release assemblies.

This method of launching requires manual hookup of the bridle or pendant and the holdback by the catapult deck crew after the aircraft has been taxied into position on the catapult. When the aircraft reaches the end of the catapult power run and the tow force decays, the bridle or pendant drops from the aircraft tow fittings and is brought to a stop on the flight deck by the bridle arrester system.

4.1 LAUNCHING EQUIPMENT.**4.1.0 GENERAL.**

4.1.0.0 CATAPULT. A direct-drive, steam-type catapult is used on all carriers. Steam, piped from the ship's boilers to a series of large steam receivers, is released suddenly to the launching engine to drive two pistons. The pistons are directly connected to the catapult tow fitting through the slotted cylinder walls. Retraction is accomplished by a separate hydro-pneumatic retracting engine.

4.1.0.1 DECK EQUIPMENT. The catapult is equipped with a shuttle which moves along the catapult track during the launching operation and transmits the catapulting force from the catapult engine to the aircraft through the launch bar, bridle or pendant. A ramp, secured to the catapult tow fitting, enables the aircraft to roll over the tow fitting. For bridle-launched aircraft, the holdback deck cleat provides numerous anchorage points for the holdback and release assembly. This deck cleat is located aft of, and on the centerline of the catapult track. The bridle arrester stops and retains the bridle or pendant after it is shed from the aircraft. Finally, the deck edge control panel provides the primary control power for operating the catapult.



LAUNCH WIND OVER DECK

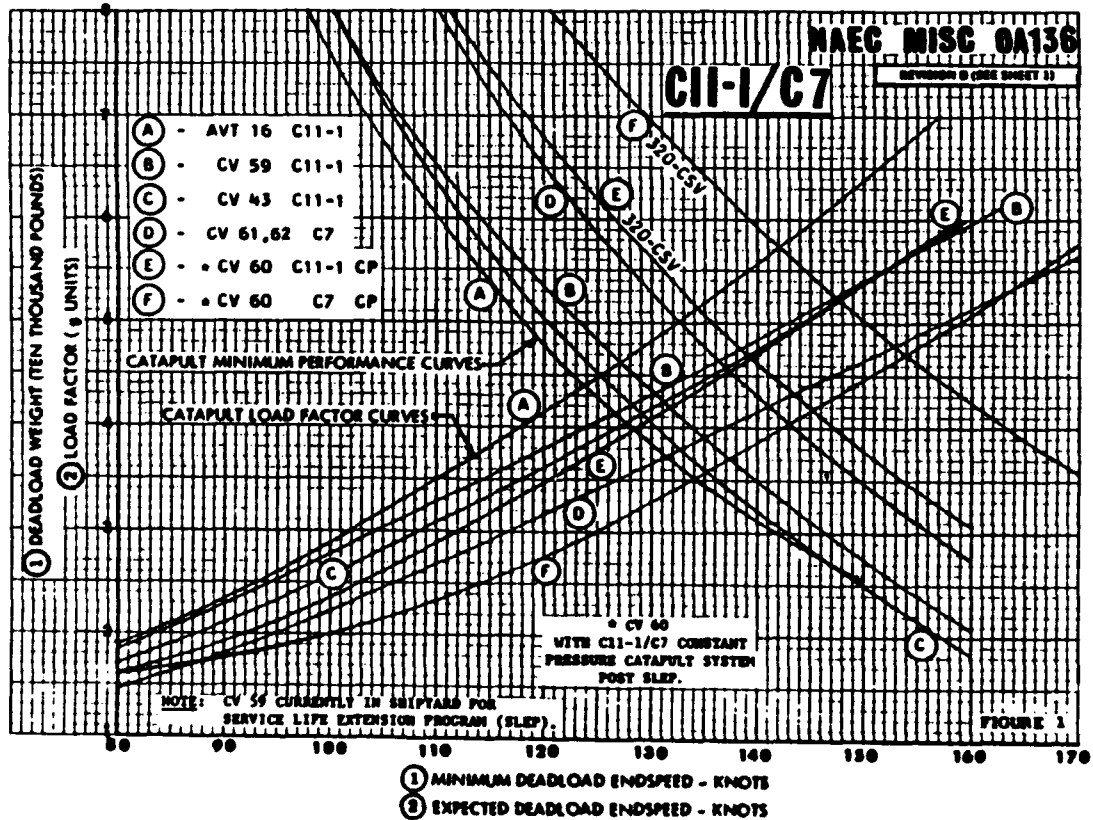
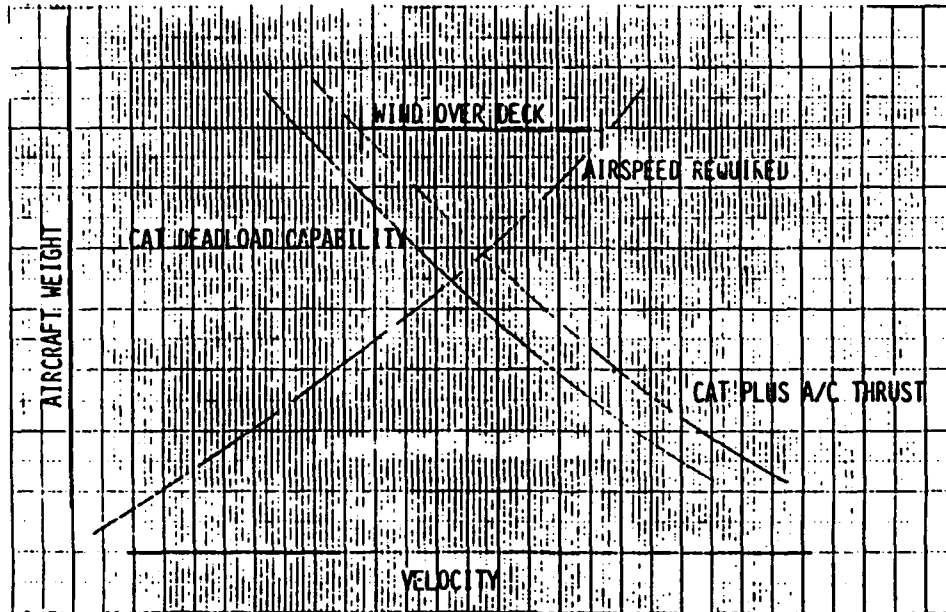
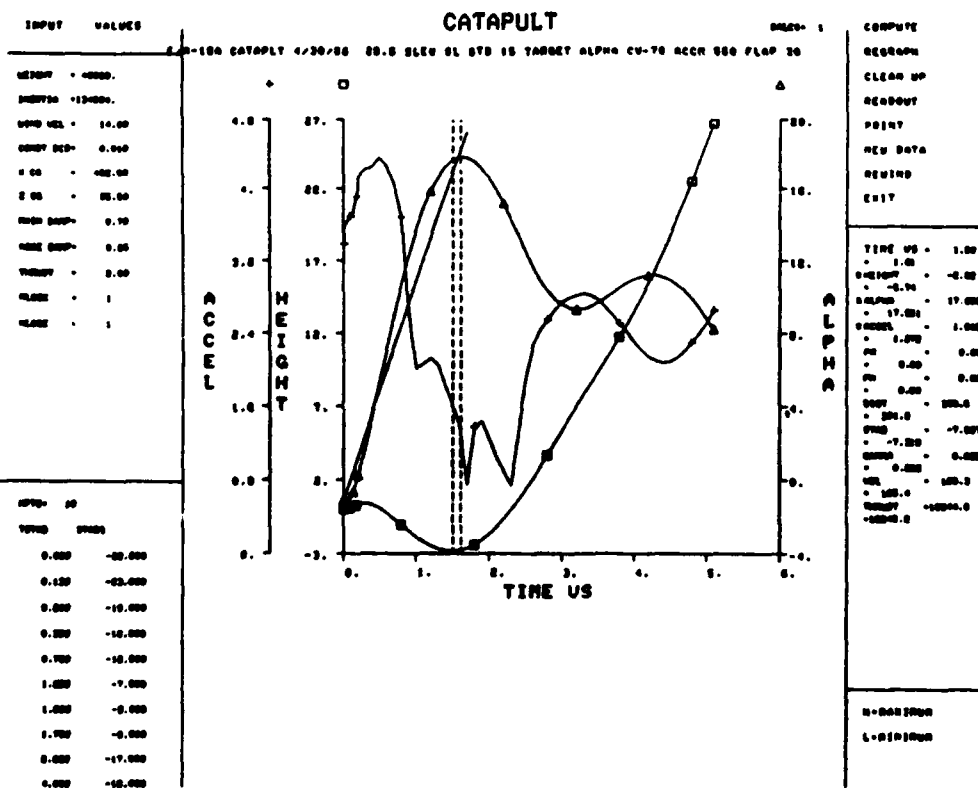
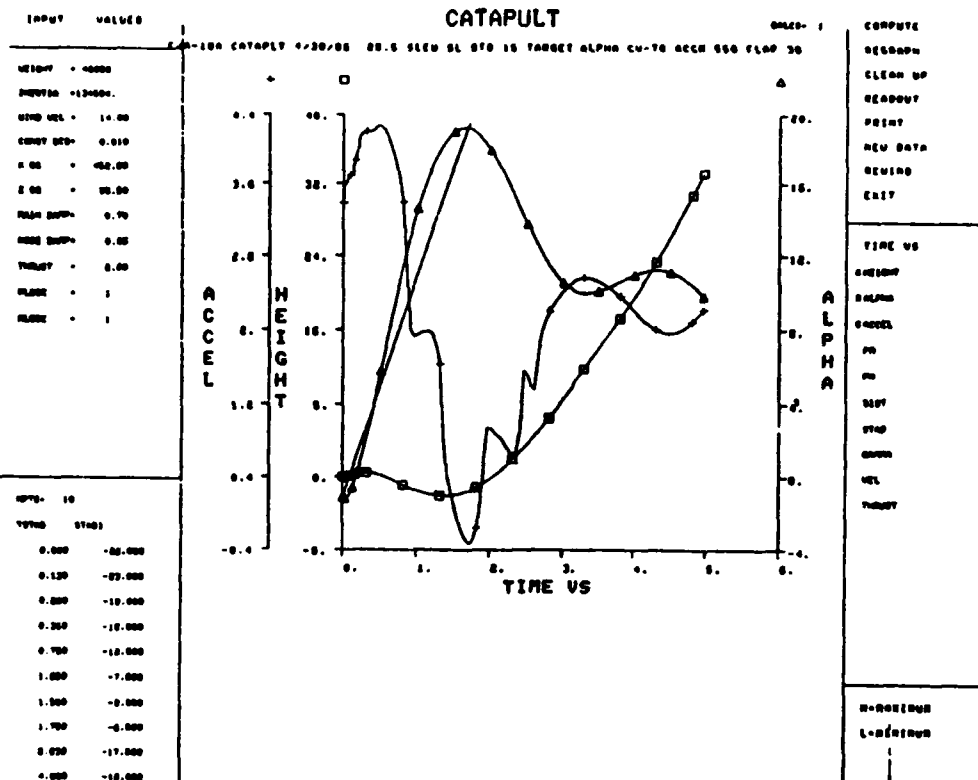


Figure 4-40. Shipboard Catapult Minimum Performance and Load Factors (Sheet 3 of 7)



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THE CATAPULT LAUNCH SIMULATION CONSISTS OF FIVE PHASES

- STATIC BALANCE
- HOLDBACK
- CATAPULT STROKE
- DECK RUN
- FLYAWAY

THE CATAPULT LAUNCH SIMULATION INCLUDES:

- CATAPULT FORCES
- HOLDBACK FORCES
- HIGH FIDELITY LANDING GEAR MODEL
- AERODYNAMIC DATA AS A FUNCTION OF
 - ANGLE OF ATTACK OR LIFT COEFFICIENT
 - NOZZLE DEFLECTION
 - THRUST COEFFICIENT OR NOZZLE PRESSURE RATIO (NPR)
 - FLAP DEFLECTION
 - PITCH TRIM SURFACE DEFLECTION
- GENERIC FLIGHT CONTROL AND STABILITY AUGMENTATION SYSTEM
- LONGITUDINAL THRUST VECTORING

PROGRAM OPTIONS

- AUTOMATIC WIND OVER DECK SOLUTION
- SOLUTION TERMINATION
- POWERSETTING
- FLAP DEFLECTION
- FLIGHT CONTROL AND STABILITY AUGMENTATION SYSTEM
- THRUST VECTORING CONTROL SYSTEM
- LANDING GEAR
- ENGINE FAILURE
- STORE JETTISON
- LANDING GEAR RETRACTION

AUTOMATIC WIND OVER DECK SOLUTION

TWO CONSTRAINTS:

- MAXIMUM SINK
- MAXIMUM ANGLE OF ATTACK
OR
MAXIMUM PITCH RATE
OR
MINIMUM LONGITUDINAL ACCELERATION

TWO VALUES DETERMINED:

- WIND OVER DECK
- STICK DISPLACEMENT (OR TAIL DEFLECTION)

SOLUTION TERMINATION

- POSITIVE TMAX
 - TIME HISTORIES STOP AT THE SPECIFIED TMAX
- NEGATIVE TMAX
 - TIME HISTORIES STOP WHEN A POSITIVE RATE OF CLIMB HAS BEEN ACHIEVED AND ANGLE OF ATTACK HAS PEAKED.
 - IF A POSITIVE RATE OF CLIMB IS NOT ACHIEVED OR ANGLE OF ATTACK IS CONTINUOUSLY INCREASING, THE TIME HISTORY WILL STOP AT THE ABSOLUTE VALUE OF TMAX.

AERODYNAMIC DATA INCLUDES PROPULSION-INDUCED EFFECTS

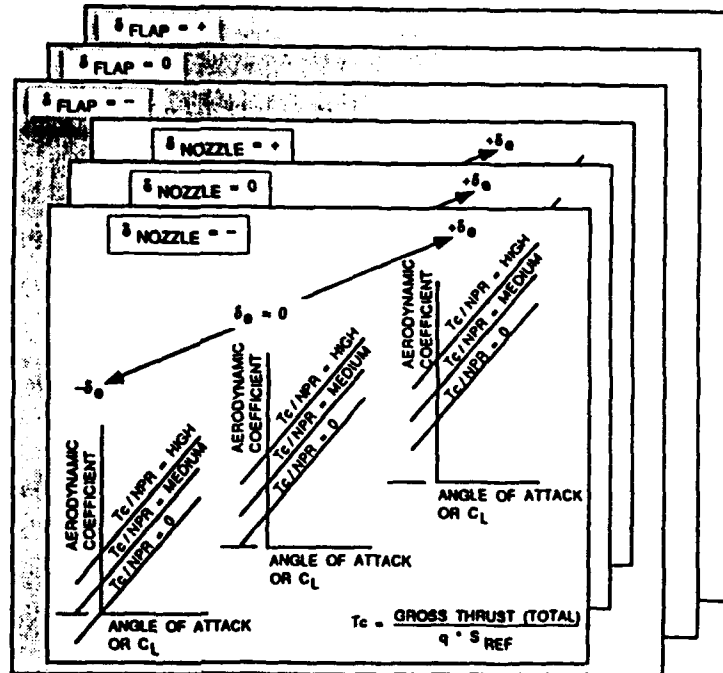
COEFFICIENTS ARE FUNCTIONS OF:

- ANGLE OF ATTACK OR LIFT COEFFICIENT
- NOZZLE DEFLECTION
- THRUST COEFFICIENT OR NPR
- FLAP DEFLECTION
- TRIM SURFACE DEFLECTION

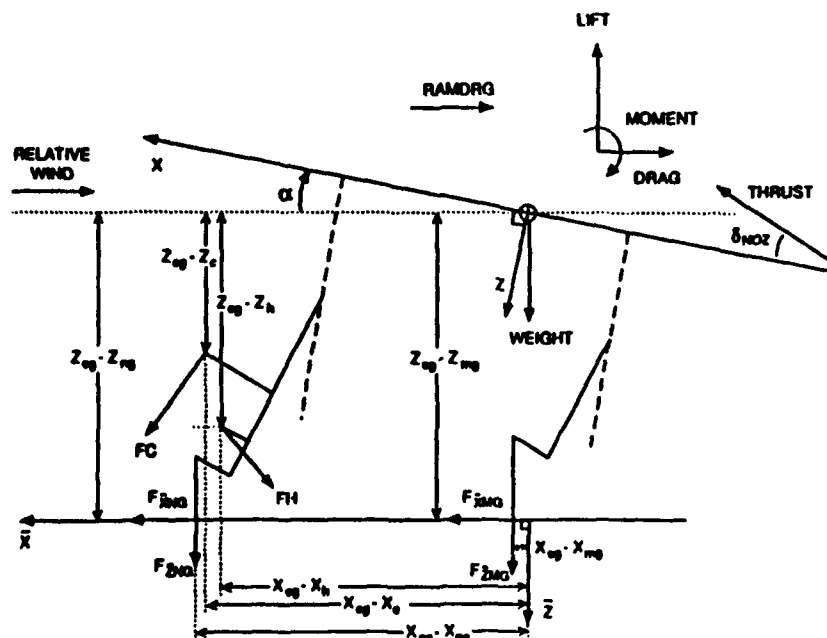
TYPICAL AERODYNAMIC DATA FOR A THRUST VECTORING CONFIGURATION

AERODYNAMIC COEFFICIENTS HAVE THE DIRECT PROPULSION EFFECTS REMOVED AND ARE FUNCTIONS OF:

- ANGLE OF ATTACK OR LIFT COEFFICIENT
- THRUST COEFFICIENT OR NPR
- NOZZLE DEFLECTION
- FLAP DEFLECTION
- TRIM SURFACE DEFLECTION



FORCES ACTING ON THE AIRCRAFT



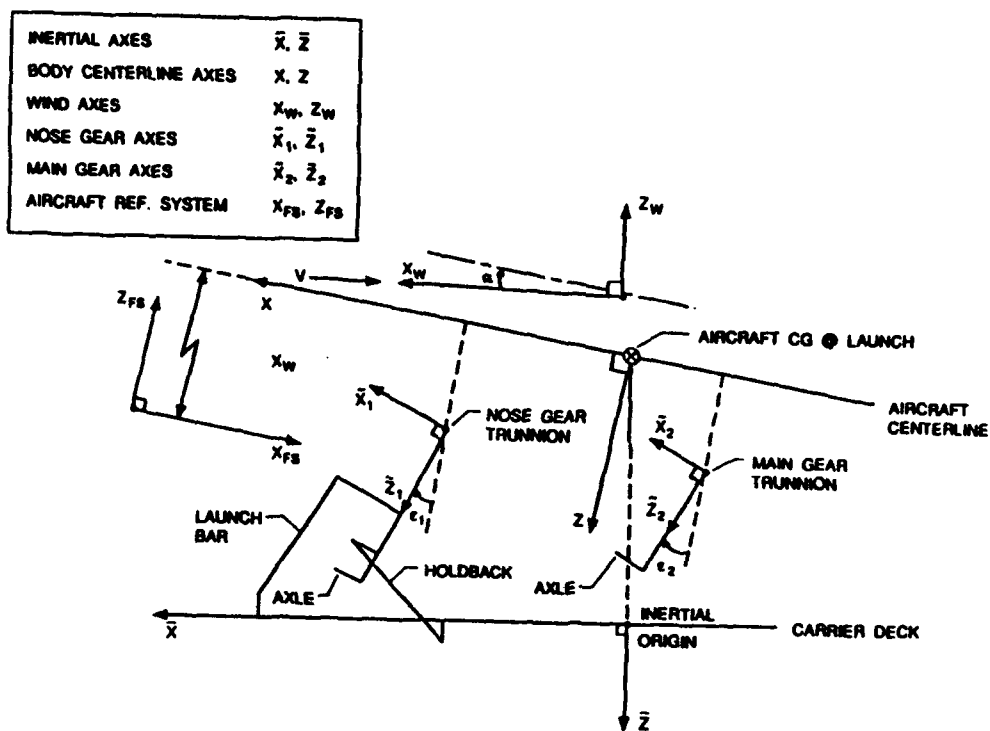


FIGURE 2-1: BODY AXES, GEAR AXES AND WIND AXES ORIENTATION WITH RESPECT TO THE INERTIAL FRAME OF REFERENCE

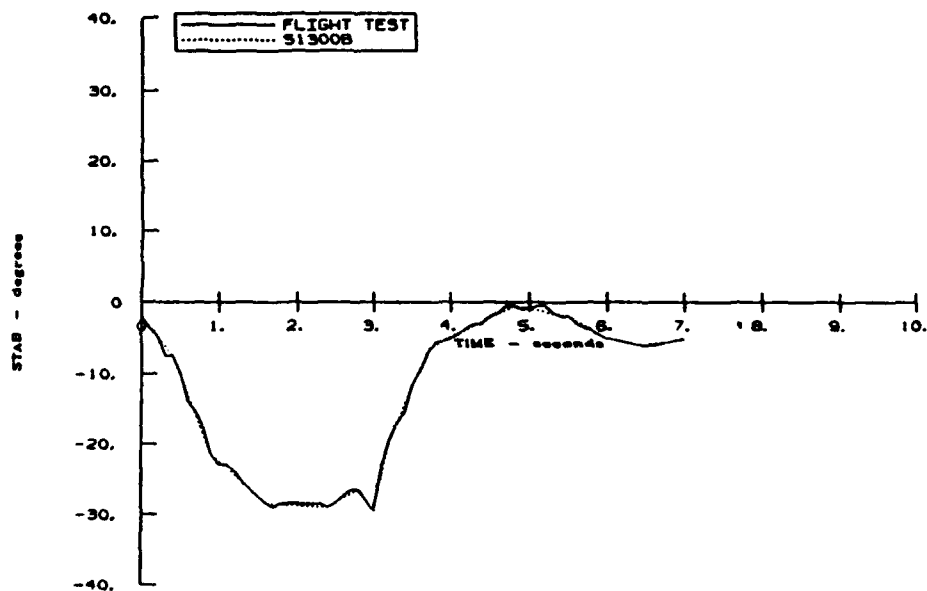


FIGURE 9a - CONTROL SURFACE DEFLECTION AS A FUNCTION OF TIME

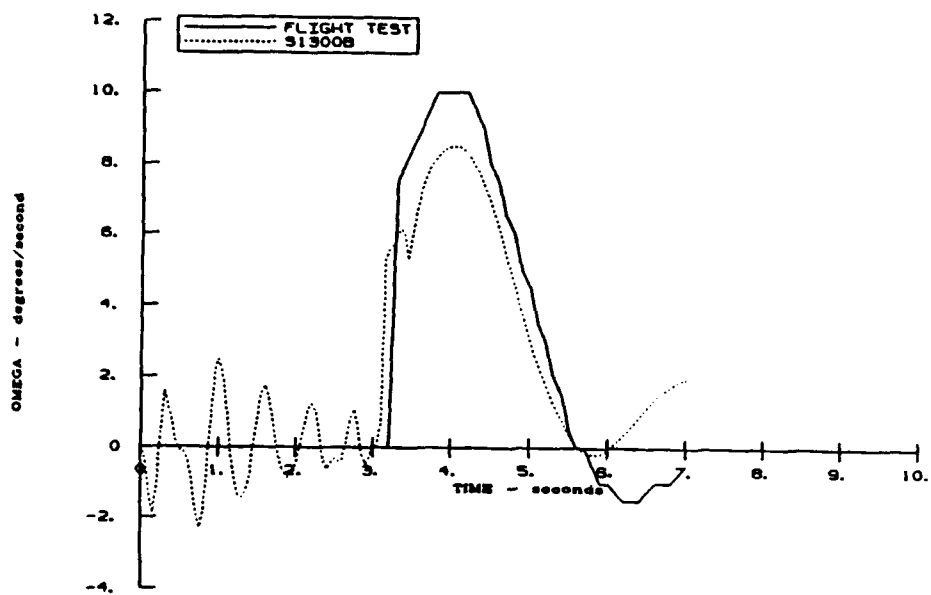


FIGURE 9b - PITCH RATE AS A FUNCTION OF TIME

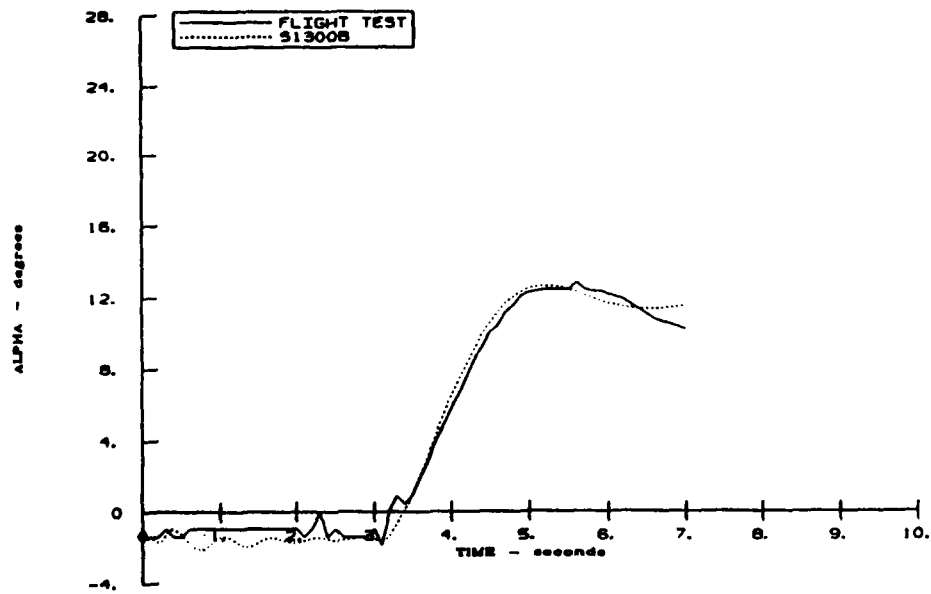


FIGURE 9c - ANGLE OF ATTACK AS A FUNCTION OF TIME

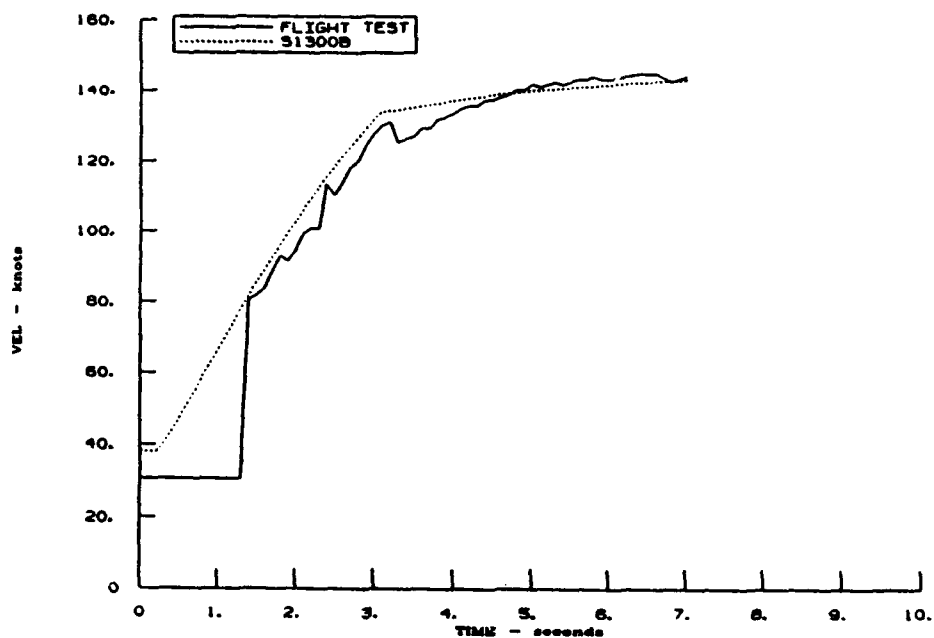
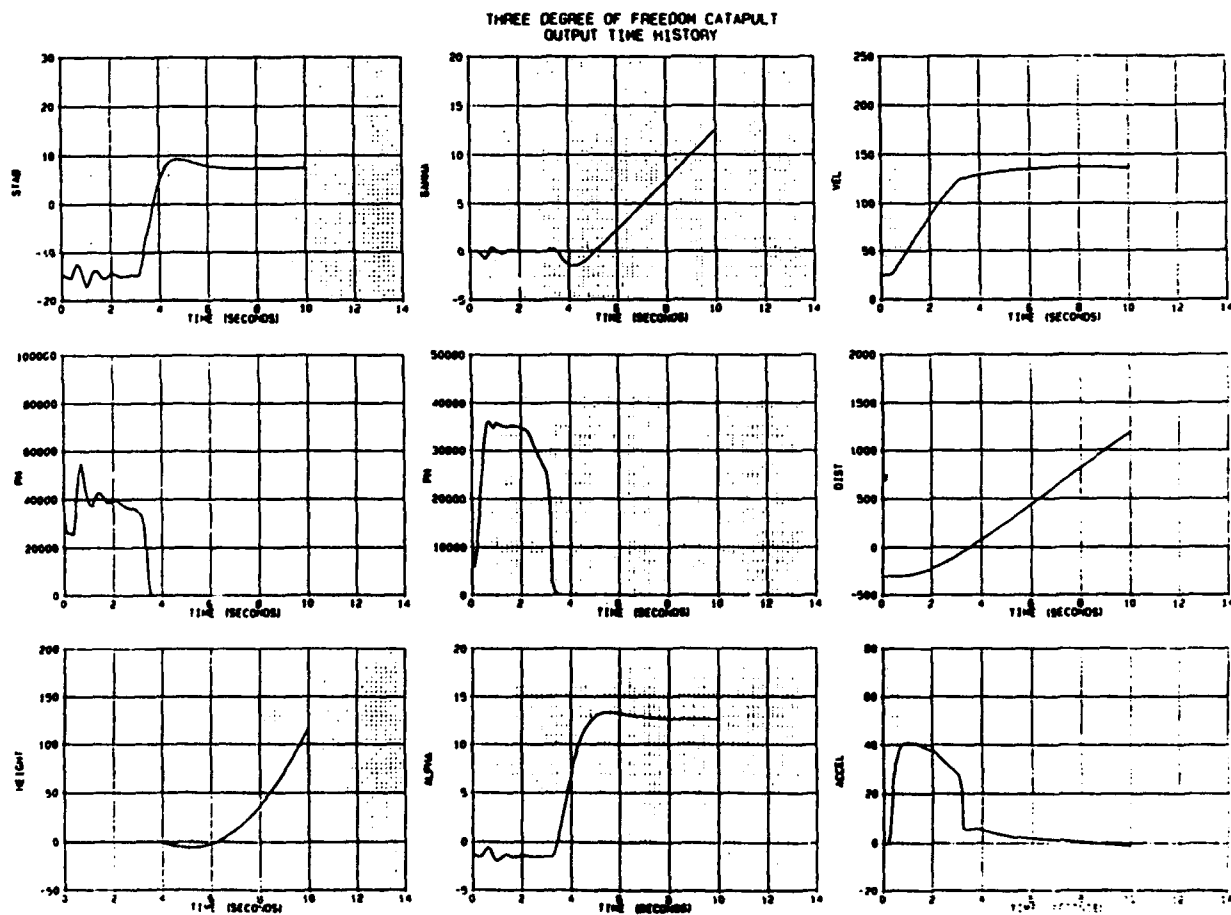
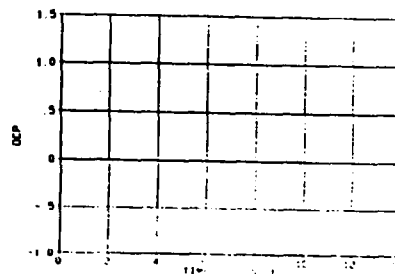
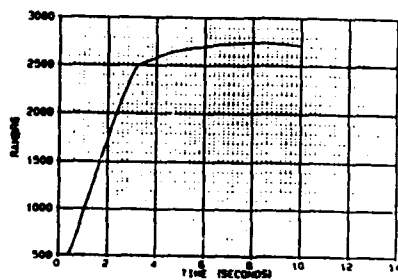
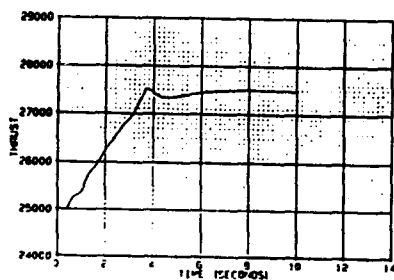
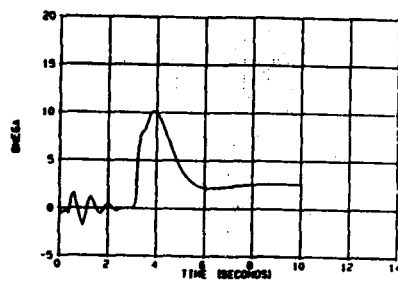
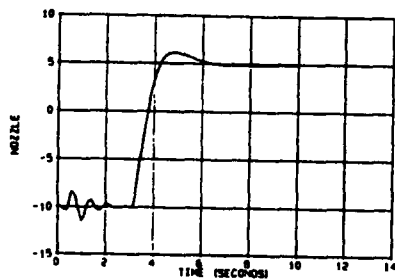


FIGURE 9d - TRUE AIRSPEED AS A FUNCTION OF TIME





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**AIRBREATHING BOOSTER PERFORMANCE OPTIMIZATION
USING MICROCOMPUTERS**

Ron Oglevie

**Irvine Aerospace Systems Co.
2001 Calle Candela,
Fullerton, CA 92633
(714) 526-6642**

**AIAA Astrodynamics Conference
Workshop on Trajectory Optimization**

10 August, 1992

- MICROCOMPUTER-BASED OPTIMIZING SIMULATION OF TRAJECTORIES (MOST)¹
 - MOTIVATION - Fill void in preliminary design tools
 - Easy to use fast running modes
 - TPBV solution for truth model
- OTIS PROGRAM OPERATION ON PC
- LOW-THRUST TRAJECTORY OPTIMIZATION PROGRAM (MICROTOP)

¹ Work performed under Air Force Contract F33615-91-C-2100.

- SUITABLE FOR RAPID PRELIMINARY DESIGN
- MICROCOMPUTER OPERATION - Run time less than 5 mins. on PC AT
- AIRBREATHING & ROCKET PROPULSION VIA TABLES AND EQUATIONS - Including realistic flight constraints
- EARTH-TO-ORBIT (ETO) FLIGHT
- PLANAR FLIGHT - Simple rotating earth model facilitates 3-D type results with minimal complexity
- EASE-OF-USE - Easy input, good graphics, & robust convergence

HYBRID APPROACH OFFERS SPEED AND PRECISION

MICROCOMPUTER-BASED OPTIMIZING SIMULATION OF TRAJECTORIES (MOST) A HYBRID APPROACH

PHASE 1

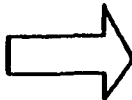
RAPID OPTIMIZATION MODEL (ROM)

FEATURES:

- 2-DIMENSIONAL FLIGHT
- AIRBREATHING AND ROCKET PROPULSION
- FLIGHT CONSTRAINTS
- MULTIPLE STAGES
- PRINTED AND GRAPHIC OUTPUT
- MULTIPLE STAGES
- FAST-RUNNING APPROXIMATE METHODS

APPLICATIONS:

- CONCEPT FEASIBILITY ASSESSMENT
- PRELIMINARY DESIGN TRADEOFFS
- PERFORMANCE ESTIMATION



PHASE 2

PRECISION OPTIMIZATION MODEL (POM)

FEATURES:

- SAME AS ROM PLUS:
- EXPLICIT INTEGRATION OF STATE EQUATIONS
- FLIGHT AND VEHICLE DESIGN PARAMETER OPTIMIZATION
- IF REQUIRED, 3-DIMENSIONAL FLIGHT SIMULATION
- RIGOROUS OPTIMIZATION

APPLICATIONS:

- ACCURATE TRAJECTORY TIMELINE GENERATION AND PERFORMANCE ANALYSIS
- SUPPORT DETAILED DESIGN ANALYSIS

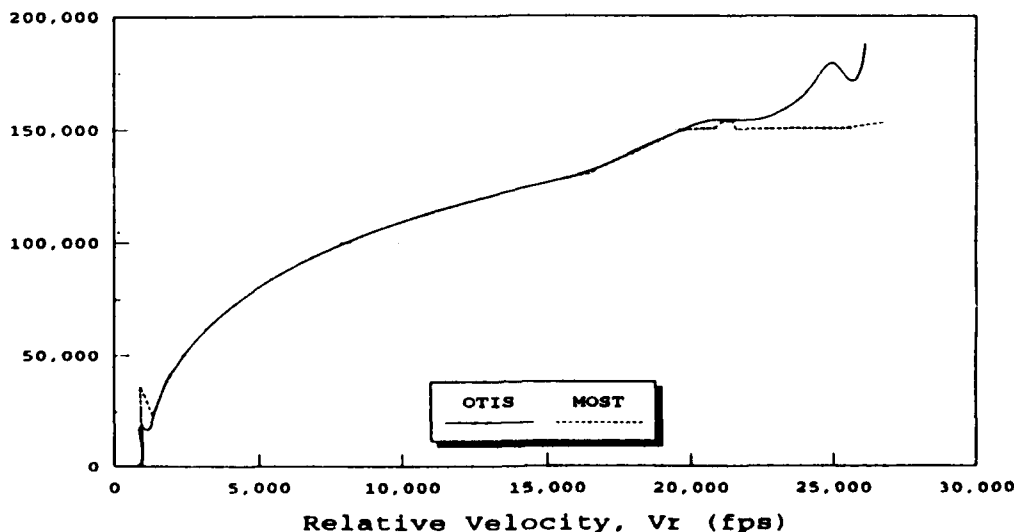
- ROM MODE RETAINED AS AN OPTION
- PROVIDES "FIRST GUESSES" TO POM SOLUTION

SINGLE-STAGE-TO-ORBIT OPTIMIZATION GOALS ACHIEVED ON PC

MOST versus OTIS H-V Flight Profile Comparison

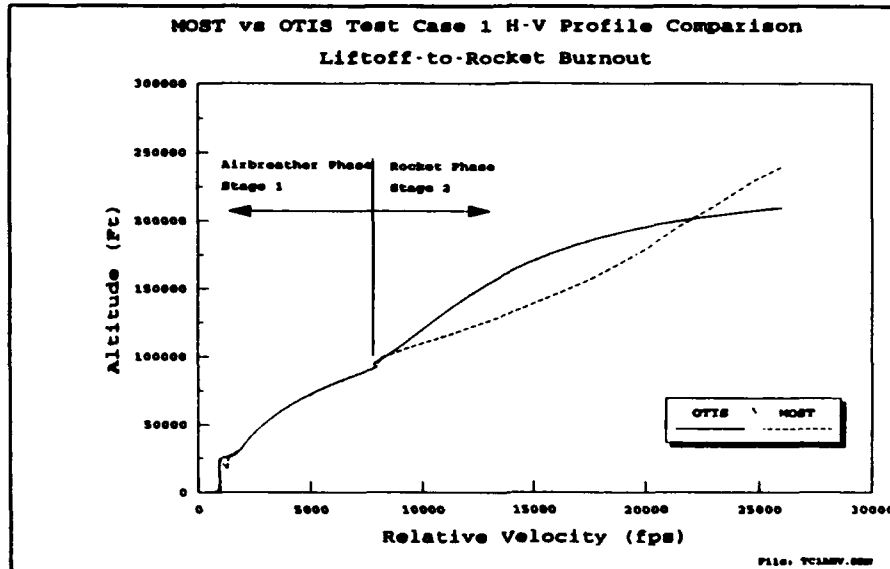
Test Case No. 2, Single-Stage-to-Orbit

Altitude, h (feet)



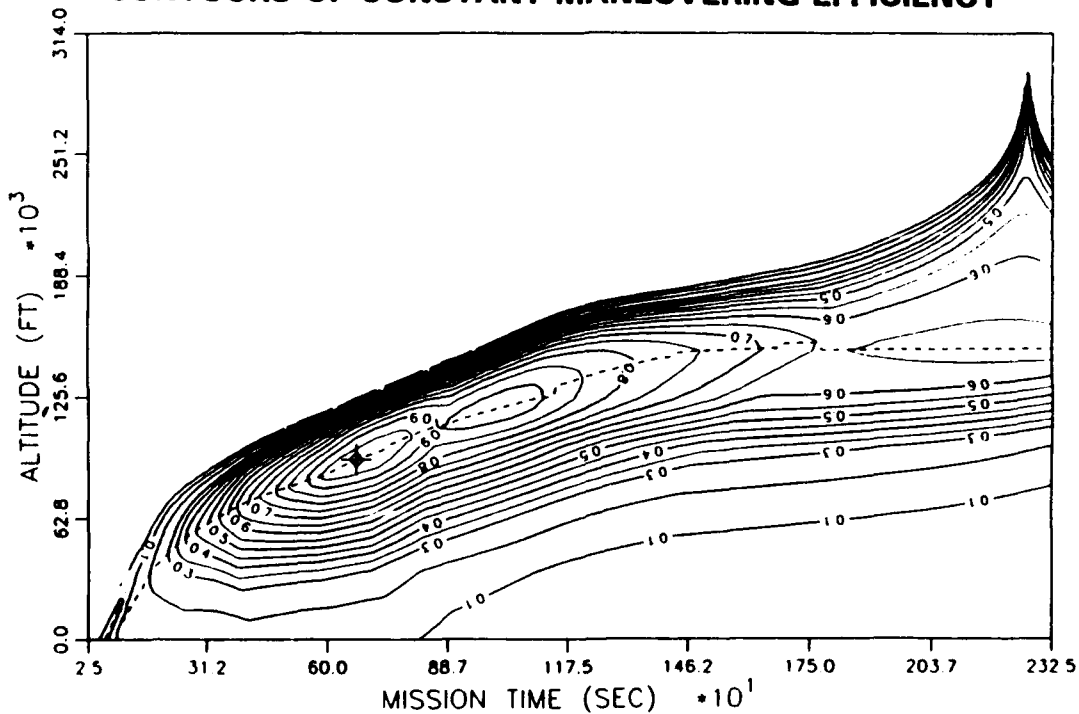
File: TC2A_NV.DRW

2 STAGES TO ORBIT OPTIMIZATION GOALS ACHIEVED ON PC



GRAPHICS ENHANCE UNDERSTANDING OF PERFORMANCE OPTIMIZATION

CONTOURS OF CONSTANT MANEUVERING EFFICIENCY



**CONCLUSIONS -
ETO PERFORMANCE OPTIMIZATION
ACHIEVED ON PERSONAL COMPUTER**

- ***MOST* - LOW COST, RAPID RESPONSE TOOL FOR PRELIM. DESIGN SUCCESSFULLY ACHIEVED**
- **ADVANTAGES OF PC DEMONSTRATED** - Low Cost, portability, and good graphics and support software (LOTUS, Harvard Graphics, Freelance, etc.)
- **USER-FRIENDLY ENVIRONMENT FOR PREPARATION OF INPUT FILES & OUTPUT DATA** - Facilitates OTIS input file preparation
- ***MOST* FAST RUNNING MODES DEMONSTRATED** - Good agreement with OTIS results. Early engineering model delivered
- **PRELIMINARY RESULTS FROM 2-D NLP/COLLOCATION ALGORITHMS (MINI-OTIS) ARE ENCOURAGING**
- **FAST RUNNING MODES FACILITATE NEW APPLICATION** - Trajectory optimizer simple and fast enough to imbed in vehicle design optimization code
- **OTIS HOSTED ON PC** - ETO flight achievable with large RAM (~40KBYTES)

An Algorithm for Trajectory Optimization on a Distributed-Memory Parallel Processor

Mark L. Psiaki and Kihong Park
Sibley School of Mechanical and Aerospace Engineering
Cornell University

Acknowledgement:

Work supported by NASA/LaRC

Continuous-Time Problem to be Solved

find: $u(t)$ and $x(t)$ for $t_0 \leq t \leq t_f$

to minimize: $J = \int_{t_0}^{t_f} L[x(t), u(t), t] dt + V[x(t_f)]$

subject to: $x(t_0)$ given

$$\dot{x} = f[x(t), u(t), t]$$
$$a_e[x(t), u(t), t] = 0$$
$$a_i[x(t), u(t), t] \leq 0$$
$$a_{ef}[x(t_f)] = 0$$
$$a_{if}[x(t_f)] \leq 0$$

Approach

- Use zero-order-hold control parameterization
- Model as a multi-stage parameter optimization problem
- Retain state variables and dynamic constraints explicitly
- Solve using a nonlinear programming algorithm that ...
 - ... has fast local and robust global convergence
 - ... allows infeasible intermediate results
 - ... parallelizes function, gradient, etc. evaluations at different time steps
 - ... exploits dynamic structure and parallelism to get search directions

Multi-Stage Nonlinear Programming Problem

find: $\mathbf{x} = \left[\mathbf{u}_0^T, \mathbf{x}_1^T, \mathbf{u}_1^T, \mathbf{x}_2^T, \dots, \mathbf{u}_{N-1}^T, \mathbf{x}_N^T \right]^T$

to minimize: $J = \sum_{k=0}^{N-1} L_k(\mathbf{x}_k, \mathbf{u}_k) + V[\mathbf{x}_N]$

subject to: $\mathbf{x}_0$ given

$$\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{u}_k) \quad \text{for } k = 0 \dots N-1$$

$$\mathbf{a}_{e_k}(\mathbf{x}_k, \mathbf{u}_k) = 0 \quad \text{for } k = 0 \dots N-1$$

$$\mathbf{a}_{i_k}(\mathbf{x}_k, \mathbf{u}_k) \leq 0 \quad \text{for } k = 0 \dots N-1$$

$$\mathbf{a}_{e_N}(\mathbf{x}_N) = 0$$

$$\mathbf{a}_{i_N}(\mathbf{x}_N) \leq 0$$

A Static/Dense Nonlinear Programming Problem

find: $\mathbf{x}$

to minimize: $J(\mathbf{x})$

subject to: $\mathbf{c}_e(\mathbf{x}) = 0$

$$\mathbf{c}_i(\mathbf{x}) \leq 0$$

Status of Project

- Parallel search direction algorithm:

FORTTRAN version tested on 32-node INTEL iPSC/2

- General static NP algorithm:

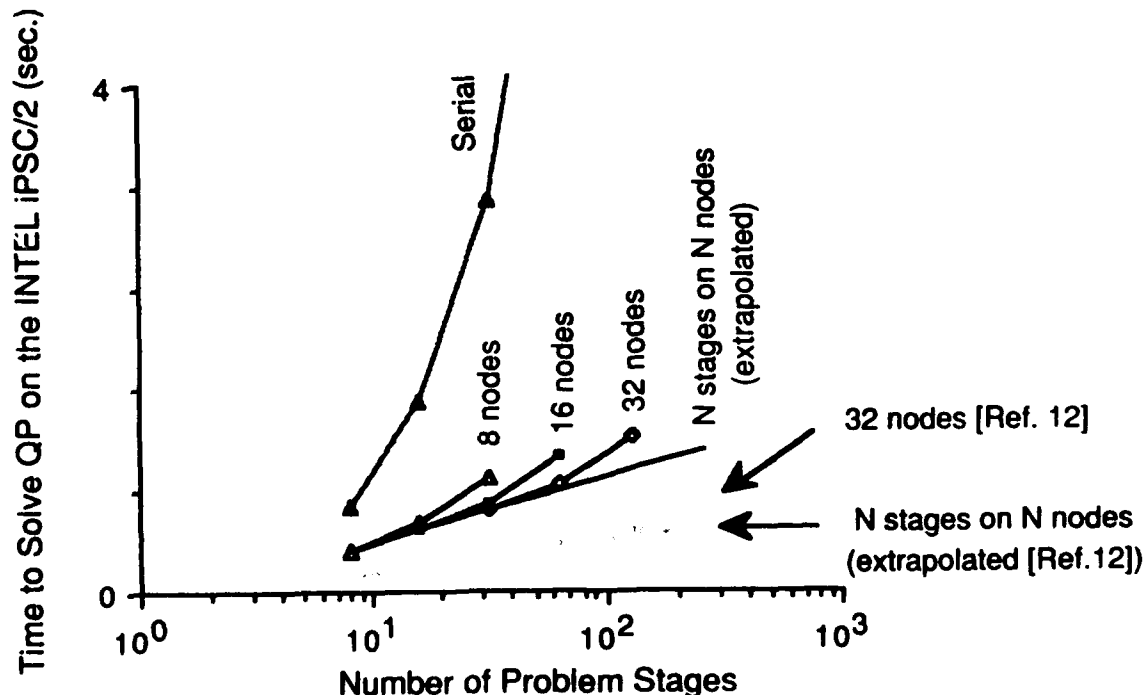
FORTTRAN version tested on 1 node of INTEL iPSC/2

Compared to NPSOL version 4.02 on static problems

- Full parallel trajectory optimization algorithm:

A "next generation" of the NP algorithm that exploits parallelism and dynamic problem structure

FORTTRAN components currently being tested on 32-node INTEL iPSC/860



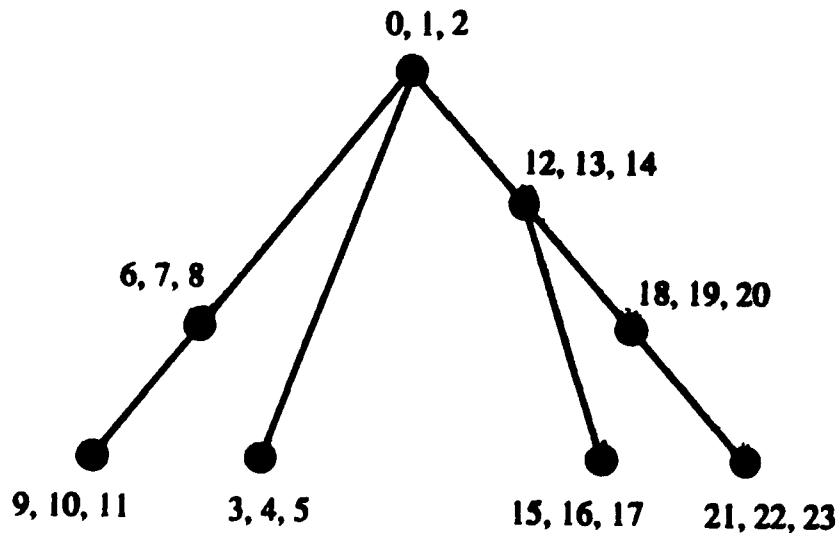
Plans

(the LORD willing)

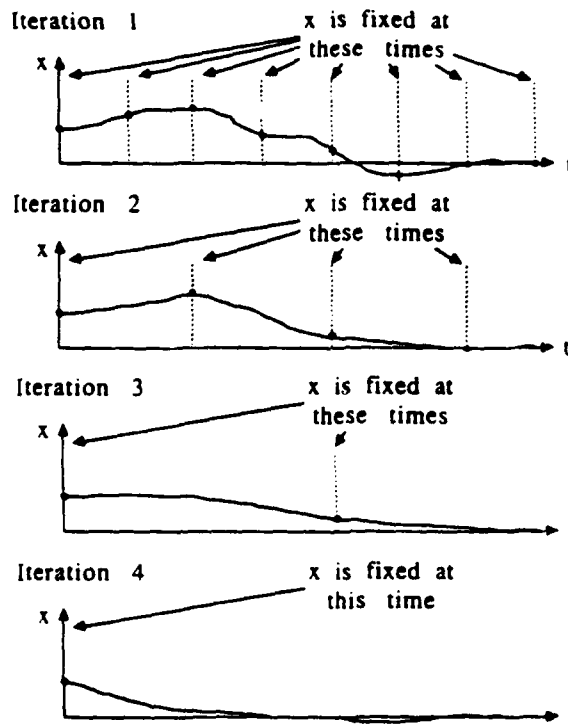
- Finish component and full algorithm testing (Present-Oct. '92?)
- Model and solve guidance problems for NASP and generic hypersonic vehicles (Oct. '92 - Dec. '93)
- Compare to existing codes (199?)
- Evaluate suitability for real-time guidance updates (199?)
- Make code user-friendly and disseminate (199?)

Distribution of Problem Stages on Parallel Processors

24 Stage Problem on 8 Processors



Divide-and-Conquer Trajectory Optimization



Test Problem 2 FOR STATIC NP ALGORITHM

find: x_1, x_2

to minimize: $J = -x_2$

subject to: $(x_1 - 1)^2 + x_2^2 + 10000(x_1^2 + x_2^2 - 1)^2 - .0625 \leq 0$

Test Results

- Augmented Lag.: 77 iterations
(52 feasibility and 25 optimality)
- NPSOL 4.02: **failed**

Test Problem 5 FOR STATIC NP ALGORITHM

find: $m_0, \Delta V_1, \Delta v, \Delta V_2$
to minimize: $J = m_0$
subject to: Newton's laws for a spherical Earth
Fixed fuel specific impulse
 $r_{LEO} - \epsilon_r \leq r_f \leq r_{LEO} + \epsilon_r$
 $V_{circ.} - \epsilon_v \leq V_f \leq V_{circ.} + \epsilon_v$
 $-\epsilon_\gamma \leq \gamma_f \leq +\epsilon_\gamma$
 $28^\circ - \epsilon_i \leq i_f \leq 28^\circ + \epsilon_i$
 $m_{empty} \leq m_f$

Test Results

- Augmented Lag.: 14/22 iterations
 $P_{\Sigma NST} = 1000$
 $P_{\Sigma NST} = 100$ 3/3 feas. 11/19 optim.
- NPSOL 4.02: 9 iterations

Aero-Assisted Orbital Maneuvering Example

(taken from Miele, 1989 ACC)

Problem: Minimize Fuel for GEO to LEO transfer with +28° inclination change

$$\mathbf{x} = [V, \gamma, \psi, r, \phi, \theta]^T$$

$$\mathbf{u} = [V_c, \gamma_c, \psi_c]^T \text{ or } [C_L, \sigma, \tau]^T$$

Constraint: Heating rate ≤ 100 watts/cm²

LQR-like problem derivation: Linear-quadraticize about guessed solution

