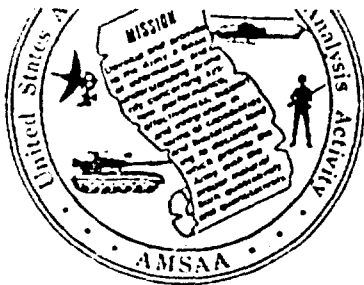


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TECHNICAL REPORT NO. 531

PREDICTION OF INPUT CONTROL FOR TIME INVARIANT
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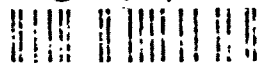
HERBERT E. COHEN

NOVEMBER 1992

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U. S. ARMY MATERIEL SYSTEMS ANALYSIS ACTIVITY
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Author: Herbert E. Cohen

PREDICTION OF INPUT CONTROL FOR TIME INVARIANT OPEN LOOP COMBAT-CONTROL SYSTEM

The analysis of the dynamical control system under investigation provides a commander with a direct analog for determining combat resupply requirements for achieving a desired goal by a specified time. It is shown that the introduction of the adjoint equation provides a means for determining the input requirements for an open loop linear piecewise time invariant control system such that any one of the states can be driven to a prescribed value.

Consider a linear piecewise time invariant dynamical system given by

$$\begin{aligned} \dot{x} &= Ax + Bu \quad (t_0 \leq t \leq t_f) \\ x(t_0) \end{aligned} \tag{1}$$

where x is the state variable with $x \in \mathbb{R}^n$, u is the external scalar input, A is a piecewise constant $n \times n$ matrix, $x(t_0)$ is the value of x at time t_0 , and B is an $n \times 1$ column vector. The adjoint equation for the homogeneous equation is given by

$$\dot{y} = -A^T y \quad (t_0 \leq t \leq t_f) \tag{2}$$

with

$$y(t_f)$$

the value of $y(t)$ at $t=t_f$.

Then it can be shown that

$$\frac{d}{dt} (y^T x) = y^T B u$$

so that

$$y^T(t_f) x(t_f) - y^T(t_0) x(t_0) = \int_{t_0}^{t_f} y^T B u \, d\xi \tag{3}$$

Let $u(t) = c g(t)$ where $g(t)$ represents the time behavior of $u(t)$ of amplitude c . Substituting into (3) and solving for c we have

$$c = \frac{y^T(t_f) x(t_f) - y^T(t_o) x(t_o)}{\int_{t_o}^{t_f} y^T B g(\xi) d\xi} \quad (4)$$

for any time dependent function $g(t)$. For the present discussion we will take $g(t)=1$ so that

$$c = \frac{y^T(t_f) x(t_f) - y^T(t_o) x(t_o)}{\int_{t_o}^{t_f} y^T B d\xi} \quad (5)$$

We now prescribe the value of $y^T(t_f)$ such that

$$y^T(t_f) x(t_f) = 0 \quad (6)$$

This being the case (5) becomes

$$c = \frac{-y^T(t_o) x(t_o)}{\int_{t_o}^{t_f} y^T(\xi) B d\xi} \quad (7)$$

This is the value of c under the condition of equation (6)

$$y^T(t_f) x(t_f) = 0$$

This equation allows us to prescribe a component of the state variable x_i to be driven to zero ($x_i = 0$) at $t = t_f$ by selecting at t_f

$$y(t_f) = (0 \ 0 \ \dots \ 0, \underset{\substack{\uparrow \\ (i^{th} \text{ component})}}{1}, 0 \ \dots \ 0)$$

that is $y(t_f)$ has all components zero except at the i^{th} component where it has the value 1.

For the special case where

$$g(t) = \delta(t-t_o) \text{ or } u = c \delta(t-t_o)$$

eq (4) reduces to

$$c = \frac{-y^T(t_0)x(t_0)}{y^T(t_0)B} \quad (8)$$

We will now consider the special case where A is of the form

$$A = \begin{bmatrix} Q & -E \\ -D & Q \end{bmatrix}$$

where $A = [a_{ij}]$, $0 \leq |a_{ij}| < 1$. In the absence of a control ($u = 0$), we have

$$\dot{x} = Ax \quad (9)$$

with initial condition at $t_0 = 1$ given by $x(1)$.

For a specified piecewise time invariant A and a given initial condition $x(1)$, x is given as shown in Figure 1 over a time interval $[1, 5]$. In Figure 1, the upper group of state variables are those state variables we wish by the application of the control to degrade while the lower group we wish to enhance. The x_i component for this study is in the upper group of state variables. The interpretation of these groups of state variables will be presented later in the report.

We now inquire as to the value of "u" that needs to be applied at time t_0 such that a specified x_i component will be driven to zero at a specified time t_f . Accordingly, for $u = c$ (a constant) from $[t_0, t_f]$ we have by Eq (7), with $t_0 = 2$, and $t_f = 4$

$$c = \frac{-y^T(2)x(2)}{\int_2^4 y^T(\xi)Bd\xi}$$

where we use quadrature integration methods to numerically integrate the denominator and where $y(t_f)$ associated with the adjoint equation is given by $y(t_f) = (0, 0 \dots 0, 1, 0 \dots 0)$. The result is given in Figures 2 and 3 where we see the $x_i(t_f)$ component at t_f is zero as was expected. Here, Figure 2 is the upper group of state variables identified in Figure 1 while Figure 3 is the lower group of state variables shown in Figure 1. We thus have demonstrated that a component x_i of the state variable x can be driven to a specified value (here considered zero) if the control is selected from Eq (4) or Eq (7) if Eq (6) is used.

This analysis of a dynamical control system has an immediate application to linear combat model described by piecewise time-invariant Lanchester equations. The equation described in Eq (1) can be interpreted as a Lanchester Equation of combat with resupply given by the Bu term where u is the total force structure and B is the distribution of types of weapons making up u . Most important is that c is the total force strength needed to reduce or annihilate a component of an opposing force by a specified time. Thus a commander in the field threatened by an opposing force has the potential of determining the total force strength needed, with a prescribed distribution B , that would reduce or annihilate the opposing force to an ineffective level. Figures 2 and 3 can now be interpreted as the force strength of red and blue forces, respectively, due to resupply of blue forces during combat.

In conclusion, it has been shown that the solution to the adjoint equation Eq (2), plays a key role in establishing the needed magnitude of the external input u in order to drive a component of the state to a prescribed value by a designated time. In addition, it should be noted that in the absence of an external input, $u=0$, Eq (3) reduces to the inner product criteria reported in reference (1).

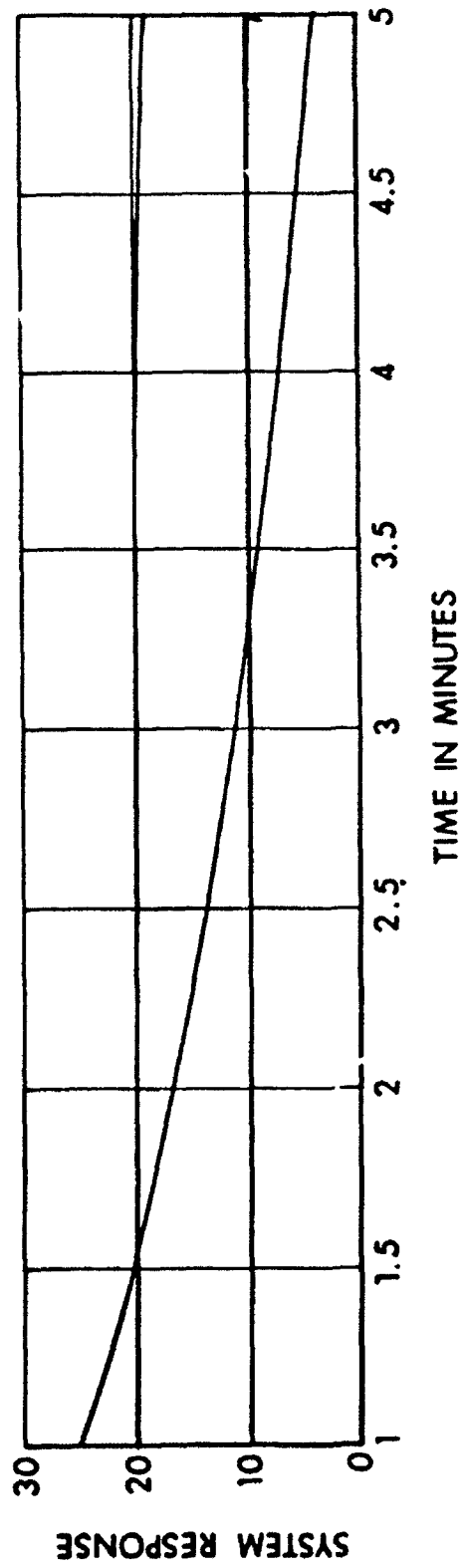
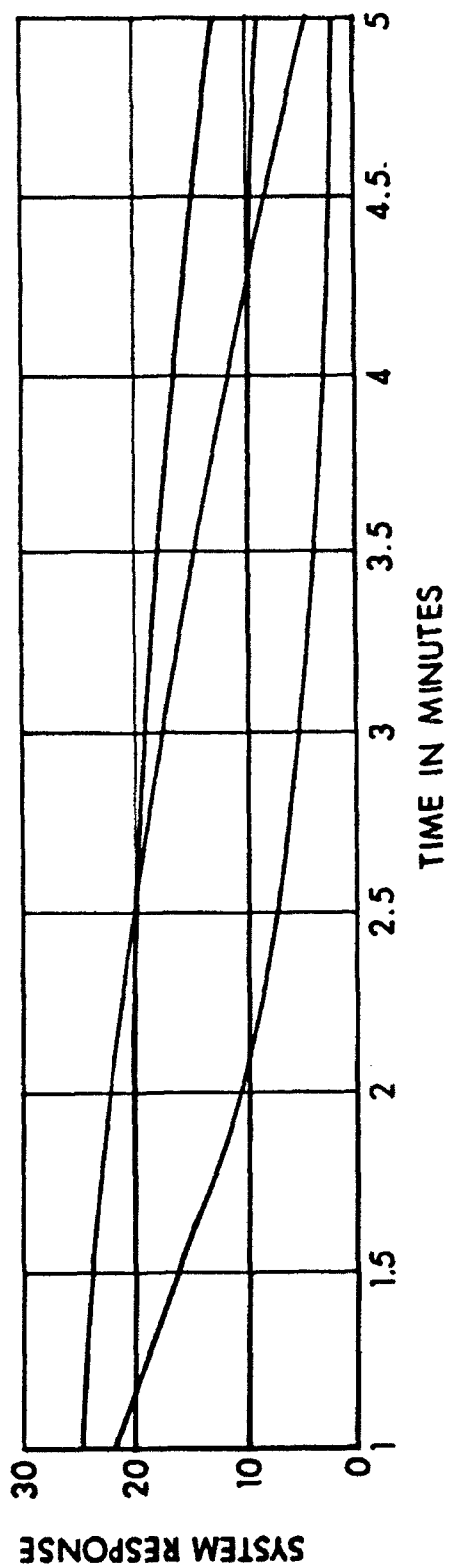


Figure 1. System Response With Control $u = 0$

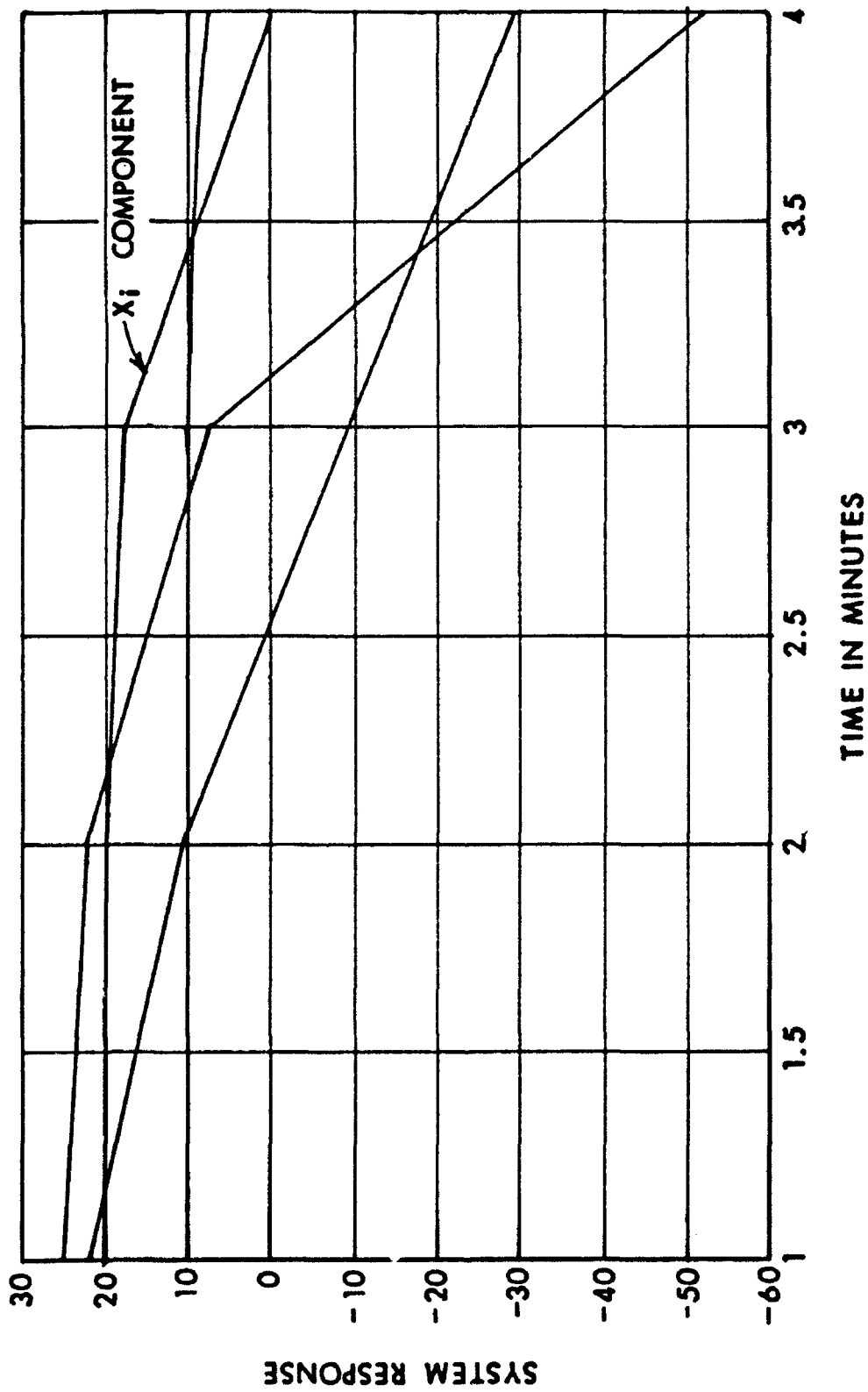


Figure 2. System Response With Control $u > 0$

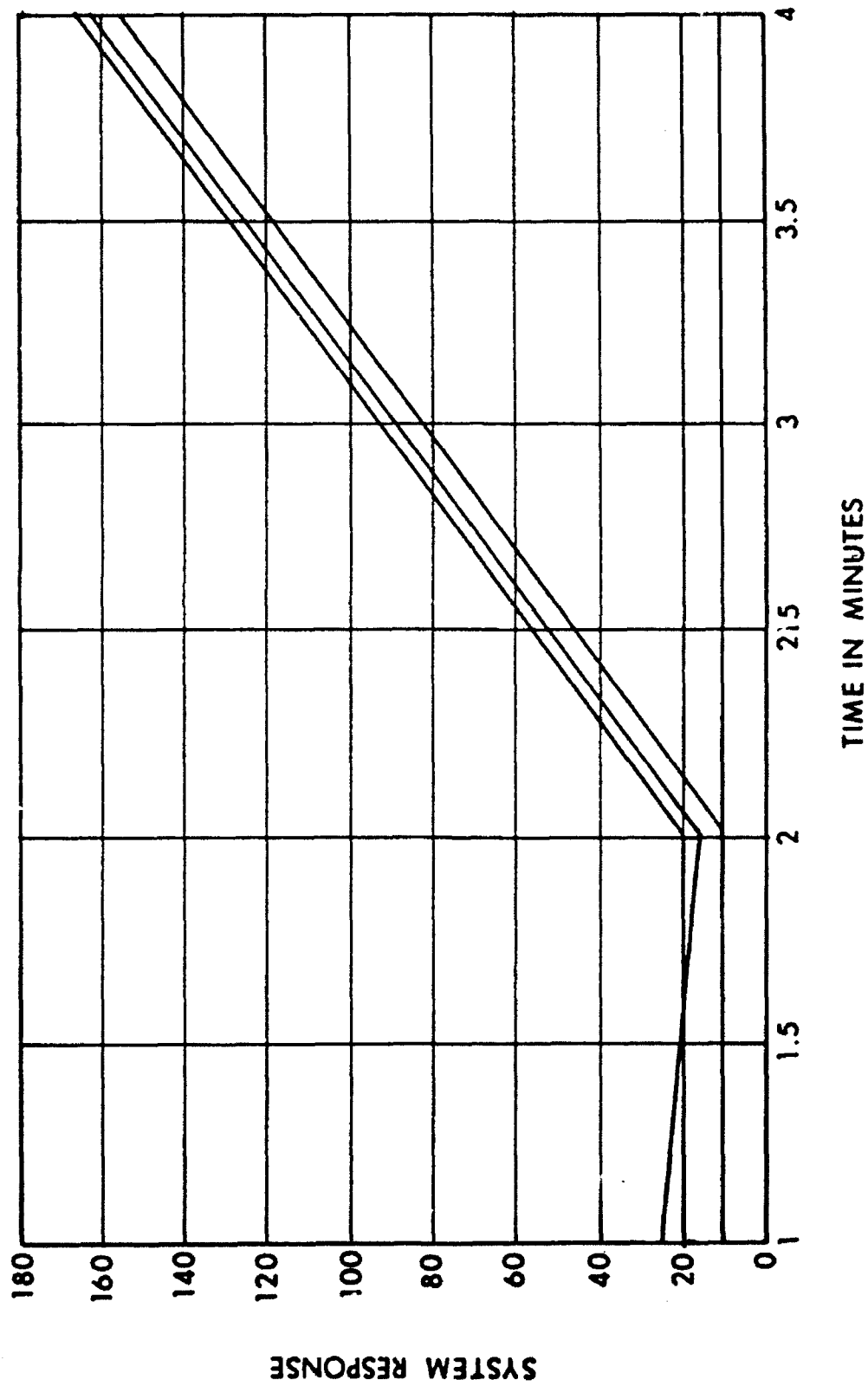


Figure 3. System Response With Control $u > 0$.

REFERENCE

1. Cohen, Herbert E (February 1991), AMSAA Technical Report No. 489, "Inner Product Performance Criteria for Evaluating Combat Models."

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PRINCIPAL FINDINGS: Developed a set of equations that can be used to predict resupply requirements for achieving a desired goal in a specified time.

MAIN ASSUMPTIONS: Linear time invariant control system

PRINCIPAL LIMITATIONS: None

SCOPE OF THE EFFORT: Develop solutions for open loop control requirements and introduce the role of the adjoint differential equation.

OBJECTIVE: Investigate role of the adjoint equation in yielding understanding of the resupply requirements for combat commander.

BASIC APPROACH: Solve vector linear time invariant system describing the control system.

REASON FOR PERFORMING THE STUDY OR ANALYSIS: Investigate properties of dynamical control equation and its relationship to equations of combat

IMPACT OF THE STUDY: For time invariant systems, resupply requirements can be determined to assist commander in combat in achieving a desired goal.

SPONSOR: None

PRINCIPAL INVESTIGATOR: Herbert E. Cohen

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