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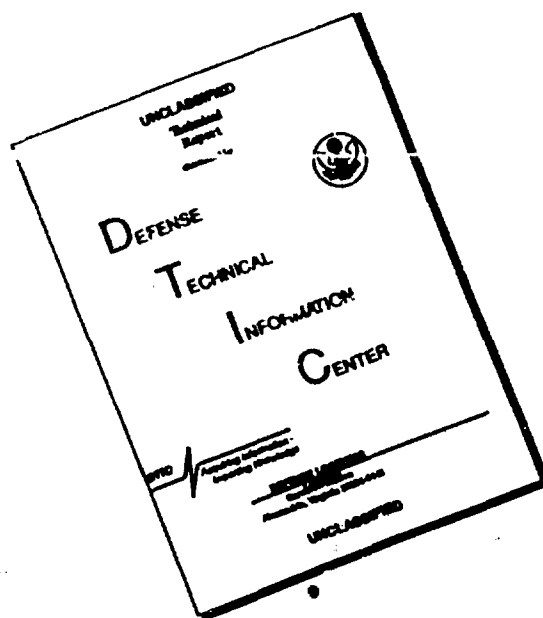
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**ANNUAL TECHNICAL REPORT  
OFFICE OF NAVAL RESEARCH**

**“ Nonlinear Problems in  
Fluid Dynamics and Inverse Scattering”**

**PROPOSAL # N00014-91-J-4037**

**M.J. Ablowitz, G. Beylkin, D.P. Sather**

**Program in Applied Mathematics  
University of Colorado—Boulder  
Boulder, CO 80309-0526**

**for period  
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## NONLINEAR PROBLEMS IN FLUID DYNAMICS AND INVERSE SCATTERING

### ABSTRACT

The inverse scattering of a class of differential-difference equations and multidimensional operators has been constructed. Solutions of nonlinear wave equations arising in fluid dynamics and plasma physics were found and analyzed. Novel and fast computational algorithms were developed and in relevant nonlinear problems wavelet bases with controlled localization properties in the time-frequency domain have proven to be an effective tool. Landau type amplitude equations governing the small gap Taylor problem have been obtained and analyzed. Detailed project summaries and research activities are given in the attached document. A list of published papers, preprints, and invited lectures are included.

# **Inverse Scattering and Nonlinear Waves**

## **M.J. Ablowitz**

### **Abstract**

Research investigations in the study of inverse scattering and nonlinear waves continues with a number of significant results obtained. Solutions and properties of physically significant multidimensional nonlinear wave equations and associated inverse scattering problems have been obtained and analyzed. Novel computational methods to solve nonlinear systems have been developed and in certain cases underlying numerical chaos has been identified and described analytically. An important new class of applicable multidimensional nonlinear equations whose solutions become unbounded in finite time has been analyzed.

### **Overview**

The past year has been a very active period for our research program. A total of eight papers have been published, one book which summarizes the recent developments in the field has been printed by Cambridge University Press and eight preprints have either been accepted or have been submitted for publication. Our preprints are also issued as reports of the Program in Applied Mathematics at the University of Colorado before they are published. An overview and brief description of our research investigations is given below. Complete details may be found in our research papers. A list of recent reprints and preprints is compiled.

### **Research Activities**

#### **Multidimensional Nonlinear Wave Equations and Inverse Scattering**

Our investigations of physically interesting nonlinear wave equations continues. The deep connection between integrable nonlinear wave equations and inverse scattering allows us to analyze and solve nonlinear systems which otherwise would be intractable. One example of a particularly natural and significant system is the two space-one time extension of the well-known Toda lattice. Using inverse scattering methods we have been able to solve this system in a variety of cases. In the case where the linearized system is analogous to an elliptic ODE we can formulate and solve the relevant boundary value problem. In one case we have discovered a nonlinear analogue of the Sommerfeld radiation condition (frequently required in linear problems) which must be imposed in order to obtain a unique solution. It is necessary to impose such a radiation condition in a much wider class of problems.

Reductions of certain physically interesting multidimensional systems has led us to study a class of nonlinear ODE's which have novel features. The prototypical situation is characterized by a system of three coupled ODE's first discovered by Halphen in 1881. We transform

this system to a third order scalar equation studied by Chazy in 1910. We have shown that Chazy's classical solution can in fact be expressed in terms of automorphic functions, and in particular well-known modular forms. We have also found generalizations of this system for which the solution can be expressed in terms of more general automorphic functions. As such these ODE's provide an important new class of "special functions" arising in integrable systems which are sure to be heavily studied by researchers in this field.

### **Computational Algorithms Instabilities and Numerical Chaos**

We continue our studies which have already led to a number of valuable discoveries. We have shown that different numerical discretizations of integrable nonlinear wave equations can lead to dramatically different results, even though the discretizations differ slightly. For example modifications of how one discretizes the nonlinear terms in the system can lead to joint spatial and temporal numerical chaos. Based upon extensive numerical experiments we have identified two excellent numerical schemes. One of them is a differential-difference equation suggested by inverse scattering analysis. The other is the well-known Fourier split step algorithm. The differential-difference algorithm is easy to implement, is effective and is considerably faster than Fourier algorithms. Nevertheless, even with the "best" schemes available we find that associated with a typical class of initial conditions, temporal chaos can be excited by roundoff errors. In fact the underlying solutions are linearly unstable in this regime of parameter space and this causes miniscule errors to grow rapidly. The errors eventually become as large as the main wave even though the schemes preserve the conserved quantities quite well. These ideas are being examined in the context of many other physically interesting nonlinear equations.

### **Focusing Singularities in Multidimensional Nonlinear Wave Equations**

We have demonstrated that a certain class of multidimensional extensions of the well-known Korteweg-deVries equations, often referred to as higher nonlinear Kadomtsev-Petviashvili (KP) equations, have solutions which will become infinite in finite time. This phenomenon is sometimes referred to as "wave collapse." We have examined the behavior of the generalized KP system both analytically and numerically, and have shown that above a certain threshold for the power of the relevant nonlinear term there can be a focusing singularity; e.g. this occurs for the so called modified KP equation which arises in plasma physics. This system is yet another well-known nonlinear wave equation which possesses blow up singularities. Other multidimensional nonlinear wave equations which have this property include the two dimensional nonlinear Schrödinger equation and the Davey-Stewartson system. Each of these equations have been extensively studied by many researchers in a variety of fields of application including nonlinear optics, fluid dynamics, and plasma physics.

## BOOKS

1. Solitons, Nonlinear Evolution Equations and Inverse Scattering, M.J. Ablowitz and P.A. Clarkson, London Mathematical Society Lecture Notes Series #149, Cambridge University Press, New York, N.Y., 1992.

## PUBLICATIONS

1. One Dimensional Reductions of Self-Dual Yang-Mills Fields and Classical Equations, S. Chakravarty, M.J. Ablowitz and P.A. Clarkson, Recent Advances in Relativity- Essays in Honor of Ted Newman, Ed. Allan I. Janis and John R. Porter (1992), 60-71, Parker and Birkhauser Press.
2. On Reductions of Self-Dual Yang Mills Equations, S. Chakravarty and M.J. Ablowitz, Painlevé Transcendents, Ed. D. Levi and P. Winternitz, 331-343, (1992), Plenum Press, New York.
3. On the Method of Solution to the  $2 + 1$  Toda Equation, J. Villarroel and M.J. Ablowitz, *Phys. Lett. A* **163**, (1992), 293-296.
4. A New Hamiltonian Amplitude Equation Governing Modulated Wave Instabilities, M. Wadati, H. Segur and M.J. Ablowitz, *Journal of the Physical Society of Japan* **61** (1992), 1187-1193.
5. Self Dual Yang-Mills Equation and New Special Functions in Integrable Systems, S. Chakravarty, M.J. Ablowitz and L.A. Takhtajan, Nonlinear Equations and Dynamical Systems, 3-11, Eds. M. Boiti, L. Martina and F. Pempinelli, World Scientific 1992.
6. Numerical Homoclinic Instabilities in the Sine-Gordon Equation, B.M. Herbst and M.J. Ablowitz, *Quaestiones Math.* **15**, 1992, 345-363.
7. Stable, Multi-State, Time-Reversible Cellular Automata with Rich Particle Content, M.J. Ablowitz, J.M. Keiser and L.A. Takhtajan, *Quaestiones Math.* **15**, 1992, 325-343.
8. Numerical Chaos, Symplectic Integrators, and Exponentially Small Splitting Distances, B.M. Herbst and M.J. Ablowitz, *Journal of Computational Physics* **105**, 1993, 122-132.

## PREPRINTS

1. On the Inverse Scattering Transform of the  $2+1$  Toda Equation, J. Villarroel and M.J. Ablowitz, PAM<sup>†</sup>#123 (April 1992), accepted *Physica D*.
2. Soliton Eigenvalue Decomposition Using Frequency Domain Data, A.F. Benner, M.J. Ablowitz and J.R. Sauer, PAM<sup>†</sup>#128 (May 1992).
3. Interaction Effects on Wavelength Multiplexed Soliton Data Packets, A.F. Benner, J.R. Sauer and M.J. Ablowitz, PAM<sup>†</sup>#129 (May 1992).
4. A Self-Dual Yang-Mills Hierarchy and its Reductions to Integrable Systems in  $1 + 1$  and  $2 + 1$  Dimensions, M.J. Ablowitz, S. Chakravarty and L.A. Takhtajan, PAM<sup>†</sup>#132 (August 1992), to appear *Commun. Math. Phys.*
5. Rapidly Forced Burgers Equation, M.J. Ablowitz and S. DeLillo, PAM<sup>†</sup>#136 (October 1992).
6. On the Elliptic  $2 + 1$  Toda Equation, J. Villarroel and M.J. Ablowitz, PAM<sup>†</sup>#140 (November 1992).

7. Wave Collapse and Instability of Solitary Waves of a Generalized Nonlinear Kadomtsev-Petviashvili Equation, X.P. Wang, M.J. Ablowitz, and H. Segur, PAM<sup>†</sup> #146 (March 1993).
8. Numerical Chaos, Roundoff Errors and Unstable Wave Processes, M.J. Ablowitz, C. Schober, B.M. Herbst, PAM<sup>†</sup> #147 (March 1993).
9. Parametric Forcing, Bound States and Solutions of a Nonlinear Schrödinger Type Equation, M.J. Ablowitz and S. De Lillo, PAM<sup>†</sup> #157 (May 1993).

PAM<sup>†</sup> : Program in Applied Mathematics report.

#### INVITED LECTURES

1. University of Kansas, Mathematics Department, "Chaos in the Nonlinear Schrodinger Equation: Real or Fake," Jan. 30, 1992; "On Reductions of the Self-Dual Yang-Mills Equations," Jan. 31, 1992.
2. Carlton University, Department of Mathematics, Ottawa, Canada, "Solitons, Computation and Chaos," April 11, 1992.
3. Kansas State University, Midwest Geometry Conference, "Reductions of the Self-Dual Yang-Mills Equations and New Integrable Systems," May 1-3, 1992.
4. SIAM Forum on Applied Mathematics, Moderator of Panel: "Training in Applied Mathematics/Preparation for Academic Careers," May 15-17, 1992.
5. Canadian Applied Math. Society, University of Alberta, Edmonton, Canada, Conference on Wave Phenomena, "Nonlinear Waves, Integrable Systems and Modular Forms," June 15-18, 1992.
6. NATO Advanced Research Workshop, Exeter University, Exeter U.K.; "Chaos in the Nonlinear Schrodinger Equation—Can it be Avoided?," July 14-18, 1992. University of Minnesota, "Reductions of the Self Dual Yang-Mills System and Modular Forms", October 8-9, 1992.
7. SUNY Stony Brook, Department of Mathematics, "Nonlinear Waves, Integrable Systems and a Novel Class of ODE's", December 3, 1992.
8. Challenge of Teaching Science and Mathematics, "Coherence and Chaos", Graduate Teacher Program, University of Colorado at Boulder, January 11, 1993.
9. Colorado School of Mines, Golden, Colorado, Sigma Xi lecture, "Solitons and Coherent Structures, March 30, 1993.
10. International Conference on Inverse Scattering, Bonn, Germany, "Inverse Scattering and Nonlinear Wave Equations,  $1 + 1$ ,  $2 + 1$  and Higher Dimensions", May 17-20, 1993.
11. NATO Lectures, Bilkent University, Department of Mathematics and Faculty of Science, Ankara, Turkey, "Coherence and Chaos—Illustrative Case Studies"; "Reductions of Self Dual Yang Mills and New Special Functions in Integrable Systems", May 20-25, 1993.

# **Propagation and capturing of singularities in problems of fluid dynamics and inverse scattering**

## **Gregory Beylkin**

### **Abstract**

The study and development of the fast adaptive algorithms for solving nonlinear equations using orthonormal bases with controlled localization in the time-frequency domain (e.g. wavelets) has been successful in establishing an approach for the use of the wavelet bases in numerical computations. This approach is novel and involves the use of sparse exponentials for the evolution in time as well as two key algorithms for adaptive differentiation and multiplication in the wavelets bases. Several additional algorithms relevant to solving the nonlinear equations and capturing the singularities (shocks) are at various stages of development. An overview and a brief description of the research effort is given below. The details may be found in the research reprints and preprints compiled for this report.

### **Research Activities**

During this year the research and development of numerical algorithms for solving nonlinear equations grew into a sizable effort. I work now with two graduate students, James Keiser and Robert Cramer. With Robert Cramer we are working on the multidimensional algorithms for the fast evaluation of a class of integral operators on functions. Such algorithm will be useful in evaluating integral operators which appear in fluid dynamics (e.g. Biot-Savart law relating vorticity and velocity of the fluid). With James Keiser we are developing adaptive algorithms for the evolution of nonlinear equations. So far we have worked on equations in one spatial dimension in order to test our approach and evaluate its advantages/disadvantages in comparison with other methods. We pay special attention to ensure that the algorithms we are developing generalize to multiple dimensions in an effective manner. In addition to this effort, several algorithms were developed (jointly with M.E. Brewster) which use wavelet bases on the interval, thus allowing us to incorporate boundary conditions properly.

### **Numerical Solution of Nonlinear Equations in Wavelet Bases**

On one hand, solving partial differential equations in wavelet bases has some features similar to those of pseudo-spectral methods, multigrid methods and adaptive grid methods. On the other hand, solving in wavelet bases has many new features that permit fast and accurate numerical algorithms. We have developed several algorithms which allow us to solve a class of nonlinear partial differential equations.

A fast adaptive algorithm for computing the pointwise product of functions represented in wavelet bases has been developed. The number of operations is proportional to the

complexity of the representation of the function in a wavelet basis, e.g. for smooth functions with a finite number of singularities the algorithm requires  $O(\log N)$  operations, where  $\log N$  is the number of dyadic scales. The constants in the complexity estimate are reasonable and the algorithm is easily extendable to the multidimensional case.

A fast adaptive algorithm for computing the derivatives in wavelet bases using the non-standard form has been developed. This algorithm is applicable to a class of operators which includes some singular integral operators such as the Hilbert Transform.

For evolution in time, we compute the exponential of an operator (e.g. of the second derivative operator) in wavelet bases. Such exponentials are sparse in wavelet bases and result in an unconditionally stable schemes. The accuracy of this approach is superior to other time stepping schemes since those attempt to approximate the exponential.

So far we used these algorithms to solve Burger's equation. We plan to apply this approach to several nonlinear equations in order to evaluate the performance of our method.

### **Fast Algorithms for Evaluation of Multidimensional Integral Operators**

We are implementing a fast algorithm for evaluation of a class of multidimensional integral operators on functions, where special care has been taken to assure small constants in the complexity estimates. We expect to compare the speed of the new algorithm with that of the Fast Multipole Method.

### **Two-point Boundary Value Problems**

A hybrid method for solving a two-point boundary value problem has been developed. The problem is formulated using ordinary finite difference approximation and then is solved in a wavelet basis. The main tool in this approach is the diagonal preconditioning available for the periodized differential operator in the wavelet bases. As a result, the fundamental solution of the two-point boundary value problem for the elliptic differential operators has been computed in  $O(N)$  operations.

Availability of the fundamental solution as a sparse  $O(N)$  matrix allows us to take a different approach to the implicit schemes for solving evolution equations (e.g. Crank-Nicolson) by inverting the operator (matrix) and, thus, making them explicit. We are looking at several applications to take advantage of such a procedure.

### **Wavelet Bases on the Interval, Multiresolution Numerical Algorithms and Homogenization**

We are starting to apply the wavelet bases on the interval as a means to solve the boundary value problems. We expect to construct high-order multiresolution methods for evolution in time. We are also looking into the extension of functions from the interval and splitting/patching of functions defined on an interval with possible signal processing

applications.

Jointly with Mary E. Brewster, we have constructed a numerical scheme for homogenization of linear differential and difference equations via multiresolution analysis. Though this work is in its beginning, it has a potential for a significant impact by permitting us to construct the equations for computing the projection on a sparse scale of the solution of equations given on the very fine scales. Our homogenization method may be used as part of a multiresolution strategy to obtain the Green's function. In this approach the Green's function is constructed on successively finer scales. Combining the multiresolution strategy with wavelet bases on the interval results in a new class of high-order methods for evolution equations.

## BOOKS

1. Co-Editor, *Wavelets and Their Applications*, Jones and Bartlett, 1992. ISBN 0-86720-225-4

## PUBLICATIONS

1. "Wavelet-Like Bases for the Fast Solution of Second-Kind Integral Equation", B. Alpert, G. Beylkin, R.R. Coifman and V. Rokhlin, *SIAM Journal on Scientific and Statistical Computing*, v. 14, (1), pp. 159-184, 1993
2. "On the representation of operators in bases of compactly supported wavelet", G. Beylkin, *SIAM Journal on Numerical Analysis*, 29, (6), pp. 1716-1740, 1992.
3. "Wavelets in Numerical Analysis", G. Beylkin, R.R. Coifman and V. Rokhlin, in *Wavelets and Their Applications*, Jones and Bartlett, 1992.
4. "Fast wavelet transforms and numerical algorithms I.", G. Beylkin, R.R. Coifman and V. Rokhlin, *Communication on Pure and Applied Mathematics* 44, 141-183, 1991.

## PREPRINTS

1. "On the fast algorithm for multiplication of functions in the wavelet bases", to appear in *Proceedings of the conference on application of wavelets in Toulouse*, June 1992. PAM<sup>†</sup> #152, August 1992.
2. "On wavelet-based algorithms for solving differential equations", Chapter in the book to be published by CRC Press. PAM<sup>†</sup> #153, Dec. 1992.
3. "Inversion and Application of the Generalized Radon Transform", to appear in *Proceedings of Conference "75 years of Radon Transform"*, Vienna, Austria, Aug. 30 - Sept. 5, 1992. PAM<sup>†</sup> #154, Jan. 1993.
4. "Multiresolution representations using the auto-correlation functions of compactly supported wavelets", (with N. Saito), submitted to *IEEE Transactions on Signal Processing*.
5. "Wavelets and Fast Numerical Algorithms", Short Course, AMC-93, April 1993, preprint.

PAM<sup>†</sup> : Program in Applied Mathematics report.

## INVITED LECTURES

1. ICASE/LaRC Short Course, 4 lectures, "Wavelets and Numerical Algorithms", Feb. 22-26, 1993.
2. AMS Short Course, "Wavelets and Fast Numerical Algorithms", January 1993.
3. Meeting of SPIE, "Wavelets, their autocorrelation functions, and multiresolution representation of signals", Boston, Nov. 15- Nov. 20, 1992.
4. Fourth Annual Symposium on Frontiers of Science, National Academy Of Science, "Wavelets and Fast Numerical Algorithms", Newport, CA Nov. 5-7, 1992.
5. Conference "75 years of Radon Transform", "Inversion and Applications of the Generalized Radon Transform", Vienna, Austria, 1992.

6. "On wavelet-based algorithms for solving differential equations", AMS Conference on Wavelets, June 30, 1992.
7. International Conference Wavelets and Applications, "On the fast algorithm for multiplication of functions in the wavelet bases", Toulouse, France, 1992.
8. Workshop at AFIT, "Multiresolution representations using the auto-correlation functions of compactly supported wavelets", March 1992.
9. AAAS Meeting, "Wavelets in Numerical Analysis", Chicago, Feb. 7, 1992.
10. Carl Heiland Lecture, "Wavelets and their applications", Jan. 27, 1992, CSM.
11. ICIAM Meeting, "Representations of Differential Operators and Fast Numerical Algorithms", Washington D.C., July 1991.
12. Fermilab, "Applications of Wavelets", March 6, 1991.

# Langmuir Circulation and Spiral Flows

## Duane P. Sather

### I. Summary of Research Activities.

(1) Landau type amplitude equations for the small-gap Taylor problem were derived and analyzed in [2] (see the list of publications in II). The equations obtained are global and are more complete than those determined by a formal two-timing analysis. Transition solutions (i.e., heteroclinic orbits) connecting the trivial Couette flow with bifurcating steady flows were obtained by solving singular evolution equations in infinite-dimensional spaces. The results obtained lead to the first analytic description of the difference between primary and secondary flows of a viscous fluid and provide a more detailed description of the transition solutions than that obtained by the use of formal two-timing methods or center manifold theory.

The work in (2)–(4) is joint with George H. Knightly of the University of Massachusetts.

(2) The bifurcation and stability properties of spiral flows for rotating plane Couette flow problems were obtained. These are the first complete results on the bifurcation and stability of viscous spiral flows. The final version of these results is published in [3].

(3) Periodic waves were shown to exist and their expansions obtained in [1;4] for various Couette–Poiseuille problems for viscous spiral flows. The existence of periodic waves in such problems has been conjectured but never proved.

(4) A continuum of periodic waves bifurcating supercritically from the basic spiral flow was obtained in [5] for rotating plane Couette flow. Results of this type provide a simple explanation for the occurrence of turbulent-like flows in spiral flow problems and also the first analytic description of such turbulent-like flows. The usual Hopf bifurcation theory does not apply to such problems and new perturbation methods using singular operators were developed.

(5) The basic equations for Langmuir circulations in upper-ocean mixing problems when the Stokes drift has a cross-wind component were developed in [6]; the set of partial differential equations are derived from a rational model that describes the development of mean currents and Langmuir circulations as a single system driven by a prescribed wind stress and a prescribed surface wave field. In physical situations where there is no cross-wind component of the Stokes drift and one seeks only roll-like solutions independent of the wind direction, the equations in [6] reduce to those derived previously by other investigators. The equations in [6] are derived by the use of multiple time scales related to the interaction of surface waves, wind and currents. The results in [6] provide the only rational theoretical explanation to date of observed phenomena in Langmuir circulations such as cross-drift of Langmuir cells and the rapid breakdown of Langmuir cells under a change in wind direction.

(6) A mathematical analysis of the equations derived in [6] is developed in [7]. The usual methods of Hopf bifurcation provide only partial results here and new perturbation methods related to those in (4) using singular operators were developed to treat Langmuir circulation problems. Such methods provide a more detailed description of the resultant periodic waves than those obtained by the use of formal two-timing methods or center manifold theory. Other problems for Langmuir circulations involving phenomena such as changes in the number of cells and patterns other than rolls also can be treated using the basic model equations derived in [6]. Such problems will be considered in a series of papers in preparation.

## II. Publications and Papers in Preparation.

The following papers were completed and accepted for publication during the period of the Progress Report. Preprints and reprints have been forwarded to ONR.

- [1] "Time-periodic states in problems containing a structure parameter," (with George H. Knightly), Progress in Partial Differential Equations (J. Bandle, et al., eds.), Pitman Research Notes in Mathematics Series, John Wiley & Sons, Inc., New York, 1992, pp. 34-92.
- [2] "Transition solutions in the Taylor problem," Arch. Rational Mech. Anal. 121 (1992), 267-301.
- [3] "Periodic waves in rotating plane Couette flow," (with George H. Knightly), Z. Angew. Math. Phys. 44, 1-16.
- [4] "Symmetry in rotating plane Couette-Poiseuille flow," (with George H. Knightly), Lectures in Applied Mathematics, American Mathematical Society, accepted for publication, refereed article, 11 pages.

The following papers were completed and submitted for publication during the period of the Progress Report. Preprints have been forwarded to ONR.

- [5] "Continua of periodic waves in rotating plane Couette flow," (with George H. Knightly), J. Diff. Eqns., submitted for publication, 34 page manuscript.
- [6] "Langmuir circulations when the Stokes drift has a cross-wind component," (with George H. Knightly), J. Fluid Mech., submitted for publication, 26 page manuscript.

The following paper is in the final stages of preparation. A preprint will be forwarded to ONR when it is available.

- [7] "Periodic traveling waves in Langmuir circulations," (with George H. Knightly), to be submitted to Arch. Rational. Mech. Anal.

### III. Invited Symposia Lectures.

A \* denotes invitations accepted

- (1) Applied Dynamics and Bifurcation Theory, Oberwolfach, Germany, January 12-18, 1992.
- (2) First World Congress of Nonlinear Analysis, Florida Inst. of Technology, Melbourne, August 19-26, 1992.
- (3)\* Summer Seminar in Applied Mathematics, "Exploiting Symmetry in Applied and Numerical Analysis," sponsored by AMS-SIAM, Fort Collins, July 26-August 1, 1992.
- (4)\* Chapman Conference on "Double-Diffusive Convection," sponsored by the American Geophysical Union, Phoenix, November 3-6, 1993.