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Grant Title: Automated Theorem Proving in the Theory of Approximate Reasoning, with Applications to Logic Programming and Expert Systems

Grant Number: N00014-87-J-1219
Reporting Period: 1 Oct 90 – 30 Sept 91

PRODUCTIVITY MEASURES

[Note: This grant was given an additional no-funds extension for the period May 1, 1991 through March 31, 1992. This was done in anticipation of releasing $10,000 of formerly withheld funds, for purposes of supporting a summer trip to Japan. However, it was subsequently decided to fund the Japan trip through a separate grant. Thus some of the work reported here was actually supported by the latter grant, which ran from May 1, 1991 through August 31, 1991. Reporting this work here seems appropriate inasmuch as it all pertains to the same ongoing research project. A final report on the Japan trip is currently in progress.]

Refereed papers submitted but not yet published: 2

Refereed papers published: 4

Unrefereed reports and articles: 3 (masters student project reports)

Contributed presentations: 2 (included above in "refereed papers published", i.e., all were published in conference proceedings and were presented at the indicated conferences.)

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DETAILED SUMMARY OF TECHNICAL RESULTS

Abstract

The primary objective of this project is to develop a new model of natural human reasoning with imprecise linguistic information. Key to this model is a collection of abstraction mechanisms based on the concept of a *linguistic variable*, which was first introduced for this purpose within the context of a semantics based on fuzzy sets. The present approach differs from the earlier one, however, in that (i) it does not require the use of fuzzy sets for the interpretation of linguistic terms, and (ii) the meanings of logical inferences are given as algorithms which act directly on terms themselves, rather than on their underlying interpretations.

Thus this work constitutes a return to the more purely *symbolic* or *axiomatic* representations of logical deduction, whereas the fuzzy-sets model concerns *denotational* or *semantic* representations. The new model should not be viewed as a negation of the earlier approaches, however, but as an augmentation of them. The present work is intended as the beginning of a larger system which encompasses both styles of reasoning.

Also underway is development of a prototype expert system shell which implements the new model, together with some of the earlier forms of fuzzy inference. This part of the work is being undertaken in part to identify the problems associated with the task of implementation and in this manner demonstrate that implementation is indeed possible. Development of the prototype is also desired so as to experiment with the model and determine its viability as a basis for reasoning in specific problem domains.

1. Introduction

A central problem addressed by the theory of approximate reasoning is how to model natural human reasoning with imprecise linguistic information. Briefly, it is desired to find an effective means for representing inferences such as "MOST EXTREMELY HIGH-YIELD investments are VERY RISKY; XYZ is a HIGH-YIELD investment; therefore it is VERY LIKELY that XYZ is AT LEAST MODERATELY RISKY." Credit for giving the first concise statement of this problem in its present form, together with identification of its principal subproblems, must go to L. A. Zadeh, whose papers papers on fuzzy (or possibilistic) reasoning have come to provide the core body of literature on the subject. Zadeh's interest in this came initially from issues in automatic control theory, but the ongoing research more recently has been fueled by the prospect of using it in the field of expert systems.

The aim of the present work is to develop a new approach to this same set of problems. Some differences between the new model and those that have preceded it are as follows. In the fuzzy-sets model, general concepts like Height (of people) are represented as linguistic variables which range over a collection of terms like TALL, VERY TALL, SHORT, NOT TALL AND NOT SHORT, etc., where the meanings of the terms are given as fuzzy subsets of an underlying measurement domain, e.g., the set of numerical heights from 0 to 8 feet. The terms themselves are generated from a set of atomic terms, like SHORT, MEDIUM, and TALL, in accordance with a context-free grammar; each of the atomic terms are given meanings as fuzzy subsets of the measurement domain; and the meanings of composite terms are defined in terms of operators on fuzzy sets. To this end there are specific operators corresponding to each of the hedges, VERY, MODERATELY, etc., and to the logical connectives, NOT, OR, and AND. Reasoning with logical combinations of terms from multiple linguistic variables, e.g., as in VERY TALL AND RATHER OLD, is then carried out as operations on fuzzy subsets of the cartesian products of the corresponding measurement domains, and logical inference, in particular, is frequently given as a matrix operation known as the compositional rule of inference. Thus this approach to approximate reasoning is purely *denotational*: all reasoning is done strictly as operations performed on the fuzzy-set denotations of linguistic terms. Stated another way, the aim of such models has been to formulate a model of *semantic* inference.

The new model, on the other hand, represents a return to the more *symbolic* approach to formal reasoning, such as employed in Prolog. Here the objective is to define the logical operations, including inference, as being operations which act more directly on the linguistic terms. The reasons for moving in this direction are varied. First was the discovery of various complexity problems surrounding efforts to implement the fuzzy-sets model. These issues seem to forbid effective uses of this model except on machines at least as fast as current supercomputers. Hence there was a desire to find a means of simplification. Second was the philosophical view that there is a distinct power of reasoning which is captured only at the symbolic level. This is supported by a full century of research in symbolic logics. Third was the belief that humans actually do perform much of their reasoning more or less symbolically, for which reason there naturally arose the interest in finding out if the development of a symbolic approach to fuzzy reasoning was mathematically feasible. Fourth was the observation that, if one could devise such a system of symbolic reasoning, then one could in this way circumvent another troubling, albeit lessor, drawback of the fuzzy-sets model, namely, that one is frequently required to manufacture measurement domains in order to represent linguistic variables, e.g., Preference or Risk, for which no such domains naturally occur. While there is certainly no harm in introducing some nominal scale, say the interval $[0, 1]$, for this purpose, doing so seems artificial and pragmatically awkward. Fifth was the discovery that, by means of a simple term algebra (defined in the following), one could conveniently represent further natural language notions not so easily represented in the fuzzy-sets approach. These include modifiers like AT LEAST and AT MOST, and several different forms of logical negation.

The following briefly describes two new modes of inference that have been developed to date, and it discusses recent work toward formulating the higher-order concepts of fuzzy quantification, fuzzy temporality (or usuality), and fuzzy likelihood.

2. Inference Modes I and II

Both forms of inference are built upon the concept of a *linguistic variable*, which may be described informally as a variable which takes its values from among the expressions in a natural or artificial language [Zadeh]. In practice, such a variable represents a general property or attribute of individuals, e.g., Height of mountains, while its values express more specific properties or attributes of the same individuals, e.g., RATHER TALL. Here a linguistic variable V will be represented formally as a triple $(\mathrm{T}, \mathrm{D}, \mathrm{M})$, where T is a set of *linguistic terms*, D is a *measurement domain*, and M is a *meaning assignment*, each described below.

The set T of linguistic terms is of the form $\mathrm{E} \cup \mathrm{S}$, where E is a set of *elementary terms* and S is a possibly empty set of *synonyms* for elementary terms. For each V, the set E is understood to contain a unique *primary term* τ_p. For example a natural choice of primary term for the linguistic variable Height would be TALL. Let $\mathrm{ant}(\tau_p)$ represent the antonym of τ_p, and let $\mathrm{med}(\tau_p)$ represent an intermediate term. Let r, v, and e be abbreviations for the linguistic hedges RATHER, VERY, and EXTREMELY. For the present exposition, it will be assumed that the set of elementary terms for any linguistic variable must have one of the forms:

F1: $\{\mathrm{ant}(\tau_p),\ \tau_p\}$
F2: $\{\mathrm{ant}(\tau_p),\ \mathrm{med}(\tau_p),\ \tau_p\}$
F3: $\{\mathrm{ant}(\tau_p),\ \text{r-ant}(\tau_p),\ \mathrm{med}(\tau_p),\ \text{r-}\tau_p,\ \tau_p\}$
F4: $\{\text{v-ant}(\tau_p),\ \mathrm{ant}(\tau_p),\ \mathrm{med}(\tau_p),\ \tau_p,\ \text{v-}\tau_p\}$
F5: $\{\text{v-ant}(\tau_p),\ \mathrm{ant}(\tau_p),\ \text{r-ant}(\tau_p),\ \mathrm{med}(\tau_p),\ \text{r-}\tau_p,\ \tau_p,\ \text{v-}\tau_p\}$
F6: $\{\text{e-ant}(\tau_p),\ \text{v-ant}(\tau_p),\ \mathrm{ant}(\tau_p),\ \text{r-ant}(\tau_p),\ \mathrm{med}(\tau_p),\ \text{r-}\tau_p,\ \tau_p,\ \text{v-}\tau_p,\ \text{e-}\tau_p\}$

Whichever form is used, we shall assume that the indicated terms are ordered by a relation $<$ in the order shown. The restriction to a maximum of nine elementary terms is based on the observation that, in ordinary human discourse, one seldom needs more than nine levels of distinction. Discussions of the sets S and D, and of the mapping M, may be found in the author's publications.

Inference Mode I is intended to produce results that are analogous to those of Zadeh's compositional rule of inference, but in a computationally simpler manner. The key idea underlying the new mode may be illustrated as follows. Consider an elementary decision rule having the form

$$\tau_1, \ldots, \tau_n \Rightarrow \tau,$$

where $\tau_1, \ldots, \tau_n$, and τ are given as linguistic terms (from typically distinct linguistic variables) thought of as unary relations all of the same individual variable. To illustrate, where E=EXPERIENCED, A=AMBITIOUS, C=CREATIVE, and S=SUITABLE are terms from four appropriately chosen linguistic variables, the inference

$$\mathrm{E}(X),\ \mathrm{A}(X),\ \mathrm{C}(X) \Rightarrow \mathrm{S}(X),$$

might be used to determine the suitability of an arbitrary individual X for an employment position. The deduction algorithm is defined in such a way that an individual's having a strong rating along one premise will counterbalance that individual's having a weak rating along another. For example, even though individual A might be only RATHER EXPERIENCED, if A is VERY AMBITIOUS and CREATIVE, then A should be SUITABLE. This notion of counterbalancing has been adapted from well-known methods of multi-criteria decision making. It is captured here by assigning a numerical *rank* to the terms for linguistic variables, defining a *distance* between terms as a difference between ranks, and then computing the concluding term for an individual A as the sum of the distances between the terms attributed to A and the corresponding terms appearing in the premises of the given inference.

This method has been extended to include terms which represent n-ary relations for $n \geq 0$. Variants on the above are obtained by modifying the ranking scheme and the summation routine. In this manner one can model several rather complex forms of logical reasoning.

For Inference Mode II, the notion of a linguistic variable must be expanded to include a set of operators defined generally for all linguistic variables. Let V be a linguistic variable with term set T. The *expressions* of V are defined as follows: (i) terms in T are expressions of V, (ii) if τ is a term in T, then all of $\text{NOT}_\text{o}\ \tau$, $\text{NOT}_\text{s}\ \tau$, $\text{NOT}_\text{a}\ \tau$, $\text{NOT}_\text{v}\ \tau$, AT LEAST τ, and AT MOST τ are expressions of V, (iii) if ε and ε' are expressions of V, then (ε OR ε'), (ε AND ε'), and $\text{NOT}_\text{t}\ \varepsilon$ are expressions of V.

Each expression ε of V is provided with a *relative meaning* $\rho(\varepsilon)$, given as a subset of T. Here "relativity" is with respect to the set T. For arbitrary $\tau \in \text{T}$, set

$$\rho(\tau) = \{\tau\}.$$

Thus the relative meaning of any term is just the singleton containing the term itself. For arbitrary expressions $\varepsilon, \varepsilon'$ of V set

$$\rho(\text{NOT}_\text{t}\ \varepsilon) = \text{T} - \rho(\varepsilon),$$
$$\rho(\varepsilon \text{ OR } \varepsilon') = \rho(\varepsilon) \cup \rho(\varepsilon'), \text{ and}$$
$$\rho(\varepsilon \text{ AND } \varepsilon') = \rho(\varepsilon) \cap \rho(\varepsilon'),$$

where $-$, $\cup$, and $\cap$ are the ordinary set operations. It follows that any expression made up in this manner of OR's, AND's, and NOT_t's is representable as a unique subset of T. The special case that $\rho(\varepsilon) = \emptyset$ is taken as saying that ε is *contradictory.*

The particular form of NOT defined above is called *total negation* and is interpreted as expressing "anything except ε." The other modifiers are defined analogously in terms of subsets of T. To illustrate, consider the *ordered negation,* a common form of negation wherein NOT τ means "something less than τ" if τ is above $\text{med}(\tau_p)$, and means "something more than τ" if τ is below $\text{med}(\tau_p)$. This may be represented here as an operator NOT_o defined by

$$\rho(\text{NOT}_\text{o}\ \tau) = \begin{cases} \{\tau' | \tau' < \tau\}, & \text{if } \tau > \text{med}(\tau_p); \\ \{\tau' | \tau' > \tau\}, & \text{if } \tau < \text{med}(\tau_p); \\ \text{undefined}, & \text{if } \tau = \text{med}(\tau_p). \end{cases}$$

Deductions under Inference Mode II have the same syntactic form as for Mode I, with the exception that the premises and conclusions may be expressions. Thus, where T =

TALL and S = SHORT are from the terms set for Height, and where A = ACCEPTABLE, one may have inferences such as

$$(\text{NOT}_o \text{ T AND NOT}_o \text{ S})(X) \Rightarrow \text{AT\_LEAST A}(X).$$

The associated inference algorithm is as follows. Suppose individual A has been attributed a certain height, given as an expression ε from the expression set for Height. We say that A *satisfies* the above premise if $\rho(\varepsilon) \subset \rho(\text{NOT}_o \text{ T AND NOT}_o \text{ S})$. The rule of deduction is simply that we conclude AT_LEAST A(A) just in case the premise is in this sense satisfied.

This naturally extends to multiple premises and to linguistic variables with all possible arities. Inference Mode II thus enables one to express many rather sophisticated relationships among any given collection of linguistic variables.

3. Fuzzy Quantification, Usuality, and Linguistic Likelihood

Several issues remaining to be resolved with the above reasoning system have been turned over to graduate students, and my own work has turned to the problem of overlay this system with a means for dealing with concepts like MOST, MANY, FEW (quantification), USUALLY, OFTEN, SELDOM (usuality), and LIKELY, UNCERTAIN, UNLIKELY (likelihood). It has been determined that there is a natural correspondence between these three notions, as illustrated by the following table.

Quantification	*Usuality*	*Likelihood*
all	always	certainly
most	usually	likely
many	frequently/often	uncertain
few/some	occasionally/seldom	unlikely
no	never	certainly not

For example, fuzzy quantification is related to linguistic likelihood in the sense portrayed by the sylogism "*Most* birds can fly, Tweety is a bird, therefore it is *likely* that Tweety can fly." Work is currently underway to capture this form of reasoning in a rigorous formalism. Insofar as this will embody a formalization of natural human reasoning with likelyhood-related terminology, it will yield a more qualitative way of dealing with uncertainty.

Recent progress has been made toward formalizing fuzzy likelihood as a modal extension of classical two-valued logic. It is believed that this will provide the basis for similar formalizations of both fuzzy quantification and usuality, as well as a means for expressing the interrelations between the three. Once such an extension of classical logic is established, it will then be undertaken to apply the same techniques to a similar extension of the foregoing fuzzy reasoning system.

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PUBLICATIONS, PRESENTATIONS, AND REPORTS

Publications and Presentations

1. Chung, H.T. and Schwartz, D.G., Design of an inference engine for reasoning with imprecise information, *Proceedings of the Florida AI Research Symposium, FLAIRS-91*, Cocoa Beach, Florida, April 2–5, 1991, pp. 22–26.

2. A system for reasoning with imprecise linguistic information, *International Journal of Approximate Reasoning*, 5 (1991) 463–488.

3. Tamir, D.E., Schwartz, D.G., and Kandel, A., A pattern recognition interpretation of implications, *Information Sciences*, 57–58 (1991) 197–215.

4. Two interpretations of fuzzy likelihood, *Proceedings of the Japan Society for Fuzzy Theory and Systems*, Nagoya, Japan, June 12–14, 1991, pp. 221-224.

5. Qualitative representations of fuzzy likelihood, *Proceedings of the World Congress on Expert Systems*, Orlando, Florida, December 16–19, 1991.

6. A min-max semantics for fuzzy likelihood, *Proceedings of the IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, March 8–12, 1992, San Diego, California, in review.

Reports

1. Yeh, Y.L. and Schwartz, D.G., *LVfms: A Linguistic Variable File Management System for a Fuzzy Expert System Shell*, Masters Degree Project Report, Technical Report No. 91-041, Department of Computer Science, Florida State University, Tallahassee, Florida, April, 1991.

2. Tsai, C.H. and Schwartz, D.G., *IRfms: An Inference Rule File Management System for a Fuzzy Expert System Shell*, Masters Degree Project Report, Technical Report No. 91-042, Department of Computer Science, Florida State University, Tallahassee, Florida, April, 1991.

3. Ko, M.H. and Schwartz, D.G., *IFfms: An Individual Fact File Management System for a Fuzzy Expert System Shell*, Masters Degree Project Report, Technical Report

No. 91-043, Department of Computer Science, Florida State University, Tallahassee, Florida, April, 1991.

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April 15, 1993

Office of Naval Research
Resident Representative
Attn: Cynthia Sloan
Georgia Institute of Technology
206 O'Keefe Building
Atlanta, GA 30332-0490

Dear Representative,

Enclosed herewith is the final technical report for Grant Number N00014-87-J-1219.

Even though the budget period for this grant extended to April 30, 1992, and the enclosed report covers the period ending April 30, 1991, *this is the final report.* An explanation is as follows.

The grant was initially a 3-year grant, issued in 1987, and extending through April 30, 1990. In accordance with the Florida Demonstration Project, a no-cost extension was granted during 1990, extending the ending date to April 30, 1991. Sometime in 1989 or 1990, while the original grant was still in effect, the scientific officer, Allan Meyrowitz, had called me and said that due to budget cuts at ONR they would like to withhold $10,000, and asked me if I could do without it. I said that I could, and by April 30, 1991, all funds except for the $10,000 were spent.

During the Fall of 1990, however, the ONR program director, Andrè van Tilborg, had discussed with me the possibility of sending me to Japan for a special fact finding trip, and in Spring 1991 it was decided that I would go. In order to simplify book keeping, he initially proposed to simply release the $10,000 formerly withheld under the existing grant. It was for this purpose that an *additional* no-cost extension, until April 30, 1992, was requested and obtained.

Then, shortly thereafter, it was determined that the Japan trip would actually require $20,000, and that a new grant would have to be issued after all. This was grant number N00014-91-J-1933, which covered the summer of 1991 and extended till Spring of 1992.

Thus all funds associated with grant number N00014-87-J-1219 were expended as of April 30, 1991, the project effectively terminated as of that date, and the enclosed report documents the work that took place up until that time.

Please accept my apologies for the delay and confusion associated with this filing. If you have any further questions, please call me at the number above. Thank you.

Yours sincerely,

D. G. Schwartz

Daniel G. Schwartz, Ph.D.
Associate Professor

Copies to:
- Defense Technical Information Center
- Director, Naval Research Laboratory
- Scientific Officer (LCDR Robert Powell)
- FSU Contracts and Grants

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TRANSITIONS AND DOD INTERACTIONS

None to date, other than a few paper requests.

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SOFTWARE AND HARDWARE PROTOTYPES

This project has also come to involve development of a prototype expert system shell which implements the new model of logical reasoning. An important design principle is to define data structures and algorithms at a sufficiently high level of abstraction that the system can conveniently be expanded at will to include a wide variety of modes of inference. In particular, the system should allow one to include the variant of the compositional rule of inference which is employed by the new Japanese fuzzy-logic controllers.

Work on this part is being executed piecemeal by masters students as MS degree programming projects. This excepts the inference engine, whose more complete version is being designed by a doctoral student. Parts of the system, formerly written in have now been revised and expanded, and rewritten in C. The final system will run under UNIX on SUN workstations and compatible machines. It will consist of six major components: (i) LVfms, a file management system for storing linguistic variables, (ii) IRfms, a file management system for storing inference rules, with references back into the LVdb to identify the linguistic variable being employed in a particular rule, (iii) IFfms, a file management system for recording facts about individuals, also referring back to specific linguistic variables, (iv) ESfms, a data base for storing complete expert systems, where each such system is recorded as a set of rules (stored in the rule base) and a set of facts (stored in the fact base), (v) KTree, a program which, when given the identifier of a specific expert system, extracts all the relevant information from the foregoing files and organizes it in memory into a data structure known as a knowledge tree, and (vi) IE, an inference engine, which answers user queries via reference to the knowledge tree.

Preliminary versions of the LVfms, IRfms, and IFfms have now been completed. The ESfms and KTree are in progress and should be finished by the end of Spring 1992. A simplified, first-cut version of an inference engine was written by a masters student some time ago, and the design and implementation of a full inference engine, complete with forward- and backward-chaining, has been undertaken by a doctoral student. This hopefully will also be completed in 1992.

The system as it is currently planned will have a rather primitive user interface, and it will be used only for purposes of testing the model for its applicability in certain kinds of reasoning tasks. It will also serve as the basis for tackling future research problems, such as (i) fuzzy information storage and retrieval, (ii) integration of fuzzy rule-based reasoning with other reasoning paradigms (in particular, semantic nets and case-based reasoning), and (iii) maintaining consistency in a forward chaining systems that allow non-monotonic reasoning. Forward chaining is especially important in problems of automated control.

The programming so far has been done by essentially free labor. Only the doctoral student has been funded by the grant. A future proposal may request additional funding to speed this part of the work.