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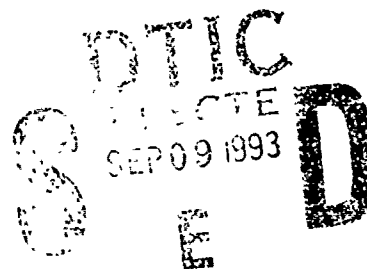


User-Defined Subroutine for Implentation of Higher-Order Shell Element in ABAQUS

Erik Saether and Alexander Tessler

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13. ABSTRACT (Maximum 200 words) A user-defined subroutine has been developed to implement a higher-order three-node facet shell finite element in the commercial finite element program ABAQUS. The element, designated HOT3, is based on a {1,2}-order plate theory specifically formulated to model thin and thick composite laminates and exhibits improved accuracy compared to conventional shear-deformable shell finite elements. This report describes the use of HOT3 in ABAQUS for static and vibration analyses by presenting the required element input data together with numerical examples. Sample input and output datasets and the complete FORTRAN subroutine which performs all computations for the HOT3 element are included.				
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Introduction

Research at Army Research Laboratory-Watertown has led to the development of a higher-order plate theory [1,2] which represents the state of the art in modeling thick composites using an equivalent single-layer type formulation. The theory has been incorporated into a 3-node anisoparametric facet shell finite element designated HOT3. The HOT3 element is a versatile displacement-based finite element incorporating higher-order kinematic expansions for use within the conventional 'h-version' finite element framework where nodal degrees of freedom are restricted to displacements and rotations. The higher-order field representation in HOT3 provides greater accuracy in the thick regime when compared to Reissner-Mindlin shear deformable elements with virtually the same computational cost.

This report outlines the use of the HOT3 element in ABAQUS via a user-defined element subroutine. A discussion of the HOT3 element and ABAQUS support of user-defined elements is presented in the following sections followed by two numerical examples illustrating the use of HOT3 in ABAQUS for static and dynamic analysis. Sample input and output datasets and the complete FORTRAN source code supporting the HOT3 element in ABAQUS are contained in separate appendices.

HOT3 Shell Element

HOT3 has been formulated as a triangular shell element incorporating transverse shear and transverse normal deformation modes. As shown in Figure 1, HOT3 has three vertex nodes with all 6 degrees of freedom defined together with an internal node which represents two constant element degrees of freedom, w_1 and w_2 , which represent the higher-order transverse displacement variables. In elastostatic analysis these two element degrees of freedom are statically condensed out at the element level in the computation of the element stiffness matrix and the internal node is not defined by the user. In dynamic analysis, however, the inertial contributions of these higher-order functions are not condensed out and an internal node must be defined and assigned two degrees of freedom. The location of this node is arbitrary and is used to represent the stiffness and mass associated with the higher-order element variables which are internally stored as the first two degrees of freedom. The remaining four degrees of freedom at the interior node - which physically do not exist - must therefore be restrained through single-point boundary constraints when using HOT3 in a dynamic analysis. Complete details of the element formulation may be found in [2,3,4].

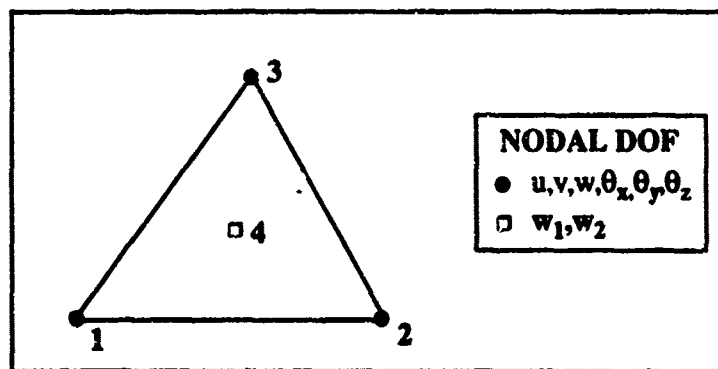


Figure 1. HOT3 : A facet shell element incorporating transverse shear and transverse normal deformation modes.

HOT3 may be defined arbitrarily in space using a counterclockwise node numbering convention. The element coordinate system (x', y', z') is shown in Figure 2. The x' axis is defined along the element side from node 1 to node 2, with the y' axis perpendicular to x' and lying in the element plane. The transverse axis z' is defined as normal to the element plane. A local coordinate system (x'', y'', z'') is also defined with the origin at node 1 and coordinate axes aligned with the global coordinate system (x, y, z). In static analysis, element output can be selected in either the local or element coordinate system.

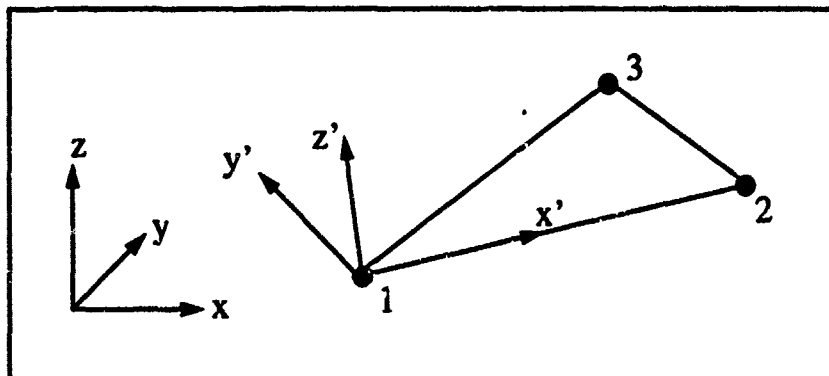


Figure 2. HOT3 element coordinate system.

For complicated geometries where the material axes do not correspond to the global coordinate system, a material coordinate system may be defined for each HOT3 element by specifying the longitudinal ($M1$) and transverse ($M2$) material axes. The normal material axis ($M3$) is assumed perpendicular to the plane formed by $M1$ and $M2$. The material coordinate system is centered at the origin of the element local coordinate system and the vector components of the material axes are computed in the local system. This is depicted in figure 3.

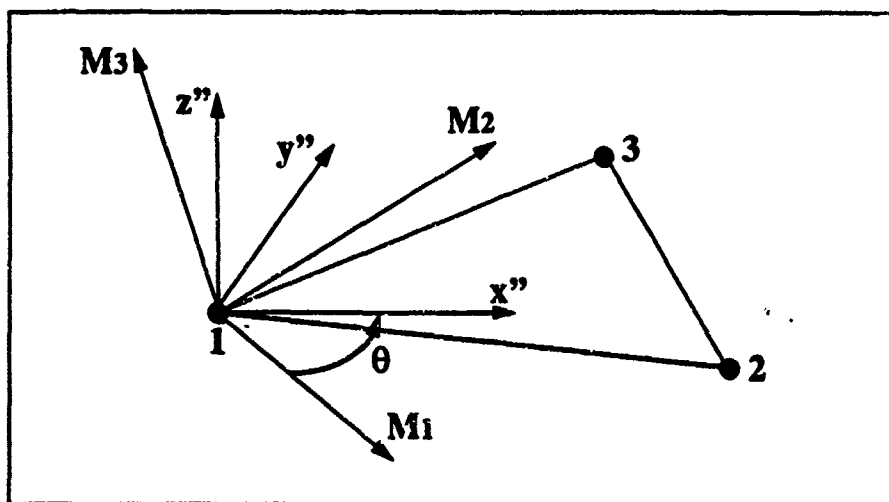


Figure 3. Specification of material axes in HOT3.

User-Defined Elements in ABAQUS

New finite elements may be implemented in ABAQUS by developing a subroutine denoted UEL (for User Element) which performs the necessary element computations and interfaces with the main ABAQUS program through a standardized parameter list in the subroutine call statement.

The *USER SUBROUTINE statement in the input deck alerts ABAQUS to the presence of user-defined subroutines which either immediately follow this data entry or are contained in a separate file. These subroutines are then compiled and linked to the main ABAQUS executable prior to job execution. A complete description of this and other user-defined capabilities in ABAQUS may be found in reference [5].

Shown in Figure 4 is the basic format of the UEL subroutine with the argument list used by ABAQUS to pass into the user-defined subroutine all the necessary information needed to compute element stiffness and mass matrices. Once computed, these matrices are then passed back to ABAQUS for global assembly and problem solution. In static analysis, data recovery is performed during a second pass through the user-defined subroutine after the global solution has been obtained. During this phase, ABAQUS passes in the nodal displacements for the current element from which all element stresses, strains and forces can be computed. In dynamic analysis, frequencies and mode shapes are computed within ABAQUS; however, individual eigenvectors are not passed back into the user-defined subroutine and, hence, modal stresses and strains are not available. It should be noted that, in addition to linear static and dynamic analysis, the information passed into the UEL subroutine is sufficient to support material and geometric nonlinear analysis.

The complete source code for the HOT3 element supporting linear static and dynamic analysis in ABAQUS is listed in appendix A.

```
SUBROUTINE UEL ( RHS, AMATRX, SVARS, ENERGY, NDOFEL, NRHS
1      NSVARS, PROPS, NPROPS, COORDS, NCRD, NNODE,
2      U, DU, V, A, JTYPE, TIME, DTIME, KSTEP, KINC,
3      JELEM, PARAMS, MDLOAD, JDLTYP, ADLMAG
4      PREDEF, NPREDF, LFLAGS, MLVARX, DDLMAG,
5      MDLOAD, PNEWDT )
C
C      REAL DOUBLE PRECISION ( A-H, O-Z )
C
C      DIMENSION RHS(MLVARX,*),AMATRX(NDOFEL,NDOFEL), SVARS(1),
1      ENERGY(8),PROPS(*), COORDS(MCRD,NNODE),
2      U(NDOFEL), DU(NDOFEL), V(NDOFEL), A(NDOFEL),
3      TIME(2), PARAMS(*), JDLTYP(MDLOAD,*), LFLAGS(5),
4      ADLMAG(MDLOAD,*), DDLMAG(MDLOAD,*),
5      PREDEF(2,NPREDF,NNODE)

      * Source code for the HOT3 element *

RETURN
END
```

Figure 4. Format of the user-defined subroutine UEL supporting the HOT3 element in ABAQUS.

Use of HOT3 in ABAQUS

Three sets of statements are used to describe HOT3 in the ABAQUS input deck. Each of these statement sets may be identified in the sample input files presented in Appendices B and C. The first set, *USER ELEMENT, defines the basic parameters of the HOT3 element. All parameters are mandatory and the user must set the N and M values in the NODES and PROPERTIES parameters as described below.

Statement set I:

- (i) *USER ELEMENT, NODES=N, TYPE=U1, PROPERTIES=M, COORDINATES=6
- (ii) 1,2,3,4,5,6

where the various input parameters are:

- Card (i): NODES=3 specifies that only 3 corner nodes are being defined for static analysis. NODES=4 specifies that an additional internal node is being defined for dynamic analysis.
TYPE=U1 specifies the internal designation of the HOT3 element in the user-defined subroutine as U1.
PROPERTIES=M specifies that a user-defined property list of length M is to be established for each HOT3 element as explained below.
COORDINATES=6 indicates that cartesian coordinates are assumed for the element together with possible specification of direction cosines for a normal vector to the element plane at each node.
- Card (ii): Specifies that all 6 degrees of freedom are defined as active at each HOT3 node. If an internal node is defined, degrees of freedom 3 through 6 must be constrained at this node.

The second entry, *UEL PROPERTY, is the primary list of input data used to compute element quantities for each HOT3 element. The size of this list is determined by the user as a function of the total number of layers in the element. Only a single layer would be specified for a homogeneous plate whereas, for a composite, the number of layers would correspond to the number of plies in the laminate. ABAQUS requires for each line in the property list that all quantities be written as real numbers in free format with eight entries per line - missing entries are simply treated as zeros. In the format of the property list shown below, the total length of the property list is calculated as $8*(2*NLAY+3)$. This length is entered as a parameter on the *USER ELEMENT entry.

Statement set II:

- (i) *UEL PROPERTY, ELSET=NM
- (ii) NLAY, RHO, L1, L2, L3, OUTPUT, NPTS
- (iii) E1, E2, E3, G12, G23, G31
- (iv) NU12, NU23, NU31, THICK, THETA
- (v) S1, S2, S3, D1, D2, D3
- (vi) NSOL, NSYS, M1x, M1y, M1z, M2x, M2y, M2z

where the input parameters are defined as:

- Card (i): NM is the set Id of HOT3 elements for which the following properties are to be used.
- Card (ii): NLAY is the number of layers in the element.
RHO is the material density.
L1-L3 are the nodal magnitudes of linearly distributed loads over the element in the order in which the element nodes are specified on the *ELEMENT input data entry.
OUTPUT is an output control flag for HOT3 element data;
 OUTPUT=0 for suppression of all HOT3 element output.
 OUTPUT=1 for stresses, strains and displacements through the element thickness, grid point forces and element strain energy.
 OUTPUT=2 for element stiffness matrix and consistent load vector only.
 OUTPUT=3 for all of the above element data
NPTS is the number of equally spaced recovery points through the thickness.
- Card (iii): E1-G31 are the layer normal and shear moduli.
- Card (iv): NU12-THETA are the layer Poisson ratios, thickness and orientation with respect to the global or material coordinate system. Note that the two material property cards, (iii) and (iv), are repeated NLAY times in order to specify material properties for each layer in the element.
- Card (v): S1-S3 and D1-D3 are the locations for stress and displacement recovery given in local triangular coordinates where the vertices correspond to the node order specified on the *ELEMENT bulk data entry.
- Card (vi): NSOL = 1 or 2 for static and dynamic analysis respectively.
NSYS = 1 for element output in the global coordinate system
 = 2 for element output in the element coordinate system.
M1x,M1y,M1z = vector components defining the longitudinal material axis
M2x,M2y,M2z = vector components defining the transverse material axis.

The last entry is the *USER SUBROUTINE statement. As stated above, this alerts ABAQUS to the presence of source code which is to be included together with the main executable code prior to running the requested job. This data statement is depicted below.

Statement set III:

(i) *USER SUBROUTINE, INPUT=uel_hot3.f

where the optional parameter INPUT specifies the name of the external file containing the source code for the HOT3 element. If this parameter is omitted, ABAQUS assumes that the source code immediately follows this statement.

Demonstration Problem I - Static Analysis

In Reference [2] the problem of cylindrical bending of a thick composite laminate has been solved using the HOT3 element. As shown in figure 5, the finite element model consists of a single strip of HOT3 elements spanning one quarter of the loading wavelength ($L/2$) in which appropriate multipoint constraints are applied to simulate the cylindrical bending features of this infinite plate. The composite selected is a carbon/epoxy symmetric angle-ply $([30/-30]_s)$ laminate subjected to a sinusoidal transverse pressure. The ply material properties are taken as

$$E_l = 25 \times 10^6 \text{ psi}, \quad E_t = 10^6 \text{ psi}$$

$$G_{lt} = .5 \times 10^6 \text{ psi}, \quad G_{tt} = .2 \times 10^6 \text{ psi}$$

$$\nu_{lt} = \nu_{tt} = .25$$

where l and t denote the longitudinal and transverse ply material directions, respectively. A sample input dataset showing the use of the various data statements defining the HOT3 element geometry, material properties, and distributed loading, together with the complete ABAQUS output are summarized in Appendix B.

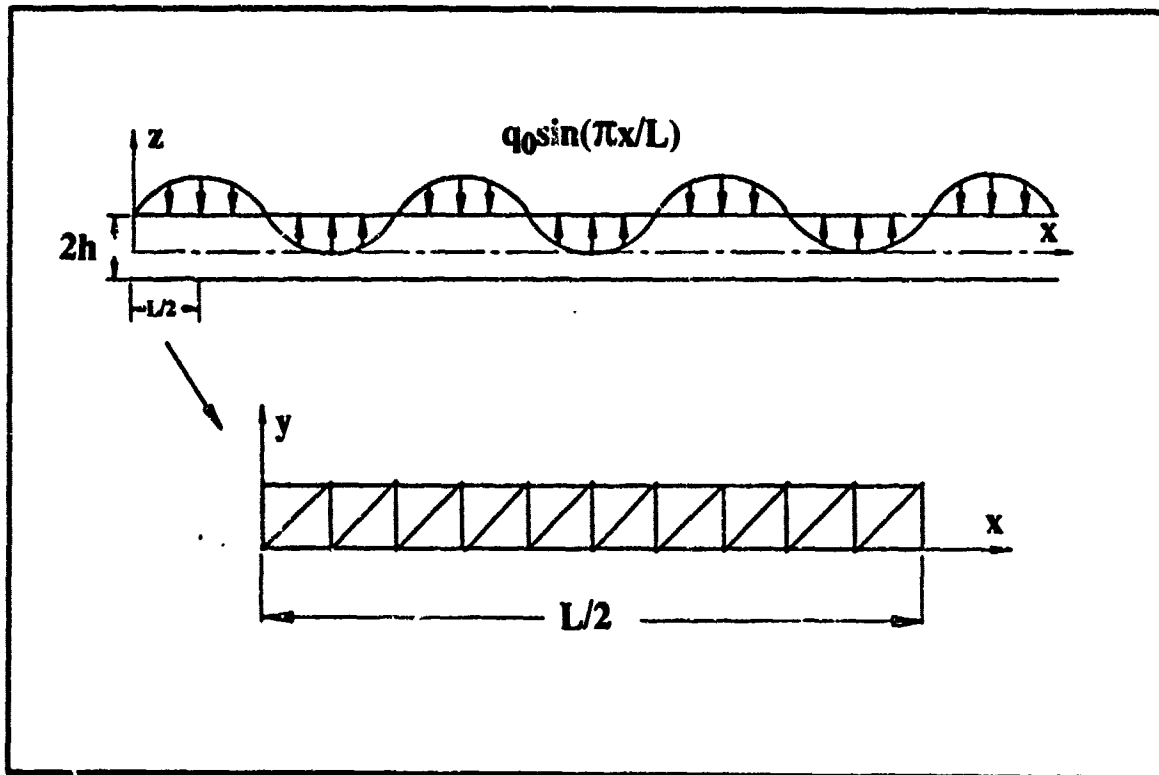


Figure 5. Finite element discretization of a carbon/epoxy $[30/-30]_s$ laminate subject to sinusoidal pressure in cylindrical bending.

Figure 6 shows the percent error of the maximum midplane transverse displacement as a function of the loading-to-thickness ratio ($L/2h$). In this and subsequent figures, results are shown variously as HOT_ANALYTIC when obtained analytically by way of the higher-order theory, as EXACT using an exact elasticity approach [6], as CPT using classical laminate plate theory, and as HOT_FEM results from the present finite element analysis using HOT3 in ABAQUS. Also, in Figure 6, results corresponding to standard shear-deformable plate theory are presented and labeled as SHEAR_DEF. All displacement and stress quantities have been normalized in accordance with [2].

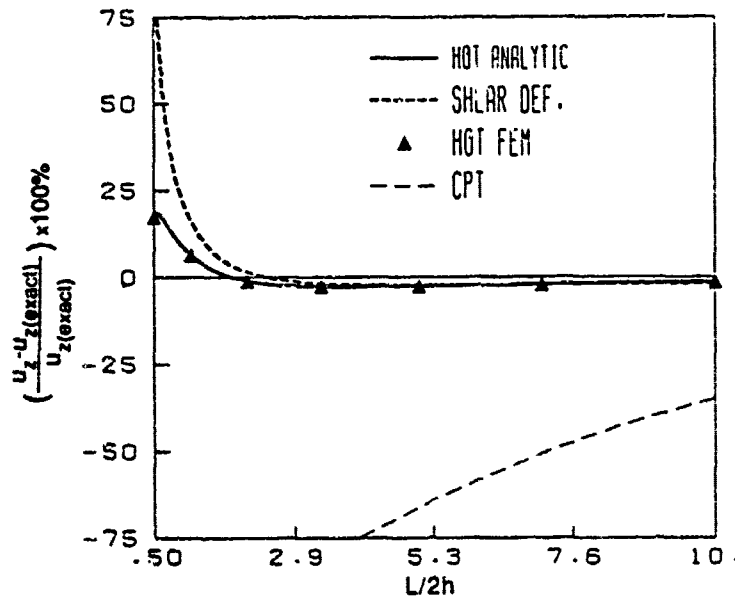


Figure 6. Percent error of maximum midplane deflection versus $L/2h$ ratio

Figure 7 depicts the through-thickness distributions of the maximum normal (ϵ_{xx}) and inplane shear (γ_{xy}) strains. As noted in reference [2], close agreement with the exact solution is obtained for $L/2h = 10$. In the thick regime with $L/2h = 4$ the departure from the exact solution is due to the linear approximations made for the inplane u_x and u_y displacements which, as evidenced by the elasticity solution, become increasingly nonlinear particularly in the outer plies.

Figure 8 shows the through-thickness distributions of the transverse shear (γ_{xz}) and transverse normal (ϵ_{zz}) strains. It may be noted that the transverse shear strain distribution compares favorably with the exact solution in a resultant sense; the assumed parabolic distribution in the theory cannot match the exact distribution for general laminates. The assumed cubic variation in the transverse normal strain is seen to compare satisfactorily with the exact solution even in the thick regime for the particular laminate selected.

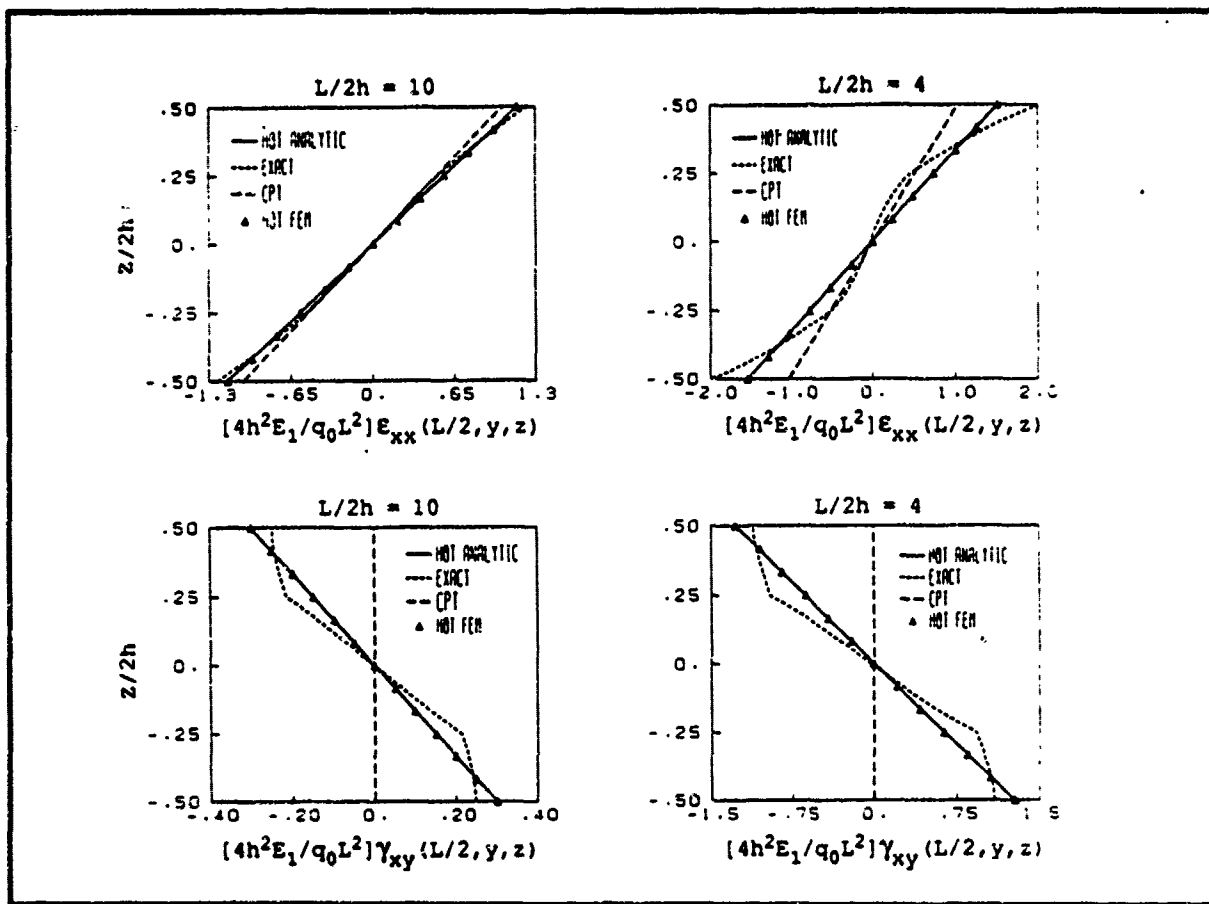


Figure 7. Distribution of inplane strains across thickness for $L/2h = 10$ and 4 .

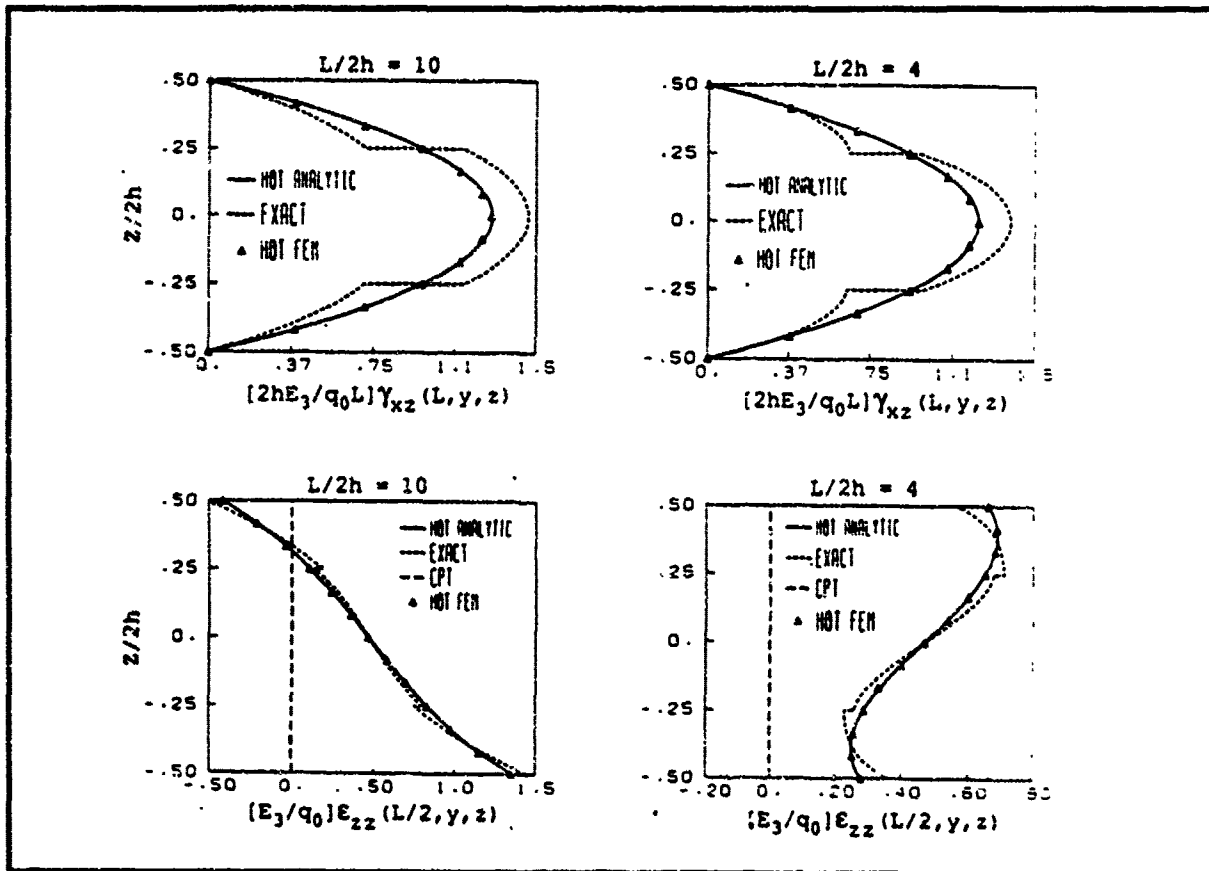


Figure 8. Distribution of transverse strains across thickness for $L/2h = 10$ and 4 .

Demonstration Problem II - Dynamic Analysis

The problem of determining the natural vibrational frequencies of a simply supported plate as shown in Figure 9 is considered. Modes associated with the free vibration of plates may be designated as symmetric or antisymmetric depending on the relative motion of the inplane displacements with respect to the midplane. Specific modes are identified as combinations of bending, stretching, and shearing motions and are ranked with an order of harmonics denoted here as modal sets (m,n) . In order to demonstrate the accuracy of the higher-order theory, analytic results from Ref. [7] for natural frequencies of orthotropic and laminated composite plates are presented followed by a discussion on the use of HOT3 with ABAQUS in dynamic analysis. Figure 10 shows the six lowest modes associated with symmetric and antisymmetric vibration corresponding to the lowest harmonic, $(m,n) = (1,1)$ which are closely correlated with exact modes.

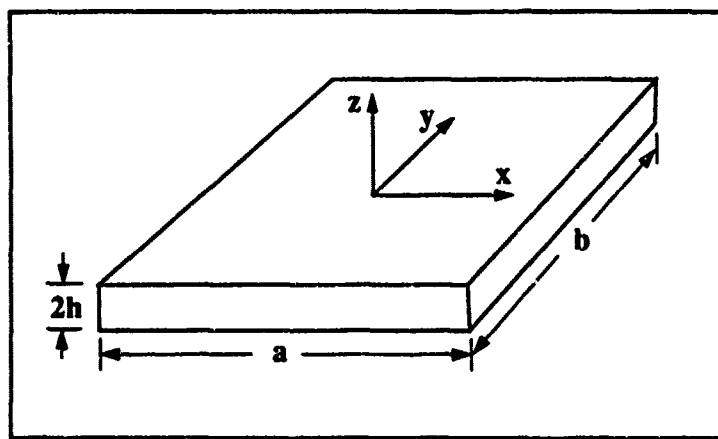


Figure 9. Simply-supported rectangular plate used in vibration analysis.

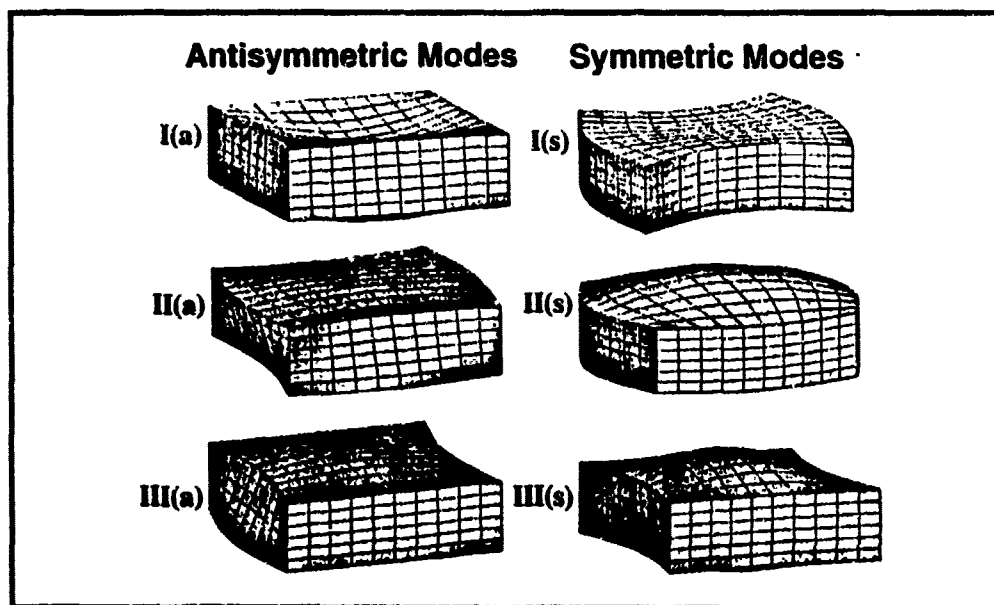


Figure 10. Mode shapes for thick, simply-supported square plate, $(m,n) = (1,1)$.

Tables 1 and 2 show normalized antisymmetric and symmetric natural frequencies of an orthotropic plate for various modal numbers. HOT3 is capable of predicting seven thickness modes - three symmetric and four antisymmetric modes. The symmetric modes are the lowest frequency inplane stretch and thickness stretch modes denoted by I(s), II(s), and III(s) respectively. The antisymmetric modes are the fundamental bending mode I(a), the two lowest frequency thickness shear modes, II(a) and III(a), and the second thickness-stretch mode V(a). For each mode, the higher-order theory results, labeled 'HOT', are compared with exact solutions derived from three-dimensional elasticity. The material properties used are given by the following ratios of the elastic stiffness coefficients:

$$C_{22}/C_{11} = 0.543103, \quad C_{33}/C_{11} = 0.530172, \quad C_{12}/C_{11} = 0.233190$$

$$C_{13}/C_{11} = 0.010776, \quad C_{23}/C_{11} = 0.098276, \quad C_{44}/C_{11} = 0.266810$$

$$C_{55}/C_{11} = 0.159914, \quad C_{66}/C_{11} = 0.262931.$$

Table 1. Comparison of nondimensionalized natural frequencies, $\lambda = \omega(2h)(\rho/C_{11})^{1/2}$ of orthotropic square plate ($2h/L=0.1$) for antisymmetric modes I(a), II(a), III(a), V(a).

Wave Number		Mode I(a)		Mode II(a)		Mode III(a)		Mode V(a)	
m	n	Exact	HOT	Exact	HOT	Exact	HOT	Exact	HOT
1	1	0.0474	0.0474	1.3077	1.3078	1.6530	1.6530	4.5625	4.5763
3	1	0.2180	0.2173	1.5777	1.5780	1.7334	1.7336	4.5907	4.5790
3	3	0.3320	0.3309	1.5737	1.5737	1.9289	1.9296	4.5064	5.5874
4	1	0.3319	0.3301	1.7179	1.7179	1.8458	1.8554	4.6178	4.5813
4	2	0.3707	0.3687	1.6940	1.6940	1.9447	1.9453	4.5775	4.5845

Table 2. Comparison of nondimensionalized natural frequencies, $\lambda = \omega(2h)(\rho/C_{11})^{1/2}$ of orthotropic square plate ($2h/L=0.1$) for symmetric modes I(s), II(s), III(s).

Wave Number		Mode I(s)		Mode II(s)		Mode III(s)	
m	n	Exact	HOT	Exact	HOT	Exact	HOT
1	1	0.2170	0.2170	0.3941	0.3941	2.2722	2.2879
3	1	0.5029	0.5029	0.9731	0.9728	2.2396	2.2880
3	3	0.6504	0.6503	1.1814	1.1813	2.2273	2.2918
4	1	0.6591	0.6591	1.2795	1.2794	2.2346	2.2880
4	2	0.7028	0.7027	1.3453	1.3452	2.2319	2.2895

Tables 3, 4 and 5 compare the fundamental bending frequencies for various thickness ratios in a symmetric [0/90/90/0], unbalanced [0/90], and an antisymmetric angle-ply [45/-45] laminates. Several analytic plate theories and the exact elasticity solution when available are included for comparison. These theories are: The higher-order theory as described in Reference [7] designated 'HOT', higher-order shear-deformable theory of Reddy and Phan [8] designated 'HSDPT', first-order shear-deformable theory designated 'FSDPT', and the classical plate theory designated 'CPT'. The ply material properties are taken as:

$$E_1 = 1.0 \times 10^6 \text{ psi}, \quad E_t = 0.025 \times 10^6 \text{ psi}$$

$$G_{lt} = .0125 \times 10^6 \text{ psi}, \quad G_{tt} = .015 \times 10^6 \text{ psi}$$

$$\nu_{lt} = \nu_{tt} = .25$$

Table 3. Comparison of nondimensionalized fundamental frequencies, $\lambda = \omega(L^2/2h)(\rho/E_2)^{1/2}$, for symmetric, cross-ply, [0/90]_s, square plates.

2h/L	Exact	HOT	HSDPT	FSDPT	CPT
0.50	5.2600	5.4914	5.576	5.4926	15.830
0.25	9.2242	9.4401	9.497	9.4231	17.907
0.20	10.748	10.910	10.989	10.892	18.215
0.10	15.149	15.173	15.270	15.161	18.652
0.08	16.187	16.194	16.276	16.185	18.707
0.05	17.626	17.623	17.668	17.618	18.767
0.04	18.023	18.019	18.050	18.016	18.780
0.02	18.605	18.598	18.606	18.597	18.799
0.01	18.753	18.753	18.755	18.752	18.804

Table 4. Comparison of nondimensionalized fundamental frequencies, $\lambda = \omega(L^2/2h)(\rho/E_2)^{1/2}$, for antisymmetric, cross-ply, [0/90], square plates.

2h/L	Exact	HOT	HSDPT	FSDPT	CPT
0.50	4.7040	5.1798	5.699	5.1672	8.4987
0.25	7.3447	7.9764	8.294	7.9534	10.292
0.20	8.1859	8.7590	9.010	8.7385	10.584
0.10	10.088	10.356	10.449	10.347	11.011
0.08	10.435	10.623	10.686	10.617	11.066
0.05	10.859	10.942	10.968	10.939	11.125
0.04	10.966	11.020	11.037	11.018	11.139
0.02	11.113	11.128	11.132	11.127	11.158
0.01	11.151	11.155	11.156	11.155	11.163

Table 5. Comparison of nondimensionalized fundamental frequencies, $\lambda = \omega(L^2/2h)(\rho/E_2)^{1/2}$, for antisymmetric, angle-ply, [45/-45], square plates.

2h/L	HOT	HSDPT	FSDPT	CPT
0.50	5.4871	6.283	5.4900	6.2832
0.25	9.1439	9.759	9.1270	12.566
0.20	10.322	10.840	10.304	13.885
0.10	13.038	13.263	13.028	14.439
0.08	13.546	13.704	13.539	14.510
0.05	14.177	14.246	14.174	14.587
0.04	14.338	14.383	14.335	14.605
0.02	14.561	14.572	14.560	14.630
0.01	14.618	14.621	14.618	14.636

To demonstrate the use of HOT3 for dynamic analysis, a particular example from Table 3 for a [0/90]_s composite plate with a thickness ratio equal to 0.2 is examined. The results using HOT3 are compared with those obtained using ABAQUS STRI3 discrete Kirchhoff and S4R first-order shear-deformable elements. The thickness ratio used was selected to demonstrate element behavior in the thick regime. Figure 11 shows typical discretizations of the plate using 3-noded HOT3 elements. Figure 12 depicts convergence of the fundamental bending frequency with mesh refinement. The STRI3 element converges to a frequency approximately 15% above the exact solution showing the effect of neglecting transverse shear and normal deformation at this thickness ratio. The S4R shear-deformable element is seen to converge below the exact solution which is due to a combination of excluding transverse normal deformation and the use of an empirically set shear stiffness relaxation parameter in the element formulation. A sample ABAQUS input and output file corresponding to the 8-element model shown in Figure 11 is included in Appendix C to show the required input format and typical output.

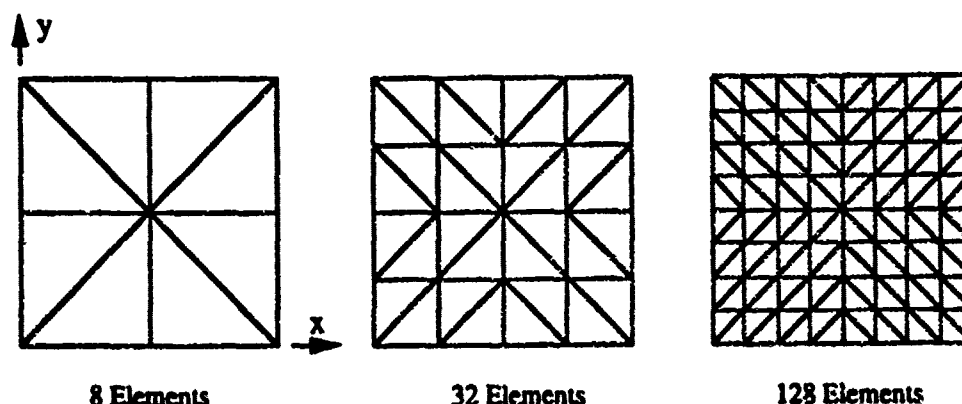


Figure 11. Typical finite element discretizations of a square plate with triangular elements.

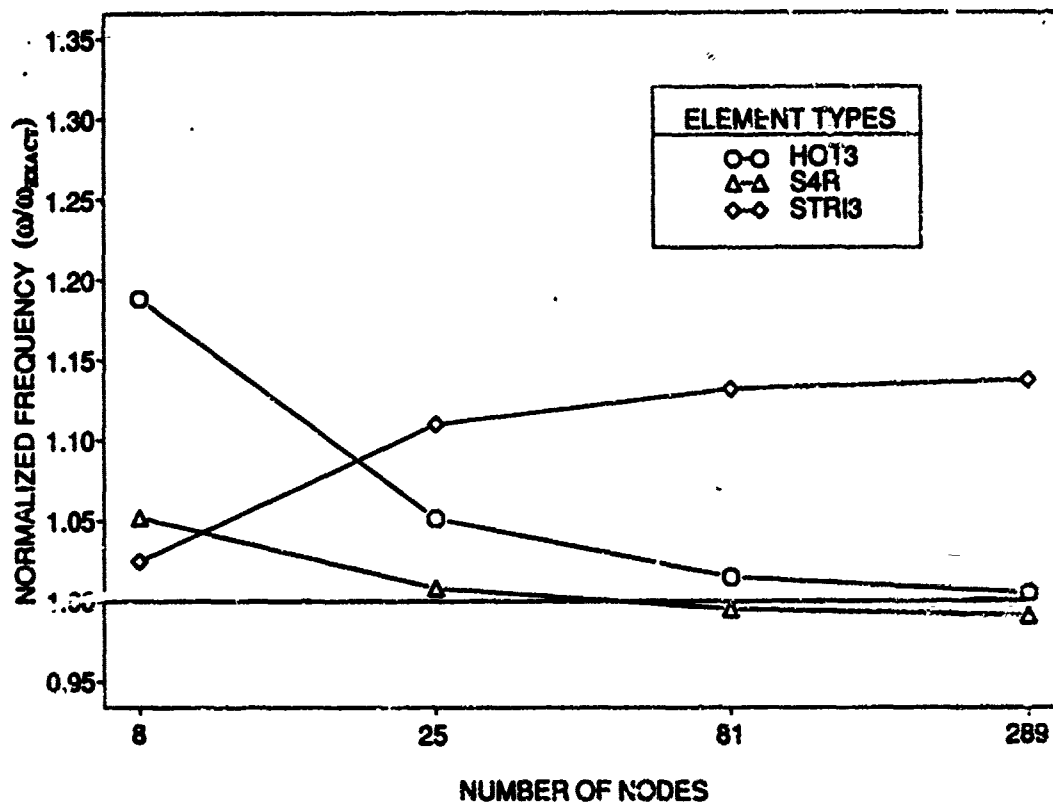


Figure 12. Convergence of the fundamental bending frequency of HOT3 , ABAQUS S4R and STRI3 elements for a square [0/90]_s laminated composite plate.

Concluding Remarks

HOT3 is a higher-order three-node facet shell element incorporating both transverse shear and transverse normal strain and stress effects. The element provides improved accuracy when compared with conventional shear-deformable finite elements with virtually the same computational cost. A user-defined subroutine has been developed allowing the user of the commercial finite element code ABAQUS to obtain the computational benefits of the HOT3 element. The complete source code which performs all computations for HOT3 in ABAQUS is contained in this report.

Acknowledgement

The authors wish to thank Colin Freese for restructuring the user-defined subroutine UEL presented in appendix A for more efficient operation within ABAQUS.

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APPENDIX A

**Source code listing of subroutine UEL
supporting the HCT3 element in
ABAQUS.**

```

C *****
C
C      SUBROUTINE UEL ( RHS,   AMATRX, SVARS, ENERGY, NDOFEL, NRHS,
&      NSVARS, PROPS,  NPROPS, COORDS, MCRD,  NNODE,
&      U,      DU,    V,    AG,    JTYPE, TIME,
&      DTIME, KSTEP, KINC,  JKLEN, PARAMS, MDLOAD,
&      JDLTYP, ADLMAG, PREDEF, NPPEDF, LFLAGS, MLVARX,
&      DDLMAG, MDLOAD, PNEWDT )
C
C      USER DEFINED ELEMENT SUBROUTINE FOR SUPPORTING THE
C      HOT3 HIGHER-ORDER TRIANGULAR SHELL ELEMENT IN THE
C      ABAQUS CODE. LINEAR STATIC AND DYNAMIC CAPABILITIES
C      ARE SUPPORTED
C
C      HOT3 SOFTWARE FLOW :
C
C      UEL                ABAQUS user element
C      UHOTTR             coordinate transformation
C      UHOTMT             hot3 material
C      UTMCMX             c-matrix
C      UMTDMX             d-matrix
C      UMTTMX             t-matrix
C      UHOTSF             hot3 stiffness
C      USFMEM             membrane stiffness
C      UBXMEM             membrane b-matrix
C      USFBND             bending stiffness
C      UBXBND             bending b-matrix
C      USFCPL             coupling stiffness
C      USXMEM             membrane b-matrix
C      UBXBND             bending b-matrix
C      USFSHR             shear stiffness
C      UINTPT             integration points
C      UBXSHR             shear b-matrix
C      UFCSHR             shear factor
C      UHOTMM             hot3 mass matrix
C      UHOTLD             hot3 load
C      UHOTCD             hot3 condensation
C      UHOTFX             additional element computation
C      UHOTIO             hot3 output
C      UHOTBX             hot3 b-matrix
C      UBXMEM             membrane b-matrix
C      UBXBND             bending b-matrix
C      UBXSHR             shear b-matrix
C
C      NOTE: ON THE *USER ELEMENT DATA ENTRY THE FOLLOWING
C      PARAMETERS VALUES ARE MANDITORY:

```

* SET NODES=3 FOR STATIC ANALYSIS OR FOR DYNAMIC ANALYSIS IN WHICH THE INERTIA ASSOCIATED WITH THE HIGHER-ORDER KINEMATIC TERMS MAY BE NEGLECTED.

* SET NODES=4 FOR DYNAMIC ANALYSIS IN WHICH THE DEGREES OF FREEDOM OF THE INTERNAL NODE ARE THE TWO HIGHER-ORDER KINEMATIC VARIABLES.

* SET COORDINATES=6 TO ALLOW FOR POSSIBLE DIRECTION COSINES DESCRIBING THE UNIT NORMAL OF THE ELEMENT.

* DEFAULT MATERIAL COORDINATE SYSTEM IS THE GLOBAL SYSTEM.

HOT3 PROPERTY LIST FORMAT:

- 1) NLAY,RHO,Q1,Q2,Q3,NOTYPE,NOUT,NMAT
- 2) E1,E2,E3,G12,G23,G31
- 3) V12,V23,V31,PLY,FIB
(REPEAT LINES 2 AND 3 FOR EACH LAYER)
- M) PSI(1),PSI(2),PSI(3),PSU(1),PSU(2),PSU(3)
- M+1) NSOL,NSYS,MVC(1),MVC(2),MVC(3)

WHERE

NLAY = NUMBER OF LAYERS IN LAMINATE

RHO = MATERIAL DENSITY

Qi = DISTRIBUTED LOAD INTENSITIES AT NODES

NOTYPE = ELEMENT OUTPUT REQUEST;

= 0 SUPPRESS ALL ELEMENT OUTPUT

= 1 OUTPUT LAYER STRESSES AT USER SELECTED NUMBER OF THICKNESS COORDINATES AND ELEMENT FORCES

= 2 OUTPUT ELEMENT STIFFNESS MATRIX AND CONSISTENT LOAD VECTOR ONLY

= 3 OUTPUT ELEMENT LAYER STRESSES, STRAINS AND INCLUDE STIFFNESS MATRIX AND LOAD VECTOR

NOUT = NUMBER OF EQUALLY SPACED OUTPUT POINTS THROUGH THE ELEMENT THICKNESS

NMAT = HIGHER-ORDER THEORY SELECTION FLAG;

= 1 FOR STRAIN-BASED

= 2 FOR STRESS-BASED (VER I)

= 3 FOR STRESS-BASED (VER II)

Ei = YOUNGS MODULI

Gij = SHEAR MODULI

Vij = POISSON RATIOS

```

C      PLY      = LAYER THICKNESS                      *
C      FIB      = LAYER ORIENTATION                    *
C      PSI(I)   = LOCATION FOR STRESS CALCULATION      *
C      PSU(I)   = LOCATION FOR DISPLACEMENT CALCULATION *
C      NSOL     = 1 FOR STATIC ANALYSIS                *
C              = 2 FOR DYNAMIC ANALYSIS                *
C              = 3 FOR DYNAMIC ANALYSIS NEGLECTING INERTIA *
C              ASSOCIATED WITH THE HIGHER-ORDER        *
C              KINEMATIC FUNCTIONS                     *
C      NSYS     = COORDINATE SYSTEM FOR ELEMENT OUTPUT  *
C              = 1 FOR GLOBAL COORDINATE SYSTEM        *
C              = 2 FOR LOCAL COORDINATE SYSTEM         *
C      MVC1(I)  = MATERIAL ORIENTATION VECTOR FOR DIRECTION 1 *
C      MVC2(I)  = MATERIAL ORIENTATION VECTOR FOR DIRECTION 2 *
C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)              *
C      PARAMETER(NPRECD=2)                             *
C
C      VARIABLE AND ARRAY DECLARATIONS FOR UEL/ABAQUS INTERFACE *
C
C      DIMENSION RHS(MLVARX,*), AMATRX(NDOFEL,NDOFEL), PROPS(*), *
C      &          SVARS(1), ENERGY(8), COORDS(MCRD,NNODE), U(NDOFEL), *
C      &          DU(MLVARX), V(NDOFEL), AG(NDOFEL), TIME(2), *
C      &          PARAMS(*), JDLTYP(MDLOAD,*), ADLMAG(MDLOAD,*), *
C      &          DDLMAG(MDLOAD,*), PREDEF(2,NPREDF,NNODE), LFLAGS(5) *
C
C      VARIABLE AND ARRAY DECLARATIONS FOR THE HOT3 ELEMENT *
C
C      DIMENSION ELTL0D(17), ELTSTF(17,17), AMASS(17,17), AMAT(4,4), *
C      &          BMAT(4,4), DMAT(4,4), GMAT(2,2), PLY(125), ZL(126), *
C      &          E1(125), E2(125), E3(125), V12(125), V23(125), *
C      &          V13(125), G12(125), G23(125), G31(125), X(3), Y(3), *
C      &          Z(3), C(6,6,125), Q(3), FIB(125), PSI(3), PSU(3), *
C      &          A(3), B(3), TRI(3,3), TRANS(24,24), RETN(24), *
C      &          PETN(24,24), STRN(6,6), ETRN(6,6), SMTN(6,6), *
C      &          EMTN(6,6), AMV1(3), AMV2(3) *
C      * * * * *
C
C      IF ( JTYPE .EQ. 1 ) THEN
C
C      COMPUTE THE ELEMENT STIFFNESS AND MASS MATRIX AND
C      CONSISTENT LOAD VECTOR FOR THE HOT3 TRIANGULAR
C      FACET SHELL ELEMENT BASED ON HIGHER-ORDER TESSLER
C      PLATE THEORY
C
C      DO I = 1, 3
C          X(I) = CUORDS(1,I)

```

```

      Y(I) = COORDS(2,I)
      Z(I) = COORDS(3,I)
END DO

```

```

C
C
C  READ ELEMENT DATA OFF PROPERTY LIST

```

```

      THICK = 0.0
      NLAY = INT(PROPS(1))
      RHO = PROPS(2)
      Q(1) = PROPS(3)
      Q(2) = PROPS(4)
      Q(3) = PROPS(5)
      NOTYPE = INT(PROPS(6))
      NOUT = INT(PROPS(7))
      NMAT = INT(PROPS(8))

```

```

C
DO I = 1, NLAY
      E1(I) = PROPS(16*(I-1)+9)
      E2(I) = PROPS(16*(I-1)+10)
      E3(I) = PROPS(16*(I-1)+11)
      G12(I) = PROPS(16*(I-1)+12)
      G23(I) = PROPS(16*(I-1)+13)
      G31(I) = PROPS(16*(I-1)+14)
      V12(I) = PROPS(16*(I-1)+17)
      V23(I) = PROPS(16*(I-1)+18)
      V13(I) = PROPS(16*(I-1)+19)
      PLY(I) = PROPS(16*(I-1)+20)
      FIB(I) = PROPS(16*(I-1)+21)
      THICK = THICK + PLY(I)
END DO

```

```

C
DO I = 1, 3
      PSI(I) = PROPS(16*NLAY+ 8+I)
      PSU(I) = PROPS(16*NLAY+11+I)
END DO
      NSOL = DINT(PROPS(16*NLAY+17))
      NSYS = DINT(PROPS(16*NLAY+18))

```

```

C
DO I = 1, 3
      AMV1(I) = PROPS(16*NLAY+16+I)
      AMV2(I) = PROPS(16*NLAY+21+I)
END DO

```

```

      IF ( NSYS .EQ. 0 ) NSYS = 2

```

```

C
C
C  SET DEFAULTS ON MATERIAL ORIENTATION VECTOR
C  IF NONE HAVE BEEN SPECIFIED

```

```

      IF ( AMV1(1) .EQ. 0.0 .AND. AMV1(2) .EQ. 0.0 .AND.

```

```

&      AMV1(3)..EQ. 0.0 ) THEN
      AMV1(1) = 1.0
END IF
IF ( AMV2(1) .EQ. 0.0 .AND. AMV2(2) .EQ. 0.0 .AND.
&      AMV2(3) .EQ. 0.0 ) THEN
      AMV2(2) = 1.0
END IF

C
ZL(1) = -THICK / 2.0

C
DO K = 1, NLAY
  ZL(K+1) = ZL(K) + PLY(K)
END DO

C
C      COMPUTE ELEMENT COORDINATE TRANSFORMATION MATRICES
C
CALL UHOTTR ( X, Y, Z, AMV1, AMV2, A, B, TRI, STRN, ETRN,
&      SMTN, EMTN, TRANS )

C
C      COMPUTE ELEMENT MATERIAL PROPERTY MATRICES
C
CALL UHOTMT ( E1, E2, E3, G12, G23, G31, V12, V23, V13,
&      FIB, ZL, C, SMTN, AMAT, BMAT, DMAT, GMAT, NLAY,
&      NMAT )

C
IF ( NSOL .EQ. 1 .OR. LFLAGS(3) .EQ. 2 ) THEN

C
C      COMPUTE ELEMENT STIFFNESS MATRIX
C
CALL UHOTSF ( ELTSTF, AMAT, BMAT, DMAT, GMAT, A, B, THICK,
&      RELAX )

C
ELSE IF ( LFLAGS(3) .EQ. 4 ) THEN

C
C      COMPUTE ELEMENT MASS MATRIX
C
CALL UHOTMM ( AMASS, A, B, THICK, NSOL, RHO )

C
END IF

C
C      TEST IF A DISTRIBUTED LOAD HAS BEEN APPLIED
C
CALL UHOTLD ( Q, A, B, ELTL0D )

C
IF (NSOL .EQ. 1 .OR. (NSOL .EQ. 3 .AND. LFLAGS(3) .EQ. 2)) THEN

C
C      PERFORM STATIC CONDENSATION
C

```

```

      CALL UHOTCD ( ELTSTF, ELTL0D )

C
END IF

C
C   RESTORE STIFFNESS AND LOAD VECTORS TO ACCOUNT FOR
C   DIFFERENT DOF ARRANGEMENT IN ABAQUS.
C

CALL DSCPY ( 0.0D0,   RHS, NDOFEL,   1 )
CALL DSCPY ( 0.0D0, AMATRX, NDOFEL, NDCFEL )

C
NBC = 0

C
DO I = 1, 5
  DO J = 1, 3
    NAC      = I + 6 * ( J - 1 )
    NBC      = NBC + 1
    RHS(NAC,1) = ELTL0D(NBC)
    NBR      = 0
    DO K = 1, 5
      DO L = 1, 3
        NAR = K + 6 * ( L - 1 )
        NBR = NBR + 1
      DO
        TEST FOR STIFFNESS OR MASS STORAGE IN AMATRX
      DO
        IF ( NSOL .EQ. 1 ) AMATRX(NAR,NAC) = ELTSTF(NBR,NBC)
        IF ( NSOL .EQ. 2 .OR. NSOL .EQ. 3 ) THEN
          IF ( LFLAGS(3) .EQ. 2 ) THEN
            AMATRX(NAR,NAC) = ELTSTF(NBR,NBC)
          ELSE IF ( LFLAGS(3) .EQ. 4 ) THEN
            AMATRX(NAR,NAC) = AMASS(NBR,NBC)
          END IF
        END IF
      END DO
    END DO
  END DO
END DO
END DO
END DO

C
C   FOR DYNAMIC ANALYSIS APPEND W1 AND W2 STIFFNESS
C   OR MASS MATRIX PARTITIONS TO AMATRX
C

IF ( NSOL .EQ. 2 ) THEN
  NB = 0
  DO I = 1, 5
    DO J = 1, 3
      NA = I + 6 * ( J - 1 )
      NB = NB + 1
      DO II = 16, 17

```

```

      NII = II + 3

      TEST FOR STIFFNESS OR MASS STORAGE IN AMATRX

      IF (LFLAGS(3) .EQ. 2 ) THEN
        AMATRX(NII,NA) = ELTSTF(II,NB)
        AMATRX(NA,NII) = ELTSTF(NB,II)
      END IF
      IF( LFLAGS(3) .EQ. 4 ) THEN
        AMATRX(NII,NA) = AMASS(II,NB)
        AMATRX(NA,NII) = AMASS(NB,II)
      END IF
    END DO
  END DO
END DO

DO I = 16, 17
  NI = I + 3
  DO J = 16, 17
    NJ = J + 3
    IF ( LFLAGS(3) .EQ. 2 ) AMATRX(NI,NJ) = ELTSTF(I,J)
    IF ( LFLAGS(3) .EQ. 4 ) AMATRX(NI,NJ) = AMASS(I,J)
  END DO
END DO
END IF

ACCOUNT FOR DIFFERENT SIGN CONVENTION IN ABAQUS FOR THETA X
AND ADD ARTIFICIAL STIFFNESS OR MASS TO BE ASSOCIATED WITH
THETA Z

CALL UHOTFX ( AMATRX, NDOFEL )

TRANSFORM STIFFNESS OR MASS MATRICES AND ELEMENT
LOAD VECTOR TO GLOBAL COORDINATES

      T          T          T
[K] = [T][K'] [T] ; [M] = [T][M'] [T] ; {R} = [T]{R'}

NDOF = NDOFEL
DO I = 1, NDOF
  RETN(I) = RHS(I,1)
END DO
CALL DGMMUL ( AMATRX,NDOF,TRANS,24,PETN,24,NDOF,NDOF,NDOF )
CALL DGMATB ( TRANS,24,PETN,24,AMATRX,NDOF,NDOF,NDOF,NDOF )
CALL DGMMUL ( TRANS,24,RETN,24,RHS,NDOF,NDOF,NDOF,1 )

IF ( NSOL .EQ. 1 ) THEN

```

```

C
C      PERFORM REQUESTED ELEMENT DATA RECOVERY
C
C      TRANSFORM GLOBAL DISPLACEMENTS INTO ELEMENT COORDINATES
C
C      CALL DGMMUL ( TRANS,24,DU,MLVARX,PETN,24,NDOF,NDOF,1)
C      CALL DSTRN ( PETN, DU, NDOF )
C
C      REINSTATE ORIGINAL SIGN OF THETA X FOR DATA RECOVERY
C
C      DO I = 1, 3
C          NN = (I-1)*6 + 4
C          DU(NN) = -DU(NN)
C      END DO
C
C      CALL UHOTIO ( ELTSTF, ELTL0D, C, LFLAGS, ZL, DU, ENERGY,
C      &              A, B, PSI, PSU, AMAT, BMAT, DMAT,  GMAT,
C      &              RELAX, NLAY, NOTYPE, NOUT , JELEM,  STRN,
C      &              ETRN, TRI, NSYS, NMAT, MLVARX
C
C      END IF
C
C      HOT3 ELEMENT PROCESSING COMPLETED
C
C      END IF
C
C      IF ( JTYPE .EQ. NUEL ) THEN
C
C          *****
C          *   INSERT MAIN PROGRAMS FOR OTHER USER DEFINED ELEMENTS *
C          *   SELECTED BY JTYPE HERE                               *
C          *****
C
C      END IF
C
C      RETURN
C      END
C
C *****
C
C      SUBROUTINE UHOTTR ( X, Y, Z, AMV1, AMV2, A, B, TRI, STRN, ETRN,
C      &                  SMTN, EMTN, TRANS )
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C
C      CALCULATE TRANSFORMATION MATRICES FOR CONVERTING QUANTITIES
C      BETWEEN LOCAL AND GLOBAL COORDINATE SYSTEMS
C
C *****

```

```

      DIMENSION X(3), Y(3), Z(3), XP(3), YP(3), ZP(3), EO(3,3),      *
&      EP(3,3), VC12(3), VC13(3), A(3), B(3), AMV1(3),      *
&      AMV2(3), TRI(3,3), TMI(3,3), EM(3,3), STRN(6,6),      *
&      ETRN(6,6), SMTN(6,6), EMTN(6,6), TRANS(24,24)      *
C
C * * * * *
C
C      UNIT VECTORS IN GLOBAL SYSTEM
C
      EO(1,1) = 1.0
      EO(1,2) = 0.0
      EO(1,3) = 0.0
      EO(2,1) = 0.0
      EO(2,2) = 1.0
      EO(2,3) = 0.0
      EO(3,1) = 0.0
      EO(3,2) = 0.0
      EO(3,3) = 1.0
C
C      LOCAL ELEMENT COORDINATE VECTORS
C
      AL = SQRT( (X(2)-X(1))**2+(Y(2)-Y(1))**2+(Z(2)-Z(1))**2 )
      VC12(1) = (X(2)-X(1))/AL
      VC12(2) = (Y(2)-Y(1))/AL
      VC12(3) = (Z(2)-Z(1))/AL
      AL = SQRT( (X(3)-X(1))**2+(Y(3)-Y(1))**2+(Z(3)-Z(1))**2 )
      VC13(1) = (X(3)-X(1))/AL
      VC13(2) = (Y(3)-Y(1))/AL
      VC13(3) = (Z(3)-Z(1))/AL
C
      EP(1,1) = VC12(1)
      EP(1,2) = VC12(2)
      EP(1,3) = VC12(3)
C
      AI = VC12(2)*VC13(3)-VC12(3)*VC13(2)
      AJ = VC12(3)*VC13(1)-VC12(1)*VC13(3)
      AK = VC12(1)*VC13(2)-VC12(2)*VC13(1)
      AL = SQRT( AI**2 + AJ**2 + AK**2 )
      EP(3,1) = AI/AL
      EP(3,2) = AJ/AL
      EP(3,3) = AK/AL
C
      AI = EP(3,2)*EP(1,3)-EP(3,3)*EP(1,2)
      AJ = EP(3,3)*EP(1,1)-EP(3,1)*EP(1,3)
      AK = EP(3,1)*EP(1,2)-EP(3,2)*EP(1,1)
      AL = SQRT( AI**2 + AJ**2 + AK**2 )
      EP(2,1) = AI/AL
      EP(2,2) = AJ/AL

```

```

EP(2,3) = AK/AL
C
C   CONSTRUCT COORDINATE TRANSFORMATION MATRIX
C
C                                     T
C   {X'} = [TRI]{X};   {X} = [TRI]{X'}
C
C
DO I = 1, 3
  DO J = 1, 3
    TRI(I,J) = 0.0
    DO K = 1, 3
      TRI(I,J) = TRI(I,J) + EP(I,K)*EO(J,K)
    END DO
  END DO
END DO

C
CALL DSCPY ( 0.000, TRANS, 24, 24 )
C
DO I = 1, 6
  NF = 3*(I-1)
  DO J = 1, 3
    DO K = 1, 3
      TRANS(NF+J,NF+K) = TRI(J,K)
    END DO
  END DO
  TRANS(18+I,18+I) = 1.0
END DO

C
DO I = 1, 3
  XP(I) = TRI(1,1)*X(I)+TRI(1,2)*Y(I)+TRI(1,3)*Z(I)
  YP(I) = TRI(2,1)*X(I)+TRI(2,2)*Y(I)+TRI(2,3)*Z(I)
  ZP(I) = TRI(3,1)*X(I)+TRI(3,2)*Y(I)+TRI(3,3)*Z(I)
END DO

C
A(1) = XP(3) - XP(2)
A(2) = XP(1) - XP(3)
A(3) = XP(2) - XP(1)

C
B(1) = YP(2) - YP(3)
B(2) = YP(3) - YP(1)
B(3) = YP(1) - YP(2)

C
C   CONSTRUCT STRESS AND STRAIN TRANSFORMATION MATRICES
C   TO CONVERT BETWEEN LOCAL ELEMENT COORDINATE SYSTEM
C   AND GLOBAL SYSTEM
C
C   [Ts] = STRESS TRANSFORMATION

```

```

C      [Te] = STRAIN TRANSFORMATION
C
C      {E'} = [Te]{E};   {S'} = [Ts]{S}
C
C      T      T
C      {E} = [Ts]{E'}; {S} = [Te]{S'}
C
C      ( CONVENTION: [Ex,Ey,Ez,Gyz,Gxz,Gxy] )
C
C      CALL TENSRT ( TRI, STRN, ETRN )
C
C      CONSTRUCT TRANSFORMATION MATRIX TO ORIENT THE MATERIAL
C      PROPERTIES DEFINED IN THE GLOBAL SYSTEM TO THE LOCAL
C      ELEMENT COORDINATE SYSTEM
C
C      [Ts] = STRESS TRANSFORMATION FOR MATERIAL PROPERTIES
C      [Te] = STRAIN TRANSFORMATION FOR MATERIAL PROPERTIES
C
C      T      T
C      [C'] = [Ts][C][Ts]   [C] = [Te][C'] [Te]
C
C      CONVENTION: [Ex,Ey,Ez,Gyz,Gxz,Gxy]
C
C      DO I = 1, 3
C         EM(1,I) = AMV1(I)/SQRT(AMV1(1)**2+AMV1(2)**2+AMV1(3)**2)
C         EM(2,I) = AMV2(I)/SQRT(AMV2(1)**2+AMV2(2)**2+AMV2(3)**2)
C      END DO
C      AI = EM(1,2)*EM(2,3)-EM(1,3)*EM(2,2)
C      AJ = EM(1,3)*EM(2,1)-EM(1,1)*EM(2,3)
C      AK = EM(1,1)*EM(2,2)-EM(1,2)*EM(2,1)
C      AL = SQRT( AI**2 + AJ**2 + AK**2 )
C      EM(3,1) = AI/AL
C      EM(3,2) = AJ/AL
C      EM(3,3) = AK/AL
C
C      DO I = 1, 3
C         DO J = 1, 3
C            TMI(I,J) = 0.0
C            DO K = 1, 3
C               TMI(I,J) = TMI(I,J) + EP(I,K)*EM(J,K)
C            END DO
C         END DO
C      END DO
C
C      CALL TENSRT ( TMI, SMTN, EMTN )
C
C      RETURN
C      END

```

```

C
C * * * * *
C
C      SUBROUTINE TENSRT ( T, TS, TE )
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C
C      COMPUTE STRESS AND STRAIN TRANSFORMATION MATRICES
C
C      DIMENSION T(3,3), TS(6,6), TE(6,6)
C
C * * * * *
C
C      CONSTRUCT TRANSFORMATION MATRICES TO CONVERT
C      BETWEEN REFERENCE COORDINATE SYSTEMS
C
C      [Ts] = STRESS TRANSFORMATION
C      [Te] = STRAIN TRANSFORMATION
C
C      {E'} = [Te]{E};   {S'} = [Ts]{S}
C
C      T           T
C      {E} = [Ts]{E'}; {S} = [Te]{S'}
C
C      CONVENTION: [Ex,Ey,Ez,Gyz,Gxz,Gxy] )
C
C      . . . STRESS TRANSFORMATION MATRIX, [Ts]:
C
C      TS(1,1) = T(1,1)**2
C      TS(1,2) = T(1,2)**2
C      TS(1,3) = T(1,3)**2
C      TS(1,4) = 2*T(1,2)*T(1,3)
C      TS(1,5) = 2*T(1,3)*T(1,1)
C      TS(1,6) = 2*T(1,1)*T(1,2)
C      TS(2,1) = T(2,1)**2
C      TS(2,2) = T(2,2)**2
C      TS(2,3) = T(2,3)**2
C      TS(2,4) = 2*T(2,2)*T(2,3)
C      TS(2,5) = 2*T(2,3)*T(2,1)
C      TS(2,6) = 2*T(2,1)*T(2,2)
C      TS(3,1) = T(3,1)**2
C      TS(3,2) = T(3,2)**2
C      TS(3,3) = T(3,3)**2
C      TS(3,4) = 2*T(3,2)*T(3,3)
C      TS(3,5) = 2*T(3,3)*T(3,1)
C      TS(3,6) = 2*T(3,1)*T(3,2)
C      TS(4,1) = T(2,1)*T(3,1)
C      TS(4,2) = T(2,2)*T(3,2)

```

```

TS(4,3) = T(2,3)*T(3,3)
TS(4,4) = T(2,2)*T(3,3) + T(3,2)*T(2,3)
TS(4,5) = T(2,3)*T(3,1) + T(3,3)*T(2,1)
TS(4,6) = T(2,1)*T(3,2) + T(3,1)*T(2,2)
TS(5,1) = T(3,1)*T(1,1)
TS(5,2) = T(3,2)*T(1,2)
TS(5,3) = T(3,3)*T(1,3)
TS(5,4) = T(3,2)*T(1,3) + T(1,2)*T(3,3)
TS(5,5) = T(3,3)*T(1,1) + T(1,3)*T(3,1)
TS(5,6) = T(3,1)*T(1,2) + T(1,1)*T(3,2)
TS(6,1) = T(1,1)*T(2,1)
TS(6,2) = T(1,2)*T(2,2)
TS(6,3) = T(1,3)*T(2,3)
TS(6,4) = T(1,2)*T(2,3) + T(2,2)*T(1,3)
TS(6,5) = T(1,3)*T(2,1) + T(2,3)*T(1,1)
TS(6,6) = T(1,1)*T(2,2) + T(2,1)*T(1,2)

```

```

C
C STRAIN TRANSFORMATION MATRIX, [Te]:
C

```

```

DO I = 1, 3
  DO J = 1, 3
    TE(I,J) = TS(I,J)
    TE(I,J+3) = TS(I,J+3)/2.0
    TE(I+3,J) = TS(I+3,J)/0.5
    TE(I+3,J+3) = TS(I+3,J+3)
  
```

```

    END DO
  END DO

```

```

C
RETURN
END

```

```

C
C *****
C SUBROUTINE USFMEM ( SMX, A, B, THICK, DMX )
C

```

```

  IMPLICIT REAL*8 (A-H,O-Z)

```

```

  CALCULATE MEMBRANE EFFECTS OF THE HOT3 TRIANGULAR PLATE
  ELEMENT INCORPORATING HIGHER ORDER KINEMATIC EXPANSIONS

```

```

  DIMENSION DMX(4,4), BMX(4,17), SMX(17,17), BTD(17,4), A(3), B(3)

```

```

  AREA = ( A(3) * B(2) - A(2) * B(3) ) / 2.0

```

```

  CALL DSCPY ( 0.00, BMX, 4, 17)

```

```

CALL UBXMEM ( BMX, 4, A, B, THICK )
CALL DMATB ( BMX, DMX, BTD, 17, 4, 4 )
CALL DMMUL ( BTD, BMX, SMX, 17, 4, 17 )
CALL DSMUL ( AREA, SMX, SMX, 17, 17)

```

```

C
RETURN
END

```

```

C
C *****
C
SUBROUTINE USFBND ( SMX, A, B, THICK, DMX )

```

```

C
IMPLICIT REAL*8 (A-H,O-Z)

```

```

C
CALCULATE BENDING STIFFNESS MATRIX OF THE HOT3 TRIANGULAR PLATE
C ELEMENT INCORPORATING HIGHER ORDER KINEMATIC EXPANSIONS

```

```

C
DIMENSION DMX(4,4), BMX(4,17), SMX(17,17), BTD(17,4), A(3), B(3)

```

```

C *****
C
AREA = ( A(3) * B(2) - A(2) * B(3) ) / 2.0

```

```

C
CALL DSCPY ( 0.DO, BMX, 4, 17)

```

```

C
CALL UBXBND ( BMX, 4, A, B, THICK )
CALL DMATB ( BMX, DMX, BTD, 17, 4, 4 )
CALL DMMUL ( BTD, BMX, SMX, 17, 4, 17 )
CALL DSMUL ( AREA, SMX, SMX, 17, 17)

```

```

C
RETURN
END

```

```

C
C *****
C
SUBROUTINE USFCPL ( SMX, A, B, THICK, DMX )

```

```

C
IMPLICIT REAL*8 (A-H,O-Z)

```

```

C
CALCULATE COUPLING EFFECTS OF THE HOT3 TRIANGULAR PLATE
C ELEMENT INCORPORATING HIGHER ORDER KINEMATIC EXPANSIONS

```

```

C
DIMENSION DMX(4,4), BMX(4,17), SMX(17,17), BTD(17,4), BTDB(17,17),
* BTDTB(17,17), A(3), B(3)

```

```

C *****
C
AREA = ( A(3) * B(2) - A(2) * B(3) ) / 2.0

```

```

C      CALL DSCPY ( 0.0DO,   BMX,    4, 17)
      CALL UBXMEN (   BMX,    4,    A, B, THICK )
      CALL DMATB (   BMX,   DMX,   BTDB, 17, 4, 4 )
      CALL DSCPY ( 0.0DO,   BMX,    4, 17)
      CALL UBXBND (   BMX,    4,    A, B, THICK )
      CALL DMMUL (   BTDB,   BMX, BTDB, 17, 4, 17 )
      CALL DMTRN (   BTDB, BTDTB,   17, 17)
      CALL DMADD (   BTDB, BTDTB,   SMX, 17, 17 )
      CALL DSMUL (   AREA,   SMX,   SMX, 17, 17 )

C
      RETURN
      END

C
C *****
C
C      SUBROUTINE USFSHR ( SMX, A, B, DMX )
C
C      PARAMETER ( KORD = 7 )
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C
C      CALCULATE SHEAR STIFFNESS OF THE HOT3 TRIANGULAR PLATE
C      ELEMENT INCORPORATING HIGHER ORDER KINEMATIC EXPANSIONS
C
C      DIMENSION DMX(2,2), BMX(2,17), SMX(17,17), BTDB(17,17),*
C      &          A(3), B(3)
C
C *****
C
C      AREA = ( A(3) * B(2) - A(2) * B(3) ) / 2.0
C
C      CALL DSCPY ( 0.DO, SMX, 17, 17 )
      CALL DSCPY ( 0.DO, BMX, 2, 17 )
C
      DO INT= 1, KORD
        CALL UINPT ( X1, X2, X3, WEIGHT, KORD, INT )
        CALL UBXSHR ( BMX, 2, A, B, X1, X2, X3 )
        CALL DMATB ( BMX, DMX, BTDB, 17, 2, 2 )
        CALL DMMUL ( BTDB, BMX, BTDB, 17, 2, 17)
        CALL DMASB ( SMX, AREA*WEIGHT, BTDB, SMX, 17, 17 )
      END DO
C
      RETURN
      END

C
C *****
C

```

```

SUBROUTINE UBXMEN ( BMX, LR, A, B, THICK )
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C
C      CALCULATE MEMBRANE B-MATRIX
C
C      DIMENSION BMX(LR,17), A(3), B(3)
C
C      * * * * *
C * * * * *
C
C      AREA = ( A(3) * B(2) - A(2) * B(3) ) / 2.0
C
C      BMX( 1, 1) = B(1) / ( 2. * AREA )
C      BMX( 1, 2) = B(2) / ( 2. * AREA )
C      BMX( 1, 3) = B(3) / ( 2. * AREA )
C
C      BMX( 2, 4) = A(1) / ( 2. * AREA )
C      BMX( 2, 5) = A(2) / ( 2. * AREA )
C      BMX( 2, 6) = A(3) / ( 2. * AREA )
C
C      BMX( 3,16) = 2.0 / THICK
C
C      BMX( 4, 1) = A(1) / ( 2. * AREA )
C      BMX( 4, 2) = A(2) / ( 2. * AREA )
C      BMX( 4, 3) = A(3) / ( 2. * AREA )
C      BMX( 4, 4) = B(1) / ( 2. * AREA )
C      BMX( 4, 5) = B(2) / ( 2. * AREA )
C      BMX( 4, 6) = B(3) / ( 2. * AREA )
C
C      RETURN
C      END
C
C      * * * * *
C * * * * *
C
C      SUBROUTINE UBXBND ( BMX, LR, A, B, THICK )
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C
C      CALCULATE BENDING B-MATRIX
C
C      DIMENSION BMX(LR,17), A(3), B(3)
C
C      * * * * *
C * * * * *
C
C      AREA = ( A(3) * B(2) - A(2) * B(3) ) / 2.0
C
C      BMX( 1,13) = B(1) / ( 2. * AREA )
C      BMX( 1,14) = B(2) / ( 2. * AREA )

```

```

      BMX( 1,15) = B(3) / ( 2. * AREA )
C
      BMX( 2,10) = A(1) / ( 2. * AREA )
      BMX( 2,11) = A(2) / ( 2. * AREA )
      BMX( 2,12) = A(3) / ( 2. * AREA )
C
      BMX( 3,17) = 2.0 / THICK
C
      BMX( 4,10) = B(1) / ( 2. * AREA )
      BMX( 4,11) = B(2) / ( 2. * AREA )
      BMX( 4,12) = B(3) / ( 2. * AREA )
      BMX( 4,13) = A(1) / ( 2. * AREA )
      BMX( 4,14) = A(2) / ( 2. * AREA )
      BMX( 4,15) = A(3) / ( 2. * AREA )
C
      RETURN
      END
C
C * * * * *
C
      SUBROUTINE UBXSHR ( BMX, LR, A, B, X1, X2, X3 )
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      CALCULATE SHEAR B-MATRIX
C
      DIMENSION BMX(LR,17), A(3), B(3)
C
C * * * * *
C
      AREA = ( A(3) * B(2) - A(2) * B(3) ) / 2.0
C
      BMX(1, 7) = A(1) / ( 2. * AREA )
      BMX(1, 8) = A(2) / ( 2. * AREA )
      BMX(1, 9) = A(3) / ( 2. * AREA )
      BMX(1,10) = ( B(3) * ( X2 * A(1) + X1 * A(2) )
&                - B(2) * ( X3 * A(1) + X1 * A(3) ) ) / (4.*AREA) + X1
      BMX(1,11) = ( B(1) * ( X3 * A(2) + X2 * A(3) )
&                - B(3) * ( X2 * A(1) + X1 * A(2) ) ) / (4.*AREA) + X2
      BMX(1,12) = ( - B(1) * ( X3 * A(2) + X2 * A(3) )
&                + B(2) * ( X3 * A(1) + X1 * A(3) ) ) / (4.*AREA) + X3
      BMX(1,13) = ( A(2) * ( X3 * A(1) + X1 * A(3) )
&                - A(3) * ( X2 * A(1) + X1 * A(2) ) ) / (4.*AREA)
      BMX(1,14) = ( - A(1) * ( X3 * A(2) + X2 * A(3) )
&                + A(3) * ( X2 * A(1) + X1 * A(2) ) ) / (4.*AREA)
      BMX(1,15) = ( A(1) * ( X3 * A(2) + X2 * A(3) )
&                - A(2) * ( X3 * A(1) + X1 * A(3) ) ) / (4.*AREA)
C

```

```

BMX(2, 7) = B(1) / ( 2. * AREA )
BMX(2, 8) = B(2) / ( 2. * AREA )
BMX(2, 9) = B(3) / ( 2. * AREA )
BMX(2,10) = ( B(3) * ( X2 * B(1) + X1 * B(2) )
&          - B(2) * ( X3 * B(1) + X1 * B(3) ))/(4.*AREA)
BMX(2,11) = ( B(1) * ( X3 * B(2) + X2 * B(3) )
&          - B(3) * ( X2 * B(1) + X1 * B(2) ))/(4.*AREA)
BMX(2,12) = ( - B(1) * ( X3 * B(2) + X2 * B(3) )
&          + B(2) * ( X3 * B(1) + X1 * B(3) ))/(4.*AREA)
BMX(2,13) = ( A(2) * ( X3 * B(1) + X1 * B(3) )
&          - A(3) * ( X2 * B(1) + X1 * B(2) ))/(4.*AREA) + X1
BMX(2,14) = ( - A(1) * ( X3 * B(2) + X2 * B(3) )
&          + A(3) * ( X2 * B(1) + X1 * B(2) ))/(4.*AREA) + X2
BMX(2,15) = ( A(1) * ( X3 * B(2) + X2 * B(3) )
&          - A(2) * ( X3 * B(1) + X1 * B(3) ))/(4.*AREA) + X3

```

C

```

RETURN
END

```

C

C * * * * *

C

```

SUBROUTINE UFCSHR ( STFSHR, STFBND, RELAX )

```

C

```

FCSHR COMPUTES THE PENALTY FACTOR RELAX FOR THE SHEAR STIFFNESS
COMPONENTS TO ENFORCE KIRCHHOFF BEHAVIOR IN THE THIN LIMIT

```

C

```

IMPLICIT REAL*8 (A-H,O-Z)

```

C

```

DIMENSION STFBND(17,17), STFSHR(17,17)

```

C

C * * * * *

C

```

SBEND = 0.0D0
SHEAR = 0.0D0

```

C

```

DO I = 10, 15
  SBEND = SBEND + STFBND(I,I)
  SHEAR = SHEAR + STFSHR(I,I)
END DO

```

C

```

IF ( SBEND .EQ. 0.0 ) THEN
  RELAX = 0.0
ELSE
  SCALE = 2.0D0
  RATIO = SBEND / SHEAR
  RELAX = SCALE * RATIO / ( 1.0D0 + SCALE * RATIO )
END IF

```

C

RETURN
END

```

C
C * * * * *
C
C      SUBROUTINE UHOTBX ( BMX, A, B, X1, X2, X3, THICK )
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C
C      CALCULATE THE STRAIN-DISPLACEMENT MATRIX
C      FOR THE HOT3 ELEMENT
C
C      DIMENSION BMX(10,17), A(3), B(3)
C
C * * * * *
C
C      CALL DSCPY ( 0.0D+0, BMX, 10, 17 )
C
C      CALL UBXMEM ( BMX( 1,1), 10, A, B, THICK )
C      CALL UBXBND ( BMX( 5,1), 10, A, B, THICK )
C      CALL UBXSHR ( BMX( 9,1), 10, A, B, X1, X2, X3 )
C
C      RETURN
C      END
C
C * * * * *
C
C      SUBROUTINE UHOTSF ( ELTSTF, AMAT, BMAT, DMAT, GMAT, A, B, THICK,
C      &                      RELAX )
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C
C      CALCULATE THE ELASTIC STIFFNESS MATRIX FOR THE HOT3 ELEMENT
C
C      DIMENSION ELTSTF(17,17), STFMEM(17,17), STFBND(17,17),
C      &          STFCPL(17,17), STFSHR(17,17), AMAT(4,4), BMAT(4,4),
C      &          DMAT(4,4), GMAT(2,2), A(3), B(3)
C
C * * * * *
C
C      CALL DSCPY ( 0.0D0, ELTSTF, 17, 17 )
C
C      CALL USFMEM ( STFMEM, A, B, THICK, AMAT )
C      CALL DMADD ( STFMEM, ELTSTF, ELTSTF, 17, 17 )
C      CALL USFBND ( STFBND, A, B, THICK, DMAT )
C      CALL DMADD ( STFBND, ELTSTF, ELTSTF, 17, 17 )
C      CALL USFCPL ( STFCPL, A, B, THICK, BMAT )
C      CALL DMADD ( STFCPL, ELTSTF, ELTSTF, 17, 17 )

```

```

C      CALL USFSHR ( STFSHR, A, B, GMAT )
C
C      CALL UFCSHR ( STFSHR, STFBND, RELAX )
C      CALL DMASB ( ELTSTF, RELAX, STFSHR, ELTSTF, 17, 17 )
C
C      RETURN
C      END
C
C *****
C
C      SUBROUTINE UHOTMM ( AMASS, A, B, THK, NSOL, RHO )
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C
C      CALCULATE HOT3 ELEMENT CONSISTENT MASS MATRIX
C
C      DIMENSION AMASS(17,17), SHAPE(3,17),  A(3),  B(3),
C      &          TEMP(17,17),  PSI(3), XML(3), XMM(3), XMN(3)
C *****
C
C      CALL DSCPY ( 0.0D0, AMASS, 17, 17 )
C      CALL DSCPY ( 0.0D0, SHAPE, 3, 17 )
C
C      H = THK/2.0
C
C      AREA = ( A(3) * B(2) - A(2) * B(3) ) / 2.0
C
C      NSOL FLAG USED TO INCLUDE/EXCLUDE MASS ASSOCIATED
C      WITH HIGHER-ORDER FUNCTIONS W1 & W2;
C
C      BETA = 1.0
C      IF ( NSOL .EQ. 3 ) BETA = 0.0
C
C      SET ORDER OF INTEGRATION ACROSS THICKNESS
C
C      NORD = 4
C
C      SET ORDER OF INTEGRATION OVER AREA
C
C      KORD = 7
C
C      DO 100 INT = 1, KORD
C
C          INVOKE GUASS INTEGRATION FOR A TRIANGLE
C
C          CALL UINTPT ( X1, X2, X3, WGHT1, KORD, INT )
C

```

PSI(1) = X1
PSI(2) = X2
PSI(3) = X3

C
C
C
ANISOPARAMETRIC SHAPE FUNCTIONS

XMN(1) = 4.0*X1*X2
XMN(2) = 4.0*X2*X3
XMN(3) = 4.0*X3*X1

C
XML(1) = B(3)/8.0*XMN(1)-B(2)/8.0*XMN(3)
XML(2) = B(1)/8.0*XMN(2)-B(3)/8.0*XMN(1)
XML(3) = -B(1)/8.0*XMN(2)+B(2)/8.0*XMN(3)

C
XMM(1) = A(2)/8.0*XMN(3)-A(3)/8.0*XMN(1)
XMM(2) = -A(1)/8.0*XMN(2)+A(3)/8.0*XMN(1)
XMM(3) = A(1)/8.0*XMN(2)-A(2)/8.0*XMN(3)

C
DO 110 IZ = 1, NORD

C
CALL UIGSS (NORD, IZ, CEE, WGHT2)

C
DO I = 1, 3
SHAPE(1,I) = PSI(I)
SHAPE(1,I+12) = H*CEE*PSI(I)
SHAPE(2,I+3) = PSI(I)
SHAPE(2,I+9) = H*CEE*PSI(I)
SHAPE(3,I+6) = PSI(I)
SHAPE(3,I+9) = XML(I)
SHAPE(3,I+12) = XMM(I)

END DO

SHAPE(3,16) = BETA*CEE

SHAPE(3,17) = BETA*(CEE**2 - 0.2)

C
C
C
NUMERICALLY INTEGRATE MASS MATRIX

CALL DMATB (SHAPE, SHAPE, TEMP, 17, 3, 17)
FAC = RHO*H*AREA*WGHT1*WGHT2

C
DO I = 1, 17
DO J = 1, 17
AMASS(I,J) = AMASS(I,J) + FAC*TEMP(I,J)
END DO
END DO

C
110 CONTINUE

C
100 CONTINUE

```

C      RETURN
C      END

C      * * * * *
C      SUBROUTINE UHOTFX ( AMATRX, NDOF )
C      IMPLICIT REAL*8 (A-H,O-Z)
C      CHANGE SIGN OF THETA X TO ACCOUNT FOR DIFFERENT SIGN
C      CONVENTION IN ABAQUS AND ADD ARTIFICIAL STIFFNESS OR
C      MASS TO BE ASSOCIATED WITH THETA Z
C      DIMENSION AMATRX(NDOF,NDOF)
C      * * * * *
C      DO I = 1, 3
C        NN = (I-1)*6 + 4
C        DO J = 1, NDOF
C          AMATRX(NN,J) = -AMATRX(NN,J)
C          AMATRX(J,NN) = -AMATRX(J,NN)
C        END DO
C      END DO
C      SET SCALE FACTOR FOR MINIMUM DIAGONAL TERM
C      SCALE = 1.0E-5
C      SMIN = AMATRX(1,1)
C      DO I = 1, 3
C        DO J = 1, 5
C          N = 6*(I-1) + J
C          SMIN = MIN( AMATRX(N,N), SMIN )
C        END DO
C      END DO
C      DO I = 1, 3
C        N = 6*I
C        AMATRX(N,N) = SCALE * SMIN
C      END DO
C      RETURN
C      END

C      * * * * *
C      SUBROUTINE UHOTLD ( Q, A, B, ELTLOD )

```

```

C
C HTLOD COMPUTES THE CONSISTENT LOAD VECTOR FOR
C THE HOT3 ELEMENT BASED ON HIGHER ORDER TESSLER
C PLATE THEORY
C
C IMPLICIT REAL*8 (A-H,O-Z)
C
C HOT3 IS FORMULATED TO ACCEPT LINEARLY VARYING LOAD
C DISTRIBUTIONS TO BE APPLIED ON BOTH THE UPPER AND
C LOWER SURFACES. CURRENT RESTRICTION IN THE INPUT
C MODULE LIMITS DISTRIBUTED LOAD INPUT TO ONLY ONE
C SURFACE WHICH IS ASSUMED TO BE THE TOP SURFACE
C
C DIMENSION ELTLOD(17), A(3), B(3), Q(3)
C
C * * * * *
C
C CALL DSCPY ( 0.0D+0, ELTLOD, 17, 1)
C
C IF (( Q(1).NE.0.0 ).OR.( Q(2).NE.0.0 ).OR.( Q(3).NE.0.0 )) THEN
C
C     AREA      = ( A(3) * B(2) - A(2) * B(3) ) / 2.0
C
C     ELTLOD( 7) = AREA * ( 2.*Q(1) + Q(2) + Q(3) ) / 12.0D0
C     ELTLOD( 8) = AREA * ( Q(1) + 2.*Q(2) + Q(3) ) / 12.0D0
C     ELTLOD( 9) = AREA * ( Q(1) + Q(2) + 2.*Q(3) ) / 12.0D0
C     ELTLOD(10) = AREA * ( ( 2.*Q(1) + 2.*Q(2) + Q(3) ) * B(3)
C     &              -( 2.*Q(1) + Q(2) + 2.*Q(3) ) * B(2) ) /
C     &              120.0D0
C     ELTLOD(11) = AREA * ( ( Q(1) + 2.*Q(2) + 2.*Q(3) ) * B(1)
C     &              -( 2.*Q(1) + 2.*Q(2) + Q(3) ) * B(3) ) /
C     &              120.0D0
C     ELTLOD(12) = AREA * ( ( 2.*Q(1) + Q(2) + 2.*Q(3) ) * B(2)
C     &              -( Q(1) + 2.*Q(2) + 2.*Q(3) ) * B(1) ) /
C     &              120.0D0
C     ELTLOD(13) = AREA * ( ( 2.*Q(1) + Q(2) + 2.*Q(3) ) * A(2)
C     &              -( 2.*Q(1) + 2.*Q(2) + Q(3) ) * A(3) ) /
C     &              120.0D0
C     ELTLOD(14) = AREA * ( ( 2.*Q(1) + 2.*Q(2) + Q(3) ) * A(3)
C     &              -( Q(1) + 2.*Q(2) + 2.*Q(3) ) * A(1) ) /
C     &              120.0D0
C     ELTLOD(15) = AREA * ( ( Q(1) + 2.*Q(2) + 2.*Q(3) ) * A(1)
C     &              -( 2.*Q(1) + Q(2) + 2.*Q(3) ) * A(2) ) /
C     &              120.0D0
C     ELTLOD(16) = AREA * ( Q(1) + Q(2) + Q(3) ) / 3.0D0
C     ELTLOD(17) = AREA * ( Q(1) + Q(2) + Q(3) ) / 2.5D0
C END IF
C

```

RETURN
END

```

C
C * * * * *
C
C      SUBROUTINE UHOTMT( E1, E2, E3, G12, G23, G31, V12, V23, V13,
&      FIB, Z, C, STRN, AMAT, BMAT, DMAT, GMAT,
&      NLAY, NMAT
C
C      MATPRP COMPUTES THE EQUIVALENT COMPOSITE A,B,D AND
C      G MATRICES FOR COMPOSITE PLATE PROPERTIES IN THE
C      HOT3 ELEMENT
C
C      IMPLICIT REAL*8 (A-H,O-Z)
C
C      DIMENSION AMAT(4,4), BMAT(4,4), DMAT(4,4), GMAT(2,2),
&      ABAR(6,6), BBAR(6,6), DBAR(6,6), C(6,6,125),
&      E1(1), E2(1), E3(1), V12(1), V23(1), V13(1),
&      G12(1), G23(1), G31(1), FIB(1), Z(1), TEMP(6,6),
&      PII(6), PIZ(6), PIP(6,6), STRN(6,6), C1(6,6,125),
&      S33(25),RA13(25),RA23(25),RA63(25),CHH(25,3),
&      AAK(25,4),CH(25,7),BK(25,3),D3(25),ABC(25,4)
C
C * * * * *
C
C      OPTION FLAG TO SIMULATE FIRST ORDER SHEAR-DEFORMABLE THEORY
C
C      NSDF = 0
C
C      DO K = 1, NLAY
C
C          IF ( NSDF .EQ. 1 ) THEN
C
C              SIMULATE FIRST-ORDER SHEAR DEFORMABLE THEORY:
C
C              V13(K) = 0.0
C              V23(K) = 0.0
C              E3(K) = 1.0E6*E1(K)
C
C          END IF
C
C          CALL UMTCMX ( E1(K), E2(K), E3(K), V12(K), V23(K), V13(K),
&          G12(K), G23(K), G31(K), FIB(K), C1(1,1,K) )
C
C      TRANSFORM MATERIAL PROPERTY MATRIX TO LOCAL COORDINATE SYSTEM
C
C          T      T      T

```

```

C      [C'] = [Ts][C][Ts] ; [C'] = [Ts][C][Ts]
C
C      CALL DMMUL( STRN, C1(1,1,K), TEMP, 6, 6, 6 )
C      CALL DMABT( TEMP, STRN, C(1,1,K), 6, 6, 6 )
C
C      END DO
C
C      PI = ACOS(-1.0DO)
C
C      HALF = ( Z(NLAY+1) - Z(1) ) / 2.0
C      THK = 2*HALF
C      AK = SQRT(5.DO/6.DO)
C      AKZ = SQRT(42.DO/85.DO)
C
C      INITIALIZE MATRICES
C
C      CALL DSCPY ( 0.0DO, ABAR, 6, 6 )
C      CALL DSCPY ( 0.0DO, BBAR, 6, 6 )
C      CALL DSCPY ( 0.0DO, DBAR, 6, 6 )
C      CALL DSCPY ( 0.0DO, GMAT, 2, 2 )
C
C      IF ( NMAT .EQ. 1 ) THEN
C
C          COMPUTE LAMINATE CONSTITUTIVE MATRICES USING
C          STRAIN-BASED VERSION OF HIGHER-ORDER THEORY
C
C          H1 = ( HALF )
C          H2 = ( HALF**2 )
C          H3 = ( HALF**3 )
C          H4 = ( HALF**4 )
C          H5 = ( HALF**5 )
C          H6 = ( HALF**6 )
C
C          FORM CONSTANTS TO ENFORCE ZERO SHEAR BOUNDARY
C          CONDITION AT THE TOP AND BOTTOM SURFACES
C
C          R13 = C(1,3,1) / C(3,3,1) - C(1,3,NLAY) / C(3,3,NLAY)
C          R23 = C(2,3,1) / C(3,3,1) - C(2,3,NLAY) / C(3,3,NLAY)
C          R36 = C(3,6,1) / C(3,3,1) - C(3,6,NLAY) / C(3,3,NLAY)
C          S13 = C(1,3,1) / C(3,3,1) + C(1,3,NLAY) / C(3,3,NLAY)
C          S23 = C(2,3,1) / C(3,3,1) + C(2,3,NLAY) / C(3,3,NLAY)
C          S36 = C(3,6,1) / C(3,3,1) + C(3,6,NLAY) / C(3,3,NLAY)
C
C          A1 = - ( HALF / 510.0 ) * S13
C          B1 = - ( HALF / 510.0 ) * 85. * R13
C          A2 = - ( HALF / 510.0 ) * S23
C          B2 = - ( HALF / 510.0 ) * 85. * R23
C          A6 = - ( HALF / 510.0 ) * S36

```

```

B6 = - ( HALF / 510.0 ) * 85. * R36
A3 = 42.D0/85.D0

```

COMPUTE LAMINATE PROPERTY MATRICES

```

DO K= 1, NLAY

```

```

      Z1 = ( Z(K+1)      - Z(K)      )
      Z2 = ( Z(K+1)**2 - Z(K)**2 )
      Z3 = ( Z(K+1)**3 - Z(K)**3 )
      Z4 = ( Z(K+1)**4 - Z(K)**4 )
      Z5 = ( Z(K+1)**5 - Z(K)**5 )
      Z6 = ( Z(K+1)**6 - Z(K)**6 )
      Z7 = ( Z(K+1)**7 - Z(K)**7 )

```

EXPLICIT INTEGRATION OF PHI TERMS (OBTAINED FROM MACSYMA) :

INT[PHI(I)]DZ

```

PII(1) = ( 105*A1*Z4-2*B1*H1*Z3-120*A1*H2*Z2+2*B1*H3*Z1 ) / (4*H3)
PII(2) = ( 105*A2*Z4-2*B2*H1*Z3-120*A2*H2*Z2+2*B2*H3*Z1 ) / (4*H3)
PII(3) = ( - 5*A3*Z4          + 30*A3*H2*Z2          ) / (8*H3)
PII(6) = ( 105*A6*Z4-2*B6*H1*Z3-120*A6*H2*Z2+2*B6*H3*Z1 ) / (4*H3)

```

INT[PHI(I)Z]DZ

```

PIZ(1) = ( 168*A1*Z5-3*B1*H1*Z4-160*A1*H2*Z3+2*B1*H3*Z2 ) / (8*H3)
PIZ(2) = ( 168*A2*Z5-3*B2*H1*Z4-160*A2*H2*Z3+2*B2*H3*Z2 ) / (8*H3)
PIZ(3) = ( - A3*Z5          + 5*A3*H2*Z3          ) / (2*H3)
PIZ(6) = ( 168*A6*Z5-3*B6*H1*Z4-160*A6*H2*Z3+2*B6*H3*Z2 ) / (8*H3)

```

INT[PHI(I)PHI(J)]DZ

```

PIP(1,1) = (( 31500*A1*A1          ) *Z7 +
&          (          -1050*A1*B1          )*H1*Z6 +
&          (-50400*A1*A1          + 9*B1*B1 )*H2*Z5 +
&          (          1425*A1*B1          )*H3*Z4 +
&          ( 24000*A1*A1          -10*B1*B1 )*H4*Z3 +
&          (          - 600*A1*B1          )*H5*Z2 +
&          (          5*B1*B1          )*H6*Z1 ) / (20*H6)
PIP(1,2) = (( 63000*A1*A2          ) * Z7 +
&          (          -1050*(A1*B2+B1*A2) )*H1*Z6 +
&          (-100800*A1*A2          +18*B1*B2)*H2*Z5 +
&          (          1425*(A1*B2+B1*A2) )*H3*Z4 +
&          ( 48000*A1*A2          -20*B1*B2)*H4*Z3 +
&          (          -600*(A1*B2+B1*A2) )*H5*Z2 +
&          (          10*B1*B2)*H6*Z1 ) / (40*H6)
PIP(1,3) = (( -300*A3*A1          ) * Z7 +

```

```

&      (      5*A3*B1)*H1*Z6 +
&      (      1500*A3*A1      )*H2*Z5 +
&      (      -25*A3*B1)*H3*Z4 +
&      (      -1200*A3*A1      )*H4*Z3 +
&      (      15*A3*B1)*H5*Z2 ) / ( 8*H6)
PIP(1,6) = (( 63000*A1*A6      )* Z7 +
&      (      -1050*(A1*B6+B1*A6) )*H1*Z6 +
&      (-100800*A1*A6      +18*B1*B6)*H2*Z5 +
&      (      1425*(A1*B6+B1*A6) )*H3*Z4 +
&      (      48000*A1*A6      -20*B1*B6)*H4*Z3 +
&      (      -600*(A1*B6+B1*A6) )*H5*Z2 +
&      (      10*B1*B6)*H6*Z1 ) / (40*H6)
PIP(2,2) = (( 31500*A2*A2      ) *Z7 +
&      (      -1050*A2*B2      )*H1*Z6 +
&      (-50400*A2*A2      + 9*B2*B2 )*H2*Z5 +
&      (      1425*A2*B2      )*H3*Z4 +
&      (      24000*A2*A2      -10*B2*B2 )*H4*Z3 +
&      (      - 600*A2*B2      )*H5*Z2 +
&      (      5*B2*B2      )*H6*Z1 ) / (20*H6)
PIP(2,3) = (( -300*A3*A2      )* Z7 +
&      (      5*A3*B2)*H1*Z6 +
&      (      1500*A3*A2      )*H2*Z5 +
&      (      -25*A3*B2)*H3*Z4 +
&      (      -1200*A3*A2      )*H4*Z3 +
&      (      15*A3*B2)*H5*Z2 ) / ( 8*H6)
PIP(2,6) = (( 63000*A2*A6      )* Z7 +
&      (      -1050*(A2*B6+B2*A6) )*H1*Z6 +
&      (-100800*A2*A6      +18*B2*B6)*H2*Z5 +
&      (      1425*(A2*B6+B2*A6) )*H3*Z4 +
&      (      48000*A2*A6      -20*B2*B6)*H4*Z3 +
&      (      -600*(A2*B6+B2*A6) )*H5*Z2 +
&      (      10*B2*B6)*H6*Z1 ) / (40*H6)
PIP(3,3) = (( 25*A3*A3      ) *Z7 +
&      (      -210*A3*A3      )*H2*Z5 +
&      (      525*A3*A3      )*H4*Z3 ) / (28*H6)
PIP(3,6) = (( -300*A3*A6      )* Z7 +
&      (      5*A3*B6)*H1*Z6 +
&      (      1500*A3*A6      )*H2*Z5 +
&      (      -25*A3*B6)*H3*Z4 +
&      (      -1200*A3*A6      )*H4*Z3 +
&      (      15*A3*B6)*H5*Z2 ) / ( 8*H6)
PIP(6,6) = (( 31500*A6*A6      ) *Z7 +
&      (      -1050*A6*B6      )*H1*Z6 +
&      (-50400*A6*A6      + 9*B6*B6 )*H2*Z5 +
&      (      1425*A6*B6      )*H3*Z4 +
&      (      24000*A6*A6      -10*B6*B6 )*H4*Z3 +
&      (      - 600*A6*B6      )*H5*Z2 +
&      (      5*B6*B6      )*H6*Z1 ) / (20*H6)

```

COMPUTE THE LAMINATE ABAR, BBAR AND DBAR MATRICES

CALL DMASB (ABAR, Z1, C(1,1,K), ABAR, 6, 6)

```

BBAR(1,1) = BBAR(1,1) + C(1,1,K) * ( Z2 / 2.0 + PII(1) )
BBAR(1,2) = BBAR(1,2) + C(1,2,K) * ( Z2 / 2.0 + PII(2) )
BBAR(1,3) = BBAR(1,3) + C(1,3,K) * ( PII(3) )
BBAR(1,6) = BBAR(1,6) + C(1,6,K) * ( Z2 / 2.0 + PII(6) )
BBAR(2,1) = BBAR(2,1) + C(2,1,K) * ( Z2 / 2.0 + PII(1) )
BBAR(2,2) = BBAR(2,2) + C(2,2,K) * ( Z2 / 2.0 + PII(2) )
BBAR(2,3) = BBAR(2,3) + C(2,3,K) * ( PII(3) )
BBAR(2,6) = BBAR(2,6) + C(2,6,K) * ( Z2 / 2.0 + PII(6) )
BBAR(3,1) = BBAR(3,1) + C(3,1,K) * ( Z2 / 2.0 + PII(1) )
BBAR(3,2) = BBAR(3,2) + C(3,2,K) * ( Z2 / 2.0 + PII(2) )
BBAR(3,3) = BBAR(3,3) + C(3,3,K) * ( PII(3) )
BBAR(3,6) = BBAR(3,6) + C(3,6,K) * ( Z2 / 2.0 + PII(6) )
BBAR(6,1) = BBAR(6,1) + C(6,1,K) * ( Z2 / 2.0 + PII(1) )
BBAR(6,2) = BBAR(6,2) + C(6,2,K) * ( Z2 / 2.0 + PII(2) )
BBAR(6,3) = BBAR(6,3) + C(6,3,K) * ( PII(3) )
BBAR(6,6) = BBAR(6,6) + C(6,6,K) * ( Z2 / 2.0 + PII(6) )

```

```

DBAR(1,1) = DBAR(1,1) + C(3,3,K) * PIP(1,1) + C(1,1,K) * Z3/3.0
& + C(1,3,K) * PIZ(1) + C(1,3,K) * PIZ(1)
DBAR(1,2) = DBAR(1,2) + C(3,3,K) * PIP(1,2) + C(1,2,K) * Z3/3.0
& + C(2,3,K) * PIZ(1) + C(1,3,K) * PIZ(2)
DBAR(1,3) = DBAR(1,3) + C(3,3,K) * PIP(1,3) + C(1,3,K) * PIZ(3)
DBAR(1,6) = DBAR(1,6) + C(3,3,K) * PIP(1,6) + C(1,6,K) * Z3/3.0
& + C(3,6,K) * PIZ(1) + C(1,3,K) * PIZ(6)
DBAR(2,2) = DBAR(2,2) + C(3,3,K) * PIP(2,2) + C(2,2,K) * Z3/3.0
& + C(2,3,K) * PIZ(2) + C(2,3,K) * PIZ(2)
DBAR(2,3) = DBAR(2,3) + C(3,3,K) * PIP(2,3) + C(2,3,K) * PIZ(3)
DBAR(2,6) = DBAR(2,6) + C(3,3,K) * PIP(2,6) + C(2,6,K) * Z3/3.0
& + C(3,6,K) * PIZ(2) + C(2,3,K) * PIZ(6)
DBAR(3,3) = DBAR(3,3) + C(3,3,K) * PIP(3,3)
DBAR(3,6) = DBAR(3,6) + C(3,3,K) * PIP(3,6) + C(3,6,K) * PIZ(3)
DBAR(6,6) = DBAR(6,6) + C(3,3,K) * PIP(6,6) + C(6,6,K) * Z3/3.0
& + C(6,3,K) * PIZ(6) + C(6,3,K) * PIZ(6)

```

INT[P2BAR]DZ

P2B2I = (45 * Z5 - 150 * H2 * Z3 + 225 * H4 * Z1) / (144 * H4)

```

GMAT(1,1) = GMAT(1,1) + C(4,4,K) * P2B2I
GMAT(1,2) = GMAT(1,2) + C(4,5,K) * P2B2I
GMAT(2,2) = GMAT(2,2) + C(5,5,K) * P2B2I
GMAT(2,1) = GMAT(1,2)

```

```

END DO

C
ELSE IF ( NMAT .EQ. 2 ) THEN
C
C   COMPUTE LAMINATE CONSTITUTIVE MATRICES USING
C   STRESS-BASED VERSION OF HIGHER-ORDER THEORY
C   (VERSION I)
C
C   INITIALIZE VARIABLES
C
C   AT1 = 0.0
C   AT2 = 0.0
C   AT12 = 0.0
C   AT3 = 0.0
C
C   A44 = 0.0
C   A45 = 0.0
C   A55 = 0.0
C
DO 100 K=1, NLAY
C
C   HK = Z(K)
C   HKP1 = Z(K+1)
C
C   ZOI = HKP1 - HK
C
C   AT1 = -(ZOI*C(1,3,K)/C(3,3,K))/THK + AT1
C   AT2 = -(ZOI*C(2,3,K)/C(3,3,K))/THK + AT2
C   AT12 = -(ZOI*C(3,6,K)/C(3,3,K))/THK + AT12
C   AT3 = (ZOI/C(3,3,K))/THK + AT3
C
C   AAK(K,1) = -C(1,3,K)/C(3,3,K)
C   AAK(K,2) = -C(2,3,K)/C(3,3,K)
C   AAK(K,3) = 1.00/C(3,3,K)
C   AAK(K,4) = -C(3,6,K)/C(3,3,K)
C
C   CH(K,1) = C(1,1,K) + C(1,3,K)*AAK(K,1)
C   CH(K,2) = C(1,2,K) + C(1,3,K)*AAK(K,2)
C   CH(K,3) = C(1,6,K) + C(1,3,K)*AAK(K,4)
C   CH(K,4) = C(1,3,K)*AAK(K,3)
C   CH(K,5) = C(2,2,K) + C(2,3,K)*AAK(K,2)
C   CH(K,6) = C(2,6,K) + C(3,6,K)*AAK(K,2)
C   CH(K,7) = C(6,6,K) - C(3,6,K)**2/C(3,3,K)
C
C   C44 = C(K,4,4)
C   C45 = C(K,4,5)
C   C55 = C(K,5,5)
C

```

SS44 = C55/(C44*C55-C45**2)
 SS55 = C44/(C44*C55-C45**2)
 SS45 = -C45/(C44*C55-C45**2)

C

1 AFACTOR = -(4*HKP1**3-3*THK**2*HKP1 -
 4*HK**3+3*THK**2*HK)/THK**3/2.0

A44 = A44 + AFACTOR*SS44
 A45 = A45 + AFACTOR*SS45
 A55 = A55 + AFACTOR*SS55

C

100 CONTINUE

C

DENN= 1.0/(A44*A55-A45**2)
 B44 = (5.0/4.0)*A44*DENN
 B45 = -(5.0/4.0)*A45*DENN
 B55 = (5.0/4.0)*A55*DENN

C

AR13 = AT1/AT3
 AR23 = AT2/AT3
 AR123= AT12/AT3

C

DO 200 K = 1, NLAY

C

HK = Z(K)
 HKP1 = Z(K+1)

C

ZOI = HKP1 - HK

C

Z1I = (HKP1**2 - HK**2) / 2.0
 Z2I = (HKP1**3 - HK**3) / 3.0
 Z3I = -.02941176470588236*(14.0*HKP1**4-21.0*THK**2*HKP1**2
 1 -14.0*HK**4 + 21.0*THK**2*HK**2)/THK**2

C

1 Z4I = (560.0*HKP1**7-1176.0*THK**2*HKP1**5
 1 +735.0*THK**4*HKP1**3-560.0*HK**7
 2 +1176.0*THK**2*HK**5-735.0*THK**4*HK**3)
 3 /THK**4/1445.0

C

BK(K,1) = AAK(K,1) - AAK(K,3)*AR13
 BK(K,2) = AAK(K,2) - AAK(K,3)*AR23
 BK(K,3) = AAK(K,4) - AAK(K,3)*AR123

C

D3(K) = AAK(K,3)/AT3

C

TB = CH(K,4) + BK(K,1)
 ABC(K,1)= TB*AR13
 ABC(K,2)= TB*AR23

ABC(K,4)= TB*AR123

ABC(K,3)= TB/AT3

CHH(K,1) = CH(K,1)-ABC(K,1)

CHH(K,2) = CH(K,2)-ABC(K,2)

CHH(K,3) = CH(K,3)-ABC(K,4)

COMPUTE THE LAMINATE ABAR, BBAR AND DBAR MATRICES

ABAR(1,1) = ABAR(1,1) + CHH(K,1) * ZOI

ABAR(1,2) = ABAR(1,2) + CHH(K,2) * ZOI

ABAR(1,3) = ABAR(1,3) + ABC(K,3) * ZOI

ABAR(1,6) = ABAR(1,6) + CHH(K,3) * ZOI

ABAR(2,2) = ABAR(2,2)+(CH(K,5)-AR23*(BK(K,2)-AAK(K,2)))*ZOI

ABAR(2,3) = ABAR(2,3) - AR23 * ZOI/(C(K,3,3)*AT3)

ABAR(2,6) = ABAR(2,6) + (C(K,2,6)
+ (AR123*AR23-C(K,2,3)*C(K,3,6))/C(K,3,3))* ZOI

BBAR(1,1) = BBAR(1,1) + CH(K,1) * Z1I - ABC(K,1)*Z3I

BBAR(1,2) = BBAR(1,2) + CH(K,2) * Z1I - ABC(K,2)*Z3I

BBAR(1,3) = BBAR(1,3) + 2.0*ABC(K,3)* Z3I

BBAR(1,6) = BBAR(1,6) + CH(K,3) * Z1I - ABC(K,4)*Z3I

BBAR(2,1) = BBAR(2,1) + CH(K,2) * Z1I + AR13*AR23*Z3I/C(K,3,3)

BBAR(2,2) = BBAR(2,2) + CH(K,5) * Z1I + AR23*AR23*Z3I/C(K,3,3)

BBAR(2,3) = BBAR(2,3) - 2.0*AR23* Z3I/(AT3*C(K,3,3))

BBAR(2,6) = BBAR(2,6) + CH(K,6) * Z1I+ AR123*AR23*Z3I/C(K,3,3)

FC1 = 1.0/(C(K,3,3)*AT3**2)

ABAR(3,3) = ABAR(3,3) + ZOI*FC1

ABAR(3,6) = ABAR(3,6) - ZOI*AT12*FC1

BBAR(3,1) = BBAR(3,1) - AT1*Z3I*FC1

BBAR(3,2) = BBAR(3,2) - AT2*Z3I*FC1

BBAR(3,3) = BBAR(3,3) + 2.0*Z3I*FC1

BBAR(3,6) = BBAR(3,6) - AT12*Z3I*FC1

FC2 = - AR123/C(K,3,3)

ABAR(6,6) = ABAR(6,6) + (CH(K,7) - AR123*FC2)*ZOI

BBAR(6,1) = BBAR(6,1) + CH(K,3) * Z1I - AR13*Z3I*FC2

BBAR(6,2) = BBAR(6,2) + CH(K,6) * Z1I - AR23*Z3I*FC2

BBAR(6,3) = BBAR(6,3) + 2.0* FC2* Z3I/AT3

BBAR(6,6) = BBAR(6,6) + CH(K,7) * Z1I - AR123*Z3I*FC2

```

FC3 = AR13/C(K,3,3)
DBAR(1,1) = DBAR(1,1) + CH(K,1) * Z2I + AR13*FC3*Z4I
DBAR(1,2) = DBAR(1,2) + CH(K,2) * Z2I + AR23*FC3*Z4I
DBAR(1,3) = DBAR(1,3) - 2.0*FC3*Z4I/AT3
DBAR(1,6) = DBAR(1,6) + CH(K,3) * Z2I + AR123*FC3*Z4I

```

C

```

FC4 = AR23/C(K,3,3)

```

C

```

DBAR(2,2) = DBAR(2,2) + CH(K,5) * Z2I + AR23*FC4*Z4I
DBAR(2,3) = DBAR(2,3) - 2.0*FC4*Z4I/AT3
DBAR(2,6) = DBAR(2,6) + CH(K,6) * Z2I + AR123*FC4*Z4I

```

C

```

DBAR(3,3) = DBAR(3,3) + 4.0*D3(K)*Z4I/AT3
DBAR(3,6) = DBAR(3,6) - 2.0*AR123*Z4I/(AT3*C(K,3,3))

```

C

```

DBAR(6,6) = DBAR(6,6) + Z2I*CH(K,7) + AR123**2*Z4I*AAK(K,3)

```

C

```

ABAR(2,1) = ABAR(1,2)
ABAR(3,1) = ABAR(1,3)
ABAR(3,2) = ABAR(2,3)
ABAR(6,1) = ABAR(1,6)
ABAR(6,2) = ABAR(2,6)
ABAR(6,3) = ABAR(3,6)

```

C

```

FACTOR = (48*HKP1**5-40*tHK**2*HKP1**3+15*tHK**4*HKP1-
1      48*HK**5+40*tHK**2*HK**3-15*tHK**4*HK)/tHK**4/15.0

```

C

```

C44 = C(K,4,4)
C45 = C(K,4,5)
C55 = C(K,5,5)

```

C

```

1 GCAP11 = (25*A45**2*C55+50*A44*A45*C45+25*A44**2*C44)/(C44*
2      (16*A44**2*A55**2-32*A44*A45**2*A55+16*A45**4)*C55+
3      C45**2*(-16*A44**2*A55**2+32*A44*A45**2*A55-
      16*A45**4))

```

C

```

1 GCAP12 = -(25*A45*A55*C55+C45*(25*A44*A55+25*A45**2)+
2      25*A44*A45*C44)/(C44*(16*A44**2*A55**2-32*A44*
3      A45**2*A55+16*A45**4)*C55+C45**2*(-16*A44**2*
      A55**2+32*A44*A45**2*A55-16*A45**4))

```

C

```

1 GCAP22 = (25*A55**2*C55+50*A45*A55*C45+25*A45**2*C44)/(C44*
2      (16*A44**2*A55**2-32*A44*A45**2*A55+16*A45**4)*C55+
      C45**2*(-16*A44**2*A55**2+32*A44*A45**2*A55-16*A45**4))

```

C

```

GMAT(2,2) = GMAT(2,2) + GCAP11*FACTOR
GMAT(1,2) = GMAT(1,2) + GCAP12*FACTOR
GMAT(1,1) = GMAT(1,1) + GCAP22*FACTOR

```

```

      GMAT(2,1) = GMAT(1,2)
C
200 CONTINUE
C
C      SCALING TO THE ORIGINAL LEGENDRE FORMULATION
C      WITH THE USE OF (3/2) FACTOR AND HALF THICKNESS
C
      SCALE = (3.0/2.0)/(thk/2.0)
C
      DO 99 I = 1, 3
        BBAR(I,3) = BBAR(I,3)*SCALE
        DBAR(I,3) = DBAR(I,3)*SCALE
99    CONTINUE
C
      DBAR(3,6) = DBAR(3,6)*SCALE
      BBAR(6,3) = BBAR(6,3)*SCALE
      DBAR(3,3) = DBAR(3,3)*SCALE
C
      DBAR(2,1) = DBAR(1,2)
      DBAR(3,1) = DBAR(1,3)
      DBAR(3,2) = DBAR(2,3)
      DBAR(6,1) = DBAR(1,6)
      DBAR(6,2) = DBAR(2,6)
      DBAR(6,3) = DBAR(3,6)
C
      ELSE IF ( NMAT .EQ. 3 ) THEN
C
      COMPUTE LAMINATE CONSTITUTIVE MATRICES USING
      STRESS-BASED VERSION OF HIGHER-ORDER THEORY
      (VERSION II)
C
      INITIALIZE VARIABLES
C
      S11 = 0.0
      S12 = 0.0
      S22 = 0.0
      T11 = 0.0
      T12 = 0.0
      T16 = 0.0
      T13 = 0.0
      TH11 = 0.0
      TH12 = 0.0
      TH16 = 0.0
      TH13 = 0.0
      T21 = 0.0
      T22 = 0.0
      T26 = 0.0
      T23 = 0.0

```

TH21 = 0.0
 TH22 = 0.0
 TH26 = 0.0
 TH23 = 0.0

C

A11 = 0.0
 A12 = 0.0
 A21 = 0.0
 A22 = 0.0
 B11 = 0.0
 B22 = 0.0
 A44 = 0.0
 A45 = 0.0
 A55 = 0.0

C

HL = HALF

C

DO K = 1, NLAY

C

HK = Z(K)
 HKP1 = Z(K+1)
 ZOI = HKP1 - HK
 Z1I = (HKP1**2 - HK**2) / 2.0

C

1 FINT = -(HKP1-HK)*(HKP1+HK)*(-3*THK**2+2*HKP1**2+
 2*HK**2)/THK**3/3.0
 1 FINT2 = (320*HKP1**7-672*THK**2*HKP1**5+420*THK**4*HKP1**3-
 320*HK**7+672*THK**2*HK**5-420*THK**4*HK**3)/
 2 THK**6/315.0
 ZFINT = -(8*HKP1**5-10*THK**2*HKP1**3-8*HK**5+
 1 10*THK**2*HK**3)/THK**3/15.0
 PINT = -(4*HKP1**5-3*THK**2*HKP1**3-4*HK**3+3*THK**2*HK)/
 1 THK**2/3.0
 PINT2 = (48*HKP1**5-40*THK**2*HKP1**3+15*THK**4*HKP1-
 1 48*HK**5+40*THK**2*HK**3-15*THK**4*HK)/THK**4/15.0

C

S33(K) = 1.0 / C(3,3,K)
 RA13(K) = C(1,3,K) / C(3,3,K)
 RA23(K) = C(2,3,K) / C(3,3,K)
 RA63(K) = C(6,3,K) / C(3,3,K)

C

C1(1,1,K) = C(1,1,K) - RA13(K) * C(1,3,K)
 C1(1,2,K) = C(1,2,K) - RA13(K) * C(2,3,K)
 C1(1,6,K) = C(1,6,K) - RA13(K) * C(6,3,K)
 C1(2,2,K) = C(2,2,K) - RA23(K) * C(2,3,K)
 C1(2,6,K) = C(2,6,K) - RA23(K) * C(6,3,K)
 C1(6,6,K) = C(6,6,K) - RA63(K) * C(6,3,K)

C

S11 = S11 + ZOI * S33(K)**2
 S12 = S12 + FINT * S33(K)**2
 S22 = S22 + FINT2 * S33(K)**2

C

T11 = T11 + ZOI * C(1,3,K)* S33(K)**2
 T12 = T12 + ZOI * C(2,3,K)* S33(K)**2
 T16 = T16 + ZOI * C(3,6,K)* S33(K)**2
 T13 = T13 + ZOI * S33(K)

C

T21 = T21 + FINT * C(1,3,K)* S33(K)**2
 T22 = T22 + FINT * C(2,3,K)* S33(K)**2
 T26 = T26 + FINT * C(3,6,K)* S33(K)**2
 T23 = T23 + FINT * S33(K)

C

TH11 = TH11 + Z1I * C(1,3,K)* S33(K)**2
 TH12 = TH12 + Z1I * C(2,3,K)* S33(K)**2
 TH16 = TH16 + Z1I * C(3,6,K)* S33(K)**2
 TH13 = TH13 + 2.0 * Z1I * S33(K)

C

TH21 = TH21 + ZFINT * C(1,3,K)* S33(K)**2
 TH22 = TH22 + ZFINT * C(2,3,K)* S33(K)**2
 TH26 = TH26 + ZFINT * C(3,6,K)* S33(K)**2
 TH23 = TH23 + 2.0 * ZFINT * S33(K)

C

C44 = C(4,4,K)
 C45 = C(4,5,K)
 C55 = C(5,5,K)

C

S44 = C55 / (C44*C55-C45**2)
 S55 = C44 / (C44*C55-C45**2)
 S45 = -C45 / (C44*C55-C45**2)

C

A11 = A11 + FINT2 * S55**2
 A12 = A12 + PINT2 * S45*S55
 A21 = A21 + PINT2 * S45*S44
 A22 = A22 + PINT2 * S44**2
 B11 = B11 + PINT * S55
 B22 = B22 + PINT * S44

C

AFACTOR = -(4*HKP1**3-3*THK**2*HKP1-4*HK**3+3*THK**2*HK)/
 THK**3/2.0

1

C

A44 = A44 + AFACTOR * S44
 A45 = A45 + AFACTOR * S45
 A55 = A55 + AFACTOR * S55

C

END DO

C

```

B44 = (5.0/4.0) * A44 / (A44*A55-A45**2)
B45 = -(5.0/4.0) * A45 / (A44*A55-A45**2)
B55 = (5.0/4.0) * A55 / (A44*A55-A45**2)

```

AVERAGE COEFFICIENTS

```

R11 = S22 / (S11*S22 - S12**2)
R22 = S11 / (S11*S22 - S12**2)
R12 = -S12 / (S11*S22 - S12**2)

```

```

Q11 = R11 * T11 + R12 * T21
Q21 = R12 * T11 + R22 * T21
Q12 = R11 * T12 + R12 * T22
Q22 = R12 * T12 + R22 * T22
Q13 = R11 * T13 + R12 * T23
Q23 = R12 * T13 + R22 * T23
Q16 = R11 * T16 + R12 * T26
Q26 = R12 * T16 + R22 * T26

```

```

QH11 = R11 * TH11 + R12 * TH21
QH21 = R12 * TH11 + R22 * TH21
QH12 = R11 * TH12 + R12 * TH22
QH22 = R12 * TH12 + R22 * TH22
QH16 = R11 * TH16 + R12 * TH26
QH26 = R12 * TH16 + R22 * TH26
QH13 = R11 * TH13 + R12 * TH23
QH23 = R12 * TH13 + R22 * TH23

```

```

D11 = A22 * B11 / (A11*A22 - A12*A21)
D12 = -A12 * B22 / (A11*A22 - A12*A21)
D21 = -A21 * B11 / (A11*A22 - A12*A21)
D22 = A11 * B22 / (A11*A22 - A12*A21)

```

SCALE = (3.0 / 2.0) / HL

DO K=1, NLAY

```

HK = Z(K)
HKP1 = Z(K+1)
ZOI = HKP1 - HK
Z1I = (HKP1**2 - HK**2) / 2.0
Z2I = (HKP1**3 - HK**3) / 3.0

```

```

PINT2 = (48*HKP1**5-40*THK**2*HKP1**3+15*THK**4*HKP1-
1      48*HK**5+40*THK**2*HK**3-15*THK**4*HK)/
2      THK**4/15.0

```

FINT = -(HKP1-HK)*(HKP1+HK)*(-3*THK**2+2*HKP1**2+

4 Q26*(Q21*(-320*HK**7+672*THK**2*HK**5-420*THK**4*
 5 HK**3)+210*Q11*THK**3*HK**4-315*Q11*THK**5*HK**2)+
 6 Q21*(210*Q16*THK**3*HK**4-315*Q16*THK**5*HK**2)-
 7 315*Q11*Q16*THK**6*HK)/THK**6/315.0

C

P5 = (320*Q22**2*HKP1**7-672*Q22**2*THK**2*HKP1**5-
 1 420*Q12*Q22*THK**3*HKP1**4+420*Q22**2*THK**4*
 2 HKP1**3+630*Q12*Q22*THK**5*HKP1**2+315*Q12**2*
 3 THK**6*HKP1+Q22**2*(-320*HK**7+672*THK**2*HK**5-
 4 420*THK**4*HK**3)+Q12*Q22*(420*THK**3*HK**4-
 5 630*THK**5*HK**2)-315*Q12**2*THK**6*HK)/
 6 THK**6/315.0

C

P6 = (HKP1**2*(315*Q12*Q23*THK**5+315*Q13*Q22*THK**5)+
 1 HKP1**4*(-210*Q12*Q23*THK**3-210*Q13*Q22*THK**3)+
 2 320*Q22*Q23*HKP1**7-672*Q22*Q23*THK**2*HKP1**5+
 3 420*Q22*Q23*THK**4*HKP1**3+315*Q12*Q13*THK**6*HKP1+
 4 Q23*(Q22*(-320*HK**7+672*THK**2*HK**5-420*THK**4*
 5 HK**3)+210*Q12*THK**3*HK**4-315*Q12*THK**5*HK**2)+
 6 Q22*(210*Q13*THK**3*HK**4-315*Q13*THK**5*HK**2)-
 7 315*Q12*Q13*THK**6*HK)/THK**6/315.0

C

P7 = (HKP1**2*(315*Q12*Q26*THK**5+315*Q16*Q22*THK**5)+
 1 HKP1**4*(-210*Q12*Q26*THK**3-210*Q16*Q22*THK**3)+
 2 320*Q22*Q26*HKP1**7-672*Q22*Q26*THK**2*HKP1**5+
 3 420*Q22*Q26*THK**4*HKP1**3+315*Q12*Q16*THK**6*HKP1+
 4 Q26*(Q22*(-320*HK**7+672*THK**2*HK**5-420*THK**4*
 5 HK**3)+210*Q12*THK**3*HK**4-315*Q12*THK**5*HK**2)+
 6 Q22*(210*Q16*THK**3*HK**4-315*Q16*THK**5*HK**2)-
 7 315*Q12*Q16*THK**6*HK)/THK**6/315.0

C

P8 = (320*Q23**2*HKP1**7-672*Q23**2*THK**2*HKP1**5-
 1 420*Q13*Q23*THK**3*HKP1**4+420*Q23**2*THK**4*
 2 HKP1**3+630*Q13*Q23*THK**5*HKP1**2+315*Q13**2*
 3 THK**6*HKP1+Q23**2*(-320*HK**7+672*THK**2*HK**5-
 4 420*THK**4*HK**3)+Q13*Q23*(420*THK**3*HK**4-
 5 630*THK**5*HK**2)-315*Q13**2*THK**6*HK)/
 6 THK**6/315.0

C

P9 = (HKP1**2*(315*Q13*Q26*THK**5+315*Q16*Q23*THK**5)+
 1 HKP1**4*(-210*Q13*Q26*THK**3-210*Q16*Q23*THK**3)+
 2 320*Q23*Q26*HKP1**7-672*Q23*Q26*THK**2*HKP1**5+
 3 420*Q23*Q26*THK**4*HKP1**3+315*Q13*Q16*THK**6*HKP1+
 4 Q26*(Q23*(-320*HK**7+672*THK**2*HK**5-420*THK**4*
 5 HK**3)+210*Q13*THK**3*HK**4-315*Q13*THK**5*HK**2)+
 6 Q23*(210*Q16*THK**3*HK**4-315*Q16*THK**5*HK**2)-
 7 315*Q13*Q16*THK**6*HK)/THK**6/315.0

C

P10 = (320*Q26**2*HKP1**7-672*Q26**2*THK**2*HKP1**5-
 1 420*Q16*Q26*THK**3*HKP1**4+420*Q26**2*THK**4*
 2 HKP1**3+630*Q16*Q26*THK**5*HKP1**2+315*Q16**2*
 3 THK**6*HKP1+Q26**2*(-320*HK**7+672*THK**2*HK**5-
 4 420*THK**4*HK**3)+Q16*Q26*(420*THK**3*HK**4-
 5 630*THK**5*HK**2)-315*Q16**2*THK**6*HK)/THK**6/315.0

C

PH1 = (HKP1**2*(315*Q11*QH23*THK**5+315*Q21*QH13*THK**5)+
 1 HKP1**4*(-210*Q11*QH23*THK**3-210*Q21*QH13*THK**3)+
 2 320*Q21*QH23*HKP1**7-672*Q21*QH23*THK**2*HKP1**5+
 3 420*Q21*QH23*THK**4*HKP1**3+315*Q11*QH13*THK**6*
 4 HKP1+QH23*(Q21*(-320*HK**7+672*THK**2*HK**5-
 5 420*THK**4*HK**3)+210*Q11*THK**3*HK**4-315*Q11*
 6 THK**5*HK**2)+Q21*(210*QH13*THK**3*HK**4-
 7 315*QH13*THK**5*HK**2)-315*Q11*QH13*THK**6*HK)/
 8 THK**6/315.0

C

PH2 = (HKP1**2*(315*Q12*QH23*THK**5+315*QH13*Q22*THK**5)+
 1 HKP1**4*(-210*Q12*QH23*THK**3-210*QH13*Q22*THK**3)+
 2 320*Q22*QH23*HKP1**7-672*Q22*QH23*THK**2*HKP1**5+
 3 420*Q22*QH23*THK**4*HKP1**3+315*Q12*QH13*THK**6*
 4 HKP1+QH23*(Q22*(-320*HK**7+672*THK**2*HK**5-
 5 420*THK**4*HK**3)+210*Q12*THK**3*HK**4-315*Q12*
 6 THK**5*HK**2)+Q22*(210*QH13*THK**3*HK**4-
 7 315*QH13*THK**5*HK**2)-315*Q12*QH13*THK**6*HK)/
 8 THK**6/315.0

C

PH3 = (HKP1**2*(315*QH11*Q23*THK**5+315*Q13*QH21*THK**5)+
 1 HKP1**4*(-210*QH11*Q23*THK**3-210*Q13*QH21*THK**3)+
 2 320*Q23*QH21*HKP1**7-672*Q23*QH21*THK**2*HKP1**5+
 3 420*Q23*QH21*THK**4*HKP1**3+315*Q13*QH11*THK**6*
 4 HKP1+QH21*(Q23*(-320*HK**7+672*THK**2*HK**5-
 5 420*THK**4*HK**3)+210*Q13*THK**3*HK**4-315*Q13*
 6 THK**5*HK**2)+Q23*(210*QH11*THK**3*HK**4-
 7 315*QH11*THK**5*HK**2)-315*Q13*QH11*THK**6*HK)/
 8 THK**6/315.0

C

PH4 = (HKP1**2*(315*QH12*Q23*THK**5+315*Q13*QH22*THK**5)+
 1 HKP1**4*(-210*QH12*Q23*THK**3-210*Q13*QH22*THK**3)+
 2 320*Q23*QH22*HKP1**7-672*Q23*QH22*THK**2*HKP1**5+
 3 420*Q23*QH22*THK**4*HKP1**3+315*Q13*QH12*THK**6*
 4 HKP1+QH22*(Q23*(-320*HK**7+672*THK**2*HK**5-
 5 420*THK**4*HK**3)+210*Q13*THK**3*HK**4-315*Q13*
 6 THK**5*HK**2)+Q23*(210*QH12*THK**3*HK**4-
 7 315*QH12*THK**5*HK**2)-315*Q13*QH12*THK**6*HK)/
 8 THK**6/315.0

C

PH5 = (HKP1**2*(315*QH13*Q23*THK**5+315*Q13*QH23*THK**5)+

```

1      HKP1**4*(-210*QH13*Q23*THK**3-210*Q13*QH23*THK**3)+
2      S20*Q23*QH23*HKP1**7-672*Q23*QH23*THK**2*HKP1**5+
3      420*Q23*QH23*THK**4*HKP1**3+315*Q13*QH13*THK**6*.
4      HKP1+QH23*(Q23*(-320*HK**7+672*THK**2*HK**5-
5      420*THK**4*HK**3)+210*Q13*THK**3*HK**4-315*Q13*
6      THK**5*HK**2)+Q23*(210*QH13*THK**3*HK**4-
7      315*QH13*THK**5*HK**2)-315*Q13*QH13*THK**6*HK)/
8      THK**6/315.0

```

C

```

      PH6 = (HKP1**2*(315*QH16*Q23*THK**5+315*Q13*QH26*THK**5)+
1      HKP1**4*(-210*QH16*Q23*THK**3-210*Q13*QH26*THK**3)+
2      320*Q23*QH26*HKP1**7-672*Q23*QH26*THK**2*HKP1**5+
3      420*Q23*QH26*THK**4*HKP1**3+315*Q13*QH16*THK**6*
4      HKP1+QH26*(Q23*(-320*HK**7+672*THK**2*HK**5-
5      420*THK**4*HK**3)+210*Q13*THK**3*HK**4-315*Q13*
6      THK**5*HK**2)+Q23*(210*QH16*THK**3*HK**4-
7      315*QH16*THK**5*HK**2)-315*Q13*QH16*THK**6*HK)/
8      THK**6/315.0

```

C

```

      PH7 = (HKP1**2*(315*QH13*Q26*THK**5+315*Q16*QH23*THK**5)+
1      HKP1**4*(-210*QH13*Q26*THK**3-210*Q16*QH23*THK**3)+
2      320*Q26*QH23*HKP1**7-672*Q26*QH23*THK**2*HKP1**5+
3      420*Q26*QH23*THK**4*HKP1**3+315*Q16*QH13*THK**6*
4      HKP1+QH23*(Q26*(-320*HK**7+672*THK**2*HK**5-
5      420*THK**4*HK**3)+210*Q16*THK**3*HK**4-315*Q16*
6      THK**5*HK**2)+Q26*(210*QH13*THK**3*HK**4-
7      315*QH13*THK**5*HK**2)-315*Q16*QH13*THK**6*HK)/
8      THK**6/315.0

```

C

```

ABAR(1,1) = ABAR(1,1) + ZOI * C1(1,1,K) + P1 * S33(K)
ABAR(1,2) = ABAR(1,2) + ZOI * C1(1,2,K) + P2 * S33(K)
ABAR(1,3) = ABAR(1,3) + P3 * S33(K)
ABAR(1,6) = ABAR(1,6) + ZOI * C1(1,6,K) + P4 * S33(K)
ABAR(2,2) = ABAR(2,2) + ZOI * C1(2,2,K) + P5 * S33(K)
ABAR(2,3) = ABAR(2,3) + P6 * S33(K)
ABAR(2,6) = ABAR(2,6) + ZOI * C1(2,6,K) + P7 * S33(K)
ABAR(3,3) = ABAR(3,3) + P8 * S33(K)
ABAR(3,6) = ABAR(3,6) + P9 * S33(K)
ABAR(6,6) = ABAR(6,6) + ZOI * C1(6,6,K) + P10* S33(K)

```

C

```

      PG1 = ZOI * Q11 * QH11 + FINT * (QH11 * Q21 + Q11 * QH21)
&          + FINT2 * QH21 * Q21
      PG2 = ZOI * Q11 * QH12 + FINT * (QH12 * Q21 + Q11 * QH22)
&          + FINT2 * QH22 * Q21
      PG3 = ZOI * Q11 * QH13 + FINT * (QH13 * Q21 + Q11 * QH23)
&          + FINT2 * QH23 * Q21
      PG4 = ZOI * Q11 * QH16 + FINT * (QH16 * Q21 + Q11 * QH26)
&          + FINT2 * QH26 * Q21

```

```

& PG5 = ZOI * Q12 * QH11 + FINT * (QH11 * Q22 + Q12 * QH21)
&      + FINT2 * QH21 * Q22
& PG6 = ZOI * Q12 * QH12 + FINT * (QH12 * Q22 + Q12 * QH22)
&      + FINT2 * QH22 * Q22
& PG7 = ZOI * Q12 * QH13 + FINT * (QH13 * Q22 + Q12 * QH23)
&      + FINT2 * QH23 * Q22
& PG8 = ZOI * Q12 * QH16 + FINT * (QH16 * Q22 + Q12 * QH26)
&      + FINT2 * QH26 * Q22
& PG9 = ZOI * Q13 * QH11 + FINT * (QH11 * Q23 + Q13 * QH21)
&      + FINT2 * QH21 * Q23
& PG10 = ZOI * Q13 * QH12 + FINT * (QH12 * Q23 + Q13 * QH22)
&      + FINT2 * QH22 * Q23
& PG11 = ZOI * Q13 * QH13 + FINT * (QH13 * Q23 + Q13 * QH23)
&      + FINT2 * QH23 * Q23
& PG12 = ZOI * Q13 * QH16 + FINT * (QH16 * Q23 + Q13 * QH26)
&      + FINT2 * QH26 * Q23
& PG13 = ZOI * Q16 * QH11 + FINT * (QH11 * Q26 + Q16 * QH21)
&      + FINT2 * QH21 * Q26
& PG14 = ZOI * Q16 * QH12 + FINT * (QH12 * Q26 + Q16 * QH22)
&      + FINT2 * QH22 * Q26
& PG15 = ZOI * Q16 * QH13 + FINT * (QH13 * Q26 + Q16 * QH23)
&      + FINT2 * QH23 * Q26
& PG16 = ZOI * Q16 * QH16 + FINT * (QH16 * Q26 + Q16 * QH26)
&      + FINT2 * QH26 * Q26

```

C

```

BBAR(1,3) = BBAR(1,3) + SCALE * PG3 * S33(K)
BBAR(2,3) = BBAR(2,3) + SCALE * PG7 * S33(K)
BBAR(3,1) = BBAR(3,1) + PG9 * S33(K)
BBAR(3,2) = BBAR(3,2) + PG10 * S33(K)
BBAR(3,3) = BBAR(3,3) + SCALE * PG11 * S33(K)
BBAR(3,6) = BBAR(3,6) + PG12 * S33(K)
BBAR(6,3) = BBAR(6,3) + SCALE * PG15 * S33(K)

```

C

```

BBAR(1,1) = BBAR(1,1) + Z1I * C1(1,1,K) + PG1 * S33(K)
BBAR(1,2) = BBAR(1,2) + Z1I * C1(1,2,K) + PG2 * S33(K)
BBAR(1,6) = BBAR(1,6) + Z1I * C1(1,6,K) + PG4 * S33(K)
BBAR(2,1) = BBAR(2,1) + Z1I * C1(1,2,K) + PG5 * S33(K)
BBAR(2,2) = BBAR(2,2) + Z1I * C1(2,2,K) + PG6 * S33(K)
BBAR(2,6) = BBAR(2,6) + Z1I * C1(2,6,K) + PG8 * S33(K)
BBAR(6,1) = BBAR(6,1) + Z1I * C1(1,6,K) + PG13 * S33(K)
BBAR(6,2) = BBAR(6,2) + Z1I * C1(2,6,K) + PG14 * S33(K)
BBAR(6,6) = BBAR(6,6) + Z1I * C1(6,6,K) + PG16 * S33(K)

```

C

```

& PF1 = ZOI * QH11**2 + 2.0 * FINT * QH11 * QH21 +
&      FINT2 * QH21**2

```

C

```

& PF2 = ZOI * QH11 * QH12 + FINT * (QH11 * QH22 + QH21 * QH12)
&      + FINT2 * QH21 * QH22

```

```

C      PF3 = ZOI * QH11 * QH13 + FINT * (QH21 * QH13 + QH11 * QH23)
      &      + FINT2 * QH21 * QH23
C
C      PF4 = ZOI * QH11 * QH16 + FINT * (QH11 * QH26 + QH21 * QH16)
      &      + FINT2 * QH21 * QH26
C
C      PF5 = ZOI * QH12**2 + 2.0 * FINT * QH12 * QH22 +
      &      FINT2 * QH22**2
C
C      PF6 = ZOI * QH12 * QH13 + FINT * (QH12 * QH23 + QH22 * QH13)
      &      + FINT2 * QH22 * QH23
C
C      PF7 = ZOI * QH12 * QH16 + FINT * (QH12 * QH26 + QH22 * QH16)
      &      + FINT2 * QH22 * QH26
C
C      PF8 = ZOI * QH13**2 + 2.0 * FINT * QH13 * QH23 +
      &      FINT2 * QH23**2
C
C      PF9 = ZOI * QH13 * QH16 + FINT * (QH13 * QH26 + QH23 * QH16)
      &      + FINT2 * QH23 * QH26
C
C      PF10= ZOI * QH16**2 + 2.0 * FINT * QH16 * QH26 +
      &      FINT2 * QH26**2
C
C      DBAR(1,1) = DBAR(1,1) + Z2I * C1(1,1,K) + PF1 * S33(K)
C      DBAR(1,2) = DBAR(1,2) + Z2I * C1(1,2,K) + PF2 * S33(K)
C      DBAR(1,3) = DBAR(1,3) +          SCALE * PF3 * S33(K)
C      DBAR(1,6) = DBAR(1,6) + Z2I * C1(1,6,K) + PF4 * S33(K)
C      DBAR(2,2) = DBAR(2,2) + Z2I * C1(2,2,K) + PF5 * S33(K)
C      DBAR(2,3) = DBAR(2,3) +          SCALE * PF6 * S33(K)
C      DBAR(2,6) = DBAR(2,6) + Z2I * C1(2,6,K) + PF7 * S33(K)
C      DBAR(3,3) = DBAR(3,3) +          SCALE**2 * PF8 * S33(K)
C      DBAR(3,6) = DBAR(3,6) +          SCALE * PF9 * S33(K)
C      DBAR(6,6) = DBAR(6,6) + Z2I * C1(6,6,K) + PF10* S33(K)
C
C      INT[P2BAR]DZ
C
C      P2B2I = (AK**4)*(9.*(HKP1**5-HK**5)-30.*HL**2*(HKP1**3-
1      HK**3)+45.*HL**4*(HKP1-HK))/(20.*HL**4)
C
C      GMAT(1,1) = GMAT(1,1) + C(4,4,K) * P2B2I
C      GMAT(1,2) = GMAT(1,2) + C(4,5,K) * P2B2I
C      GMAT(2,2) = GMAT(2,2) + C(5,5,K) * P2B2I
C      GMAT(2,1) = GMAT(1,2)
C
C      END DO
C

```

```

    ABAR(2,1) = ABAR(1,2)
    ABAR(3,1) = ABAR(1,3)
    ABAR(3,2) = ABAR(2,3)
    ABAR(6,1) = ABAR(1,6)
    ABAR(6,2) = ABAR(2,6)
    ABAR(6,3) = ABAR(3,6)
    DBAR(2,1) = DBAR(1,2)
    DBAR(3,1) = DBAR(1,3)
    DBAR(3,2) = DBAR(2,3)
    DBAR(6,1) = DBAR(1,6)
    DBAR(6,2) = DBAR(2,6)
    DBAR(6,3) = DBAR(3,6)
C
    END IF
C
C    APPLY CORRECTION FACTORS TO LAMINATE MATRICES
C
    ACSHR = PI**2/10.0
    ACTHK = PI/SQRT(12.0)
    ACHMD = PI*SQRT(17.0/252.0)
C
    DO I = 1, 6
        ABAR(I,3) = ABAR(I,3) * ACTHK
        ABAR(3,I) = ABAR(3,I) * ACTHK
        BBAR(3,I) = BBAR(3,I) * ACTHK
        BBAR(I,3) = BBAR(I,3) * ACHMD
        DBAR(3,I) = DBAR(3,I) * ACHMD
        DBAR(I,3) = DBAR(I,3) * ACHMD
    END DO
C
    DO I = 1, 2
        DO J = 1, 2
            GMAT(I,J) = GMAT(I,J) * ACSHR
        END DO
    END DO
C
C    REORDER STORAGE OF A, B AND D MATRICES FOR CALLING PROGRAM
C
    DO I = 1, 4
        DO J = I, 4
            N = I
            M = J
            IF ( I .EQ. 4 ) N = 6
            IF ( J .EQ. 4 ) M = 6
            AMAT(I,J) = ABAR(N,M)
            AMAT(J,I) = ABAR(N,M)
            BMAT(I,J) = BBAR(N,M)
            BMAT(J,I) = BBAR(M,N)

```

```

      DMAT(I,J) = DBAR(N,M)
      DMAT(J,I) = DBAR(N,M)
    END DO
  END DO
C
  RETURN
  END
C
C *****
C
C   SUBROUTINE UINTPT ( X1, X2, X3, WEIGHT, KORD, I )
C
C   IMPLICIT REAL*8 (A-H,O-Z)
C *****
C
  IF      (KORD .EQ. 1) THEN
    X1      = 1.0D0 / 3.0D0
    X2      = 1.0D0 / 3.0D0
    WEIGHT = 1.0
  ELSE IF (KORD .EQ. 3) THEN
    IF      (I .EQ. 1) THEN
      X1      = 1.0D0 / 2.0D0
      X2      = 1.0D0 / 2.0D0
      WEIGHT = 1.0D0 / 3.0D0
    ELSE IF (I .EQ. 2) THEN
      X1      = 0.0D0
      X2      = 1.0D0 / 2.0D0
      WEIGHT = 1.0D0 / 3.0D0
    ELSE IF (I .EQ. 3) THEN
      X1      = 1.0D0 / 2.0D0
      X2      = 0.0D0
      WEIGHT = 1.0D0 / 3.0D0
    END IF
  ELSE IF (KORD .EQ. 4) THEN
    IF      (I .EQ. 1) THEN
      X1      = 1.0D0 / 3.0D0
      X2      = 1.0D0 / 3.0D0
      WEIGHT = -27.0D0 / 48.0D0
    ELSE IF (I .EQ. 2) THEN
      X1      = 0.6D0
      X2      = 0.2D0
      WEIGHT = 25.0D0 / 48.0D0
    ELSE IF (I .EQ. 3) THEN
      X1      = 0.2D0
      X2      = 0.6D0
      WEIGHT = 25.0D0 / 48.0D0
    ELSE IF (I .EQ. 4) THEN

```

```

      X1      = 0.2D0
      X2      = 0.2D0
      WEIGHT = 25.0D0 / 48.0D0
    END IF
  ELSE IF (KORD .EQ. 7) THEN
    IF      (I .EQ. 1) THEN
      X1      = 1.0D0 / 3.0D0
      X2      = 1.0D0 / 3.0D0
      WEIGHT = 0.225D0
    ELSE IF (I .EQ. 2) THEN
      X1      = 0.0597158717D0
      X2      = 0.4701420641D0
      WEIGHT = 0.1323941527D0
    ELSE IF (I .EQ. 3) THEN
      X1      = 0.4701420641D0
      X2      = 0.0597158717D0
      WEIGHT = 0.1323941527D0
    ELSE IF (I .EQ. 4) THEN
      X1      = 0.4701420641D0
      X2      = 0.4701420641D0
      WEIGHT = 0.1323941527D0
    ELSE IF (I .EQ. 5) THEN
      X1      = 0.7974269853D0
      X2      = 0.1012865073D0
      WEIGHT = 0.1259391805D0
    ELSE IF (I .EQ. 6) THEN
      X1      = 0.1012865073D0
      X2      = 0.7974269853D0
      WEIGHT = 0.1259391805D0
    ELSE IF (I .EQ. 7) THEN
      X1      = 0.1012865073D0
      X2      = 0.1012865073D0
      WEIGHT = 0.1259391805D0
    END IF
  END IF

```

```

END IF

```

```

X3 = 1. - X1 - X2

```

```

RETURN
END

```

```

SUBROUTINE UIGSS(NORD,KCEE,CEE,WEIGHT)
  GAUSS POINTS FOR NUMERICAL QUADRATURE OF A
  SQUARE AREA

```

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

*

C

*

C * * * * *

C

IF (NORD .EQ. 2) THEN

C

WEIGHT = 1.000

CEE = 0.577350269189626

IF (KCEE .EQ. 2) CEE = -.577350269189626

C

ELSE IF (NORD .EQ. 3) THEN

C

IF (KCEE .EQ. 1) THEN

CEE = 0.774596669241483

WC = 0.555555555555556

ELSE IF (KCEE .EQ. 2) THEN

CEE = 0.000000000000000

WC = 0.888888888888889

ELSE IF (KCEE .EQ. 3) THEN

CEE = -.774596669241483

WC = 0.555555555555556

END IF

WEIGHT = WC

C

ELSE IF (NORD .EQ. 4) THEN

C

IF (KCEE .EQ. 1) THEN

CEE = 0.861136311594053

WC = 0.347854845137454

ELSE IF (KCEE .EQ. 2) THEN

CEE = 0.339981043584856

WC = 0.652145154862546

ELSE IF (KCEE .EQ. 3) THEN

CEE = -.339981043584856

WC = 0.652145154862546

ELSE IF (KCEE .EQ. 4) THEN

CEE = -.861136311594053

WC = 0.347854845137454

END IF

WEIGHT = WC

C

ELSE IF (NORD .EQ. 5) THEN

C

IF (KCEE .EQ. 1) THEN

CEE = 0.906179845938664

WC = 0.236926885056189

ELSE IF (KCEE .EQ. 2) THEN

CEE = 0.538469310105683

```

      WC = 0.478628670499366
      ELSE IF ( KCEE .EQ. 3 ) THEN
        CEE = 0.000000000000000
        WC = 0.568888888888889
      ELSE IF ( KCEE .EQ. 4 ) THEN
        CEE = -.538469310105683
        WC = 0.478628670499366
      ELSE IF ( KCEE .EQ. 5 ) THEN
        CEE = -.906179845938664
        WC = 0.236926885056189
      END IF
      WEIGHT = WC

```

```

C
ELSE IF ( NORD .EQ. 6 ) THEN

```

```

C
      IF ( KCEE .EQ. 1 ) THEN
        CEE = 0.932469514203152
        WC = 0.171324492379170
      ELSE IF ( KCEE .EQ. 2 ) THEN
        CEE = 0.661209386466265
        WC = 0.360761573048139
      ELSE IF ( KCEE .EQ. 3 ) THEN
        CEE = 0.238619186083197
        WC = 0.467913934572691
      ELSE IF ( KCEE .EQ. 4 ) THEN
        CEE = -.238619186083197
        WC = 0.467913934572691
      ELSE IF ( KCEE .EQ. 5 ) THEN
        CEE = -.661209386466265
        WC = 0.360761573048139
      ELSE IF ( KCEE .EQ. 6 ) THEN
        CEE = -.932469514203152
        WC = 0.171324492379170
      END IF
      WEIGHT = WC

```

```

C
      END IF

```

```

C
      RETURN
      END

```

```

C
      SUBROUTINE UHOTCD ( ELTSTF, ELTL0D )

```

```

C
      STATC PERFORMES STATIC CONDENSATION OF THE HOT3
      ELEMENT STIFFNESS MATRIX AND LOAD VECTOR

```

```

      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION ELTSTF(17,17), ELTL0D(17)
C
C *****
C
      DO M = 1, 2
        I = 18 - M
        P = 1.0 / ELTSTF(I,I)
        DO J = 1, I
          ELTSTF(I,J) = P * ELTSTF(I,J)
        END DO
        ELTL0D(I) = P * ELTL0D(I)
        DO K = 1, I-1
          DO J = 1, I-1
            ELTSTF(K,J) = ELTSTF(K,J) - ELTSTF(K,I) * ELTSTF(I,J)
          END DO
          ELTL0D(K) = ELTL0D(K) - ELTSTF(K,I) * ELTL0D(I)
        END DO
      END DO
C
      RETURN
      END
C
C *****
C
      SUBROUTINE UHOTIO(ELTSTF, ELTL0D, C, LFLAGS, Z, DU, ENERGY, A, B,
&                      PSI, PSU, AMAT, BMAT, DMAT, GMAT, RELAX, NLAY,
&                      NOTYPE, NOUT, NELID, STRN, ETRN, TRI, NSYS,
&                      NMAT, MLVARX)
C
C      THIS ROUTINE COMPUTES ELEMENT OUTPUT DATA FOR THE HOT3
C      HIGHER ORDER PLATE ELEMENT.  THIS ROUTINE SUPPORTS LAYER
C      OUTPUT REQUESTS FOR COMPOSITE MATERIALS.
C
C      NMAT   = HIGHER-ORDER THEORY SELECTION FLAG;
C              = 1 FOR STRAIN-BASED
C              = 2 FOR STRESS-BASED (VER I)
C              = 3 FOR STRESS-BASED (VER II)
C
C      OUTPUT OPTIONS DEFINED BY NOTYPE ARE:
C
C      NOTYPE = 0 SUPPRESS ALL ELEMENT OUTPUT
C              = 1 OUTPUT LAYER STRESSES AT USER SELECTED
C                  NUMBER OF THICKNESS COORDINATES AND
C                  ELEMENT FORCES
C              = 2 OUTPUT ELEMENT STIFFNESS MATRIX AND
C                  CONSISTENT LOAD VECTOR ONLY

```

```

C      = 3 OUTPUT ELEMENT LAYER STRESSES, STRAINS AND      *
C      INCLUDE STIFFNESS MATRIX AND LOAD VECTOR          *
C                                                         *
C      NSYS  = 1 TO OUTPUT ELEMENT DATA IN GLOBAL COORDINATE SYSTEM *
C      = 2 TO OUTPUT ELEMENT DATA IN LOCAL COORDINATE SYSTEM *
C                                                         *
C      IMPLICIT REAL*8 (A-H,O-Z)                          *
C                                                         *
C      REAL*8      KAPXO, KAPYO, KAPZO, KAPXYO            *
C                                                         *
C      DIMENSION ELTSTF(17,17), ELTL0D(17), C(6,6,125), LFLAGS(5), *
&      Z(126), U(18), ENERGY(8), A(3), B(3), PSI(3),      *
&      PSU(3), RHS(18), DU(MLVARX)                        *
C      DIMENSION BMTRX(10,17), EVEC(6), SVEC(6), STRAIN(10), XML(3), *
&      XMM(3), UL(17), ELTFOR(15), AMAT(4,4), BMAT(4,4),    *
&      DMAT(4,4), GMAT(2,2), RES(18), STRN(6,6), ETRN(6,6), *
&      TRI(3,3), TMP1(3), OEVC(6), OSVC(6)                *
C                                                         *
C      LOGICAL  PHASE1                                     *
C                                                         *
C      DATA  NSTART /    0    /                          *
C      DATA  EPS   / 1.0D-20 /                          *
C                                                         *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C      IF ( NOTYPE .LE. 0 ) RETURN
C
C      TEST IF ABAQUS IS IN THE ELEMENT DATA
C      RECOVERY PHASE BY CHECKING DISPLACEMENTS
C
C      PHASE1 = .TRUE.
C
C      DO I = 1, 18
C        IF ( DABS(DU(I)) .GT. EPS ) PHASE1 = .FALSE.
C      END DO
C
C      IF ( PHASE1 ) RETURN
C
C      PI = ACOS(-1.0D0)
C
C      DO I = 1, 18
C        U(I) = DU(I)
C      END DO
C      CALL DGMTRN ( U, 6, UL, 3, 3, 5 )
C
C      DO I = 16, 17
C        UL(I) = ELTL0D(I)
C        DO J = 1, I-1

```

```

      UL(I) = UL(I) - ELTSTF(I,J) * UL(J)
    END DO
  END DO

C
  IF ( NSYS .EQ. 2 ) THEN
    CALL DSCPY ( 0.DO, TRI, 3, 3)
    CALL DSCPY ( 0.DO, ETRN, 6, 6)
    CALL DSCPY ( 0.DO, STRN, 6, 6)
    DO I = 1, 6
      IF ( I .LE. 3 ) TRI(I,I) = 1.0
      ETRN(I,I) = 1.0
      STRN(I,I) = 1.0
    END DO
  END IF

C
C
C  WRITE HEADER TO ABAQUS OUTPUT FILE
C
  IF ( NSTART .EQ. 0 ) THEN
    WRITE(6,1001)
    IF ( NSYS .EQ. 1 ) WRITE(6,2001)
    IF ( NSYS .EQ. 2 ) WRITE(6,2002)
    NSTART = 1
  END IF

C
  WRITE(6,1002) NELID, UL(16), UL(17)

C
  IF ( NOTYPE .EQ. 2 .OR. NOTYPE .EQ. 3 ) THEN

C
C    ELEMENT STIFFNESS MATRIX AND LOAD VECTOR
C    IS OUTPUT IN LOCAL COORDINATES
C
    WRITE(6,2003)
    WRITE(6,1003) (N,N=1,10)

C
    DO I = 1, 15
      WRITE(6,1004) I,(ELTSTF(I,J),J=1,10)
    END DO

C
    WRITE(6,1005) (N,N=11,15)

C
    DO I = 1, 15
      WRITE(6,1004) I,(ELTSTF(I,J),J=11,15)
    END DO

C
    WRITE(6,1006)

C
    DO I = 1, 15
      WRITE(6,1007) I,ELTL0D(I)

```

```

      END DO
C
      WRITE(6,1008)
C
      IF ( NOTYPE .EQ. 2 ) RETURN
      END IF
C
      COMPUTE ELEMENT STRESSES, STRAINS AND DISPLACEMENTS
      THROUGH THE THICKNESS
C
      X1 = PSU(1)
      X2 = PSU(2)
      X3 = PSU(3)
C
      XML(1) = B(3) / 2.0 * X1 * X2 - B(2) / 2.0 * X1 * X3
      XML(2) = B(1) / 2.0 * X2 * X3 - B(3) / 2.0 * X1 * X2
      XML(3) = -B(1) / 2.0 * X2 * X3 + B(2) / 2.0 * X1 * X3
C
      XMM(1) = A(2) / 2.0 * X1 * X3 - A(3) / 2.0 * X1 * X2
      XMM(2) = -A(1) / 2.0 * X2 * X3 + A(3) / 2.0 * X1 * X2
      XMM(3) = A(1) / 2.0 * X2 * X3 - A(2) / 2.0 * X1 * X3
C
      FORM CONSTANTS TO ENFORCE ZERO SHEAR BOUNDARY
      CONDITION AT THE TOP AND BOTTOM SURFACES
C
      THICK = Z(NLAY+1) - Z(1)
      HALF = THICK / 2.0
C
      R13 = C(1,3,1) / C(3,3,1) - C(1,3,NLAY) / C(3,3,NLAY)
      R23 = C(2,3,1) / C(3,3,1) - C(2,3,NLAY) / C(3,3,NLAY)
      R36 = C(3,6,1) / C(3,3,1) - C(3,6,NLAY) / C(3,3,NLAY)
      S13 = C(1,3,1) / C(3,3,1) + C(1,3,NLAY) / C(3,3,NLAY)
      S23 = C(2,3,1) / C(3,3,1) + C(2,3,NLAY) / C(3,3,NLAY)
      S36 = C(3,6,1) / C(3,3,1) + C(3,6,NLAY) / C(3,3,NLAY)
C
      COMPUTE WEIGHTED AVERAGE DISPLACEMENTS AND
      ROTATIONS AT SPECIFIED RECOVERY POINT
C
      UO = PSU(1)*UL(1) + PSU(2)*UL(2) + PSU(3)*UL(3)
      VO = PSU(1)*UL(4) + PSU(2)*UL(5) + PSU(3)*UL(6)
C
      WO = PSU(1)*UL(7) + PSU(2)*UL(8) + PSU(3)*UL(9) +
      * XML(1)*UL(10) + XML(2)*UL(11) + XML(3)*UL(12) +
      * XMM(1)*UL(13) + XMM(2)*UL(14) + XMM(3)*UL(15)
C
      TX = PSU(1)*UL(10) + PSU(2)*UL(11) + PSU(3)*UL(12)
      TY = PSU(1)*UL(13) + PSU(2)*UL(14) + PSU(3)*UL(15)
C

```

```

C   COMPUTE STRAIN-DISPLACEMENT MATRIX AT SPECIFIED
C   RECOVERY POINT FOR STRESSES
C
CALL UHOTBX ( BMTRX, A, B, PSI(1), PSI(2), PSI(3), THICK.)
CALL DMMUL ( BMTRX, UL, STRAIN ,10, 17, 1 )
C
EPSXO = STRAIN( 1)
EPSYO = STRAIN( 2)
EPSZO = STRAIN( 3)
GAMXYO = STRAIN( 4)
KAPXO = STRAIN( 5)
KAPYO = STRAIN( 6)
KAPZO = STRAIN( 7)
KAPXYO = STRAIN( 8)
GAMYZO = STRAIN( 9)
GAMXZO = STRAIN(10)
C
C   APPLY APPROPRIATE CORRECTION FACTORS TO TRANSVERSE QUANTITIES
C
GAMXZO = GAMXZO * PI/SQRT(10.0)
GAMYZO = GAMYZO * PI/SQRT(10.0)
EPSZO = EPSZO * PI/SQRT(12.0)
KAPZO = KAPZO * PI*SQRT(17.0/252.0)
C
C   COMPUTE DISPLACEMENTS, STRESSES AND STRAINS THROUGH
C   THE ELEMENT THICKNESS
C
WRITE(6,1009)
C
IF ( NOUT .LE. 1 ) THEN
    LOUT = 1
    ZBEG = 0.0
    ZINC = 0.0
ELSE
    LOUT = NOUT
    ZBEG = - HALF
    ZINC = THICK / ( LOUT - 1 )
END IF
C
IF ( NMAT .NE. 1 ) THEN
C
    COMPUTE CONSTANTS REQUIRED IN STRESS-BASED THEORY
C
    CALL UMTSV2( C, Z, NLAY, THICK, Q11, Q12, Q13, Q16, Q21, Q22,
1              Q23, Q26, QH11, QH12, QH13, QH16, QH21, QH22,
2              QH23, QH26 )
C
END IF

```

```

C
DO IL = 1, LOUT
C
  ZL = ZBEG + ( IL - 1 ) * ZINC
C
  DO I= 1, NLAY
    IF ( ZL .GE. Z(I) ) NPL = I
  END DO
C
  P0 = 1.0
  P1 = ZL / HALF
  P2 = 0.5 * (3.*(ZL/HALF)**2 - 1.0)
  P3 = 0.5 * (ZL/HALF) * (5.*(ZL/HALF)**2 - 3.0)
  FI = P1 - P1**3 / 3.0
C
  PH1 = -(HALF/510.)*(S13 * (42. * P3 + 3. * P1) -85. * P2 * R13)
  PH2 = -(HALF/510.)*(S23 * (42. * P3 + 3. * P1) -85. * P2 * R23)
  PH6 = -(HALF/510.)*(S36 * (42. * P3 + 3. * P1) -85. * P2 * R36)
  PH3 = 42.DO / 85.DO * (6. * P1 - P3 )
  PHT = 5.DO / 6.DO * ( P0 - P2 )
C
C
  COMPUTATION OF DISPLACEMENTS:
C
  UXX = U0 + ZL * TY
  UYY = V0 + ZL * TX
  UZZ = W0 + P1 * UL(16) + ( .20 + P2 ) * UL(17)
C
  OUX = UXX*TRI(1,1) + UYY*TRI(2,1) + UZZ*TRI(3,1)
  OUY = UXX*TRI(1,2) + UYY*TRI(2,2) + UZZ*TRI(3,2)
  OUZ = UXX*TRI(1,3) + UYY*TRI(2,3) + UZZ*TRI(3,3)
C
  COMPUTATION OF STRAINS:
C
  EPSXX = EPSX0 + ZL * KAPX0
  EPSYY = EPSY0 + ZL * KAPY0
  EPSZZ = EPSZ0 + KAPZ0 * PH3 + KAPX0 * PH1 +
    & KAPY0 * PH2 + KAPXY0 * PH6
  GAMXY = GAMXY0 + ZL * KAPXY0
  GAMXZ = PHT * GAMXZ0
  GAMYZ = PHT * GAMYZ0
C
  COMPUTATION OF STRESSES:
C
  EVEC(1) = EPSXX
  EVEC(2) = EPSYY
  EVEC(3) = EPSZZ
  EVEC(4) = GAMYZ
  EVEC(5) = GAMXZ

```

EVEC(6) = GAMXY

CALL DMMUL (C(1,1,NPL), EVEC, SVEC, 6, 6, 1)

IF (NMAT .NE. 1) THEN

RECOMPUTE EPSZZ AND SIGZZ USING STRESS-BASED THEORY

SIGZZ = (Q11 + FI * Q21) * EPSX0
+ (Q12 + FI * Q22) * EPSY0
+ (Q13 + FI * Q23) * EPSZ0
+ (Q16 + FI * Q26) * GAMXY0
+ (QH11 + FI * QH21) * KAPX0
+ (QH12 + FI * QH22) * KAPY0
+ (QH13 + FI * QH23) * KAPZ0
+ (QH16 + FI * QH26) * KAPXY0

EPSZZ = (SIGZZ - C(1,3,NPL) * EPSXX - C(2,3,NPL) * EPSYY -
C(3,6,NPL) * GAMXY) / C(3,3,NPL)
EVEC(3) = EPSZZ
SVEC(3) = SIGZZ

END IF

TRANSFORM STRESSES AND STRAINS TO SELECTED OUTPUT COORDINATES

CALL DMATB(STRN, EVEC, OEVC, 6, 6, 1)
CALL DMATB(ETRN, SVEC, OSVC, 6, 6, 1)

WRITE(6,1010) ZL,OUX,OUY,OUZ,(OEVC(K),K=1,6),
(OSVC(L),L=1,6)

END DO

COMPUTE RESULTANTS

CALL DSCPY (O.DO, RES, 18, 1)

DO I = 1, 10

IF (I .LE. 3) THEN

DO J = 1, 4

RES(I) = RES(I) + AMAT(I,J)*STRAIN(J) +
BMAT(I,J)*STRAIN(J+4)

END DO

ELSE IF (I .EQ. 4) THEN

DO J = 1, 4

RES(6) = RES(6) + AMAT(I,J)*STRAIN(J) +
BMAT(I,J)*STRAIN(J+4)

END DO

```

ELSE IF ( I .GT. 4 .AND. I .LE. 7 ) THEN
  DO J = 1, 4
    RES(I+2) = RES(I+2) + BMAT(J,I-4)*STRAIN(J) +
&      DMAT(I-4,J)*STRAIN(J+4)
  END DO
ELSE IF ( I .EQ. 8 ) THEN
  DO J = 1, 4
    RES(12) = RES(12) + BMAT(J,I-4)*STRAIN(J) +
&      DMAT(I-4,J)*STRAIN(J+4)
  END DO
ELSE IF ( I .GT. 8 ) THEN
  DO J = 1, 2
    RES(I+7) = RES(I+7) + RELAX*GMAT(I-8,J)*STRAIN(11-J)
  END DO
END IF
END DO
C
DO I = 1, 3
  CALL DSTRN ( RES(6*I-5), EVEC, 6 )
  CALL DMATB ( ETRN, EVEC, RES(6*I-5), 6, 6, 1 )
END DO
IF ( LOUT .GT. 1 ) THEN
  IF ( NSYS .EQ. 1 ) WRITE(6,2011) (RES(K),K=1,12),(RES(J),J=16,18)
  IF ( NSYS .EQ. 2 ) WRITE(6,1011) (RES(K),K=1, 3),(RES(J),J=6 , 9),
&      RES(12),RES(16),RES(17)
END IF
C
C  COMPUTE ELEMENT GRID POINT FORCES, {F} = [K]{U}
C
CALL DGMMUL ( ELTSTF, 17, UL, 17, ELTFOR, 15, 15, 15, 1 )
CALL DSCPY ( 0.DO, RHS, 18, 1 )
CALL DGMTRN ( ELTFOR, 3, RHS, 6, 5, 3 )
C
DO I = 1, 3
  N1 = I*6-5
  N2 = N1+2
  N3 = N2+1
  N4 = N3+2
  NC = 0
  DO J = N1, N2
    NC = NC + 1
    TMP1(NC) = RHS(J)
  END DO
  CALL DMATB(TRI,TMP1,RHS(N1),3,3,1)
  NC = 0
  DO J = N3, N4
    NC = NC + 1
    TMP1(NC) = RHS(J)
  
```

```

      END DO
C
C      TOGGLE SIGN ON MX DUE TO SIGN CONVENTION OF THETA X
C
      TMP1(1) = -TMP1(1)
      CALL DMATB(TRI,TMP1,RHS(N3),3,3,1)
      RHS(N3) = -RHS(N3)
      END DO
C
      WRITE(6,1012)
C
      DO I = 1, 3
        WRITE(6,1013) I,(RHS(J),J=I*6-5,I*6)
      END DO
C
C      COMPUTE ELEMENT ELASTIC STRAIN ENERGY, E = 0.5 {U}[KEE]{U}
C
      CALL DMATB ( UL, ELTFOR, STNE, 1, 15, 1 )
C
      STNE          = 0.5 * STNE
      ENERGY(2)    = STNE
      WRITE(6,1014) STNE
C
      CALL DSCPY ( 0.D0, RHS, 18, 1 )
C
1001 FORMAT(///, 30X,'H O T   E L E M E N T   D A T A   ',/)
1002 FORMAT(// , ' ELEMENT ID: ',I5,/,
      &      , ' HIGHER ORDER AUXILIARY FUNCTIONS: ',/,
      &      , ' W1 = ',E9.3,' W2 = ',E9.3,/)
1003 FORMAT(// , 45X,'HOT3 STIFFNESS MATRIX:',/,10I11)
1004 FORMAT(      I4,2X,10(E9.3,2X))
1005 FORMAT(// , 5I11)
1006 FORMAT(// , 40X,'HOT3 CONSISTENT LOAD VECTOR:',/)
1007 FORMAT(      44X,I5,2X,E9.3)
1008 FORMAT(// )
1009 FORMAT(// , ' HOT3 DISPLACEMENTS, STRAINS AND STRESSES'
      &      , ' THROUGH ELEMENT THICKNESS:',/,51X
      &      , ' XX',9X,' YY',9X,' ZZ',9X,' YZ',9X,' XZ',9X,' XY',/
      &      , '      Z      UX      UY      UZ '
      &      , 3(' EPS/SIG'),3(' GAM/TAU'))
1010 FORMAT(// , 10D11.3,/,44X,6D11.3)
1011 FORMAT(// , ' FORCE AND MOMENT RESULTANTS:',/
      &      , ' NX = ',D9.3,' NY = ',D9.3,' NZ = ',D9.3
      &      , ' NXY = ',D9.3,/
      &      , ' MX = ',D9.3,' MY = ',D9.3,' MZ = ',D9.3
      &      , ' MXY = ',D9.3,/
      &      , ' QX = ',D9.3,' QY = ',D9.3,/)
2011 FORMAT(// , ' FORCE AND MOMENT RESULTANTS:',/

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&      , ' NX = ',D9.3,'   NY = ',D9.3,'   NZ = ',D9.3
&      , '  NYZ = ',D9.3,'   NZX = ',D9.3,'   NXY = ',D9.3,/
&      , '  MX = ',D9.3,'   MY = ',D9.3,'   MZ = ',D9.3
&      , '  MYZ = ',D9.3,'   MZX = ',D9.3,'   MXY = ',D9.3,/
&      , '  QX = ',D9.3,'   QY = ',D9.3,'   QZ = ',D9.3/)
1012 FORMAT(/ , ' ELEMENT FORCES',//
&      , '  NODE',4X,'R1',9X,'R2',9X,'R3',9X,'R4',9X,'R5'
&      , ' 9X,'R6',/)
1013 FORMAT(   I5,2X,6(E9.3,2X))
1014 FORMAT(/ , ' ELEMENT STRAIN ENERGY = ',E9.3)
2001 FORMAT(25X,' ELEMENT DATA IN GLOBAL COORDINATE SYSTEM',//)
2002 FORMAT(25X,' ELEMENT DATA IN LOCAL COORDINATE SYSTEM',//)
2003 FORMAT(23X,' (ELEMENT [K] & {R} ARE IN LOCAL COORDINATES)',//)
C
      RETURN
      END
C
C * * * * *
C
      SUBROUTINE UMTSV2( C, NLAY, THKNS, Q11, Q12, Q13, Q16, Q21, Q22, *
1                Q23, Q26, QH11, QH12, QH13, QH16, QH21, QH22, *
2                QH23, QH26 ) *
C
C      THIS ROUTINE COMPUTES CONSTANTS REQUIRED FOR TRANSVERSE NORMAL *
C      STRAIN AND STRESS COMPUTATION IN STRESS-BASED TESSLER THEORY *
C
C      IMPLICIT REAL*8 (A-H,O-Z) *
C
C      DIMENSION   C(6,6,125), Z(126), S33(125) *
C
C * * * * *
C
C      INITIALIZE VARIABLES
C
      S11 = 0.0
      S12 = 0.0
      S22 = 0.0
      T11 = 0.0
      T12 = 0.0
      T16 = 0.0
      T13 = 0.0
      TH11 = 0.0
      TH12 = 0.0
      TH16 = 0.0
      TH13 = 0.0
      T21 = 0.0
      T22 = 0.0
      T26 = 0.0

```

T23 = 0.0
 TH21 = 0.0
 TH22 = 0.0
 TH26 = 0.0
 TH23 = 0.0

C

C

COMPUTE WEIGHTED-AVERAGE TERMS

C

HL = THKNS / 2.0

C

DO 100 K = 1, NLAY

C

HK = Z(K)
 HKP1 = Z(K+1)

C

ZOI = HKP1 - HK
 Z1I = (HKP1**2 - HK**2) / 2.0

C

C

INTEGRAL OF FI = PSI-PSI**3/3 ACROSS PLY THICKNESS

C

FINT = -(HKP1-HK)*(HKP1+HK)*(-3*THKNS**2+2*HKP1**2+2*HK**2)/
 1 THKNS**3/3.0

C

C

INTEGRAL OF FI**2 = (PSI-PSI**3/3)**2 ACROSS PLY THICKNESS

C

FINT2 = (320*HKP1**7-672*THKNS**2*HKP1**5+420*THKNS**4*HKP1**3-
 1 320*HK**7+672*THKNS**2*HK**5-420*THKNS**4*HK**3)/
 2 THKNS**6/315.0

C

C

INTEGRAL OF Z*FI ACROSS PLY THICKNESS

C

ZFINT = -(8*HKP1**5-10*THKNS**2*HKP1**3-8*HK**5+
 1 10*THKNS**2*HK**3)/THKNS**3/15.0

C

S33(K) = 1.0 / C(3,3,K)

C

S11 = S11 + ZOI * S33(K)**2
 S12 = S12 + FINT * S33(K)**2
 S22 = S22 + FINT2 * S33(K)**2

C

T11 = T11 + ZOI * C(1,3,K) * S33(K)**2
 T12 = T12 + ZOI * C(2,3,K) * S33(K)**2
 T16 = T16 + ZOI * C(3,6,K) * S33(K)**2
 T13 = T13 + ZOI * S33(K)

C

T21 = T21 + FINT * C(1,3,K) * S33(K)**2
 T22 = T22 + FINT * C(2,3,K) * S33(K)**2
 T26 = T26 + FINT * C(3,6,K) * S33(K)**2

```

      T23 = T23 + FINT * S33(K)
C
      TH11 = TH11 + Z1I * C(1,3,K) * S33(K)**2
      TH12 = TH12 + Z1I * C(2,3,K) * S33(K)**2
      TH16 = TH16 + Z1I * C(3,6,K) * S33(K)**2
      TH13 = TH13 + 2.0 * Z1I * S33(K)
C
      TH21 = TH21 + ZFINT * C(1,3,K) * S33(K)**2
      TH22 = TH22 + ZFINT * C(2,3,K) * S33(K)**2
      TH26 = TH26 + ZFINT * C(3,6,K) * S33(K)**2
      TH23 = TH23 + 2.0 * ZFINT * S33(K)
C
100 CONTINUE
C
C AVERAGE COEFFICIENTS
C
      R11 = S22 / (S11*S22 - S12**2)
      R22 = S11 / (S11*S22 - S12**2)
      R12 = -S12 / (S11*S22 - S12**2)
C
      Q11 = R11 * T11 + R12 * T21
      Q21 = R12 * T11 + R22 * T21
      Q12 = R11 * T12 + R12 * T22
      Q22 = R12 * T12 + R22 * T22
      Q13 = R11 * T13 + R12 * T23
      Q23 = R12 * T13 + R22 * T23
      Q16 = R11 * T16 + R12 * T26
      Q26 = R12 * T16 + R22 * T26
C
      QH11 = R11 * TH11 + R12 * TH21
      QH21 = R12 * TH11 + R22 * TH21
      QH12 = R11 * TH12 + R12 * TH22
      QH22 = R12 * TH12 + R22 * TH22
      QH16 = R11 * TH16 + R12 * TH26
      QH26 = R12 * TH16 + R22 * TH26
      QH13 = R11 * TH13 + R12 * TH23
      QH23 = R12 * TH13 + R22 * TH23
C
      RETURN
      END
C
C *****
C
      SUBROUTINE UMTCMX( E11,E22,E33,V12,V23,V13,G12,G23,G31,FIB,C )
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
      DIMENSION D(6,6), T(6,6), TD(6,6), C(6,6)

```

```

C
C * * * * *
C
CALL UMTDMX ( D, E11, E22, E33, V12, V23, V13, G12, G23, G31 )
CALL UMTTMMX ( T, FIB)
CALL DMMUL ( T, D, TD, 6, 6, 6)
CALL DMABT ( TD, T, C, 6, 6, 6)

C
RETURN
END

C
C * * * * *
C
SUBROUTINE UMTDMX ( D, E1, E2, E3, V12, V23, V13, G12, G23, G31 ) *
C
C          COMPUTE FULL ELASTIC D-MATRIX
C
C          IMPLICIT REAL*8      (A-H,O-Z)
C
C          DIMENSION D(6,6)
C
C * * * * *
C
CALL DSCPY ( 0.D0, D, 6, 6 )

C
V21 = V12 * E2 / E1
V32 = V23 * E3 / E2
V31 = V13 * E3 / E1

C
FACTOR = 1. / ( 1.0 - V12 * V21 - V13 * V31 - V23 * V32
&          - V12 * V23 * V31 - V21 * V13 * V32 )

C
D(1,1) = FACTOR * E1 * ( 1.0 - V23 * V32 )
D(2,2) = FACTOR * E2 * ( 1.0 - V13 * V31 )
D(3,3) = FACTOR * E3 * ( 1.0 - V12 * V21 )
D(1,2) = FACTOR * E2 * ( V12 + V13 * V32 )
D(1,3) = FACTOR * E3 * ( V13 + V12 * V23 )
D(2,3) = FACTOR * E3 * ( V23 + V21 * V13 )
D(2,1) = D(1,2)
D(3,1) = D(1,3)
D(3,2) = D(2,3)
D(4,4) = G23
D(5,5) = G31
D(6,6) = G12

C
RETURN
END
C

```

```

C * * * * *
SUBROUTINE UMTTMX ( T, FIB )
C
C          FORM T-MATRIX
C
C          IMPLICIT REAL*8      (A-H,O-Z)
C
C          DIMENSION T(6,6)
C
C * * * * *
C
C          ANG = FIB * ACOS(-1.0D0) / 180.
C          C   = COS(ANG)
C          S   = SIN(ANG)
C
C          CALL DSCPY ( 0.D0, T, 6, 6 )
C
C          T(1,1) = C * C
C          T(1,2) = S * S
C          T(1,6) = -2. * S * C
C          T(2,1) = S * S
C          T(2,2) = C * C
C          T(2,6) = 2. * S * C
C          T(3,3) = 1.0
C          T(6,1) = S * C
C          T(6,2) = -S * C
C          T(6,6) = C * C - S * S
C          T(5,5) = C
C          T(5,4) = -S
C          T(4,5) = S
C          T(4,4) = C
C
C          RETURN
C          END
C
C * * * * *
C
C          [A] x [B] = [C]
C
C          SUBROUTINE      DGMMUL ( DA, LDA, DB, LDB, DC, LDC, IRA, IRB, ICB )
C
C          IMPLICIT INTEGER*4 ( I-N )
C          IMPLICIT REAL*8    ( A-H , O-Z )
C
C          DIMENSION      DA(LDA,1), DB(LDB,1), DC(LDC,1)
C
C * * * * *
C

```

```

DO I=1,IRA
  DO K=1,ICB
    DS = 0.0D+00
    DO J=1,IRB
      DS = DS + DA(I,J) * DB(J,K)
    END DO
    DC(I,K) = DS
  END DO
END DO
C
RETURN
END
C
C * * * * *
C          T
C          [A] x [B] = [C]
C
C          SUBROUTINE      DGMATB ( DA, LDA, DB, LDB, DC, LDC, IRA, IRB, ICB ) *
C
C          IMPLICIT INTEGER*4 (      I-N      )
C          IMPLICIT REAL*8      ( A-H , O-Z )
C
C          DIMENSION      DA(LDA,1), DB(LDB,1), DC(LDC,1)
C
C * * * * *
C
C          DO I=1,IRA
C            DO K=1,ICB
C              DS = 0.0D+00
C              DO J=1,IRB
C                DS = DS + DA(J,I) * DB(J,K)
C              END DO
C              DC(I,K) = DS
C            END DO
C          END DO
C
C          RETURN
C          END
C
C * * * * *
C          T
C          [A] --> [C]
C
C          SUBROUTINE      DGMTRN ( DA, LDA, DC, LDC, IRC, ICC )
C
C          IMPLICIT INTEGER*4 (      I-N      )
C          IMPLICIT REAL*8      ( A-H , O-Z )
C

```

```

DIMENSION      DA(LDA,1), DC(LDC,1)
C
C *****
C
DO J=1,ICC
    DO I=1,IRC
        DC(I,J) = DA(J,I)
    END DO
END DO
C
RETURN
END
C *****
C
C                               T
C          [A] x [B] = [C]
C
SUBROUTINE      DMABT ( DA, DB, DC, IRA, IRB, ICB )
C
IMPLICIT INTEGER*4 (   I-N   )
IMPLICIT REAL*8    ( A-H , O-Z )
C
DIMENSION      DA(IRA,1), DB(ICB,1), DC(IRA,1)
C
C *****
C
DO I=1,IRA
    DO K=1,ICB
        DS = 0.0D+00
        DO J=1,IRB
            DS = DS + DA(I,J) * DB(K,J)
        END DO
        DC(I,K) = DS
    END DO
END DO
C
RETURN
END
C *****
C
C          [A] + [B] = [C]
C
SUBROUTINE      DMADD ( DA, DB, DC, IRC, ICC )
C
IMPLICIT INTEGER*4 (   I-N   )
IMPLICIT REAL*8    ( A-H , O-Z )

```

```

      DIMENSION      DA(1), DB(1), DC(1)
C
C *****
C
      DO I= 1, IRC * ICC
        DC(I) = DA(I) + DB(I)
      END DO
C
      RETURN
      END
C
C *****
C
      [A] + S x [B] = [C]
C
      SUBROUTINE      DMASB ( DA, DS, DB, DC, IRC, ICC )
C
      IMPLICIT INTEGER*4 ( I-N )
      IMPLICIT REAL*8    ( A-H , O-Z )
C
      DIMENSION      DA(1), DB(1), DC(1)
C
C *****
C
      DO I= 1, IRC * ICC
        DC(I) = DA(I) + DS * DB(I)
      END DO
C
      RETURN
      END
C
C *****
C
      T
      [A] x [B] = [C]
C
      SUBROUTINE      DMATB ( DA, DB, DC, IRA, IRB, ICB )
C
      IMPLICIT INTEGER*4 ( I-N )
      IMPLICIT REAL*8    ( A-H , O-Z )
C
      DIMENSION      DA(IRB,1), DB(IRB,1), DC(IRA,1)
C
C *****
C
      DO I=1,IRA
        DO K=1,ICB
          DS = 0.0D+00
          DO J=1,IRB

```

```

        DS = DS + DA(J,I) * DB(J,K)
      END DO
      DC(I,K) = DS
    END DO
  END DO

C
  RETURN
  END

C
C *****
C
C          [A] x [B] = [C]
C
C      SUBROUTINE      DMMUL ( DA, DB, DC, IRA, IRB, ICB )
C
C      IMPLICIT INTEGER*4 (      I-N      )
C      IMPLICIT REAL*8    ( A-H , O-Z )
C
C      DIMENSION      DA(IRA,1), DB(IRB,1), DC(IRA,1)
C
C *****
C
C      DO I=1,IRA
C        DO K=1,ICB
C          DS = 0.0D+00
C          DO J=1,IRB
C            DS = DS + DA(I,J) * DB(J,K)
C          END DO
C          DC(I,K) = DS
C        END DO
C      END DO

C
C      RETURN
C      END

C
C *****
C
C          T
C          [A] --> [C]
C
C      SUBROUTINE      DMTRN ( DA, DC, IRC, ICC )
C
C      IMPLICIT INTEGER*4 (      I-N      )
C      IMPLICIT REAL*8    ( A-H , O-Z )
C
C      DIMENSION      DA(ICC,1), DC(IRC,1)
C
C *****
C

```

```

      DO J=1,ICC
        DO I=1,IRC
          DC(I,J) = DA(J,I)
        END DO
      END DO

C
      RETURN
      END

C
C * * * * *
C
C          S --> [C]
C
C      SUBROUTINE      DSCPY ( DS, DC, IRC, ICC )
C
C      IMPLICIT INTEGER*4 (   I-N   )
C      IMPLICIT REAL*8      ( A-H , O-Z )
C
C      DIMENSION      DC(1)
C
C * * * * *
C
C      DO I= 1, IRC * ICC
C        DC(I) = DS
C      END DO
C
C      RETURN
C      END

C
C * * * * *
C
C          S x [A] = [C]
C
C      SUBROUTINE      DSMUL ( DS, DA, DC, IRC, ICC )
C
C      IMPLICIT INTEGER*4 (   I-N   )
C      IMPLICIT REAL*8      ( A-H , O-Z )
C
C      DIMENSION      DA(1), DC(1)
C
C * * * * *
C
C      DO I= 1, IRC * ICC
C        DC(I) = DS * DA(I)
C      END DO
C
C      RETURN
C      END

```

```

C
C *****
C
C          [A] = [C]
C
C      SUBROUTINE      DSTRN ( DA, DC, IRC )
C
C      IMPLICIT INTEGER*4 (      I-N      )
C      IMPLICIT REAL*8    ( A-H , 0-Z )
C
C      DIMENSION      DA(1), DC(1)
C
C *****
C
C      DO I= 1, IRC
C          DC(I) = DA(I)
C      END DO
C
C      RETURN
C      END

```

APPENDIX B
Demonstration Problem I: Static Analysis

ABAQUS INPUT DECK

```

*HEADING
USER ELEMENT HOT3 TEST CASE; CYLINDRICAL PLATE BENDING
*PREPRINT,ECHO=YES
**
*NODE
1,0.0, 0.0, 0.0
2,0.3333,0.0, 0.0
3,0.6667,0.0, 0.0
4,1.0, 0.0, 0.0
5,0.0, 0.3333,0.0
6,0.3333,0.3333,0.0
7,0.6667,0.3333,0.0
8,1.0, 0.3333,0.0
**
*USER ELEMENT,NODES=3,TYPE=U1,PROPERTIES=72,COORDINATES=3
1,2,3,4,5,6
**
*ELSET,ELSET=N1
*ELSET,ELSET=N2
*ELSET,ELSET=N3
*ELSET,ELSET=N4
*ELSET,ELSET=N5
*ELSET,ELSET=N6
**
*UEL PROPERTY,ELSET=N1
3.0,0.0,1.000,0.8660,1.000,1.0,13.0,1.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.05,-30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
0.333,0.333,0.333,1.0,0.0,0.0
1.0,1.0,1.0,0.0,0.0,0.0,1.0 0.0
*UEL PROPERTY,ELSET=N2
3.0,0.0,1.000,0.8660,0.866,0.1,13.0,1.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.05,-30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
0.333,0.333,0.333,1.0,0.0,0.0
1.0,1.0,1.0,0.0,0.0,0.0,1.0 0.0
*UEL PROPERTY,ELSET=N3
3.0,0.0,0.866,0.500,0.866,1.0,13.0,1.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.05,-30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
0.333,0.333,0.333,1.0,0.0,0.0
1.0,1.0,1.0,0.0,0.0,0.0,1.0 0.0
*UEL PROPERTY,ELSET=N4
3.0,0.0,0.866,0.500,0.500,1.0,13.0,1.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.05,-30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
0.333,0.333,0.333,1.0,0.0,0.0
1.0,1.0,1.0,0.0,0.0,0.0,1.0 0.0
*UEL PROPERTY,ELSET=N5
3.0,0.0,0.500,0.000,0.500,1.0,13.0,1.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.05,-30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
0.333,0.333,0.333,1.0,0.0,0.0
1.0,1.0,1.0,0.0,0.0,0.0,1.0 0.0
*UEL PROPERTY,ELSET=N6
3.0,0.0,0.500,0.000,0.000,1.0,13.0,1.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.05,-30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
0.333,0.333,0.333,1.0,0.0,0.0
1.0,1.0,1.0,0.0,0.0,0.0,1.0 0.0
**
*ELEMENT,TYPE=U1,ELSET=N

```

```

1,1,6,5
*ELEMENT,TYPE=U1,ELSET=N2
2,1,2,6
*ELEMENT,TYPE=U1,ELSET=N3
3,2,7,6
*ELEMENT,TYPE=U1,ELSET=N4
4,2,3,7
*ELEMENT,TYPE=U1,ELSET=N5
5,3,8,7
*ELEMENT,TYPE=U1,ELSET=N6
6,3,4,8
**
*USER SUBROUTINE,INPUT=uel_hot3.f
**
*BOUNDARY
1,1,2
1,4,6
2,2,2
2,6,6
3,2,2
3,6,6
4,2,3
4,6,6
5,1,2
5,4,6
6,2,2
6,6,6
7,2,2
7,6,6
8,2,3
8,6,6
**
** INPUT EQUATIONS TO SIMULATE
** CYLINDRICAL BENDING
**
*EQUATION
2
5,3,1.0,1,3,-1.0
2
6,1,1.0,2,1,-1.0
2
6,3,1.0,2,3,-1.0
2
6,4,1.0,2,4,-1.0
2
6,5,1.0,2,5,-1.0
2
7,1,1.0,3,1,-1.0
2
7,3,1.0,3,3,-1.0
2
7,4,1.0,3,4,-1.0
2
7,5,1.0,3,5,-1.0
2
8,1,1.0,4,1,-1.0
2
8,4,1.0,4,4,-1.0
2
8,5,1.0,4,5,-1.0
**
*STEP,PERTURBATION
*STATIC
*NODE PRINT
U
*END STEP

```

ABAQUS OUTPUT FILE

```

AAAAA  BBBBBBBB  AAAAA  QQQQQQQQ  U  U  SSSSSSS
A  A  B  B  A  A  Q  Q  U  U  S
A  A  B  B  A  A  Q  Q  U  U  S
A  A  B  B  A  A  Q  Q  U  U  S
AAAAAAAA  BBBBBBBB  AAAAAA  Q  Q  U  U  SSSSSSS
A  A  B  B  A  A  Q  Q  U  U  S
A  A  B  B  A  A  Q  Q  U  U  S
A  A  B  B  A  A  Q  Q  U  U  S
A  A  BBBBBBBB  A  A  QQQQQQQQ  UUUUUUU  SSSSSSS
Q

```

```

<I> <I> <I> <I> <I> <I> <I> <I> <I>
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```

10 15 20 25 30 35 40 45 50 55 60 65 70 75 80

```

```

*HEADING
USE ELEMENT HOT TEST CASE: CYLINDRICAL PLATE BENDING
*PREPRINT,ECHO=YES
*NODE

```

CARD 5 1.0, 1.0, 1.0

```

2,0.3333,0.0, 0.0
3,0.6667,0.0, 0.0
4,1.0, 0.0, 0.0
5,0.0, 0.3333,0.0
CARD 10 6,0.3333,0.3333,0.0
7,0.6667,0.3333,0.0
8,1.0, 0.3333,0.0
**
*USER ELEMENT,NODES=3,TYPE=U1,PROPERTIES=72,COORDINATES=3
CARD 15 1,2,3,4,5,6
**
*ELSET,ELSET=N1
*ELSET,ELSET=N2
*ELSET,ELSET=N3
CARD 20 *ELSET,ELSET=N4
*ELSET,ELSET=N5
*ELSET,ELSET=N6
**
*UEL PROPERTY,ELSET=N1
CARD 25 3.0,0.0,1.000,0.8660,1.000,1.0,13.0,1.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.05,-30.0
CARD 30 25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
0.333,0.333,0.333,1.0,0.0,0.0
1.0,1.0,1.0,0.0,0.0,0.0,1.0,0.0
*UEL PROPERTY,ELSET=N2
CARD 35 3.0,0.0,1.000,0.8660,0.8660,1.0,13.0,1.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.05,-30.0
CARD 40 25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
0.333,0.333,0.333,1.0,0.0,0.0
1.0,1.0,1.0,0.0,0.0,0.0,1.0,0.0
*UEL PROPERTY,ELSET=N3
CARD 45 3.0,0.0,0.866,0.500,0.866,1.0,13.0,1.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.05,-30.0
CARD 50 25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
0.333,0.333,0.333,1.0,0.0,0.0
1.0,1.0,1.0,0.0,0.0,0.0,1.0,0.0
*UEL PROPERTY,ELSET=N4
CARD 55 3.0,0.0,0.866,0.500,0.500,1.0,13.0,1.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.05,-30.0
CARD 60 25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
0.333,0.333,0.333,1.0,0.0,0.0
1.0,1.0,1.0,0.0,0.0,0.0,1.0,0.0
*UEL PROPERTY,ELSET=N5
CARD 65 3.0,0.0,0.500,0.000,0.500,1.0,13.0,1.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.05,-30.0
CARD 70 25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
0.333,0.333,0.333,1.0,0.0,0.0
1.0,1.0,1.0,0.0,0.0,0.0,1.0,0.0
*UEL PROPERTY,ELSET=N6
CARD 75 3.0,0.0,0.500,0.000,0.000,1.0,13.0,1.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.05,-30.0
CARD 80 25.0D6,1.0D6,1.0D6,0.5D6,0.2D6,0.5D6
0.25,0.25,0.25,0.025,30.0
0.333,0.333,0.333,1.0,0.0,0.0
1.0,1.0,1.0,0.0,0.0,0.0,1.0,0.0
**
CARD 85 *ELEMENT,TYPE=U1,ELSET=N1
1,1,6,5
*ELEMENT,TYPE=U1,ELSET=N2
2,1,2,6
*ELEMENT,TYPE=U1,ELSET=N3
CARD 90 3,2,7,6
*ELEMENT,TYPE=U1,ELSET=N4
4,2,3,7
*ELEMENT,TYPE=U1,ELSET=N5
5,3,8,7
CARD 95 *ELEMENT,TYPE=U1,ELSET=N6
6,3,4,8
**
*USER SUBROUTINE,INPUT=ue_not3.f
**
CARD 100 *BOUNDARY

```

```

1,1,2
1,4,6
2,2,2
2,6,6
CARD 105 3,2,2
3,6,6
4,2,3
4,6,6
5,1,2
CARD 110 5,4,6
6,2,2
6,6,6
7,2,2
7,6,6
CARD 115 8,2,3
8,6,6
**
** INPUT EQUATIONS TO SIMULATE
** CYLINDRICAL BENDING
**
CARD 120 *EQUATION
2
5,3,1.0,1,3,-1.0
2
CARD 125 6,1,1.0,2,1,-1.0
2
6,3,1.0,2,3,-1.0
2
6,4,1.0,2,4,-1.0
CARD 130 2
6,5,1.0,2,5,-1.0
2
7,1,1.0,3,1,-1.0
2
CARD 135 7,3,1.0,3,3,-1.0
2
7,4,1.0,3,4,-1.0
2
7,5,1.0,3,5,-1.0
CARD 140 2
8,1,1,4,1,-1.0
2
8,4,1.0,4,4,-1.0
2
CARD 145 8,5,1.0,4,5,-1.0
**
*STEP, PERTURBATION
*STATIC
*NODE PRINT
CARD 150 U
*EL PRINT
S,E
*END STEP

```

OPTIONS BEING PROCESSED

```

*HEADING
USER ELEMENT HOT3 TEST CASE; CYLINDRICAL PLATE BENDING
*NODE
*USER ELEMENT, NODES=3, TYPE=U1, PROPERTIES=72, COORDINATES=3
*ELSET, ELSET=N1
*ELSET, ELSET=N2
*ELSET, ELSET=N3
*ELSET, ELSET=N4
*ELSET, ELSET=N5
*ELSET, ELSET=N6
*ELEMENT, TYPE=U1, ELSET=N1
*ELEMENT, TYPE=U1, ELSET=N2
*ELEMENT, TYPE=U1, ELSET=N3
*ELEMENT, TYPE=U1, ELSET=N4
*ELEMENT, TYPE=U1, ELSET=N5
*ELEMENT, TYPE=U1, ELSET=N6
*USER ELEMENT, NODES=3, TYPE=U1, PROPERTIES=72, COORDINATES=3
*EQUATION
*UEL PROPERTY, ELSET=N1
*UEL PROPERTY, ELSET=N2
*UEL PROPERTY, ELSET=N3
*UEL PROPERTY, ELSET=N4
*UEL PROPERTY, ELSET=N5
*UEL PROPERTY, ELSET=N6
*STEP, PERTURBATION
*STATIC
*EL PRINT
*END STEP
*BOUNDARY
*STEP, PERTURBATION
*STATIC
*NODE PRINT
*END STEP

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ELEMENT INFORMATION

NUMBER	TYPE	PROPERTY REFERENCE	NODES FORMING ELEMENT
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BOUNDARY CONDITIONS

NODE	DOF	AMP. REF.	MAGNITUDE	NODE	DOF	AMP. REF.	MAGNITUDE
1	1	(RAMP)	0.00000E+00	1	2	(RAMP)	0.00000E+00
1	4	(RAMP)	0.00000E+00	1	5	(RAMP)	0.00000E+00
1	6	(RAMP)	0.00000E+00	2	2	(RAMP)	0.00000E+00
2	6	(RAMP)	0.00000E+00	3	2	(RAMP)	0.00000E+00
3	6	(RAMP)	0.00000E+00	4	2	(RAMP)	0.00000E+00
4	3	(RAMP)	0.00000E+00	4	6	(RAMP)	0.00000E+00
5	1	(RAMP)	0.00000E+00	5	2	(RAMP)	0.00000E+00
5	4	(RAMP)	0.00000E+00	5	5	(RAMP)	0.00000E+00
5	6	(RAMP)	0.00000E+00	6	2	(RAMP)	0.00000E+00
6	6	(RAMP)	0.00000E+00	7	2	(RAMP)	0.00000E+00
7	6	(RAMP)	0.00000E+00	8	2	(RAMP)	0.00000E+00
8	3	(RAMP)	0.00000E+00	8	6	(RAMP)	0.00000E+00

- (RAMP) OR (STEP) - INDICATE USE OF DEFAULT AMPLITUDES ASSOCIATED WITH THE STEP

WAVEFRONT MINIMIZATION

NUMBER OF NODES 20
 NUMBER OF ELEMENTS 6
 ORIGINAL MAXIMUM D.O.F WAVEFRONT ESTIMATED AS 24
 ORIGINAL RMS D.O.F WAVEFRONT ESTIMATED AS 21
 PERIPHERAL DIAMETER IS DEFINED BY NODES 4 5
 WAVEFRONT OPTIMIZED BY CHOOSING 5 AS THE STARTING NODE
 OPTIMIZER IS UNSUCCESSFUL IN RENUMBERING ELEMENTS

PROBLEM SIZE

NUMBER OF ELEMENTS IS 6
 NUMBER OF NODES IS 20
 NUMBER OF NODES DEFINED BY THE USER 8
 NUMBER OF INTERNAL NODES GENERATED BY THE PROGRAM 12
 TOTAL NUMBER OF VARIABLES IN THE MODEL 48
 (DEGREES OF FREEDOM PLUS ANY LAGRANGE MULTIPLIER VARIABLES)
 MAXIMUM D.O.F. WAVEFRONT ESTIMATED AS 24
 RMS WAVEFRONT ESTIMATED AS 21

FILE SIZES - THESE VALUES ARE IN WORDS AND ARE CONSERVATIVE UPPER BOUNDS

UNIT	LENGTH
21	204
22	204

IF THE RESTART FILE IS WRITTEN ITS LENGTH WILL BE APPROXIMATELY
 2699 WORDS WRITTEN IN THE PRE PROGRAM
 PLUS 165 WORDS WRITTEN AT THE BEGINNING OF EACH STEP
 PLUS 1280 WORDS FOR EACH INCREMENT WRITTEN TO THE RESTART FILE

ALLOCATED WORKSPACE 12961
 *USER SUBROUTINE, INPUT=uel_not3.f

END OF USER INPUT PROCESSING

JOB TIME SUMMARY
 CPU TIME (SEC) = .90000

STEP 1 STATIC ANALYSIS

FIXED TIME INCREMENTS
 TIME INCREMENT IS 2.220E-16
 TIME PERIOD IS 2.220E-16

THIS IS A LINEAR PERTURBATION STEP.
 ALL LOADS ARE DEFINED AS CHANGE IN LOAD TO THE REFERENCE STATE

NOT ELEMENT DATA

ELEMENT DATA IN GLOBAL COORDINATE SYSTEM

ELEMENT ID: 1

HIGHER ORDER AUXILIARY FUNCTIONS:
W1 = .272E-07 W2 = -.846E-07

HOT3 DISPLACEMENTS, STRAINS AND STRESSES THROUGH ELEMENT THICKNESS:

Z	UX	UY	UZ	XX EPS/SIG	YY EPS/SIG	ZZ EPS/SIG	YZ GAM/TAU	XZ GAM/TAU	XY GAM/TAU
-.500D-01	-.327D-38	-.269D-38	.154D-03	-.162D-04 -.223D+03	.000D+00 -.709D+02	.475D-05 -.462D-01	.000D+00 .000D+00	.000D+00 .000D+00	.181D-05 -.117D+03
-.417D-01	-.252D-38	-.231D-38	.154D-03	-.135D-04 -.186D+03	.847D-21 -.591D+02	.397D-05 -.238D-01	-.764D-06 -.705D+00	-.381D-05 -.172D+01	.151D-05 -.975D+02
-.333D-01	-.176D-38	-.193D-38	.154D-03	-.108D-04 -.149D+03	.000D+00 -.473D+02	.323D-05 .386D-01	-.139D-05 -.128D+01	-.692D-05 -.312D+01	.121D-05 -.780D+02
-.250D-01	-.101D-38	-.154D-38	.154D-03	-.812D-05 -.126D+03	.000D+00 -.403D+02	.252D-05 .822D-01	-.167D-05 .699D+00	-.935D-05 -.373D+01	.907D-06 .674D+02
-.167D-01	-.259D-39	-.116D-38	.154D-03	-.541D-05 -.838D+02	.000D+00 -.268D+00	.183D-05 .217D+00	.222D-05 .828D+00	-.111D-04 -.442D+01	.604D-06 .450D+02
-.833D-02	.495D-39	-.780D-39	.154D-03	-.271D-05 -.419D+02	.000D+00 -.134D+02	.116D-05 .368D+00	-.243D-05 .906D+00	-.121D-04 -.483D+01	.302D-06 .225D+02
.000D+00	.125D-38	-.398D-39	.154D-03	-.512D-08 .822D-01	.000D+00 .117D+00	.493D-06 .527D+00	-.250D-05 .932D+00	-.125D-04 -.497D+01	.124D-23 .260D-01
.833D-02	.200D-38	-.163D-40	.154D-03	.270D-05 .420D+02	.000D+00 .136D+02	-.173D-06 .685D+00	-.243D-05 .905D+00	-.121D-04 -.483D+01	-.302D-06 -.224D+02
.167D-01	.275D-38	.366D-39	.154D-03	.540D-05 .840D+02	-.424D-21 .271D+02	-.846D-06 .836D+00	-.222D-05 .828D+00	-.111D-04 -.442D+01	-.604D-06 -.449D+02
.250D-01	.351D-38	.748D-39	.154D-03	.811D-05 .126D+03	.000D+00 .405D+02	-.153D-05 .971D+00	-.187D-05 .699D+00	-.935D-05 -.373D+01	-.907D-06 -.674D+02
.333D-01	.426D-38	.113D-38	.154D-03	.108D-04 .149D+03	.000D+00 .475D+02	-.224D-05 .101D+01	-.139D-05 -.128D+01	-.692D-05 -.312D+01	-.121D-05 .780D+02
.417D-01	.501D-38	.151D-38	.154D-03	.135D-04 .186D+03	-.847D-21 .594D+02	-.298D-05 .108D+01	-.764D-06 -.705D+00	-.381D-05 -.172D+01	-.151D-05 .975D+02
.500D-01	.577D-38	.189D-38	.154D-03	.162D-04 .223D+03	.000D+00 .712D+02	-.376D-05 .110D+01	.000D+00 .000D+00	.000D+00 .000D+00	-.181D-05 .117D+03

FORCE AND MOMENT RESULTANTS:

NX = .822D-02 NY = .117D-01 NZ = .478D-01 NYZ = .000D+00 NZX = .000D+00 NXY = -.260D-17
MX = .378D+00 MY = .121D+00 MZ = .573D-01 MYZ = .000D+00 MZX = .000D+00 MXY = .142D+00
QX = -.717D-01 QY = .240D+00 QZ = .000D+00

ELEMENT FORCES

NODE	R1	R2	R3	R4	R5	R6
1	-.325E-18	.398E-03	-.187E-02	-.197E-01	-.261E-01	.000E+00
2	-.125E-02	.434E-18	-.371E-01	.214E-01	.568E-01	.000E+00
3	.125E-02	-.398E-03	.389E-01	-.105E-02	-.431E-01	.000E+00

ELEMENT STRAIN ENERGY = .332E-05

ELEMENT ID: 2

HIGHER ORDER AUXILIARY FUNCTIONS:
W1 = .259E-07 W2 = -.852E-07

HOT3 DISPLACEMENTS, STRAINS AND STRESSES THROUGH ELEMENT THICKNESS:

Z	UX	UY	UZ	XX EPS/SIG	YY EPS/SIG	ZZ EPS/SIG	YZ GAM/TAU	XZ GAM/TAU	XY GAM/TAU
-.500D-01	-.327D-38	-.269D-38	.154D-03	-.162D-04 -.223D+03	-.156D-37 -.709D+02	.475D-05 -.436D-01	.000D+00 .000D+00	.000D+00 .000D+00	.181D-05 -.117D+03
-.417D-01	-.252D-38	-.231D-38	.154D-03	-.135D-04 -.186D+03	-.156D-37 -.591D+02	.397D-05 -.221D-01	-.382D-05 -.135D+01	-.228D-05 -.147D+01	.151D-05 .975D+02
-.333D-01	-.176D-38	-.193D-38	.154D-03	-.108D-04 -.149D+03	-.156D-37 -.473D+02	.323D-05 .373D-01	-.694D-05 -.245D+01	-.415D-05 -.266D+01	.121D-05 -.780D+02
-.250D-01	-.101D-38	-.154D-38	.154D-03	-.812D-05 -.126D+03	-.156D-37 -.403D+02	.252D-05 .763D-01	-.937D-05 -.185D+01	-.560D-05 -.116D+01	.907D-06 .674D+02
-.167D-01	-.259D-39	-.116D-38	.154D-03	-.541D-05 -.838D+02	-.156D-37 -.268D+02	.187D-05 .205D+00	-.111D-04 -.219D+01	-.564D-05 -.139D+01	.604D-06 .450D+02

-.833D-02	.495D-39	-.780D-39	.154D-03	-.271D-05 -.419D+02	-.156D-37 -.134D+02	.114D-05 .350D+00	-.122D-04 -.240D+01	-.727D-05 -.151D+01	.302D-06 .225D+02
.000D+00	.125D-38	-.398D-39	.154D-03	-.512D-08 .749D-01	-.156D-37 .111D+00	.470D-06 .502D+00	-.125D-04 -.247D+01	-.746D-05 -.155D+01	.592D-38 .266D-01
.833D-02	.200D-38	-.163D-40	.154D-03	.270D-05 .420D+02	-.156D-37 .136D+02	-.202D-06 .654D+00	-.122D-04 -.240D+01	-.726D-05 -.151D+01	-.302D-06 -.224D+02
.167D-01	.275D-38	.366D-39	.154D-03	.540D-05 .840D+02	-.156D-37 .270D+02	-.881D-06 .799D+00	-.111D-04 -.219D+01	-.664D-05 -.138D+01	-.604D-06 -.449D+02
.250D-01	.351D-38	.748D-39	.154D-03	.811D-05 .126D+03	-.156D-37 .405D+02	-.157D-05 .928D+00	-.937D-05 -.185D+01	-.560D-05 -.116D+01	-.907D-06 -.674D+02
.333D-01	.426D-38	.113D-38	.154D-03	.108D-04 .149D+03	-.156D-37 .475D+02	-.229D-05 .967D+00	-.694D-05 -.245D+01	-.415D-05 -.266D+01	-.121D-05 .780D+02
.417D-01	.501D-38	.151D-38	.154D-03	.135D-04 .166D+03	-.156D-37 .593D+02	-.303D-05 .103D+01	-.382D-05 -.135D+01	-.228D-05 -.147D+01	-.151D-05 .975D+02
.500D-01	.577D-38	.189D-38	.154D-03	.162D-04 .223D+03	-.156D-37 .712D+02	-.381D-05 .105D+01	.000D+00 .000D+00	.000D+00 .000D+00	-.191D-05 .117D+03

FORCE AND MOMENT RESULTANTS:

NX = .749D-02	NY = .111D-01	NZ = .455D-01	NYZ = .000D+00	NZX = .000D+00	NXY = .291D-32
MX = .378D+00	MY = .120D+00	MZ = .546D-01	MYZ = .000D+00	MZX = .000D+00	MYX = .142D+00
QX = -.482D-01	QY = -.208D+00	QZ = .000D+00			

ELEMENT FORCES

NODE	R1	R2	R3	R4	R5	R6
1	.125E-02	-.485E-33	.160E-01	-.254E-01	-.656E-01	.000E+00
2	-.125E-02	.398E-03	.501E-02	.187E-02	.382E-01	.000E+00
3	.485E-33	-.398E-03	-.210E-01	.165E-01	.221E-01	.000E+00

ELEMENT STRAIN ENERGY = .331E-05

ELEMENT ID: 3

HIGHER ORDER AUXILIARY FUNCTIONS:

W1 = .212E-07 W2 = -.613E-07

HOT3 DISPLACEMENTS, STRAINS AND STRESSES THROUGH ELEMENT THICKNESS:

Z	UX	UY	UZ	XX EPS/SIG	YY EPS/SIG	ZZ EPS/SIG	YZ GAM/TAU	XZ GAM/TAU	XY GAM/TAU
-.500D-01	-.541D-05	.604D-06	.134D-03	-.119D-04 -.163D+03	.169D-20 -.519D+02	.347D-05 -.367D-01	.000D+00 .000D+00	.000D+00 .000D+00	.133D-05 -.856D+02
-.417D-01	-.451D-05	.504D-06	.134D-03	-.990D-05 -.136D+03	.169D-20 -.433D+02	.291D-05 -.191D-01	-.514D-05 -.257D+01	-.888D-05 -.444D+01	.111D-05 -.714D+02
-.333D-01	-.361D-05	.403D-06	.134D-03	-.792D-05 -.109D+03	.000D+00 -.346D+02	.237D-05 .295D-01	-.935D-05 -.467D+01	-.161D-04 -.808D+01	.885D-06 -.571D+02
-.250D-01	-.270D-05	.302D-06	.134D-03	-.594D-05 -.920D+02	.847D-21 -.295D+02	.185D-05 .659D-01	-.126D-04 -.640D+00	-.218D-04 -.762D+01	.664D-06 .494D+02
-.167D-01	-.180D-05	.201D-06	.134D-03	-.396D-05 -.613D+02	.424D-21 -.196D+02	.135D-05 .170D+00	-.150D-04 -.758D+00	-.258D-04 -.903D+01	.442D-06 .329D+02
-.833D-02	-.903D-06	.101D-06	.134D-03	-.198D-05 -.306D+02	.212D-21 -.977D+01	.865D-06 .287D+00	-.164D-04 -.829D+00	-.283D-04 -.988D+01	.221D-06 .165D+02
.000D+00	-.171D-08	.310D-24	.134D-03	-.350D-08 .712D-01	.000D+00 .937D-01	.384D-06 .410D+00	-.168D-04 -.853D+00	-.291D-04 -.102D+02	-.217D-24 .165D-01
.833D-02	.899D-06	-.101D-06	.134D-03	.198D-05 .308D+02	-.212D-21 .995D+01	-.972D-07 .533D+00	-.164D-04 -.829D+00	-.283D-04 -.988D+01	-.221D-06 -.164D+02
.167D-01	.180D-05	-.201D-06	.134D-03	.395D-05 .615E+02	-.424D-21 .198D+02	-.584D-06 .650D+00	-.150D-04 -.758D+00	-.258D-04 -.903D+01	-.442D-06 -.329D+02
.250D-01	.270D-05	-.302D-06	.134D-03	.593D-05 .922D+02	-.424D-21 .297D+02	-.108D-05 .754D+00	-.126D-04 -.640D+00	-.218D-04 -.762D+01	-.664D-06 -.493D+02
.333D-01	.360D-05	-.403D-06	.134D-03	.791D-05 .109D+03	-.847D-21 .348D+02	-.160D-05 .791D+00	-.935D-05 -.467D+01	-.161D-04 -.808D+01	-.885D-06 .571D+02
.417D-01	.450D-05	-.504D-06	.134D-03	.989D-05 .136D+03	.000D+00 .435D+02	-.214D-05 .839D+00	-.514D-05 -.257D+01	-.888D-05 -.444D+01	-.111D-05 .713D+02
.500D-01	.540D-05	-.604D-06	.134D-03	.119D-04 .164D+03	-.169D-20 .521D+02	-.271D-05 .857D+00	.000D+00 .000D+00	.000D+00 .000D+00	-.191D-05 .117D+03

FORCE AND MOMENT RESULTANTS:

NX = .712D-02 NY = .937D-02 NZ = .372D-01 NYZ = .000D+00 NZX = .000D+00 NXY = -.499D-17
 MX = .277D+00 MY = .882D-01 MZ = .446D-01 MYZ = .000D+00 MZX = .000D+00 MXY = .104D+00
 QX = -.300D+00 QY = .597D+00 QZ = .000D+00

ELEMENT FORCES

NODE	R1	R2	R3	R4	R5	R6
1	-.542E-19	.272E-03	.178E-01	-.176E-01	-.231E-01	.000E+00
2	-.853E-03	.379E-18	-.802E-01	.117E-01	.327E-01	.000E+00
3	.853E-03	-.272E-03	.624E-01	.555E-04	-.364E-01	.000E+00

ELEMENT STRAIN ENERGY = .210E-05

ELEMENT ID: 4

HIGHER ORDER AUXILIARY FUNCTIONS:
 W1 = .177E-07 W2 = -.630E-07

HOT3 DISPLACEMENTS, STRAINS AND STRESSES THROUGH ELEMENT THICKNESS:

Z	UX	UY	UZ	XX EPS/SIG	YY EPS/SIG	ZZ EPS/SIG	YZ GAM/TAU	XZ GAM/TAU	XY GAM/TAU
-.560D-01	-.541D-05	.604D-06	.134D-03	-.119D-04 -.163D+03	-.987D-38 -.519D+02	.348D-05 -.297D-01	.000D+00 .000D+00	.000D+00 .000D+00	.133D-05 -.856D+02
-.417D-01	-.451D-05	.504D-06	.134D-03	-.990D-05 -.136D+03	-.987D-38 -.433D+02	.291D-05 -.145D-01	-.738D-05 -.304D+01	-.776D-05 -.426D+01	.111D-05 -.714D+02
-.333D-01	-.361D-05	.403D-06	.134D-03	-.792D-05 -.109D+03	-.987D-38 -.346D+02	.236D-05 .260D-01	-.134D-04 -.552D+01	-.141D-04 -.774D+01	.885D-06 -.571D+02
-.250D-01	-.270D-05	.302D-06	.134D-03	-.594D-05 -.920D+02	-.987D-38 -.295D+02	.184D-05 .501D-01	-.181D-04 -.251D+01	-.190D-04 -.574D+01	.664D-06 .494D+02
-.167D-01	-.180D-05	.201D-06	.134D-03	-.396D-05 -.613D+02	-.987D-38 -.196D+02	.132D-05 .139D+00	-.215D-04 -.297D+01	-.226D-04 -.681D+01	.442D-06 .329D+02
-.833D-02	-.903D-06	.101D-06	.134D-03	-.198D-05 -.306D+02	-.987D-38 -.978D+01	.820D-06 .238D+00	-.235D-04 -.325D+01	-.247D-04 -.744D+01	.221D-06 .165D+02
.000D+00	-.171D-08	.157D-38	.134D-03	-.350D-08 .511D-01	-.987D-38 .756D-01	.321D-06 .343D+00	-.241D-04 -.334D+01	-.254D-04 -.766D+01	-.116D-38 .182D-01
.833D-02	.899D-06	-.101D-06	.134D-03	.198D-05 .308D+02	-.987D-38 .993D+01	-.177D-06 .447D+00	-.235D-04 -.325D+01	-.247D-04 -.744D+01	-.221D-06 -.164D+02
.167D-01	.180D-05	-.201D-06	.134D-03	.395D-05 .615D+02	-.987D-38 .198D+02	-.681D-06 .547D+00	-.215D-04 -.297D+01	-.226D-04 -.681D+01	-.442D-06 -.329D+02
.250D-01	.270D-05	-.302D-06	.134D-03	.593D-05 .921D+02	-.987D-38 .296D+02	-.119D-05 .636D+00	-.181D-04 -.251D+01	-.190D-04 -.574D+01	-.664D-06 -.493D+02
.333D-01	.360D-05	-.403D-06	.134D-03	.791D-05 .109D+03	-.987D-38 .348D+02	-.172D-05 .660D+00	-.134D-04 -.552D+01	-.141D-04 -.774D+01	-.885D-06 .571D+02
.417D-01	.450D-05	-.504D-06	.134D-03	.989D-05 .136D+03	-.987D-38 .434D+02	-.227D-05 .700D+00	-.738D-05 -.304D+01	-.776D-05 -.426D+01	-.111D-05 .713D+02
.500D-01	.540D-05	-.604D-06	.134D-03	.119D-04 .164D+03	-.987D-38 .521D+02	-.284D-05 .715D+00	.000D+00 .000D+00	.000D+00 .000D+00	-.133D-05 .856D+02

FORCE AND MOMENT RESULTANTS:

NX = .511D-02 NY = .756D-02 NZ = .311D-01 NYZ = .000D+00 NZX = .000D+00 NXY = -.570D-33
 MX = .277D+00 MY = .882D-01 MZ = .373D-01 MYZ = .000D+00 MZX = .000D+00 MXY = .104D+00
 QX = -.225D+00 QY = -.365D+00 QZ = .000D+00

ELEMENT FORCES

NODE	R1	R2	R3	R4	R5	R6
1	.853E-03	.949E-34	.648E-01	-.226E-01	-.569E-01	.000E+00
2	-.853E-03	.272E-03	-.303E-01	.219E-02	.231E-01	.000E+00
3	-.949E-34	-.272E-03	-.345E-01	.892E-02	.122E-01	.000E+00

ELEMENT STRAIN ENERGY = .208E-05

ELEMENT ID: 5

HIGHER ORDER AUXILIARY FUNCTIONS:
 W1 = .947E-08 W2 = -.216E-07

HOT3 DISPLACEMENTS, STRAINS AND STRESSES THROUGH ELEMENT THICKNESS:

XX YY ZZ YZ XZ XY

Z	UX	UY	UZ	EPS/SIG	EPS/SIG	EPS/SIG	GAM/TAU	GAM/TAU	GAM/TAU
-.500D-01	-.937D-05	.105D-05	.771D-04	-.435D-05 -.598D+02	.127D-20 -.190D+02	.127D-05 -.172D-01	.000D+00 .000D+00	.000D+00 .000D+00	.486D-06 -.313D+02
-.417D-01	-.781D-05	.873D-06	.771D-04	-.362D-05 -.499D+02	.106D-20 -.158D+02	.106D-05 -.926D-02	-.814D-05 -.374D+01	-.116D-04 -.597D+01	.405D-06 -.261D+02
-.333D-01	-.625D-05	.698D-06	.772D-04	-.290D-05 -.399D+02	.635D-21 -.127D+02	.868D-06 .126D-01	-.148D-04 -.680D+01	-.210D-04 -.109D+02	.324D-06 -.209D+02
-.250D-01	-.469D-05	.524D-06	.772D-04	-.217D-05 -.337D+02	.635D-21 -.108D+02	.585D-06 .321D-01	-.200D-04 -.181D+01	-.284D-04 -.947D+01	.243D-06 .181D+02
-.167D-01	-.312D-05	.349D-06	.772D-04	-.145D-05 -.224D+02	.424D-21 -.718D+01	.509D-06 .780D-01	-.237D-04 -.214D+01	-.337D-04 -.112D+02	.162D-06 .120D+02
-.833D-02	-.156D-05	.175D-06	.772D-04	-.725D-06 -.112D+02	.159D-21 -.357D+01	.339D-06 .129D+00	-.259D-04 -.234D+01	-.368D-04 -.123D+02	.810D-07 .602D+01
.000D+00	-.287D-08	.207D-24	.772D-04	-.937D-09 .411D-01	-.414D-24 .449D-01	.172D-06 .184D+00	-.266D-04 -.241D+01	-.379D-04 -.126D+02	-.619D-24 .248D-01
.833D-02	.156D-05	-.175D-06	.772D-04	.723D-06 .113D+02	-.159D-21 .366D+01	.440D-08 .238D+00	-.259D-04 -.234D+01	-.368D-04 -.123D+02	-.810D-07 -.602D+01
.167D-01	.312D-05	-.349D-06	.772D-04	.145D-05 .225D+02	-.318D-21 .727D+01	-.166D-06 .290D+00	-.237D-04 -.214D+01	-.337D-04 -.112D+02	-.162D-06 -.120D+02
.250D-01	.468D-05	-.524D-06	.772D-04	.217D-05 .338D+02	-.635D-21 .109D+02	-.341D-06 .375D+00	-.200D-04 -.181D+01	-.284D-04 -.947D+01	-.243D-06 -.181D+02
.333D-01	.624D-05	-.698D-06	.772D-04	.290D-05 .400D+02	-.635D-21 .128D+02	-.524D-06 .355D+00	-.148D-04 -.680D+01	-.210D-04 -.109D+02	-.324D-06 .209D+02
.417D-01	.780D-05	-.873D-06	.772D-04	.362D-05 .499D+02	-.127D-20 .159D+02	-.718D-06 .377D+00	-.814D-05 -.374D+01	-.116D-04 -.597D+01	-.405D-06 .261D+02
.500D-01	.936D-05	-.105D-05	.772D-04	.434D-05 .599D+02	-.127D-20 .191D+02	-.924D-06 .385D+00	.000D+00 .000D+00	.000D+00 .000D+00	-.486D-06 .313D+02

FORCE AND MOMENT RESULTANTS:

NX = .411D-02 NY = .449D-02 NZ = .167D-01 NYZ = .000D+00 NZX = .000D+00 NXY = .422D-19
 MX = .101D+00 MY = .323D-01 MZ = .200D-01 MYZ = .000D+00 MZX = .000D+00 MXY = .382D-01
 QX = -.447D+00 QY = .794D+00 QZ = .000D+00

ELEMENT FORCES

NODE	R1	R2	R3	R4	R5	R6
1	-.136E-19	.729E-04	.326E-01	-.108E-01	-.139E-01	.000E+00
2	-.228E-03	-.176E-18	-.102E+00	-.122E-02	-.100E-03	.000E+00
3	.228E-03	-.729E-04	.692E-01	.114E-02	-.199E-01	.000E+00

ELEMENT STRAIN ENERGY = .865E-06

ELEMENT ID: 6

HIGHER ORDER AUXILIARY FUNCTIONS:

W1 = .474E-08 W2 = -.239E-07

HOT3 DISPLACEMENTS, STRAINS AND STRESSES THROUGH ELEMENT THICKNESS:

Z	UX	UY	UZ	XX EPS/SIG	YY EPS/SIG	ZZ EPS/SIG	YZ GAM/TAU	XZ GAM/TAU	XY GAM/TAU
-.500D-01	-.937D-05	.105D-05	.771D-04	-.435D-05 -.598D+02	-.203D-38 -.190D+02	.128D-05 -.693D-02	.000D+00 .000D+00	.000D+00 .000D+00	.486D-06 -.313D+02
-.417D-01	-.781D-05	.873D-06	.771D-04	-.362D-05 -.499D+02	-.203D-38 -.158D+02	.107D-05 -.304D-02	-.896D-05 -.391D+01	-.112D-04 -.591D+01	.405D-06 -.261D+02
-.333D-01	-.625D-05	.698D-06	.772D-04	-.290D-05 -.399D+02	-.203D-38 -.127D+02	.863D-06 .771D-02	-.163D-04 -.712D+01	-.203D-04 -.107D+02	.324D-06 -.209D+02
-.250D-01	-.469D-05	.524D-06	.772D-04	-.217D-05 -.337D+02	-.203D-38 -.108D+02	.664D-06 .104D-01	-.220D-04 -.249D+01	-.274D-04 -.878D+01	.243D-06 .181D+02
-.167D-01	-.312D-05	.349D-06	.772D-04	-.145D-05 -.225D+02	-.203D-38 -.719D+01	.469D-06 .353D-01	-.261D-04 -.295D+01	-.325D-04 -.104D+02	.162D-06 .120D+02
-.833D-02	-.156D-05	.175D-06	.772D-04	-.725D-06 -.112D+02	-.203D-38 -.338D+01	.277D-06 .629D-01	-.285D-04 -.323D+01	-.355D-04 -.114D+02	.810D-07 .602D+01
.000D+00	-.287D-08	.119D-38	.772D-04	-.937D-09 .137E-01	-.203D-38 .253E-01	.861D-07 .919D-01	-.293D-04 -.332D+01	-.365D-04 -.117D+02	-.255D-38 .488E-02
.833D-02	.156D-05	-.175D-06	.772D-04	.723D-06 .112D+02	-.203D-38 .362D+01	-.105D-06 .121D+00	-.285D-04 -.323D+01	-.355D-04 -.114D+02	-.810D-07 -.602D+01

.167D-01	.312D-05	-.349D-06	.772D-04	.145D-05	.203D-38	-.297D-06	-.261D-04	-.325D-04	-.162D-06
				.225D+02	.723D+01	.148D+09	-.295D+01	-.104D+02	-.120D+02
.250D-01	.468D-05	-.524D-06	.772D-04	.217D-05	-.203D-38	-.492D-06	-.220D-04	-.274D-04	-.263D-06
				.337D+02	.108D+02	.173D+00	-.249D+01	-.879D+01	-.181D+02
.333D-01	.624D-05	-.698D-06	.772D-04	.290D-05	-.203D-38	-.691D-06	-.163D-04	-.203D-04	-.324D-06
				.399D+02	.127D+02	.176D+00	-.712D+01	-.107D+02	.209D+02
.417D-01	.780D-05	-.873D-06	.772D-04	.362D-05	-.203D-38	-.895D-06	-.896D-05	-.112D-04	-.405D-06
				.499D+02	.159D+02	.187D+00	-.391D+01	-.591D+01	.261D+02
.500D-01	.936D-05	-.105D-05	.771D-04	.434D-05	-.203D-38	-.111D-05	.000D+00	.000D+00	-.486D-06
				.599D+02	.190D+02	.191D+00	.000D+00	.000D+00	.313D+02

FORCE AND MOMENT RESULTANTS:

NX = .137D-02	NY = .203D-02	NZ = .833D-02	NYZ = .000D+00	NZX = .000D+00	NXZ = -.125E-32
MX = .101D+00	MY = .323D-01	MZ = .100D-01	MYZ = .000D+00	MZX = .000D+00	MAX = .363D-01
QX = -.342D+00	QY = -.424D+00	QZ = .000D+00			

ELEMENT FORCES

NODE	R1	R2	R3	R4	R5	R6
1	.228E-03	.209E-33	.962E-01	-.138E-01	-.329E-01	.000E+00
2	-.228E-03	.729E-04	-.574E-01	.193E-02	.187E-02	.000E+00
3	-.209E-33	-.729E-04	-.388E-01	-.109E-02	-.103E-02	.000E+00

ELEMENT STRAIN ENERGY = .855E-06

INCREMENT 1 SUMMARY

TIME INCREMENT COMPLETED	2.220E-16,	FRACTION OF STEP COMPLETED	1.00
STEP TIME COMPLETED	2.220E-16,	TOTAL TIME COMPLETED	0.000E+00

N O D E O U T P U T

THE FOLLOWING TABLE IS PRINTED FOR ALL NODES

NODE FOOT-NOTE	U1	U2	U3	UR1	UR2	UR3
1	0.0000E+00	0.0000E+00	1.5436E-04	0.0000E+00	0.0000E+00	0.0000E+00
2	-1.7058E-09	0.0000E+00	1.3368E-04	1.2089E-05	1.0813E-04	0.0000E+00
3	-2.8716E-09	0.0000E+00	7.7172E-05	2.0940E-05	1.8731E-04	0.0000E+00
4	-3.1838E-09	0.0000E+00	0.0000E+00	2.4178E-05	2.1628E-04	0.0000E+00
5	0.0000E+00	0.0000E+00	1.5436E-04	0.0000E+00	0.0000E+00	0.0000E+00
6	-1.7058E-09	0.0000E+00	1.3368E-04	1.2089E-05	1.0813E-04	0.0000E+00
7	-2.8716E-09	0.0000E+00	7.7172E-05	2.0940E-05	1.8731E-04	0.0000E+00
8	-3.1838E-09	0.0000E+00	0.0000E+00	2.4178E-05	2.1628E-04	0.0000E+00
MAXIMUM AT NODE	0.0000E+00 1	0.0000E+00 1	1.5436E-04 1	2.4178E-05 4	2.1628E-04 4	0.0000E+00 1
MINIMUM AT NODE	-3.1838E-09 4	0.0000E+00 1	0.0000E+00 4	0.0000E+00 1	0.0000E+00 1	0.0000E+00 1

THE ANALYSIS HAS BEEN COMPLETED

APPENDIX C

Demonstration Problem II: Dynamic Analysis

ABAQUS INPUT DECK

```

*HEADING
EIGENANALYSIS OF A SQUARE PLATE USING 8 HOT3 ELEMENTS
*PREPRINT,ECHO=NO
**
*NODE
1,0.0,0.0,0.0
2,0.5,0.0,0.0
3,1.0,0.0,0.0
4,0.0,0.5,0.0
5,0.5,0.5,0.0
6,1.0,0.5,0.0
7,0.0,1.0,0.0
8,0.5,1.0,0.0
9,1.0,1.0,0.0
10, 0.1666667, 0.1666667, 0.0
11, 0.3333333, 0.3333333, 0.0
12, 0.6666667, 0.3333333, 0.0
13, 0.8333333, 0.1666667, 0.0
14, 0.1666667, 0.8333333, 0.0
15, 0.3333333, 0.6666667, 0.0
16, 0.6666667, 0.6666667, 0.0
17, 0.8333333, 0.8333333, 0.0
**
*NSET,NSET=NOUT,GENERATE
1,9,1
*NSET,NSET=NCEN,GENERATE
10,17,1
**
*USER ELEMENT,NODES=4,TYPE=U1,PROPERTIES=72,COORDINATES=6
1,2,3,4,5,6
*ELSET,ELSET=N1
**
*UEL PROPERTY,ELSET=N1
3.0,1.0,0.000,0.000,0.000,1.0,3.0,1.0
1.0E6,0.025E6,0.025E6,0.015E6,0.0125E6,0.015E6
0.25,0.25,0.25,0.05,0.0
1.0E6,0.025E6,0.025E6,0.015E6,0.0125E6,0.015E6
0.25,0.25,0.25,0.1,90.0
1.0E6,0.025E6,0.025E6,0.015E6,0.0125E6,0.015E6
0.25,0.25,0.25,0.05,0.0
0.333,0.333,0.333,1.0,0.0,0.0
2.0,0.0,1.0,0.0,0.0,0.0,1.0,0.0
**
*ELEMENT,TYPE=U1,ELSET=N1
1,1,5,4,11
2,1,2,5,10
3,2,3,5,12
4,3,6,5,13
5,4,5,7,14
6,5,8,7,15
7,5,9,8,17
8,5,6,9,16
**
*USER SUBROUTINE,INPUT=uel_hot3.f
**
*BOUNDARY
NOUT,6,6
NCEN,3,6
1,1,5
2,1,1
2,3,3
2,5,5
3,1,5
4,2,4
6,2,4
7,1,5
8,1,1
8,3,3
8,5,5
9,1,5
**
*STEP,PERTURBATION:
*FREQUENCY
5
*NODE PRINT, NSET=NOUT
!
*END STEP

```

ABAQUS OUTPUT FILE

```

AAAAAA  BBBBBBBB  AAAAAA  QQQQQQQQ  U  U  SSSSSSSS
A  A  B  B  A  A  Q  Q  U  U  S
A  A  B  B  A  A  Q  Q  U  U  S
A  A  B  B  A  A  Q  Q  U  U  S
AAAAAA  BBBBBBBB  AAAAAA  Q  Q  Q  Q  U  U  SSSSSSSS
A  A  B  B  A  A  Q  Q  Q  Q  U  U  S
A  A  B  B  A  A  Q  Q  Q  Q  U  U  S
A  A  B  B  A  A  Q  Q  Q  Q  U  U  S
A  A  BBBBBBBB  A  A  QQQQQQQQ  UUUUUUUU  SSSSSSSS

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1080 MAIN STREET
PAWTUCKET, R.I. 02860

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PLEASE MAKE SURE YOU ARE USING VERSION 5.2 MANUALS
PLUS THE NOTES ACCOMPANYING THIS RELEASE. THESE NOTES
CAN BE OBTAINED BY USING THE INFORMATION OPTION ON THE
ABAQUS COMMAND LINE.

OPTIONS BEING PROCESSED

*HEADING SIGANALYSIS OF A SQUARE PLATE USING 6 NOD3 ELEMENTS
*NODE
*NSET, NSET1*NOU7.GENERATI

*NSET,NSET=NCEN,GENERATE
 *USER ELEMENT,NODES=4,TYPE=U1,PROPERTIES=72,COORDINATES=6
 *ELSET,ELSET=N1
 *ELEMENT,TYPE=U1,ELSET=N1
 *USER ELEMENT,NODES=4,TYPE=U1,PROPERTIES=72,COORDINATES=6
 *UEL PROPERTY,ELSET=N1
 *STEP,PERTURBATION
 *FREQUENCY
 *END STEP
 *BOUNDARY
 *STEP,PERTURBATION
 *FREQUENCY
 *NODE PRINT, NSET=NOUT
 *END STEP

ELEMENT DEFINITIONS

NUMBER	TYPE	PROPERTY REFERENCE	NODES FORMING ELEMENT					
1	U1	1	1	5	4	11		
2	U1	1	1	2	5	10		
3	U1	1	2	3	5	12		
4	U1	1	3	6	5	13		
5	U1	1	4	5	7	14		
6	U1	1	5	8	7	15		
7	U1	1	5	9	8	17		
8	U1	1	5	6	9	16		

USER ELEMENTS

ELEMENT TYPE U1
 NUMBER OF NODES 4
 NUMBER OF COORDINATES 6
 NUMBER OF PROPERTIES 72
 NUMBER OF VARIABLES 1

DEGREES OF FREEDOM
 NODE D.O.F.
 1 1 2 3 4 5 6
 2 1 2 3 4 5 6
 3 1 2 3 4 5 6
 4 1 2 3 4 5 6

USER ELEMENT PROPERTY

PROPERTY NUMBER 1
 PROPERTIES

3.000	1.000	0.0000E+00	0.0000E+00	0.0000E+00	1.000	3.000	1.000
1.0000E+06	2.5000E+04	2.5000E+04	1.5000E+04	1.2500E+04	1.5000E+04	0.0000E+00	0.0000E+00
.2500	.2500	.2500	5.0000E-02	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
1.0000E+06	2.5000E+04	2.5000E+04	1.5000E+04	1.2500E+04	1.5000E+04	0.0000E+00	0.0000E+00
.2500	.2500	.2500	.1000	90.00	0.0000E+00	0.0000E+00	0.0000E+00
1.0000E+06	2.5000E+04	2.5000E+04	1.5000E+04	1.2500E+04	1.5000E+04	0.0000E+00	0.0000E+00
.2500	.2500	.2500	5.0000E-02	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
.3330	.3330	.3330	1.000	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2.000	0.0000E+00	1.000	0.0000E+00	0.0000E+00	0.0000E+00	1.000	0.0000E+00

ELEMENT SETS

SET	N1	MEMBERS	1	2	3	4	5	6	7	8
-----	----	---------	---	---	---	---	---	---	---	---

NODE SETS

SET	NOUT	MEMBERS	1	2	3	4	5	6	7	8	9
SET	NCEN	MEMBERS	10	11	12	13	14	15	16	17	

NODE DEFINITIONS

NODE NUMBER	COORDINATES						NORMAL	SINGLE POINT CONSTRAINTS		
								TYPE	PLUS	DOF
1	C.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00				1 2 3 4 5 6
2	.50000	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00				1 3 4 5 6
3	1.0000	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00				1 3 4 5 6
4	C.00000E+00	.50000	C.00000E+00	C.00000E+00	C.00000E+00	C.00000E+00				1 4 5 6
5	.50000	.50000	C.00000E+00	C.00000E+00	C.00000E+00	C.00000E+00				1 4 5 6
6	1.0000	.50000	C.00000E+00	C.00000E+00	C.00000E+00	C.00000E+00				1 4 5 6
7	C.00000E+00	1.0000	C.00000E+00	C.00000E+00	C.00000E+00	C.00000E+00				1 3 4 5 6
8	.50000	1.0000	C.00000E+00	C.00000E+00	C.00000E+00	C.00000E+00				1 3 4 5 6
9	1.0000	1.0000	C.00000E+00	C.00000E+00	C.00000E+00	C.00000E+00				1 3 4 5 6

10	.16667	.16667	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	3 4 5 6
11	.33333	.33333	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	3 4 5 6
12	.66667	.33333	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	3 4 5 6
13	.83333	.16667	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	3 4 5 6
14	.16667	.83333	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	3 4 5 6
15	.33333	.66667	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	3 4 5 6
16	.66667	.66667	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	3 4 5 6
17	.83333	.83333	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	3 4 5 6

STEP 1 CALCULATION OF EIGENVALUES
FOR NATURAL FREQUENCIES

NUMBER OF EIGENVALUES 5
HIGHEST FREQUENCY OF INTEREST .10000E+19
MAXIMUM NUMBER OF ITERATIONS 30
NUMBER OF VECTORS IN ITERATION 10

THIS IS A LINEAR PERTURBATION STEP.
ALL LOADS ARE DEFINED AS CHANGE IN LOAD TO THE REFERENCE STATE

EXTRAPOLATION WILL NOT BE USED

PRINT OF INCREMENT NUMBER, TIME, ETC., EVERY 1 INCREMENTS

NODE PRINT

THE FOLLOWING TABLE IS PRINTED FOR NODESET NOUT AT EVERY 1 INCREMENT

SUMMARIES WILL BE PRINTED

THIS DATA IS PRINTED FOR ALL EIGENVALUE MODES

TABLE 1 U1 U2 U3 UR1 UR2 UR3							
BOUNDARY CONDITIONS							
NODE	DOF	AMP. REF.	MAGNITUDE	NODE	DOF	AMP. REF.	MAGNITUDE
1	6	(STEP)	0.00000E+00	1	1	(STEP)	0.00000E+00
1	2	(STEP)	0.00000E+00	1	3	(STEP)	0.00000E+00
1	4	(STEP)	0.00000E+00	1	5	(STEP)	0.00000E+00
2	6	(STEP)	0.00000E+00	2	1	(STEP)	0.00000E+00
2	3	(STEP)	0.00000E+00	2	5	(STEP)	0.00000E+00
3	6	(STEP)	0.00000E+00	3	1	(STEP)	0.00000E+00
3	2	(STEP)	0.00000E+00	3	3	(STEP)	0.00000E+00
3	4	(STEP)	0.00000E+00	3	5	(STEP)	0.00000E+00
4	6	(STEP)	0.00000E+00	4	2	(STEP)	0.00000E+00
4	3	(STEP)	0.00000E+00	4	4	(STEP)	0.00000E+00
5	6	(STEP)	0.00000E+00	6	6	(STEP)	0.00000E+00
6	2	(STEP)	0.00000E+00	6	3	(STEP)	0.00000E+00
6	4	(STEP)	0.00000E+00	7	6	(STEP)	0.00000E+00
7	1	(STEP)	0.00000E+00	7	2	(STEP)	0.00000E+00
7	3	(STEP)	0.00000E+00	7	4	(STEP)	0.00000E+00
7	5	(STEP)	0.00000E+00	8	6	(STEP)	0.00000E+00
8	1	(STEP)	0.00000E+00	8	3	(STEP)	0.00000E+00
8	5	(STEP)	0.00000E+00	9	6	(STEP)	0.00000E+00
9	1	(STEP)	0.00000E+00	9	2	(STEP)	0.00000E+00
9	3	(STEP)	0.00000E+00	9	4	(STEP)	0.00000E+00
9	5	(STEP)	0.00000E+00	10	3	(STEP)	0.00000E+00
10	4	(STEP)	0.00000E+00	10	5	(STEP)	0.00000E+00
10	6	(STEP)	0.00000E+00	11	3	(STEP)	0.00000E+00
11	4	(STEP)	0.00000E+00	11	5	(STEP)	0.00000E+00
11	6	(STEP)	0.00000E+00	12	3	(STEP)	0.00000E+00
12	4	(STEP)	0.00000E+00	12	5	(STEP)	0.00000E+00
12	6	(STEP)	0.00000E+00	13	3	(STEP)	0.00000E+00
13	4	(STEP)	0.00000E+00	13	5	(STEP)	0.00000E+00
13	6	(STEP)	0.00000E+00	14	3	(STEP)	0.00000E+00
14	4	(STEP)	0.00000E+00	14	5	(STEP)	0.00000E+00
14	6	(STEP)	0.00000E+00	15	3	(STEP)	0.00000E+00
15	4	(STEP)	0.00000E+00	15	5	(STEP)	0.00000E+00
15	6	(STEP)	0.00000E+00	16	3	(STEP)	0.00000E+00
16	4	(STEP)	0.00000E+00	16	5	(STEP)	0.00000E+00
16	6	(STEP)	0.00000E+00	17	3	(STEP)	0.00000E+00
17	4	(STEP)	0.00000E+00	17	5	(STEP)	0.00000E+00
17	6	(STEP)	0.00000E+00				

- (RAMP) OR (STEP) - INDICATE USE OF DEFAULT AMPLITUDES ASSOCIATED WITH THE STEP

WAVEFRONT MINIMIZATION

NUMBER OF NODES 17
NUMBER OF ELEMENTS 6
ORIGINAL MAXIMUM D.O.F WAVEFRONT ESTIMATED AS 30
ORIGINAL RMS D.O.F WAVEFRONT ESTIMATED AS 29

PERIPHERAL DIAMETER IS DEFINED BY NODES 10 11
 WAVEFRONT OPTIMIZED BY CHOOSING 11 AS THE STARTING NODE
 OPTIMIZER IS UNSUCCESSFUL IN RENUMBERING ELEMENTS

P R O B L E M S I Z E

NUMBER OF ELEMENTS IS 8
 NUMBER OF NODES IS 17
 NUMBER OF NODES DEFINED BY THE USER 17
 NUMBER OF INTERNAL NODES GENERATED BY THE PROGRAM 0
 TOTAL NUMBER OF VARIABLES IN THE MODEL 102
 (DEGREES OF FREEDOM PLUS ANY LAGRANGE MULTIPLIER VARIABLES)
 MAXIMUM D.O.F. WAVEFRONT ESTIMATED AS 30
 RMS WAVEFRONT ESTIMATED AS 29

FILE SIZES - THESE VALUES ARE IN WORDS AND ARE CONSERVATIVE UPPER BOUNDS

UNIT	LENGTH
1	2792
21	272
22	272
30	96

IF THE RESTART FILE IS WRITTEN ITS LENGTH WILL BE APPROXIMATELY
 3317 WORDS WRITTEN IN THE PRE PROGRAM
 PLUS 149 WORDS WRITTEN AT THE BEGINNING OF EACH STEP
 PLUS 2340 WORDS FOR EACH INCREMENT WRITTEN TO THE RESTART FILE

JOB TIME SUMMARY
 CPU TIME (SEC) = .63000
 SEC) = .63000

S T E P 1 C A L C U L A T I O N O F E I G E N V A L U E S F O R N A T U R A L F R E Q U E N C I E S

NUMBER OF EIGENVALUES 5
 HIGHEST FREQUENCY OF INTEREST .10000E+19
 MAXIMUM NUMBER OF ITERATIONS 30
 NUMBER OF VECTORS IN ITERATION 10

THIS IS A LINEAR PERTURBATION STEP.
 ALL LOADS ARE DEFINED AS CHANGE IN LOAD TO THE REFERENCE STATE

TOTAL MASS OF MODEL ASSOCIATED WITH EACH DIRECTION
 (MASS MAY BE DIFFERENT IN DIFFERENT DIRECTIONS IF POINT MASSES HAVE BEEN INTRODUCED)
 DIRECTION 1 DIRECTION 2 DIRECTION 3

.267 .248 .227

LOCATION OF THE CENTER OF MASS OF THE MODEL
 5.000E-01 5.000E-01 0.000E+00

MOMENTS OF INERTIA ABOUT THE ORIGIN
 I (XX) I (YY) I (ZZ)
 8.863E-02 8.248E-02 .150

PRODUCTS OF INERTIA ABOUT THE ORIGIN
 I (XY) I (XZ) I (YZ)
 6.667E-02 0.000E+00 8.674E-19

MOMENTS OF INERTIA ABOUT THE CENTER OF MASS
 I (XX) I (YY) I (ZZ)
 2.196E-02 2.048E-02 3.667E-02

PRODUCTS OF INERTIA ABOUT THE CENTER OF MASS
 I (XY) I (XZ) I (YZ)
 1.000E-02 0.000E+00 8.674E-19

E I G E N V A L U E O U T P U T

MODE NO	EIGENVALUE	FREQUENCY	GENERALIZED MASS	COMPOSITE MODAL DAMPING
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		(RAD/TIME)	(CYCLES/TIME)		
1	1.61171E+05	401.46	63.895	3.85765E-02	0.00000E+00
2	1.79837E+05	424.07	67.493	6.63065E-02	0.00000E+00
3	1.79837E+05	424.07	67.493	6.63065E-02	0.00000E+00
4	1.34118E+06	1158.1	184.32	4.52038E-04	0.00000E+00
5	2.06608E+06	1437.4	228.77	3.47775E-04	0.00000E+00

PARTICIPATION FACTORS

MODE NO	X-COMPONENT	Y-COMPONENT	Z-COMPONENT	X-ROTATION	Y-ROTATION	Z-ROTATION
1	6.93378E-15	.24951	1.8768	.93838	-.93838	.12475
2	-1.28000E-05	1.5045	1.09180E-15	5.59230E-16	-2.81353E-16	.75221
3	1.5045	1.28000E-05	1.27514E-15	6.36317E-16	-5.37526E-16	-.75223
4	-1.80703E-05	-1.52750E-07	5.28742E-07	4.5466	-8.15872E-07	6.43465E-02
5	-3.07371E-04	-1.20822E-06	2.64896E-05	1.26053E-05	4.2705	-.45501

EFFECTIVE MASS

MODE NO	X-COMPONENT	Y-COMPONENT	Z-COMPONENT	X-ROTATION	Y-ROTATION	Z-ROTATION
1	1.85465E-30	2.40152E-03	.13588	3.39691E-02	3.39691E-02	6.00381E-04
2	1.08637E-11	.15009	7.90392E-32	2.07366E-32	5.24850E-33	3.75173E-02
3	.15009	1.08637E-11	1.07814E-31	2.68474E-32	1.91582E-32	3.75198E-02
4	1.47607E-13	1.05472E-17	1.26375E-16	9.34449E-03	3.00897E-16	1.87165E-06
5	3.28567E-11	5.07680E-16	2.44033E-13	5.52591E-14	6.34246E-03	7.20003E-05
TOTAL	.15009	.15249	.13588	4.33136E-02	4.03116E-02	7.57113E-02

EIGENVALUE NUMBER 1

NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODESET NOUT

NODE FOOT-NOTE	U1	U2	U3	UR1	UR2	UR3
2	0.0000E+00	0.0000E+00	0.0000E+00	2.223	0.0000E+00	0.0000E+00
4	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	-.5488	0.0000E+00
5	0.0000E+00	0.0000E+00	1.000	-9.2600E-11	-2.2083E-11	0.0000E+00
6	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	.5488	0.0000E+00
8	0.0000E+00	0.0000E+00	0.0000E+00	-2.223	0.0000E+00	0.0000E+00
MAXIMUM AT NODE	0.0000E+00 1	0.0000E+00 1	1.000 5	2.223 2	.5488 6	0.0000E+00 1
MINIMUM AT NODE	0.0000E+00 1	0.0000E+00 1	0.0000E+00 1	-2.223 8	-.5488 4	0.0000E+00 1

EIGENVALUE NUMBER 2

NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODESET NOUT

NODE FOOT-NOTE	U1	U2	U3	UR1	UR2	UR3
2	0.0000E+00	.9928	0.0000E+00	-9.1922E-15	0.0000E+00	0.0000E+00
4	-8.4463E-06	0.0000E+00	0.0000E+00	0.0000E+00	1.1201E-14	0.0000E+00
5	-8.5077E-06	1.000	0.0000E+00	1.1074E-14	1.8538E-15	0.0000E+00
6	-8.4463E-06	0.0000E+00	0.0000E+00	0.0000E+00	-6.3215E-14	0.0000E+00
8	0.0000E+00	.9928	0.0000E+00	8.7601E-14	0.0000E+00	0.0000E+00
MAXIMUM AT NODE	0.0000E+00 1	1.000 5	0.0000E+00 1	8.7601E-14 8	1.1201E-14 4	0.0000E+00 1
MINIMUM AT NODE	-8.5077E-06 5	0.0000E+00 1	0.0000E+00 1	-9.1922E-15 2	-6.3215E-14 6	0.0000E+00 1

EIGENVALUE NUMBER 3

NODE OUTPUT

THE FOLLOWING TABLE IS PRINTED FOR NODESET NOUT

NODE FOOT-NOTE	U1	U2	U3	UR1	UR2	UR3
2	0.0000E+00	1.4463E-06	0.0000E+00	4.0088E-15	0.0000E+00	0.0000E+00
4	.9928	0.0000E+00	0.0000E+00	0.0000E+00	7.4105E-15	0.0000E+00
5	1.000	8.5077E-06	0.0000E+00	-1.3079E-15	4.6538E-16	0.0000E+00
6	.9928	0.0000E+00	0.0000E+00	0.0000E+00	-2.8555E-14	0.0000E+00
8	0.0000E+00	1.4463E-06	0.0000E+00	-1.8408E-14	0.0000E+00	0.0000E+00

MAXIMUM	1.000	8.5077E-06	0.0000E+00	9.0088E-15	7.4105E-15	0.0000E+00
AT NODE	5	5	1	2	4	1
MINIMUM	0.0000E+00	0.0000E+00	0.0000E+00	-1.8408E-14	-2.8555E-14	0.0000E+00
AT NODE	1	1	1	8	6	1

E I G E N V A L U E N U M B E R 4
N O D E O U T P U T

THE FOLLOWING TABLE IS PRINTED FOR NODESET NOUT

NODE FOOT- NOTE	U1	U2	U3	UR1	UR2	UR3
2	0.0000E+00	-3.9328E-08	0.0000E+00	-.3312	0.0000E+00	0.0000E+00
4	5.5276E-08	0.0000E+00	0.0000E+00	0.0000E+00	-1.9931E-07	0.0000E+00
5	-2.4498E-09	-1.0240E-08	-4.4542E-08	1.000	-2.3596E-08	0.0000E+00
6	-4.7208E-08	0.0000E+00	0.0000E+00	0.0000E+00	9.5804E-07	0.0000E+00
8	0.0000E+00	7.3736E-08	0.0000E+00	-.3312	0.0000E+00	0.0000E+00
MAXIMUM	5.5276E-08	7.3736E-08	0.0000E+00	1.000	9.5804E-07	0.0000E+00
AT NODE	4	8	1	5	6	1
MINIMUM	-4.7208E-08	-3.9328E-08	-4.4542E-08	-.3312	-1.9931E-07	0.0000E+00
AT NODE	6	2	5	8	4	1

E I G E N V A L U E N U M B E R 5
N O D E O U T P U T

THE FOLLOWING TABLE IS PRINTED FOR NODESET NOUT

NODE FOOT- NOTE	U1	U2	U3	UR1	UR2	UR3
2	0.0000E+00	-1.8815E-06	0.0000E+00	1.1580E-05	0.0000E+00	0.0000E+00
4	-7.3756E-08	0.0000E+00	0.0000E+00	0.0000E+00	.7977	0.0000E+00
5	-3.8900E-07	-2.0010E-07	-1.2175E-06	-2.5895E-06	1.0000	0.0000E+00
6	1.3821E-06	0.0000E+00	0.0000E+00	0.0000E+00	.7978	0.0000E+00
8	0.0000E+00	2.5497E-06	0.0000E+00	-2.9885E-05	0.0000E+00	0.0000E+00
MAXIMUM	1.3821E-06	2.5497E-06	0.0000E+00	1.1580E-05	1.0000	0.0000E+00
AT NODE	6	8	1	2	5	1
MINIMUM	-3.8900E-07	-1.8815E-06	-1.2175E-06	-2.9885E-05	0.0000E+00	0.0000E+00
AT NODE	5	2	5	8	1	1

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