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Research of Richard E. Barlow:
The principle of indifference has been used to analyze problems in survival analysis. In *De Finetti-Type Representations for Life Distributions*, Barlow and Mendel (1992) used this principle to derive so-called generalized gamma and generalized Weibull likelihood models with applications to survival analysis. Beginning with a finite population of units and the judgment of exchangeability for units with respect to lifetime, we argue that measures of similarity lead to the appropriate probabilistic models for aging. This in turn implies that Schur-concavity of the joint probability function (or more generally, the joint survival distribution) provides the correct probabilistic description of aging.

Research of William S. Jewell:
Recent research on systems reliability and inference has focussed on four areas: the prediction of unreported events and costs from partial information on claims with delayed reporting; models of life testing with censored or truncated incomplete data; make or buy decisions of production with random yields; and investigation of natural conjugate priors for familiar two-parameter distributions.

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RESEARCH OF RICHARD E. BARLOW

SUMMARY

We have used the principle of indifference to analyze problems in survival analysis. In *De Finetti-Type Representations for Life Distributions*, Barlow and Mendel (1992) used this principle to derive so-called generalized gamma and generalized Weibull likelihood models with applications to survival analysis. Beginning with a finite population of units and the judgment of exchangeability for units with respect to lifetime, we argue that measures of similarity lead to the appropriate probabilistic models for aging. This in turn implies that Schur-concavity of the joint probability function (or more generally, the joint survival distribution) provides the correct probabilistic description of aging. Following this argument and using the Principle of Indifference, we argue that the appropriate probability models for life distributions conditional on average life are in a family of distributions which we call the generalized gamma distributions. If, on the other hand, we are interested in probabilistic models for aging conditional on average lifetime maintenance cost, it follows from our development that generalized Weibull distributions are appropriate.

Similarity as a Probabilistic Characteristic of Aging, by Barlow and Mendel (1993), is an expository paper presenting a Bayesian approach to survival analysis and basic definitions associated with that approach. Relative to this approach, the concept of aging relative to "wear-out" is characterized for finite (as opposed to conceptually infinite) populations of units. The concept of no aging relative to finite populations is characterized by the Schur-constant joint survival distributions which are then determined up to a mixing distribution.

In *Schur-Concave Survival Functions and Survival Analysis*, Barlow and Spizzichino (1993), we consider an N-tuple of exchangeable non-negative random variables, which can e. g. be interpreted as

lifetimes of N similar units, and we assume that the joint survival function

$$\overline{F}_N(x_1, \ldots, x_N) = P\{X_1 > x_1, \ldots, X_N > x_N\}$$

is, in particular, Schur-concave. This condition is relevant since, as it was shown in the papers discussed above, it provides a probabilistic model for aging in the subjectivist set-up. In this paper we analyze general properties of Schur-concave survival functions and give representation theorems. In particular, we study properties of Schur-concave survival distributions which are a finite population version of time-transformed exponential distributions. These distribution models are of interest in analyzing life data.

Strength of Materials

Reliability theory has until very recently been mostly relevant to problems involving electrical or electronic systems as opposed to problems in materials science and mechanical engineering. Using the principle of indifference, a new methodology for deriving probability distribution functions has been developed which, for the first time, can provide a relevant statistical methodology for problems in materials science and mechanical engineering. The most important fact about these derived distributions is that they are fully grounded in well-known theoretical principles. By developing probability models based on physical principles, probability models should not only be more relevant but also statisticians should be better able to communicate with engineers and scientists.

Current probability models in common use for analyzing data relevant for research in the physical sciences (e. g. Walodie Weibull's distribution for strength of materials) lack a theoretical basis. Weibull realized, shortly after this original derivation was published, that an assumption he was required to make in his original argument destroys the theoretical basis for his derived distribution. We begin with well known deterministic theory such as the von Mises criterion used to predict the yield strength of isotropic linear elastic materials (or Hooke's law in the simplest case). Then we focus on a parameter of interest such as the distortion energy relative to the yielding of a material. Using the principle of indifference in a Bayesian context, we determine conditional probability models for relevant quantities which can be measured either experimentally or in the field. A paper in progress, *Strength of Materials and the Weibull Distribution*, by Barlow, Mendel and Lindquist details these ideas.

A. Publication in Reviewed Journals

Barlow, R. E., 1991, Computing the optimal design for a calibration experiment, **Journal of Statistical Planning and Inference**, 29, pp. 5-19

Barlow, R. E. and Pereira, C., 1991, Medical diagnosis using influence diagrams, **Networks**, 20, pp. 565-577

Barlow, R. E. and M. B. Mendel, 1992, De Finetti-type Representations or Life Distributions. **Journal of the American Statistical Association**, Vol. 87, pp. 1116-1122.

Barlow, R. E. and F. Spizzichino, 1993, Schur-concave survival functions and survival analysis, **Journal of Computational and Applied Mathematics**, 40, To appear.

B. Books or Book Chapters Published

Barlow, R. E. and C. Pereira, 1991, Conditional independence and probabilistic influence diagrams. In **Topics in Statistical Dependence**, Institute of Mathematical Statistics, Hayward California, pp. 19-33, edited by H. W. Block, A. R. Sampson and T. H. Savits.

Barlow, R. E., 1992, Introduction to de Finetti (1937) Foresight: its logical laws, its subjective sources. In **Breakthroughs in statistics, Volume I**, eds. Johnson and Kotz, pp. 127-133, Springer-Verlag, New York.

Barlow, R. E. and T. Irony, 1992, Foundations of statistical quality control. **Current Issues in Statistical Inference: essays in honor of D. Basu**, M. Ghosh, ed., IMS Lecture Notes Monograph Series (#17), Hayward California.

Barlow, R. E., T. Irony and C. A. Pereira, 1992. Bayesian Models for Quality Assurance. In **Bayesian Statistics 4**, ed. by J. M. Bernardo, J. O. Berger, A. P. Dawid and A. F. M. Smith, Oxford University Press, pp. 675-688, Oxford, England.

Barlow, R. E. and M. B. Mendel, 1993, Similarity as a Probabilistic Characteristic of Aging. To appear in **RELIABILITY AND DECISION MAKING**, edited by R. E. Barlow, C. Clarotti and F. Spizzichino, Elsevier.

RESEARCH OF WILLIAM S. JEWELL

SUMMARY

Recent research on systems reliability and inference has focussed on four areas: the prediction of unreported events and costs from partial information on claims with delayed reporting; models of life testing with censored or truncated incomplete data; make or buy decisions of production with random yields; and investigation of natural conjugate priors for familiar two-parameter distributions.

DETAILED DESCRIPTION

1) PREDICTION IN DELAYED REPORTING MODELS

An interesting partial information problem occurs when an unknown number of samples is generated from a point process and the reporting of the events is *delayed*, with both the process and delay parameters assumed to be unknown, a priori. The problem is to predict the number of *unreported* events still outstanding, based only upon the partial number of events reported during some small observation interval. This model describes, *inter alia*: the delayed reporting of effects from some catastrophe or environmental insult (Chernobyl, asbestosis, etc.); the financially important problem of estimating IBNYR (Incurred But Not Yet Reported) claims in insurance; problems in predicting with "incomplete returns" in sample surveys, voting; *etc.*

Previous research considered a basic model in which: a random Poisson number of events are generated; reporting delays are exponentially distributed; and the information reported consists of exact measurements of various combinations of (i) occurence dates, and/or reporting dates for the observed events. We showed how the exact predictive distribution for the unreported event count can be obtained in a recursive form that involves the ratio of certain integrals that can be accurately approximated. The effect of different data types and the length of observation were examined in computational detail.

A second report considered the important practical model variation in which the dates are subject to quantized reporting during some common accounting interval. The main emphasis was to compare quantized reporting with continuous data, especially the loss of prediction accuracy with increasingly longer reporting cycles. A third report investigated the effect of including *cohort data* from *several* exposure intervals (the so-called *IBNR Triangle* data); this model is of great practical interest, particularly where quantized counts are reported.

Insurance claims reporting also includes *costs* along with the event/delay data. In recent research, the concepts and models necessary to extend the Bayesian approach to predict the *ultimate total costs* of an IBNR claim (including both the further development of reported claims, as well as the costs of IBNYR claims) were developed. This framework was then presented to a 1982 actuarial conference (see below), together with a survey of the previous work on event/delay prediction. A qualilitative version of this talk appeared in the conference proceedings (Jewell, 1992), and it is planned to develop these ideas further for scientific publication.

2) PREDICTION IN LIFE TESTING WITH CENSORED INCOMPLETE DATA

Incompete information also arises in life testing applications; for example, the standard experimental setup for unreliable components is the *censored* test, terminated at some fixed time T with certain of the components having failed but others still functioning. Accurate estimation and prediction use not only completed failure lifetimes, but also the number of remaining components still *on test*. For exponential lifetimes, this leads to the classic "total-time-on-test" statistic and estimator. The effect of censored samples with other lifetime densities and models is analytically difficult because of the presence of the survival distribution in the likelihood, but research shows that a Gammoid approximation is useful for developing good predictive formulae.

Truncated tests are also of interest in other applications. For example, in the pricing of *stop-loss* coverage in reinsurance, only the the *excess losses* above a certain, usually high, limit are insured by the primary carrier. Because of sampling costs, values falling below this limit are often not reported, leaving actuaries in the unenviable position of estimating the tail of a distribution only from the observed largest samples, with no information about the bulk of the distribution ! This appplication was described in a previous report.

Our recent research has focussed on understanding *why* the censored test of exponential lifetimes gives an exact credibility-type prediction formula (*i.e.*, the predictive mean is linear in the familiar total-time-on-test statistic) and all other cases are so difficult. For arbitrary lifetimes, one can see that a natural conjugate prior must include a term with the survival distribution, plus the usual terms conjugate to the density. For the exponential family of lifetime distributions, we have been able to show that certain *combinations* of predictions of *mean life and mean excess life* are linear in the data (*i.e.*, are of credibility form). In this context, it follows that *any* (exponential-type) lifetime density in which the excess mean life is a certain multiple of the mean life also has the simplicity of the pure exponential! This work is currently being written up (Jewell, 1993a).

3) PREDICTING PRODUCTION OUTPUT WITH RANDOM YIELDS

With the assistance of Dr. Candace Yano, we have investigated the following random yield problem: suppose a machine can only produce a perfect part at known cost with *unknown yield probability*, π ; this difficult-to-produce part can also be purchased from an outside vendor at a much higher cost. Thus we have a "make or buy" decision under uncertainty. What sequential strategy of production attempts followed by "give up" outside sourcing minimizes the total expected cost of producing N perfect items in T trials, taking into account *learning* about the yield? This problem is easy to formulate and compute as a dynamic program; recently we have been successful in characterizing the optimal strategy in analytic form. (Jewell,1993b). Further elaboration is underway.

4) NATURAL CONJUGATE PRIORS FOR TWO-PARAMETER FAMILIES

Recently, the author and Margaret Lin have been exploring the two-dimensional behaviour of natural conjugate priors for familiar two-parameter distributions. The easiest example is the normal with unknown mean and precision, whose prior is well-known and well-behaved. In analyzing two lesser-known papers that defined various "Gamcon" priors for the gamma model with unknown shape and scale parameters, we discovered some very interesting graphical and analytic properties of these priors, including hyperparameters that must be eliminated if certain prior beliefs are held (Jewell & Lin, 1993). This work was presented at a recent international research meeting. Our investigations into other "interesting" two-parameter families, including models for which sufficient statistics of one or both parameters do not exist.

SCIENTIFIC PAPERS PRESENTED

(Jewell, 1992) was presented at the Casualty Loss Reserve Seminar of the American Academy of Actuaries, Denver, Colorado, September, 1992, and was published in the Seminar Proceedings.

(Jewell & Lin, 1993) was presented at the International Meeting on Risk Theory, sponsored by the Federal Institute of Technology, Zürich, Switzerland, and held in Ascona, Switzerland in March, 1993.

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Margaret Lin's thesis, "Bayesian Estimation and Prediction in a Model of Survey Response" was accepted by the IEOR Department, U.C., Berkeley, and she was awarded the Ph.D. degree in May, 1993.

PUBLICATIONS SUPPORTED BY AFOSR-90-0087

Jewell, W.S. (1992). Loss Emergence: Predicting IBNYR and IBNFR Delays, Events, and Costs. Seminar Proceedings.

Jewell, W.S. (1993a). Natural Conjugate Priors and Credibility for Censored Lifetimes. In preparation.

Jewell, W.S. (1993b). How to Make or Buy N Perfect Parts with Uncertain Yields. In preparation.

Jewell, W.S. and M. Lin (1993). Properties of Gamcon Priors for Gamma Models with Unknown Scale and Shape Parameters. Submitted for publication.

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William S. Jewell