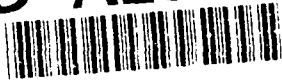


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ARMY RESEARCH LABORATORY



# Helicopter Forced Response Vibration Analysis Method RTVIB20

Joseph Fries

ARL-TR-75

February 1993

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156

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# TABLE OF CONTENTS

|                                                                                                                                        | <u>Page</u> |
|----------------------------------------------------------------------------------------------------------------------------------------|-------------|
| 1. INTRODUCTION .....                                                                                                                  | 1           |
| 2. NOMENCLATURE .....                                                                                                                  | 3           |
| 3. DERIVATION OF AERODYNAMIC TERMS .....                                                                                               | 4           |
| 4. APPLICATION OF THE ANALYSIS FOR A UH-60A HELICOPTER WITH A<br>DAMAGED MAIN ROTOR BLADE .....                                        | 14          |
| 5. DISCUSSION OF RESULTS .....                                                                                                         | 15          |
| 6. CONCLUSIONS .....                                                                                                                   | 18          |
| 7. REFERENCES .....                                                                                                                    | 19          |
| APPENDIX: DESCRIPTION OF THE COMPUTER PROGRAM TO PREDICT<br>HELICOPTER VIBRATIONS FOR UNDAMAGED AND DAMAGED<br>MAIN ROTOR BLADES ..... | 21          |
| DISTRIBUTION LIST .....                                                                                                                | 31          |

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## 1. INTRODUCTION

This report describes an analytical method and its associated computer program which can be used to predict helicopter vibrations for undamaged and damaged main rotor blades. The analysis is generic and can be used to analyze any helicopter with a minimal amount of physical input data.

The model consists of independent main rotor blade flapping-torsional degrees of freedom and helicopter longitudinal-vertical degrees of freedom. Level flight trimmed conditions are determined first, after which damage is imposed onto the main rotor blades. Vibration levels are calculated in the cockpit both for the undamaged and damaged cases. The model considers aerodynamic forces and moments using blade element theory wherein each harmonic is calculated individually.

This program calculates cockpit vibrations for a bare helicopter with no vibration absorber devices present, and, in conjunction with input of human tolerance to vibration data, the code can be used to predict aircraft kill categories.

There are other sources contributing to helicopter vulnerability due to main rotor blade damage besides human vibration tolerance, which can also place a helicopter in jeopardy. These include loss of main rotor blade structural integrity, vibrations transmitted to other critical structural components on the aircraft, and lack of pilot control authority to compensate for blade damage. These additional factors are not included in this analysis.

The aerodynamic terms are developed using blade element theory in Section 3 and are referenced to Figure 1. The basic theory is from Barnes (1967) and Chopra (1992). In this analysis, the aerodynamic forcing functions are separated into individual harmonics so as to be able to trace the frequency filtering effect from the rotating blade into the fixed (fuselage) system and to isolate and identify the fundamental physical parameter contributions to the aerodynamic forcing functions.

Section 4 describes results of application of the analysis to a UH-60A helicopter in which one blade in the main rotor is considered to be damaged. The damage consists of the removal of various amounts of the outboard end (tip) portions of one rotor blade. Section 5 discusses the results of the damaged rotor blade analyses, and Section 6 draws conclusions from the results.

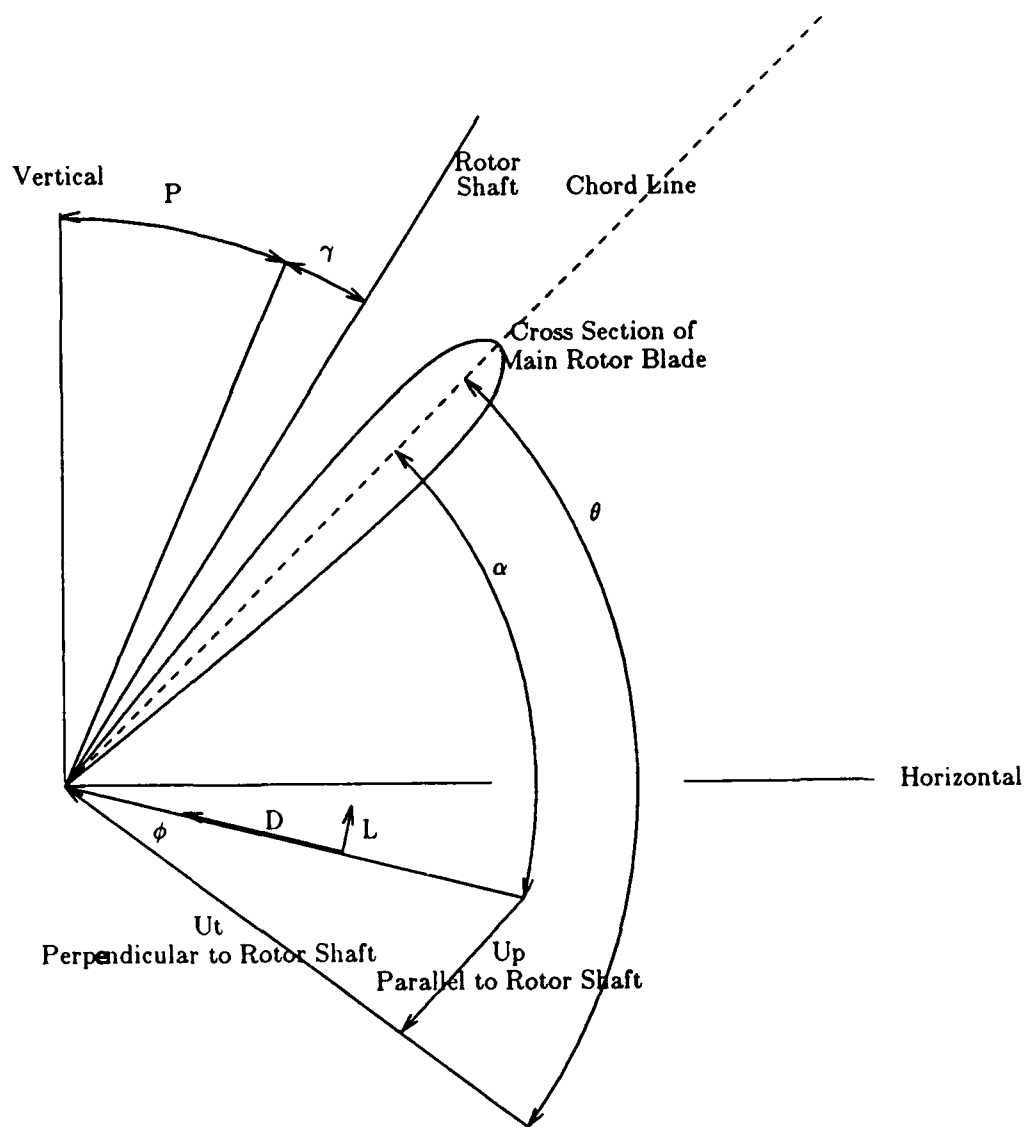


Figure 1. Aerodynamic airfoil definitions.

The Appendix contains a description of the code, the input variables required, a description of the computer code output, and a list of output variables.

## 2. NOMENCLATURE

|          |                                                                         |
|----------|-------------------------------------------------------------------------|
| $\alpha$ | Angle of attack                                                         |
| $a$      | Lift slope                                                              |
| $\beta$  | Flap angle                                                              |
| $c$      | Chord                                                                   |
| $C_d$    | Blade drag coefficient                                                  |
| $C_l$    | Blade lift coefficient                                                  |
| $C_m$    | Blade torsional moment coefficient                                      |
| $L$      | Lift                                                                    |
| $D$      | Drag                                                                    |
| $e_a$    | Blade torsional elastic axis,<br>+ measured from PA toward leading edge |
| $F$      | Force parallel to rotor shaft                                           |
| $F_o$    | Steady value of $F$                                                     |
| $\gamma$ | Rotor shaft built in tilt angle + forward                               |
| $\Omega$ | Rotor speed                                                             |
| $P$      | Helicopter pitch angle + into the wind                                  |
| $PA$     | Blade pitch axis, + measured from leading edge                          |
| $\Pi$    | $\Pi = 3.14$                                                            |
| $\Psi$   | Rotor azimuth angle                                                     |
| $r$      | Variable blade radius                                                   |
| $R$      | Blade radius                                                            |
| $\rho$   | Air density                                                             |
| $t$      | Time                                                                    |
| $\theta$ | Pitch angle                                                             |
| $U$      | Average downwash velocity                                               |
| $X$      | Helicopter longitudinal displacement                                    |
| $Z$      | Helicopter vertical displacement                                        |
| $N$      | Number of blades per rotor                                              |
| $Q$      | Coefficient of $\alpha^2$ in $C_m$ equation                             |
| $K$      | Harmonic Number                                                         |

### 3. DERIVATION OF AERODYNAMIC TERMS

The rotor blade lift and drag per unit length is defined as\*

$$\frac{dL}{dr} = \frac{1}{2} \rho C_L c (U_T^2 + U_P^2) \quad (1)$$

and

$$\frac{dD}{dr} = \frac{1}{2} \rho C_D c (U_T^2 + U_P^2), \quad (2)$$

where

$$C_L = \alpha a, \quad \alpha = \theta - \phi, \quad (3)$$

$$\phi = \tan^{-1} \left( \frac{U_P}{U_T} \right), \quad (4)$$

$$U_T = \Omega r + \left( \frac{dX}{dt} \cos(P + \gamma) - \frac{dZ}{dt} \sin(P + \gamma) \sin \Psi \right), \quad (5)$$

$$U_P = U + \frac{dX}{dt} \sin(P + \gamma) + \frac{dZ}{dt} \cos(P + \gamma) + \frac{r d\beta}{dt}, \quad (6)$$

$$\theta = \sum_{i=0}^K \theta_{ic} \cos i\Psi + \theta_{is} \sin i\Psi, \quad (7)$$

and

$$\beta = \sum_{i=0}^K \beta_{ic} \cos i\Psi + \beta_{is} \sin i\Psi. \quad (8)$$

Resolving  $dL/dr$  and  $dD/dr$  parallel to the rotor shaft,

$$\frac{dF}{dr} = \frac{dL}{dr} \cos \phi - \frac{dD}{dr} \sin \phi. \quad (9)$$

---

\* Equations 1 to 4 are standard blade element theory equations from Barnes (1967) and Chopra (1992). The equations 5 to 42 were developed by the author.

For high-speed level flight,  $\phi = U_P/U_T$ , then

$$\frac{dL}{dr} = \frac{1}{2} \rho a c \left( U_T^2 + U_P^2 \right) \left( \theta - \frac{U_P}{U_T} \right). \quad (10)$$

Also for level flight, let  $\cos\phi = 1$  and  $\sin\phi = \phi$ , so that

$$\frac{dF}{dr} = \frac{1}{2} \rho a c \left[ \theta \left( U_T^2 + U_P^2 \right) - U_P U_T \right] - \frac{1}{2} \rho C_D c U_P U_T, \quad (11)$$

where terms of third order are neglected

$$\frac{U_P^3}{U_T}.$$

Collecting terms

$$\frac{dF}{dr} = \frac{1}{2} \rho c \left[ a \theta \left( U_T^2 + U_P^2 \right) - U_T U_P (a + C_D) \right]. \quad (12)$$

$U_T$  and  $U_P$  are written as

$$U_T = \Omega r + \left( \frac{dX}{dt} - \frac{dZ}{dt} (P + \gamma) \right) \sin \Psi \quad (13)$$

and

$$U_P = U + \frac{dX}{dt} (P + \gamma) + \frac{dZ}{dt} + \frac{r d\beta}{dt}. \quad (14)$$

The downwash velocity is calculated as

$$U = \sqrt{\frac{N F_o}{2 \rho \Pi R^2}}. \quad (15)$$

For level high-speed flight,  $U_P \ll U_T$ , so Equation 12 can be written as

$$\frac{dF}{dr} = \frac{1}{2} \rho c \left[ a \theta U_T^2 - U_T U_P (a + C_D) \right]. \quad (16)$$

Define the following parameters:

$$AU = \frac{dX}{dt} - \frac{dZ}{dt} (P + \gamma), \quad (17)$$

$$BU = \Omega^2 r^2 + \frac{(AU)^2}{2}, \quad (18)$$

$$DU = 2\Omega r(AU), \quad (19)$$

$$EU = -\frac{(AU)^2}{2}, \quad (20)$$

and

$$GU = U + \frac{dX}{dt} (P + \gamma) + \frac{dZ}{dt}. \quad (21)$$

This allows

$$U_T^2 = (BU) + (DU) \sin \Psi + (EU) \cos 2\Psi \quad (22)$$

and

$$U_T U_P = \Omega r(GU) + \Omega r^2 \frac{d\beta}{dt} + (AU)(GU) \sin \Psi + (AU)r \frac{d\beta}{dt} \sin \Psi. \quad (23)$$

Performing a radial integration of the terms containing  $r$ , evaluating the trigonometric product terms, and substituting in Equation 16, we obtain new harmonic terms that are sums and differences of the original harmonic products. This is called harmonic cross coupling.

$$F = \frac{1}{2} \rho c \left( a \Omega^2 \frac{R^3}{3} + (AU)^2 \frac{R}{2} \right) \sum_{i=0}^K \theta_{ic} \cos i\Psi$$

$$\begin{aligned}
& + a \left( \Omega^2 \frac{R^3}{3} + (AU)^2 \frac{R}{2} \right) \sum_{i=0}^K \theta_{is} \sin i\Psi \\
& + a \Omega R^2 \frac{(AU)}{2} \sum_{i=0}^K \theta_{ic} [\sin(i+1)\Psi - \sin(i-1)\Psi] \\
& + a \Omega R^2 \frac{(AU)}{2} \sum_{i=0}^K \theta_{is} [\cos(i-1)\Psi - \cos(i+1)\Psi] \\
& - a (AU)^2 \frac{R}{4} \sum_{i=0}^K \theta_{ic} [\cos(i+2)\Psi + \cos(i-2)\Psi] \\
& - a (AU)^2 \frac{R}{4} \sum_{i=0}^K \theta_{is} [\sin(i+2)\Psi + \sin(i-2)\Psi] \\
& - (a + C_D) \frac{\Omega}{2} R^2 (GU) \\
& - (a + C_D) R^3 \frac{\Omega^2}{3} \sum_{i=0}^K i (-\beta_{ic} \sin i\Psi + \beta_{is} \cos i\Psi) \\
& - (a + C_D) (AU) (GU) R \sin \Psi \\
& - (a + C_D) (AU) \frac{R^2}{2} \Omega \sum_{i=0}^K \frac{-i\beta_{is}}{2} [\cos(i-1)\Psi - \cos(i+1)\Psi] \\
& - (a + C_D) (AU) \frac{R^2}{2} \Omega \sum_{i=0}^K \frac{i\beta_{ic}}{2} [\sin(i+1)\Psi - \sin(i-1)\Psi] .
\end{aligned} \tag{24}$$

For torsion, the moment per unit span is

$$\frac{dM}{dr} = \frac{1}{2} \rho C_M c^2 (U_T^2 + U_P^2) , \tag{25}$$

where

$$C_M = C_{M0} + \gamma \alpha + Q \alpha^2, \quad (26)$$

$$C_M = C_{M0} + \gamma \left( \theta - \frac{U_P}{U_T} \right) + Q \left( \theta - \frac{U_P}{U_T} \right)^2, \quad (27)$$

and

$$\frac{dM}{dr} = \frac{1}{2} \rho c^2 \left( C_{M0} U_T^2 + \gamma \theta U_T^2 - \gamma U_T U_P + Q \theta^2 U_T^2 - 2Q \theta U_T U_P \right). \quad (28)$$

After integrating radially along the blade and separating out harmonics similar to the harmonic cross coupling that was obtained for the force terms F of Equation 24.

$$\begin{aligned} M = & \frac{1}{2} \rho c^2 C_{M0} \left[ \left( \Omega^2 \frac{R^3}{3} + (AU)^2 \frac{R}{2} \right) + \Omega R^2 (AU) \sin \Psi - (AU)^2 \frac{R}{2} \cos 2\Psi \right] \\ & + \frac{1}{2} \rho c^2 \gamma \left[ \left( \Omega^2 \frac{R^3}{3} + (AU)^2 \frac{R}{2} \right) \sum_{i=0}^K \theta_{ic} \cos i\Psi \right] \\ & + \frac{1}{2} \rho c^2 \gamma \left[ \left( \Omega^2 \frac{R^3}{3} + (AU)^2 \frac{R}{2} \right) \sum_{i=0}^K \theta_{is} \sin i\Psi \right] \\ & + \frac{1}{2} \rho c^2 \gamma \left( \Omega R^2 \frac{(AU)}{2} \sum_{i=0}^K \theta_{ic} [\sin(i+1)\Psi - \sin(i-1)\Psi] \right) \\ & + \frac{1}{2} \rho c^2 \gamma \left( \Omega R^2 \frac{(AU)}{2} \sum_{i=0}^K \theta_{is} [\cos(i-1)\Psi - \cos(i+1)\Psi] \right) \\ & - \frac{1}{2} \rho c^2 \gamma \left( [AU]^2 \frac{R}{4} \sum_{i=0}^K \theta_{ic} [\cos(i+2)\Psi + \cos(i-2)\Psi] \right) \end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2} \rho c^2 \gamma \left( (AU)^2 \frac{R}{4} \sum_{i=0}^K \theta_{is} [SIN(i+2)\Psi + SIN(i-2)\Psi] \right) \\
& -\frac{1}{2} \rho c^2 \gamma \left( \Omega \frac{R^2}{2} (GU) + R^3 \frac{\Omega^2}{3} \sum_{i=0}^K i \left( -\beta_{ic} SIN i \Psi + \beta_{is} COS i \Psi \right) \right) \\
& -\frac{1}{2} \rho c^2 \gamma (AU) (GU) R SIN \Psi \\
& -\frac{1}{2} \rho c^2 \gamma (AU) R^2 \frac{\Omega}{2} \sum_{i=0}^K \frac{-i \beta_{ic}}{2} ([COS(i-1)\Psi - COS(i+1)\Psi]) \\
& -\frac{1}{2} \rho c^2 \gamma (AU) R^2 \frac{\Omega}{2} \sum_{i=0}^K i \beta_{is} [SIN(i+1)\Psi - SIN(i-1)\Psi] \\
& +\frac{1}{2} \rho c^2 Q \left( \Omega^2 \frac{R^3}{3} + (AU)^2 \frac{R}{2} \right) \sum_{i=0}^K \frac{\theta_{ic}^2}{2} (COS 2i \Psi + 1) \\
& +\frac{1}{2} \rho c^2 Q \left( \Omega^2 \frac{R^3}{3} + (AU) \frac{2R}{2} \right) \sum_{i=0}^K \theta_{ic} \theta_{is} SIN 2i \Psi \\
& +\frac{1}{2} \rho c^2 Q \left( \Omega^2 \frac{R^3}{3} + (AU)^2 \frac{R}{2} \right) \sum_{i=0}^K \frac{\theta_{is}^2}{2} (1 - COS 2i \Psi) \\
& +\frac{1}{2} \rho c^2 Q \Omega R^2 \frac{(AU)}{2} \sum_{i=0}^K \theta_{ic}^2 \left( \frac{1}{2} [SIN(2i+1)\Psi - SIN(2i-1)\Psi] + SIN \Psi \right) \\
& +\frac{1}{2} \rho c^2 Q \Omega R^2 \frac{(AU)}{2} \sum_{i=0}^K \theta_{ic} \frac{\theta_{is}}{2} [COS(2i-1)\Psi - COS(2i+1)\Psi] \\
& +\frac{1}{2} \rho c^2 Q \Omega R^2 \frac{(AU)}{2} \sum_{i=0}^K \theta_{is}^2 \left( SIN \Psi - \frac{1}{2} [SIN(2i+1)\Psi - SIN(2i-1)\Psi] \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2} \rho c^2 Q(AU)^2 \frac{R}{4} \sum_{i=0}^K \theta_{ic}^2 \left( \frac{1}{2} [\cos(2i+2)\Psi + \cos(2i-2)\Psi] + \cos 2\Psi \right) \\
& -\frac{1}{2} \rho c^2 Q(AU)^2 \frac{R}{2} \sum_{i=0}^K \frac{\theta_{ic} \theta_{is}}{2} [\sin(2i+2)\Psi + \sin(2i-2)\Psi] \\
& -\frac{1}{2} \rho c^2 Q(AU)^2 \frac{R}{4} \sum_{i=0}^K \theta_{is}^2 \left( \cos 2\Psi - \frac{1}{2} \cos(2i+2)\Psi + \cos(2i-2)\Psi \right) \\
& -\frac{1}{2} \rho c^2 Q \frac{\Omega R^2}{2} (GU) \sum_{i=0}^K (\theta_{ic} \cos i\Psi + \theta_{is} \sin i\Psi) \\
& -\frac{1}{2} \rho c^2 Q \Omega^2 \frac{R^3}{3} \sum_{i=0}^K \theta_{ic} \sum_{p=0}^W -\frac{p\beta_{pc}}{2} [\sin(i+p)\Psi - \sin(i-p)\Psi] \\
& -\frac{1}{2} \rho c^2 Q \Omega^2 \frac{R^3}{3} \sum_{i=0}^K \theta_{ic} \sum_{p=0}^W \frac{p\beta_{ps}}{2} [\cos(i+p)\Psi - \cos(i-p)\Psi] \\
& -\frac{1}{2} \rho c^2 Q \Omega^2 \frac{R^3}{3} \sum_{i=0}^K \theta_{is} \sum_{p=0}^W -\frac{p\beta_{pc}}{2} [\cos(i-p)\Psi - \cos(i+p)\Psi] \\
& -\frac{1}{2} \rho c^2 Q \Omega^2 \frac{R^3}{3} \sum_{i=0}^K \theta_{is} \sum_{p=0}^W \frac{p\beta_{ps}}{2} [\sin(i+p)\Psi + \sin(i-p)\Psi] \\
& -\frac{1}{2} \rho c^2 Q(AU)(GU)R \sum_{i=0}^K \frac{\theta_{ic}}{2} [\sin(i+1)\Psi - \sin(i-1)\Psi] \\
& -\frac{1}{2} \rho c^2 Q(AU)(GU)R \sum_{i=0}^K \frac{\theta_{is}}{2} [\cos(i-1)\Psi - \cos(i+1)\Psi] \\
& -\frac{1}{2} \rho c^2 \frac{2Q(AU)R^2}{2} \Omega \sum_{i=0}^K \theta_{ic} \sum_{p=0}^W \frac{-p\beta_{pc}}{4} [\cos(i+p-1)\Psi + \cos(i-p+1)\Psi]
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{2} \rho c^2 \frac{2Q(AU)R^2}{2} \Omega \sum_{i=0}^K \theta_{ic} \sum_{p=0}^W \frac{p\beta_{pc}}{4} [\cos(i+p+1)\Psi + \cos(i-p-1)\Psi] \\
& -\frac{1}{2} \rho c^2 \frac{2Q(AU)R^2}{2} \Omega \sum_{i=0}^K \theta_{ic} \sum_{p=0}^W \frac{p\beta_{ps}}{4} [\sin(i+p+1)\Psi - \sin(i-p-1)\Psi] \\
& -\frac{1}{2} \rho c^2 \frac{2Q(AU)R^2}{2} \Omega \sum_{i=0}^K \theta_{ic} \sum_{p=0}^W \frac{-p\beta_{ps}}{4} [\sin(i+p-1)\Psi - \sin(i-p+1)\Psi] \\
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& -\frac{1}{2} \rho c^2 \frac{2Q(AU)R^2}{2} \Omega \sum_{i=0}^K \theta_{is} \sum_{p=0}^W [\sin(i+p+1)\Psi + \sin(i-p-1)\Psi] \\
& -\frac{1}{2} \rho c^2 \frac{2Q(AU)R^2}{2} \Omega \sum_{i=0}^K \theta_{is} \sum_{p=0}^W [\cos(i-p-1)\Psi - \cos(i+p+1)\Psi] \\
& -\frac{1}{2} \rho c^2 \frac{2Q(AU)R^2}{2} \Omega \sum_{i=0}^K \theta_{is} \sum_{p=0}^W \frac{-p\beta_{ps}}{4} [\cos(i-p+1)\Psi - \cos(i+p-1)\Psi]. \quad (29)
\end{aligned}$$

In addition, there is an effect in torsion when the pitch and elastic axes are different than quarter chord, where the aerodynamic force acts. The torsional moment due to the pitch and elastic offset axes is given as

$$\frac{dM_{e_a}}{dr} = - \left( \frac{C}{4} - PA + e_a \right) \left[ \frac{dL}{dr} + \frac{dD}{dr} \left( \theta - \frac{U_P}{U_T} \right) \right] \quad (30)$$

and

$$\frac{dM_{e_a}}{dr} = - \left( \frac{C}{4} - PA + e_a \right) \frac{1}{2} \rho c (a + C_D) (\theta U_T^2 - U_T U_P). \quad (31)$$

Performing the radial integration, again separating harmonics and letting  $(C/4 - PA + e_a) = CB$ ,

$$\begin{aligned}
M_{e_a} = & -CB \frac{1}{2} \rho c (a + C_D) \left( \Omega^2 \frac{R^3}{3} + (AU)^2 \frac{R}{2} \right) \sum_{i=0}^K \theta_{ic} \cos i \Psi \\
& - CB \frac{1}{2} \rho c (a + C_D) \left( \Omega^2 \frac{R^3}{3} + (AU)^2 \frac{R}{2} \right) \sum_{i=0}^K \theta_{is} \sin i \Psi \\
& - CB \frac{1}{2} \rho c (a + C_D) \frac{\Omega R^2}{2} (AU) \sum_{i=0}^K \theta_{ic} [\sin (i+1) \Psi - \sin (i-1) \Psi] \\
& - CB \frac{1}{2} \rho c (a + C_D) \frac{\Omega R^2}{2} (AU) \sum_{i=0}^K \theta_{is} [\cos (i-1) \Psi - \cos (i+1) \Psi] \\
& + CB \frac{1}{2} \rho c \frac{(a + C_D)}{4} (AU)^2 R \sum_{i=0}^K \theta_{ic} [\cos (i+2) \Psi + \cos (i-2) \Psi] \\
& + CB \frac{1}{2} \rho c \frac{(a + C_D)}{4} (AU)^2 R \sum_{i=0}^K \theta_{is} [\sin (i+2) \Psi + \sin (i-2) \Psi] \\
& + CB \frac{1}{2} \rho c (a + C_D) \Omega^2 \frac{R^2}{2} (GU) \\
& + CB \frac{1}{2} \rho c (a + C_D) R^3 \frac{\Omega^2}{3} \sum_{i=0}^K i (-\beta_{ic} \sin i \Psi + \beta_{is} \cos i \Psi) \\
& + CB \frac{1}{2} \rho c (a + C_D) (AU) (GU) R \sin \Psi \\
& + CB \frac{1}{2} \rho c (a + C_D) (AU) R^2 \frac{\Omega}{2} \sum_{i=0}^K \frac{-i \beta_{ic}}{2} [\cos (i-1) \Psi - \cos (i+1) \Psi] \\
& + CB \frac{1}{2} \rho c (a + C_D) (AU) R^2 \frac{\Omega}{2} \sum_{i=0}^K \frac{i \beta_{is}}{2} [\sin (i+1) \Psi - \sin (i-1) \Psi] . \quad (32)
\end{aligned}$$

The rotor blade flapping equation of motion is

$$I_B \frac{d^2 \beta}{dt^2} + C_B \frac{d\beta}{dt} + K_B \beta = \sum_{i=0}^K H_{ic} \cos i\Psi + H_{is} \sin i\Psi, \quad (33)$$

where  $I_B$  = flap 2nd moment,  $C_B$  = flap damping,  $K_B$  = flap spring, and  $H_{ic}$  and  $H_{is}$  = flap moment forcing (cosine and sine).

Let

$$\beta = \sum_{i=0}^K \beta_{ic} \cos i\Psi + \beta_{is} \sin i\Psi, \quad (34)$$

$$\frac{d\beta}{dt} = \sum_{i=0}^K -i\Omega \beta_{ic} \sin i\Psi + i\Omega \beta_{is} \cos i\Psi, \quad (35)$$

Substituting in Equation 3 and equating *SIN* and *COS* terms on both sides of the equal sign,

$$-I_B \beta_{ic} \Omega^2 + C_B \beta_{is} i\Omega + K_B \beta_{ic} = H_{ic} \quad (37)$$

and

$$-I_B \beta_{is} \Omega^2 + C_B \beta_{ic} i\Omega + K_B \beta_{is} = H_{is}. \quad (38)$$

and

$$\frac{d^2 \beta}{dt^2} = \sum_{i=0}^K (-i\Omega)^2 \beta_{ic} \cos i\Psi - (-i\Omega)^2 \beta_{is} \sin i\Psi. \quad (36)$$

Let

$$H_{ic} = H_{ic-} + \beta_{is} LA_{ic} \quad (39)$$

and

$$H_{is} = H_{is-} + \beta_{ic} LA_{is}, \quad (40)$$

where  $\beta_{is}(LA)_{ic}$  = aero coefficient of the *COS* $i\Psi$  terms and  $\beta_{ic}(LA)_{is}$  = aero coefficient of the *SIN* $i\Psi$  terms.

Taking these terms over to the left-hand side in Equations 37 and 38 results in

$$- I_{\beta} \beta_{ic} \Omega^2 + (C_{\beta} i \Omega - L A_{ic}) \beta_{is} + K_{\beta} \beta_{ic} = H_{ic} - \quad (41)$$

and

$$- I_{\beta} \beta_{is} \Omega^2 + (- C_{\beta} i \Omega - L A_{is}) \beta_{ic} + K_{\beta} \beta_{is} = H_{is} - \quad (42)$$

From Equations 41 and 42, the blade flapping responses  $\beta_{ic}$  and  $\beta_{is}$  can be determined.

Notice that the dynamic blade flap response feeds back into Equation 6. This means that the blade flap response changes the aerodynamic forcing function which in turn means that the dynamic blade flap motion will change. Therefore, there is nonlinear coupling continually going on, and the analysis should be iterated until a solution is obtained. The analysis does indeed perform this iterative process during the trim routine.

The dynamic blade equations of motion in torsion have the same form as that for flap, and elastic blade pitch occurs, having harmonic terms  $\theta_{eic}$  and  $\theta_{eis}$ . These elastic blade pitch motions feed back into Equation 7 and also alter the aerodynamic forcing functions. In the case of torsion, it is more powerful than the flap contribution because it gets directly into the angle of attack, so that small torsional deflections can cause large changes in the blade aerodynamic forces.

#### 4. APPLICATION OF THE ANALYSIS FOR A UH-60A HELICOPTER WITH A DAMAGED MAIN ROTOR BLADE

For the purposes of this report, the harmonic forcing functions derived in Section 3 are applied at the hub of the main rotor of a UH-60A helicopter, and the helicopter's dynamic response is calculated. Vibrations are calculated in the cockpit, assuming the fuselage to be a rigid body with six degrees of freedom (i.e., longitudinal, lateral, vertical, roll, pitch, and yaw motions of the helicopter about its center of gravity).

The sources of the rotor blade and fuselage characteristics data come from a combination of special dynamics data requested from the manufacturer and from general technical manual data from

the manufacturer. There is no one place from which an analyst can obtain the data, but it is a matter of digging and searching into all kinds of documents or phoning the manufacturer for information.

The damage that is assumed is the removal of sections of the blade tip ranging from 0% to 48% of the blade. The loss of blade sections results in shortening, reduction of weight, reduction of the blade's first and second moments, and the increase of the blade's first flap frequency. Table 1 shows the changes of blade properties as a function of blade loss. The damage is applied to one of the four-bladed rotor sets with the helicopter flying at 160 knots.

The cockpit vibrations that result from the blade damage defined from Table 1 are shown in Figure 2. These vibrations are resultant vibrations, the vector sum of vertical, longitudinal, and lateral vibrations.

It is seen in Figure 2 that the cockpit vibrations vary up to 2 Gs at a frequency equal to the rotor RPM, called 1/REV, in the cockpit when 48% of one blade is lost. Reduced amounts of 2/REV, 3/REV, 4/REV, and 5/REV vibrations are present. 1/REV means a vibration equal to the rotor speed, 2/REV means a vibration at two times the rotor speed, and so on up the line to the higher /REV values.

## 5. DISCUSSION OF RESULTS

These results show the significance of an unbalanced rotor on vibrations in the cockpit of a helicopter. When a rotor is balanced, inherent vibrations are transmitted into the fixed system fuselage at frequencies that are integer multiples of the number of blades in the rotor. This means for the case of the Black Hawk helicopter with four blades, only 4/REV, 8/REV, and 12/REV vibrations can be transmitted into the fuselage for an undamaged rotor. Transmission of all other frequencies is zero.

For the Black Hawk, the 4/REV frequency with no damage is .072 Gs, which is insignificant on the scale of vibrations shown in Figure 2. The 8/REV frequency vibrations are even less.

For blade damage, however, loss of blade sections causes a steady unbalanced load in the rotating system, which transfers into the fixed fuselage system as a 1/REV. When 48% of the outboard section of one blade is removed, the vibrations are 2 Gs, a very severe condition for human tolerance. The

Table 1. Rotor Blade Physical Properties

| Tip Loss (%) | Blade Radius (ft) | Weight (lb) | 1st Moment (lb-sec <sup>2</sup> ) | 2nd Moment (lb-ft-sec <sup>2</sup> ) |
|--------------|-------------------|-------------|-----------------------------------|--------------------------------------|
| 0            | 26.83             | 211.00      | 87.90                             | 1,572                                |
| 2            | 26.29             | 206.78      | 84.42                             | 1,479                                |
| 4            | 25.75             | 202.56      | 81.01                             | 1,391                                |
| 6            | 25.22             | 198.34      | 77.67                             | 1,305                                |
| 8            | 24.68             | 194.12      | 74.40                             | 1,224                                |
| 10           | 24.14             | 189.90      | 71.20                             | 1,146                                |
| 12           | 23.61             | 185.68      | 68.07                             | 1,071                                |
| 14           | 23.07             | 181.46      | 65.01                             | 1,000                                |
| 16           | 22.53             | 177.24      | 62.02                             | 931                                  |
| 18           | 22.00             | 173.02      | 59.10                             | 866                                  |
| 20           | 21.46             | 168.80      | 56.25                             | 805                                  |
| 22           | 20.92             | 164.58      | 53.48                             | 746                                  |
| 24           | 20.39             | 160.36      | 50.77                             | 690                                  |
| 26           | 19.85             | 156.14      | 48.13                             | 637                                  |
| 28           | 19.31             | 151.92      | 45.57                             | 586                                  |
| 30           | 18.78             | 147.70      | 43.07                             | 539                                  |
| 32           | 18.24             | 143.48      | 40.64                             | 494                                  |
| 34           | 17.70             | 139.26      | 38.29                             | 452                                  |
| 36           | 17.17             | 135.04      | 36.00                             | 412                                  |
| 38           | 16.63             | 130.82      | 33.79                             | 374                                  |
| 40           | 16.09             | 126.60      | 31.64                             | 339                                  |
| 42           | 15.56             | 122.38      | 29.57                             | 306                                  |
| 44           | 15.02             | 118.16      | 27.56                             | 276                                  |
| 46           | 14.48             | 113.94      | 25.63                             | 247                                  |
| 48           | 13.95             | 109.72      | 23.76                             | 221                                  |

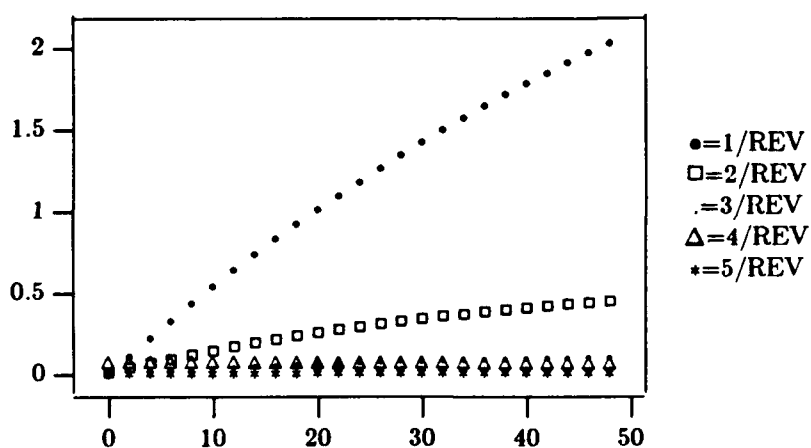


Figure 2. Cockpit vibrations (Gs) vs. percentage of blade lost.

higher harmonics are less severe in magnitude but may be just as effective in debilitating a human if human organs are resonant to these higher frequencies (Linder 1959).

With blade damage, the 1/REV, 2/REV, and 3/REV vibrations are all higher than the ambient 4/REV vibrations (no damage).

The computer program is an ongoing in-house development program conducted by the author. Several papers have been presented by the author wherein results were generated using this computer program (Fries 1990a, 1990b, 1990c).

The computer program runs by establishing a steady-state trim condition and the damage is imposed on the main rotor. The vibrations that arise are instantaneous in nature, of just one rotor revolution duration after the damage occurs. The program does not analyze the helicopter vibrations after the damage for long periods of time. To do this, we know that the damage will cause the helicopter to go out of trim and the pilot would have to compensate with the controls in some manner.

The program is of first order accuracy and is considered to be a first order approximation to the vibrations that exist for a helicopter with a damaged rotor. The aerodynamics are of first order, the second order terms are neglected, and the airframe is a rigid body not considering flexibility or natural modes. The program is the first attempt at understanding this vibration problem and ongoing work is needed to enhance the program. The advantage of the program is that it is relatively small and easy to use to get quick results as opposed to very large analyses existing at universities and helicopter manufacturers that require years of experience and many months to exercise a specific helicopter model.

The program has approximately 700 lines of code in BASIC language that runs on the author's Northgate Elegance - 425i personal computer. It is available from the author to anyone who wishes to use it.

## 6. CONCLUSIONS

A workable vibration analysis that can quantify aircraft vibrations for a variety of main rotor damages is available. For this report, only one type of damage was considered, that of the loss of various amounts of the outboard sections of one blade.

This analysis brings up the question of human tolerance to vibrations and the effect on a pilot's ability to fly the aircraft from a physiological point of view.

Another question is the control authority remaining to the pilot with blade loss, and then there is the further question of the aircraft's stability characteristics after damage.

This computer program is a useful tool for vulnerability analysis and may be used to investigate the many factors that could lead to helicopter attrition caused by combat damage.

## 7. REFERENCES

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**APPENDIX:**

**DESCRIPTION OF THE COMPUTER PROGRAM TO PREDICT HELICOPTER  
VIBRATIONS FOR UNDAMAGED AND DAMAGED MAIN ROTOR BLADES**

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## A-1. DESCRIPTION OF THE RTVIB20 COMPUTER PROGRAM

The computer code first performs a trim analysis for forward speed level flight by computing the aerodynamic forces in the manner described in Section 3. From these forces, a balance is struck in the vertical and horizontal directions so that level flight is maintained at a constant forward speed. This balance is maintained by adjusting the blade collective pitch and the aircraft pitch attitude by iteration until a balanced condition is obtained.

The physical properties input into the program are listed in Section A-2 of this appendix, along with the computer code line numbers. The numerical values listed are for the UH-60A helicopter and come from such a wide variety of sources that they are not listed. They come by word of mouth through telephone conversations with the manufacturer, or by special correspondence, or by digging out data from the manufacturer's technical manuals. Once a trimmed condition is achieved, the program goes on to calculate fixed system vibrations at a position on the fuselage as specified by the user. In this report, the cockpit location was chosen.

Two iterative trim loops start at lines 521 and 530 of the code and finish at lines 1320 and 2350. Within the inner loop, the blade collective pitch is varied, and in the outer loop the aircraft's pitching attitude is changed until a vertical and horizontal force balance is achieved.

When the helicopter is flying straight and level, the program will perform either an undamaged or damaged main rotor vibration analysis. Whichever path is taken depends on the physical input data on lines 3060 to 3090. On these input lines there are inputs R, IB, SB, W, A, CD, EB, WN, ITH, CM, A1, and Q variables on N separate lines to represent individual blades. That is, on line 3060 we have this data for blade 1, line 3060 for blade 2, and so on for a four-bladed rotor. The last line is line 3090. For an undamaged blade, the input values on these lines will be identical to the corresponding values input to lines 50, 300, 360, 440, or 500, as described in Section A-2.

For an undamaged blade, all the input parameters for one variable on each of these lines (3060-3090 for a four-bladed rotor) are identical. When one of the blades is damaged in some way, the physical properties will change and this will be reflected in a change in one or more of the input parameters on one of these particular lines. For instance, a hole through one blade will require IB,

SB, W, A, CD, WN, ITH, CM, A1, and Q on one particular line for the blade to be changed to account for damage (see input variable definitions).

The fixed system cockpit vibrations produced by blade shears and moments for undamaged and damaged blades are calculated using the computer code and are reported in Section A-4 for the example of the UH-60A helicopter.

## A-2. INPUT VARIABLES OF THE COMPUTER CODES

| <u>Variable<br/>Inputs</u> | <u>Variable<br/>Input Name</u>                    | <u>Unit of<br/>Measurement</u> | <u>Line<br/>No.</u> | <u>Numerical<br/>Input</u> |
|----------------------------|---------------------------------------------------|--------------------------------|---------------------|----------------------------|
| AT                         | Pressure altitude                                 | ft                             | 50                  | 4,000                      |
| T                          | Outside temperature                               | deg F                          | 50                  | 95                         |
| CH                         | Blade chord = C                                   | inches                         | 300                 | 20.88                      |
| R                          | Blade tip radius                                  | ft                             | 300                 | 26.83                      |
| TT                         | Blade geometric twist                             | deg/ft                         | 300                 | .671                       |
| OM                         | Rotor speed                                       | rpm                            | 300                 | 258.                       |
| V                          | Aircraft forward speed                            | knots                          | 300                 | 160.                       |
| VC                         | Vertical climb rate                               | knots                          | 300                 | 0.                         |
| GA                         | Rotor shaft tilt = G,<br>+ into wind              | deg                            | 300                 | 3                          |
| PI                         | Aircraft pitch,<br>+ into wind. Note:<br>not 3.14 | deg                            | 300                 | 10.49                      |
| CD                         | Blade drag coefficient,<br>steady                 | nd                             | 300                 | .04                        |
| GW                         | Aircraft gross weight                             | lb                             | 360                 | 16,260.                    |
| TO                         | Blade collective pitch                            | deg                            | 360                 | 37.29                      |
| T1                         | Blade lateral cyclic pitch                        | deg                            | 360                 | .88                        |
| T2                         | Blade longitudinal cyclic<br>pitch                | deg                            | 360                 | -10.79                     |
| N                          | Number of blades<br>per rotor                     | nd                             | 360                 | 4                          |
| FE                         | Aircraft profile flat<br>plate area               | ft <sup>2</sup>                | 360                 | 74.55                      |
| IB                         | Blade 2nd flap moment                             | lb-sec <sup>2</sup> -ft        | 440                 | 1,570                      |
| SB                         | Blade 1st flap moment                             | lb-sec <sup>2</sup>            | 440                 | 87.90                      |
| W                          | Blade weight                                      | lb                             | 440                 | 211                        |
| EB                         | Blade flap offset from<br>center of rotation      | in                             | 440                 | 21.98                      |
| PA                         | Blade pitch axis from<br>leading edge             | in                             | 500                 | 5.22                       |

| <u>Variable<br/>Inputs</u> | <u>Variable<br/>Input Name</u>                                                              | <u>Unit of<br/>Measurement</u> | <u>Line<br/>No.</u> | <u>Numerical<br/>Input</u> |
|----------------------------|---------------------------------------------------------------------------------------------|--------------------------------|---------------------|----------------------------|
| WN                         | Blade 1st torsion natural<br>frequency normalized<br>by rotor speed                         | nd                             | 500                 | 3.728                      |
| ITH                        | Blade pitch inertia                                                                         | lb-sec <sup>2</sup> -ft        | 500                 | .865                       |
| CM                         | Blade aerodynamic pitch<br>coefficient steady                                               | nd                             | 500                 | -.04                       |
| MC                         | Blade chordwise<br>mass center from l.e.                                                    | in                             | 500                 | 0                          |
| A1                         | Blade linear aerodynamic<br>pitch coefficient, multiplies<br>angle of attack                | nd                             | 500                 | -.08                       |
| EA                         | Blade elastic axis<br>from l.e.                                                             | in                             | 500                 | 5.22                       |
| Q                          | Blade non-linear<br>aerodynamic pitch<br>coefficient, multiplies<br>angle of attack squared | nd                             | 500                 | 0                          |
| IY                         | Aircraft pitch inertia                                                                      | lb-sec <sup>2</sup> -ft        | 5680                | 60,731                     |
| IX                         | Aircraft roll inertia                                                                       | lb-sec <sup>2</sup> -ft        | 5680                | 4,118                      |
| XX                         | Longitudinal distance from<br>rotor hub to the place<br>where vibrations<br>are calculated  | in                             | 5680                | 134                        |
| YY                         | Lateral distance to<br>vibration location                                                   | in                             | 5680                | 20                         |
| ZZ                         | Vertical distance to<br>vibration location<br>as above                                      | in                             | 5680                | -100                       |
| EX                         | Longitudinal distance<br>from the rotor hub to<br>the aircraft c.g.                         | in                             | 5680                | 11                         |
| EZ                         | Vertical distance from<br>rotor hub to the aircraft<br>c.g.                                 | in                             | 5680                | 63                         |

### A-3. DESCRIPTION OF COMPUTER PROGRAM OUTPUT

Only pertinent outputs will be given. There is much output which is intermediate in nature so excerpts will be presented.

After the program iterates on a trim condition, a final set of values of blade collective pitch, aircraft pitch attitude, vertical force, and downwash velocity are produced. The results are given in

Section A-4 for the undamage case for the UH-60A flying at 160 knots. Also presented in Section A-4 are the results of calculations of the vertical fixed system hub shears and the in-plane hub shears. The example case is for an undamaged rotor condition, in which only the four/rev shears are significant for a four-bladed rotor. For a damaged case rotor blade, all the harmonics may be significant. The 1/REV in-plane shears only are given, and we see that for the undamaged case these shears are small; however, for a damaged case, the steady damage in the rotating system translates into 1/REV fixed system shears that are significant.

Fixed system 4/REV hub moments are given in Section A-4. Again, only the 4/REV values are listed because only these are transmitted into the fixed system for an undamaged rotor. Also, in Section A-4 are listed the quasi-theoretical moments based upon the expected output at specific azimuth positions. These are used as a check against the values that are calculated in the stepwise incrementing around the azimuth. These theoretical moments come from the harmonic closed form hand calculations from which the magnitudes at specific azimuth positions are known. For instance, at 1/REV, we know the cosine amplitude at zero azimuth is either maximum or minimum and the sine amplitudes are either maximum or minimum at the azimuth position of 90 degrees.

The final output consists of cockpit vibrations which are also listed in Section A-4. These are the base case vibrations for an undamaged rotor against which damage cases can be compared. They are broken down into cosine and sine components of vibration in the longitudinal, lateral, and vertical directions. Again, here only the 4/REV vibrations are transmitted from the blade rotating system into the fuselage fixed system because of a balanced rotor when no damage is present.

Note that some of the variable output names in Section A-4 are identical in physical meaning, but only in a programming sense. That is, they are variables in the code that are printed out sequentially in time and are reused at a later point in the execution of the program. The line number on which the variable rests determines the physical quantity that is printed.

#### A-4. OUTPUT VARIABLES OF THE COMPUTER CODE

| <u>Variable</u><br><u>Output</u> | <u>Variable</u><br><u>Output Name</u> | <u>Unit of</u><br><u>Measurement</u> | <u>Line</u><br><u>No.</u> | <u>Numerical</u><br><u>Outputs</u> |
|----------------------------------|---------------------------------------|--------------------------------------|---------------------------|------------------------------------|
| TO                               | Blade collective<br>pitch             | deg                                  | 1171<br>1241              | 38.57<br>38.57                     |
| PI                               | Aircraft pitch<br>into wind           | deg                                  | 1171<br>1241              | 8.32<br>8.32                       |
| FV*                              | Vertical force                        | lb                                   | 1271                      | 15,948.81                          |
| GW                               | Gross weight                          | lb                                   | 1271                      | 16,260.                            |
| U                                | Downwash velocity                     | ft/sec                               | 1271                      | 49.71                              |

#### Fixed System Vertical Forces

|              |                              |     |      |         |
|--------------|------------------------------|-----|------|---------|
| I<br>(4/REV) | Harmonic number              | nd  | 4680 | 1 to 6  |
| ACX          | Max. cosine amplitude        | lb  | 4680 | 212.70  |
| ZCX          | Azimuthal position<br>of ACX | deg | 4680 | 45      |
| ACN          | Min. cosine amplitude        | lb  | 4680 | -212.70 |
| ZCN          | Azimuthal position<br>of ACN | deg | 4680 | 0       |
| ASX          | Max. sine amplitude          | lb  | 4690 | 5.87    |
| ZSX          | Azimuthal position<br>of ASX | deg | 4690 | 22.5    |
| ASN          | Min. sine amplitude          | lb  | 4690 | -5.87   |
| ZSN          | Azimuthal position<br>of ASN | deg | 4690 | 67.5    |

#### In-Plane Fixed System Forces, 1/REV

|     |                                                                              |     |      |       |
|-----|------------------------------------------------------------------------------|-----|------|-------|
| FXC | Last longitudinal<br>cosine force calculated<br>about the azimuthal<br>steps | lb  | 4990 | .5019 |
| ACX | Max. longitudinal<br>cosine force                                            | lb  | 4990 | .753  |
| XCX | Azimuthal position of<br>deg ACX                                             | deg | 4990 | 49.09 |
| ACN | Min. longitudinal<br>cosine force                                            | lb  | 4990 | -.753 |
| XCN | Azimuthal position<br>of ACN                                                 | deg | 4990 | 212.7 |

$$*FV = N*TH*\cos(GA+PI)$$

| <u>Variable<br/>Output</u> | <u>Variable<br/>Output Name</u>                                       | <u>Unit of<br/>Measurement</u> | <u>Line<br/>No.</u> | <u>Numerical<br/>Outputs</u> |
|----------------------------|-----------------------------------------------------------------------|--------------------------------|---------------------|------------------------------|
| FYS                        | Last lateral sine<br>force calculated<br>about the azimuthal<br>steps | lb                             | 5000                | -.585                        |
| ASX                        | Max. lateral sine<br>force                                            | lb                             | 5000                | .753                         |
| XSX                        | Azimuthal position<br>of ASX                                          | deg                            | 5000                | 130.9                        |
| ASN                        | Min. lateral sine<br>force                                            | lb                             | 5000                | -.765                        |
| XSN                        | Azimuthal position<br>of ASN                                          | deg                            | 5000                | 310.9                        |

Fixed System Moments

|              |                                                       |       |      |           |
|--------------|-------------------------------------------------------|-------|------|-----------|
| I<br>(4/REV) | Harmonic number                                       | nd    | 5520 | 1 to 6    |
| XSX          | Max. sine<br>rolling moment                           | ft-lb | 5520 | 1,014.56  |
| PSX          | Azimuth position<br>of XSX                            | deg   | 5520 | 67.5      |
| XSN          | Min. sine<br>rolling moment                           | ft-lb | 5520 | -1,014.56 |
| PSN          | Azimuth position<br>of XSN                            | deg   | 5520 | 22.5      |
| MXS          | Last calculated<br>sine moment about<br>the azimuth   | ft-lb | 5520 | -.0081    |
| XCX          | Max. cosine<br>rolling moment                         | ft-lb | 5530 | 2,485.99  |
| PCX          | Azimuth position<br>of XCX                            | deg   | 5530 | 0         |
| XCN          | Min. cosine<br>rolling moment                         | ft-lb | 5530 | -2,485.99 |
| PCN          | Azimuth position<br>of XCN                            | deg   | 5530 | 45        |
| MXC          | Last calculated<br>cosine moment about<br>the azimuth | ft-lb | 5530 | 2,485.99  |
| YSX          | Max. sine<br>pitching moment                          | ft-lb | 5540 | 2,611.93  |
| QSX          | Azimuth position<br>of YSX                            | deg   | 5540 | 22.5      |
| YSN          | Min. sine<br>pitching moment                          | ft-lb | 5540 | -2,611.93 |

| <u>Variable<br/>Output</u> | <u>Variable<br/>Output Name</u>                                 | <u>Unit of<br/>Measurement</u> | <u>Line<br/>No.</u> | <u>Numerical<br/>Outputs</u> |
|----------------------------|-----------------------------------------------------------------|--------------------------------|---------------------|------------------------------|
| QSN                        | Azimuth position<br>of YSN                                      | deg                            | 5540                | 67.5                         |
| MYS                        | Last calculated<br>sine pitching<br>moment about the<br>azimuth | ft-lb                          | 5540                | .208                         |
| YCX                        | Max. cosine<br>pitching moment                                  | ft-lb                          | 5550                | 1,083.36                     |
| QCX                        | Azimuth position<br>of YCX                                      | deg                            | 5550                | 0                            |
| YCN                        | Min. cosine<br>pitching moment                                  | ft-lb                          | 5550                | -1,083.36                    |
| QCN                        | Azimuth position<br>of YCN                                      | deg                            | 5550                | 45                           |
| MYC                        | Last calculated<br>cosine pitching<br>moment about<br>azimuth   | ft-lb                          | 5550                | 1,083.36                     |

#### Theoretical Fixed System Moment Values

|              |                           |       |      |           |
|--------------|---------------------------|-------|------|-----------|
| I<br>(4/REV) | Harmonic number           | nd    | 5630 | 1 to 9    |
| XMS          | Sine rolling<br>moment    | ft-lb | 5630 | -1,014.56 |
| YMS          | Sine pitching<br>moment   | ft-lb | 5630 | 2,611.93  |
| XMC          | Cosine rolling<br>moment  | ft-lb | 5630 | 2,485.99  |
| YMC          | Cosine pitching<br>moment | ft-lb | 5630 | 1,083.36  |

#### Cockpit Vibrations

|              |                     |    |      |         |
|--------------|---------------------|----|------|---------|
| I<br>(4/REV) | Harmonic number     | nd | 5800 | 1 to 10 |
| X2C          | Longitudinal cosine | Gs | 5800 | .00140  |
| X2S          | Longitudinal sine   | Gs | 5800 | .00412  |
| Y2C          | Lateral cosine      | Gs | 5800 | .0578   |
| Y2S          | Lateral sine        | Gs | 5800 | -.0235  |
| Z2C          | Vertical cosine     | Gs | 5800 | .0367   |
| Z2S          | Vertical sine       | Gs | 5800 | .00342  |

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