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DYNAMIC STRAINS IN AN ORTHOTROPICALLY-WRAPPED GUN TUBE PART I - THEORETICAL

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The axial symmetric equations of motion for a cylinder wrapped with an orthotropic jacket are used to compute the effect of jacket properties on the lowest axial symmetric critical velocity. It is shown that an orthotropic jacket, often used to strengthen a cylinder, may actually result in a critical velocity lower than that of the cylinder without the jacket unless the jacket stiffness to mass properties are sufficiently high.

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INTRODUCTION AND BACKGROUND

In previous reports [1,2,3] the theoretical prediction and experimental verification of strain amplification due to critical velocity effects in gun tubes have been given. In particular, equation (19a) of reference [1] shows the importance of the shell bending stiffness, D, and the shell membrane stiffness, M, in determining the projectile velocity that causes resonant flexural vibrations *in a thin-walled* gun tube. i.e.,

$$V_{cr}^2 = \frac{2\sqrt{D}\sqrt{M}}{h\rho} \tag{1}$$

where

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

and

$$M = \frac{Eh}{R^2}$$

E and ν are Young's modulus and Poisson's ratio, respectively and R is the mean radius of a tube having wall thickness h and mass density ρ.

Increasing either D or M thus increases the critical velocity of a thin-walled gun tube. Theories that account for substantial wall thickness and/or bore eccentricity predict critical velocities somewhat lower than that predicted by equation (1) above. The essential result, however, is that projectiles fired at velocities sufficiently less than the critical value ,V_{cr}, do not significantly amplify stresses within the tube wall.

Conceptually, at least, a lightweight material wrapped around and bonded to the tube to form a jacket is one way of increasing the overall stiffness and hence the critical velocity of a given gun tube. In this report, the development of the necessary theory to predict the critical velocity of an orthotropically-wrapped gun tube is given and applied to a particular composite material.

THEORY

A uniform, isotropic cylinder (a gun tube) is assumed to be infinite in length and is wrapped with an orthotropic layer. A cross section of this composite cylinder is shown in figure 1. Donnell shell theory [7] forms the basis for the theoretical development. In the following, all displacements and strains refer to the exterior surface of the jacket. From reference [4] the relevant equations for the free motion of the wrapped cylinder when written in terms of these displacements are:

$$L_{11}u + L_{12}v + L_{13}w - \rho_T \ddot{u} = 0$$

$$L_{12}u + L_{22}v + L_{23}w - \rho_T \ddot{v} = 0 \tag{2}$$

$$L_{13}u + L_{23}v + L_{33}w + \rho_T \ddot{w} = 0$$

- where u,v, and w are the axial, tangential and radial displacements, respectively, of the exterior surface of the jacket. For axisymmetric motions, these displacements are functions of the axial coordinate x and the time t. ρ_T is the density parameter:

$$\rho_T = \rho^{(1)} h_1 + \rho^{(2)} (h_2 - h_1)$$

In keeping with the notation of reference [4], the superscript (1) refers to jacket properties and the superscript (2) to properties of the isotropic cylinder. Note, however, that while h_1 refers to the jacket thickness, h_2 is the *combined* thickness of the tube and jacket. (cf. fig 1)

The L_{ij} operators are defined as follows:

$$\begin{aligned} L_{11}(\ ) &= A_{11}(\ )_{,xx} \\ L_{12}(\ ) &= 0 \\ L_{13}(\ ) &= -D^{*}_{11}(\ )_{,xxx} - (A_{12}/a)(\ )_{,x} \\ L_{22}(\ ) &= A_{66}(\ )_{,xx} \\ L_{23}(\ ) &= 0 \\ L_{33}(\ ) &= D_{11}(\ )_{,xxxx} + (2/a)D^{*}_{12}(\ )_{,xx} + (A_{22}/a^2)(\ ) \end{aligned} \tag{3}$$

where

$$\begin{aligned} A_{11} &= C^{(1)}_{11} h_1 + C^{(2)}_{11} (-h_1 + h_2) \\ A_{12} &= C^{(1)}_{12} h_1 + C^{(2)}_{12} (-h_1 + h_2) \\ A_{22} &= C^{(1)}_{22} h_1 + C^{(2)}_{22} (-h_1 + h_2) \\ D^{*}_{11} &= \frac{1}{2}\{C^{(1)}_{11} h_1^2 + C^{(2)}_{11} (-h_1^2 + h_2^2)\} \\ D^{*}_{12} &= \frac{1}{2}\{C^{(1)}_{12} h_1^2 + C^{(2)}_{12} (-h_1^2 + h_2^2)\} \\ D^{*}_{22} &= \frac{1}{2}\{C^{(1)}_{22} h_1^2 + C^{(2)}_{22} (-h_1^2 + h_2^2)\} \\ D_{11} &= \frac{1}{3}\{C^{(1)}_{11} h_1^3 + C^{(2)}_{11} (-h_1^3 + h_2^3)\} \\ D_{12} &= \frac{1}{3}\{C^{(1)}_{12} h_1^3 + C^{(2)}_{12} (-h_1^3 + h_2^3)\} \\ D_{22} &= \frac{1}{3}\{C^{(1)}_{22} h_1^3 + C^{(2)}_{22} (-h_1^3 + h_2^3)\} \\ A_{66} &= C^{(1)}_{66} h_1 + C^{(2)}_{66} (-h_1 + h_2) \\ D^{*}_{66} &= \frac{1}{2}\{C^{(1)}_{66} h_1^2 + C^{(2)}_{66} (-h_1^2 + h_2^2)\} \end{aligned} \tag{4}$$

$$D_{66} = \frac{1}{3}\{C_{66}^{(1)}h_1^3 + C_{66}^{(2)}(-h_1^3 + h_2^3)\}$$

and, for j = 1,2 :

$$C_{11}^{(j)} = \frac{E_x^{(j)}}{\{1 - \nu_{x\theta}^{(j)}\nu_{\theta x}^{(j)}\}}$$
$$C_{22}^{(j)} = \frac{E_\theta^{(j)}}{\{1 - \nu_{x\theta}^{(j)}\nu_{\theta x}^{(j)}\}} \tag{5}$$
$$C_{12}^{(j)} = \frac{E_\theta^{(j)}\nu_{x\theta}}{\{1 - \nu_{x\theta}^{(j)}\nu_{\theta x}^{(j)}\}}$$
$$C_{66}^{(j)} = G_{x\theta}^{(j)}$$

—where for example, $E_x^{(1)}$ and $\nu_{x\theta}^{(1)}$ are Young's modulus and Poisson's ratio, respectively, of the orthotropic jacket in the directions indicated. (In $\nu_{x\theta}^{(1)}$ for example, the stress is applied in the x-direction and the lateral deformation takes place in the θ-direction.) $G_{x\theta}^{(j)}$ is the torsional shear modulus. The same definitions apply to the isotropic cylinder when the superscript is (2), but these of course reduce to just two independent elastic constants because of the isotropy. The axisymmetric displacements of interest have the general form of travelling waves:

$$u = Ue^{i\alpha(x-vt)}$$
$$v = Ve^{i\alpha(x-vt)} \tag{6}$$
$$w = We^{i\alpha(x-vt)}$$

— where v is the phase velocity of the wave and α is the wave number.

These displacements, when substituted into equations (2), give the following three homogeneous equations for the determination of the coefficients U,V, and W:

$$\frac{iA_{12}W\alpha}{a} - A_{11}U\alpha^2 + -iD_{11}^{*}W\alpha^3 + U\alpha^2\rho_T v^2 = 0$$
$$-(A_{66}V\alpha^2) + V\alpha^2\rho_T v^2 = 0 \tag{7}$$
$$\frac{A_{22}W}{a^2} + \frac{iA_{12}U\alpha}{a} - \frac{2D_{12}^{*}W\alpha^2}{a} + -iD_{11}^{*}U\alpha^3 + D_{11}W\alpha^4 - W\alpha^2\rho_T v^2 = 0$$

The second of equations (7) is uncoupled from the others and gives a value for v independent of α. This is the dispersion relation for torsional waves showing that torsional waves are uncoupled from the axial and radial motions and are nondispersive. These waves are of no interest to this study, and the second of equations (7) is therefore eliminated from the system leaving two homogeneous equations in the unknowns U and W. In order for these equations to yield non-zero solutions, the determinant of the coefficients of U and W must vanish. This gives the dispersion relation for the axisymmetric flexural waves assumed in equation (6).

This dispersion relation is:

$$\frac{A_{12}{}^2}{a^2} - \frac{A_{11}A_{22}}{a^2} - \frac{2A_{12}D_{11}^{*}\alpha^2}{a} + \frac{2A_{11}D_{12}^{*}\alpha^2}{a}$$
$$-A_{11}D_{11}\alpha^4 + D_{11}^{*}{}^2\alpha^4 + \frac{A_{22}v^2\rho_T}{a^2} \tag{8}$$
$$+A_{11}\alpha^2v^2\rho_T - \frac{2D_{12}^{*}\alpha^2v^2\rho_T}{a} + D_{11}\alpha^4v^2\rho_T - \alpha^2v^4\rho_T{}^2 = 0$$

Since the dispersion relation is biquadratic in the phase velocity, v, there are four solutions or branches $v_j(\alpha)$, $j = 1,4$. Of these, the branch v_1 - which contains the lowest possible phase velocity - is of primary interest. As shown in reference [1], waves having a phase velocity v are only excited in a gun tube when the projectile velocity is also v. Usually, the lowest branch of the dispersion curve includes all of the possible phase velocities of which a *conventional* gun tube is capable. Moreover, thin shell theories, such as the Donnell theory used herein, do not usually predict the higher branches accurately.

APPLICATION: 60-MM GUN TUBE WITH A GLASS/EPOXY JACKET

The main purpose of this experiment was to test the applicability of the forgoing theory rather than to make a serious attempt toward a superior design. In fact, as shown, the jacket thickness of the glass/epoxy material utilized theoretically does not have a sufficient strength-to-weight ratio to raise the critical velocity above that of the unjacketed tube. Its main attributes were availability and ease of construction as well as being representative of materials that have seen use in tube components and appendages.

The 60-mm gun tube referred to in previous studies [2] was wound with a continuous glass filament for a distance of 48 inches rearward of the muzzle. The filament windings were precoated with epoxy resin and wound at angles of $+60°$ and $-60°$ with respect to the tube axis, the sign of the angle alternating with each layer. Eighteen layers complete the jacket with a final measured wall thickness of 0.151 inch. The jacketed portion of the steel gun tube has a uniform wall thickness of 0.12 inch. Prior to winding, strain gages were bonded to the exterior surface of the gun tube at several locations. After the jacket was completed, additional gages were placed on the exterior jacket surface directly over those on the gun tube, as shown in Figure 1. Figure 2 is a photograph of the jacketed gun tube.

Knowledge of the material properties of the gun tube and the jacket are needed to evaluate the coefficients in the dispersion relation, Eq.(8). For the steel gun tube:

$E^{(2)} = 3.03 * 10^7$psi $\qquad \nu^{(2)} = 0.3 \qquad \rho^{(2)} = 0.0007365$ lb-sec^2/in.

By substituting these values in Equation (5), the $C_{ij}^{(2)}$ can be determined:

i.e., $\qquad C_{11}^{(2)} = C_{22}^{(2)} = 33.3 * 10^6$psi $\qquad$ and $\qquad C_{12}^{(2)} = 9.99 * 10^6$psi

(Note that $C_{66}^{(j)}$ is only needed for the torsional equation of motion which was eliminated previously.)

The $C_{ij}^{(1)}$ for the jacket can be determined theoretically from the material properties of the filament and matrix materials or by direct measurement of a sample of the composite jacket material. Both procedures were used in this study. A computer program [5] was used to theoretically predict the elastic moduli and Poisson's ratios of the orthotropic jacket. The resulting $C_{ij}^{(1)}$ were computed using equations (5) and compared with those obtained from test measurements. The Appendix describes how the $C_{ij}^{(1)}$ were established using these test measurements. For unknown reasons, the agreement between the predicted elastic properties of the jacket and those resulting from these direct test measurements was poor. Some of the theoretically predicted elastic moduli were nearly twice those deduced from the test measurements.

Considering the substantial difference between the measured elastic properties and those calculated using the computer program, one can expect a considerable difference in the corresponding dispersion relations. The *measured* $C_{ij}^{(1)}$ are as follows:

$C_{11}^{(1)} = 1.60 * 10^6$psi $\quad$ $C_{22}^{(1)} = 3.39 * 10^6$psi $\quad$ $C_{12}^{(1)} = 0.987 * 10^6$psi

$\rho^{(1)} = 0.0001972$ lb-sec^2/in. was obtained from the measured dimensions and weight of a jacket sample.

From figure 1 we have that:

$a =$(bore radius)$+h_2 = 1.1811 + (0.151 + 0.12) = 1.4521$ in.

Substituting all numerical values into equation (8) results in the following biquadratic equation for $v(\alpha)$:

$$-8.19447 * 10^{12} - 5.71641 * 10^{10}\alpha^2 - 3.99518 * 10^{10}\alpha^4 + 252.551v^2 + 457.619\alpha^2v^2 + 21.8019\alpha^4v^2 - 1.39611 * 10^{-8}\alpha^2v^4 = 0 \tag{9}$$

Of the four roots to this equation, the lowest branch is: $v_1(\alpha) =$

$$\left[1.63891 + \frac{0.90448}{\alpha^2} + 0.0780809\alpha^2 - \sqrt{2.78631 + \frac{0.818084}{\alpha^4} - \frac{2.90477}{\alpha^2} + 0.227318\alpha^2 + 0.00609662\alpha^4}\right]^{.5} \tag{10}$$

The *predicted* set of elastic constants, computed by K. Miner of Benet Laboratories using the computer program previously mentioned [5], has the following values:

$E_x^{(1)} = 2.87 * 10^6$ $\quad$ $E_\theta^{(1)} = 5.23 * 10^6$ $\quad$ $\nu_{x\theta}^{(1)} = 0.24$ $\quad$ $\nu_{\theta x}^{(1)} = 0.437$

Substituting these values in equations (5):

$C_{11}^{(1)} = 3.21 * 10^6$ psi $\quad$ $C_{22}^{(1)} = 5.84 * 10^6$ psi $\quad$ $C_{12}^{(1)} = 1.40 * 10^6$psi

The jacket wall thickness, h_1, as computed by the program, is 0.144 in. The density of the jacket is not computed by the program, however, and the measured value was therefore used (0.0001972 lb-sec^2/in.). The corresponding dispersion equation (equation (8)) is:

$$-9.38486*10^{12} - 4.61712*10^{10}\alpha^2 - 5.70712*10^{10}\alpha^4 + 273.676v^2 + 484.3\alpha^2v^2 + 20.5909\alpha^4v^2 - 1.39611*10^{-8}\alpha^2v^4 = 0 \tag{11}$$

Plots of the dispersion relation $v_1(\alpha)$ corresponding to the measured and predicted values of the $C_{ij}^{(1)}$ appear in figure 3 along with that for the tube without a jacket. The minimum phase velocity of each curve defines a critical velocity. The dispersion curve obtained using the measured elastic properties shows a critical velocity of 37260 in./sec (3105 fps), whereas the curve obtained using the predicted values shows a critical velocity of 43258 in./sec (3605 fps). The tube without any jacket has a critical velocity of 46618 in./sec (3885 fps). It is notable that both of the critical velocities corresponding to the jacketed tubes are *lower* than that for the bare tube. Evidently, the additional mass of the jacket has a stronger influence than the additional stiffness. As suggested from the thin-walled case (equation (1)), increased mass (the ρh divisor) lowers the critical velocity.

THE MORE GENERAL SITUATION —

As the example application of the previous section has demonstrated, adding a glass/epoxy jacket to a gun tube does not necessarily raise its critical velocity, since the effect of the increased mass may outweigh the effect of increased stiffness. (This is not the case for the tube without a jacket, whereby an increase in the tube wall thickness always results in an increase in critical velocity.) It is of interest to know if the critical velocity of the jacketed tube might have turned out to be greater than that of the bare tube had the jacket thickness been made sufficiently large. Figure 4 shows that if the *measured* values of the elastic constants were used in the analysis even a very substantial jacket thickness would not result in a critical velocity greater than that of the bare tube. On the other hand, if the *predicted* values of the elastic constants were used, a jacket thickness in excess of 0.254 inch would result in a critical velocity greater than that for the bare tube.

Usually it is desirable that the gun tube be designed so that its critical velocity is well above any anticipated projectile velocity while keeping the weight of the tube at a minimum - especially in those regions near the muzzle. The question arises as to whether these goals can be better achieved by simply increasing the wall thickness of the tube itself and eliminating the glass/epoxy jacket altogether. Figure 5 shows that this is indeed the case, at least for the 60-mm tube in question. The curves clearly show that adding a given amount of mass by increasing the wall thickness of the unjacketed tube results in a greater increase in critical velocity than by increasing the thickness of a glass/epoxy jacket. (Predicted values of the elastic constants were used in computing the effect of adding jacket mass - these are assumed to represent the highest stiffness possible for the glass/epoxy jacket as designed.) On the other hand, there may be instances when it is actually desirable to lower the critical velocity of a gun tube. This can arise when the projectile velocity exceeds the critical velocity of the tube despite all practical efforts to the contrary. As shown in references [1,2], a projectile velocity *sufficiently* greater than the critical value will excite dynamic strains. which at worst approach only twice those predicted by the Lamé formula.* Thus, dynamic tube strains might actually be lessened by *lowering* the critical velocity as much as possible. A thinner tube wall and a glass/epoxy (or similar) jacket to replace lost tube strength might be used to this end.

* Strictly speaking this applies only to *steady-state* strains.

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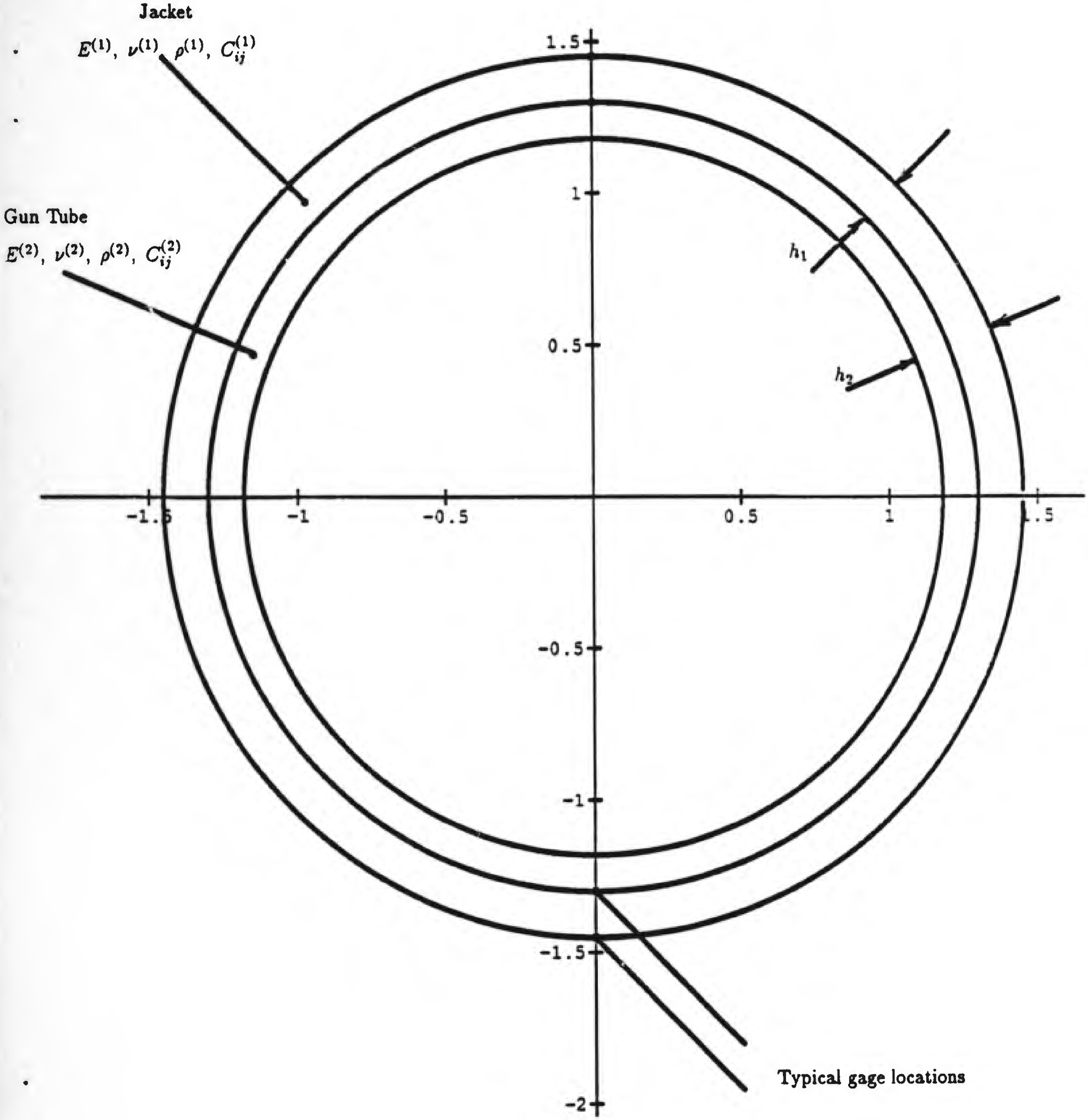

Figure 1 - Cross Section of Jacketed Gun Tube

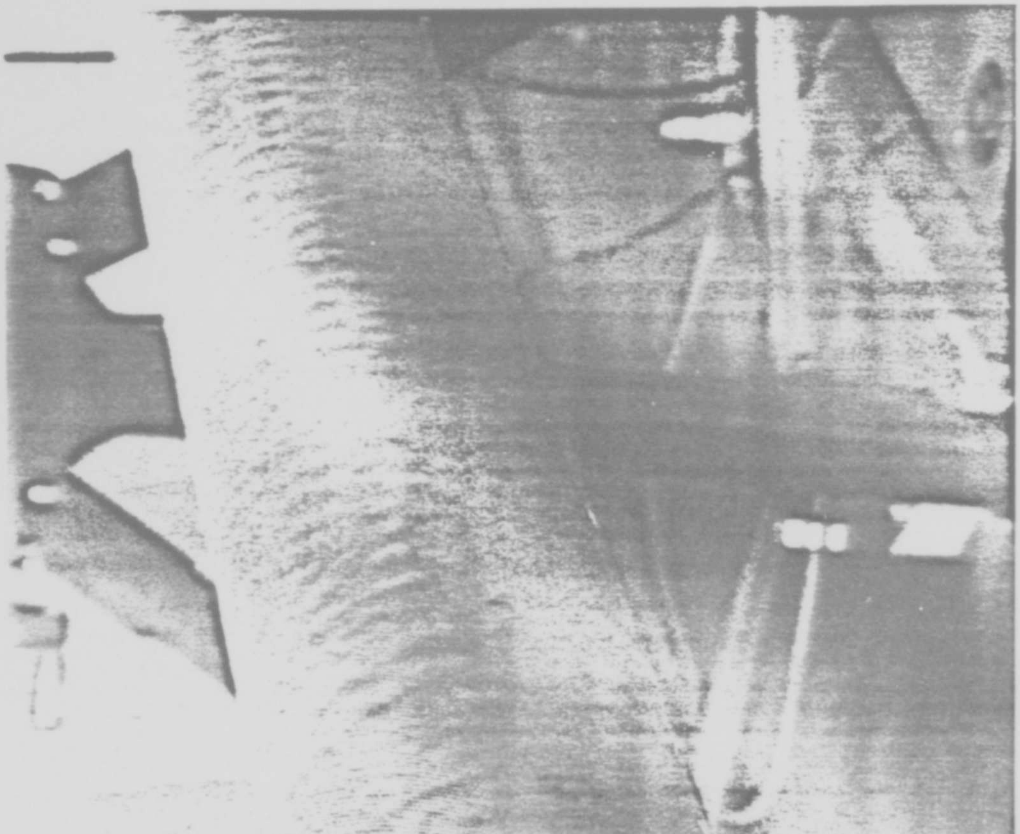

Figure 2 - The 60-mm Tube and Jacket

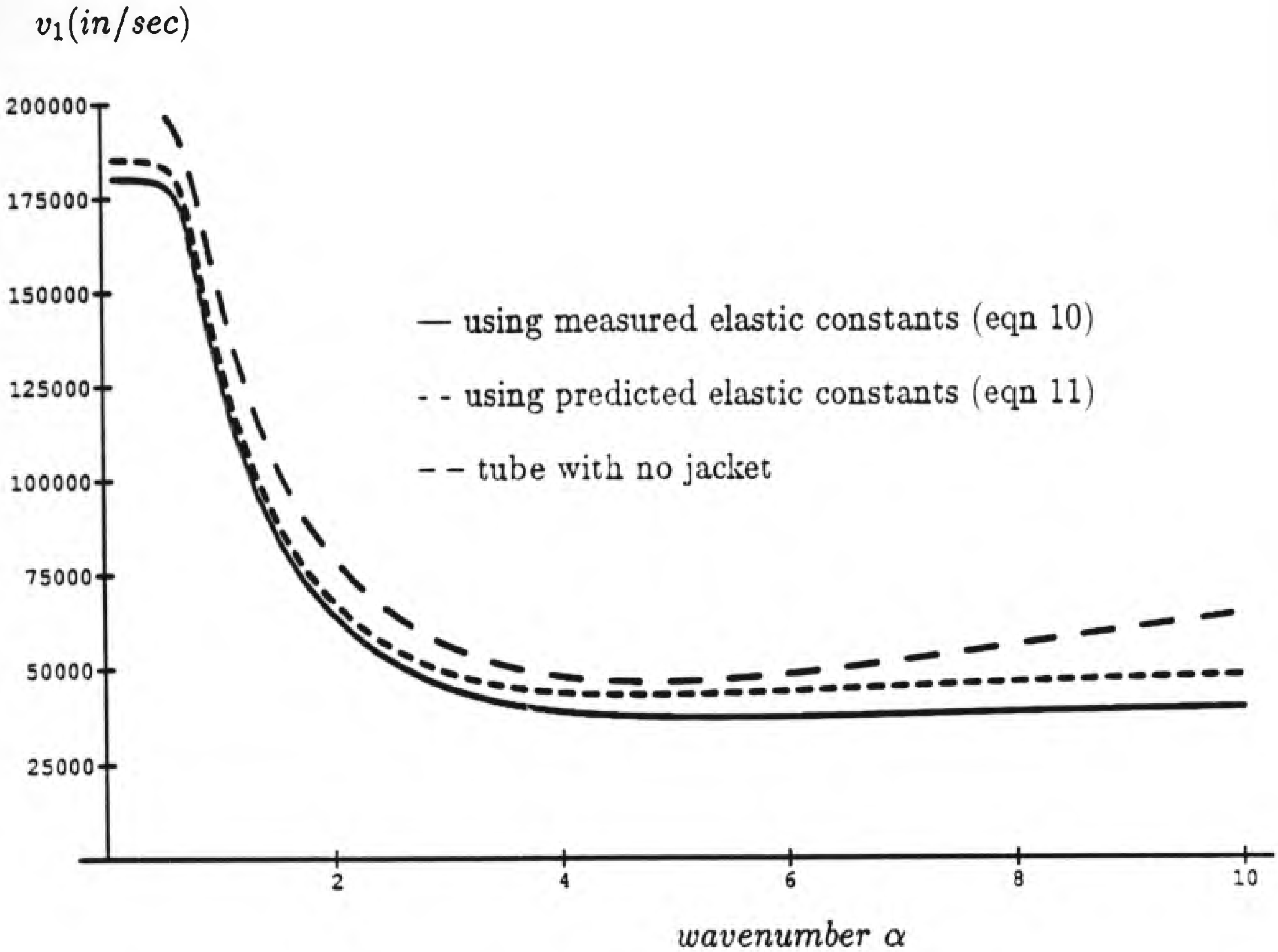

Figure 3 — Comparison of Relevant Dispersion Curves

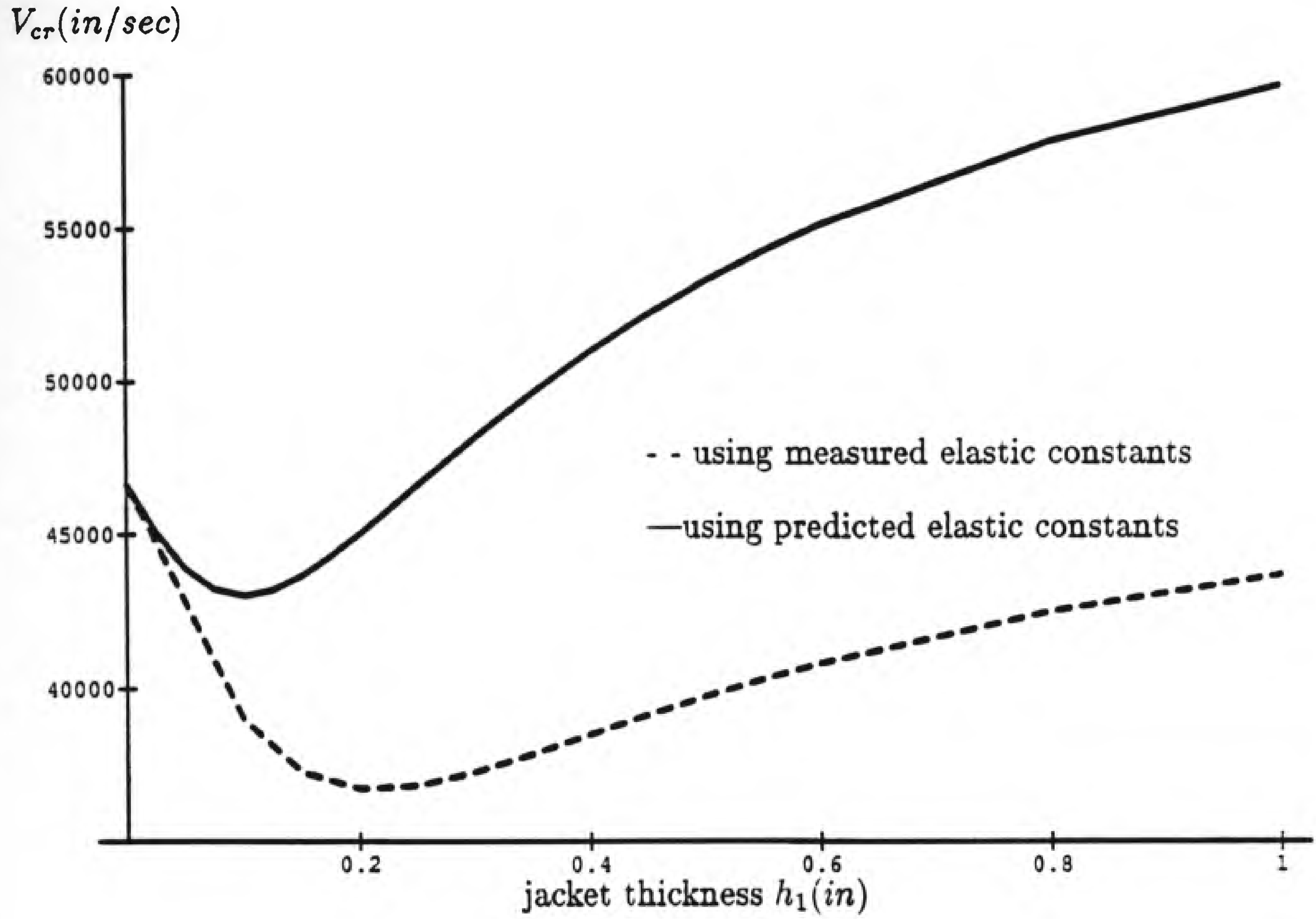

Figure 4 — Effect of Jacket Thickness on Critical Velocity

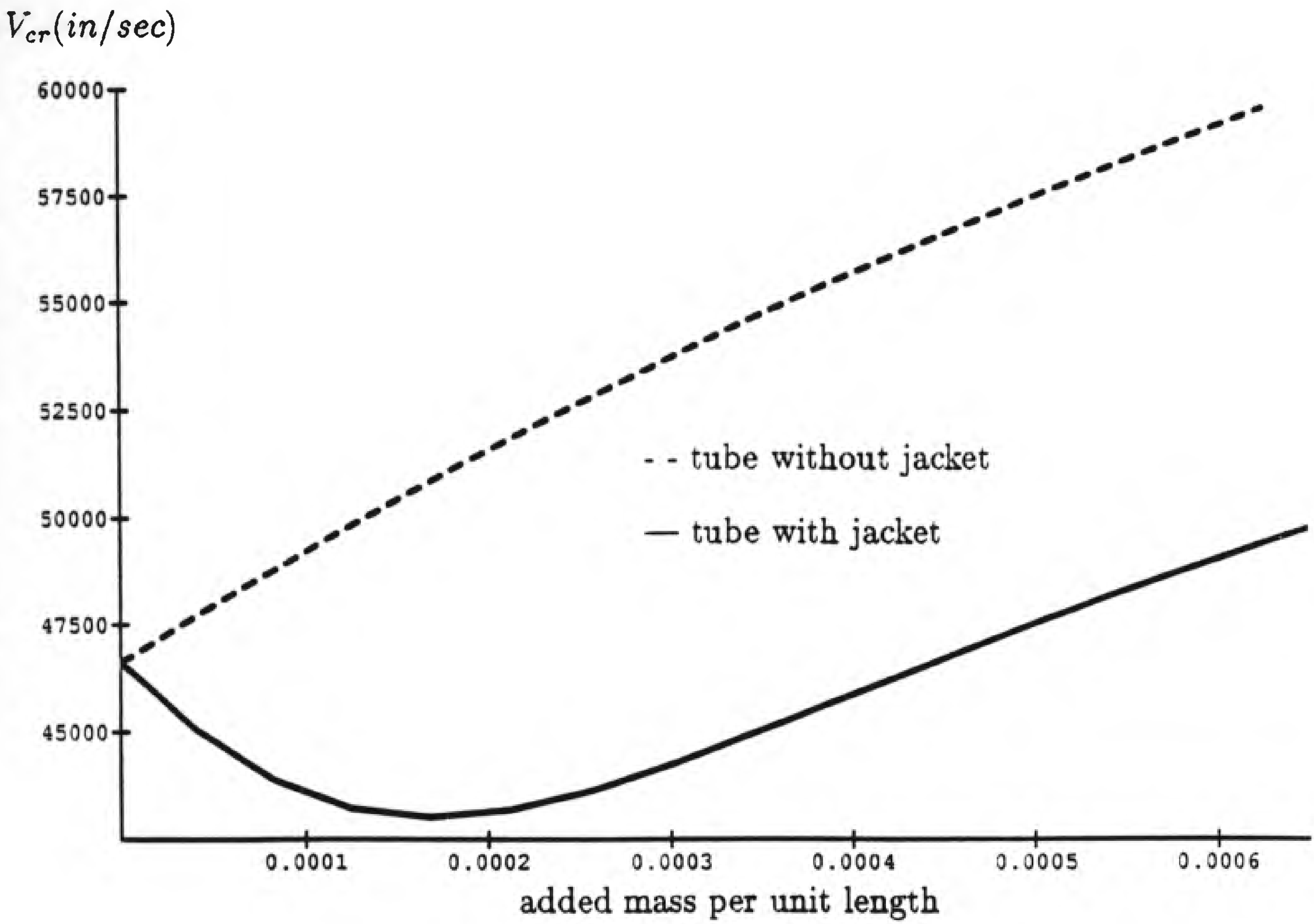

Figure 5 — Effects of Adding Mass to Tube or Jacket

Appendix

For an orthotropic cylindrical shell (the jacket), the stress-strain relations are:

$$\epsilon_{xx} = \frac{C_{22}\sigma_{xx}}{d} - \frac{C_{12}\sigma_{\theta\theta}}{d}$$
$$\epsilon_{\theta\theta} = \frac{-C_{12}\sigma_{xx}}{d} + \frac{C_{11}\sigma_{\theta\theta}}{d} \qquad (A1)$$

—where $d = C_{11}C_{22} - C_{12}^2$

Relations (A1) can be used with experimental measurements of the strains to compute the $C_{ij}^{(1)}$.

1. Uniaxial compression test — ϵ_{xx}^c and $\epsilon_{\theta\theta}^c$ are measured in response to the application of a uniform end load F. The superscript c indicates the strains measured during this compression test. Then $\sigma_{xx}^c = \frac{F}{A}$, where A is the cross-sectional area of the jacket. $\sigma_{\theta\theta}^c = \tau_{x\theta}^c = 0$. Equations (A1) then give two equations in the three unknowns C_{11}, C_{12} and C_{22}.

2. Hydraulic pressure test — $\epsilon_{\theta\theta}^p$ is measured upon application of an internal pressure p_i. In this test $\sigma_{xx}^p = 0$. For moderately thin orthotropic shells, work by G.P. O'Hara [6] shows that the Lame´ formula can be used with accuracy: —i.e.

$$\sigma_{\theta\theta}^p = \frac{2}{w^2 - 1}p_i \qquad (A2)$$

—where w is the wall ratio of the outer and inner radii of the jacket. Applying the second of equations (A1) then gives a third equation in the same unknowns. Thus the first two of equations (A1) applied to the compression test and the second of equations (A1) applied to the pressure test yield the following expressions for the $C_{ij}^{(1)}$:

$$C_{11}^{(1)} = \frac{\epsilon_{\theta\theta}^p {\sigma_{xx}^c}^2}{\epsilon_{\theta\theta}^p \epsilon_{xx}^c \sigma_{xx}^c - {\epsilon_{\theta\theta}^c}^2 \sigma_{\theta\theta}^p}$$
$$C_{12}^{(1)} = -\frac{\epsilon_{\theta\theta}^c \sigma_{xx}^c \sigma_{\theta\theta}^p}{\epsilon_{\theta\theta}^p \epsilon_{xx}^c \sigma_{xx}^c - {\epsilon_{\theta\theta}^c}^2 \sigma_{\theta\theta}^p} \qquad (A3)$$
$$C_{12}^{(1)} = -\frac{\epsilon_{xx}^c \sigma_{xx}^c \sigma_{\theta\theta}^p}{\epsilon_{\theta\theta}^p \epsilon_{xx}^c \sigma_{xx}^c - {\epsilon_{\theta\theta}^c}^2 \sigma_{\theta\theta}^p}$$

The data actually recorded during the compression tests were F vs ϵ_{xx}^c and F vs $\epsilon_{\theta\theta}^c$, whereas that recorded during the hydraulic pressure tests was p_i vs $\epsilon_{\theta\theta}^p$. i.e.,

$$F = m_x^c \epsilon_{xx}^c$$
$$F = m_\theta^c \epsilon_{\theta\theta}^c$$
$$p_i = m_\theta^p \epsilon_{\theta\theta}^p \qquad (A4)$$

—where, for example, m_x^c is the empirically derived slope of the average F vs ϵ_{xx}^c data.

In addition we have that:

$$\sigma_{xx}^c = \frac{F}{A} = m_x F = 0.7646F \qquad (A5)$$

and

$$\sigma_{\theta\theta}^p = \frac{2}{w^2 - 1}p_i = m_\theta p_i = 8.1565p_i \qquad (A6)$$

where m_x is simply the reciprocal of the measured cross-sectional area A of the jacket and, from equation (A2), $m_\theta = \frac{2}{w^2-1}$ where w is the measured wall ratio. Thus, from equations (A5) and (A6) we have the stress quantities in terms of the measured compressive force F and the measured hydraulic pressure p_i and (A4) gives F and p_i in terms of the measured strains.

Empirical values for the average slopes were established using a least squares fit and resulted in the following:

$$m_\theta^c = -5.87822 * (10^6)\frac{lb}{\frac{in}{in}}$$

$$m_x^c = 1.71309 * (10^6)\frac{lb}{\frac{in}{in}} \qquad (A7)$$

$$m_\theta^p = 0.340432 * (10^6)\frac{psi}{\frac{in}{in}}$$

Substituting equations (A5), (A6), and (A7) into (A3) gives the $C_{ij}^{(1)}$ in terms of the empirical slope quantities:

$$C_{11}^{(1)} = 1.59745 * (10^6)$$

$$C_{22}^{(1)} = 3.38646 * (10^6) \qquad (A8)$$

$$C_{12}^{(1)} = 0.986916 * (10^6)$$

From equation (5) one can also deduce that these correspond to:

$$E_x^{(1)} = 1.30983 * (10^6)$$

$$E_\theta^{(1)} = 2.776734 * (10^6) \qquad (A9)$$

$$\nu_{\theta x}^{(1)} = 0.61781$$

$$\nu_{x\theta}^{(1)} = 0.29143$$

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