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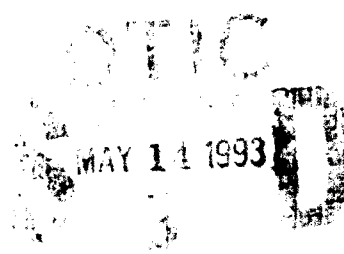
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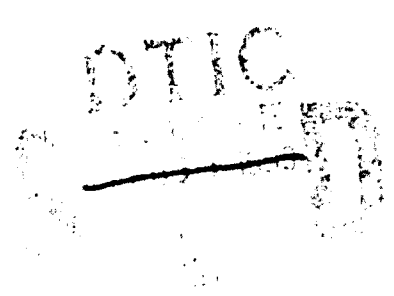
W.J. SCHAFFER ASSOCIATES, INC.

# SBS Modeling — Status and Assessment

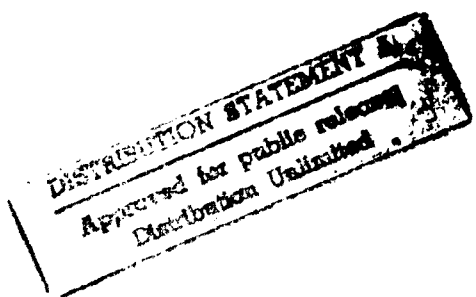


Eugene J. Sigal

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W.J. Schafer Associates, Inc.  
1901 N. Ft. Myer Dr.  
Arlington, VA 22209



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## 1 Introduction

This report describes the status of the Detailed SBS Model<sup>1</sup> being developed by WJSA in the Arlington, Va. office. Once completed, the WJSA Detailed SBS Model will be used for predicting the performance of SBS cell designs being considered for SBL applications.

The goal of the effort has been to develop a full three-dimensional (two transverse and one longitudinal) wave optics SBS model with diffraction capable of convergence to a solution using reasonable computer resources. Previous efforts have not been entirely successful in the regime of interest (i.e. high reflectivity) due to convergence instability and due to excessive computer costs in memory and time. Although the Rocketdyne GLAD SBS model (see Ref. [1]) is a full three-dimensional treatment with physical optics, it uses excessive computer time in the few cases for which it does converge. Until recently, the most successful model was the Lehmborg BOUNCE code, from which the WJSA/Albuquerque single transverse dimension model and the TRW BRIWON model were derived. Although BOUNCE does not require excessive computer resources (primarily because of the single transverse dimension), it does have convergence problems for cases with reflectivities higher than about 15%, and there is some skepticism regarding the validity of the results since the code is limited to a single transverse dimension. BOUNCE is, however, the current industry standard.

The most successful model to date has been developed at Hughes Research Laboratories (see Ref. [2]) who abandoned their efforts of trying to solve a set of steady state SBS equations and turned to a time-dependent problem. They, just like other developers, including WJSA, encountered serious convergence difficulties with the many numerical methods devised for the steady state SBS equations. Brown and Wandzura developed a four-dimensional SBS code which they successfully tested on a number of SBS problems. Unfortunately, in its current form, the Hughes code can not treat the cases of interest for SBL systems. The code does not possess the capability to model focused SBS geometries. The numerical methods experience instabilities when four-dimensional SBS with high (> 80%) reflectivities is modeled, and it requires excessive amounts of computing resources even if run on a CRAY computer. However, the Hughes approach shows a lot of promise because it is based on the time-dependent theory of SBS.

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<sup>1</sup>The model is being implemented with WJSA's  $\Omega$  4-D diffractive optics code.



these common-path phase aberrations.

The SBL configurations of interest may include multiline and multi-amplifier chemical laser systems with multiple SBS cells for conjugation. Some of the potential configurations are shown on Fig. 2. These systems are described in Ref. [5].

## 2 Numerical Methods for Steady State SBS

In the initial stages of the model development WJSA performed a comprehensive overview of documented numerical methods available for modeling SBS interactions. Descriptions of the only two documented SBS modeling approaches (Lehmberg's iteration procedure and Rocketdyne's shooting method) along with a variety of other pertinent numerical techniques can be found in Ref. [6].<sup>2</sup> These existing SBS models were fully investigated in order to take advantage of their strengths and to avoid their weaknesses. WJSA's Douglas Crawford and Eugene Sigal met with Robert Lehmberg (see Ref. [7]) and with the members of the Rocketdyne team to facilitate the evaluation process as well as to obtain additional insight to the very complex problem of SBS modeling. After the evaluation it was concluded that the approach taken by R. H. Lehmberg was the most successful, and it was decided that an extension of this approach could yield the best results.

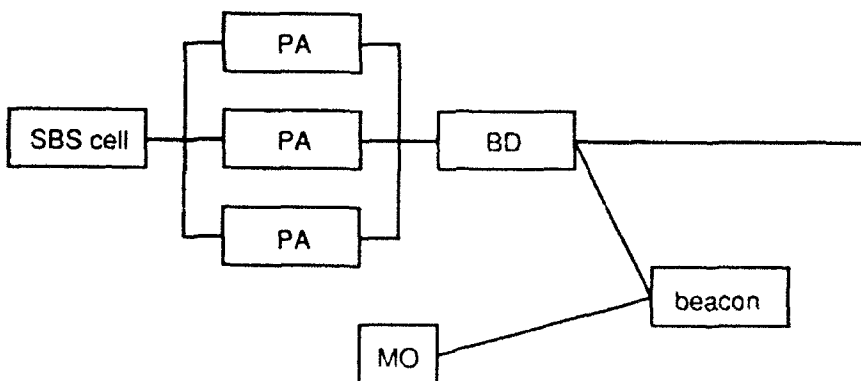
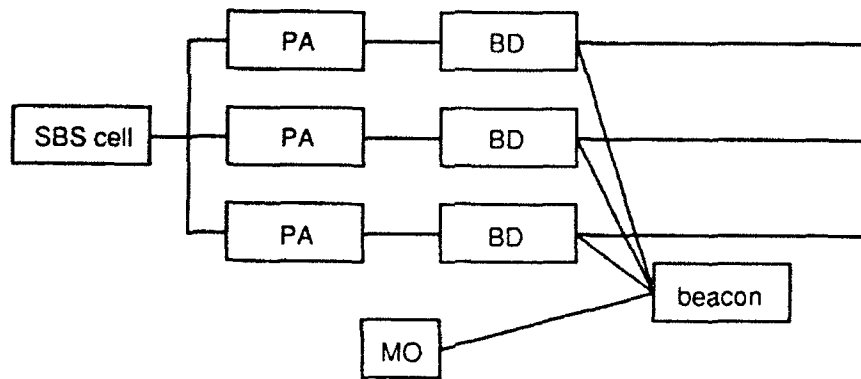
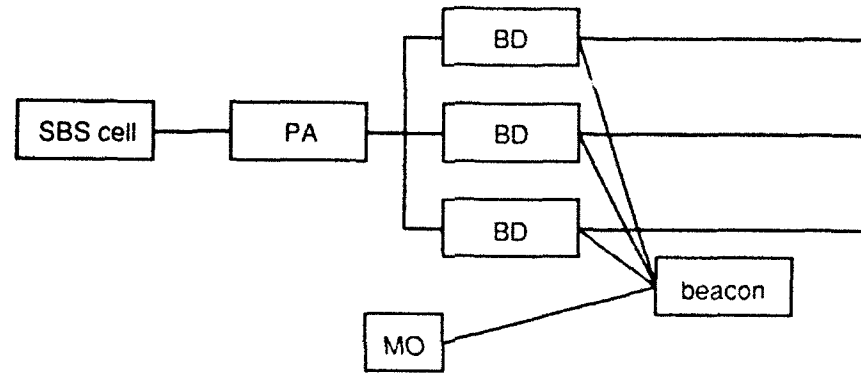
Recognizing some of the shortcomings of Lehmberg's method and the instabilities that Lehmberg encountered in SBS modeling WJSA developed a numerical procedure called PPIM (Propagating Pulse Iteration Method) (see Ref. [6]) which was designed to reach stable solutions based on the physical situation of an incident pump beam propagating through the interaction region. A number of other improvements to Lehmberg's method were also made in the implementation of PPIM.

The stability issue was addressed by WJSA through a thorough investigation into the stability properties of the SBS equations, and very important discoveries were made (See Ref. [7]). First it was learned that for moderate reflectivity cases ( $> 10\%$ ) the solutions were very sensitive to the boundary conditions and to the initial conditions when one-way propagations were carried out. Second, for the simplified set of equations (without diffractive

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<sup>2</sup>Since that time Hughes Research Laboratories published the results of their work including the description of their methods.

Figure 2: Proposed SBL system configurations



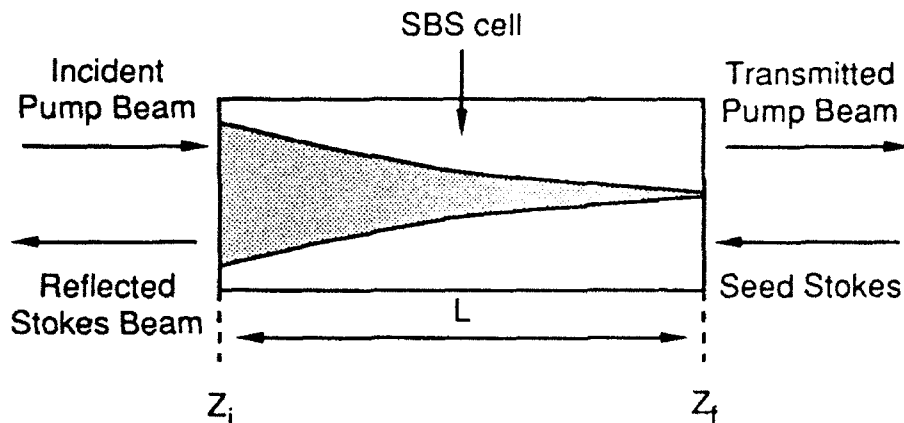
terms) it was shown that the range of the solutions was limited even if the boundary conditions, gain, and interaction length were allowed to span their (mathematically) allowable ranges from zero to infinity. Consequently, in either Lehmberg's or Rocketdyne's approach, some care had to be taken in selecting the guesses for the fields both at the start and during the iteration process. As mentioned earlier, neither Lehmberg nor the Rocketdyne team were aware of the sensitivity or the stability properties of the SBS equations, or at least they did not believe that these conditions could pose serious difficulties .

Consequently, during the iteration procedure (either Lehmberg's or Rocketdyne's) if the estimate of the pump beam at the end of the cell was taken to be too large then the Stokes beam would be amplified beyond its correct value. This error poses no difficulty if the consequent iterations force the value of the Stokes beam to the correct value. However, if the solution to the initial value problem is very sensitive to changes in the pump intensity at the end of the cell (and they are *very sensitive* for high reflectivities) then the exponentially amplified Stokes beam is able not only to exceed its correct value by orders of magnitude, but it can grow beyond the floating point bounds of the computer.

Recognizing this potential instability WJSA took great care in controlling the magnitude of the pump beam at the end of the cell during the iteration procedure. Furthermore, in the process of implementing the needed improvements to the code, WJSA developed a much more efficient and accurate three-dimensional SBS model. This process was long and difficult because it involved constant revisions and improvements of the latest techniques to combat stability, convergence, and accuracy. WJSA went through seven different variations of the iteration procedure in the effort to devise the most reliable, efficient, and accurate one. The latest model was no longer a mere improved extension of Lehmberg's iteration procedure; it was a genuinely original method that incorporates elements of shooting methods and Lehmberg's iteration method. It will be referenced here as a Direction-Alternating Shooting Iteration (DASI) method.

In order to provide a reference to existing methods for the DASI method it is appropriate to briefly summarize the essential elements of Lehmberg's iteration procedure and the Rocketdyne shooting method. The geometry of the problem is shown on Fig. 3. Both methods are suitable for solving the governing SBS equations that represent a longitudinal boundary value problem for a system of two complex nonlinear partial differential equa-

Figure 3: Basic SBS cell geometry



tions,

$$\frac{\partial E_1}{\partial z} = -\frac{i}{2k} \nabla_{\perp}^2 E_1 - \frac{1}{2} g E_1 E_2 E_2^* \quad (1a)$$

$$\frac{\partial E_2}{\partial z} = \frac{i}{2k} \nabla_{\perp}^2 E_2 - \frac{1}{2} g E_2 E_1 E_1^* \quad (1b)$$

where  $E_1$  and  $E_2$  are the complex electric fields<sup>3</sup> of the counterpropagating pump and Stokes beams,  $g$  is the SBS gain, and  $k$  is the wave number of the beams (a very small frequency shift between the two beams is neglected). The boundary conditions for these equations are

$$E_1 \Big|_{z=z_i} = E_{1i} \quad (2a)$$

$$E_2 \Big|_{z=z_f} = E_{2f} \quad (2b)$$

This system of equations describes the basic steady state SBS process including diffraction effects. For the derivation of these equations and for a discussion of the SBS theory see, Refs. [8, 9, 10].

Taken individually, Eqs. (1a) and (1b) can be solved using FFT methods in conjunction with the Split-Operator technique, or using finite difference methods which are well suited for these initial value problems (see Ref. [6] for a description of these and other methods, and further references). The complexity in solving the above system of equations lies in

<sup>3</sup>The electric fields are normalized such that  $E^2 = I$ , the intensity.

the fact that they are coupled, and that the beams are *counterpropagating*, thus representing a boundary value problem. A standard technique for solving boundary value problems is the shooting method. When using the shooting method the equations are transformed to an initial value problem by assuming some arbitrary values for unknown quantities at either boundary, i.e.  $E_2|_{z=z_i} = E_{2i}$  or  $E_1|_{z=z_f} = E_{1f}$ . Then the initial value problem can be readily solved, and the errors between the resulting electric fields at the second boundary and the desired boundary conditions can be calculated. The error information is used to update the initial guesses for the values of the fields, and the procedure is repeated until convergence is reached.

The technique employed by Lehmborg was drastically different. In Lehmborg's iteration procedure one of the fields is assumed to be known (i.e. stored as a two-dimensional array) throughout the interaction region at a prescribed number of longitudinal locations  $z_n$ . If  $E_1$  is assumed known and fixed then Eqs. (1b) and (2b) can be solved, and the values of  $E_2$  are stored at the  $z_n$ 's. With  $E_2$  now known and assumed fixed Eqs. (1b) and (2b) can be solved, and the values of  $E_1$  at the  $z_n$ 's updated. The procedure is then repeated until convergence is reached.

The advantages and difficulties associated with both of these methods are discussed in detail in Ref. [6], so it would be redundant to fully explain them here. But in short, the shooting method poses a problem with updating the successive approximations of the initial conditions for even moderate resolution transverse field arrays, and Lehmborg's iteration procedure requires the storage of many transverse field arrays. The DASI method attempts to capitalize on the advantages of each method while circumventing the difficulties.

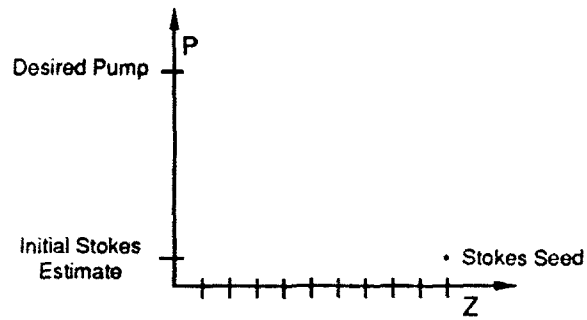
DASI is an iteration procedure that attempts to use pump loading to control the instabilities as illustrated on Fig. 4. The process begins by defining the initial conditions for the pump and Stokes beams at  $z_i$  and  $z_f$  respectively and the guesses for the reflected Stokes beam and the pump beam at  $z_i$ . The guesses for the pump and Stokes beams are copropagated in the forward direction to  $z_f$  with the pump intensities being stored at predetermined longitudinal locations. At  $z_f$  the Stokes field is replaced with the correct value (i.e. the initial condition), and the Stokes beam is propagated in the reverse direction to  $z_i$  with the pump beam remaining fixed. At  $z_i$  a point by point comparison between the initial guess for the Stokes beam and the updated Stokes beam is made. If the intensity at any point of the updated array exceeds the corresponding intensity in the guessed array by

a predetermined amount (say 1%) then the guess for the pump is too high. Otherwise the pump guess is too low. This procedure of adjusting the pump level is referred to as pump loading. The proper level of the pump is achieved by using the bisection method which is illustrated on Fig. 5. Once the proper level of the pump is achieved, the Stokes value at  $z_i$  is updated, and the entire process is repeated until the pump level reaches the desired value (i.e. initial condition at  $z_i$ ).

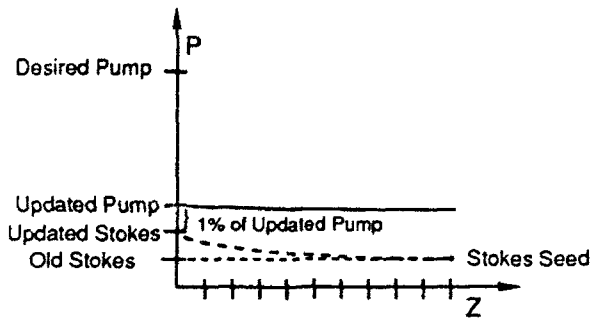
At the time of this report, DASI had been fully implemented. It was tested for cases where the diffractive effects were absent, and the exact solutions were known. The method performed as expected, and the correct field solutions were obtained. However, when the diffraction was included, the results turned discouraging. The iteration procedure converged to reasonable results for only one case of a supergaussian pump beam with a flat phase. For this case the reflectivity at the center of the beam was about 15%. The results of this simulation are presented in Appendix A. When higher resolutions, higher reflectivities, or aberrated beams were tried the solutions converged, but exhibited large numerical round-off errors. Such results were unexpected and had not been observed in earlier WJSA methods.

Further investigation in to the problem led WJSA to the conclusion that such large round-off error could be contributed to the nature of the numerical method and the precision of a computer. Initially, the round-off errors were being introduced into the calculations in the 4th or 5th significant digit by the numerical algorithms of FFT's. Such errors are inevitable and can not be eliminated. Once introduced, these error were exponentially amplified on each successive iteration. The number of iteration was large enough to allow these errors to grow to significant levels and to corrupt the final results. WJSA identified actions that could be taken to minimize the growth of numerical error such as analyzing FFT routines in order to minimize the errors there, or transporting the code to a CRAY where the calculations could be carried with 64 bits of precision versus the current 32 bits of WJSA's MicroVAX. But before these alternatives were analyzed further, WJSA became aware of the results reported by Hughes Research Laboratories which offered perhaps a better alternative.

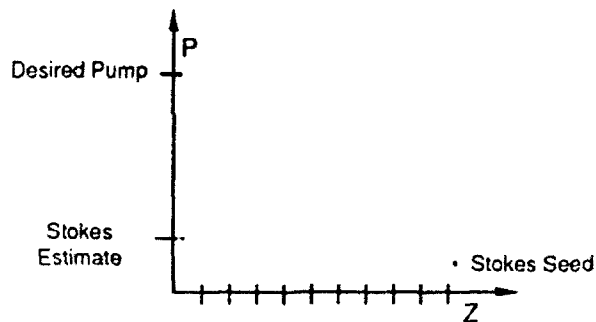
Figure 4: Pump loading technique for the DASI method



1. Define initial Stokes.



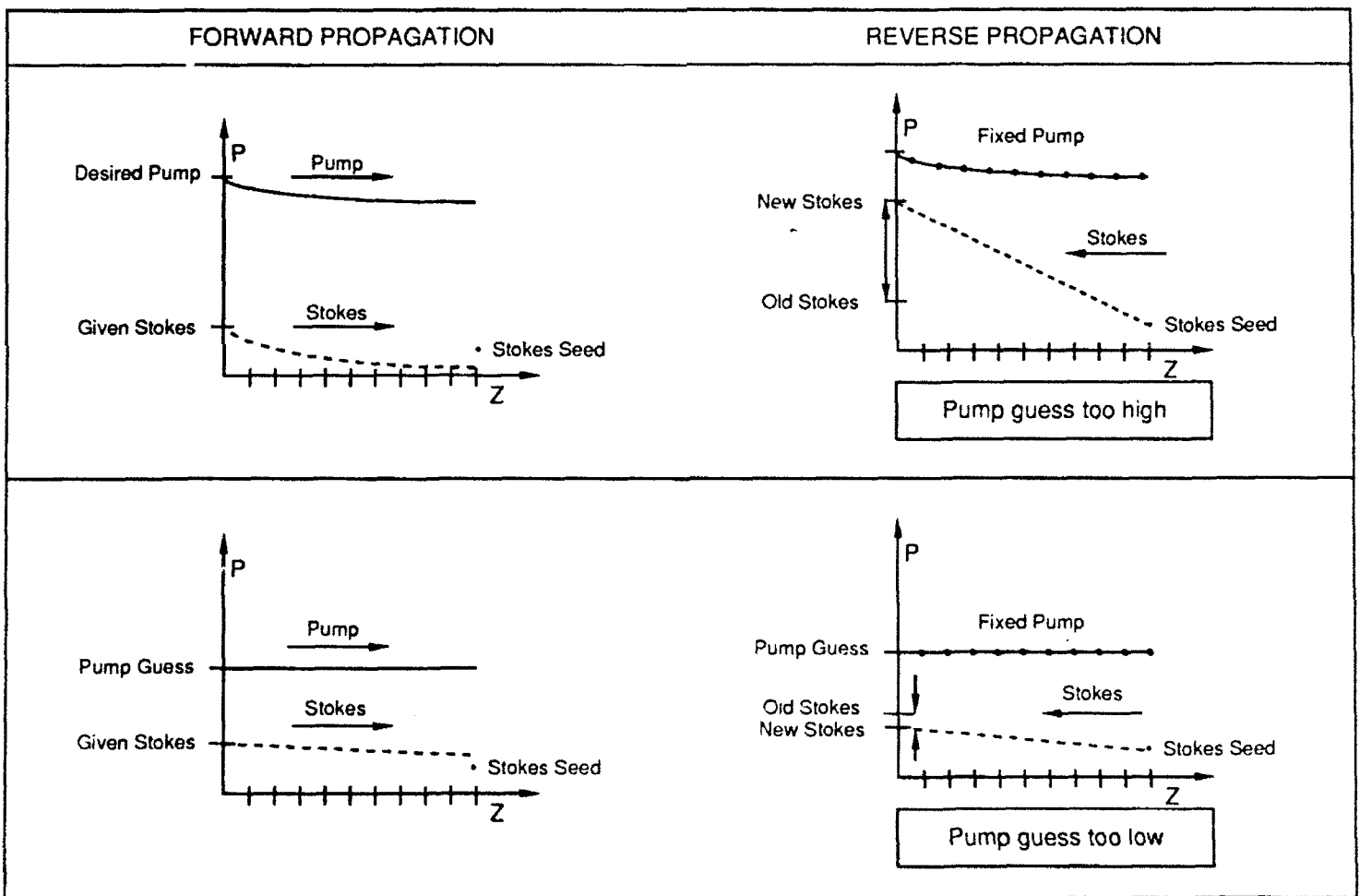
2. Using Bisection Method load pump power until the intensity of at least one point in the Stokes array reaches 1% of the corresponding pump intensity.



3. Save new value of the Stokes field.

4. Repeat steps 2 and 3.

Figure 5: Bisection method for determining pump load level



### 3 Time-Dependent Approach to SBS Problem

As mentioned in the introduction, recently Brown, Wandzura, and others at Hughes Research Laboratory reported on their work in modeling time-dependent SBS equations (see Ref. [2]). Having learned of Hughes' work Sigal of WJSA arranged the meeting with Brown and Wandzura to discuss their findings and conclusions. Presented below is the synopsis of their conclusions.

1. Conventional Stimulated Brillouin scattering does not attain steady state because the process is driven by time-dependent phonon noise.
2. In bulk media (the case of interest for SBL's) steady state can be attained if a very small portion ( $1 \times 10^{-6}$ ) of the reflected Stokes is fed back as Stokes seed. They think this situation occurs in TRW's experiments. Feedback does not produce steady state in a fiber or long lightguide.
3. Amplitude, phase, and fidelity fluctuations of the Stokes wave are correlated [with each other]. (No mention is made of possible correlation with the noise source.)
4. Time-averaged results agree very well with the steady state theory.

The first conclusion is drastically different from the accepted belief in the community because the steady state SBS has been demonstrated experimentally. However, the second conclusion resolves this issue. The correlation between amplitude, phase, and fidelity fluctuations is not surprising, but no mention is made of possible correlation to the noise source which may be extremely important. Finally, time averaged results agree very well with the steady state theory which is gratifying, but it does not minimize the importance of time-dependent solutions. Even if on the average the solutions behave in the steady state manner, large temporal fluctuations may still exist and may adversely impact the SBL system.

Hughes' results are also important from the numerical standpoint. In the first part of their contract Brown unsuccessfully tried to model steady-state SBS for one and a half years. He tried iteration schemes a la Lehmberg, but sufficiently different from Lehmberg's work. He observed the kinds of instabilities the author has been observing in WJSA codes, which also treated the steady state equations. In the second phase of their contract the Hughes

team changed the direction of its effort and tried modeling a complete set of time-dependent SBS equations including random phonon source. This approach worked much better as they were able to converge to solutions with reflectivities as high as 80% with diffraction and 99% without diffraction. In addition, from the computational standpoint, stepping in time may require no more resources than an iteration procedure for steady state equations, and time stepping makes more sense physically.

However, to be totally objective the following observations must be made. Hughes' code requires enormous amounts of computer memory to model four-dimensional SBS (up to 100 million words on a CRAY to run a 128 by 128 simulation) as they store beam arrays at up to 1600 longitudinal locations. All the results they presented in their final report are based on simulations with no more than only 1 transverse dimension. In addition, they are still encountering instabilities in the solutions as Stokes reflectivities exceed 80%. Moreover, they have not implemented focused geometry; therefore, it is not clear how focusing will effect the stability.

After evaluating Hughes' findings WJSA came to a conclusions that solving the time-dependent SBS problem was beneficial from both physical and numerical standpoints. Furthermore, WJSA decided that some of the shortcomings of the Hughes' code can be overcome using the expertise acquired in the course of modeling the steady state problem. For example, the storage can be reduced through clever step selection and disk storage. The instabilities may be due to the numerical techniques from improper treatment of the sensitivity issue. Focused geometry may pose no problems at all if a proper coordinate transformation is chosen. Consequently, WJSA decided to change the approach to the SBS problem from the steady state to time-dependent treatment.

## 4 Summary and Conclusions

As part of developing the steady state SBS model WJSA performed a thorough investigation of the numerical methods available, identified their strengths and weaknesses, and developed a variety of methods in attempts to solve the SBS problem. The WJSA results were consistent with findings of other researches who found the steady state equations to be numerically unstable. WJSA investigated possible sources of the instabilities, identified and studied one

such instability which has not been mentioned by other model developers, and attempted to eliminate this instability through careful treatment of the pump solutions in the focal region of the SBS cell. The results have been encouraging but not totally successful as another problem, numerical round-off errors, arose.

WJSA also kept in contact with the most recent developments in the SBS modeling arena and recently learned of the success of Brown, Wandzura, and others at Hughes Research Laboratories who developed a four-dimensional SBS code. WJSA reviewed their work and conclusions, found them to be very interesting and useful, but at the same time learned of important limitations of Hughes' code which currently make it inapplicable to the SBL SBS problem. Consequently, WJSA decided to extend its efforts and attempt to model the time-dependent set of the SBS equations using its own expertise and drawing on the work at Hughes. WJSA is currently studying the physics of the time-dependent SBS process and evaluating the techniques necessary to solve the time-dependent SBS equations.

## Acknowledgment

The author would like to thank Douglas Crawford and David Schwamb for numerous and helpful discussions and advice.

## Appendix

### A Simulation Results

The figures in this section are the results of a single successful simulation of the collimated SBS interaction using WJSA 3-D code. The wavelength of the pump and Stokes beams was  $\lambda = 2.8 \mu\text{m}$ , the incident pump intensity was  $I_p = 1 \times 10^{13} \text{ W/m}^2$ , the seed Stokes intensity was  $I_s = 0.153 \text{ W/m}^2$ , the cell length was  $L = 1 \text{ cm}$ , the pump beam diameter was  $D_p = 2 \text{ mm}$ , the SBS gain was  $g = 0.035 \text{ cm/MW}$ , the transverse resolution of the field arrays was 32 by 32, and the number of longitudinal storage locations was 15. Physically the interaction of this type could be taking place in the focal region the SBS cell.

Figure 6: The pump power shows well-behaved convergence as the number of iterations increases.

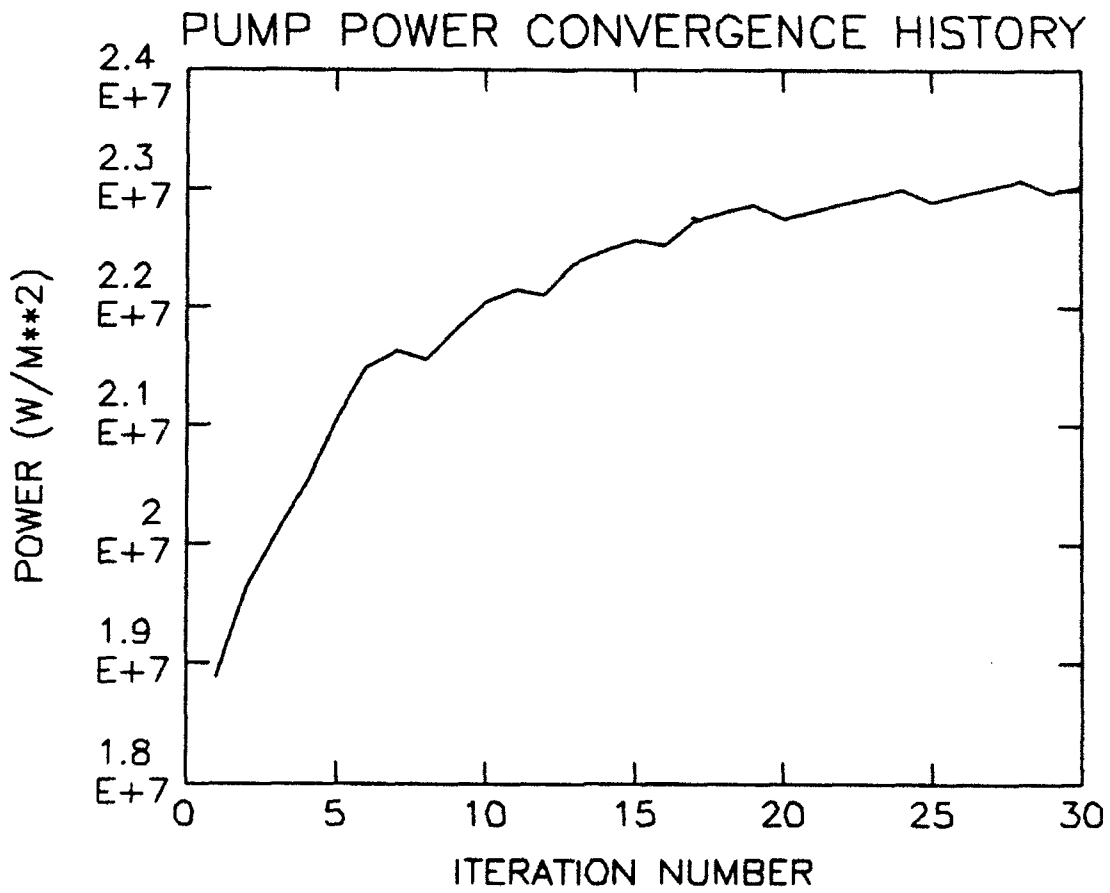


Figure 7: The Stokes power shows well-behaved convergence as the number of iterations increases.

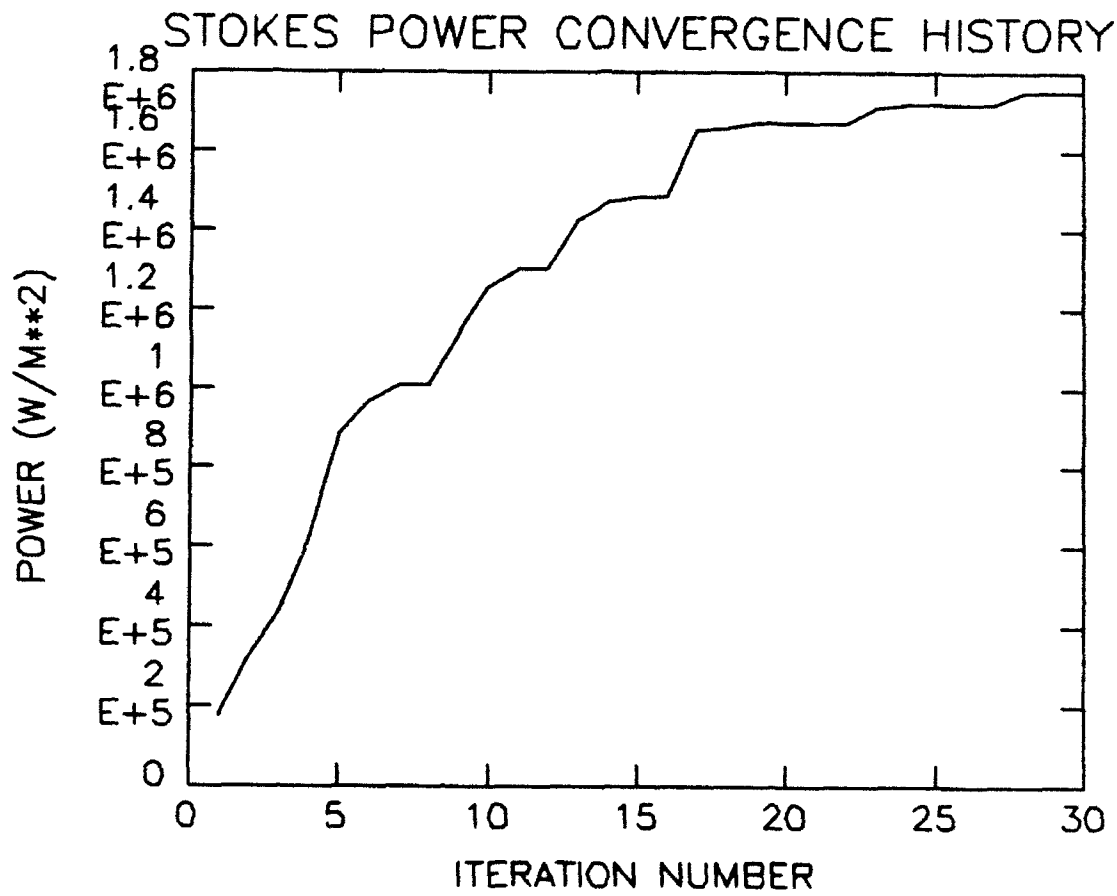


Figure 8: The Stokes fidelity shows well-behaved convergence as the number of iterations increases.

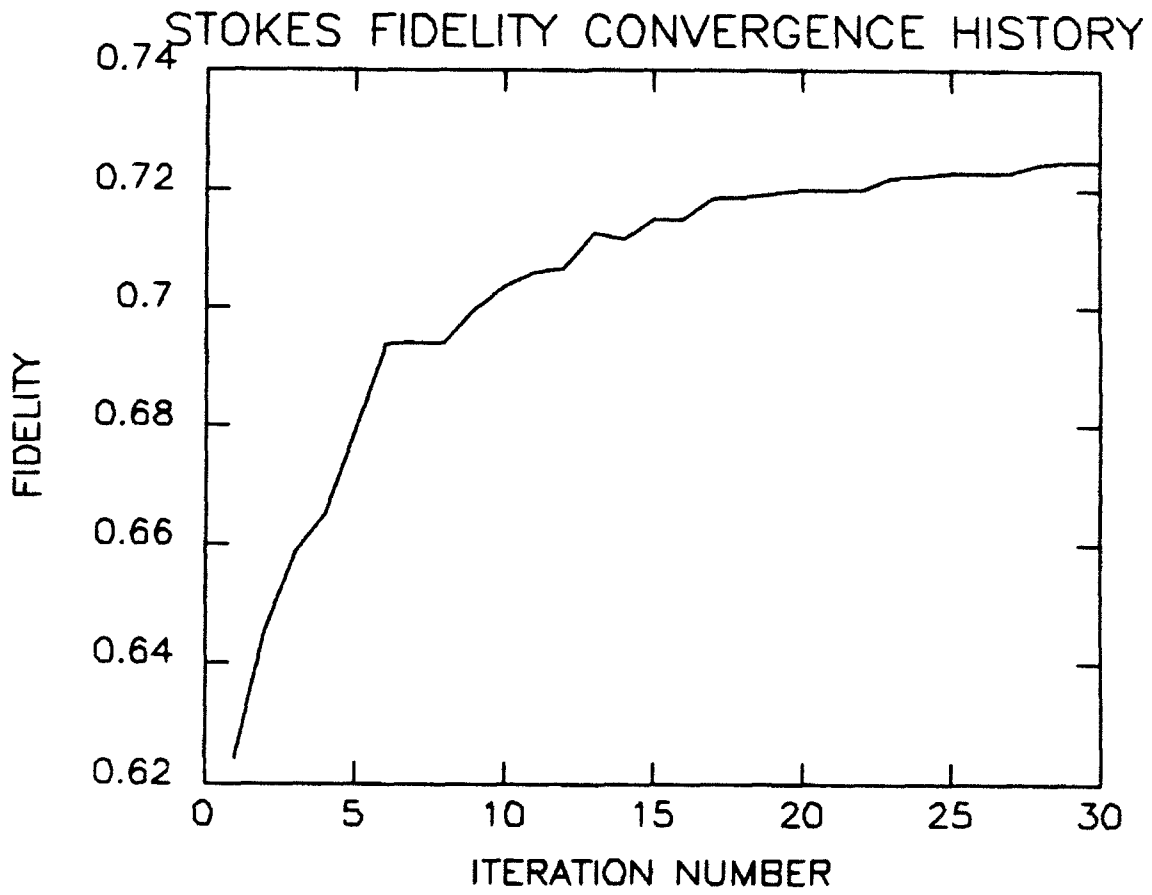


Figure 9: The Stokes reflectivity shows well-behaved convergence as the number of iterations increases.

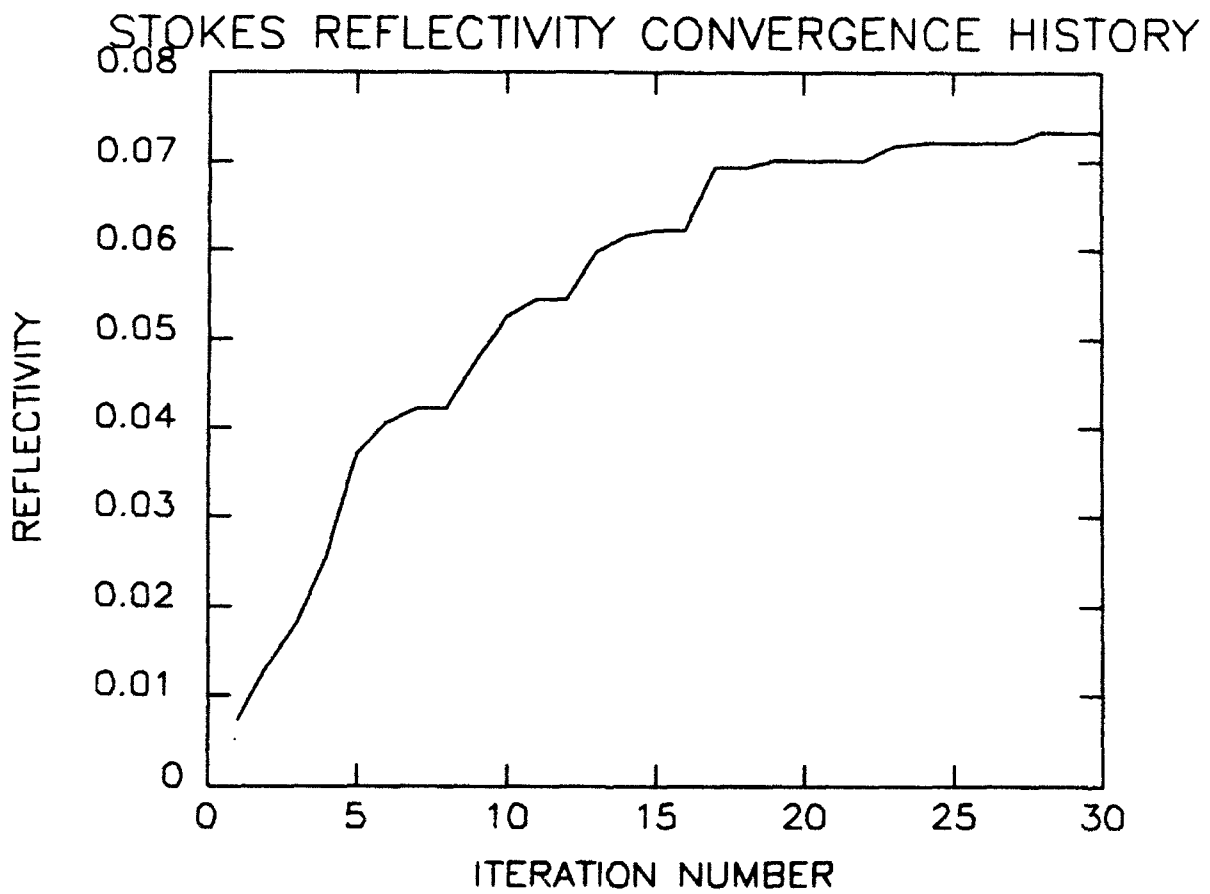


Figure 10: The pump global error decreases monotonically as the number of iterations increases.

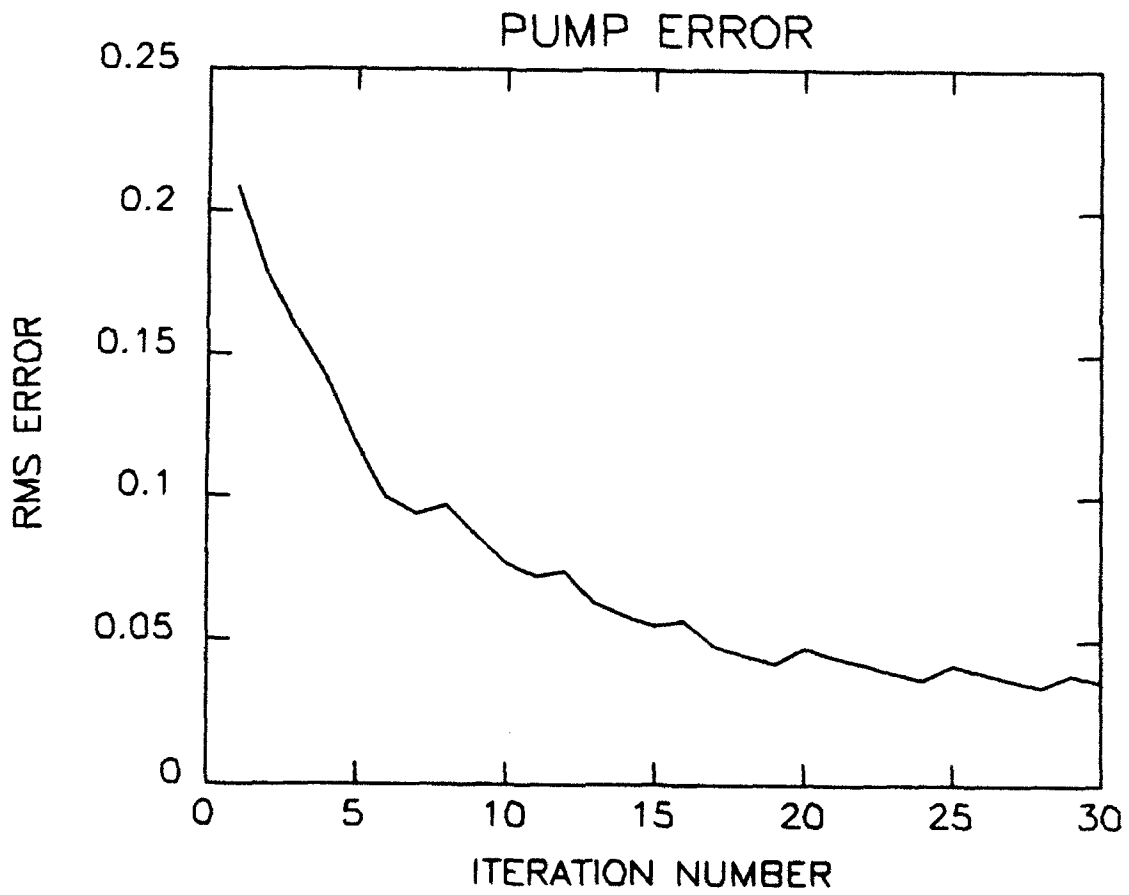


Figure 11: The Stokes local error approaches zero as the number of iterations increases.

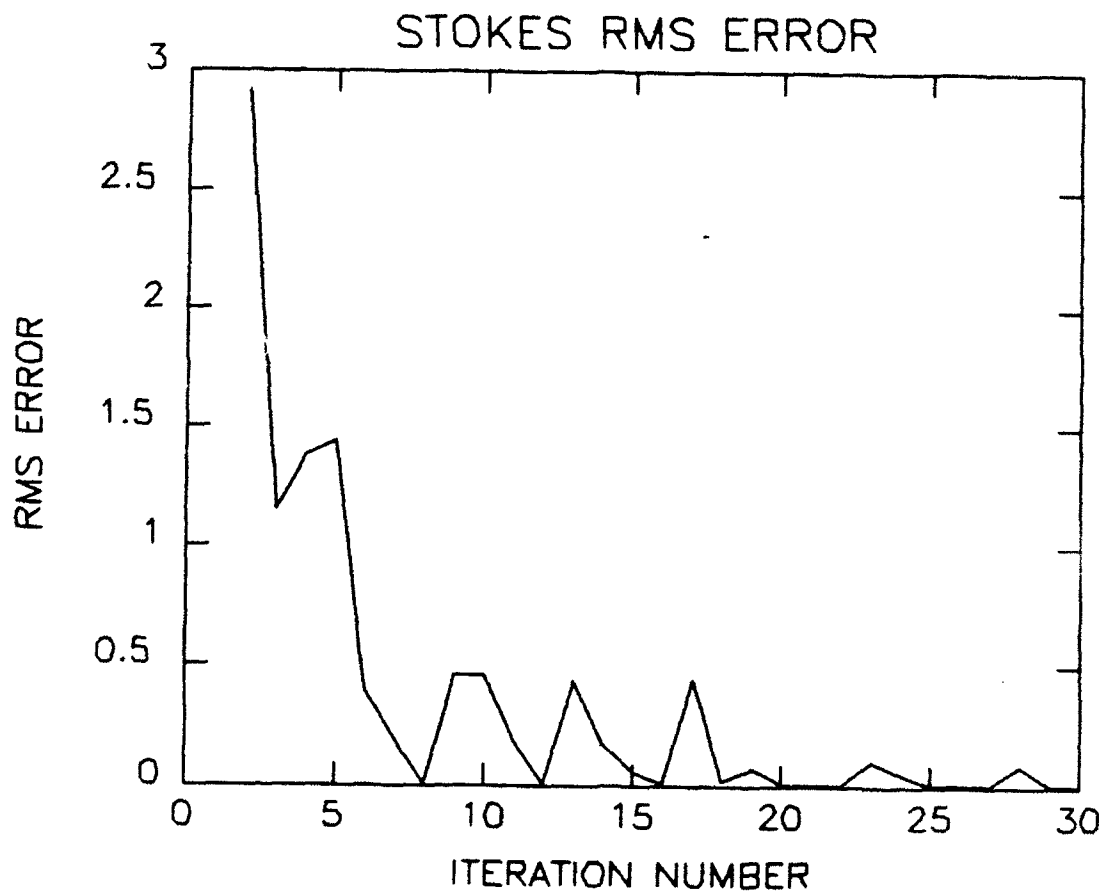


Figure 12: Power conversion curves for pump and Stokes beams as functions of longitudinal distance. The total reflectivity is about 9%, while the reflectivity in the center of the Stokes beam is about 15%.

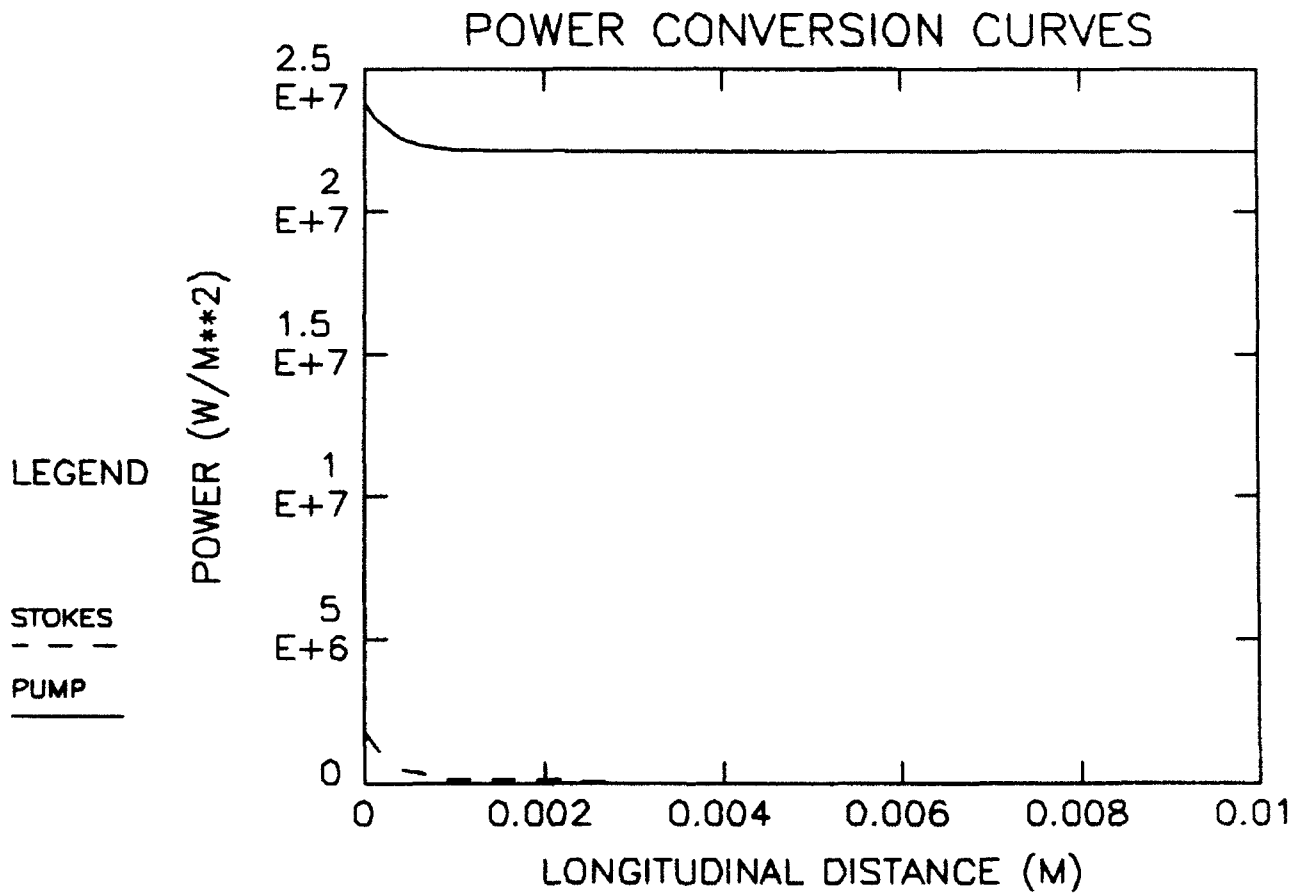


Figure 13: Power conversion curves for pump and Stokes beams as functions of longitudinal distance on a logarithmic scale.

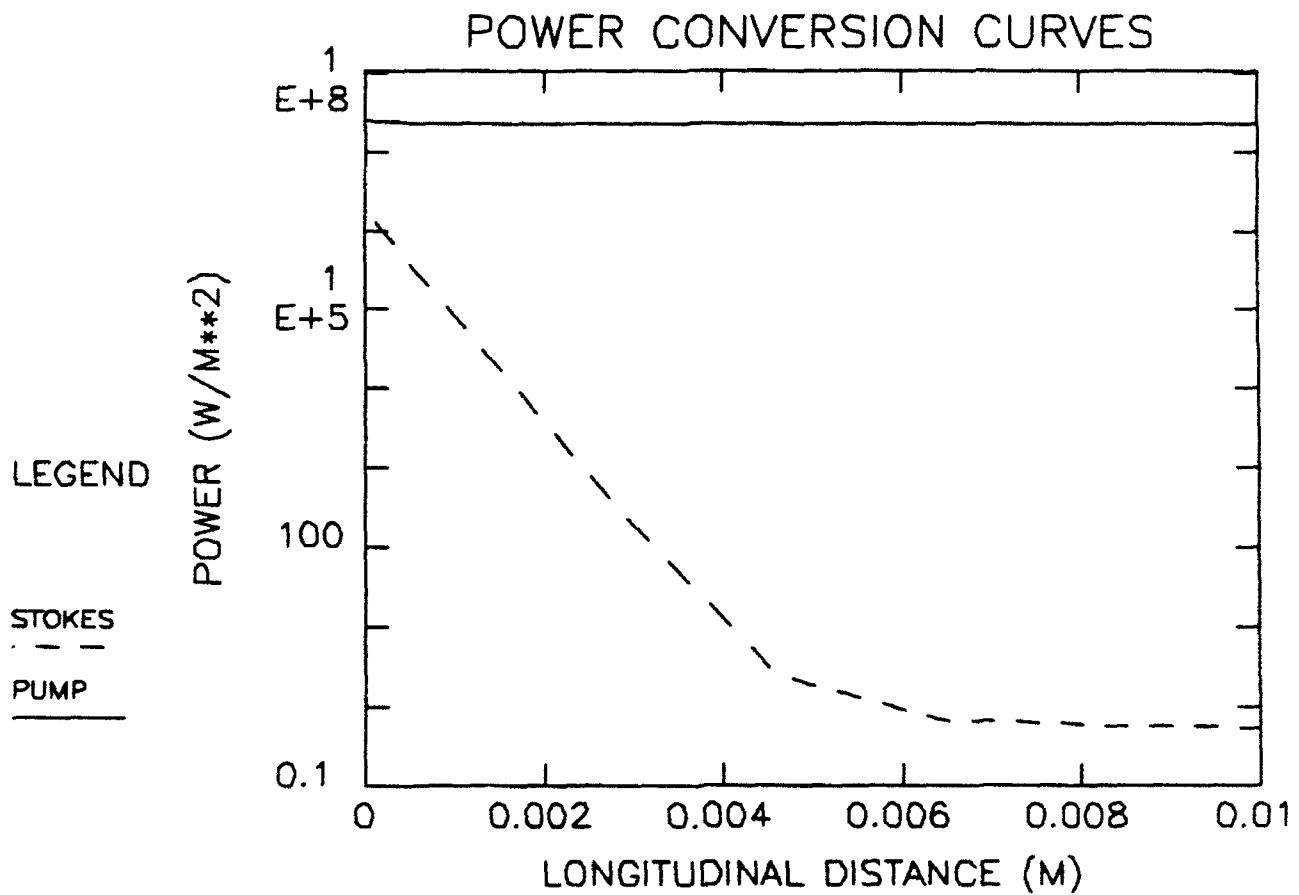


Figure 14: Incident pump intensity cross section.

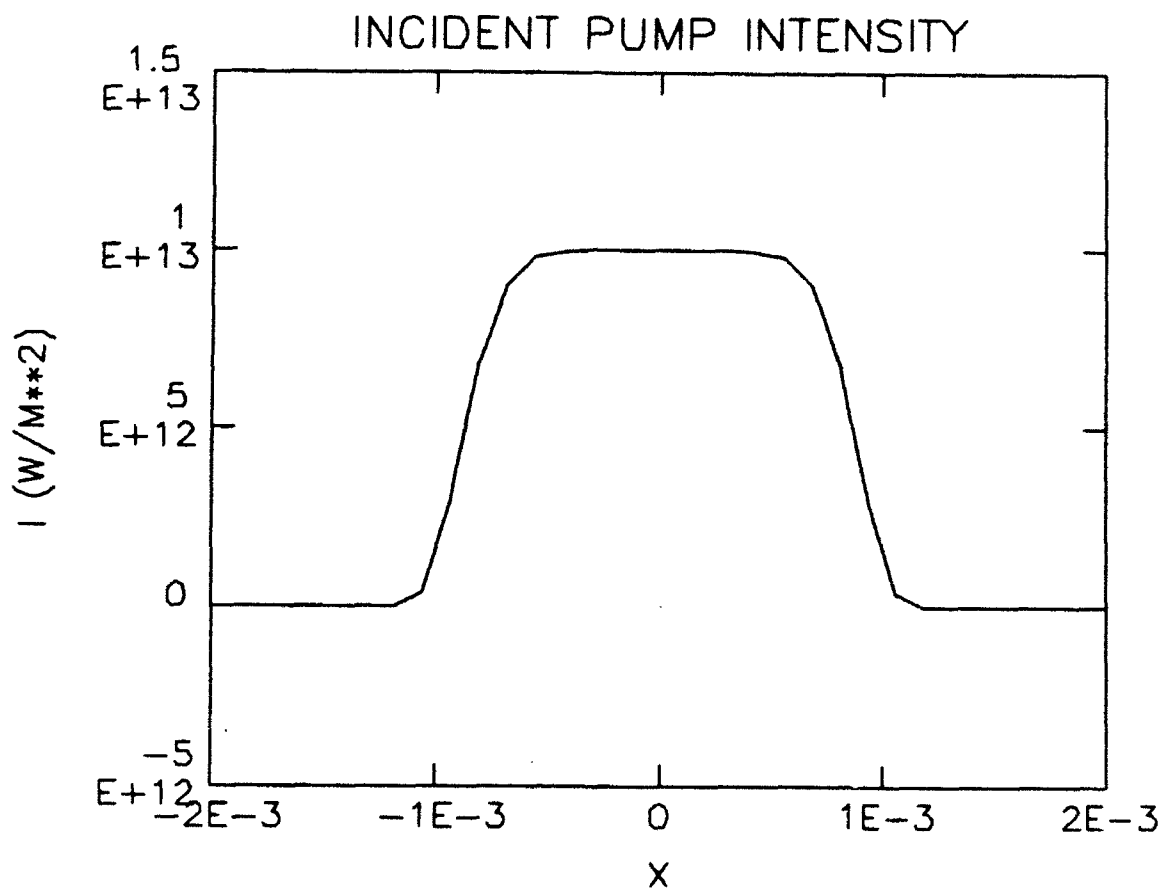


Figure 15: Incident pump phase cross section.

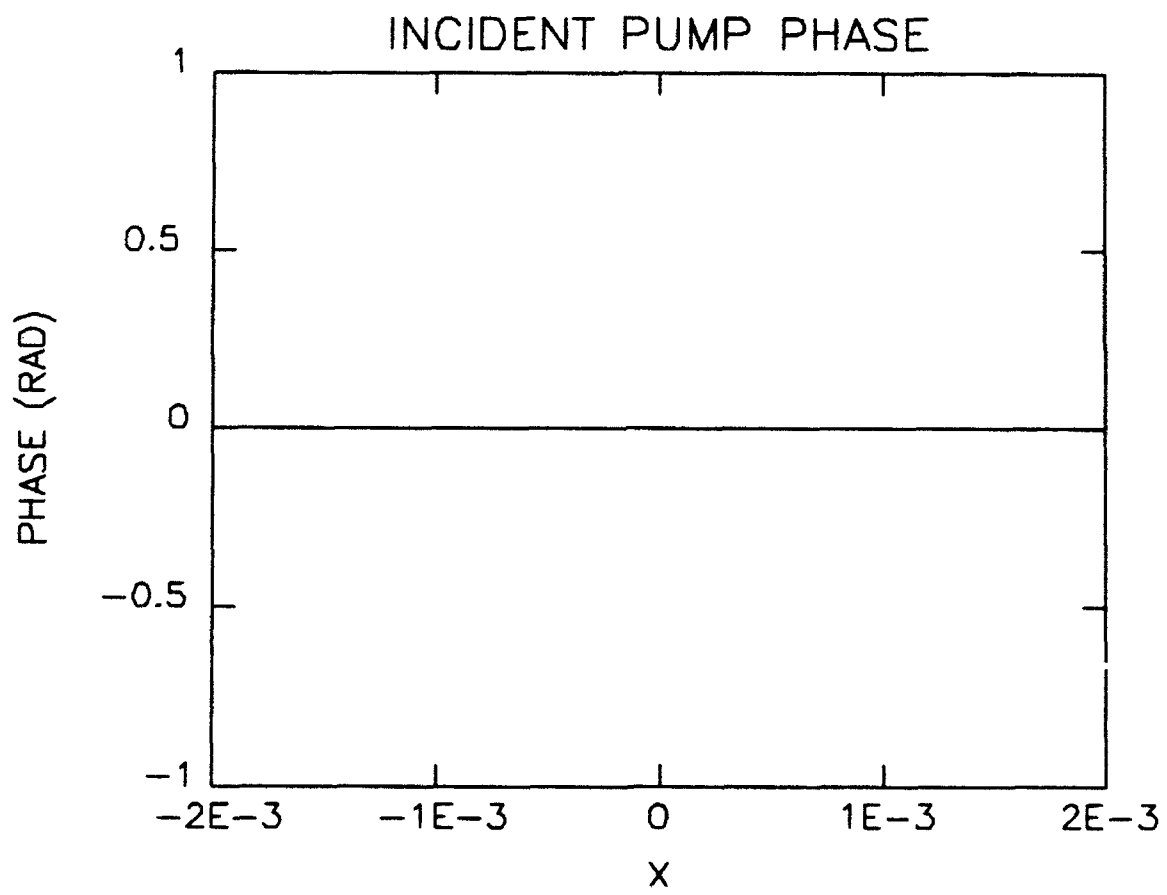


Figure 16: Reflected Stokes intensity cross section shows expected spatial narrowing due to the fact that pump intensity is below threshold at the edges of the beam.

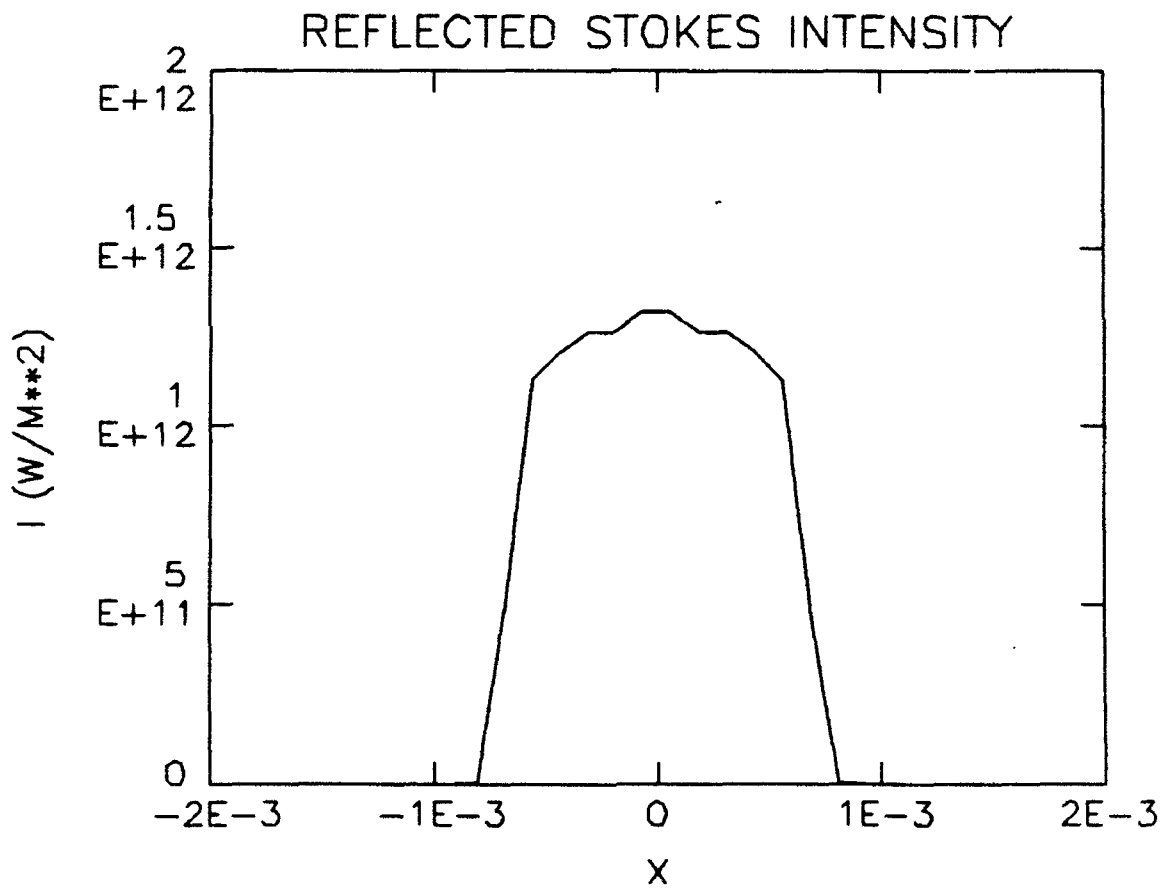


Figure 17: Reflected Stokes phase cross section is approximately flat in the center. The highly oscillatory behavior is in the guard-band region where the intensity is negligible.

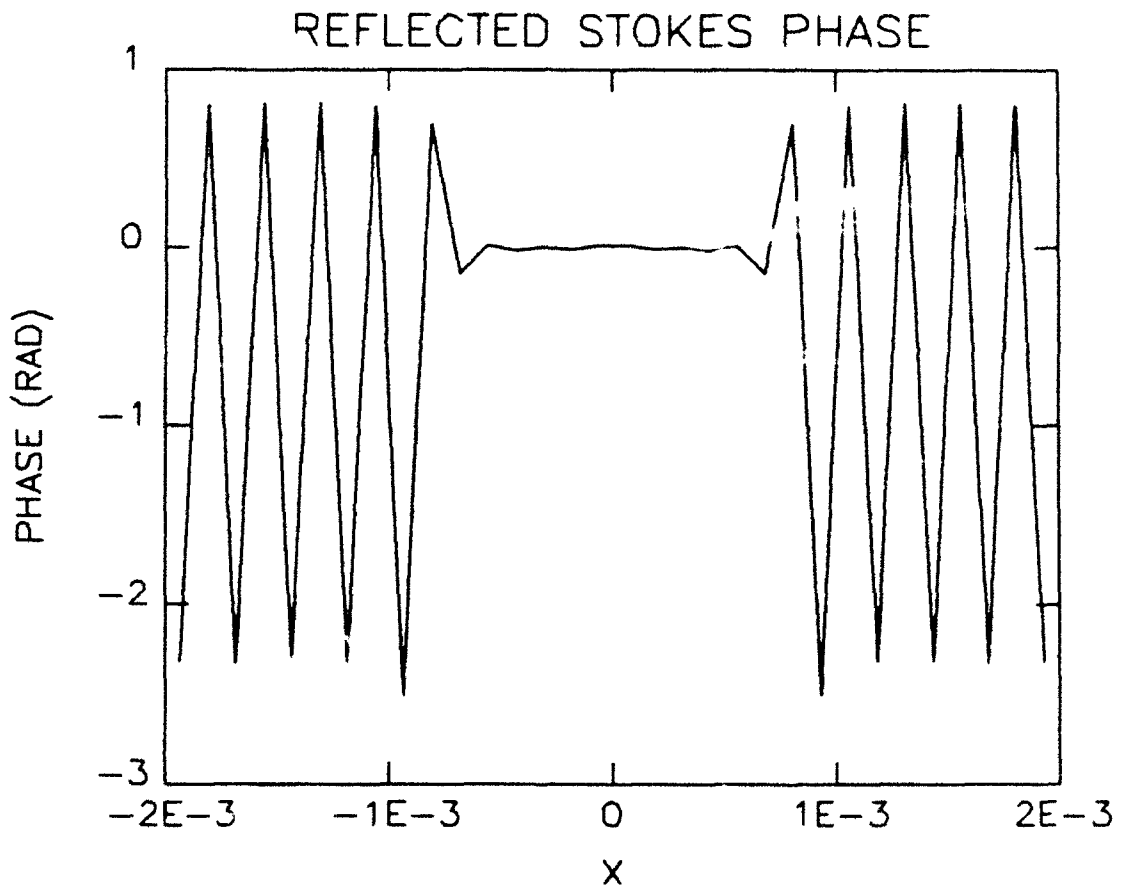


Figure 18: Incident pump intensity contour and surface plots.

INCIDENT PUMP INTENSITY (W/M\*\*2)

MIN VAL = 2E-10

MAX VAL = 1.00000E+13

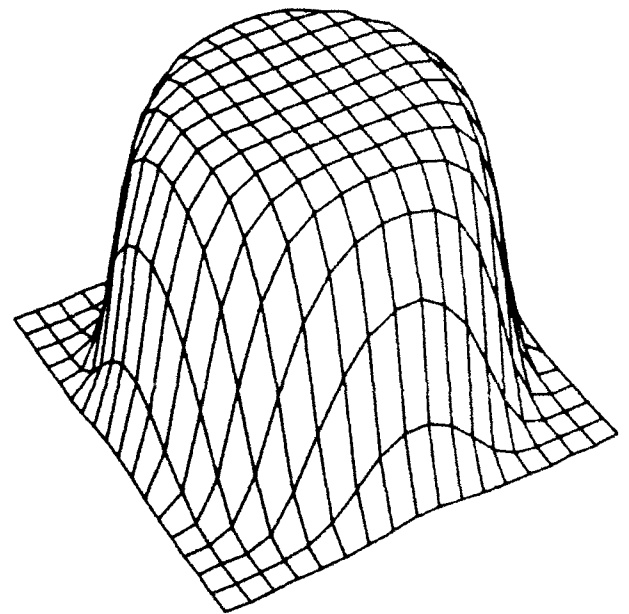
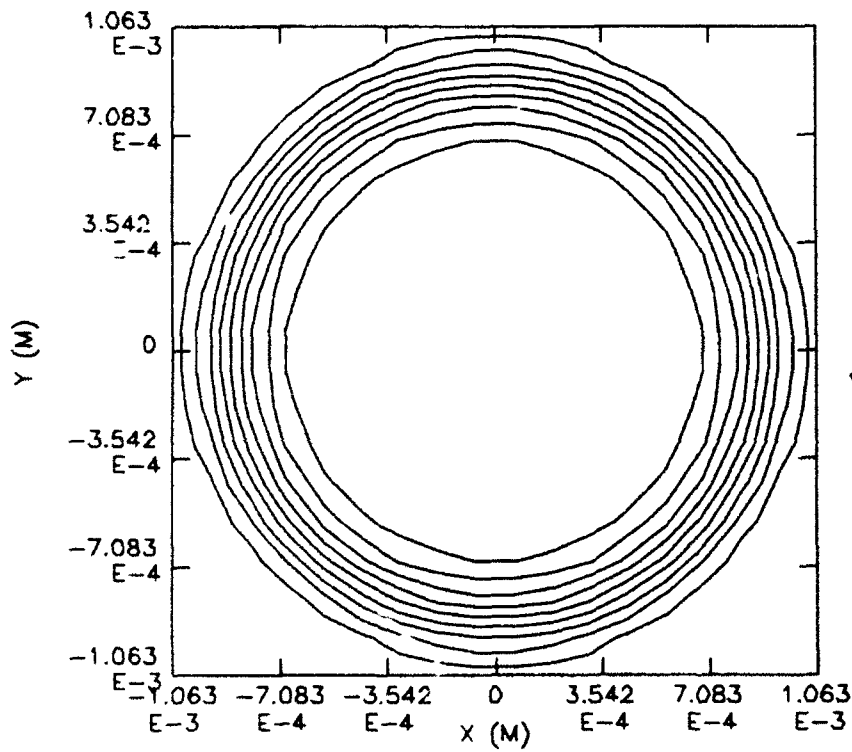


Figure 19: Incident pump phase contour and surface plots.

INCIDENT PUMP PHASE (RAD)

MIN VAL = 0.0000

MAX VAL = 0E+00

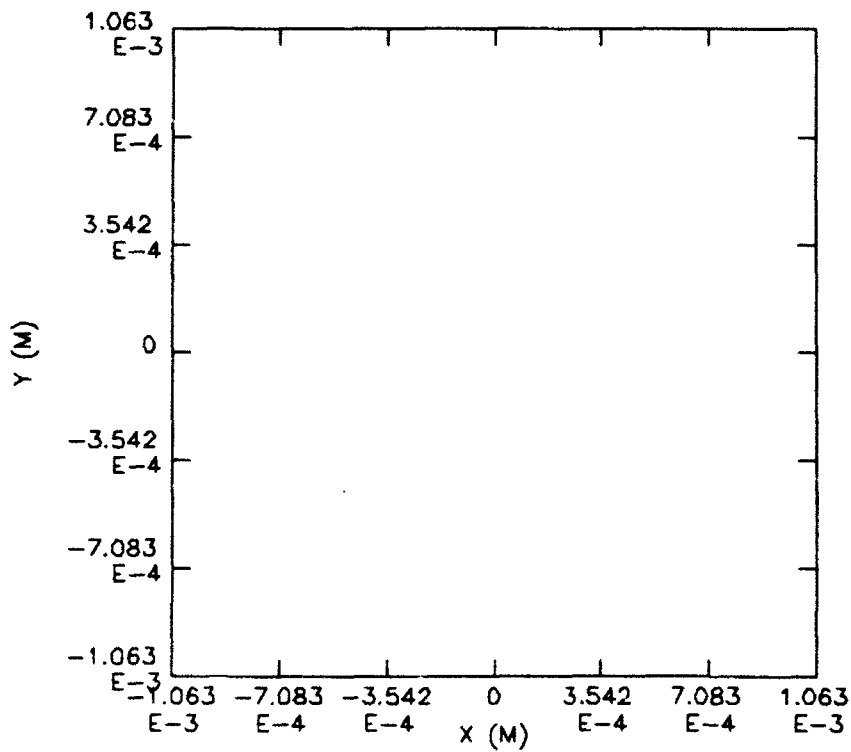


Figure 20: Reflected Stokes intensity contour and surface plots.

REFLECTED STOKES INTENSITY (W/M\*\*2)

MIN VAL = 1.10679E+05

MAX VAL = 1.32397E+12

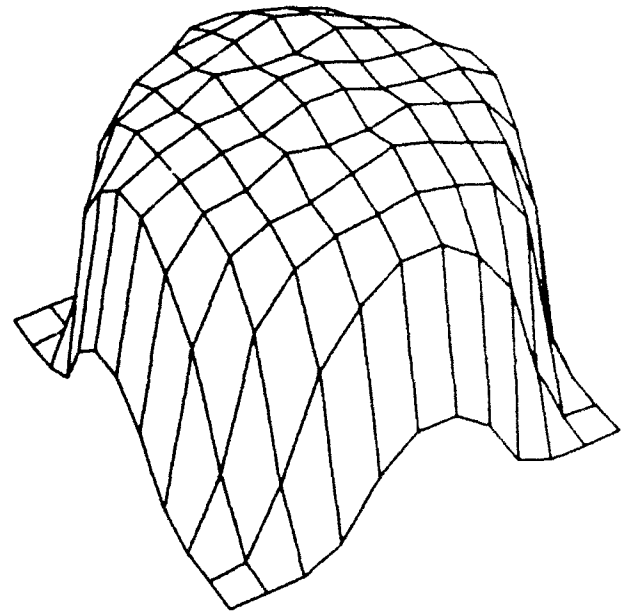
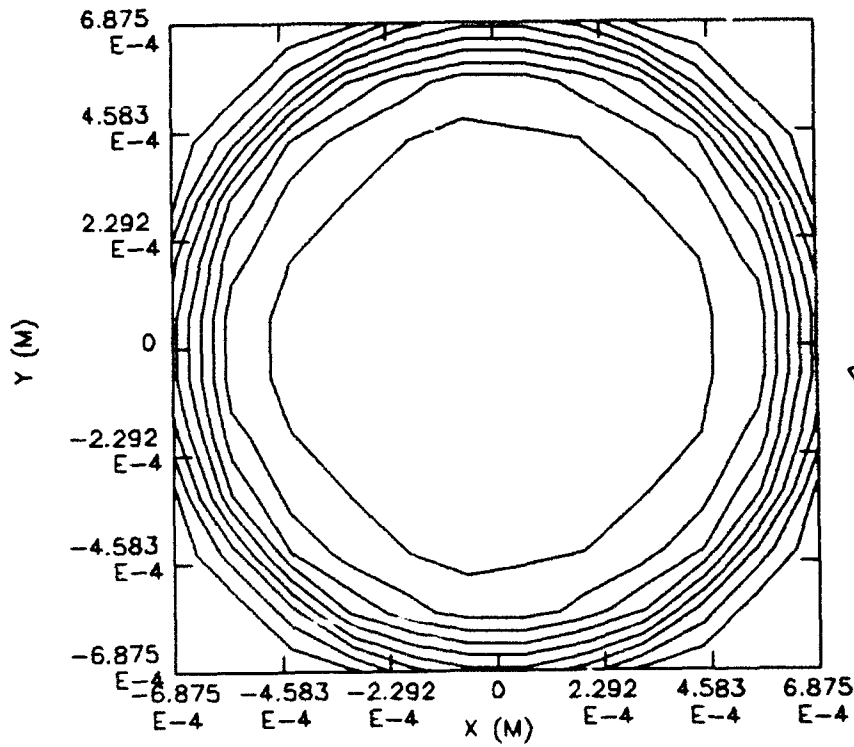
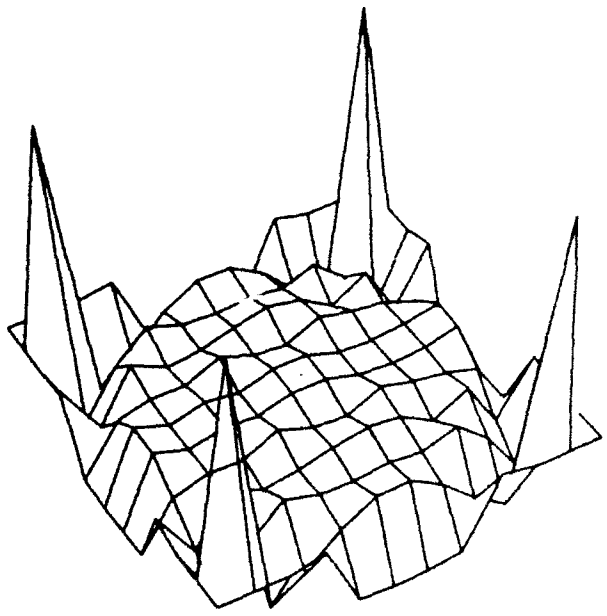
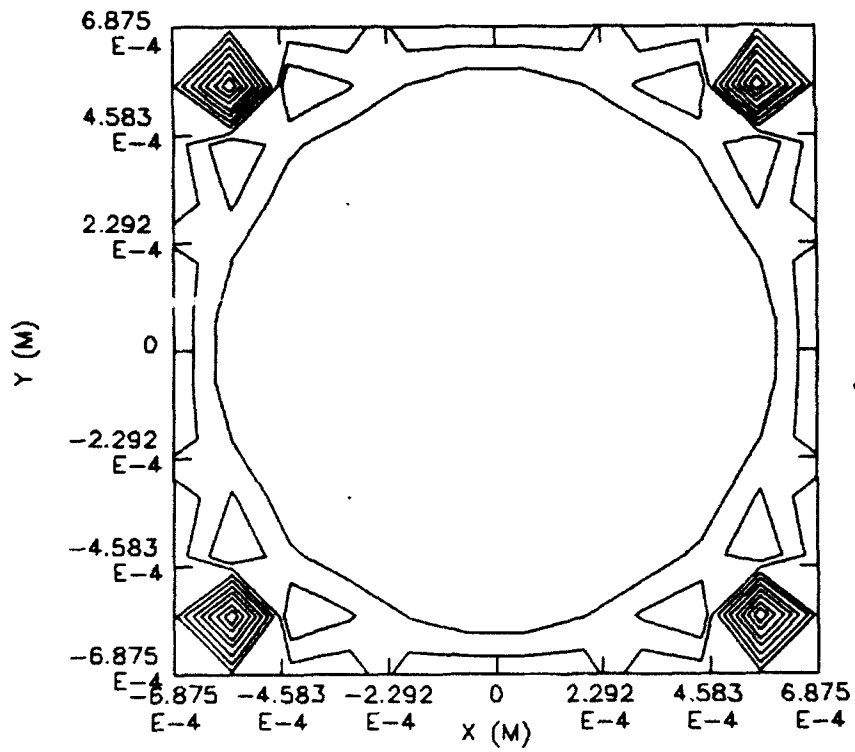


Figure 21: Reflected Stokes phase contour and surface plots.

REFLECTED STOKES PHASE (RAD)

MIN VAL = -0.15900

MAX VAL = 0.45300



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