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THESIS

A CRITICAL ANALYSIS OF A RANGE DEPENDENT
PROBABILITY OF KILL FOR THE NATO SEASPARROW
MISSILE SYSTEM

by

Lt. Curt William Steigers
March 1993

Thesis Advisor

Prof. W. Max Woods

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A CRITICAL ANALYSIS OF A RANGE DEPENDENT PROBABILITY OF
KILL FOR THE NATO SEASPARROW MISSILE SYSTEM

by

Curt William Steigers
Lieutenant, United States Navy
B.S. Naval Architecture, U.S. Naval Academy, 1985

Submitted in partial fulfillment
of the requirements for the degree of

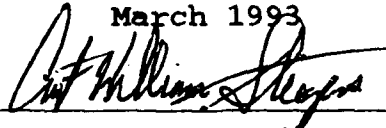
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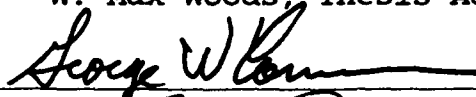


Curt William Steigers

Approved by:



W. Max Woods, Thesis Advisor



George Conner, Second Reader



Peter Purdue, Chairman

Department of Operations Research

ABSTRACT

This thesis provides a method to utilize existing digital simulations and missile reliability data to determine the probability of kill (Pk) versus range for the Seasparrow for each system state. A procedure is presented that uses the Pk versus range curve and the firing doctrine to determine the probability of defeating an antiship cruise missile attack for input into a baseline system effectiveness model. A method to optimize the firing doctrine using linear programming techniques is presented. Implemented using the General Algebraic Modeling System (GAMS), the optimized firing doctrine provides a significant enhancement to the probability of defeating an ASCM attack. The thesis enhances the Baseline Systems Effectiveness model for the NATO Seasparrow Missile System (NSSMS) developed by Professor W.M. Woods of the Naval Postgraduate School for the Naval Warfare Assessment Center, Corona, California.

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I INTRODUCTION

The NATO Seasparrow Missile System (NSSMS) is the primary point defense system for the United States Navy. Like many modern weapon systems, the Seasparrow is designed with a variety of alternate operating modes for use in the event of component failures. Although this is desirable from an operational standpoint, it creates difficulties in assessing the utility of the system. The Navy has traditionally tracked system availability by recording what percentage of the time a system is fully operational, usually referred to as A_0 . This method has several drawbacks when applied to the NSSMS. The major drawback is the lack of accounting for degraded modes of operations. Additionally, it provides no measure of the systems capability, only reliability and maintainability. To address these concerns, system effectiveness models were developed. In essence, a system effectiveness model sums, for all operating states, the product of the expected proportion of time a system will be in a given state (system state availability), the probability the system transitions to a different state during an engagement, and the capability of the system for the system state in which the system is operating at the time of missile launch. This paper provides

an analytical framework to determine the probability of the NSSMS defeating an antiship cruise missile (ASCM) attack, for input into a NSSMS system effectiveness model developed for the Naval Warfare Assessment Center, Corona, CA (NWAC). This probability is one factor in the NSSMS system effectiveness equations.

A. PURPOSE

A method is developed to determine the probability of the NSSMS defeating an antiship cruise missile (ASCM) attack based on the operating state of the NSSMS at the time the missiles are launched. The thesis will explore the use of existing digital simulations developed by the Naval Air Warfare Center, China Lake, CA, (NAWC) to develop an estimate of the single shot probability of kill (P_k) for the Seasparrow when the NSSMS is in system state S_i at the time of launch. This estimate will be combined with missile reliability data and used in conjunction with the firing doctrine to evaluate the overall probability of kill (P_k) for the NSSMS against a representative subsonic ASCM. Finally, a method to use linear programming techniques to optimize the firing doctrine versus a single target will be examined.

II. BACKGROUND

In an effort to improve the way in which the reliability of the NATO Seasparrow Missile System (NSSMS) is tracked, the Naval Warfare Assessment Center (NWAC) funded Professor W.M. Woods of the Naval Postgraduate School to develop a baseline systems effectiveness model for the system [Ref. 1]. The model he developed is based on methodology developed by the Weapon Systems Effectiveness Industrial Advisory Committee [Ref. 2]. The baseline model recognizes seven system states, S_1 to S_7 , and is written as

$$SE = \sum_{i=1}^7 \sum_{j=1}^7 AS_i DS_{i,j} CS_j \quad (2.1)$$

where AS_i is the operational availability of system state S_i , $DS_{i,j}$ is the probability the system transitions from state S_i to state S_j during an engagement, and CS_j is the capability of the system when in state S_j . This paper provides a method to determine the value of CS_j using digital simulation and missile reliability data.

CS_j is defined as the capability of the system to defeat the chosen threat given eight missiles are loaded at the beginning of the scenario [Ref. 1]. In the baseline model, CS_j is defined as the probability of success given that two missiles are fired at the inbound target. This is written as

$$CS_j = P_L(1) Pk_j(1) + P_L(2) Pk_j(2) \quad (2.2)$$

where $P_L(n)$ is the probability of successfully firing n missiles and $Pk_j(n)$ is the probability of successfully destroying the target given that n missiles were fired in system state S_j . It should be noted that equation (2.2) incorporates the missile reliability term, R_m , in the original CS_j formula into the $Pk_j(n)$ term for brevity. The formulation and calculation of CS_j will be further discussed in Chapters IV and V.

A. SYSTEM CONFIGURATION

The NSSMS comes in two basic configurations: one fire control director or two fire control directors. Each director has its own transmitter, receiver, signal data processor (SDP), and radar system controller (RSC). An eight cell box launcher is used in both configurations for launching the missiles. For purposes of systems effectiveness, if a two director configured ship has a casualty in only one director, the system is considered to be fully functional.

B. SYSTEM STATES

The baseline system effectiveness model for the NSSMS recognizes seven different system states. Although there are more than seven possible degradations to the system, only seven were considered to occur with sufficient frequency, or

have a sufficient impact on probability of success, to justify inclusion in the model. The system states and their associated equipment casualties are listed in Table 1 [Ref. 1,p. 6]:

Table 1: NSSMS OPERATING STATES

System State	Operating Mode	Equipment Casualty
S ₁	Normal	All up
S ₂	Normal	All up except fault code 100: Transmitter power output degraded.
S ₃	Normal	All up except fault code 106: Signal Data Processor range signal degraded.
S ₄	Normal	All up except fault code 112: Range deviation too large.
S ₅	Normal	All up except FOC/RSC which has only control power sufficient to assign, tune, and fire missile in a normal manner.
S ₆	SI (Slaved Illuminator) or LLLTV	All up except SDP or receiver.
S ₇	Computer complex Casualty (CCC)	All up except digital computer or signal data converter (SDC).

C. MISSILE COMMANDS

The RIM-7 Seasparrow missile is a derivative of the AIM-7 Sparrow Air-to-Air missile. It has been optimized for the point defense role for use against antiship cruise missiles. The missile employs semi-active radar homing and pre-launch missile commands cannot be updated in flight. Degraded missile commands are the primary effect of equipment degradations in system states S₃, S₄, S₅, and S₆. The seven missile commands are

listed below with one system command, launcher super-elevation, which is also affected by system degradations that may affect missile performance.

- a. Seeker Antenna (Head: Aim(pitch and yaw))
- b. English Bias
- c. Sweep Select
- d. Sweep Control
- e. Video Inject
- f. Range at Launch
- g. Channel Designate/HOH
- h. Launcher Super-Elevation

Reference 3 provides a complete discussion of these missile commands. In the event of a system casualty that prevents the fire control system from providing inputs to the missile, state S_6 for example, default values are utilized. Of these commands, Range at Launch is evaluated as having the greatest effect on missile performance.

D. DIGITAL SIMULATIONS

Due to the limited number of actual firings, it is necessary to conduct simulations to determine the missiles probability of success. The NATO Seasparrow program office (NSPO), maintains a network of simulations which are available throughout the Seasparrow technical community [Ref. 4]. The

Navy organization responsible for the Seasparrow simulations is the Naval Air Warfare Center, China Lake, CA (NAWC). NAWC maintains two Seasparrow flight simulations that offer sufficiently accurate modelling to give valid results: a six degree of freedom digital simulation and a six degree of freedom, hardware-in-the-loop simulation. Due to the order of magnitude different in cost between the digital simulation and the hardware-in-the-loop simulation, the digital simulation is the preferred choice. In addition to the flight simulation, NAWC maintains several endgame lethality simulations that will be utilized, including a target detecting device (TDD) simulation, a warhead fragmentation pattern simulation, and a target vulnerability model.

The four simulations are used sequentially to determine the probability of success for each simulation trial. The flight simulation is used to determine the flight path of the missile from launch to intercept. The last few seconds of the missile and target flight paths are then entered into the TDD simulation to calculate the warhead detonation point. After determining the warhead detonation point, the relative positions and velocities of the missile and target are fed into the warhead simulation to determine how many fragments struck the target. The estimated fragmentation damage to the target is then compared with the target vulnerability model to determine the probability of the target being destroyed.

III. SIMULATION PLAN

Due to the limited number of actual missile firings versus ASCMs, simulations are required to determine the impact of range and system degradations on the probability of kill (Pk) for the Seasparrow in each system state. Digital simulations offer the advantage of substantially lower cost and significantly shorter turn around time, when compared to hardware-in-the-loop simulations.

In any systems effectiveness model, a specific scenario is required to determine the probability of success of the system. The baseline systems effectiveness model [Ref. 1] provides three Seasparrow missiles configurations and three possible ASCMs to be evaluated. The simulation plan in this thesis is developed for the RIM-7P configuration of the Seasparrow versus the Exocet cruise missile. This selection was chosen since all elements required (flight simulation, TDD simulation, warhead simulation, and target vulnerability model) are readily available at NAWC China Lake [Ref. 5].

A. BACKGROUND

Initially, it was assumed that the probability of kill for the Seasparrow could be represented by a single value for the entire range of the missile. Conversations with the technical community [Refs. 5&6] indicated that this would be an over

simplification. This would be especially true of any degraded modes of operation for the system. It is expected that P_k will increase as the velocity of the Seasparrow increases, and decrease when the velocity of the missile decreases. This effect should be especially noticeable at or near maximum range when the missile is coasting and has very little energy available for terminal maneuvers. It is anticipated that the P_k versus range curve will look roughly as shown in Figure 1.

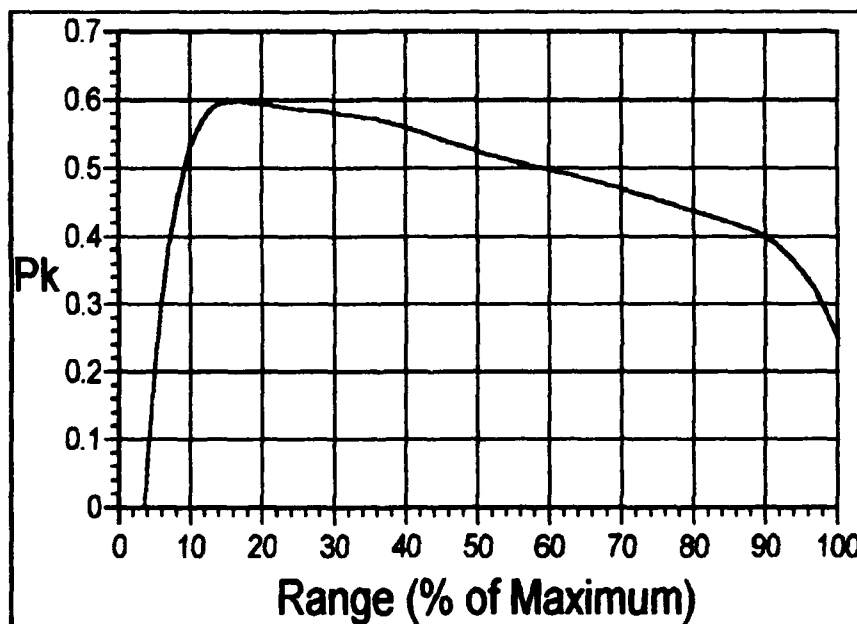


Figure 1: Expected P_k vs Range Curve

It should be noted the values for P_k indicated above are for illustrative purposes only and do not represent the actual probability of kill for the Seasparrow missile.

B. REQUIREMENTS

The simulation plan must conform to several constraints. The plan must cover the full range of the Seasparrow, have a sufficient number of test points to provide good shape definition for the PK versus Range curve, have enough trials at each point to provide a reasonable confidence interval for Pk, and meet the cost constraints discussed in Reference 1.

C. ASSUMPTIONS

As a result of limitations of the digital simulations, several assumptions were made when the simulation plan was created. First, the engagement is in an electronic countermeasures (ECM) clear environment, at least in the spectrum covered by the NSSMS director. The engagement is limited to a single inbound ASCM, and each missile is assumed to act independently from all others. Finally, the firing ship is the target for the ASCM, and the Exocet flies its optimum profile, detecting the firing ship at maximum range and commences sea skimming as soon as possible.

D. SYSTEM STATES

Due to the scenario chosen, RIM-7P versus Exocet in a ECM clear environment, the seven system states can be reduced to five states by combining states S_1 , S_5 and S_7 . Degradations to the FOC, S_5 , are only important in an ECM environment, while a computer or signal data converter failure, S_7 , only affects

the ability to acquire the target and should have negligible effect on the probability of success.

E. SIMULATION PLAN

To explore the Pk versus range dependency, thirteen intercept points were chosen, with the points spaced closest at the extremes of range. Intercepts closer than five percent of the maximum range are not evaluated as this is inside the range of the close in weapons systems. A sample size of fifty was adopted as a compromise between cost and confidence limit precision. Complete details of the Simulation Plan can be found in Appendix A.

IV. DATA ANALYSIS

Once the simulations are completed, the resultant data and the missile's reliability data needs to be analyzed and combined to determine the P_k versus range values. Once these values are determined, they can be compared to the firing doctrine to find $P_k(n)_j$, the probability of killing the target given n missiles are successfully fired in system state S_j , and $P_L(n)$, the probability of successfully firing n missiles during an engagement.

A. ESTIMATING PROBABILITY OF KILL

An estimate of the probability of kill for any single missile is given by combining the probability that the missile guides correctly once it launches, R_{MG} , the probability that the missile doesn't fail in flight, R_{MF} , and the probability of success given that first two conditions are met, P_{k_s} . The reliability information can be derived from missile flight data available from the flight analysis department at NWAC, while the probability of success is derived from the simulations.

1. Determining P_{k_s}

The fifty values at each simulation point constitute fifty independently, identically distributed (IID) estimates,

$\hat{p}_{k_{si}}$, $i=1,2,\dots,50$, of the probability of kill, p_{k_s} . By the Central Limit theorem, the mean, \bar{x} , of the fifty estimates should have a nearly normal distribution with mean p_{k_s} . If σ denotes the standard deviation of the probability distribution for p_{k_s} , then the quantity

$$\frac{(\bar{x}-p_{k_s})}{(\sigma/\sqrt{n})} \quad (4.1)$$

has a nearly standard normal distribution. Since the standard deviation, σ , is not known, the sample standard deviation, s , is substituted for σ in equation (4.1). The probability distribution of the resulting expression is approximated by a t-distribution with $n-1$ degrees of freedom. The resulting formula for a 95% lower confidence limit, $\hat{p}_{k_{s(L)}}$, for p_{k_s} is

$$\hat{p}_{k_{s(L)}} = \bar{x} - \frac{s}{\sqrt{n}} t_{(n-1),0.95}, \quad (4.2)$$

where

$$\bar{x} = \frac{1}{n} \sum_1^n \hat{p}_{k_{si}}, \quad (4.3)$$

$$s = \frac{1}{n-1} \sum_1^n (\hat{p}_{k_{si}} - \bar{x})^2, \quad (4.4)$$

and n is the number of trials (replications of simulations) and $t_{(n-1),0.95}$ is the 95th percentile of a t probability distribution with $n-1$ degrees of freedom .

2. Missile Reliability

After Pk_s is determined, it must be multiplied by the two missile reliability terms to determine the value for Pk at any intercept range. The first term, the probability that the missile guides once it is launched, R_{MG} , is simply the probability that all components of the missile function properly at launch. The second term, the probability of no in-flight failures, R_{MF} , varies with the duration of the flight [Ref. 7]. Missile reliability can be adequately represented by an exponential life distribution reliability model, where the probability of a successful flight is defined as

$$P(\text{Reliable flight}) = R_{MG}R_{MF} \quad (4.5)$$

where R_{MG} is the probability that all missile components are functioning correctly at launch and R_{MF} is the probability that a missile functions correctly until intercept. R_{MF} has an exponential life distribution.

a. Estimating R_{MG}

A point estimate, \hat{R}_{MG} , of R_{MG} the probability of the missile to correctly guide once launched, is the ratio of guidance successes divided by the total number of launches. A guidance failure is defined as a missile that launched but failed to guide properly because not all components functioned at launch. The number of successes is the number of tests

minus the number of guidance failures. A $100(1-\alpha)\%$ lower confidence limit, $\hat{R}_{MG(L)}$, for R_{MG} can be obtained by looking up the appropriate value in a table of confidence limits for binomial attribute data using existing tables such as Reference 8. Alternatively,

$$\hat{R}_{MG(L)} = \frac{n-f}{(n-f) + (f+1) F_{1-\alpha}(2(n-f+1), 2f)} \quad (4.6)$$

where n is the number of launches, f is the number of failures, and $F_{1-\alpha}$ is the $100(1-\alpha)\%$ percentile point of the F-distribution with $2(n-f+1)$ and $2f$ degrees of freedom. If RIM-7M and RIM-7P missiles have sufficient commonality of components, the number of test firings could be pooled for determining \hat{R}_{MG} and $\hat{R}_{MG(L)}$.

b. Estimating R_{MF}

The second component of missile reliability, R_{MF} , involves failures that occur in flight. Due to the limited number of failures that could be attributed to in-flight failures, as opposed to undiscovered failures that occurred prior to launch, a qualitative statement on the distribution of in-flight failures is impossible. For simplicity, the failure rate, λ , is assumed to be constant, which implies an exponential life distribution for the in-flight time to failure. The resultant term for in-flight reliability is

$$R_{MF} = e^{-\lambda t} \quad (4.7)$$

and its estimate \hat{R}_{MF} is

$$\hat{R}_{MF} = e^{-\lambda t} \quad (4.8)$$

where

$$\hat{\lambda} = \frac{f_{if}}{T_f}, \quad (4.9)$$

t is the time of flight to the intercept point, f_{if} is the number of in-flight failures, and T_f is the total flight time for all missile test firings. Again, if the RIM-7M and RIM-7P have sufficient commonality, the test results can be pooled.

3. Estimating Pk_i

The estimate of the probability of kill derived from the simulations is multiplied by the estimate of the probability that the missile guides correctly, and the estimate of the missiles in-flight reliability for each of the thirteen sample points; that is

$$\hat{P}k_i = \hat{R}_{MG} \hat{R}_{MF} \hat{P}k_s, \quad i=1,2,\dots,13 \quad (4.10)$$

where \hat{R}_{MF} is recalculated for each intercept point. The result is an estimate for Pk at each of the thirteen intercept ranges. It is then necessary to fit a curve to the points to allow estimation of Pk at any possible intercept range.

4. Curve Fitting

There are variety of commercially available graphics programs that perform curve fitting. Given the expected shape of the Pk versus range curve, a high order, seventh to tenth,

polynomial curve should best fit the data. Figure 2 below shows a ninth order polynomial curve fit generated by the commercially available program DeltaGraph Professional, but many other graphics programs support curve fitting.

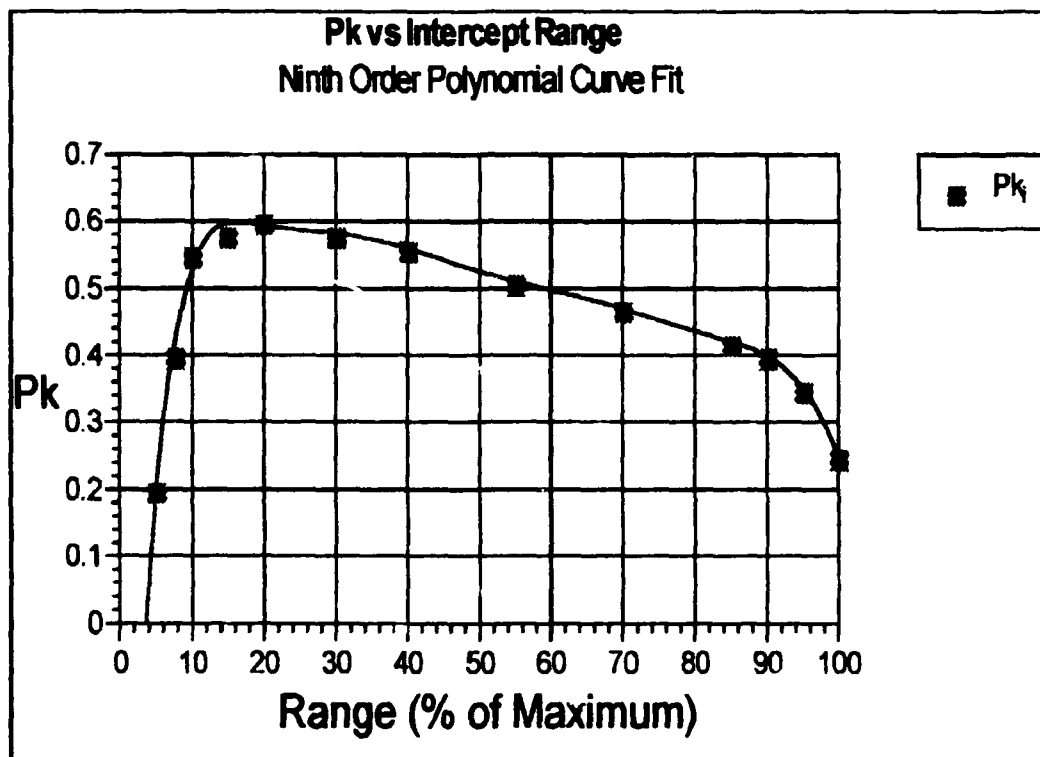


Figure 2: Sample Curve Fitting

When the best fit has been determined for the data, the formula used to generate the curve provides an estimate for P_k for any intercept range.

B. ESTIMATING R_{ML}

Once the value of P_k have been calculated for all possible intercept ranges, the next component required to calculate CS_j

is $P_L(n)$, the probability of being able to launch n missiles. The factor required to calculate $P_L(n)$ is R_{ML} , the probability that the missile successfully launches. A success is defined as a missile that, after the firing signal was generated, launched. A failure is when the launch signal is generated, and the missile does not leave the launch rail. Since the NSSMS detects misfires and automatically attempts to fire the next available missile, these failures should not significantly affect the probability of kill, but do impact the probability of being able to launch a specified number of missiles.

A point estimate, \hat{R}_{ML} , of R_{ML} , the probability of the missile to correctly launch, is the ratio of the number of successful launches divided by the total number of attempted launches. A $100(1-\alpha)\%$ lower confidence limit, $\hat{R}_{ML(L)}$, for \hat{R}_{ML} can be obtained by using tables, as for $\hat{R}_{MG(L)}$. Alternatively,

$$\hat{R}_{ML(L)} = \frac{n_{SL}}{n_{SL} + (n_{AL} - n_{SL} + 1) F_{1-\alpha}(2(n_{SL}+1), 2(n_{AL}-n_{SL}))} \quad (4.11)$$

where n_{SL} is the number of successful launches, n_{AL} is the number of attempted launches, and $F_{1-\alpha}$ is the $100(1-\alpha)\%$ percentile point of the F-distribution with $2(n_{SL}+1)$ and $2(n_{AL}-n_{SL})$ degrees of freedom. As with the other reliability terms, if the RIM-7M and RIM-7P missiles have sufficient commonality of components, the number of attempted firings could be pooled for determining \hat{R}_{ML} and $\hat{R}_{ML(L)}$.

V. CAPABILITY OF SYSTEM

Once the estimates for the value for P_k at any intercept range, \hat{P}_{k_i} , and the reliability of the missile to launch, \hat{R}_{ML} , have been made, it is possible to estimate CS_j , the capability of the system in state S_j . In the baseline model CS_j was defined as the probability of success given two missiles were fired. Since the NSSMS has sufficient range to allow a shoot-shoot-look-shoot-shoot doctrine against most threats, CS_j has been redefined as the probability of success given up to four missiles are fired during an engagement. This change allows CS_j to more accurately reflect the expected employment of the system versus subsonic cruise missiles. CS_j is made up of two components, $P_k(n)_j$, the probability of success given that n missiles are fired in state S_j , and $P_L(n)$, the probability of successfully firing n missiles given eight missiles are available at the beginning of the engagement. The new equation for CS_j is written as

$$CS_j = \sum_{n=1}^4 P_L(n) P_k(n)_j, \quad (5.1)$$

where n is the number of missiles fired during the engagement.

A. ESTIMATING $P_L(n)$

To estimate the probability of firing one, two, three, or four missiles, the missile launch reliability term, \hat{R}_{ML} , is required. For the case of one, two, or three missiles fired, the binomial formula provides the probability of each occurrence.

$$\hat{P}_L(n) = \binom{8}{n} \hat{R}_{ML}^n (1 - \hat{R}_{ML})^{8-n}, \quad n \leq 3. \quad (5.2)$$

$\hat{P}_L(4)$ includes the probability that four or more missiles are able to be fired, and is written as

$$\hat{P}_L(4) = \sum_{n=4}^8 \binom{8}{n} \hat{R}_{ML}^n (1 - \hat{R}_{ML})^{8-n}. \quad (5.3)$$

To simplify the calculations, $\hat{P}_L(4)$ can be thought of as one minus the probability that n is less than four, which is written as

$$\hat{P}_L(4) = 1 - \left(\sum_{n=1}^3 \hat{P}_L(n) \right) + (1 - \hat{R}_{ML})^8. \quad (5.4)$$

Once each term for $\hat{P}_L(n)$ has been estimated, it is multiplied by the appropriate $P_k(n)_j$ term to find CS_j .

B. ESTIMATING $P_k(n)_j$

To estimate the value of $P_k(n)_j$, the firing doctrine is reviewed to determine at what range the intercepts would occur, and the appropriate values of $\hat{P}k_i$ are determined from the P_k versus range curve. The formula for $\hat{P}k(n)_j$ is then

$$\hat{P}k(n)_j = 1 - \prod_{i=1}^n (1 - (\hat{P}k_i | RI_i)) \quad (5.5)$$

with RI_i being the estimated range at intercept for missile i . Once the value of $\hat{P}k(n)_j$ has been determined for all values of n , the values can be multiplied by their respective $\hat{P}_L(n)$ terms and summed to find CS_j . When estimating $\hat{P}k(n)_j$ each missile is assumed to act independently, and there is no accumulation of damage to the target. This simplifies the calculation and provides the most conservative estimate of success.

For system state S_6 , SI/LLTV mode, on those ships that do not have a three dimensional radar to provide positioning data to the NSSMS director, replenishment ships for example, the low light level television is used for tracking, and the maximum range of detection, and thus intercept, must be shortened accordingly. Specifications for LLLTV expected detection ranges can be found in Reference 9.

C. SEARCH RADARS

The detection range plays an important role in determining the capability of the system. This is especially true for the Computer Complex Casualty mode, state S_7 , which relies on manual acquisition of the target with the NSSMS fire control radar. Adding search radar performance and acquisition times to the baseline system effectiveness model would enhance the accuracy of the baseline model, but would entail a substantial increase in the complexity of the model. Although not covered in detail, a method to incorporate search radar performance is provided as a basis for future enhancements to the baseline system effectiveness model.

1. Modifications to Baseline Model

The baseline system effectiveness model is modified to account for search radar performance and reliability by introducing an additional factor in each term of the original formula. The system effectiveness equation becomes

$$SE = \sum_{k=1}^m \sum_{j=1}^7 \sum_{i=1}^7 AC_k AS_i DS_{i,j} CS_{j,k} \quad (5.7)$$

where AC_k is the availability of each shipboard search radar configuration C_k , m is the number of possible configurations of the search radars, and $CS_{j,k}$ is the capability to defeat the threat given that the NSSMS is in state S_j , and the search

radars are in configuration C_k . The other terms remain unchanged from the baseline model.

Each search radar configuration, C_k , is determined by the search radar configuration of the ship. A Spruance class destroyer might have eight configurations as shown in table 2. A Wichita class replenishment oiler, AOR, might have only two configurations, MK23 Target Acquisition System (TAS) up and TAS down.

Table 2: DD963 SEARCH RADAR CONFIGURATIONS

C_k	Radar Status
C_1	TAS up, SPS-40 up, SPQ-9 up
C_2	TAS down, SPS-40 up, SPQ-9 up
C_3	TAS up, SPS-40 down, SPQ-9 up
C_4	TAS down, SPS-40 down, SPQ-9 up
C_5	TAS up, SPS-40 up, SPQ-9 down
C_6	TAS down, SPS-40 up, SPQ-9 down
C_7	TAS up, SPS-40 down, SPQ-9 down
C_8	TAS down, SPS-40 down, SPQ-9 down

Depending on the search radar in question, there could be several possible operating states that could effect the capability of the NSSMS to engage the target. For instance, on some ships, the TAS computer links the combat direction system and the NSSMS together [Refs. 9&10]. If the TAS computer fails, the NSSMS operators must manually acquire all targets,

increasing the acquisition time significantly. This would result in three states for TAS, all up, radar down but computer up, and all down, instead of two, and the number of configurations would increase accordingly.

The new term for the capability of the system, $CS_{j,k}$, needs to incorporate three additional components; the expected range of first detection, the acquisition time, and the expected probability of success given the target is acquired at a specific range. The addition of these terms would allow the effects of system states S_5 , FOC/RSC degraded, and S_7 , CCC mode, to be isolated from system state S_1 , all up, resulting in a better estimate of the capability of the system.

VI. OPTIMIZING FIRING DOCTRINE

Once the P_k versus range curve is known, it is possible to optimize the firing doctrine to maximize the probability of defeating the threat. Ideally, if the optimization were a linear program, one could take advantage of commercially available optimization software such as GAMS or LINDO. $P_k(n)$ is maximized by minimizing the probability of missing. The probability of missing can be written as

$$\prod_{i=1}^4 (1 - (Pk_i | RI_i)). \quad (6.1)$$

Although the probability of missing is still a non-linear equation, by taking its natural logarithm, it becomes a linear equation,

$$\sum_{i=1}^4 \ln(1 - (Pk_i | RI_i)), \quad (6.2)$$

which is readily solvable as a modified set covering problem.

A. LINEAR PROGRAM

The first requirement to allow the use of linear programming techniques is to break up the P_k vs range curve into discrete time intervals with constant P_k within the interval. The use of one second intervals based on the time of

first possible launch (maximum range intercept) until the last possible launch (minimum range intercept) results in approximately one hundred and twenty time intervals. This provides sufficient resolution to minimize the effects of discretizing the continuous data, but makes the problem small enough to be readily solvable. For each launch time interval, the appropriate intercept range as the percentage of maximum range is entered into the formula selected for the curve fitting to return the P_k for the time interval. Solving the following formulation results in the optimum firing doctrine.

1. Index

- t : The time interval in which the missile is fired. t is indexed from 1, the maximum range intercept launch, to t_{\max} , the minimum range intercept launch.
- u : An alias for t , u offers the same index for use with matrix notation.

2. Data

- P_t : The natural log of the probability of missing ($1-P_k$) for a missile fired during time interval t .
- $C_{u,t}$: A confounding matrix, a one indicates a missile launched at time interval u will be airborne during the interval t . A zero indicate a missile launched at time interval u is not airborne during time period t . An additional five seconds is added to each time of flight to allow for operator assessment of the engagement.

3. Decision Variable

- X_t : A binary variable, a one indicates a missile should be launched during time interval t , a zero indicates no missile is to be fired during time interval t . Although X_t is a binary variable, in practice it only needs to be limited to values less than or equal to one to provide binary results.

4. Formulation

$$\text{Minimize } \sum_{t=1}^{t_{\max}} P_t X_t \quad (\text{Obj})$$

Subject to:

$$\sum_{t=1}^{t_{\max}} X_t \leq 4 \quad (1)$$

$$\sum_{t=1}^{t_{\max}} C_{u,t} X_t \leq 2, \text{ for all } u \quad (2)$$

$$X_{t-1} + X_t \leq 1, 2 \leq t \leq t_{\max} \quad (3)$$

$$X_t \leq 1, \text{ for all } t \quad (4)$$

In these equations, constraint (1) limits the number of missiles fired to a maximum of 4 per engagement, constraint (2) limits the number of missiles airborne at any time to two, constraint (3) limits the rate of fire to one missile every two seconds, and constraint (4) limits the number of missiles fired during any given time period to one.

B. RESULTS

To determine the effect of using the optimized firing doctrine, the linear program was implemented using the General Algebraic Modeling System (GAMS). To avoid using classified

data, a range of ten nautical miles (NM) and a constant velocity of 600 nautical miles per hour (kts) was assumed for the Seasparrow. The Pk versus Range curve found in figures 1 and 2 was utilized, and the Exocet was assumed to have a constant speed of 600 kts. The optimized firing doctrine returned a value for $P_k(4)_j$ of 0.967. A shoot-shoot-look-shoot-shoot doctrine with the first shots at near maximum range returns a value of 0.927, a difference of 0.04. An even bigger difference is seen when the probability of destroying the target with the first two missiles is examined. With a shoot when the target is within range doctrine, the probability of destroying the target is 0.584. The optimized doctrine increases the probability of success to 0.752. This is especially significant given the limited number of missiles available in the launcher and the extended time required to reload the launcher. In all cases, the increased Pk from the optimized doctrine results from a shorter intercept range, with a corresponding increase in single round Pk.

Although these numbers are only for illustrative purposes, they show the significant increase available from an optimized firing doctrine when compared to a doctrine that assumes a constant Pk. The optimization program ran on a 80486/33MHZ computer in just under 1.5 seconds, and can be easily modified to accommodate almost any firing doctrine by simply changing constants. The GAMS model is provided in Appendix B. Before using with actual data, the missile in-flight confounding

matrix, $C(u,t)$, needs to be updated with actual time of flight data.

VII. CONCLUSIONS AND RECOMMENDATIONS

Dr. Woods' baseline system effectiveness model for the NSSMS provides the framework to assess both the availability and capability of the system. This thesis provides an enhancement to the baseline model by providing a method to determine the probability of kill for the NATO Seasparrow missile as a function of range and system state. The Pk versus range curve can be developed using existing missile simulations and, when used in conjunction with the available missile reliability data, can provide a way to estimate the capability of the NSSMS to defeat the designated threat.

Beyond enhancing the system effectiveness model, a Pk versus range curve can provide insight into the proper tactical employment of the system. Through the application of linear programming techniques, it is possible to significantly increase the probability of success by changing the firing doctrine to maximize Pk. Additionally, the development of Pk versus range curves for degraded states of operation can provide the basis for firing strategies to minimize the effects of the degradations.

Further enhancements to the baseline system effectiveness model should concentrate on incorporating the effects of search radars and acquisition times into the model, as

outlined in chapter V. The baseline model could easily be expanded to include NSSMS, search radars, and close in weapons systems (CIWS), providing an assessment of the capability of a ship to defend itself against a chosen threat.

Modern weapon systems with several degraded modes of operation make the traditional methods of tracking systems availability less suitable. The baseline systems effectiveness model for the NSSMS, with the enhancements provided by this paper, provides the technical community with a better method of measuring the utility of the NATO Seasparrow.

APPENDIX A. SIMULATION PLAN

The following simulation plan is proposed for execution by the Naval Air Warfare Center, China Lake, California, using the six degree of freedom, digital Sparrow simulation and associated endgame lethality simulations. The procedure for requesting utilization of simulations in the NSPO performance assessment models network to conduct studies is outlined in Reference 1.

Common to all cases.

Missile: RIM-7P

Target: Exocet SSM

Target Profile: Exocet profile with firing ship as the intended target. The Exocet acquires the firing ship at maximum range and commences sea skimming at the earliest opportunity.

Simulation Points:

<u>Intercept Range (% of maximum)</u>	<u>Number of runs</u>
5.0	50
7.5	50
10.0	50
15.0	50
20.0	50
30.0	50
40.0	50
55.0	50
70.0	50
85.0	50
90.0	50
95.0	50
100.0	50

All range at launch missile command degradations are created by adjusting the actual launch range, not the range at launch commands. The range at launch command should always reflect the range required for intercept at the desired range.

Case 1: NSSMS system states S_1 : All up, S_5 : FOC/RSC Degraded, and S_7 : Computer Complex Casualty Mode.

Missile Commands: Nominal
Launcher Super-Elevation: In accordance with algorithm
Transmitter Power: Nominal

Case 2: NSSMS system state S_2 : Transmitter power degraded.

Missile Commands: Nominal
Launcher Super-Elevation: In accordance with algorithm
Transmitter Power: Degraded

Case 3: NSSMS system state S_3 : SDP range signal degraded.

Missile Commands: Range at launch deviation $\pm 6\%$ - 15% from actual, all others nominal
Launcher Super-Elevation: In accordance with algorithm
Transmitter Power: Nominal

Case 4: NSSMS system state S_4 : SDP Range deviation too large.

Missile Commands: Range at launch deviation greater than $+15\%$ from actual, all others nominal
Launcher Super-Elevation: In accordance with algorithm
Transmitter Power: Nominal

Case 5: NSSMS system state S_6 : Slaved Illuminator/Low Light Level Television mode.

Missile Commands: Default
Launcher Super-Elevation: Default
Transmitter Power: Nominal

Note: Values for missile command degradations presented here and in Reference 1, Appendix F are tentative, and will require modification following the NATO Seasparrow technical community review of Reference 1.

APPENDIX B. GAMS SOURCE CODE FOR OPTIMIZED DOCTRINE

\$TITLE Thesis Formulation
 \$STITLE Optimized Firing Doctrine
 -----GAMS AND DOLLAR CONTROL OPTIONS -----

\$OFFUPPER OFFSYMLIST OFFSYMXREF

OPTIONS

LIMCOL = 0 , LIMROW = 0 , SOLPRINT = OFF , DECIMALS = 2
 RESLIM = 100 , ITERLIM = 10000 , OPTCR = 0.00 , SEED = 3141;

-----DEFINITIONS AND DATA-----

SCALAR TM total missiles available /4/
 AM missiles airborne at one time /2/;

SETS

t launch time interval /1*115/
 t1 subset of t /1*25/
 t2 subset of t /26*50/
 t3 subset of t /51*75/
 t4 subset of t /76*100/
 t5 subset of t /101*115/;

PARAMETERS

PK1(t1) negative ln of 1-Pk
 /1 0.2881
 2 0.3173
 3 0.3445
 4 0.3697
 5 0.3927
 6 0.4137
 7 0.4326
 8 0.4495
 9 0.4646
 10 0.4779
 11 0.4897
 12 0.5001
 13 0.5093
 14 0.5173
 15 0.5245
 16 0.5310
 17 0.5368
 18 0.5422
 19 0.5472
 20 0.5520
 21 0.5566

22	0.5612
23	0.5657
24	0.5702
25	0.5748/

PK2(t2) negative ln of 1-Pk

/26	0.5794
27	0.5842
28	0.5890
29	0.5939
30	0.5989
31	0.6040
32	0.6091
33	0.6142
34	0.6194
35	0.6245
36	0.6296
37	0.6347
38	0.6397
39	0.6446
40	0.6494
41	0.6541
42	0.6588
43	0.6633
44	0.6677
45	0.6720
46	0.6763
47	0.6804
48	0.6846
49	0.6887
50	0.6928/

PK3(t3) negative ln of 1-Pk

/51	0.6969
52	0.7011
53	0.7054
54	0.7097
55	0.7142
56	0.7188
57	0.7236
58	0.7285
59	0.7336
60	0.7389
61	0.7445
62	0.7502
63	0.7560
64	0.7621
65	0.7683
66	0.7747
67	0.7811
68	0.7876
69	0.7942
70	0.8007

71	0.8072
72	0.8136
73	0.8199
74	0.8259
75	0.8317/

PK4(t4) negative ln of 1-Pk

/76	0.8373
77	0.8425
78	0.8474
79	0.8519
80	0.8560
81	0.8597
82	0.8631
83	0.8661
84	0.8688
85	0.8711
86	0.8733
87	0.8753
88	0.8771
89	0.8789
90	0.8808
91	0.8828
92	0.8850
93	0.8874
94	0.8901
95	0.8930
96	0.8961
97	0.8993
98	0.9023
99	0.9050
100	0.9070/

PK5(t5) negative ln of 1-Pk

/101	0.9076
102	0.9063
103	0.9021
104	0.8941
105	0.8812
106	0.8618
107	0.8348
108	0.7985
109	0.7518
110	0.6935
111	0.6229
112	0.5396
113	0.4439
114	0.3364
115	0.2181/;

TABLE

Cl(t,t1) missile in air confounding matrix

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25


```

107 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
108 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
109 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
110 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
111 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
112 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
113 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
114 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
115 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;

```

TABLE

C2(t,t2) missile in air confounding matrix

	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
23	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
26	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
27	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
28	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
29	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
31	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
32	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
33	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
34	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
35	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
36	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
37	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
38	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
39	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1

	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
67	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
69	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
71	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
72	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
73	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
74	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
75	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
76	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
77	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
78	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
79	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
80	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
81	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
82	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
83	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
84	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
85	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
86	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
87	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
88	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
89	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
90	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
91	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
92	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
93	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
94	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
95	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
96	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
97	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
98	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
99	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
100	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
101	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
102	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
103	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
104	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1

105	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
106	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
107	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
108	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
109	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
110	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
111	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
112	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
113	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
114	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
115	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1;

TABLE

Rl(t,t1) rate of fire limitations

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
52	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
53	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
55	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
57	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
58	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
59	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
60	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
61	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
62	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
63	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
66	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
67	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
68	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
69	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
72	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
73	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
74	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
76	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
77	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
78	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
79	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
81	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
82	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
83	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
84	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
85	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
86	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
87	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
88	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

89	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
91	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
92	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
93	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
94	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
95	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
96	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
97	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
98	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
99	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
100	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
101	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
102	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
103	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
104	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
105	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
106	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
107	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
108	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
109	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
110	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
111	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
112	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
113	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0
114	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
115	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1;

-----MODEL-----

BINARY VARIABLES

X(t) indicates if firing in time period
 X1(t1)
 X2(t2)
 X3(t3)
 X4(t4)
 X5(t5);

VARIABLE

TPK total pk;

EQUATIONS

OBJ total pk
 NMIS number of missiles available
 AMIS(t) number of missiles airborne at once
 ROF(t) rate of fire limitations;

* MAXIMIZE

OBJ.. TPK =E= SUM(t1, X1(t1)*PK1(t1))+SUM(t2, X2(t2)*PK2(t2))
+SUM(t3, X3(t3)*PK3(t3))+SUM(t4, X4(t4)*PK4(t4))
+SUM(t5, X5(t5)*PK5(t5));

* subject to

NMIS.. SUM(t1, X1(t1))+SUM(t2, X2(t2))+SUM(t3, X3(t3))
+SUM(t4, X4(t4))+SUM(t5, X5(t5)) =L= TM;

AMIS(t).. SUM(t1, C1(t,t1)*X1(t1))+SUM(t2, C2(t,t2)*X2(t2))
+SUM(t3, C3(t,t3)*X3(t3))+SUM(t4, C4(t,t4)*X4(t4))
+SUM(t5, C5(t,t5)*X5(t5)) =L= AM;

ROF(t).. SUM(t1, R1(t,t1)*X1(t1))+SUM(t2, R2(t,t2)*X2(t2))
+SUM(t3, R3(t,t3)*X3(t3))+SUM(t4, R4(t,t4)*X4(t4))
+SUM(t5, R5(t,t5)*X5(t5)) =L= 1;

MODEL MIX /ALL/

SOLVE MIX USING MIP MAXIMIZING TPK;

*-----REPORTS-----

* print the optimal objective value and solution.

DISPLAY TPK.L, X1.L, X2.L, X3.L, X4.L, X5.L;

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China Lake, CA 93555
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NATO Seasparrow Project Office
Crystal Park 4, Suite 210
Washington, DC 20362
15. LT Curt W. Steigers 1
1005 S. 234 PL.
Des Moines, WA 98198