

AD-A269 171



1

DTIC
ELECTE
SEP 10 1993
S A D

UCRL-ID-114993 App B

**HEMP 3D—A Finite Difference Program
for Calculating Elastic-Plastic Flow**

Mark L. Wilkins

This document has been approved
for public release and sale; its
distribution is unlimited.

May 26, 1993

Lawrence
Livermore
National
Laboratory

This is an informal report intended primarily for internal or limited external distribution. The opinions and conclusions stated are those of the author and may or may not be those of the Laboratory.
Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

93-21030



93 9 06 046

DISCLAIMER

This document was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor the University of California nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial products, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or the University of California, and shall not be used for advertising or product endorsement purposes.

This report has been reproduced
directly from the best available copy.

Available to DOE and DOE contractors from the
Office of Scientific and Technical Information
P.O. Box 62, Oak Ridge, TN 37831
Prices available from (615) 576-8401, FTS 626-8401

Available to the public from the
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Rd.,
Springfield, VA 22161

PREFACE

The HEMP 3D numerical technique for calculating elastic-plastic flow in three space dimensions and time was developed in the late 1960s at the Lawrence Livermore National Laboratory with funding from the Material Science's Office of the Defense Advance Research Projects Agency (DARPA). The numerical scheme programmed for the CDC 6600 computer by John French was first presented at the Second International Conference on Numerical Methods in Fluid Dynamics at the University of California, Berkeley, September 15-19, 1970. The usefulness of the computer simulation program at that time was limited by the lack of adequate three dimensional graphics. With continued funding from DARPA, a production program was developed for the CDCSTAR Computer by Eugene Cronshagen using vector programming, including three dimensional graphics. This work was published in 1975 as UCRL-57574, "A Method for Computer Simulation of Problems in Solid Mechanics and Gas Dynamics in Three Dimensions and Time." The program has for many years been operating on the CRAY-1 computer at the Lawrence Livermore National Laboratory. Presented here is an update of the 1975 report that includes the sliding surface routines programmed by Robert Gulliford.

Accession For		
NTIS	CRA&I	☒
DTIC	TAB	☐
Unannounced		☐
Justification		
By		
Distribution/		
Availability Codes		
Dist	Avail and/or Special	
A-1		

HEMP 3D—A Finite Difference Program for Calculating Elastic-Plastic Flow

INTRODUCTION

The HEMP 3D program can be used to solve problems in solid mechanics involving dynamic plasticity and time dependent material behavior and problems in gas dynamics.

The equations of motion, the conservation equations, and the constitutive relations listed below are solved by finite difference methods following the format of the HEMP computer simulation program formulated in two space dimensions and time.¹

A. Equations of motion.

$$\text{i)} \quad \rho \frac{dx}{dt} = \frac{\partial \Sigma_x}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z}$$

$$\text{ii)} \quad \rho \frac{dy}{dt} = \frac{\partial T_{xy}}{\partial x} + \frac{\partial \Sigma_y}{\partial y} + \frac{\partial T_{yz}}{\partial z}$$

$$\text{iii)} \quad \rho \frac{dz}{dt} = \frac{\partial T_{xz}}{\partial x} + \frac{\partial T_{yz}}{\partial y} + \frac{\partial \Sigma_z}{\partial z}$$

B. Conservation of mass and energy.

$$\text{i)} \quad \frac{dM}{dt} = 0 \quad ; \quad M = \text{Mass element}$$

C. First law of thermodynamics.

$$\text{i)} \quad \dot{E} = -(P + q)\dot{V} + V[s_{xx}\dot{\epsilon}_{xx} + s_{yy}\dot{\epsilon}_{yy} + s_{zz}\dot{\epsilon}_{zz} + T_{xy}\dot{\epsilon}_{xy} + T_{yz}\dot{\epsilon}_{yz} + T_{xz}\dot{\epsilon}_{xz}]$$

E = Internal energy per original volume

V = Relative volume = ρ^0/ρ

ρ = actual density

ρ^0 = reference density of equation of state

D. Velocity strains.

$$\begin{array}{ll} \text{i)} & \dot{\epsilon}_{xx} = \frac{\partial \dot{x}}{\partial x} \\ \text{ii)} & \dot{\epsilon}_{yy} = \frac{\partial \dot{y}}{\partial y} \\ \text{iii)} & \dot{\epsilon}_{zz} = \frac{\partial \dot{z}}{\partial z} \\ \text{iv)} & \dot{\epsilon}_{xy} = \left(\frac{\partial \dot{x}}{\partial y} + \frac{\partial \dot{y}}{\partial x} \right) \\ \text{v)} & \dot{\epsilon}_{xz} = \left(\frac{\partial \dot{x}}{\partial z} + \frac{\partial \dot{z}}{\partial x} \right) \\ \text{vi)} & \dot{\epsilon}_{yz} = \left(\frac{\partial \dot{y}}{\partial z} + \frac{\partial \dot{z}}{\partial y} \right) \end{array}$$

E. Stress deviator tensor.

$$\begin{array}{ll} \text{i)} & \dot{s}_{xx} = 2\mu \left(\dot{\epsilon}_{xx} - \frac{1}{3} \dot{V} \right) \\ \text{ii)} & \dot{s}_{yy} = 2\mu \left(\dot{\epsilon}_{yy} - \frac{1}{3} \dot{V} \right) \\ \text{iii)} & \dot{s}_{zz} = 2\mu \left(\dot{\epsilon}_{zz} - \frac{1}{3} \dot{V} \right) \\ \text{iv)} & \dot{T}_{xy} = \mu (\dot{\epsilon}_{xy}) \\ \text{v)} & \dot{T}_{xz} = \mu (\dot{\epsilon}_{xz}) \\ \text{vi)} & \dot{T}_{yz} = \mu (\dot{\epsilon}_{yz}) \end{array}$$

μ = shear modulus

F. Pressure equation of state.

$$\text{i)} \quad P = a(\eta - 1) + b(\eta - 1)^2 + c(\eta - 1)^3 + d\eta E$$

$$\text{ii)} \quad \eta = \frac{1}{V} = \rho / \rho^0, \text{ where } a, b, c \text{ and } d \text{ are equation-of-state constants}$$

G. Total stresses.

$$\text{i)} \quad \Sigma_{xx} = -(P + q) + s_{xx}$$

$$\text{ii)} \quad \Sigma_{yy} = -(P + q) + s_{yy}$$

$$\text{iii)} \quad \Sigma_{zz} = -(P + q) + s_{zz}$$

H. Artificial viscosity.

$$i) \quad q = C_0^2 \rho L^2 \left(\frac{ds}{dt} \right)^2 + C_L \rho L a \left| \frac{ds}{dt} \right| ,$$

C_0 and C_L are constants

$$ii) \quad \frac{ds}{dt} = \text{rate of strain in the direction of acceleration}$$

L = measure of grid size

a = local sound speed

ρ = local density

I. von Mises Yield Condition

$$i) \quad \sqrt{2J} - \sqrt{\frac{2}{3}} Y \leq 0$$

Y = plastic flow stress

$$Y = a(b + \bar{\epsilon}^p)^c$$

$\bar{\epsilon}^p$ = equivalent plastic strain

$2J$ = second invariant of the deviatoric stress tensor

a , b and c are flow stress constants.

FINITE DIFFERENCE EQUATIONS FOR HEMP 3D

The finite difference equations that integrate the equations of physics are based on the divergence theorem. Partial derivatives are evaluated by summing the flux through the surface enclosing an element of mass.

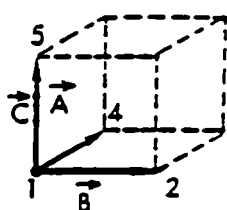
Thus, both the physics equations and the method of solving them are in a conservation form.

The physical object is divided into zones defined by eight grid points, Fig. 1. The grid, (i,j,k) moves with the material and the mass within a zone remains constant. In the notations that follow a superscript refers to the time centering of a parameter or equation and the subscript refers to the space centering.

Defining the Vectors

Three vectors are associated with each of the eight grid points, g , shown in Fig. 1.

$g = 1$



Vector

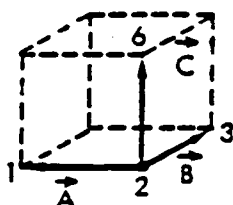
Components

$$\bar{A}: (a_i)_1 = x_4 - x_1; (a_j)_1 = y_4 - y_1; (a_k)_1 = z_4 - z_1.$$

$$\bar{B}: (b_i)_1 = x_2 - x_1; (b_j)_1 = y_2 - y_1; (b_k)_1 = z_2 - z_1.$$

$$\bar{C}: (c_i)_1 = x_5 - x_1; (c_j)_1 = y_5 - y_1; (c_k)_1 = z_5 - z_1.$$

$g = 2$



Vector

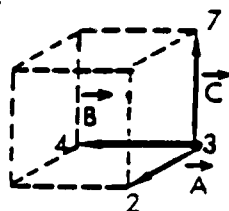
Components

$$\bar{A}: (a_i)_2 = x_1 - x_2; (a_j)_2 = y_1 - y_2; (a_k)_2 = z_1 - z_2.$$

$$\bar{B}: (b_i)_2 = x_3 - x_2; (b_j)_2 = y_3 - y_2; (b_k)_2 = z_3 - z_2.$$

$$\bar{C}: (c_i)_2 = x_5 - x_2; (c_j)_2 = y_6 - y_2; (c_k)_2 = z_6 - z_2.$$

$g = 3$



Vector

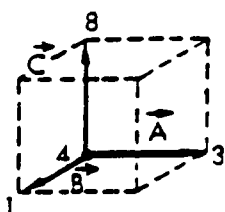
Components

$$\bar{A}: (a_i)_3 = x_2 - x_3; (a_j)_3 = y_2 - y_3; (a_k)_3 = z_2 - z_3.$$

$$\bar{B}: (b_i)_3 = x_4 - x_3; (b_j)_3 = y_4 - y_3; (b_k)_3 = z_4 - z_3.$$

$$\bar{C}: (c_i)_3 = x_7 - x_3; (c_j)_3 = y_7 - y_3; (c_k)_3 = z_7 - z_3.$$

$g = 4$



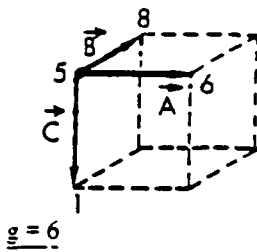
Vector

Components

$$\bar{A}: (a_i)_4 = x_2 - x_4; (a_j)_4 = y_3 - y_4; (a_k)_4 = z_3 - z_4.$$

$$\bar{B}: (b_i)_4 = x_1 - x_4; (b_j)_4 = y_1 - y_4; (b_k)_4 = z_1 - z_4.$$

$$\bar{C}: (c_i)_4 = x_5 - x_4; (c_j)_4 = y_6 - y_4; (c_k)_4 = z_6 - z_4.$$

g = 5

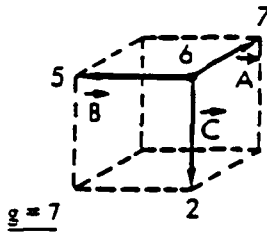
Vector

Components

$$\bar{A}: (a_i)_5 = x_6 - x_5; (a_j)_5 = y_6 - y_5; (a_k)_5 = z_6 - z_5.$$

$$\bar{B}: (b_i)_5 = x_8 - x_5; (b_j)_5 = y_8 - y_5; (b_k)_5 = z_8 - z_5.$$

$$\bar{C}: (c_i)_5 = x_7 - x_5; (c_j)_5 = y_7 - y_5; (c_k)_5 = z_7 - z_5.$$



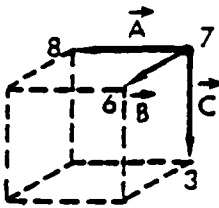
Vector

Components

$$\bar{A}: (a_i)_6 = x_7 - x_6; (a_j)_6 = y_7 - y_6; (a_k)_6 = z_7 - z_6.$$

$$\bar{B}: (b_i)_6 = x_5 - x_6; (b_j)_6 = y_5 - y_6; (b_k)_6 = z_5 - z_6.$$

$$\bar{C}: (c_i)_6 = x_8 - x_6; (c_j)_6 = y_8 - y_6; (c_k)_6 = z_8 - z_6.$$



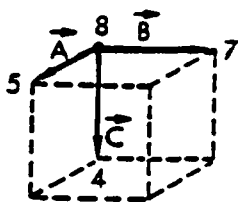
Vector

Components

$$\bar{A}: (a_i)_7 = x_8 - x_7; (a_j)_7 = y_8 - y_7; (a_k)_7 = z_8 - z_7.$$

$$\bar{B}: (b_i)_7 = x_6 - x_7; (b_j)_7 = y_6 - y_7; (b_k)_7 = z_6 - z_7.$$

$$\bar{C}: (c_i)_7 = x_5 - x_7; (c_j)_7 = y_5 - y_7; (c_k)_7 = z_5 - z_7.$$

g = 8

Vector

Components

$$\bar{A}: (a_i)_8 = x_5 - x_8; (a_j)_8 = y_5 - y_8; (a_k)_8 = z_5 - z_8.$$

$$\bar{B}: (b_i)_8 = x_7 - x_8; (b_j)_8 = y_7 - y_8; (b_k)_8 = z_7 - z_8.$$

$$\bar{C}: (c_i)_8 = x_6 - x_8; (c_j)_8 = y_6 - y_8; (c_k)_8 = z_6 - z_8.$$

Refer to Fig 1.

Calculation of the Volume of Zone ①, $v_{①}$

Refer to Fig. 1.

$$v_{①} = \frac{1}{8} \sum_{g=1}^8 [\bar{B}X\bar{A} \cdot \bar{C}]$$

$$[\bar{B}X\bar{A} \cdot \bar{C}]_{g=1}^n = \begin{vmatrix} b_i & b_j & b_k \\ a_i & a_j & a_k \\ c_i & c_j & c_k \end{vmatrix}_{g=1}^n = [b_i(a_j c_k - a_k c_j) - b_j(a_i c_k - a_k c_i) + b_k(a_i c_j - a_j c_i)]_{g=1}^n$$

Repeat for $g = 2 \rightarrow 8$

Calculation of the Mass of Zone 1, $M_{①}$

$$M_{①} = \left[\frac{\rho^0}{V^0} v^0 \right]_{①}$$

ρ^0 = reference density

V^0 = initial relative volume

v^0 = actual volume calculated from the coordinates at time $t = 0$.

Conservations of Mass

$V_{\textcircled{1}}^* = \left(\frac{\rho^0}{M} \right)_{\textcircled{1}} v_{\textcircled{1}}^*$; where $v_{\textcircled{1}}^*$ is the volume at time $t = n$ and $V_{\textcircled{1}}^*$ is the relative volume. Similarly,

$V_{\textcircled{1}}^{*+1} = \left(\frac{\rho^0}{M} \right)_{\textcircled{1}} v_{\textcircled{1}}^{*+1}$ where the volume $v_{\textcircled{1}}^{*+1}$ is calculated from the coordinates at time $n+1$.

$$V_{\textcircled{1}}^{*+1/2} = \frac{1}{2} (V_{\textcircled{1}}^{*+1} + V_{\textcircled{1}}^*) \text{ definition of relative volume at } t = n + 1/2.$$

Equations of Motion

The following acceleration equations are applied to point 0 in Fig. 2.

Mass Associated with Point (i,j,k)

$$(\Phi)_{i,j,k} = \frac{1}{8} [M_{\textcircled{1}} + M_{\textcircled{2}} + M_{\textcircled{3}} + M_{\textcircled{4}} + M_{\textcircled{5}} + M_{\textcircled{6}} + M_{\textcircled{7}} + M_{\textcircled{8}}]$$

Motion in the x Direction

$$\left(\frac{dx}{dt} \right)_{i,j,k} = \frac{1}{\rho_{i,j,k}^*} \left[\frac{\partial \Sigma_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} \right]_{i,j,k}^n, \text{ where}$$

$$\begin{aligned} \left(\frac{1}{\rho} \frac{\partial \Sigma_{xx}}{\partial x} \right)_{i,j,k}^n &= \frac{1}{4\Phi_{i,j,k}} \{ (\Sigma_{xx})_{\textcircled{1}} [(y_{VI}-y_V)(z_{IV}-z_V) - (z_{VI}-z_V)(y_{IV}-y_V)] \\ &\quad + (\Sigma_{xx})_{\textcircled{4}} [(y_{II}-y_V)(z_{VI}-z_V) - (z_{II}-z_V)(y_{VI}-y_V)] \\ &\quad + (\Sigma_{xx})_{\textcircled{2}} [(y_{IV}-y_{III})(z_{VI}-z_{III}) - (z_{IV}-z_{III})(y_{VI}-y_{III})] \\ &\quad + (\Sigma_{xx})_{\textcircled{3}} [(y_{VI}-y_{III})(z_{II}-z_{III}) - (z_{VI}-z_{III})(y_{II}-y_{III})] \\ &\quad + (\Sigma_{xx})_{\textcircled{5}} [(y_V-y_I)(z_{IV}-z_I) - (z_V-z_I)(y_{IV}-y_I)] \} \end{aligned}$$

$$\begin{aligned}
& + (\Sigma_{xx})_{\textcircled{8}} [(y_{II}-y_I)(z_V-z_I)-(z_{II}-z_I)(y_V-y_I)] \\
& + (\Sigma_{xx})_{\textcircled{6}} [(y_I-y_{III})(z_{IV}-z_{III})-(z_I-z_{III})(y_{IV}-y_{III})] \\
& + (\Sigma_{xx})_{\textcircled{7}} [(y_{II}-y_{III})(z_I-z_{III})-(z_{II}-z_{III})(y_I-y_{III})] \}^n_{i,j,k}
\end{aligned}$$

To form $(1/\rho \partial T_{xy}/\partial y)^n_{i,j,k}$, replace each Σ_{xx} in the above expression with T_{xy} , every y with the corresponding x and every z with the corresponding y .

The x -direction velocity at $n + 1/2$ and positions at times $n + 1$ and $n + 1/2$ are:

$$\dot{x}_{i,j,k}^{n+1/2} = \dot{x}_{i,j,k}^{n-1/2} + \left(\frac{d\dot{x}}{dt} \right)^n_{i,j,k} \Delta t^n$$

$$x_{i,j,k}^{n+1} = x_{i,j,k}^n + \dot{x}_{i,j,k} \Delta t^{n+1/2}$$

$$x_{i,j,k}^{n+1/2} = \frac{1}{2} (x_{i,j,k}^{n+1} + x_{i,j,k}^n)$$

Motion in the y Direction

$$\left(\frac{dy}{dt} \right)^n_{i,j,k} = \frac{1}{\rho^n_{i,j,k}} \left[\frac{\partial T_{xy}}{\partial x} + \frac{\partial \Sigma_{yy}}{\partial y} + \frac{\partial T_{yz}}{\partial z} \right]^n_{i,j,k}; \text{ where}$$

$$\left(\frac{1}{\rho} \frac{\partial T_{xy}}{\partial x} \right)^n_{i,j,k} = \text{same as } (1/\rho \partial \Sigma_{xx}/\partial x)^n_{i,j,k}, \text{ defined above, except replace each } \Sigma_{xx}$$

by the corresponding value of T_{xy} .

$$\left(\frac{1}{\rho} \frac{\partial \Sigma_{yy}}{\partial y} \right)^n_{i,j,k} = \text{same as } (1/\rho \partial T_{xy}/\partial y)^n_{i,j,k}, \text{ defined above, except replace each } T_{xy}$$

by the corresponding value of Σ_{yy} .

$$\left(\frac{1}{\rho} \frac{\partial T_{yz}}{\partial z} \right)^n_{i,j,k} = \text{same as } (1/\rho \partial T_{zx}/\partial z)^n_{i,j,k}, \text{ defined above, except replace each } T_{zx}$$

by the corresponding value of T_{yz} .

The y-direction velocity at time $n + 1$ and positions at times $n + 1$ and $n = 1/2$ are:

$$\dot{y}_{i,j,k}^{n+1/2} = \dot{y}_{i,j,k}^{n-1/2} + \left(\frac{d\dot{y}}{dt} \right)_{i,j,k}^n \Delta t^n$$

$$y_{i,j,k}^{n+1} = y_{i,j,k}^n + \dot{y}_{i,j,k}^{n+1/2} \Delta t^{n+1/2}$$

$$y_{i,j,k}^{n+1/2} = \frac{1}{2} (y_{i,j,k}^{n+1} + y_{i,j,k}^n).$$

Motion in the z Direction

$$\left(\frac{dz}{dt} \right)_{i,j,k}^n = \frac{1}{\rho_{i,j,k}^n} \left[\frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{yy}}{\partial y} + \frac{\partial \Sigma_{zz}}{\partial z} \right]_{i,j,k}^n$$

$$\left(\frac{1}{\rho} \frac{\partial T_{xx}}{\partial x} \right)_{i,j,k}^n = \text{same as } (1/\rho \partial \Sigma_{xx} / \partial x)_{i,j,k}^n, \text{ defined above, except replace each}$$

Σ_{xx} by the corresponding value of T_{xx} .

$$\left(\frac{1}{\rho} \frac{\partial T_{yy}}{\partial y} \right)_{i,j,k}^n = \text{same as } (1/\rho \partial T_{xy} / \partial y)_{i,j,k}^n, \text{ defined above; except replace each } T_{xy}$$

by the corresponding value of T_{yz} .

$$\left(\frac{1}{\rho} \frac{\partial \Sigma_{zz}}{\partial z} \right)_{i,j,k}^n = \text{same as } (1/\rho \partial T_{zz} / \partial z)_{i,j,k}^n, \text{ defined above; except replace each}$$

T_{zz} by the corresponding value of Σ_{zz} .

The z-direction velocity at time $n + 1/2$ and positions at times $n + 1$ and $n + 1/2$ are:

$$\dot{z}_{i,j,k}^{n+1/2} = \dot{z}_{i,j,k}^{n-1/2} + \left(\frac{d\dot{z}}{dt} \right)_{i,j,k}^n \Delta t^n$$

$$z_{i,j,k}^{n+1} = z_{i,j,k}^n + \dot{z}_{i,j,k}^{n+1/2} \Delta t^{n+1/2}$$

$$z_{i,j,k}^{n+1/2} = \frac{1}{2} (z_{i,j,k}^{n+1} + z_{i,j,k}^n).$$

Calculation of Incremental Strains

The finite difference mapping procedure to calculate the surface integral of zone ①, Fig. 1, covers the surface in units of triangles. The velocity associated with a given triangle is taken as the average of the velocities defined at the triangle corners. The triangular surface area vectors are calculated to point out of the zone surface. The dot product of the area vector with the direction vector multiplied by the average velocity gives the velocity flux through the surface in the given direction. The mapping procedure actually covers the zone surface area, Fig. 1, two times. The difference equations used to calculate

$$\frac{\partial \dot{x}}{\partial x}, \frac{\partial \dot{x}}{\partial y} \text{ and } \frac{\partial \dot{x}}{\partial z} \text{ are given explicitly below.}$$

The remaining velocity derivatives required to calculate the components of strain are calculated by replacing $\dot{x}$ in those equations by $\dot{y}$ and then by $\dot{z}$ so as to complete the set:

$$\begin{vmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} & \frac{\partial \dot{x}}{\partial z} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} & \frac{\partial \dot{y}}{\partial z} \\ \frac{\partial \dot{z}}{\partial x} & \frac{\partial \dot{z}}{\partial y} & \frac{\partial \dot{z}}{\partial z} \end{vmatrix}$$

Velocity Derivatives Corresponding to Zone ①, Fig. 1.

$$\left(\frac{\partial \dot{x}}{\partial x} \right)_{\text{①}}^{n+1/2} = \left(\frac{1}{12V_{\text{①}}^{n+1/2}} \right) \sum_{i=1}^8 \left[\dot{x}_{AB} (\bar{A} \times \bar{B}) \cdot \bar{i} + \dot{x}_{CA} (\bar{C} \times \bar{A}) \cdot \bar{i} + \dot{x}_{BC} (\bar{B} \times \bar{C}) \cdot \bar{i} \right]_i^{n+1/2},$$

where

$$(\dot{x}_{AB})_{g=1} = (\dot{x}_1 + \dot{x}_2 + \dot{x}_4), (\dot{x}_{CA})_{g=1} = (\dot{x}_1 + \dot{x}_4 + \dot{x}_5), (\dot{x}_{BC})_{g=1} = (\dot{x}_1 + \dot{x}_2 + \dot{x}_5).$$

$$(\bar{A} X \bar{B} \cdot \bar{i})_{g=1} = \begin{vmatrix} 1 & 0 & 0 \\ a_i & a_j & a_k \\ b_i & b_j & b_k \end{vmatrix}_{g=1} = [(a_j b_k - a_k b_j)]_{g=1}.$$

$$(\bar{C} X \bar{A} \cdot \bar{i})_{g=1} = \begin{vmatrix} 1 & 0 & 0 \\ c_i & c_j & c_k \\ a_i & a_j & a_k \end{vmatrix}_{g=1} = [(c_i a_k - c_k a_i)]_{g=1}.$$

$$(\bar{B} X \bar{C} \cdot \bar{i})_{g=1} = \begin{vmatrix} 1 & 0 & 0 \\ b_i & b_j & b_k \\ c_i & c_j & c_k \end{vmatrix}_{g=1} = [(b_j c_k - b_k c_j)]_{g=1}.$$

The above steps, written for $g = 1$, must be repeated for $g = 2 \rightarrow 8$.

$$\left(\frac{\partial \dot{x}}{\partial y}\right)_{\textcircled{1}}^{n+1/2} = \left(\frac{1}{12v_{\textcircled{1}}^{n+1/2}}\right) \sum_{g=1}^8 [\dot{x}_{AB}(\bar{A}X\bar{B}) \cdot \bar{j} + \dot{x}_{CA}(\bar{C}X\bar{A}) \cdot \bar{j} + \dot{x}_{BC}(\bar{B}X\bar{C}) \cdot \bar{j}]_g^{n+1/2},$$

where

$(\dot{x}_{AB})_{g=1}, (\dot{x}_{CA})_{g=1}$, and $(\dot{x}_{BC})_{g=1}$ are as defined above.

$$(\bar{A} X \bar{B} \cdot \bar{j})_{g=1} = \begin{vmatrix} 0 & 1 & 0 \\ a_i & a_j & a_k \\ b_i & b_j & b_k \end{vmatrix}_{g=1} = [-(a_j b_k - a_k b_j)]_{g=1}.$$

$$(\bar{C} X \bar{A} \cdot \bar{j})_{g=1} = \begin{vmatrix} 0 & 1 & 0 \\ c_i & c_j & c_k \\ a_i & a_j & a_k \end{vmatrix}_{g=1} = [-(c_i a_k - c_k a_i)]_{g=1}.$$

$$(\bar{B} X \bar{C} \cdot \bar{j})_{g=1} = \begin{vmatrix} 0 & 1 & 0 \\ b_i & b_j & b_k \\ c_i & c_j & c_k \end{vmatrix}_{g=1} = [-(b_i c_k - b_k c_i)]_{g=1}.$$

The above steps, written for $g = 1$, must be repeated for $g = 2 \rightarrow 8$.

$$\left(\frac{\partial \dot{x}}{\partial y}\right)_{\textcircled{1}}^{n+1/2} = \left(\frac{1}{12v_{\textcircled{1}}^{n+1/2}}\right) \sum_{g=1}^8 [\dot{x}_{AB}(\bar{A} X \bar{B}) \cdot \bar{j} + \dot{x}_{CA}(\bar{C} X \bar{A}) \cdot \bar{j} + \dot{x}_{BC}(\bar{B} X \bar{C}) \cdot \bar{j}]_g^{n+1/2},$$

where

$(\dot{x}_{AB})_{g=1}$, $(\dot{x}_{CA})_{g=1}$, and $(\dot{x}_{BC})_{g=1}$ are defined above.

$$(\bar{A} X \bar{B} \cdot \bar{k})_{g=1} = \begin{vmatrix} 0 & 0 & 1 \\ a_i & a_j & a_k \\ b_i & b_j & b_k \end{vmatrix}_{g=1} = [(a_i b_j - a_j b_i)]_{g=1}.$$

$$(\bar{C} X \bar{A} \cdot \bar{k})_{g=1} = \begin{vmatrix} 0 & 0 & 1 \\ c_i & c_j & c_k \\ a_i & a_j & a_k \end{vmatrix}_{g=1} = [(c_i a_j - c_j a_i)]_{g=1}.$$

$$(\bar{B} X \bar{C} \cdot \bar{k})_{g=1} = \begin{vmatrix} 0 & 0 & 1 \\ b_i & b_j & b_k \\ c_i & c_j & c_k \end{vmatrix}_{g=1} = [(b_i c_j - b_j c_i)]_{g=1}.$$

The above steps, written for $g = 1$, must be repeated for $g = 2 \rightarrow 8$.

$\frac{\partial \dot{y}}{\partial x}$ = same as $\partial \dot{x} / \partial x$ except replace $\dot{x}$ by the corresponding $\dot{y}$

$\frac{\partial \dot{z}}{\partial x}$ = same as $\partial \dot{x} / \partial x$ except replace $\dot{x}$ by the corresponding $\dot{z}$

$$\frac{\partial \dot{y}}{\partial y} = \text{same as } \partial \dot{x} / \partial y \text{ except replace } \dot{x} \text{ by the corresponding } \dot{y}$$

$$\frac{\partial \dot{z}}{\partial y} = \text{same as } \partial \dot{x} / \partial y \text{ except replace } \dot{x} \text{ by the corresponding } \dot{z}$$

$$\frac{\partial \dot{y}}{\partial z} = \text{same as } \partial \dot{x} / \partial z \text{ except replace } \dot{x} \text{ by the corresponding } \dot{y}$$

$$\frac{\partial \dot{z}}{\partial z} = \text{same as } \partial \dot{x} / \partial z \text{ except replace } \dot{x} \text{ by the corresponding } \dot{z}$$

Incremental Strains

$$(\Delta \epsilon_x)_{\textcircled{1}}^{n+1/2} = \left(\frac{\partial \dot{x}}{\partial x} \right)_{\textcircled{1}}^{n+1/2} \Delta t^{n+1/2}$$

$$(\Delta \epsilon_y)_{\textcircled{1}}^{n+1/2} = \left(\frac{\partial \dot{y}}{\partial y} \right)_{\textcircled{1}}^{n+1/2} \Delta t^{n+1/2}$$

$$(\Delta \epsilon_z)_{\textcircled{1}}^{n+1/2} = \left(\frac{\partial \dot{z}}{\partial z} \right)_{\textcircled{1}}^{n+1/2} \Delta t^{n+1/2}$$

$$(\Delta \epsilon_{xy})_{\textcircled{1}}^{n+1/2} = \left[\left(\frac{\partial \dot{x}}{\partial y} \right)_{\textcircled{1}}^{n+1/2} + \left(\frac{\partial \dot{y}}{\partial x} \right)_{\textcircled{1}}^{n+1/2} \right] \Delta t^{n+1/2}$$

$$(\Delta \epsilon_{yz})_{\textcircled{1}}^{n+1/2} = \left[\left(\frac{\partial \dot{y}}{\partial z} \right)_{\textcircled{1}}^{n+1/2} + \left(\frac{\partial \dot{z}}{\partial y} \right)_{\textcircled{1}}^{n+1/2} \right] \Delta t^{n+1/2}$$

$$(\Delta \epsilon_{xz})_{\textcircled{1}}^{n+1/2} = \left[\left(\frac{\partial \dot{z}}{\partial x} \right)_{\textcircled{1}}^{n+1/2} + \left(\frac{\partial \dot{x}}{\partial z} \right)_{\textcircled{1}}^{n+1/2} \right] \Delta t^{n+1/2}$$

$$\left(\frac{\Delta V}{V} \right)_{\textcircled{1}}^{n+1/2} = (\Delta \epsilon_x)_{\textcircled{1}}^{n+1/2} + (\Delta \epsilon_y)_{\textcircled{1}}^{n+1/2} + (\Delta \epsilon_z)_{\textcircled{1}}^{n+1/2}$$

Calculation of Stress

Stress Deviators

$$(s_{xx})_{\textcircled{1}}^{n+1} = (s_{xx})_{\textcircled{1}}^n + 2\mu \left[(\Delta \varepsilon_{xx})_{\textcircled{1}}^{n+1/2} - \frac{1}{3} \left(\frac{\Delta V}{V} \right)_{\textcircled{1}}^{n+1/2} \right] + (\delta_{xx}^n)_{\textcircled{1}},$$

$$(s_{yy})_{\textcircled{1}}^{n+1} = (s_{yy})_{\textcircled{1}}^n + 2\mu \left[(\Delta \varepsilon_{yy})_{\textcircled{1}}^{n+1/2} - \frac{1}{3} \left(\frac{\Delta V}{V} \right)_{\textcircled{1}}^{n+1/2} \right] + (\delta_{yy}^n)_{\textcircled{1}},$$

$$(s_{zz})_{\textcircled{1}}^{n+1} = (s_{zz})_{\textcircled{1}}^n + 2\mu \left[(\Delta \varepsilon_{zz})_{\textcircled{1}}^{n+1/2} - \frac{1}{3} \left(\frac{\Delta V}{V} \right)_{\textcircled{1}}^{n+1/2} \right] + (\delta_{zz}^n)_{\textcircled{1}},$$

$$(T_{xy})_{\textcircled{1}}^{n+1} = (T_{xy})_{\textcircled{1}}^n + \mu (\Delta \varepsilon_{xy})_{\textcircled{1}}^{n+1/2} + (\delta_{xy}^n)_{\textcircled{1}},$$

$$(T_{yz})_{\textcircled{1}}^{n+1} = (T_{yz})_{\textcircled{1}}^n + \mu (\Delta \varepsilon_{yz})_{\textcircled{1}}^{n+1/2} + (\delta_{yz}^n)_{\textcircled{1}},$$

$$(T_{zx})_{\textcircled{1}}^{n+1} = (T_{zx})_{\textcircled{1}}^n + \mu (\Delta \varepsilon_{zx})_{\textcircled{1}}^{n+1/2} + (\delta_{zx}^n)_{\textcircled{1}},$$

Note: The terms δ that have been added to the stress deviators are corrections for zone rotations. If a zone has rotated during the time interval $\Delta t^{n+1/2} = t^{n+1} - t^n$ the stresses at time t^n must be recalculated so that they will be referred to the x, y, z coordinate systems in their new positions.¹ The correction terms are given by

$$\delta_{xx}^n = -2 \omega_z^n T_{xy}^n + 2 \omega_y^n T_{xz}^n,$$

$$\delta_{yy}^n = +2 \omega_z^n T_{xy}^n - 2 \omega_x^n T_{yz}^n,$$

$$\delta_{zz}^n = +2 \omega_x^n T_{yz}^n - 2 \omega_y^n T_{xz}^n = -\delta_{yy}^n - \delta_{xx}^n,$$

$$\delta_{xy}^n = \omega_z^n (s_{xx}^n - s_{yy}^n) + \omega_y^n T_{yz}^n - \omega_x^n T_{xz}^n,$$

$$\delta_{yz}^n = \omega_x^n (s_{yy}^n - s_{zz}^n) + \omega_z^n T_{xz}^n - \omega_y^n T_{xy}^n,$$

$$\delta_{xx}^n = \omega_y^n (s_{xx}^n - s_{yy}^n) + \omega_x^n T_{xy}^n - \omega_z^n T_{yz}^n,$$

where

$$\omega_x^n = \frac{1}{2} \left(\frac{\partial \dot{z}}{\partial y} - \frac{\partial \dot{y}}{\partial z} \right) \Delta t^{n+1/2},$$

$$\omega_y^n = \frac{1}{2} \left(\frac{\partial \dot{x}}{\partial z} - \frac{\partial \dot{z}}{\partial x} \right) \Delta t^{n+1/2},$$

$$\omega_z^n = \frac{1}{2} \left(\frac{\partial \dot{y}}{\partial x} - \frac{\partial \dot{x}}{\partial y} \right) \Delta t^{n+1/2}.$$

Pressure Equation of State

$$P_{\textcircled{1}}^{n+1} = A(V_{\textcircled{1}}^{n+1}) + B(V_{\textcircled{1}}^{n+1}) E_{\textcircled{1}}^{n+1},$$

where A and B are functions of the volume V and E is the internal energy.

Total Stresses

$$(\Sigma_{xx})_{\textcircled{1}}^{n+1} = - (P_{\textcircled{1}}^{n+1} + q_{\textcircled{1}}^{n+1/2}) + (s_{xx})_{\textcircled{1}}^{n+1},$$

$$(\Sigma_{yy})_{\textcircled{1}}^{n+1} = - (P_{\textcircled{1}}^{n+1} + q_{\textcircled{1}}^{n+1/2}) + (s_{yy})_{\textcircled{1}}^{n+1},$$

$$(\Sigma_{zz})_{\textcircled{1}}^{n+1} = - (P_{\textcircled{1}}^{n+1} + q_{\textcircled{1}}^{n+1/2}) + (s_{zz})_{\textcircled{1}}^{n+1},$$

von Mises Yield Condition

$$2J_{\textcircled{1}}^{n+1} = \left\{ \left[(s_{xx})^2 + (s_{yy})^2 + (s_{zz})^2 \right] + 2 \left[(T_{xy})^2 + (T_{yz})^2 + (T_{zx})^2 \right] \right\}_{\textcircled{1}}^{n+1}.$$

$$K_{\textcircled{1}}^{n+1} = 2J_{\textcircled{1}}^{n+1} - \frac{2}{3}(Y_{\textcircled{1}}^n)^2.$$

If: $K_{\textcircled{1}}^{n+1} \leq 0$ use the stress deviators as defined above.

If: $K_{\textcircled{1}}^{n+1} > 0$, then multiply each of the stresses $(s_{xx})_{\textcircled{1}}^{n+1}, (s_{yy})_{\textcircled{1}}^{n+1}, (s_{zz})_{\textcircled{1}}^{n+1}, (T_{xy})_{\textcircled{1}}^{n+1}, (T_{yz})_{\textcircled{1}}^{n+1}$, and $(T_{zx})_{\textcircled{1}}^{n+1}$ by $\sqrt{2/3} Y_{\textcircled{1}}^n / \sqrt{2J_{\textcircled{1}}^{n+1}}$.

Principal Stress Deviators (for Edit Routine)

$$s_1 = 2\sqrt{-a/3} \cos \phi / 3,$$

~

$$s_2 = 2\sqrt{-a/3} \cos(\phi/3 + 2\pi/3),$$

~

$$s_3 = 2\sqrt{-a/3} \cos(\phi/3 + 4\pi/3);$$

~

where

$$a = [(s_{yy}s_{zz} + s_{xx}s_{yy} + s_{xx}s_{zz}) - (T_{yz}^2 + T_{xy}^2 + T_{zx}^2)],$$

$$b = [-s_{xx}s_{yy}s_{zz} + s_{xx}T_{yz}^2 + s_{xx}T_{xy}^2 + s_{yy}T_{zx}^2 - 2T_{yz}T_{xy}T_{zx}],$$

$$\phi = \cos^{-1} \left[+ \left(\sqrt{\frac{27}{-a}} \right) \left(\frac{b}{2a} \right) \right]. \quad s_1 \succ s_2 \succ s_3$$

$$s_1 = \max \begin{pmatrix} s_1, s_2, s_3 \\ \sim \quad \sim \quad \sim \end{pmatrix}$$

$$s_3 = \min \begin{pmatrix} s_1, s_2, s_3 \\ \sim \quad \sim \quad \sim \end{pmatrix}$$

$$s_2 = -(s_1 + s_3)$$

Artificial Viscosity

An artificial viscosity is required to permit shocks to form in the grid. The artificial viscosity, q , used here is composed of a quadratic and linear function of the rate of strain. The quadratic portion is a generalization to three dimensions of the one dimensional von Neumann q for calculating shocks.² The linear portion provides damping for oscillations that can occur behind the shock with the q method of calculating the shock front. The term ds/dt used in the q calculations here is the rate of strain in the direction of acceleration.³

$$q_1 = C_0^2 \rho L^2 \left(\frac{ds}{dt} \right)^2 + C_L \rho L a \left| \frac{ds}{dt} \right|,$$

$$q = 0 \quad \text{for } \frac{ds}{dt} \geq 0,$$

$$\begin{aligned} \frac{ds}{dt} = & \left[\frac{\partial \dot{x}}{\partial x} (A_x)^2 + \frac{\partial \dot{y}}{\partial y} (A_y)^2 + \frac{\partial \dot{z}}{\partial z} (A_z)^2 + \left(\frac{\partial \dot{x}}{\partial y} + \frac{\partial \dot{y}}{\partial x} \right) A_x A_y \right. \\ & \left. + \left(\frac{\partial \dot{x}}{\partial z} + \frac{\partial \dot{z}}{\partial x} \right) A_x A_z + \left(\frac{\partial \dot{y}}{\partial z} + \frac{\partial \dot{z}}{\partial y} \right) A_y A_z \right] \cdot \left[\frac{1}{A_x^2 + A_y^2 + A_z^2} \right]. \end{aligned}$$

A_x , A_y and A_z are the x, y, z components of acceleration respectively.

L = measure of the zone size taken here as: $\sqrt[3]{\text{zone volume}}$

a = $\sqrt{P/\rho}$

C_0 = $\simeq 2$

C_L = $\simeq 1$

The q is added to the pressure P .

Tensor Artificial Viscosity for Stabilizing the Grid

For quasi-static problems in solid mechanics nonphysical numerical oscillations can occur in the grid under certain boundary conditions. A tensor viscosity based on the rate of strain of volume elements formed by the zone corners is used to damp this type oscillation. Referring to Fig. 2 it is seen that surrounding point 0 there are eight tetrahedrons defined by the corners of the eight zones. A Navier-Stoke type tensor viscosity based on the rates of strain of the tetrahedron volumes is calculated for each tetrahedron that contains 0, Fig. 2. The details for calculating the components of viscosity for the tetrahedron in zone ① are given below.

The tetrahedron corresponding to zone ① is shown in Fig. 3. The grid numbering follows the scheme shown in Fig. 1. Here grid point 1 corresponds to point 0 of Fig. 2. The finite difference integration mapping procedure is applied to the four surfaces of the tetrahedron formed by vectors, $\bar{A}, \bar{B}, \bar{C}$, of Fig. 3.

Volume v_{ABC} formed by the vectors $\bar{A}, \bar{B}, \bar{C}$, of Fig. 3 is:

$$(v_{ABC})^{n+1} = \frac{1}{6}(\bar{B} \times \bar{A}) \cdot \bar{C} = \frac{1}{6}[b_1(a_j c_k - a_k c_j) - b_j(a_i c_k - a_k c_i) + b_k(a_i c_j - a_j c_i)]^{n+1}$$

The notation for the components of the vectors is the same as used for the vectors of the volume of zone ①.

Velocity Derivatives

Velocity derivatives corresponding to the tetrahedron, Fig. 3.

$$\left(\frac{\partial \dot{x}}{\partial x}\right)^{n+1/2} = \left(\frac{1}{6v_{ABC}^{n+1/2}}\right) [\dot{x}_{AB}(\bar{A} \times \bar{B}) \cdot \bar{i} + \dot{x}_{CA}(\bar{C} \times \bar{A}) \cdot \bar{i} + \dot{x}_{BC}(\bar{B} \times \bar{C}) \cdot \bar{i} + \dot{x}_{ED}(\bar{E} \times \bar{D}) \cdot \bar{i}]^{n+1/2}$$

where

$$\dot{x}_{AB} = (\dot{x}_1 + \dot{x}_2 + \dot{x}_4); \quad \dot{x}_{CA} = (\dot{x}_1 + \dot{x}_4 + \dot{x}_5); \quad \dot{x}_{BC} = (\dot{x}_1 + \dot{x}_2 + \dot{x}_5); \quad \dot{x}_{ED} = (\dot{x}_2 + \dot{x}_4 + \dot{x}_5);$$

$$\text{and } v_{ABC}^{n+1/2} = \frac{1}{2} [v_{ABC}^n + v_{ABC}^{n+1}].$$

This expression can be simplified by expressing vectors $\bar{D}$ and $\bar{E}$ in terms of vectors $\bar{A}$ and $\bar{B}$.

$$\left(\frac{\partial \dot{x}}{\partial x} \right)^{n+1/2} = \left(\frac{1}{6v_{ABC}^{n+1/2}} \right) \left[(\dot{x}_1 - \dot{x}_5)(\bar{A} \times \bar{B}) \cdot \bar{i} + (\dot{x}_1 - \dot{x}_2)(\bar{C} \times \bar{A}) \cdot \bar{i} + (\dot{x}_1 - \dot{x}_4)(\bar{B} \times \bar{C}) \cdot \bar{i} \right]^{n+1/2},$$

where

$$(\bar{A} \times \bar{B}) \cdot \bar{i} = (a_j b_k - a_k b_j); \quad (\bar{C} \times \bar{A}) \cdot \bar{i} = -(c_j a_k - c_k a_j), \text{ and } (\bar{B} \times \bar{C}) \cdot \bar{i} = -(b_j c_k - b_k c_j).$$

$$\left[\frac{\partial \dot{x}}{\partial y} \right]^{n+1/2} = \left(\frac{1}{6v_{ABC}^{n+1/2}} \right) \left[(\dot{x}_1 - \dot{x}_5)(\bar{A} \times \bar{B}) \cdot \bar{j} + (\dot{x}_1 - \dot{x}_2)(\bar{C} \times \bar{A}) \cdot \bar{j} + (\dot{x}_1 - \dot{x}_4)(\bar{B} \times \bar{C}) \cdot \bar{j} \right]^{n+1/2},$$

where

$$\bar{A} \times \bar{B} \cdot \bar{j} = -(a_i b_k - a_k b_i); \quad (\bar{C} \times \bar{A}) \cdot \bar{j} = -(c_i a_k - c_k a_i), \text{ and } (\bar{B} \times \bar{C}) \cdot \bar{j} = -(b_i c_k - b_k c_i).$$

$$\left[\frac{\partial \dot{x}}{\partial z} \right]^{n+1/2} = \left(\frac{1}{6v_{ABC}^{n+1/2}} \right) \left[(\dot{x}_1 - \dot{x}_5)(\bar{A} \times \bar{B}) \cdot \bar{k} + (\dot{x}_1 - \dot{x}_2)(\bar{C} \times \bar{A}) \cdot \bar{k} + (\dot{x}_1 - \dot{x}_4)(\bar{B} \times \bar{C}) \cdot \bar{k} \right]^{n+1/2},$$

where

$$\bar{A} \times \bar{B} \cdot \bar{k} = (a_i b_j - a_j b_i); \quad \bar{C} \times \bar{A} \cdot \bar{k} = (c_i a_j - c_j a_i); \quad \bar{B} \times \bar{C} \cdot \bar{k} = (b_i c_j - b_j c_i).$$

$\frac{\partial \dot{y}}{\partial x}$ and $\frac{\partial \dot{z}}{\partial x}$ are calculated in the same way as $\partial \dot{x} / \partial x$, but replace $\dot{x}$ by $\dot{y}$ and then $\dot{z}$.

$\frac{\partial \dot{y}}{\partial y}$ and $\frac{\partial \dot{z}}{\partial y}$ are calculated in the same way as $\partial \dot{x} / \partial y$, but replace $\dot{x}$ by $\dot{y}$ and then $\dot{z}$.

$\frac{\partial \dot{y}}{\partial z}$ and $\frac{\partial \dot{z}}{\partial z}$ are calculated in the same way as $\frac{\partial \dot{x}}{\partial z}$, but replace $\dot{x}$ by $\dot{y}$ and then $\dot{z}$.

Rates of Strain

Components of the rate of strain of the tetrahedron defined by vectors A, B, C, Fig. 3 are

$$\begin{aligned} \dot{\epsilon}_{xx} &= \frac{\partial \dot{x}}{\partial x}; \quad \dot{\epsilon}_{yy} = \frac{\partial \dot{y}}{\partial y}; \quad \dot{\epsilon}_{zz} = \frac{\partial \dot{z}}{\partial z}; \quad \dot{\epsilon}_{xy} = \left(\frac{\partial \dot{x}}{\partial y} + \frac{\partial \dot{y}}{\partial x} \right); \quad \dot{\epsilon}_{yz} = \left(\frac{\partial \dot{y}}{\partial z} + \frac{\partial \dot{z}}{\partial y} \right); \quad \dot{\epsilon}_{zx} = \left(\frac{\partial \dot{x}}{\partial z} + \frac{\partial \dot{z}}{\partial x} \right); \\ \frac{\dot{v}}{v} &= \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z}. \end{aligned}$$

Artificial Viscosity

Tensor artificial viscosity for tetrahedron A, B, C, Fig. 3, is

$$q_{xx}^{n+1/2} = 2\mu_1 \left[\dot{\epsilon}_{xx} - \frac{1}{3} \frac{\dot{v}}{v} \right]^{n+1/2}; \quad q_{yy}^{n+1/2} = 2\mu_1 \left[\dot{\epsilon}_{yy} - \frac{1}{3} \frac{\dot{v}}{v} \right]^{n+1/2}; \quad q_{zz}^{n+1/2} = 2\mu_1 \left[\dot{\epsilon}_{zz} - \frac{1}{3} \frac{\dot{v}}{v} \right]^{n+1/2};$$

$$q_{xy}^{n+1/2} = \mu_1 [\dot{\epsilon}_{xy}]^{n+1/2}; \quad q_{yz}^{n+1/2} = \mu_1 [\dot{\epsilon}_{yz}]^{n+1/2}; \quad q_{zx}^{n+1/2} = \mu_1 [\dot{\epsilon}_{zx}]^{n+1/2}.$$

$$\text{where } \mu_1 = \left[C_{NS} \left(\frac{\rho^0}{V} \right)_{\text{①}} \sqrt[3]{V_{ABC}} \right]^{n+1},$$

$$C_{NS} = \text{constant} \approx 10^{-2},$$

$$\rho^0 = \text{reference density of zone ①},$$

$$V = \text{relative volume of zone ①},$$

The above components of the tensor artificial viscosity are added to the corresponding components of stress tensor defined at time $n + 1$.

Increment of energy dissipated by the tensor artificial viscosity

$$\Delta W_{\textcircled{1}} = \sum_{i=1}^8 V_i [q_{xx} \dot{\epsilon}_{xx} + q_{yy} \dot{\epsilon}_{yy} + q_{zz} \dot{\epsilon}_{zz} + q_{xy} \dot{\epsilon}_{xy} + q_{xz} \dot{\epsilon}_{xz} + q_{yz} \dot{\epsilon}_{yz}]_i \Delta t$$

Here $i = 1 \rightarrow 8$ are the eight nodes that define zone $\textcircled{1}$.

Energy Equation

Distortion Energy Increment

$$\Delta z_{\textcircled{1}}^{n+1/2} = V_{\textcircled{1}}^{n+1/2} [s_{xx} \Delta \epsilon_{xx} + s_{yy} \Delta \epsilon_{yy} + s_{zz} \Delta \epsilon_{zz} + T_{xy} \Delta \epsilon_{xy} + T_{yz} \Delta \epsilon_{yz} + T_{zx} \Delta \epsilon_{zx}]_{\textcircled{1}}^{n+1/2}$$

s_{xx} , etc. and $\Delta \epsilon_{xx}$, etc. are the components of the stress tensor and increments of strain respectively defined at the zone center.

Total Internal Energy per Original Volume

$$E_{\textcircled{1}}^{n+1} = \left(\frac{E^n - \left\{ \frac{1}{2} [A(V^{n+1}) + P^n] + \bar{q} \right\} \cdot (V^{n+1} - V^n) + \Delta z_{\textcircled{1}}^{n+1/2}}{1 + \frac{1}{2} [B(V^{n+1})] \cdot (V^{n+1} - V^n)} \right)_{\textcircled{1}}$$

$$\bar{q} = \frac{1}{2} (q^{n+1/2} + q^{n-1/2})$$

Note: It has been assumed here that the pressure equation-of-state has the form $P = A(V) + B(V)E$.

Plastic Strain

In the following definitions of plastic strain the stress deviators at time (n+1) are taken as the values *after* the yield condition has been satisfied. If yielding has not occurred, these equations are bypassed.

Components of Plastic Strain Rate

$$p \dot{\epsilon}_{xx}^{n+1/2} = \dot{\epsilon}_{xx}^{n+1/2} - \frac{1}{\Delta t^{n+1/2}} \left[\frac{s_{xx}^{n+1} - s_{xx}^n - \delta_{xx}}{2\mu} + \frac{1}{3} \frac{V^{n+1} - V^n}{V^{n+1/2}} \right],$$

$$p \dot{\epsilon}_{yy}^{n+1/2} = \dot{\epsilon}_{yy}^{n+1/2} - \frac{1}{\Delta t^{n+1/2}} \left[\frac{s_{yy}^{n+1} - s_{yy}^n - \delta_{yy}}{2\mu} + \frac{1}{3} \frac{V^{n+1} - V^n}{V^{n+1/2}} \right],$$

$$p \dot{\epsilon}_{zz}^{n+1/2} = \dot{\epsilon}_{zz}^{n+1/2} - \frac{1}{\Delta t^{n+1/2}} \left[\frac{s_{zz}^{n+1} - s_{zz}^n - \delta_{zz}}{2\mu} + \frac{1}{3} \frac{V^{n+1} - V^n}{V^{n+1/2}} \right],$$

$$p \dot{\epsilon}_{xy}^{n+1/2} = \dot{\epsilon}_{xy}^{n+1/2} - \frac{1}{\Delta t^{n+1/2}} \left[\frac{T_{xy}^{n+1} - T_{xy}^n - \delta_{xy}}{\mu} \right],$$

$$p \dot{\epsilon}_{zx}^{n+1/2} = \dot{\epsilon}_{zx}^{n+1/2} - \frac{1}{\Delta t^{n+1/2}} \left[\frac{T_{zx}^{n+1} - T_{zx}^n - \delta_{zx}}{\mu} \right],$$

$$p \dot{\epsilon}_{yz}^{n+1/2} = \dot{\epsilon}_{yz}^{n+1/2} - \frac{1}{\Delta t^{n+1/2}} \left[\frac{T_{yz}^{n+1} - T_{yz}^n - \delta_{yz}}{\mu} \right].$$

$\dot{\epsilon}_{xx}^{n+1/2}$ etc. are the velocity strains in the calculation of the stress deviators.

Equivalent Plastic Strain

$$\begin{aligned}
 (\dot{\epsilon}^p)^{n+1/2} &= \frac{\sqrt{2}}{3} \left\{ (p\dot{\epsilon}_{xx} - p\dot{\epsilon}_{yy})^2 + (p\dot{\epsilon}_{yy} - p\dot{\epsilon}_{zz})^2 + (p\dot{\epsilon}_{zz} - p\dot{\epsilon}_{xx})^2 \right. \\
 &\quad \left. + \frac{3}{2} \left[(p\dot{\epsilon}_{xy})^2 + (p\dot{\epsilon}_{yz})^2 + (p\dot{\epsilon}_{zx})^2 \right] \right\}^{1/2}, \\
 (\bar{\epsilon}^p)^{n+1} &= (\bar{\epsilon}^p)^n + (\dot{\epsilon}^p)^{n+1/2} \Delta t^{n+1/2}.
 \end{aligned}$$

Flow Stress

$$Y^{n+1} = a \left[b + (\bar{\epsilon}^p)^{n+1} \right]^c.$$

Here a, b and c are material constants, not to be confused with the vector components $a_{ij,k}$ etc.

Time Step

$$(\Delta t)^{n+3/2} = 0.67 \frac{L^{n+1}}{\sqrt{a^2 + b^2}} \Big|_{\text{Min. of all zones}}$$

$$\text{and } (\Delta t)^{n+3/2} \leq 1.1 \Delta t^{n+1/2}.$$

L is the minimum zone thickness, defined as

$$L^{n+1} = \frac{v^{n+1}}{s_m^{n+1}}, \text{ where } v^{n+1} = \text{volume of zone associated with point } i,j,k \text{ at } t^{n+1}, \text{ and } s_m^{n+1} \text{ is the area of the largest side of the zone. Also, in this equation for } \Delta t,$$

a = sound speed calculated from the equation-of-state and

$$b = 8[C_0^2 + C_L]L^{n+1}\left(\left|\frac{ds}{dt}\right|\right)^{n+1/2}$$

where C_0^2 and C_L are the quadratic and the linear q constants, respectively, and $\frac{ds}{dt}$ is the rate of strain used in the calculation of q .

Further, $(\Delta t)^{n+1} = \frac{1}{2}(\Delta t^{n+3/2} + \Delta t^{n+1/2})$.

Boundary Conditions

Pseudo zones with zero mass are assumed to surround the grid that defines the physical object. Thus points associated with the surface of the physical object may be calculated without changing the logic. Normally a free surface boundary condition is provided, i.e. the pseudo zone pressures are considered always equal to zero. Pressure boundary conditions may be applied by entering the desired space-time values into the pseudo zones.

A reflection boundary condition is obtained by setting equal to zero the normal component of accelerations of a surface point when it points into the reflection surface.

CHECK PROBLEMS

Simple Harmonic Motion

The calculation of the motion of a vibrating plate, clamped at one end, provides a problem that can be readily checked by elasticity theory. Orienting the plate at an arbitrary angle in three dimensional space activates all six components of the stress tensor.

In the calculations shown in Fig. 4 an elastic plate clamped at the top is set into motion by applying a velocity $v = 10$ m/s to the lower right edge in the direction perpendicular to the edge for a time $t = 50$ μ s. After this time the applied velocity is

released, but the lower portion of the plate continues to move due to the kinetic energy. Actually upon release the end of the plate initially moves faster than the applied velocity since this velocity does not correspond to the natural frequency of the plate. Figure 4d is a time-displacement plot for a position in the geometric center of the bottom plane of the plate. It is easily verified that the calculation reproduces the fundamental frequency of the plate.

$$\begin{array}{l}
 \text{Dimensions} \left\{ \begin{array}{l} \text{length: } L = 52.5 \text{ mm} \\ \text{width: } W = 20.0 \text{ mm} \\ \text{thickness: } T = 10.0 \text{ mm} \end{array} \right. \\
 \\
 \text{Elastic constants:} \left\{ \begin{array}{l} \text{bulk modulus: } k = 188 \text{ GPa} \\ \text{shear modulus: } \mu = 81.4 \text{ GPa} \\ \text{density: } \rho^0 = 7.72 \text{ Mg/m}^3 \end{array} \right.
 \end{array}$$

The tensor artificial viscosity used in this calculation is $C_{NS} = 0.05$, more than enough to suppress grid oscillations that would otherwise occur. Figure 4d shows that the amplitude of the oscillation has not been damped or affected by the artificial viscosity.

Plasticity

The impact of a right circular cylinder on a rigid boundary provides a calculation to test the plasticity aspect of the computer program. Since this problem requires only two space dimensions it can be calculated with the HEMP program.¹ Figure 5a shows results of the HEMP calculation where cylindrical symmetry is incorporated into the fundamental equations. Figure 5b shows results of the same problem calculated with the HEMP 3D program described here. It can be seen in Figure 5b that the cylinder has been discretized with three dimensional zones. The calculated time to stop the cylinder, 30 μ s, and the final cylinder length, 19.28 mm, were the same for both HEMP and HEMP 3D. Comparison of the cylinder profiles at $t = 30 \mu$ s also showed almost identical results.

SLIDING SURFACES IN THREE DIMENSIONS

The sliding surface technique described here has evolved over several years of applications. Very good results are obtained even for severely warped surfaces.

The implementation of sliding surfaces in a three dimensional Lagrange grid i, j, k follows similar procedures as slide lines in the two dimensional problem.¹ However, instead of mapping stresses from one side of the interface to the other side the vector accelerations are added from one side of the interface to the other. (This method can also be used in two dimensions but there is no particular advantage). Interfaces are defined in i, j, k space that separates two regions. The grid points at the interface of one region slide on the surface provided by the grid points of the opposite region and vice versa. The grid points associated with one side of the interface are designated in advance as slave points while the grid points associated with the opposite side of the interface are designated master points. The calculations are symmetric in that the grid points of both regions at the interface are advanced in time in the same manner. After the grid points associated with each region have been advanced by the integration time step, the position of slave points are adjusted to lie on the surface defined by the master points when penetration of one grid surface into the opposite grid surface occurs. It has been found convenient to define a local surface at each grid point as the plane through the grid point that is perpendicular to the normal vector defined at the point. Thus, the interface between two regions is actually composed of a series of local surfaces.

All grid points at the interface of the two regions are tagged as either void open or void closed. Void open means there is a void between the point and the opposite surface and void closed means the point is in contact with the opposite surface. Void open points are advanced in time with the usual free surface calculations.

At the interface between the two regions it will be convenient to refer to the point that is currently being advanced as the "current" point. Parameters

associated with the other side of the interface that are required to advance a current point are identified by the word "opposite." The roles are then reversed after calculations have been completed for one side of the interface. The symmetry of the calculation permits sliding surfaces to be defined simultaneously in more than one direction. However, for illustration of the method, we will assume a sliding surface at a constant Lagrange coordinate j ; see Fig. 6. The letter f will be used to designate a current point.

Free surface boundary conditions are used to calculate the acceleration of point f in the x, y, z coordinate system. The components of acceleration are transformed into a coordinate system where two components are in the plane of the sliding surface interface at point f . The acceleration component normal to the interface includes a contribution of mass from the opposite grid and in addition, the normal component of acceleration of the opposite grid. The two acceleration components in the plane of the interface are unchanged by the presence of the interface. The normal component of acceleration from the opposite grid must include a contribution of mass from the present grid. Thus, the symmetric treatment of the interface calculations requires preprocessing each side of the interface. A final calculation is then made to advance in time points associated with each side of the interface.

OUTLINE OF SLIDING SURFACE CALCULATION

Step I

For every grid point on the sliding surface interface, calculate in advance the following quantities and store with the grid point.

1. The mass per unit area, m .
2. The acceleration assuming the point is on a free surface.
3. An outward pointing unit vector normal to an element of surface defined at each point on the sliding surface interface.

Step II

For each current void closed grid point f:

1. Locate the three points of the opposite grid that are closest to point f.
2. Determine by interpolation the mass per unit area of the opposite grid at the position of point f.
3. Calculate the mass weighting factor at point f (z-factor).
4. Resolve the acceleration obtained in Step I-2 in the direction of the normal vector defined in Step I-3.
5. Repeat 1-4 with the opposite grid as the current grid.

Step III

For each current void closed grid point f:

1. Locate the three points of the opposite grid that are closest to point f.
2. Determine by interpolation a value for the normal component of acceleration from the opposite grid at the position of current grid point f.
3. Add to the normal component of acceleration of the present grid from step II-4 the normal component of acceleration contributed by the opposite grid from Step 2 above.
4. Calculate new velocities and new coordinates in the x, y, z coordinate system.

5. Repeat 1-4 with the opposite grid as the current grid.

Step IV

1. Test for penetration of the current grid points into the opposite grid.
2. Adjust the velocities for all points that have penetrated the opposite grid to conserve momentum.
3. Repeat (1) and (2) with the opposite grid as the current grid.
4. Relocate slave grid points that have penetrated the master grid onto the master grid. Void open points are advanced in time in the x,y,z coordinate system using free surface boundary conditions independent of Steps I, II and III. If penetration of the opposite grid occurs from Step IV the point is relocated and labeled void closed. Void closed points are labeled void open when the distance of the point to the opposite surface is greater than 10% of the zone size.

Calculational Steps to Advance in Time Grid Points on a Sliding Surface

Step I

1. Calculate the mass per unit area for all grid points on the sliding interface. Referring to Fig. 6 assume point a is a point on the interface. The mass per unit area, m_a , is given by:

$$m_a = \frac{1}{2} \left[\frac{M_{\textcircled{1}} + M_{\textcircled{2}} + M_{\textcircled{3}} + M_{\textcircled{4}}}{A_{\textcircled{1}} + A_{\textcircled{2}} + A_{\textcircled{3}} + A_{\textcircled{4}}} \right]$$

$M_{\textcircled{1}}$ is the mass of zone ① etc. $A_{\textcircled{1}}$ is the area of the triangle in zone ① that is associated with opposite grid point a. Similar for $A_{\textcircled{2}}$, $A_{\textcircled{3}}$, and

A₄. The parameter m_a is seen to be the average of the mass per unit areas of the zones that share point a.

2. Calculate the acceleration of each point on the interface with free surface boundary conditions.

a. For a given point i, j, k calculate the acceleration $\bar{A}_{i,j,k}^*$ (see point a Fig. 6).

$$\bar{A}_{i,j,k}^* = \frac{d\dot{x}}{dt} \bar{i} + \frac{d\dot{y}}{dt} \bar{j} + \frac{d\dot{z}}{dt} \bar{k}$$

i) x direction

$$\left(\frac{d\dot{x}}{dt} \right)_{i,j,k} = \frac{1}{\rho_{i,j,k}^*} \left[\frac{\partial \Sigma_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} \right]_{i,j,k}^*, \text{ where}$$

$$\frac{1}{\rho} \left(\frac{\partial \Sigma_{xx}}{\partial x} \right)_{i,j,k}^* = \frac{1}{4\Phi_{i,j,k}} \{ (\Sigma_{xx})_{\odot} [(y_{VI} - y_V)(z_{IV} - z_V) - (z_{VI} - z_V)(y_{IV} - y_V)]$$

$$+ (\Sigma_{xx})_{\ominus} [(y_{II} - y_V)(z_{VI} - z_V) - (z_{II} - z_V)(y_{VI} - y_V)]$$

$$+ (\Sigma_{xx})_{\oplus} [(y_{IV} - y_{III})(z_{VI} - z_{III}) - (z_{IV} - z_{III})(y_{VI} - y_{III})]$$

$$+ (\Sigma_{xx})_{\otimes} [(y_{VI} - y_{III})(z_{II} - z_{III}) - (z_{VI} - z_{III})(y_{II} - y_{III})] \}$$

$$(\Phi)_{i,j,k} = \frac{1}{8} [M_{\odot} + M_{\ominus} + M_{\oplus} + M_{\otimes}]$$

To form $(1/\rho \partial T_{xy}/\partial y)_{i,j,k}^*$, replace each Σ_{xx} in the right side of the above expression with T_{xy} , every y with the corresponding z, and each z with the corresponding x.

To form $(1/\rho \partial T_{xz}/\partial z)_{i,j,k}^*$, replace each Σ_{xx} in the above expression with T_{zx} , every y with the corresponding x, and every z with the corresponding y.

ii) y direction

$$\left(\frac{dy}{dt}\right)_{i,j,k}^n = \frac{1}{\rho_{i,j,k}^n} \left[\frac{\partial T_{xy}}{\partial x} + \frac{\partial \Sigma_{yy}}{\partial y} + \frac{\partial T_{yz}}{\partial z} \right]_{i,j,k}^n ; \text{ where}$$

$$\left(\frac{1}{\rho} \frac{\partial T_{xy}}{\partial x}\right)_{i,j,k}^n = \text{same as } \left(\frac{1}{\rho} \frac{\partial \Sigma_{xx}}{\partial x}\right)_{i,j,k}^n \text{ defined above,}$$

except replace each Σ_{xx} by the corresponding value of T_{xy} .

$$\left(\frac{1}{\rho} \frac{\partial \Sigma_{yy}}{\partial y}\right)_{i,j,k}^n = \text{same as } \left(\frac{1}{\rho} \frac{\partial T_{xy}}{\partial y}\right)_{i,j,k}^n \text{ defined above,}$$

except replace each T_{xy} by the corresponding value of Σ_{yy} .

$$\left(\frac{1}{\rho} \frac{\partial T_{yz}}{\partial z}\right)_{i,j,k}^n = \text{same as } \left(\frac{1}{\rho} \frac{\partial T_{xz}}{\partial z}\right)_{i,j,k}^n \text{ defined above,}$$

except replace each T_{xz} by the corresponding value of T_{yz} .

iii) z direction

$$\left(\frac{dz}{dt}\right)_{i,j,k}^n = \frac{1}{\rho_{i,j,k}^n} \left[\frac{\partial T_{xz}}{\partial x} + \frac{\partial T_{yz}}{\partial y} + \frac{\partial \Sigma_{zz}}{\partial z} \right]_{i,j,k}^n$$

$$\left(\frac{1}{\rho} \frac{\partial T_{xz}}{\partial x}\right)_{i,j,k}^n = \text{same as } \left(\frac{1}{\rho} \frac{\partial \Sigma_{xx}}{\partial x}\right)_{i,j,k}^n \text{ defined above,}$$

except replace each Σ_{xx} by the corresponding value of T_{zx} .

$$\left(\frac{1}{\rho} \frac{\partial T_{yz}}{\partial y}\right)_{i,j,k}^n = \text{same as } \left(\frac{1}{\rho} \frac{\partial T_{xy}}{\partial y}\right)_{i,j,k}^n \text{ defined above,}$$

except replace each T_{xy} by the corresponding value of T_{yz} .

$$\left(\frac{1}{\rho} \frac{\partial \Sigma_{zz}}{\partial z} \right)_{i,j,k}^n = \text{same as } \left(\frac{1}{\rho} \frac{\partial T_{zz}}{\partial z} \right)_{i,j,k}^n \text{ defined above,}$$

except replace each T_{zz} by the corresponding value of Σ_{zz} .

Determine an outward pointing unit vector normal to an element of surface defined at each point on the sliding surface interface. Referring to Fig. 6 assume point a is any point on the sliding surface interface.

- a) Calculate the normal vectors for each of the triangular surface areas associated with point a. The normal vector corresponding to zone ① is:

$$\begin{aligned} \vec{r}_{a,v,N} &= \begin{vmatrix} i & j & k \\ x_v - x_a & y_v - y_a & z_v - z_a \\ x_N - x_a & y_N - y_a & z_N - z_a \end{vmatrix} \\ &= A_{a,v,N} \vec{i} + B_{a,v,N} \vec{j} + C_{a,v,N} \vec{k} \end{aligned}$$

where

$$\begin{aligned} A_{a,v,N} &= [(y_v - y_a)(z_N - z_a) - (z_v - z_a)(y_N - y_a)] \\ B_{a,v,N} &= -[(x_v - x_a)(z_N - z_a) - (z_v - z_a)(x_N - x_a)] \\ C_{a,v,N} &= [(x_v - x_a)(y_N - y_a) - (y_v - y_a)(x_N - x_a)] \end{aligned}$$

Similar for zones ②, ③, and ④, Fig. 6.

Note: The vector cross products must be taken so that the normal vectors point outward from the grid. A convenient way to assure that vector $\vec{r}_{a,v,N}$ points outward from the grid is to take the dot product of $\vec{r}_{a,v,N}$ with the vector formed by point a and its corresponding interior point. The interior point is called a connector point. In Fig. 6 point VI is the connector point for point a. If the product is positive reverse the sign $\vec{r}_{a,v,N}$ otherwise the vector has the correct outward direction.

- b) Calculate $\bar{r}_a$, a unit vector obtained from the average of the vectors perpendicular to the triangles that surround point a, i.e., Fig. 6 triangles (a,V,IV), (a,IV,III), (a,III,II) and (a,II,V).

$$\bar{r}_a = A_a \bar{i} + B_a \bar{j} + C_a \bar{k} / \sqrt{A_a^2 + B_a^2 + C_a^2} = l \bar{i} + m \bar{j} + n \bar{k}$$

where A_a , B_a , and C_a are the direction numbers of vector r_a and l , m and n are the direction cosines.

$$A_a = (A_{a,V,IV} + A_{a,IV,III} + A_{a,III,II} + A_{a,II,V})$$

$$B_a = (B_{a,V,IV} + B_{a,IV,III} + B_{a,III,II} + B_{a,II,V})$$

$$C_a = (C_{a,V,IV} + C_{a,IV,III} + C_{a,III,II} + C_{a,II,V})$$

Step II

1. Locate the opposite surface points associated with each current void closed point f:

- a) Calculate d_f^2 , the square of the distance from point f to successive points i, k of the opposite grid.

$$d_f^2 = (x_f - x_{i,k})^2 + (y_f - y_{i,k})^2 + (z_f - z_{i,k})^2$$

- b) Let point a, of the opposite grid be the point which has the shortest distance from point f. See Fig. 7.
- c) Project the points 1, 2, 3, 4 and f onto the local surface at point a, defined by point a and the unit vector $\bar{r}_a$ at point a. Designate as $(x^*, y^*, z^*)_i$ the coordinates of a point i and $(x, y, z)_i$ the coordinates after projection onto the local surface at point a.

$$x_i = x_i^* - l d_i$$

$$y_i = y_i^* - md_i$$

$$z_i = z_i^* - nd_i$$

where $d_i = \bar{r}_a \cdot \bar{c}_i$ $i = 1, 2, 3, 4$, and f . The vectors $\bar{c}_i$ are formed by the points i and the origin. Here l , m and n are the direction cosines of $\bar{r}_a$, the unit vector at point a calculated in Step 1.3b.

- d) Point f can be in any one of the four quadrants formed by the projections of opposite grid points surrounding point a onto the surface defined by point a and the unit vector at point a , Fig. 7. The following procedure is used to locate the quadrant that contains current point f .

- i) Referring to Fig. 7 calculate the area of triangles $\Delta_{a,2,3}$, α , β , and γ using the coordinates obtained in Step 1c above.
- ii) Point f is contained within triangle $\Delta_{a,2,3}$ if: $[\Delta_{a,2,3} - (\alpha + \beta + \gamma) < 10^{-3} \Delta_{a,2,3}]$. Here, $\Delta_{a,2,3}$, α , β , γ refer to the areas of the triangles.
- iii) Repeat step (2) for the remaining quadrants.
- iv) If all quadrant tests fail to locate point f then select the next closest point a to point f and repeat (1-3).
- v) Repeat (1-4) for all points f in the current plane.

Note: It is important that the search logic described above be conducted with the grid points projected onto the local surface (Step 1c).

- e) It is necessary to determine when a current point f is not covered by the opposite grid as shown in Fig. 8. Assume current point f has been determined closest to opposite grid point a and it is known that grid point a is on the boundary of the opposite grid, Fig. 8. The

four quadrants for the search routine described above are formulated by extending the opposite grid through point a. Referring to Fig. 8 an extension point 3 is established by calculating a vector extension.

$$i) \quad \bar{V}_3 = \bar{V}_a + k_e(\bar{V}_a - \bar{V}_1)$$

Here $\bar{V}_a$ designates the vector formed by point a and the origin. Similar for points 1 and 3. The parameter k_e provides the dimension of the extension. k_e is taken as a large number, e.g., 1000, to assure that point f is covered by the opposite grid for the search routine that locates the quadrant.

ii) Coordinates of extension point 3 (Fig. 9)

$$x_3 = x_a + k_e(x_a - x_1)$$

$$y_3 = y_a + k_e(y_a - y_1)$$

$$z_3 = z_a + k_e(z_a - z_1)$$

iii) When point f has been located in a quadrant that contains the extension point it is outside the opposite grid. If the point is more than one zone thickness off the opposite grid it is considered a free point independent of the sliding interface. If the point is less than a zone thickness off the opposite grid it is considered still on the sliding interface. To make this distinction the extension point is recalculated with a value of k_e to provide an extension of the opposite grid of approximately one zone thickness of the current grid. (The default value is $k_e = 1$, which assumes both grids are the same size).

Point e in Fig. 8 is the new extension point. The following method is used to locate the position of current point f with respect to the extension surface. Calculate:

$$\frac{\bar{S} \cdot \bar{C}_e}{A_e^2 + B_e^2 + C_e^2} = d_e$$

Here A_e , B_e and C_e are the direction numbers of vector $\bar{C}_e$.
 $\bar{S}$ is the vector from point a to current point f and $\bar{C}_e$ the vector from point a to the extension point e.

If $d_e \geq 1$, point f is beyond the extension and is accelerated with free surface boundary conditions.

If $d_e < 1$, point f is considered to be still on the sliding interface and the calculation proceeds.

2. Calculate an interpolated mass per unit area, m_f , at the position corresponding to point f. Figure 9 shows an overlay of the current grid containing point f on the opposite grid. We wish to obtain the mass per unit area of the opposite grid at the position of current point f. This mass per unit area will then be used to increase the mass associated with point f for the acceleration of point f in the direction normal to the sliding interface.

Referring to Fig. 10 the mass per unit area at point f is:

$$(m_f) = \frac{m_a\alpha + m_b\beta + m_c\gamma}{\alpha + \beta + \gamma}$$

See Step I-1 for the calculation of m_a , etc.

3. Calculate the mass weighting factor at point f. (z factor)
 - a) Let M_m be the mass due to the opposite surface that is to be included with the mass of point f.

$$M_m = (m_f) [A_{\textcircled{5}} + A_{\textcircled{6}} + A_{\textcircled{7}} + A_{\textcircled{8}}] \text{ (see Fig. 9)}$$

Here $A_{\textcircled{5}}$ is the scalar area of the triangle (f, V, IV) in zone $\textcircled{5}$ Fig. 11, and is calculated as follows:

$$2A_{\textcircled{5}} = \sqrt{(A_{f,v,N})^2 + (B_{f,v,N})^2 + (C_{f,v,N})^2}$$

where

$$A_{f,v,N} = [(y_v - y_f)(z_v - z_f) - (z_v - z_f)(y_N - y_f)]$$

$$B_{f,v,N} = [(x_v - x_f)(z_N - z_f) - (z_v - z_f)(x_N - x_f)]$$

$$C_{f,v,N} = [(x_v - x_f)(y_N - y_f) - (y_v - y_f)(x_N - x_f)]$$

Similar for $2A_{\textcircled{6}}$, $2A_{\textcircled{7}}$ and $2A_{\textcircled{8}}$.

- b) Calculate the z-factor

$$z = 1 + \frac{M_n}{\frac{1}{2}(M_{\textcircled{5}} + M_{\textcircled{6}} + M_{\textcircled{7}} + M_{\textcircled{8}})}$$

Here $M_{\textcircled{5}}$ etc. are the masses associated with point f, see Fig. 11.

4. Calculate the acceleration of grid points on the sliding surface. The acceleration normal to the sliding surface includes the mass from the opposite grid using the z-factor determined from the preceding step.

- a) Calculate $\vec{N}_f^*$, the free surface acceleration of point f resolved in the direction of the average normal, $\vec{r}_f$.

$$\vec{N}_f^* = (\vec{A}_f^* \cdot \vec{r}_f) \vec{r}_f$$

The free surface acceleration of point f, $\vec{A}_f^*$, was calculated in Step I-2. The average normal, $\vec{r}_f$, was calculated in Step I.3.b.

- b) Calculate $\bar{N}_f$, the acceleration of current point f in the direction of the average normal that includes the mass of the opposite grid.

$$\bar{N}_f = \frac{\bar{N}_f^*}{z}$$

5. Repeat 1-4 with the opposite grid as the current grid.

Note: $\bar{N}_f$ is a partial acceleration normal to the surface defined at point f that includes the mass of the opposite grid. The total normal acceleration of current point f must also include a contribution from the opposite grid and is described in Step III that follows.

Step III

1. For the current void closed point f, locate the opposite grid points that surround point f. See Step II-1 and Fig. 9.
2. From the three opposite grid points a, b, c that surround point f, Fig. 10, determine acceleration vector $\bar{N}_f^{og}$. Vector $\bar{N}_f^{og}$ is an interpolated normal component of acceleration from the opposite grid at the position of point f. The interpolation method is shown in Fig. 10.

$$\bar{N}_f^{og} = \frac{\bar{N}_a\alpha + \bar{N}_b\beta + \bar{N}_c\gamma}{\alpha + \beta + \gamma}$$

$\bar{N}_a$, $\bar{N}_b$ and $\bar{N}_c$ are the accelerations of opposite grid points a, b and c calculated in Step II-5.

Test for void opening. If $\bar{N}_f \cdot \bar{C}_f > 0$ and $\bar{N}_f^{og} \cdot \bar{C}_f < 0$ tag point f as void open. $\bar{N}_f$ is calculated in Step II-4. The vector $\bar{C}_f$ is formed by point f and the connector point associated with point f. If the above test is positive the acceleration of point f is given by $\bar{A}_f^*$, the free surface acceleration from Step I-2.

3. Calculate the total acceleration, $\bar{A}_f$, of point f.

$$\bar{A}_f = \bar{A}_f^* - \bar{N}_f^* + \bar{N}_f + \bar{N}_f^{OG} = A_x \bar{i} + A_y \bar{j} + A_z \bar{k}$$

Note: $\bar{N}_f$ must be saved for use in the interpolation procedure when the above process is reversed and the current grid becomes the opposite grid.

4. Calculate the x, y, z components of velocity and new coordinates for current point f.

- a) Velocity

$$\dot{x}_f^{n+1/2} = \dot{x}_f^{n-1/2} + \Delta t^n A_x$$

$$\dot{y}_f^{n+1/2} = \dot{y}_f^{n-1/2} + \Delta t^n A_y$$

$$\dot{z}_f^{n+1/2} = \dot{z}_f^{n-1/2} + \Delta t^n A_z$$

- b) New coordinates

$$x_f^{n+1} = x_f^n + \dot{x}_f^{n+1/2} \Delta t^{n+1/2}$$

$$y_f^{n+1} = y_f^n + \dot{y}_f^{n+1/2} \Delta t^{n+1/2}$$

$$z_f^{n+1} = z_f^n + \dot{z}_f^{n+1/2} \Delta t^{n+1/2}$$

5. Repeat 1-4 for all interface grid points.

Step IV

1. Test to see if a point f has penetrated the opposite grid. Assume f, in Fig. 7, is a point on a grid that is to be tested for penetration into the opposite grid local surface at point a.

- a. Calculate d the perpendicular distance from the point f to the local surface at point a.

$$d = [\vec{l}\vec{i} + m\vec{j} + n\vec{k}] \cdot [(x_f - x_a)\vec{i} + (y_f - y_a)\vec{j} + (z_f - z_a)\vec{k}]$$

or

$$d = +[l(x_f - x_a) + m(y_f - y_a) + n(z_f - z_a)]$$

Here l, m, n are the direction cosines of the unit vector defined at point a (see Step I.3b).

- b. If $0 < d < +\delta$, point f^{n+1} remains as calculated in Step III with the same void status as before. Here δ is a positive number equal to 0.1 times the grid spacing calculated as:

$$\delta = 0.1\sqrt{A^2 + B^2 + C^2}$$

- c. If $d > \delta$, point f^{n+1} remains as calculated in Step III and is tagged void open.
- d. If $d < 0$ point f^{n+1} has penetrated the opposite grid and is tagged void closed.
2. Adjust the velocities of all void closed points.
- a. Calculate velocities normal to the interface. Assume point f in Fig. 9 has penetrated the local surface at point a of the opposite grid, calculate the velocity components $\dot{N}_f, \dot{N}_a, \dot{N}_b$ and $\dot{N}_c$ that are normal to the surface.

$$\dot{N}_a = l\dot{x}_a + m\dot{y}_a + n\dot{z}_a$$

$$\dot{N}_b = l\dot{x}_b + m\dot{y}_b + n\dot{z}_b$$

$$\dot{N}_c = l\dot{x}_c + m\dot{y}_c + n\dot{z}_c$$

$$\dot{N}_f = l\dot{x}_f + m\dot{y}_f + n\dot{z}_f$$

- b. Calculate $\dot{N}_f^+$ the velocity of point f from the conservation of linear momentum:

$$\dot{N}_f^+ = \left[\frac{\frac{\alpha M_a \dot{N}_a + \beta M_b \dot{N}_b + \gamma M_c \dot{N}_c + M_f \dot{N}_f}{\alpha + \beta + \gamma}}{\frac{\alpha M_a + \beta M_b + \gamma M_c}{\alpha + \beta + \gamma} + M_f} \right]$$

$$M_f + \frac{1}{4}[M_{\odot} + M_{\odot} + M_{\odot} + M_{\odot}] = 2(\phi)_f$$

$$M_a + \frac{1}{4}[M_{\odot} + M_{\odot} + M_{\odot} + M_{\odot}] = 2(\phi)_a$$

similar for M_b and M_c .

Note: The mass Φ associated with a point on the interface is calculated in Step. I.

Note: To minimize the number of calculations, only the linear momentum has been considered instead of including conservation of angular and linear momentum as was done in the two dimensional problem. Actually it is the artificial viscosity, q , and the equations of motion that accomplish the conservation of momentum. Adjusting the velocities at the interface after a collision sets up the initial conditions for the artificial viscosities on each side of the interface.

- c) Calculate the x, y, z components of the new velocity of point f.

$$\dot{x}_f^{n+1/2} = \dot{x}_f^* + l \Delta \dot{N}_f$$

$$\dot{y}_f^{n+1/2} = \dot{y}_f^* + m \Delta \dot{N}_f$$

$$\dot{z}_f^{n+1/2} = \dot{z}_f^* + n \Delta \dot{N}_f$$

where

$$\Delta \dot{N}_f = (\dot{N}_f^* - \dot{N}_f)$$

Here x_f^* , y_f^* , z_f^* refers to the velocity of point f before the adjustment for conservation of momentum. The velocities of all void closed points are adjusted point by point. That is, only the velocity of the point under consideration is adjusted, point f in the example described above. The velocities of all mass points on one side of the grid are adjusted when penetration has occurred. Subsequently the velocities of all points on the opposite grid are adjusted. Thus, all of the old velocities and coordinates must be retained until all of the velocities of both sets of sliding surface points have been adjusted.

3. Declare one grid the slave grid and the grid opposite it the master grid. Relocate slave points onto the master surface for slave points where $d < -\delta$. A slave point f that has penetrated the master surface is set back to the master surface by subtracting the length d from the position of the point. For the sign convention used here d is a negative number. The direction cosines l, m, n point outward from the grid, thus the new coordinates of point f are:

$$x_f^{n+1} = x_f^* - ld$$

$$y_f^{n+1} = y_f^* - md$$

$$z_f^{n+1} = z_f^* - nd$$

Here x_f^* , y_f^* , z_f^* refers to the coordinates of point f used to determine that $d < -\delta$.

When penetration occurs new velocities are calculated on both sides of the interface, but only the position of slave points are adjusted.

APPLICATIONS OF SLIDING SURFACE ROUTINE

The major difficulty with sliding surface routines arises from failure of the search routines that must locate one grid with respect to the other. The problem becomes aggravated with curved or warped surfaces with a search routine that operates in three dimensions. The method used here projects the grid point onto local two dimensional surfaces to establish the orientation of one grid with respect to the other. With this procedure the search technique is robust even for distorted surfaces. Figure 12 shows an application with two curved surfaces. Fig. 13 shows the acceleration of a metal plate by an explosive with a sliding surface between the two materials. The explosive was detonated at nine equally spaced points on a line along the top surface of the explosive.⁴

ZONE DIMENSION CHANGE AND SUBCYCLING

It is useful to be able to change from coarse to fine zoning in a localized region and to be able to join two independent grids at an interface. The latter being especially important for constructing grids for three dimensional problems.

Zone dimension change at an interface in two dimensions

Figure 14 shows schematically a zone change from large to small zones across a Lagrange coordinate k_x . The grid with the largest zone size is chosen as the master grid and the small zone grid the slave grid. The master grid defines the interface k_x . In Fig. 14 grid points associated with the master and slave grids are shown as closed circles and open circles, respectively.

1. To advance in time master and slave points on the interface k_x , Fig. 14.
 - a. Calculate the partial acceleration of all master points j, k on k -line k_x .

Components of partial acceleration for point a, Fig. 14.

$$\left(\frac{d\dot{x}}{dt}\right)_a^{master} = \frac{1}{z} \left\{ \frac{-\Delta t^n}{2\phi_{j,k}^n} \left[(\Sigma_{xx})_{\odot}^n (y_{II}^n - y_{III}^n) + (\Sigma_{xx})_{\ominus}^n (y_{III}^n - y_{IV}^n) \right. \right. \\ \left. \left. - (T_{xy})_{\odot}^n (x_{II}^n - x_{III}^n) - (T_{xy})_{\ominus}^n (x_{III}^n - x_{IV}^n) \right] + \Delta t^n (\alpha)_{j,k}^n \right\}.$$

$$i) \quad \left(\frac{d\dot{y}}{dt}\right)_a^{master} = \frac{1}{z} \left\{ \frac{-\Delta t^n}{2\phi_{j,k}^n} \left[(\Sigma_{yy})_{\odot}^n (x_{II}^n - x_{III}^n) + (\Sigma_{yy})_{\ominus}^n (x_{III}^n - x_{IV}^n) \right. \right. \\ \left. \left. - (T_{xy})_{\odot}^n (y_{II}^n - y_{III}^n) - (T_{xy})_{\ominus}^n (y_{III}^n - y_{IV}^n) \right] + \Delta t^n (\beta)_{j,k}^n \right\}$$

$$\phi_{j,k}^n = \frac{1}{4} \left[\left(\frac{\rho^0 A^n}{V^n} \right)_{\odot} + \left(\frac{\rho^0 A^n}{V^n} \right)_{\ominus} \right]$$

$$ii) \quad \alpha_{j,k}^n = \frac{1}{4} \left\{ \left[T_{xy}^n \left(\frac{A^n}{M} \right) \right]_{\odot} + \left[T_{xy}^n \left(\frac{A^n}{M} \right) \right]_{\ominus} \right\} \\ \beta_{j,k}^n = \frac{1}{2} \left\{ \left[(\Sigma_{yy}^n - \Sigma_{\theta\theta}^n) \left(\frac{A^n}{M} \right) \right]_{\odot} + \left[(\Sigma_{yy}^n - \Sigma_{\theta\theta}^n) \left(\frac{A^n}{M} \right) \right]_{\ominus} \right\}$$

The z factor that is found in equations (i) and (ii) above is obtained by mapping the masses of the slave zones between master points II and IV onto the master grid.

- b. Calculate the partial accelerations for all slave points on k -line, k_s . The same procedure as above is used. The z factor now maps mass from the master grid onto the slave grid.
- c. For each master grid point on k -line k_s , determine a slave grid partial acceleration by interpolation. Referring to Fig. 14 the slave grid partial acceleration corresponding to master grid point a is:

$$\left(\frac{d\dot{x}}{dt}\right)_a^{slave} = \alpha \left(\frac{d\dot{x}}{dt}\right)_s^{slave} + (1 - \alpha) \left(\frac{d\dot{x}}{dt}\right)_t^{slave}$$

Here $\alpha = l_{at}/l_{st}$ where l_{at} is the distance between points a and t and l_{st} the distance between points s and t .

Similar for

$$\left(\frac{d\dot{y}}{dt}\right)_a^{slave}$$

- d. Calculate the total acceleration of all master points on k-line k_s . Referring to Fig. 14 the total acceleration of master point a (point j, k Fig. 14) is:

$$\left(\frac{d\dot{x}}{dt}\right)_{j,k} = \left(\frac{d\dot{x}}{dt}\right)_a^{master} + \left(\frac{d\dot{x}}{dt}\right)_a^{slave}$$

Similar for

$$\left(\frac{d\dot{y}}{dt}\right)_{j,k}$$

- e. Calculate new velocities for all master points on k-line k_s .

$$\dot{x}_{j,k}^{n+1/2} = \dot{x}_{j,k}^{n-1/2} + \Delta t^{n+1/2} \left(\frac{d\dot{x}}{dt}\right)_{j,k}$$

$$\dot{y}_{j,k}^{n+1/2} = \dot{y}_{j,k}^{n-1/2} + \Delta t^{n+1/2} \left(\frac{d\dot{y}}{dt}\right)_{j,k}$$

- f. Obtain new velocities for the slave grid points on k-line k_s by interpolation so that the original spacing between consecutive master points is maintained. Referring to Fig. 14 the new velocities for slave point t are:

$$\dot{x}_t^{n+1/2} = \beta \dot{x}_a^{n+1/2} + (1 - \beta) \dot{x}_b^{n+1/2}$$

$$\dot{y}_t^{n+1/2} = \beta \dot{y}_a^{n+1/2} + (1 - \beta) \dot{y}_b^{n+1/2}$$

$$\beta = \frac{l_{tb}}{l_{ab}}$$

Here β is a constant calculated when the grid is generated. l_{tb} is the distance from point t to point b and l_{ab} the distance between points a and b.

Zone dimension change at an interface in three dimensions

The same procedure is followed as described for the two dimensional case. Fig. 15 shows two grids that are to be joined together without the interface grid points of both grids being necessarily coincident.

1. To advance in time master and slave points at the interface of two grids.
 - a. Calculate the partial acceleration for all interface grid points associated with the master grid. Referring to Fig. 15 we wish to accelerate grid point a associated with master grid zones ①, ②, ③, ④.

Partial acceleration in x-direction.

$$i) \quad \left(\frac{dx}{dt} \right)_{i,j,k}^{master} = \frac{1}{\rho_{i,j,k}^n} \left[\frac{\partial \Sigma_x}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} \right]_{i,j,k}^n, \text{ where}$$

$$ii) \quad \frac{1}{\rho} \left(\frac{\partial \Sigma_x}{\partial x} \right)_{i,j,k}^n = \frac{1}{z4\Phi_{i,j,k}} (\Sigma_x)_{\odot} [(y_{VI} - y_V)(z_{IV} - z_V) - (z_{VI} - z_V)(y_{IV} - y_V)] \\ + (\Sigma_x)_{\odot} [(y_{II} - y_V)(z_{VI} - z_V) - (z_{II} - z_V)(y_{VI} - y_V)] \\ + (\Sigma_x)_{\odot} [(y_{IV} - y_{III})(z_{VI} - z_{III}) - (z_{IV} - z_{III})(y_{VI} - y_{III})] \\ + (\Sigma_x)_{\odot} [(y_{VI} - y_{III})(z_{II} - z_{III}) - (z_{VI} - z_{III})(y_{II} - y_{III})]$$

$$iii) \quad (\Phi)_{i,j,k} = \frac{1}{8} [M_{\odot} + M_{\odot} + M_{\odot} + M_{\odot}]$$

The factor z that appears in (ii) above is the weighting factor that maps the mass of the opposite grid. The remaining terms in equation (i) are composed in the usual manner and include the z factor, as shown in equation (ii), the finite difference equation for the first term in equation 1. In a similar manner the y, z components are calculated.

- b. Calculate the partial acceleration for all points associated with the slave grid on the interface. The z -factor now maps the mass from the master surface onto the slave grid-point.
- c. For each master grid point determine a slave grid partial acceleration by interpolation. Referring to Fig. 16 assume master grid point a has been found to be in the neighborhood of slave grid points f, g, h .

$$\left(\frac{d\dot{x}}{dt}\right)_a^{slave} = \frac{\alpha\left(\frac{d\dot{x}}{dt}\right)_f + \beta\left(\frac{d\dot{x}}{dt}\right)_g + \gamma\left(\frac{d\dot{x}}{dt}\right)_h}{\alpha + \beta + \gamma}$$

Here f, g , and h denotes the partial acceleration of slave grid points. Similar for the y, z components.

- d. Calculate the total acceleration for all master points on the interface. For master point a (point i, j, k , Fig. 15) the total acceleration is:

$$\left(\frac{d\dot{x}}{dt}\right)_{i,j,k} = \left(\frac{d\dot{x}}{dt}\right)^{slave} + \left(\frac{d\dot{x}}{dt}\right)^{master}$$

Similar for y, z components of acceleration.

- e. Calculate new velocities for all master points on the interface.
- f. Obtain new velocities for the slave grid points on the interface using the interpolation scheme above.

Subcycling with zone dimension change in two dimensions

The time step for a given cycle is dictated by the zone with the smallest zone dimension divided by the local sound speed. Rather than calculate the entire grid with a given time step a saving in computer time can be obtained by dividing the grid into different regions with a different time step for each region. A region with small zones is calculated for several time steps until the time step of a region that can use a larger time step is reached. The region with the larger time step is then advanced with a single time step equal to the sum of the time steps used in the region with the small zone. It is convenient to calculate the region with the largest time step first. The time steps for the regions with the smaller time step requirements are chosen so that an integral number of equal time steps can be used to reach the time step used for the largest grid.

Example for a zone size change of two to one

1. With a time step Δt calculate new velocities for all grid points of the largest grid using the stress boundary conditions provided by the small grid. Advance all points of the large grid and calculate the new zonal parameters. Save the old positions of the large grid for points on the interface between the two grids.
2. Find the new velocities of all small grid points that are on the interface by interpolation. These velocities are the boundary conditions during the subcycling of the small grid.
3. With the old interface positions, so that all of the positions of the small grid points are at the same time calculate new velocities from the acceleration equations for all small grid points except those on the interface using a time step $\Delta t/2$.
4. Calculate new coordinates for all of the small grid points including those on the interface. The points on the interface use velocities

from step b to obtain new coordinates and the remainder of the points of the small grid used in the velocities from Step c.

5. Calculate the new zonal quantities for the small grid.
6. Calculate new velocities again for all small grid points except those on the interface using time Step $\Delta t/2$.
7. Calculate new coordinates for all grid points including those on the interface. (The large grid coordinates will now coincide with the values obtained in Step 1.
8. Calculate new zonal quantities for the small grid.

The procedure is similar for three dimensions.

REFERENCES

1. M. L. Wilkins, in "Methods in Computational Physics" (B. Alder, S. Fernbach, and M. Rotenberg, Eds.), Vol. 3, pp. 211-263, Academic Press, New York, 1964; also Lawrence Livermore Laboratory Report UCRL-7322 Rev. 1, 1969.
2. J. von Neumann and R. D. Richtmyer, J. Appl. Phys. 21, 232 (1950).
3. M. L. Wilkins, "Use of Artificial Viscosity in Multidimensional Fluid Dynamic Calculations," J. Comp. Physics, Vol. 36, No. 3, July 15, 1988.
4. D. B. Tuff, C. S. Godfrey and M. L. Wilkins, UCRL-87678 preprint, November 3, 1982.

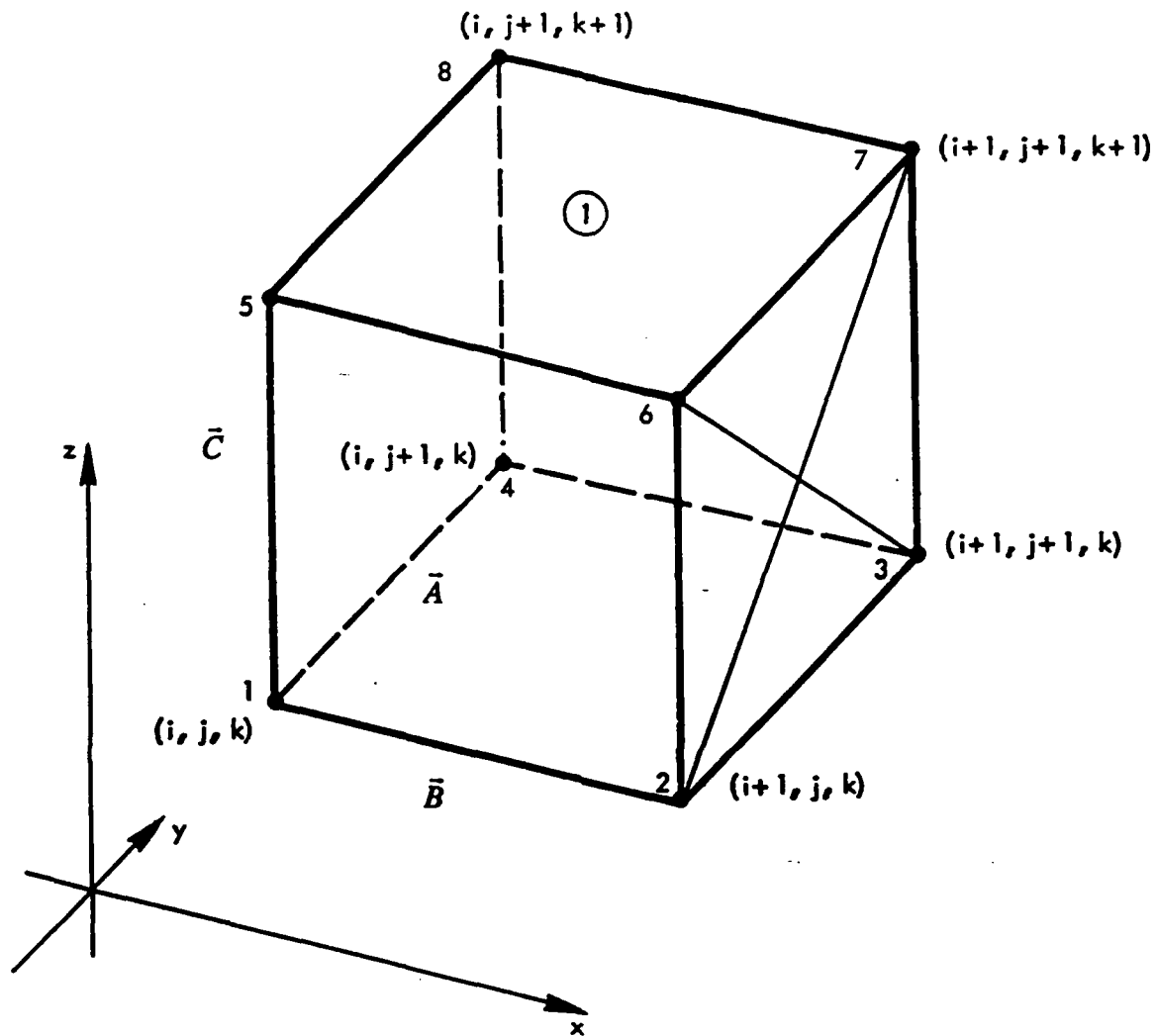


Fig. 1. Grid numbering scheme for Zone ①.

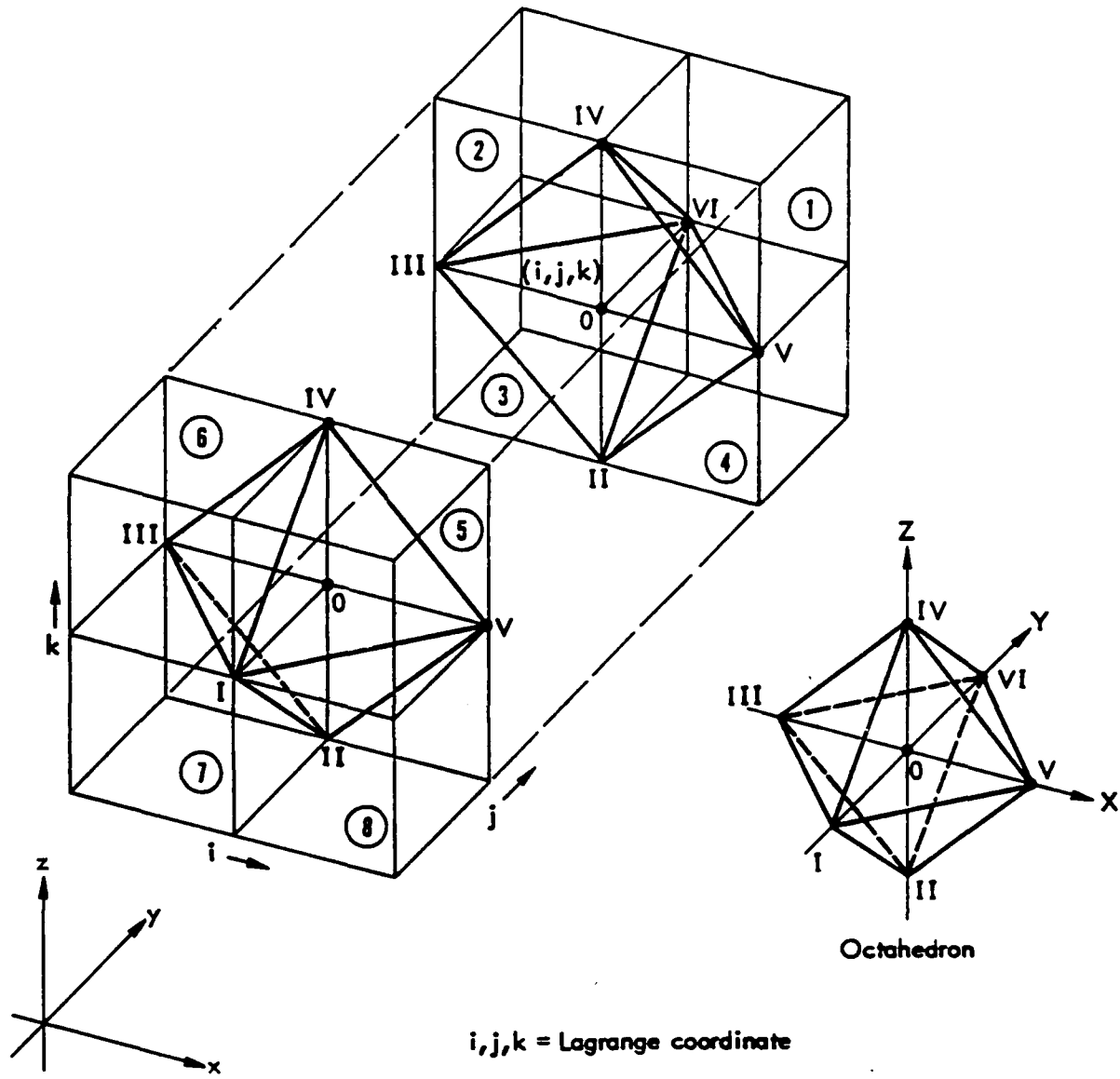


Fig. 2. Grid for accelerating point (i,j,k) .

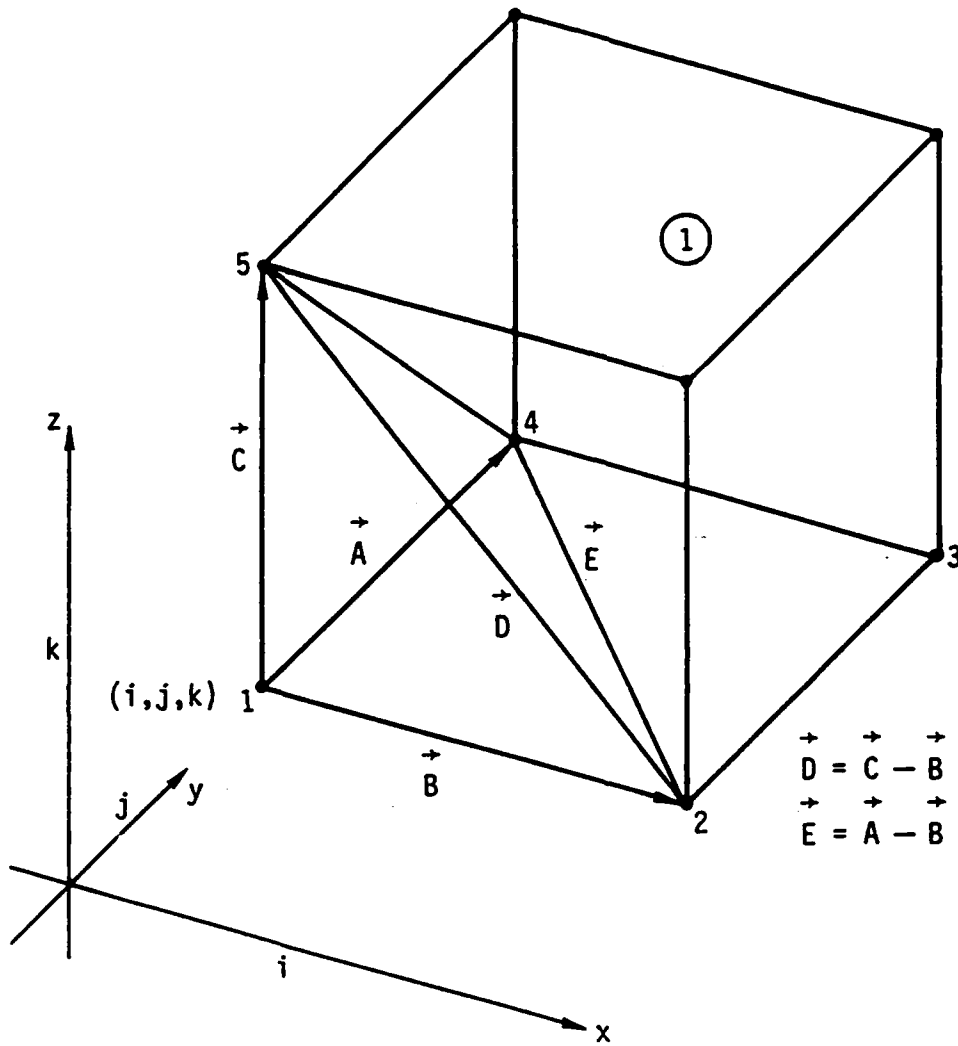


Fig. 3. Grid numbering scheme for calculating the tensor viscosity of the tetrahedron associated with zone ①.

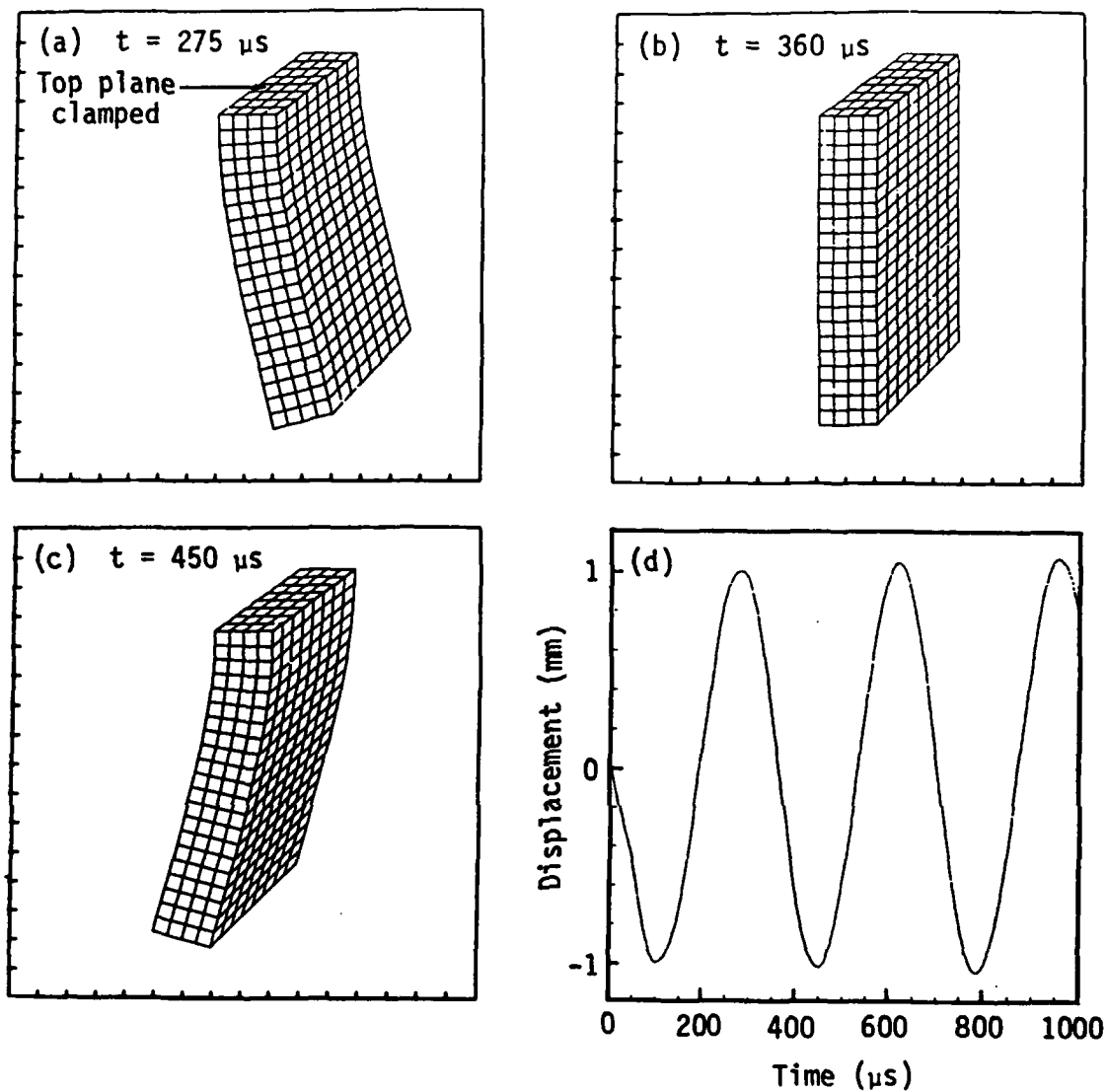


Fig. 4. Simulation of the motion of a vibrating elastic plate. (a) Position of maximum positive displacement, $t = 275 \mu s$. (b) Position of maximum kinetic energy, $t = 360 \mu s$. (c) Position of maximum negative displacement, $t = 450 \mu s$. (d) Displacement history for a point in the geometric center of the bottom plane.

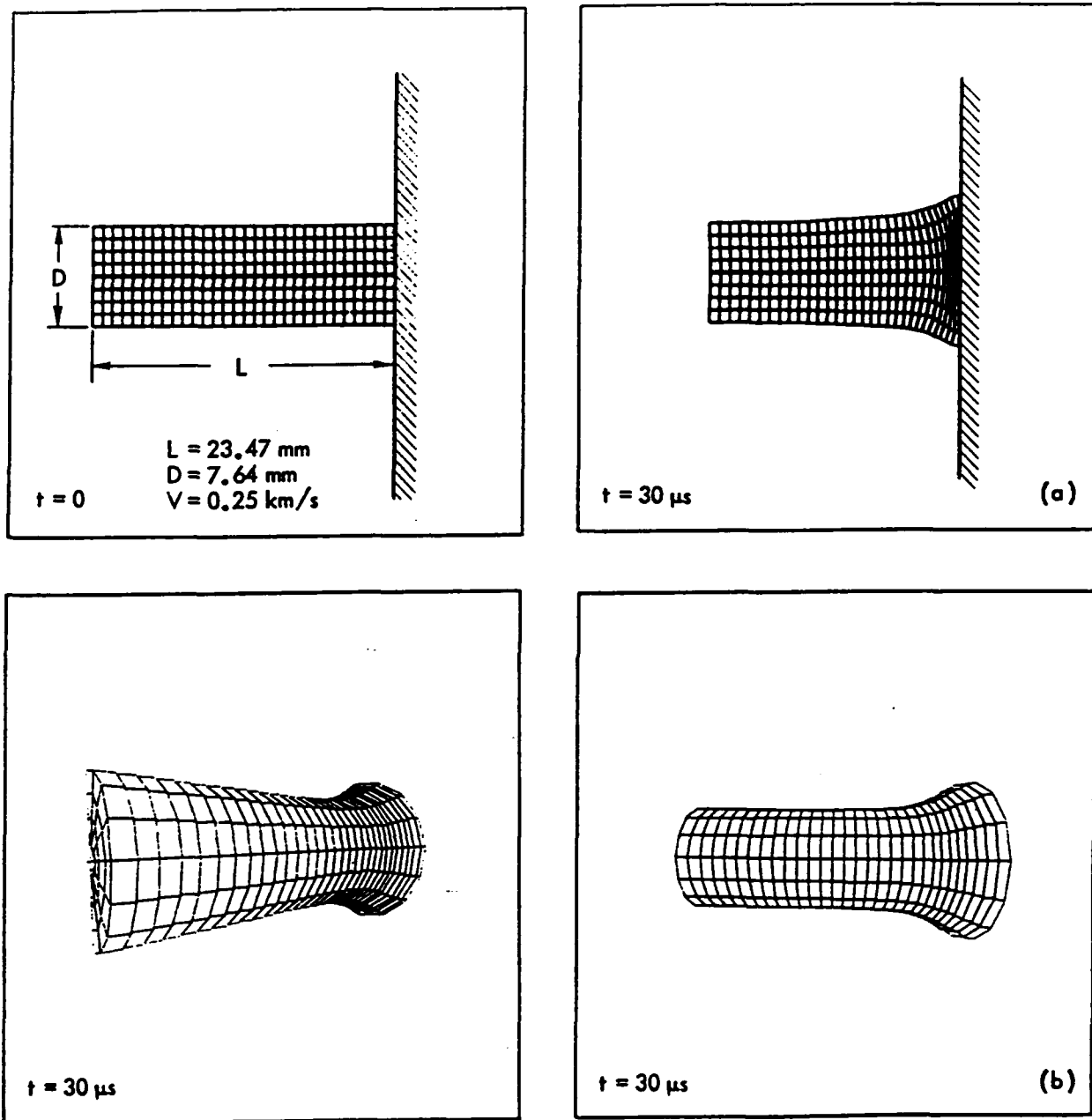


Fig. 5. Simulation of the impact of a cylinder on a rigid wall. Constitutive model:

Pressure	$P = 76 \left(\frac{\rho}{\rho^0} - 1 \right) \text{ GPa}$
Density	$\rho^0 = 2.7 \text{ Mg/m}^3$
Shear modulus	$\mu = 24.8 \text{ GPa}$
Flow stress	$Y = 0.46 (0.008 + \epsilon^p)^{0.1} \text{ GPa}$

ϵ^p is the equivalent plastic strain.

(a) Before and after views using the two-dimensional HEMP program.

(b) Two views using the HEMP 3D program.

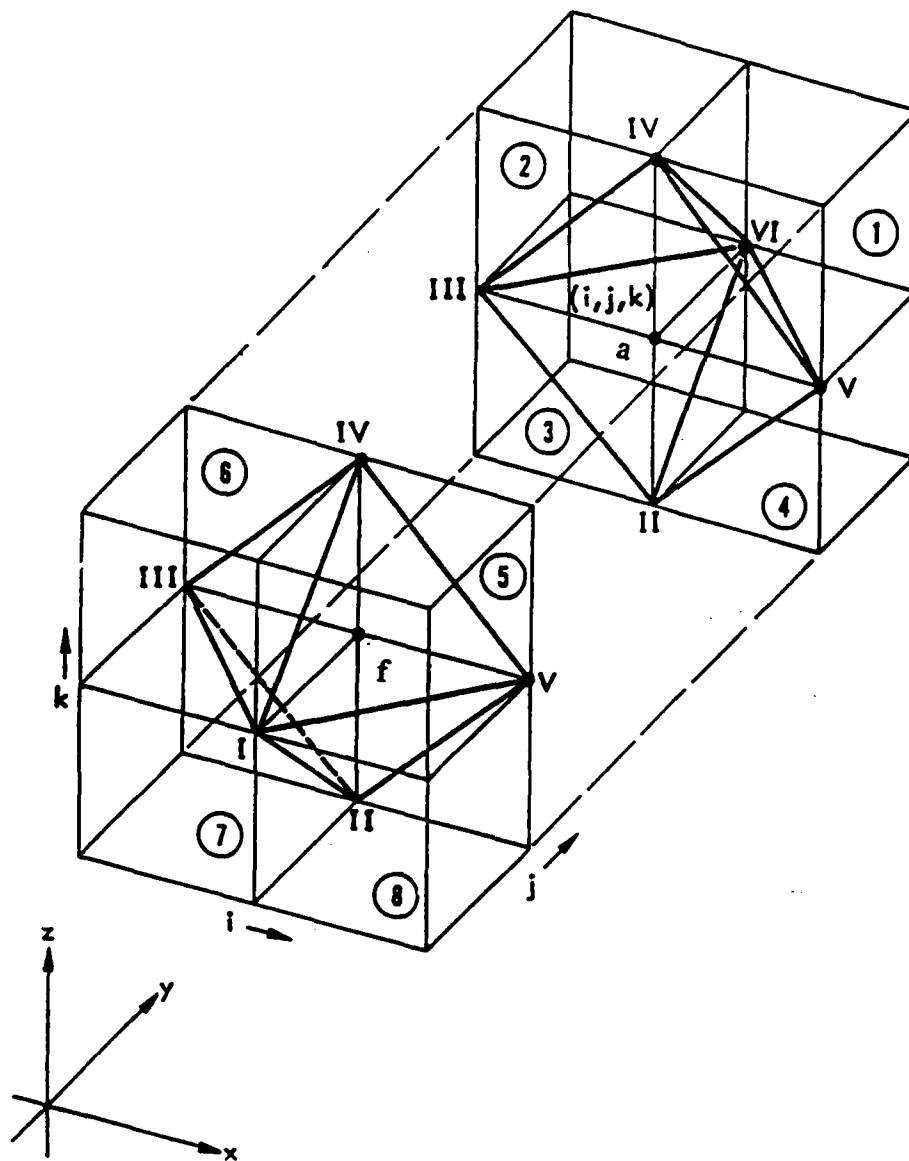


Fig. 6. Computational grid separated at a constant Lagrange coordinate j .
View of a current grid point f on the interface.

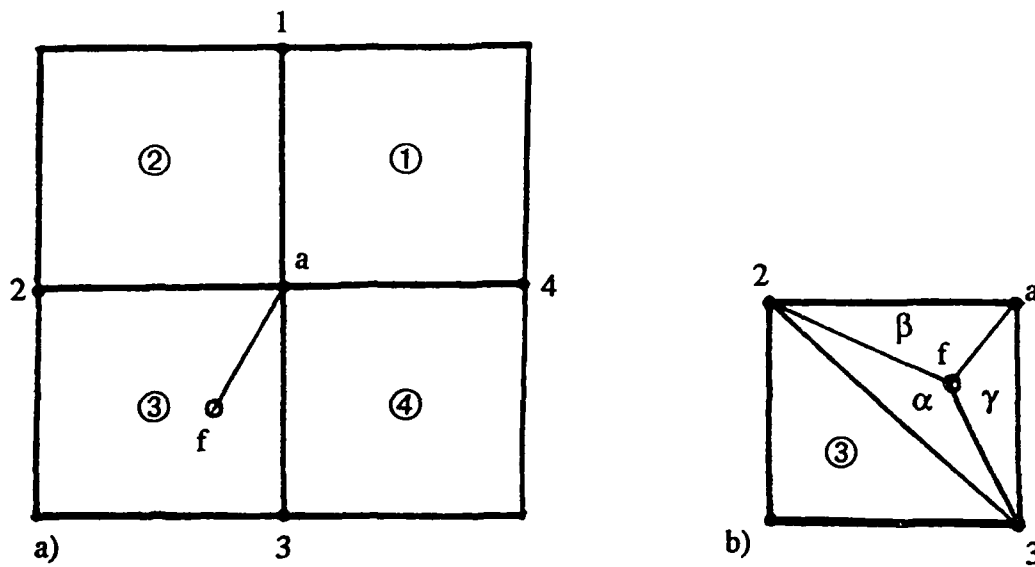


Fig. 7. Schematic grid to determine which of the four opposite grid zones covers a current point f .

- (a) Quadrants surrounding opposite grid point a with current grid point f in quadrant 3.
- (b) System of triangle shown for quadrant 3.

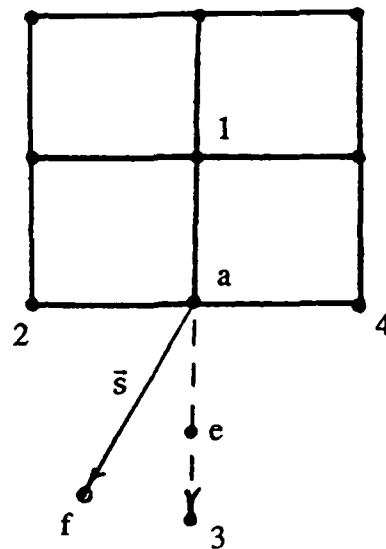


Fig. 8. Schematic grid to determine if a current point f is outside the opposite grid.

A diagram of a tetrahedron with vertices labeled a , b , and c . A point f is located inside the tetrahedron. Solid lines connect f to each vertex (af , bf , cf). Dashed lines connect the vertices to each other (ab , bc , ca). The angles at point f are labeled: α between bf and cf , β between af and cf , and γ between af and bf .

$$m_f = \frac{m_a \alpha + m_b \beta + m_c \gamma}{\alpha + \beta + \gamma}$$

Fig. 10. Weighting scheme for obtaining the value of a parameter defined at points a, b, c at position f.

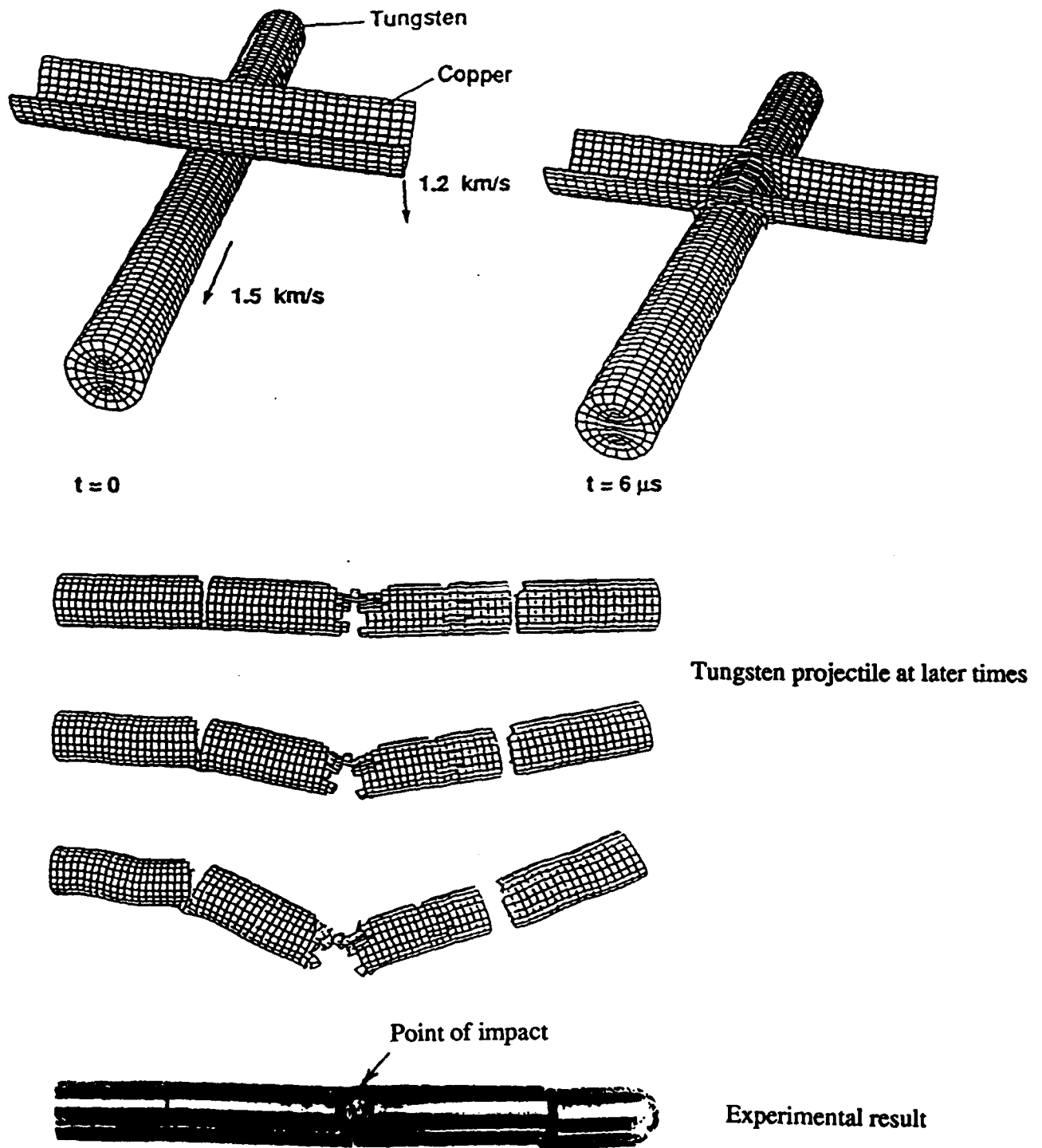


Fig. 12. Simulation of a copper plate charge striking a tungsten projectile.

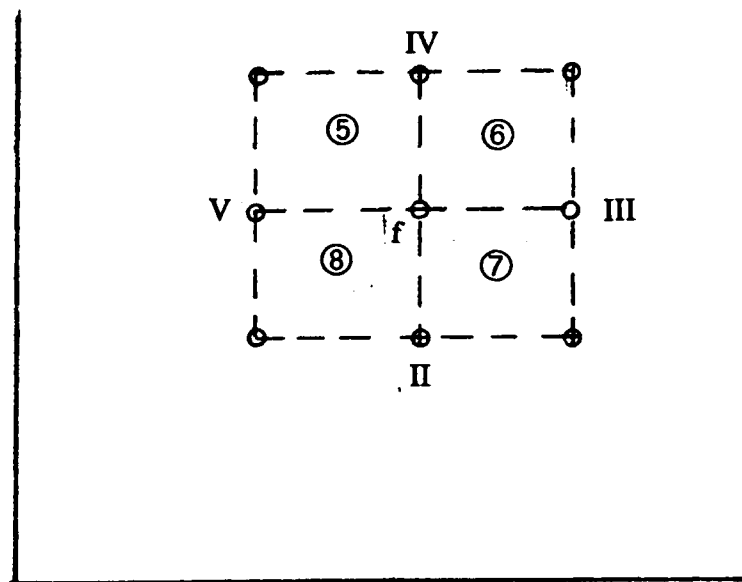


Fig. 11. Grid associated with current point f.

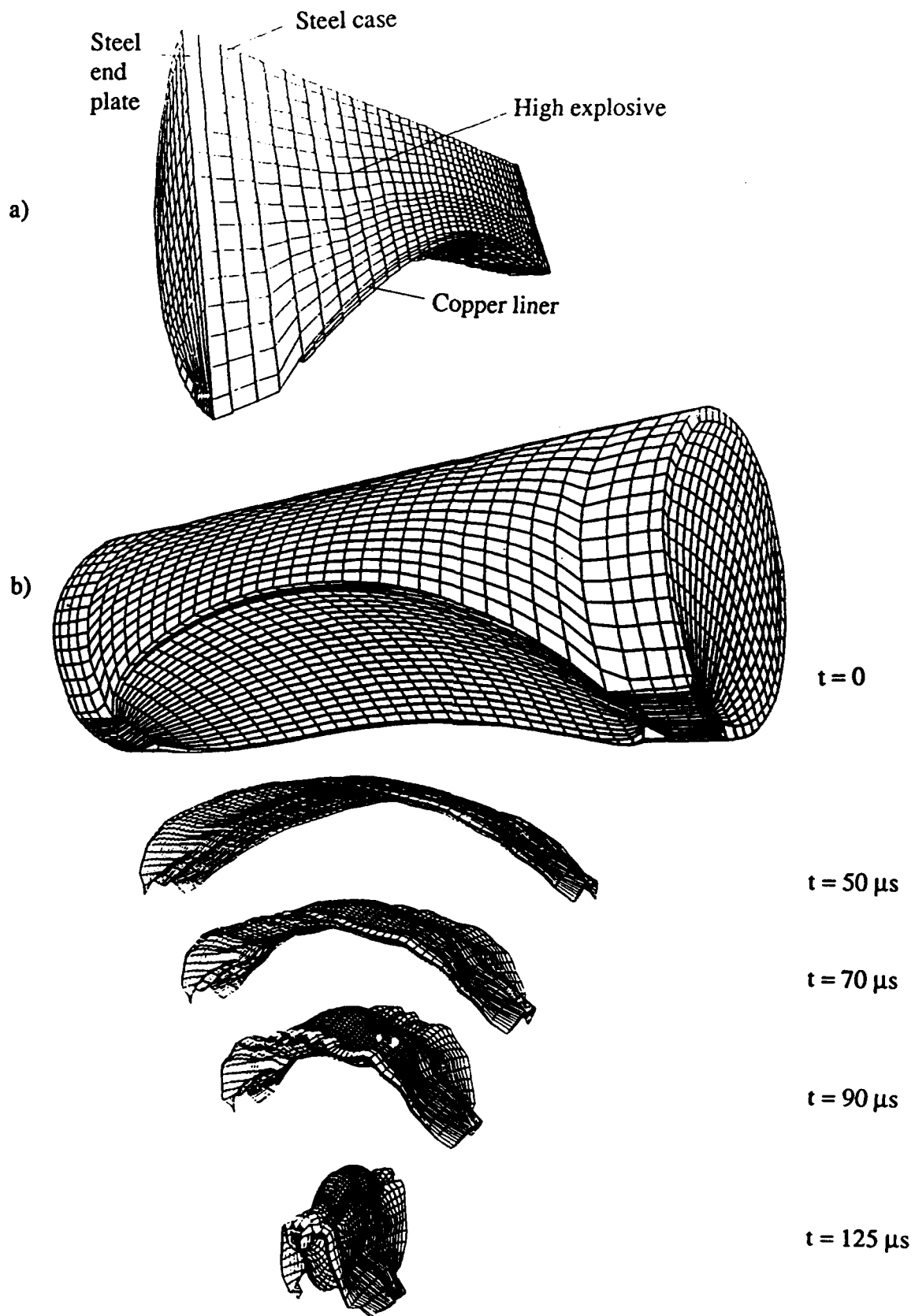


Fig. 13. Calculation of a three-dimensional implosion of a copper liner.

- (a) Section view of geometry.
(b) Time sequence of implosion.

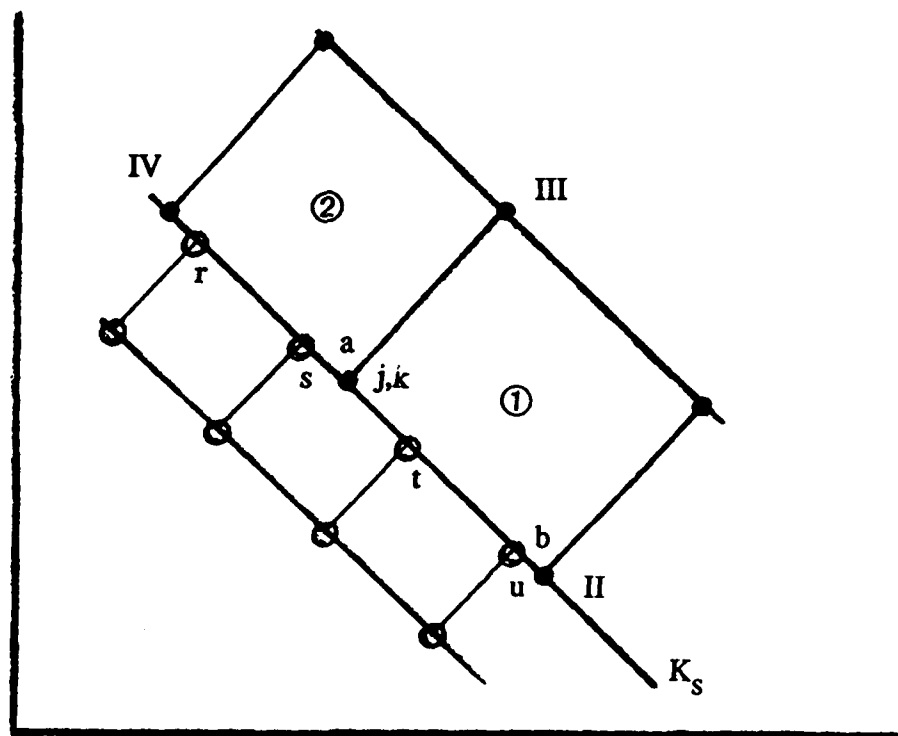


Fig. 14. Schematic of a zone dimension change at k-line k_s .

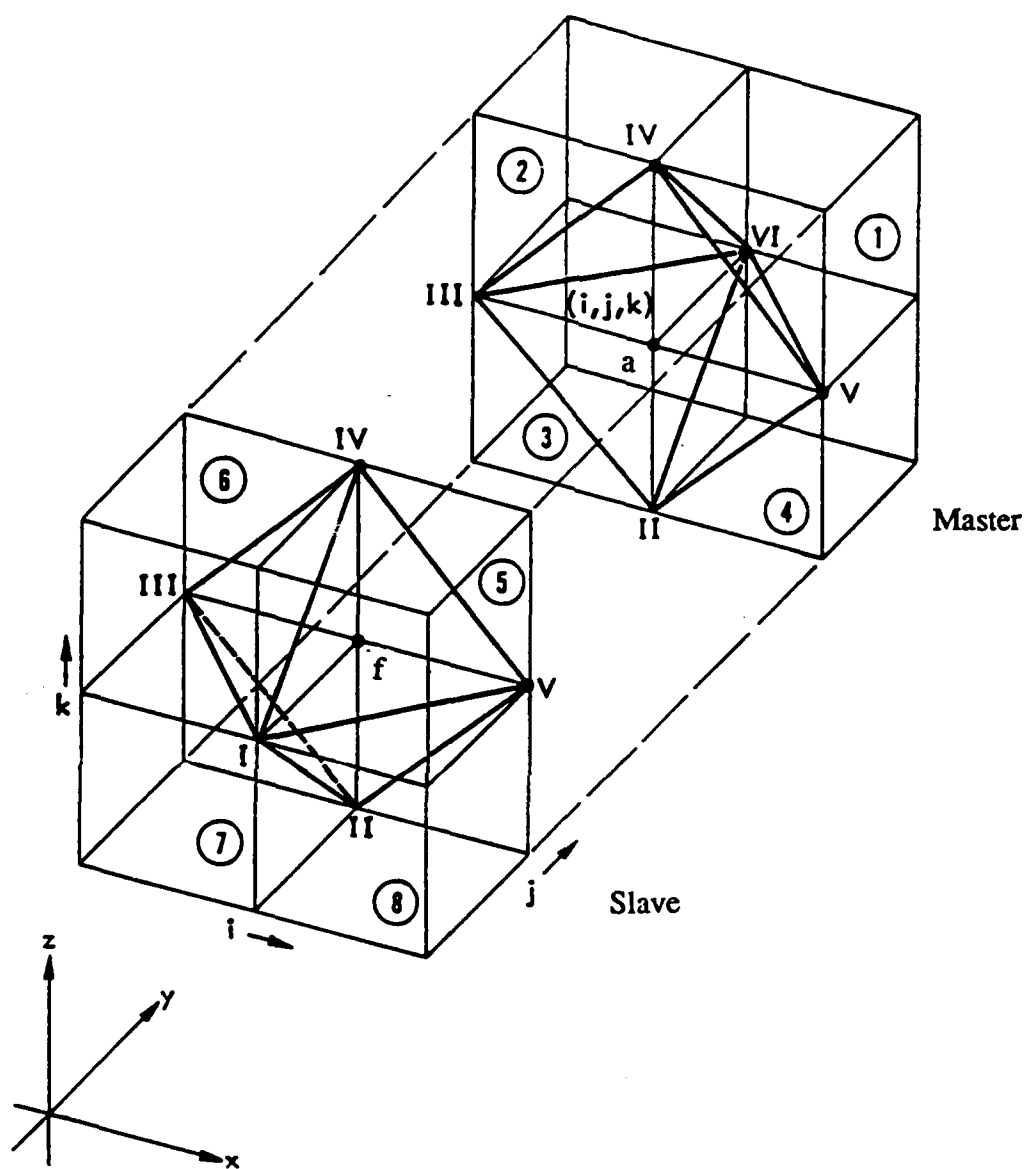


Fig. 15. Joining of two independent grids.

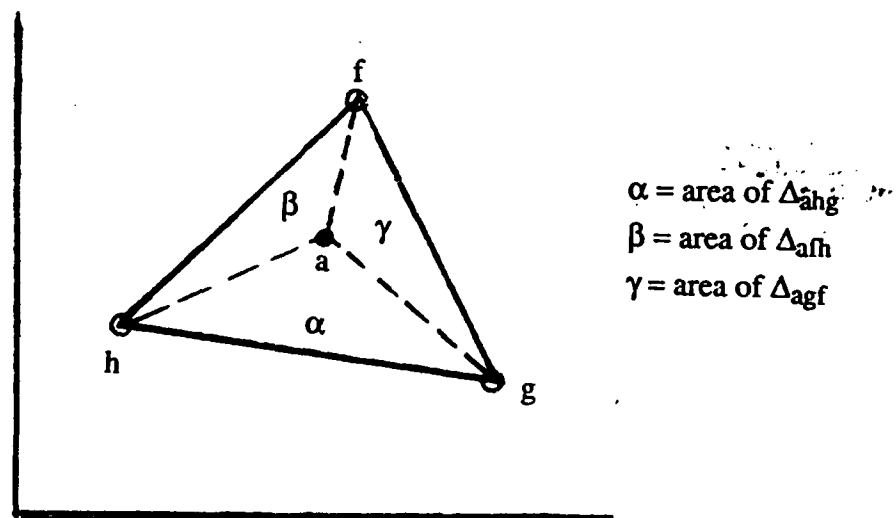


Fig. 16. Interpolation scheme for obtaining at position a information defined at positions f, g, h .