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IMPLEMENTATION OF A NONISOTHERMAL UNIFIED INELASTIC-STRAIN THEORY INTO ADINA6.0 FOR A TITANIUM ALLOY - USER GUIDE

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13. ABSTRACT *(Maximum 200 words)*

Simulations with many modern aerospace materials require realistic mechanical response models for use in isothermal and nonisothermal applications. One such model is a unified inelastic-strain theory which has been applied to capture the strain-rate sensitivity and time dependent behavior of the titanium alloy Timetal 21S (formerly ß21S). The Bodner-Partom form of unified theory satisfactorily describes the ß21S stress-strain response for a range of temperatures from 23°C to 815°C and strain rate from 10^{-3}/s to $1.X10^{-7}$/s. Practical use of the theory in finite element applications depends on advanced numerical algorithms that rapidly solve the inherently "stiff" constitutive equations. Special "user-defined" subroutines provide the framework for the incorporation of these numerical algorithms into ADINA6.0, a general purpose finite element package. This user-guide contains a brief overview of the theory, the ß21S material parameters, subroutine structure, variable nomenclature, and numerical algorithms. Test cases are described to illustrate the numerical integration schemes and provide guidance for error management. The appendices contain hard-copies of the subroutines for two- and three-dimensional elements.

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Foreword

Since the writing of this document, the material referenced within as β21S has been trademarked as TIMETAL21S. The work documented in this report has been targeted toward the use of TIMETAL21S with the Bodner-Partom theories; however, much of the final product is applicable to any material response model implemented and verified in ADINA6.0. Similarly, the algorithms presented within are explicitly written and presented for TIMETAL21S; however, they can be easily adopted for any visco-elastic-plastic material with extensive strain-rate sensitivity or time dependent behavior.

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Section 1

Introduction

To adequately describe the strain-rate sensitivity and creep behavior of β21S during isothermal and nonisothermal load conditions requires a unified inelastic-strain theory. The unified inelastic-strain theory consisting of a Bodner-Partom flow rule [1] with modifications can adequately model the response of β21S. Due to the advanced nature of these theories, many finite element programs do not include these formulations directly into their codes. However, general purpose finite element programs, such as ADINA6.0 [2], do allow user-defined subroutines that can be written to solve unified inelastic-strain theories. This manual describes this unified inelastic strain theory for β21S and provides the associated user-defined subroutines for use with ADINA6.0. The manual is designed to assist the user with implementing these subroutines into ADINA6.0 or modifying the algorithms within these subroutines for use with other material behavior models or finite element codes. These subroutines and associated data files given in the appendices are available by electric mail or on a magnetic medium from the authors.

Section 2

Bodner-Partom Constitutive Model

This section presents the Bodner-Partom constitutive equations and the material parameters that describe the β21S response. Several formulations of the Bodner-Partom model can characterize inelastic deformation under a variety of conditions which may account for anisotropic, isothermal, and/or non-isothermal material response. The formulation presented in the Section 2.1 parallels the previous isotropic nonisothermal theory of Chan, Bodner, and Lindholm [3,4,5]. The terminology is similar to that of Chan and Lindholm [6]. Section 2.2 contains the material parameters that characterize the β21S material.

2.1 Theory

The first assumption in this analysis is the decomposition of the total strain, ε_{ij}^{tot}, into elastic, thermal, and inelastic components. This decomposition is expressed as:

$$\varepsilon_{ij}^{tot} = \varepsilon_{ij}^{el} + \varepsilon_{ij}^{th} + \varepsilon_{ij}^{in}. \qquad \text{Eq. 1}$$

The elastic strains, ε_{ij}^{el}, depend on the current stress state, elastic modulus E, and Poisson's ratio ν. The thermal strain components, ε_{ij}^{th}, equal the product of the coefficient of thermal expansion and the difference between the current and reference temperatures. The Bodner-Partom flow rule governs the evolution of the inelastic strains, ε_{ij}^{in}.

As opposed to other unified inelastic strain formulations, this theory describes the directional (kinematic) hardening with a special directional hardening term. Other theories represent directional hardening phenomena with a "back-stress" modified equivalent stress [7,8,9]. Introduction of the directional hardening term alters the Bodner-Partom flow rule by replacing the previously known variable "drag-stress" with the sum of isotropic and directional hardening terms, Z^I and Z^D, respectively. These two hardening terms enter into the inelastic strain rate equation or flow law:

$$\dot{\varepsilon}_{ij}^{in} = D_0 \exp\left\{-\frac{1}{2}\left(\frac{\left(Z^I+Z^D\right)^2}{3J_2}\right)^n\right\}\frac{s_{ij}}{\sqrt{J_2}}, \qquad \text{Eq. 2}$$

where D_0 is the limiting strain rate, s_{ij} are the components of deviatoric stress, and $J_2 = s_{ij}\, s_{ij}/2$.

The evolutions of Z^I and Z^D have similar empirical forms. Each equation consists of a hardening term, a thermal recovery term, and a temperature rate term. The isotropic hardening evolution equation with these three terms is:

$$\begin{aligned}\dot{Z}^I = {} & m_1(Z_1 - Z^I)\dot{W}^{in} - A_1 Z_1\left(\frac{Z^I - Z_2}{Z_1}\right)^{r_1} \\ & + \left(\left(\frac{Z^I - Z_2}{Z_1 - Z_2}\right)\frac{\partial Z_1}{\partial T} + \left(\frac{Z_1 - Z^I}{Z_1 - Z_2}\right)\frac{\partial Z_2}{\partial T}\right)\dot{T}\end{aligned}, \qquad \text{Eq. 3}$$

where the inelastic work rate is given by:

$$\dot{W}^{in} = \sigma_{ij}\dot{\varepsilon}_{ij}^{ij}. \qquad \text{Eq. 4}$$

The initial isotopic hardening, $Z^I(0)$, is Z_0. The material parameters associated with the isotopic hardening evolution are m_1, Z_0, Z_1, Z_2, A_1, and r_1. The thermal differential terms $\frac{\partial Z_1}{\partial T}$ and $\frac{\partial Z_2}{\partial T}$ appropriately scale the isotropic hardening variable when inelastic deformation and thermal recovery do not occur during nonisothermal conditions, thus preventing Z^I from passing through maximum or minimum values with temperature changes. The treatment of these thermal differential terms is more consistent with the work of McDowell [10] and others [11,12,13,14] than those proposed by Chan, Lindholm and Bodner [4]. The thermal differential terms appearing in the user-defined subroutines of Appendices A and B reflect the fact that Z_1 is temperature independent for β21S; i.e, $\frac{\partial Z_1}{\partial T}=0$.

The scalar product of a state variable, β_{ij}, and a unit stress vector u_{ij} yields the magnitude of the directional hardening term:

$$Z^D = \beta_{ij} u_{ij}, \qquad \text{Eq. 5}$$

where:

$$\dot{\beta}_{ij} = m_2(Z_3 u_{ij} - \beta_{ij})\dot{W}^{in} - A_2 Z_1\left(\frac{\sqrt{\beta_{kl}\beta_{kl}}}{Z_1}\right)^{r_2} v_{ij} + \frac{\beta_{ij}}{Z_3}\frac{\partial Z_3}{\partial T}\dot{T}, \qquad \text{Eq. 6}$$

$$u_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{kl}\sigma_{kl}}}, \qquad \text{Eq. 7}$$

and:

$$v_{ij} = \frac{\beta_{ij}}{\sqrt{\beta_{kl}\beta_{kl}}}. \qquad \text{Eq. 8}$$

The initial directional hardening variable, $Z^D(0)$ is set to zero. The material constants associated with the directional hardening evolution equation are m_2, Z_3, A_2, and r_2. The temperature differential term $\frac{\partial Z_3}{\partial T}$ appropriately scales the directional hardening variable when inelastic deformation and thermal recovery do not occur during nonisothermal conditions. In particular, without these differential terms the directional hardening accrued at one temperature may exceed the limiting value Z_3 at another temperature, which is not physically realistic.

Table 1 summarizes the 15 material parameters that characterize the strain-rate sensitivity and time-dependent behavior of β21S. The number of material parameters is effectively less than those in Table 1 after applying the usual assumptions: $A_1 = A_2$, $r_1 = r_2$, and $Z_0 = Z_2$. The temperature dependency of these constants is specific to β21S. Other materials, such as Mar-M47 or B1900+Hf [5], require different temperature-dependent parameters to represent their particular response.

Table 1

Material Parameters in Bodner-Partom Constitutive Model

Parameter	Units	Description	Temperature Dependency
E	MPa	Elastic modulus	yes
ν	---	Poisson's ratio	no
α	1/°C	Coefficient of thermal expansion	yes
D_0	s^{-1}	Limiting shear strain rate	no
Z_0	MPa	Initial value of the isotropic hardening variable	yes
Z_1	MPa	Limiting (maximum) value of Z	no
Z_2	MPa	Fully recovered (minimum) value of Z^I	yes
Z_3	MPa	Limiting (maximum) value of Z^D	yes
m_1	MPa^{-1}	Hardening rate coefficient of Z^I	yes
m_2	MPa^{-1}	Hardening rate coefficient of Z^D	yes
n	---	Strain rate sensitivity parameter	yes
A_1	s^{-1}	Recovery coefficient for Z^I	yes
A_2	s^{-1}	Recovery coefficient for Z^D	yes
r_1	----	Recovery exponent for Z^I	yes
r_2	----	Recovery exponent for Z^D	yes

2.2 β21S Material Parameters

The material parameters for the Bodner-Partom model with directional hardening were determined from β21S using monotonic, cyclic, and creep test data [15]. The parameters are valid for a wide window of strain rates (10^{-3} to 10^{-7} 1/s) and temperatures (23° - 815°C). The strategy for determining the parameters involves a number of steps. First, the temperature-dependent parameters are identified and the values of the temperature-independent parameters are estimated. Then at each temperature where experimental data are available, the temperature-dependent parameters are determined through an iterative process to minimize the differences between the model simulations and experiments. *Mathematica* [16] is used to generate the model simulations. Similar to other inelastic strain theories, the set of material parameter for any particular load case is not unique. Thus, for a given set of experimental load responses, a range of values are suitable for each material parameter.

The resulting set of temperature-dependent parameters becomes continuous with temperature as the range of possible values is decreased for each parameter. The response can be very sensitive to small changes in some of the material parameters with temperature, especially in the transition regimes between different deformation mechanisms. For β21S, a transition region for inelastic behavior occurs between 482°C for plasticity and at 650°C for power law creep. Thus, an anomalous change in the saturated stress level can occur if linear interpolation of material parameters is used within this transition region.

Since no experimental data are available within this transition region, values for temperature-dependent parameters are chosen between 482°C and 650°C so that the resulting saturated stress is smooth and continuous with temperature, thus reducing the anomalous effects of linear interpolation. The final version of the material parameters (version 4.0) for β21S are shown in Table 2.

Table 2

Bodner-Partom Material Parameters for β21S

Temperature-Independent Constants

$m_1 = 0\ MPa^{-1}$ (no isotropic hardening)	$Z_1 = 1600\ MPa$
$r_1 = r_2 = 3$	$D_o = 1 \times 10^4\ s^{-1}$
$n = 0.34$	

Temperature-Dependent Constants

Temp.	E	α*	n	$Z_0 = Z_2$	Z_3	m_2	$A_1 = A_2$
°C	MPa	10-6/°C		MPa	MPa	MPa^{-1}	s^{-1}
23	**112000**	**6.31**	**4.8**	**1550**	**100**	**0.35**	**0**
260	**108000**	**7.26**	**3.5**	**1300**	**300**	**0.35**	**0**
315	ϕ	ϕ	◊	◊	*390*	◊	ϕ
365	ϕ	ϕ	◊	◊	*500*	◊	ϕ
415	ϕ	ϕ	◊	◊	*660*	◊	ϕ
465	ϕ	ϕ	◊	◊	*960*	◊	ϕ
482	**98100**	**8.15**	**1.7**	**1100**	**1100**	**5**	**0.0076**
500	ϕ	ϕ	*1.5*	◊	*1300*	◊	ϕ
525	ϕ	ϕ	*1.28*	◊	*1670*	◊	ϕ
550	ϕ	ϕ	*1.1*	◊	*2100*	◊	ϕ
575	ϕ	ϕ	*0.97*	◊	*2600*	◊	ϕ
600	ϕ	ϕ	**0.82**	◊	**3700**	**10**	ϕ
650	**86600**	**8.83**	**0.74**	**1000**	**3800**	**10**	**0.21**
760	**77200**	**9.27**	**0.58**	**600**	**4000**	**15**	**1.0**
815	**72000**	**9.49**	**0.55**	**300**	**4100**	**30**	**2.0**

* Secant α with $T_o = 25°C$
◊ - Linear interpolate between values given in table.
ϕ - Use functions to determine values.
Bold were values determined based on experiments.
Italics are values that describe smooth and continuous saturated stress change.

Functions

$$E = 112000 + 0.591\,T - 0.061\,T^2 \qquad T \text{ in } °C$$

$$\alpha = 6.22 \times 10^{-6} + 4.01 \times 10^{-9}\,T \qquad T \text{ in } °C$$

$$A_1 = A_2 = 5.8 \times 10^5 \exp\left(\frac{-1.37 \times 10^4}{T + 273}\right) \qquad T \text{ in } °C$$

Section 3

Installation of the Subroutines

Included with the delivered version of ADINA 6.0 are overlay files that allow users to incorporate new segments of code into certain regions within ADINA's algorithms. Two particular overlay files, ovl030u.f and ovl040u.f, contain examples of user-defined subroutines that incorporate alternative material response models into ADINA. Within these two files, the user-defined subroutines, CUSER2 and CUSER3, contain sections of code that are modified to implement the Bodner-Partom model into ADINA's two- and three-dimensional elements, respectively. The Bodner-Partom versions of the user-defined subroutines for the two-dimensional axisymmetric and three-dimensional brick elements reside in deliverable files, cuser2_dbeta.f and cuser3_dbeta.f, respectively (see Appendices A and B). The last step of the installation process is to integrate the subroutines within these two files into ADINA's numerical algorithms.

Integrating the Bodner-Partom user-defined subroutines into ADINA6.0 can be accomplished by several methods. One method is replacing the subroutines within the overlay files from ADINA with those of cuser2_dbeta.f and cuser3_dbeta.f. Another method consists of linking new object code over the pre-existing object code. In particular, subroutines of files cuser2_dbeta.f and cuser3_dbeta.f are compiled for produce two object files. These object files are then linked with ADINA's object code to generate a new executable version of ADINA that contains the Bodner-Partom algorithms. Unfortunately, each linker (or loader) is machine dependent. For example, some linkers may declare multiple subroutine declarations and cease the operation. Renaming or replacing the subroutines CUSER2 and CUSER3 in overlay files, ovl030u.f and ovl040u.f, respectively, will correct these errors. Refer to your FORTRAN77 manual to determine the details of your particular linker or segment loader.

To avoid a common error, check the precision (single or double) of the existing user-subroutines found in ADINA's overlay files (e.g., olv030u.f). Then, modify the variable precision of the subroutines in files cuser2_dbeta.f and cuser3_dbeta.f to be of the same precision as found in the overlay files.

Section 4

Structure and Algorithms

This section describes the structure and numerical algorithms of the Bodner-Partom modified subroutines which characterize the strain-rate sensitivity and creep behavior of β21S. Several tables briefly describe subroutine nomenclature and variable usage. Then, the numerical algorithm that solve the Bodner-Partom equations is outlined. Hard copies of the subroutines described in this section reside in the files cuser2_dbeta.f and cuser3_dbeta.f for the two-dimensional axisymmetric and three-dimensional brick elements, respectively. (see Appendices A and B). Subroutine structure and variables usage are identical for the two-dimensional and three-dimensional elements. Thus the details are only presented for the two-dimensional case. The names appended with 2 and 3 designate the two- and three- dimensional case, respectively. For example, CUSER2 and CUSER3 are user-defined subroutines for the two- and three-dimensional cases, respectively.

4.1 Subroutines and Common Blocks

The functionality of subroutines found in cuser2_dbeta.f are summarized in Table 3. Subroutines IUSER and USER2 appear in the overlay file ovl030u.f which is supplied with the ADINA6.0 [2]. Subroutine CUSER2 has four major functions that integrate new material behavior models into the ADINA code. The four functions include -- 1) the initialization of internal state variables, 2) the determination of new states from previous strains and current incremental strains, 3) the computation of incremental stiffness matrix, D, and 4) output of the converged solution. A minor function of CUSER2 is to retrieve and store state variables from ADINA's working variable, ARRAY. Subroutine STGET2 retrieves while STPUT2 stores these variables from ARRAY. DBODNER2 contain the numerical algorithms that solve the Bodner-Partom constitutive equations found in Section 2.1

Table 3

Subroutine Names and Functions

Subroutine*	Called From	Function
CUSER2	IUSER2 and USER2	to determine stresses from user-defined material behavior models
STGET2	CUSER2	to retrieve state variables from working variable ARRAY
STPUT2	CUSER2	to store state variables from working variable ARRAY
DBODNER2	CUSER2	to solve the constitutive equations of the Bodner Partom model

*Two-dimensional subroutines are listed. The three-dimensional subroutines are designated with a suffix 3 rather than a 2.

The two common blocks MATCONST and BPSTAT2 retain values of certain variables entering from other subroutines. MATCONST transfers material constants and associated temperature differentials between CUSER2 and DBODNER2. Table 4 lists variables that are in common block MATCONST. MATCONST is the same common block in the two-dimensional element as found in the three-dimensional elements. Common block BPSTAT2 transfers the values of the internal state variables (e.g., inelastic strain, isotropic hardening, etc.) between all the subroutines. Table 5 contains the state variables of common block BPSTAT2 and BPSTAT3.

Table 4

Internal Variables of Common Block MATCONST

Variable Name	Symbol	Variable Description
E	E	elastic modulus, CTD(1)
AN	n	kinetic parameter, CTD(2)
Z0,Z2	Z_0, Z_2	fully recovered (minimum) value of isotropic hardening, CTD(3)
Z3	Z_3	limiting (maximum) value of directional hardening, CTD(4)
AM2	m_2	hardening rate coefficient of directional hardening, CTD(5)
A1, A2	A_1, A_2	thermal recovery coefficient for hardening, (determined by CTD(6) and CTD(7))
AM1	m_1	isotropic hardening rate coefficient, CTI(1)
Z1	Z_1	limiting value of isotropic hardening, CTI(2)
R1, R2	r_1, r_2	thermal recovery exponent for hardening, CTI(3)
D0	D_0	limiting inelastic strain rate, CTI(4)
ANU	ν	Poisson's ratio, CTI(5)
G	G	shear modulus
AK	K	bulk modulus
DAM2	$\frac{\partial m_2}{\partial T}\frac{\partial T}{\partial t}dt$	differential am2 with solution increment
DAN	$\frac{\partial n}{\partial T}\frac{\partial T}{\partial t}dt$	differential an with solution increment
DZ0	$\frac{\partial Z_0}{\partial T}\frac{\partial T}{\partial t}dt$	differential Z_0 with solution increment
DZ2	$\frac{\partial Z_2}{\partial T}\frac{\partial T}{\partial t}dt$	differential Z_2 with solution increment
DZ3	$\frac{\partial Z_3}{\partial T}\frac{\partial T}{\partial t}dt$	differential Z_3 with solution increment
DA1	$\frac{\partial A_1}{\partial T}\frac{\partial T}{\partial t}dt$	differential A_1 with solution increment
DA2	$\frac{\partial A_2}{\partial T}\frac{\partial T}{\partial t}dt$	differential A_2 with solution increment
DG	$\frac{\partial G}{\partial T}\frac{\partial T}{\partial t}dt$	differential shear modulus with solution increment
DK	$\frac{\partial K}{\partial T}\frac{\partial T}{\partial t}dt$	differential bulk modulus with solution increment

Table 5

State Variables of Common Blocks BPSTAT2 and BPSTAT3

Variable Name	Symbol	Variable Description
EIN	ε_{ij}^{in}	components of inelastic strain
ZI	ZI	current isotropic drag stress
SIGEFF	$\sqrt{3J_2}$	effective stress
ZD	ZD	current magnitude of directional drag stress
BETA	β_{ij}	components of directional drag stress
EPS0	---	mechanical strain at the beginning of ADINA's time TAU

4.2 Internal Variable Names

This section summarizes the internal variables of the user-defined subroutines described in the previous section (4.1). Table 6 presents ADINA6.0 internal variables which enter into CUSER2 and CUSER3 from subroutines USER2 and USER3, respectively. Table 7 reviews the variables found within CUSER2 and CUSER3, while Table 8 lists the variables found in DBODNER2 and DBODNER3.

Some variables change values at certain locations within the numerical scheme. For example, the variable STRESS may have a different value when exiting subroutines CUSER2 than entering. The values of such variables depend on solution control variables -- TIME, TAU, INTER, and KTR. The variable TIME refers to the time at the beginning of a major time step, NSTEP. TAU is the value of time at the beginning of a sub-incremental time step. INTER and KTR are control variables for ADINA's sub-incremental solution scheme. ADINAIN3.0 manuals [17] provide details of this sub-incremental solution scheme. Variables, DT and DTAU, refer to major and sub-incremental time steps sizes, respectively.

Table 6

ADINA Variables Supplied to CUSER2 and CUSER3

Variable Name	Variable Description
KEY = 1	initialize the working arrays during input phase
KEY = 2	calculate element stresses
KEY = 3	calculate the stress/strain matrix
KEY = 4	print stresses and other desired variables during stress print-out
NG	element group number
NEL	element number
IPT	integration point number
IT2D	2d element type identifier
IT2D EQ.0	axisymmetric (in y-z plane)
IT2D EQ.1	plane strain (in y-z plane)
IT2D EQ.2	plane stress (in y-z plane)
IT2D EQ.3	plane stress (in 3/d space)
STRESS	stress components, received at time TAU and updated to time TAU+DTAU
EPS	total strain components at time TIME.
STRAIN	total strain components at time TIME+DT.
DEPS	subdivided incremental mechanical strain components (total incremental strain minus incremental thermal strains)
DEPST	components of sub incremental thermal strain
THSTR1	total thermal strain at time TAU
THSTR2	total thermal strain at time TAU+DTAU
INTER	number of subdivisions for the strain increments

Table 6 (continued)
ADINA Variables Supplied to CUSER2 and CUSER3

Variable Name	Variable Description
KTR KTR = 1 KTR = INTER	current subdivision number for first subdivision for last subdivision
SCP	solution control parameters
SCP(1)	relaxation factor
SCP(2)	Bodner-Partom convergence tolerance
SCP(3)	element number for special history output
SCP(4)	Gauss point for special history output
ARRAY	working array (real) for user-defined state variables at time TAU and updated by user-supplied coding to correspond to time TAU+DTAU
IARRAY	working array (integer) for user-defined state variables at time TAU and updated by user-supplied coding to correspond to time TAU+DTAU
D	stress/strain matrix , to be calculated by user-supplied coding
ALFA	coefficient of thermal expansion at time TAU
CTD	temperature-dependent material constants at time TAU
CTD(1)	elastic modulus, E
CTD(2)	kinetic parameter, n
CTD(3)	fully recovered (minimum) value of isotropic hardening, Z_0,Z_2
CTD(4)	limiting (maximum) value of directional hardening, Z_3
CTD(5)	hardening rate coefficient of directional hardening, m_2
ALFAA	coefficient of thermal expansion at time TAU+DTAU
CTDD	temperature-dependent material constants (same as CTD) at time TAU+DTAU
CTI	temperature-independent material constants
CTI(1)	isotropic hardening rate coefficient, m_1
CTI(2)	limiting value of isotropic hardening, Z_1
CTI(3)	recovery coefficient for drag hardening, r_1,r_2

Table 6 (continued)
ADINA Variables Supplied to CUSER2 and CUSER3

Variable Name	Variable Description
CTI(4)	limiting inelastic strain rate, D_0
CTI(5)	Poisson's ratio, ν
CTI(6)	coefficient for recover coefficient (functional form for A_1, A_2)
CTI(7)	exponential for recover coefficient (functional form for A_1, A_2)
TMP1	temperature at integration point IPT at time TAU
TMP2	temperature at integration point IPT at time TAU+DTAU
TIME	time at current step
DT	time step increment
INTEG	integration parameter for stress integration
ISUBM	flag for continuation of subdivision in time step
INDNL	flag for element formulation

Table 7
Internal Variables in CUSER2 and CUSER3

Variable Name	Variable Description
RELAX	relaxation factor, SCP(1)
TOLER	Bodner-Partom convergence tolerance, SCP(2)
HINT	Gauss point for special history output, SCP(3)
HELE	element number for special history output, SCP(4)
E	elastic modulus, CTD(1)
AN	kinetic parameter, CTD(2)
Z0,Z2	fully recovered (minimum) value of isotropic hardening, CTD(3)
Z3	limiting (maximum) value of directional hardening, CTD(4)
AM2	hardening rate coefficient of directional hardening, CTD(5)
AM1	isotropic hardening rate coefficient, CTI(1)
Z1	limiting value of isotropic hardening, CTI(2)
R1,R2	recovery exponent for hardening, CTI(3)
D0	limiting inelastic strain rate, CTI(4)
ANU	Poisson's ratio, CTI(5)
DTAU	time increment step
G	shear modulus
AK	bulk modulus
ALPHA1	biaxial model parameter (not used)
DAM2	differential am2 with solution increment
DAN	differential an with solution increment
DZ0	differential Z_0 with solution increment
DZ2	differential Z_2 with solution increment
DZ3	differential Z_3 with solution increment
DA1	differential A_1 with solution increment
DA2	differential A_2 with solution increment
DG	differential shear modulus with solution increment
DK	differential bulk modulus with solution increment

Table 8
Internal Variables in DBODNER2 and DBODNER3

Variable Name	Variable Description
DEVEPS	deviatoric strains
EPS	mechanical strains (total minus thermal)
AVGEPS	average (mean) normal strain
DDEPS	sub-incremental deviatoric strains
DEIEFF	effective inelastic strain increment
DEPS	incremental mechanical strain (total minus thermal)
DEPSI	incremental inelastic deviatoric strain
DEVSIG	deviatoric stress
ICOUNT	iteration loop counter
ISUB	number of sub-increments within DBODNER2
RELAX	relaxation factor for new stress estimate
TOLER	tolerance for convergence
SIGNEW	effective stress of previously converged stress state
SIGOLD	old estimate effective stress
SIGEST	new effective stress estimate
SIGHYD	hydrostatic (mean) stress
STRES0	stress of previously converged stress state
STRESS	current stress
TIME	current time value
EINEST	estimated inelastic strains
EIN0(4)	inelastic strains of previously converged stress state
BETADOT	directional drag stress rate vector
BETAEST	estimated directional drag stress vector
BETA0(4)	directional drag stress of previously converged stress state
U, V	directional unit vectors
ZIEST	estimated isotropic drag stress

4.3 Numerical Algorithms

The algorithms that solve the Bodner-Partom constitutive equations consist of a mixture of iteration loops and sub-incremental schemes. This hybrid combination of iterations and sub-incrementation works well for the inherently "stiff" nature of the Bodner-Partom differentials equations and β21S material parameters. The algorithms discussed in this section are found in subroutines DBODNER2 and DBODNER3.

The algorithm, as shown in Figure 1, consists of two iteration loops and a sub-incremental solution scheme. Prior to sub-incremental integration, state variable values assume their pre-incremental values; the number of sub-increments (ISUB) initializes to one; and constant rate variables get scale by TFACTOR. The integration of the sub-incremental loop begins at Step III. The primary iteration loop starts at step III.D and converges on stress, inelastic strain, isotopic hardening and the directional hardening parameters. Preliminary investigations on convergence found that the rate equations for β_{ij}, Eq. (6) strongly depend on the current value of β_{ij}, thus making convergence on β_{ij} difficult. The value for Z^D often diverges during the primary iteration loop. The second iteration loop, starting to Step III.D.3, saves computational time by determining Z^D divergence more quickly than the primary iteration loop, since the second loop avoids several numerical operations found in the primary iteration loop. Non-convergent solutions, as defined by a maximum limit set on iteration steps, return to the beginning of the sub-incremental integration (Step I) with an increase in the number of sub-increments, ISUB. When the maximum number of sub-incremental step equals 128, the solution solver ceases operation and prints a diagnostic debugging output.

I. Store variable at the beginning of solution increment
   A. state variables
   B. temperature-dependent material parameters

II. Initialize variables for sub-incremental solution step cutting of ISUB
   A. initialize estimated new values for state variables
   B. restore material parameters from Step I
   C. determine new material parameters rates by factors of ISUB
   D. restore state variables to values from Step I

III Begin sub-incremental solution integration
   A. update all temperature-dependent parameters to end of sub-incremental step
   B. update deviatoric and mean mechanical strains
   C. step iteration counter ICOUNT
   D. begin primary iteration loop
      1. estimate inelastic strain increment (engineering inelastic shear
         strains are computed)
      2. compute new stress state and inelastic work rate
      3. begin secondary iteration loop on directional hardening variable BETA
         a. compute beta rate
         b. compute new estimate for beta
         c. check for convergence of beta
            (1) if converged, then continue
            (2) if not, then increase cutting factor ISUB by factor of 2.0 and precede to Step II.A
         d. stop solution for excessive number of time cuts
      4. compute new estimate for isotropic hardening
      5. check for convergence of effective stress, incremental inelastic
         strains, isotropic and directional hardening
         a. if converged, then continue with conclude current sub-increment via. STEP III.E
         b. if not, make new estimate for effective stress and increase iteration count ICOUNT BY 1.
         c. check for excessive iteration count
            (1) if excessive, then increase cutting factor ISUB by factor of 2.0 and precede to Step II.A
            (2) if not, to Step III.D
   E. Update converged solution with estimates, then continue a STEP III.A

IV. Complete all sub-incremental cycles and then return to CUSER2 (or CUSER3)

Figure 1 Numerical Algorithm for Solving of Bodner-Partom Equations.

Section 5

Example, Verification and Errors

This section provides an example of the application of the Bodner-Partom user-defined subroutines with the β21S material parameters. The example ADINAIN3.0 input file for a simple test case. This section also presents several numerical exercises that serve as a basis for comparisons of results from an independent solution source with those from the finite element method. The difference between the results from these two solution sources are measures of accuracy for these algorithms and assoiated intergration schemes. The last part of this section presents an investigation that guilds the users of these subroutines to minimize errors effectively.

5.1 Numerical Example

The file, vcase1.in, (see Appendix C) is an example ADINAIN3.0 input file that contains the required β21S material parameters. ADINA6.0 preprocessor ADINAIN3.0 generates a data file that executes the user-defined subroutines that were implemented into ADINA6.0. The example consists of an axisymmetric element unidirectionally loaded at a constant strain rate of 833.3×10^{-6}/s and constant temperature of 25°C, which has the same conditions of verification test case 1 (discussed below). Input of the β21S material properties starts with the string "MATERIAL 1 USER ..." (see Appendix C). The cards that refer to material and solution parameters are described in Table 9 [17]. In these example analyses, the thermal expansion coefficients are set to zero, since the comparisons made here only consider mechanical strain (total strain minus thermal strain).

Table 9 [17]

Variable Names in ADINAIN3. 0 Input File

Variable Name*	Variable Description
ALFA	coefficient of thermal expansion
CTD(1)	elastic modulus, E
CTD(2)	kinetic parameter, n
CTD(3)	fully recovered (minimum) value of isotropic hardening, Z_0,Z_2
CTD(4)	limiting (maximum) value of directional hardening,Z_3
CTD(5)	hardening rate coefficient of directional hardening, m_2
CTI(1)	isotropic hardening rate coefficient, m_1
CTI(2)	limiting value of isotropic drag stress, Z_1
CTI(3)	thermal recovery exponent for hardening, r_1, r_2
CTI(4)	limiting inelastic strain rate, D_0
CTI(5)	Poisson's ratio,ν
CTI(6)	coefficient for thermal recovery's functional form of A_1 and A_2
CTI(7)	exponential for thermal recovery's functional form for A_1 and A_2
XINTER	number of incremental subdivisions of NSTEP
SCP(1)	relaxation factor
SCP(2)	Bodner Partom convergence tolerance, TOLER
SCP(3)	element number for special history output 'fort.53'
SCP(4)	Gauss point for special history output file 'fort.53'

*User-defined material parameters as described in ADINAIN3.0 manual [17]

5.2 Solution Verification

This section presents five comparisons of the finite element results with those obtained through an independent solution source. These test cases consist of three isothermal and two nonisothermal simulations. The accuracy measure for this investigation is percent deviation of the finite element stress solution from *Mathematica* [16], which has an associated error of approximately 10^{-4} percent. The percent stress deviation, %D, is written as:

$$\%D = 100\% * \left| \frac{\sigma_{FE} - \sigma_{math}}{\sigma_{math}} \right|. \qquad \text{Eq. (9)}$$

Where σ_{FE} and σ_{math} are the stresses obtained from the finite element and *Mathematica* solutions, respectively. The two- and three-dimensional finite element stress solutions are identical, thus only one finite element solution deviation is shown.

All test cases simulate monotonically loaded elements with constant mechanical strain rates of either $833.3\text{x}10^{-6}$/s or $8.33\text{e}10^{-6}$/s. Table 10 summarizes all the test conditions and location in the appendices of the *Mathematica* solution files used in the comparisons. The data in these files appear in Appendices D through H. Figures 2, 3 and 4 illustrate the stress-strain response and the finite element stress deviation from *Mathematica* for isothermal Test Cases 1, 2, and 3, respectively. Overall, the isothermal load cases show that the finite element and *Mathematica* solutions are well within 0.5 percent of predicting the same response.

Table 10

Verification Test Cases

Test Case Number	Temperature (°C)	Thermal Condition	Strain Rate (10^{-6}/s)	*Mathematica* Solution File	Appendix
1	25	isothermal	833.3	vcase1.math	D
2	650	isothermal	833.3	vcase2.math	E
3	650	isothermal	8.333	vcase3.math	F
4	25/482/25	non-isothermal	833.3	vcase4.math	G
5	650/760/650	non-isothermal	833.3	vcase5.math	H

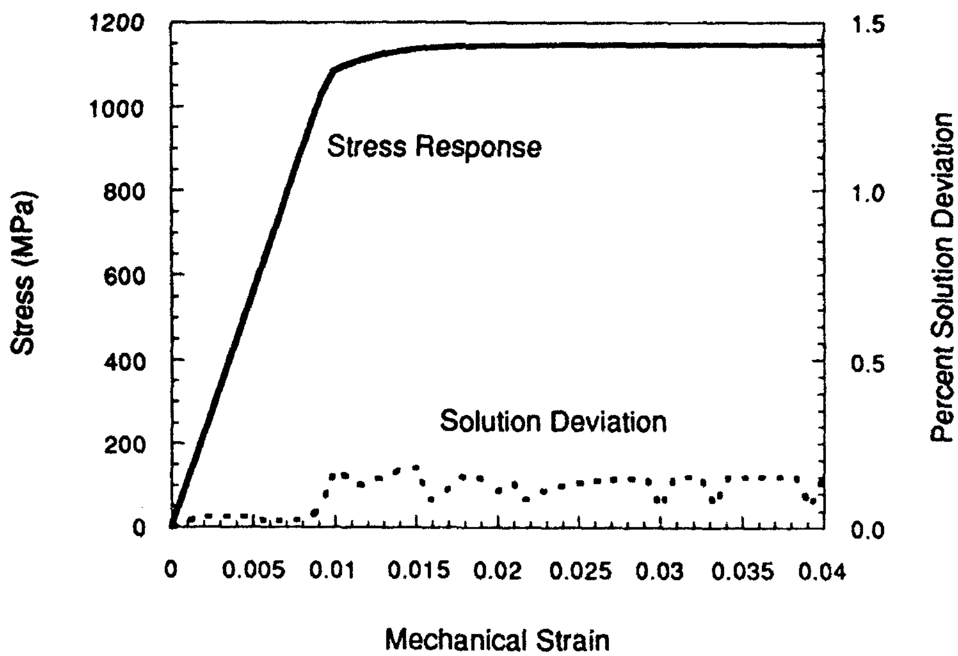

Figure 2 Stress-Strain Response and Finite Element Solution Deviation from *Mathematica* for Test Case 1

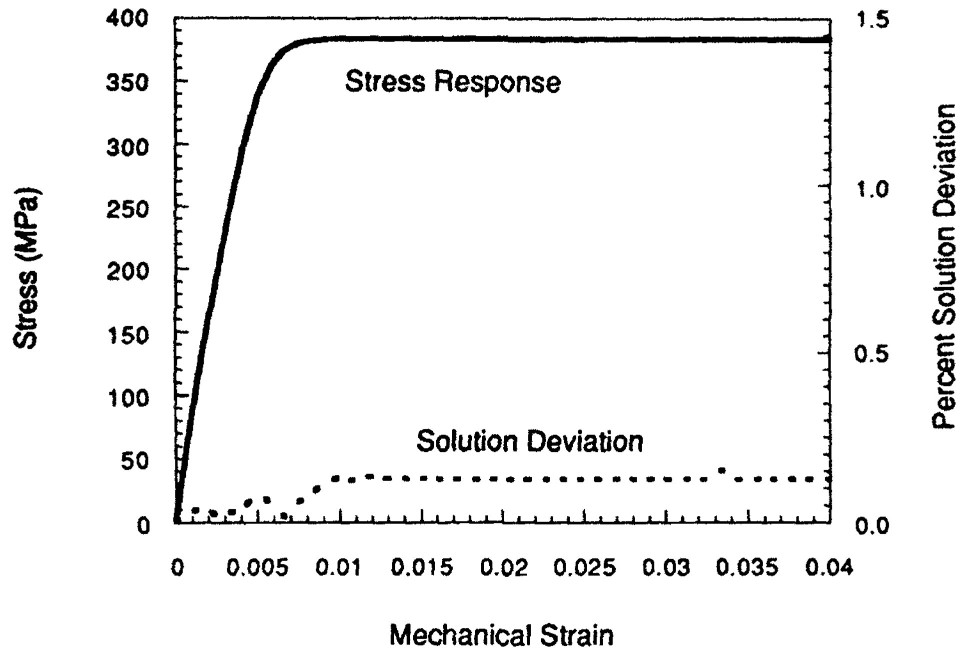

Figure 3 Stress-Strain Response and Finite Element Solution Deviation from *Mathematica* for Test Case 2

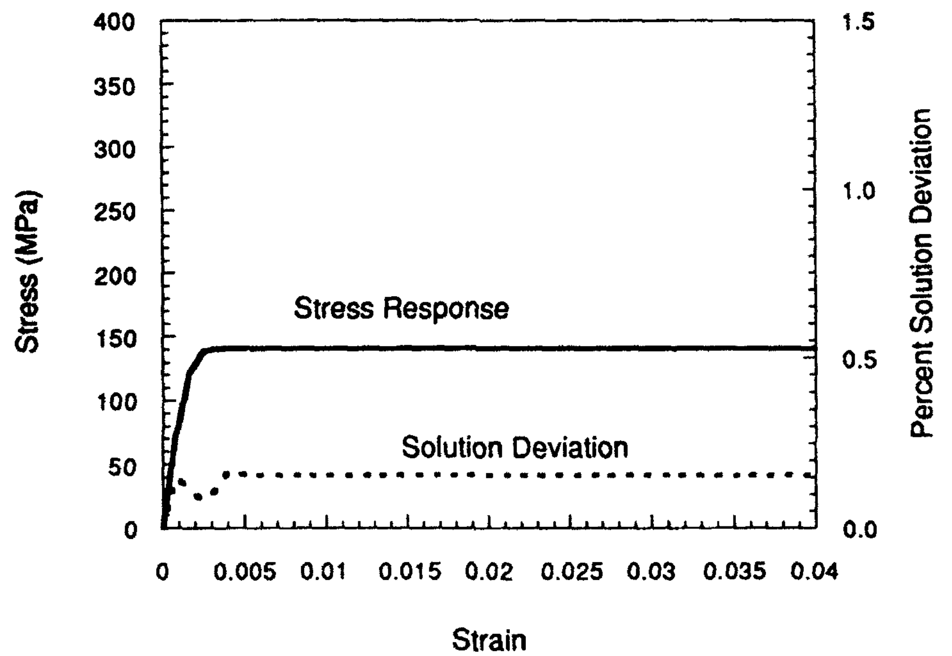

Figure 4 Stress-Strain Response and Finite Element Solution Deviation from *Mathematica* for Test Case 3

The nonisothermal test cases are constant strain-rate tensile simulations with changes in temperatures at prescribed strain levels. These non-isothermal loading profiles provide an excellent basis to evaluate the various forms of state-variable temperature differentials (e.g., $\frac{\partial Z_1}{\partial T}$, $\frac{\partial Z_2}{\partial T}$ and $\frac{\partial Z_3}{\partial T}$). The temperature profile and stress-strain response of Test Case 4 are illustrated in Figure 5 with the associated finite element solution deviation from *Mathematica* presented in Figure 6. The temperature profile and stress response for Test Case 5 are shown in Figure 7 with solution deviation given in Figure 8. Overall, the finite element and mathematica stresses are well within 0.5 percent of predicting the same response for the nonisothermal segments of the loading. During the nonisothermal periods of loading the percent deviation increases slightly, which is more evident in Test Case 5 than in of Test Case 4. For practical use, more solution increments and/or smaller convergence tolerances may be required to achieve the same level accuracy for non isothermal loading segments as those obtained during isothermal periods, especially at elevated temperatures.

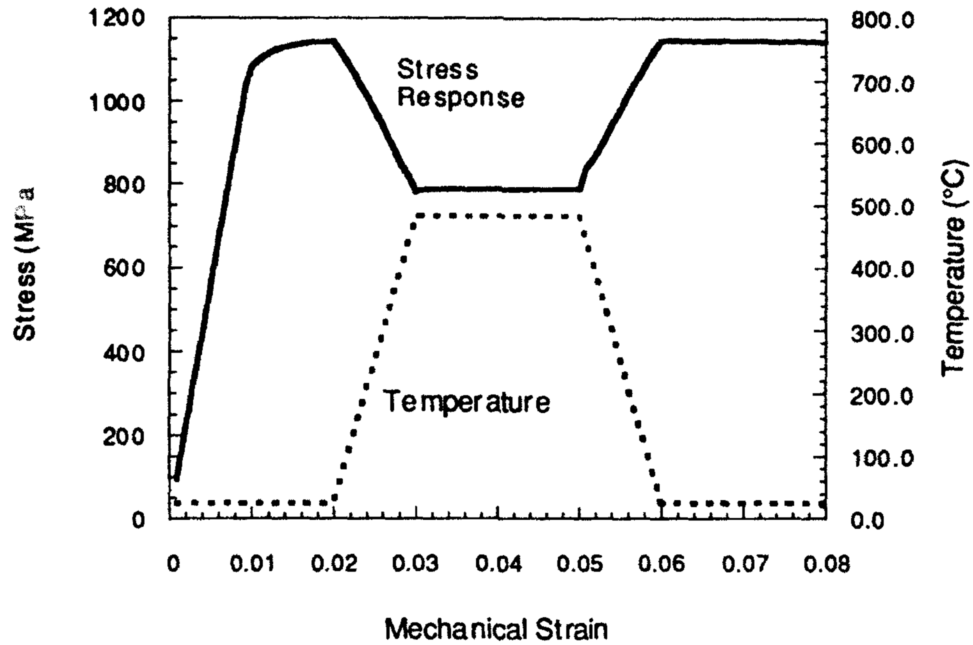

Figure 5 Temperature Profile and Stress Response for Non-Isothermal Test Case 4

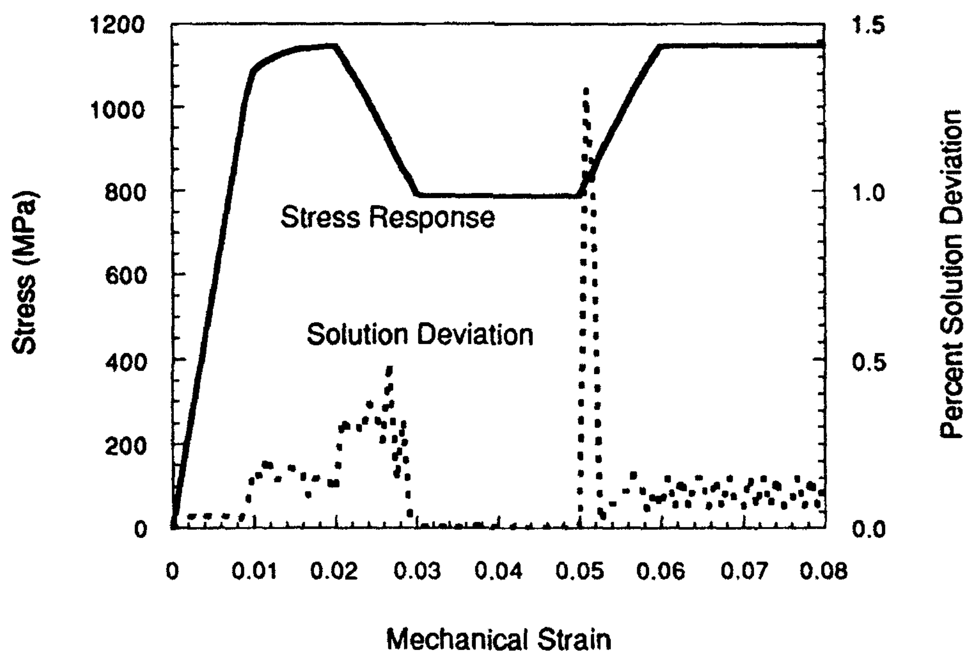

Figure 6 Stress-Strain Response and Finite Element Solution Deviation from *Mathematica* for Test Case 4

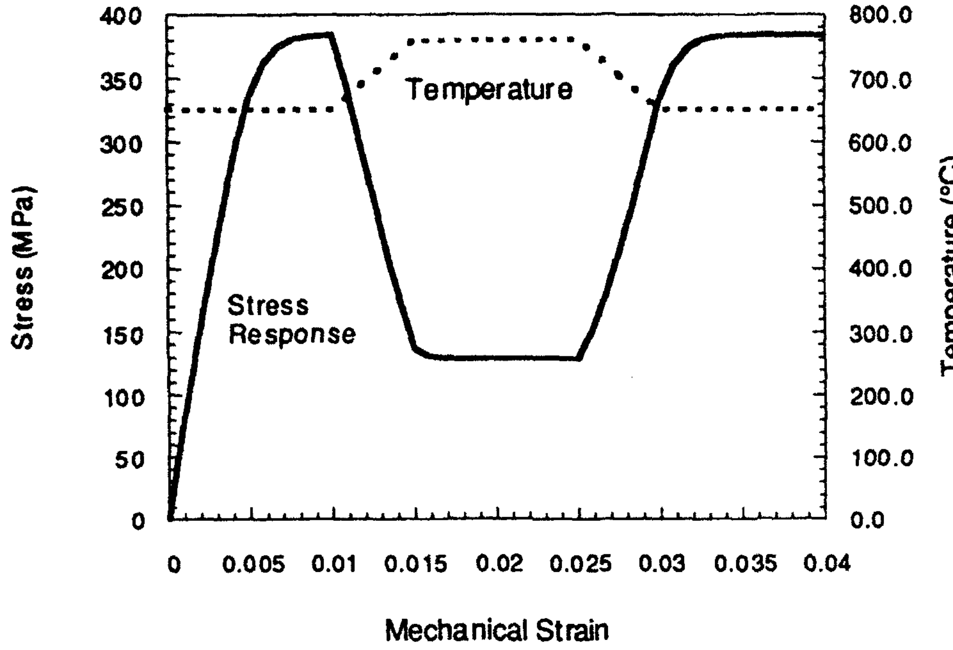

Figure 7 Temperature Profile and Stress Response for Non-Isothermal Test Case 5

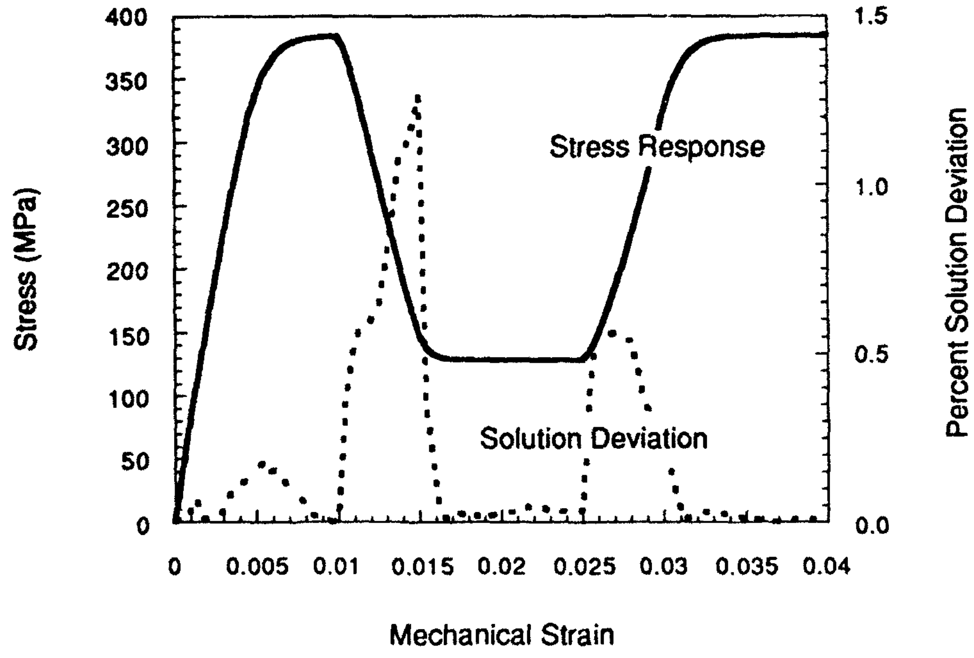

Figure 8 Stress-Strain Response and Finite Element Solution Deviation from *Mathematica* for Test Case 5

5.3 Solution Error

Preliminary investigations with Test Case 5 revealed an unacceptable deviation between finite element and *Mathematica* solutions (2.9%). The poor correlation occurred when Test Case 5 consisted of 48 solution increments, rather than the 96 increment case shown in Figure 8. The nature of the excessive solution deviation is determined as each solution parameter is systematically changed. The investigation considers two types of solution parameters -- iteration convergence tolerances and solution increment number.

Iteration convergence tolerances occur at two different levels within the numerical algorithms -- global and local. Global tolerance DNORM is a measure of convergence on displacements and forces (energy) within the entire system of nodes and elements for each major time step, NSTEP. On the local level, within the Bodner-Partom iteration schemes (see Section 4.3) the tolerance parameter TOLER measures the accuracy of the converged stress state for strain increments send to each elemental Gauss point.

The global parameter DNORM is not a direct measure of solution convergence tolerance within ADINA. Convergence occurs when the change in displacement, $\left|d\underline{u}^i\right|$, for each global iteration estimate becomes relatively small compared to DNORM, which is expressed as:

$$\frac{\left|d\underline{u}^i\right|}{\mathrm{DNORM}} \leq \mathrm{DTOL}. \qquad \text{Eq. (10)}$$

With the displacement tolerance DTOL set constant, a change in DNORM directly alters the solution tolerance. Since, DNORM parameter has no standard default value within ADINA, this investigation considers variations in DNORM rather than DTOL. The previously completed analyses of Section 5.2 used a DNORM base-line value of 1.0×10^{-5}. Values of DNORM for this investigation are 1.0×10^{-3} and 1.0×10^{-4}. The change of DNORM to 1.0×10^{-3} or 1.0×10^{-4} produces a only a small change in stress solution (less than 0.1%).

The changes in the localized tolerance parameter TOLER within the Bodner-Partom iteration loops, results in minor shift in the solution (0.25%), as illustrated in Figure 9. Overall, the tighter tolerances of DNORM and TOLER still produced unacceptable levels of solution deviation from *Mathematica*. Note that in change in TOLER from 0.001 to 0.00001 did increase the finite element solution running time from 122 to 431 CPU seconds*, respectively, as shown in Table 11. Based on the marked debit in computational efficiency with little increase in solution accuracy, increasing TOLER for a more accurate solution is not practical. However, the TOLER at and above 0.001 can produce non-convergent solutions, especially if the finite element configuration contains additional non-linearities (e.g. gap elements). A smaller values for TOLER to may be required to achieve convergence for these highly non linear solutions.

* CPU time on a Sun Sparc Station II with 4/75 processor

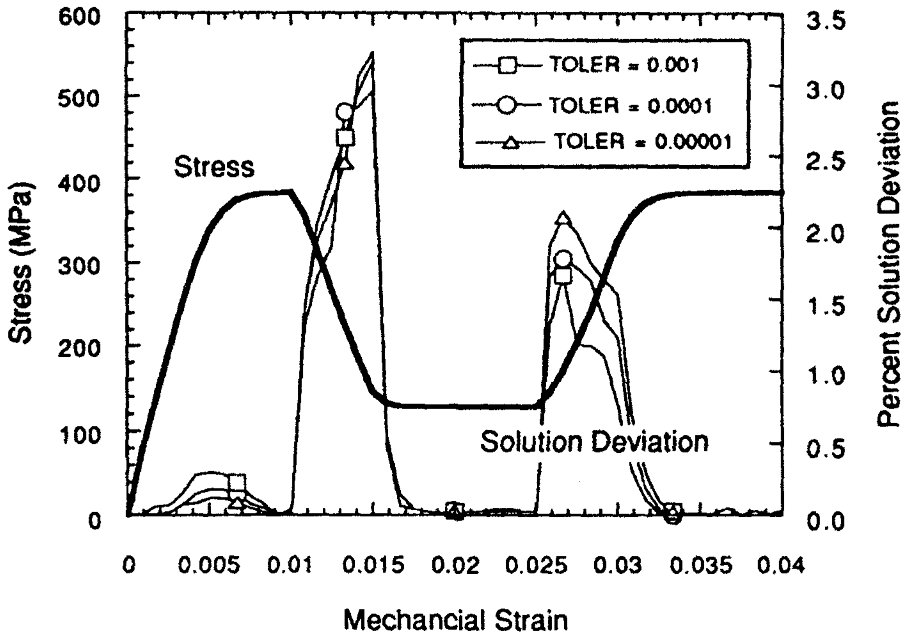

Figure 9 Effects of Localized Tolerance Parameter, TOLER, on Finite Element Solution Deviation from *Mathematica*.

ADINA's integration scheme has two different levels of incrementing the solution -- major and minor. The global convergence of displacements occurs at the major solution increment, NSTEP. The minor solution increment sub-divides the major increment by XINTER for quicker integration of the nonlinear material models. Increasing either NSTEP or XINTER decreases the solution deviation, as shown in Figures 10 and 11, respectively. The decrease of solution deviation with increases in solution time steps is more significant than changes caused by increasing the tolerance parameters.

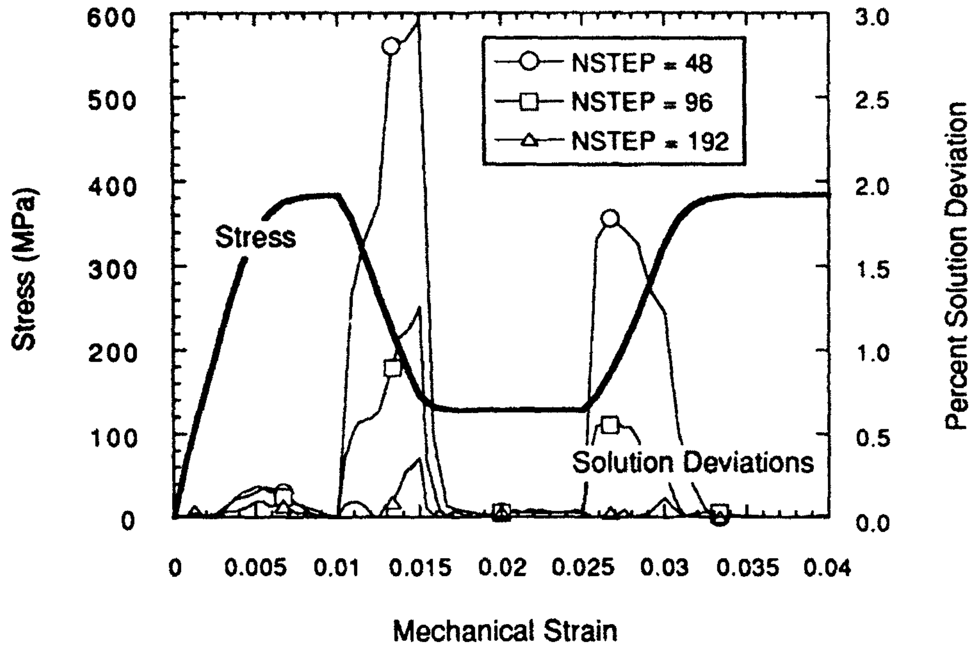

Figure 10 Effect of Major Time Step NSTEP on Finite Element Solution Deviation from *Mathematica*.

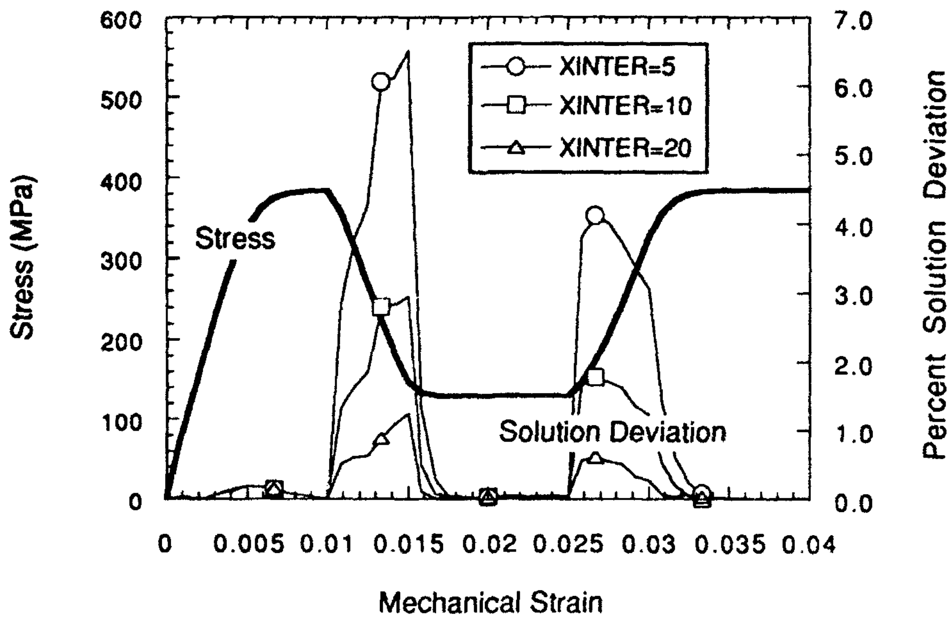

Figure 11 Effect of Sub-incremental Parameter XINTER on Finite Element Solution Deviation from *Mathematica*.

To summarize, this error investigation only considers a simple unidirectional test case during nonisothermal loading; however, knowledge of the results will save time and effort when conducting similar error investigation for more complex cases. First, note that the errors associated with isothermal conditions are quite small compared to those of nonisothermal loading periods, even with a few time steps. For the nonisothermal Test Case 5, Table 11 presents a summary of finite element solution parameters and their effects on computer CPU* time and solution deviation. Increases in solution tolerances DNORM and TOLER do not significantly change the resulting stress error; however, CPU time did increase. Increases in number of solution increments, NSTEP or XINTER, significantly decrease the solution error, while CPU time remains relatively constant. Overall, the most accurate solution per CPU time occurs with increased number of major time steps, NSTEP.

---

*CPU time on a Sun Sparc Station II with 4/75 processor

Table 11

Effects of Finite Element Solution Parameters
on CPU* Time and Solution Deviation

NSTEP	DNORM	XINTER	TOLER	CPU Time (s)	Maximum Percent Deviation
48	1.0×10^{-5}	10	0.0001	206	2.96
48	1.0×10^{-4}	10	0.0001	173	3.02
48	1.0×10^{-3}	10	0.0001	149	3.02
48	1.0×10^{-5}	5	0.0001	180	6.51
48	1.0×10^{-5}	20	0.0001	204	1.25
48	1.0×10^{-5}	10	0.001	122	3.16
48	1.0×10^{-5}	10	0.00001	432	3.23
96	1.0×10^{-5}	10	0.0001	200	1.25
192	1.0×10^{-5}	10	0.0001	179	0.362
384	1.0×10^{-5}	10	0.0001	231	0.0478
Mathematica	---	---	---	2638	---

* CPU time on a Sun Sparc Station II with 4/75 processor

Section 6

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Appendix A

User-Defined Subroutines for Two-Dimensional Axisymmetric Elements (file cuser2_dbeta.f)

```
      SUBROUTINE CUSER2 (NG,NEL,IPT,IT2D,STRESS,EPS,STRAIN,DEPS,DEPST,
     A         THSTR1,THSTR2,KTR,INTER,SCP,ARRAY,IARRAY,D,
     B         ALFA,CTD,ALFAA,CTDD,CTI,TMP1,TMP2,TIME,DTAU,
     C         PHIST,PRST,ANGLE,DPSP,INTEG,ISUBM,INDNL,KEY)
C*I
C*I
C*I
C*I THIS SUBROUTINE IS TO BE SUPPLIED BY THE USER TO CALCULATE
C*I THE STRESSES AND CONSTITUTIVE MATRIX OF A SPECIAL MATERIAL.
C*I
C*I THIS SUBROUTINE IS CALLED IN USER2 FOR EACH INTEGRATION POINT
C*I FOR EACH 2-D SOLID ELEMENT TO PERFORM THE FOLLOWING OPERATIONS :
C*I
C*I   KEY.EQ.1  INITIALIZE THE WORKING ARRAYS DURING INPUT PHASE
C*I
C*I   KEY.EQ.2  CALCULATE ELEMENT STRESSES
C*I
C*I   KEY.EQ.3  CALCULATE THE STRESS/STRAIN MATRIX
C*I
C*I   KEY.EQ.4  PRINT CALCULATED STRESSES AND OTHER DESIRED
C*I             VARIABLES DURING STRESS PRINT-OUT
C*I
C*I
C*I THE FOLLOWING VARIABLES ARE USED TO PERFORM THE ABOVE OPERATIONS :
C*I
C*I   NG         ELEMENT GROUP NUMBER
C*I
C*I   NEL        ELEMENT NUMBER
C*I
C*I   IPT        INTEGRATION POINT NUMBER
C*I
C*I   IT2D       2D ELEMENT TYPE IDENTIFIER ( NPAR(5) )
C*I              EQ.0  AXISYMMETRIC (IN Y-Z PLANE)
C*I              EQ.1  PLANE STRAIN (IN Y-Z PLANE)
C*I              EQ.2  PLANE STRESS (IN Y-Z PLANE)
C*I              EQ.3  PLANE STRESS (IN 3/D SPACE)
C*I
C*I   STRESS(4)     STRESS COMPONENTS, RECEIVED AT TIME TAU
C*I              AND UPDATED BY USER-SUPPLIED CODING TO
C*I              CORRESPOND TO TIME TAU+DTAU
C*I
C*I   EPS(4)        TOTAL STRAIN COMPONENTS AT TIME T.
C*I              IN CASE OF LARGE STRAIN FORMULATION
C*I              (U.L.H.), EPS(4) ARE ELASTIC STRAINS AT
C*I              TIME T.
```

```
C*I
C*I   STRAIN(4)        TOTAL STRAIN COMPONENTS AT TIME T+DT.
C*I                IN CASE OF U.L.H. :
C*I                 A) FOR KEY.EQ.2 - ELASTIC STRAINS
C*I                   AT TIME TAU+DTAU (FOR KTR.EQ.0 -
C*I                   TRIAL ELASTIC STRAINS)
C*I                 B) FOR KEY.GT.2 - ELASTIC STRAINS AT
C*I                   TIME T+DT
C*I
C*I   DEPS(4)          SUBDIVIDED INCREMENTAL STRAIN COMPONENTS
C*I                DEPS(I) = ( STRAIN(I) - EPS(I) )/INTER
C*I                       - DEPST(I)
C*I                ( PASSED TO SUBROUTINE CUSER2 BY THE
C*I                 PROGRAM ADINA )
C*I
C*I   DPSP(4)          INCREMENT OF INELASTIC STRAIN ( PLASTIC
C*I                AND/OR CREEP AND/OR VISCOPLASTIC, ETC.)
C*I                IN THE SUBDIVISION. CALCULATED BY USER-
C*I                SUPPLIED CODING AND PASSED TO THE PROGRAM
C*I                ADINA. USED FOR U.L.H. ONLY.
C*I
C*I   DEPST(4)         COMPONENTS OF SUBINCREMENTAL THERMAL
C*I                STRAIN
C*I                ( PASSED TO SUBROUTINE CUSER2 BY THE
C*I                 PROGRAM ADINA )
C*I
C*I   THSTR1(4)        TOTAL THERMAL STRAIN AT TIME TAU
C*I                FOR KEY.EQ.4 - THERMAL STRAIN AT TIME T
C*I
C*I   THSTR2(4)        TOTAL THERMAL STRAIN AT TIME TAU+DTAU
C*I                FOR KEY EQ.4 - THERMAL STRAIN AT TIME T+DT
C*I
C*I   INTER            NUMBER OF SUBDIVISIONS FOR THE STRAIN
C*I                INCREMENTS   (INTER = INT(PROP(123))
C*I
C*I   KTR              CURRENT SUBDIVISION NUMBER
C*I                EQ.0     CALCULATION OF TRIAL ELASTIC
C*I                         STATE, IN CASE INTEG=1
C*I                EQ.1     FOR FIRST SUBDIVISION
C*I                EQ.INTER   FOR LAST SUBDIVISION
C*I
C*I   SCP(4)           SOLUTION CONTROL PARAMETERS
C*I
C*I   ARRAY(60)        WORKING ARRAY (REAL) , RECEIVED AT
C*I                TIME TAU AND UPDATED BY USER-SUPPLIED
C*I                CODING TO CORRESPOND TO TIME TAU+DTAU
C*I
C*I   IARRAY(2)        WORKING ARRAY (INTEGER) , RECEIVED AT
C*I                TIME TAU AND UPDATED BY USER-SUPPLIED
C*I                CODING TO CORRESPOND TO TIME TAU+DTAU
C*I
C*I   D(4,4)           STRESS/STRAIN MATRIX , TO BE CALCULATED
```

```
C*I                 BY USER-SUPPLIED CODING
C*I
C*I   ALFA          COEFFICIENT OF THERMAL EXPANSION AT
C*I                 TIME TAU
C*I
C*I   CTD(5)        TEMPERATURE-DEPENDENT MATERIAL CONSTANTS
C*I                 AT TIME TAU
C*I
C*I   ALFAA         COEFFICIENT OF THERMAL EXPANSION AT
C*I                 TIME TAU+DTAU
C*I
C*I   CTDD(5)       TEMPERATURE-DEPENDENT MATERIAL CONSTANTS
C*I                 AT TIME TAU+DTAU
C*I
C*I   CTI(8)        TEMPERATURE-INDEPENDENT MATERIAL CONSTANTS
C*I
C*I   TMP1          TEMPERATURE AT INTEGRATION POINT IPT AT
C*I                 TIME TAU. FOR KEY.EQ.4 - TEMPERATURE AT
C*I                 TIME T
C*I
C*I   TMP2          TEMPERATURE AT INTEGRATION POINT IPT AT
C*I                 TIME TAU+DTAU. FOR KEY.EQ.4 - TEMPERATURE
C*I                 AT TIME T+DT
C*I
C*I   TIME          TIME AT CURRENT STEP , T+DT
C*I
C*I   DT            TIME STEP INCREMENT , DT
C*I
C*I   PHIST(3,3)    MATRIX CONTAINING DIRECTION COSINES OF
C*I                 PRINCIPAL STRETCH DIRECTIONS, IN CASE OF
C*I                 U.L.H. FORMULATION
C*I
C*I   PRST(3)       PRINCIPAL STRETCHES, IN CASE OF U.L.H.
C*I
C*I   ANGLE(2)      ANGLES OF THE FIRST AND SECOND (IN PLANE)
C*I                 PRINCIPAL STRETCH DIRECTIONS WITH RESPECT
C*I                 TO Y-AXIS, U.L.H.
C*I
C*I   INTEG         INTEGRATION PARAMETER FOR STRESS
C*I                 INTEGRATION
C*I                 EQ.0 - FORWARD INTEGRATION
C*I                 EQ.1 - BACKWARD INTEGRATION
C*I                     (RETURN MAPPING)
C*I
C*I   ISUBM         FLAG FOR CONTINUATION OF SUBDIVISION
C*I                 IN TIME STEP, APPLICABLE FOR INTEG.EQ.1
C*I                 EQ.0 -  CONTINUATION
C*I                 EQ.-1 - STOP OF SUBDIVISION
C*I                 IN THE USER-SUPPLIED CODING THE FLAG
C*I                 (INITIALLY EQ.0) MUST BE SET TO -1
C*I                 WHEN CRITERIA FOR STOPPING SUBDIVISIONS
C*I                 ARE REACHED
```

```
C*I
C*I    INDNL          FLAG FOR ELEMENT FORMULATION (NPAR(3))
C*I                   EQ.1 - MATERIALLY NONLINEAR ONLY (M.N.O.)
C*I                   EQ.2 - LARGE DISPLACEMENTS AND SMALL
C*I                          STRAINS (T.L.)
C*I                   EQ.3 - LARGE DISPLACEMENTS AND LARGE
C*I                          STRAINS (U.L.H.)
C*I
C*I
C*I    NOTE THAT THE VARIABLES PASSED TO THE SUBROUTINE WHEN KEY=3,4
C*I    ARE THESE CALCULATED LAST: I.E., CALCULATED IN THE LAST
C*I    SUBDIVISION WHEN KEY=2. HENCE THESE VARIABLES ARE NOT
C*I    CALCULATED WHEN KEY=3,4.
C*I
C*I
C*I
      IMPLICIT DOUBLE PRECISION ( A-H,O-Z )
C
      DIMENSION STRESS(4),STRAIN(4),DEPS(4),ARRAY(60),IARRAY(2),D(4,4),
     A        CTD(5),CTI(8),EPS(4),SCP(4)
      DIMENSION CTDD(5),DEPST(4),THSTR1(4),THSTR2(4)
      DIMENSION PHIST(3,3),PRST(3),ANGLE(2),DPSP(4)
C
C
      REAL*8    DTAU
C               TIME INCREMENT STEP, DT
      REAL*8    E
C               ELASTIC MODULUS
      REAL*8    EIDEFF
C               EFFECTIVE INELASTIC STRAIN RATE
      REAL*8    G
C               SHEAR MODULOUS
      INTEGER   HINT
C          INTEGRATION POINT WRITTEN TO HISTORY FILE
      INTEGER   HELE
C          ELEMENT NUMBER ASSOCIATED WITH HIPT
      INTEGER   I
C          DO LOOP COUNTER
      INTEGER   J
C          DO LOOP COUNTER
C
C DIRECTIONAL BODNER-PARTOM MATERIAL CONSTANTS:
C
C  TEMPERATURE INDEPENDENT CONSTANTS
C
       REAL*8  AM1
       REAL*8  Z1
       REAL*8  R1
       REAL*8  R2
       REAL*8  D0
       REAL*8  ALPHA1
C
```

```
C  TEMPERATURE INDEPENDENT CONSTANTS
C
   REAL*8  AN
   REAL*8  AM2
   REAL*8  Z0
   REAL*8  Z2
   REAL*8  Z3
   REAL*8  A1
   REAL*8  A2
C
C  TEMPERATURE DIFFERENTIALS
C
   REAL*8  DAM2
   REAL*8  DAN
   REAL*8  DZ0
   REAL*8  DZ2
   REAL*8  DZ3
   REAL*8  DA1
   REAL*8  DA2
C
   COMMON /MATCONST/ E,  G,  AK,  DG,  DK,
  &          AM1, AM2,  ALPHA1, DAM2,
  &          Z1, Z3, R1, R2,  D0,
  &          AN, Z0, Z2, A1,  A2,
  &          DAN, DZ0, DZ2, DZ3, DA1, DA2
C
C  COMMON BLOCK FOR STATE VARIBLES
C
   COMMON /BPSTAT2/ EIN(4), ZI, SIGEFF, BETAEFF, EPEFF,  ALPHA,
  &          ZD, BETA(4), EPS0(4)
C
C  RECALL STATE VARIBLES AT TIME TAU
C
   CALL STGET2(ARRAY)
C
   GO TO (1,2,3,4), KEY
C*I
C*I
C*I  K E Y = 1
C*I
C*I  INITIALIZE COMPONENTS OF REAL AND INTEGER WORKING ARRAYS
C*I  ( INITIALIZE ARRAY(60) AND IARRAY(2) )
C*I
 1 CONTINUE
C*I
C*I  ***  I N S E R T   U S E R - S U P P L I E D   C O D I N G
C
C
C   INITIALIZE STATE VARIBLES
C
   EIN(1) = 0.0
   EIN(2) = 0.0
```

```
      EIN(3)  = 0.0
      EIN(4)  = 0.0
      BETA(1) = 0.0
      BETA(2) = 0.0
      BETA(3) = 0.0
      BETA(4) = 0.0
      ZI      = CTD(3)
      SIGEFF  = 0.0
      EPEFF   = 0.0
      BETAEFF = 0.0
      ALPHA   = 0 0
      ZD      = 0.0
C
C     STORE STATE VARIBLES
C
      CALL STPUT2(ARRAY)
C*I
      RETURN
C*I
C*I
C*I  K E Y = 2
C*I
C*I  INTEGRATION OF ELEMENT STRESSES
C*I  ( CALCULATE STRESS(4) )
C*I
    2 CONTINUE
C*I
C*I  ***  I N S E R T   U S E R - S U P P L I E D   C O D I N G
C*I
C    B O D N E R - P A R T O M   M A T E R I A L
C
C    FOR 2-D SOLID ELEMENT
C
C    GET CURRENT VALUES OF MATERIAL CONSTANTS FOR GIVEN TEMPERATURE
C
      AM1    = CTI(1)
      Z1     = CTI(2)
      R1     = CTI(3)
      R2     = CTI(3)
      D0     = CTI(4)
      ANU    = CTI(5)
      ALPHA1 = CTI(8)
C
      A1  = CTI(6) * EXP(-CTI(7)/(TMP1+273.))
      A2  = A1
      DA1 = CTI(6) * EXP(-CTI(7)/(TMP2+273.))-A1
      DA2 = DA1
C
      G    =  CTD(1)/(2.*(1.+ ANU))
      DG   =  CTDD(1)/(2.*(1.+ ANU)) - G
C
      AK   =  CTD(1)/(1.-2.* ANU)
```

```
      DK    = CTDD(1)/(1.-2.* ANU) - AK
C
      AN = CTD(2)
      DAN = CTDD(2)-CTD(2)
C
      Z2 = CTD(3)
      DZ0 = CTDD(3)-CTD(3)
      DZ2 = CTDD(3)-CTD(3)
C
      Z3 = CTD(4)
      DZ3 = CTDD(4)-CTD(4)
C
      AM2 = CTD(5)
      DAM2 = CTDD(5)-CTD(5)
C
      RELAX = SCP(1)
      TOLER = SCP(2)
      HINT = IFIX(SCP(3))
      HELE = IFIX(SCP(4))
C
C     COMPUTE INCREMENTAL STRAINS WITH INCREMENTAL THERMAL STRAINS
C
      IF (KTR .EQ. 1) THEN
      DO 20 I = 1,4
20    EPS0(I) = EPS(I) - THSTR1(I)
      ENDIF
C
      DTAU1 = DTAU / FLOAT( INTER )
C
C     CALCULATE INCREMENTAL STRESSES AND UPDATE STRESS VECTOR
C
      CALL DBODNER2 ( DEPS, STRESS, DTAU1,
     +       NEL, IPT, KTR, TIME, INTER, RELAX, TOLER )
C
      DO 30 I = 1,4
30    EPS0(I) = EPS0(I) + DEPS(I)
C
C-----------------------------------------------------
C     STORE STATE VARIBLES AT TIME TAU + DTAU
C
      CALL STPUT2(ARRAY)
C
      RETURN
C*I
C*I
C*I
C*I  K E Y = 3
C*I
C*I  FORM CONSTITUTIVE LAW
C*I  ( CALCULATE D(4,4) )
C*I
    3 CONTINUE
```

```
C*I
C*I *** INSERT USER-SUPPLIED CODING
C
C-----------------------------------------------
C
C   UPDATE THE CONSTITUTIVE MATRIX
C
C-----------------------------------------------
C
C-----------------------------------------------
C
C THE TEMPERATURE DEPENDENT ELASTIC MATRIX IS DETERMINED
C WITHOUT A PLASTIC CORRECTION
C
      E   = CTDD(1)
      ANU =  CTI(5)
C
      A1 = E/(1.+ANU)
      C1 = A1*0.5
      A1 = A1/(1.-2.*ANU)
      B1 = A1*ANU
C
      DO 315 I=1,4
      DO 315 J=1,4
 315    D(I,J) = 0
C
      D(1,1) = A1
      D(1,2) = B1
      D(1,4) = B1
      D(2,1) = B1
      D(2,2) = A1
      D(2,4) = B1
      D(3,3) = C1
      D(4,1) = B1
      D(4,2) = B1
      D(4,4) = A1
C
C   STORE STATE VARIBLES AT TIME TAU + DTAU
C
      CALL STPUT2(ARRAY)
C
     RETURN
C
C*I
C*I  KEY=4
C*I
C*I  PRINTING OF ELEMENT RESPONSE
C*I  ( PRINT STRESS(4),STRAIN(4) )
C*I
   4 CONTINUE
C*I
C*I *** INSERT USER-SUPPLIED CODING
```

```
C*I
C*I
C
    IF((NEL.EQ.HELE).AND.(IPT.EQ.HINT)) THEN
 7006  FORMAT(2(1X,F8.1),12(1X,E11.4))
   ZMECHS = STRAIN(2) - THSTR2(2)
   ZTOT   = ZI+ZD
   WRITE(53,7006) TIME,TMP2,STRESS,EIN,ZMECHS,
  &          SIGEFF
    ENDIF
C
C    PRINT HEADING AND ELEMENT NUMBER
C
   IF(IPT.EQ.1) WRITE(6,9001) NEL
C
C    PRINT STRESSES
C
   MODE = '  ELASTIC'
   IF (EIDEFF.GT.1.E-9) MODE='INELASTIC'
   WRITE(6,9002) IPT,STRESS,SIGEFF,MODE,STRAIN,EIDEFF
C
C    FORMAT STATEMENTS
C
 9001 FORMAT (//,4X,3HNEL,4X,3HIPT,6X,9HSTRESS-YY,6X,
   +          9HSTRESS-ZZ,6X,9HSTRESS-YZ,
   +          6X,9HSTRESS-XX,4X,11HEFF. STRESS,/,
   +                  20X,9HSTRAIN-YY,6X,
   +          9HSTRAIN-ZZ,6X,9HSTRAIN-YZ,
   +          6X,9HSTRAIN-XX,4X,6HEIDEFF,//,I7)
 9002 FORMAT  (7X,I7,5(2X,1PE13.6),3X,A9,/,14X,5(2X,E13.6))
   IF((NEL.EQ.1).AND.(NG.EQ.HNG).AND.(IPT.EQ.1)) THEN
    REWIND(55)
   ENDIF
C
   CALL STPUT2(ARRAY)
C
   RETURN
   END
C
   SUBROUTINE DBODNER2 ( DEPS, STRESS,  DTAU,
   +              NEL, IPT, KTR, TIME, INTER, RELAX, TOLER)
C
   IMPLICIT DOUBLE PRECISION ( A-H,O-Z )
C-----------------------------------------------------
C
   REAL*8    DEVEPS(4)
   REAL*8    EPS(4)
C
   REAL*8    AVGEPS
C            AVERAGE NORMAL STRAIN INCREMENT AT TIME T
   REAL*8    DDEPS(4)
C             SUBINCREMENTIAL DEVITORIC STRAINS
```

```
      REAL*8    DEIEFF
C               EFFECTIVE INELASTIC STRAIN INCREMENT
      REAL*8    DEPS(4)
C               INCREMENTAL TOTAL STRAIN VECTOR
      REAL*8    DEPSI(4)
C               INELASTIC PORTION OF THE DEVIATORIC STRAIN
C               VECTOR
      REAL*8    DEVSIG(4)
C               DEVIATORIC STRESS VECTOR OF THE STRESS STATE
C               AT TIME T
      INTEGER   ICOUNT
C               ITERATION LOOP COUNTER
      REAL*8    FACT1
C               INTEGER FACTOR
      REAL*8    FACT2
C               INTEGER FACTOR
      INTEGER   INTER
C               NUMBER OF SUBINCREMENTS
      INTEGER   IPT
C               GAUSS POINT
      INTEGER   ISUB
C               NUMBER OF SUBINCREMENTS WITHIN C
      INTEGER   KTR
C         CURRENT SUBDIVISION NUMBER
C               EQ. 1 FOR FIRST SUBDIVISION
C               EQ. 2 FOR LAST SUBDIVISION
      INTEGER   NEL
C         ELEMENT NUMBER
      REAL*8    RELAX
C               RELAXATION FACTOR FOR NEW STRESS ESTIMATE
      REAL*8    TOLER
C               TOLERANCE FOR CONVERGENCE
      REAL*8    SIGNEW
C               NEW EFFECTIVE STRESS
      REAL*8    SIGOLD
C               INITIAL EFFECTIVE STRESS
      REAL*8    SIGEST
C               EFFECTIVE STRESS FOR TIME STEP SIZE OF DT/JINT
      REAL*8    SIGHYD
C               HYDROSTATIC STRESS
      REAL*8    SIGMAX
C             MAXIMUM ABSOLUTE VALUE OF EITHER SIGEFF OR SIGEFZ
C             FOR INVESTIGATING CONVERGENCE
      REAL*8    STRES0(4)
C               STRESS VECTOR AT TIME TAU
      REAL*8    STRESS(4)
C               STRESS VECTOR AT TIME TAU+DTAUT
      REAL*8    TIME
C               CURRENT TIME VALUE
      REAL*8    EINEST(4)
C               ESTIMATED PLASTIC STRAINS
      REAL*8    EIN0(4)
```

```
C             PLASTIC STRAINS AT TIME TAU
      REAL*8  BETADOT(4)
C             BETA RATE VECTOR
      REAL*8  BETAEST(4)
C             ESTIMATED BETA VECTOR
      REAL*8  BETA0(4)
C             BETA VECTOR AT TIME TAU
      REAL*8  U(4), V(4)
C             TEMPERATURE VECTOR
      REAL*8  ALEST
C             ESTIMATED ALPHA
      REAL*8  ZIEST
C             ESTIMATED ISOTROPIC DRAG STRESS
C
C-----------------------------------------------------------------
C
      COMMON /MATCONST/ E, G, AK, DG, DK,
     &          AM1, AM2, ALPHA1, DAM2,
     &          Z1, Z3, R1, R2, D0,
     &          AN, Z0, Z2, A1, A2,
     &          DAN, DZ0, DZ2, DZ3, DA1, DA2
C
      COMMON /BPSTAT2/ EIN(4), ZI, SIGEFF, BETAEFF, EPEFF, ALPHA,
     &          ZD, BETA(4), EPS0(4)
C
      SQRT3 = SQRT(3.)
C
C   STORE OLD STATE VARIBLES
C
      DO 10 I=1,4
      STRES0(I) = STRESS(I)
      EIN0(I)  = EIN(I)
      BETA0(I) = BETA(I)
10    CONTINUE
C
      ZI0    = ZI
      ALPHA0  = ALPHA
      ZD0    = ZD
C
C  STORE OLD MATERIAL CONSTANTS
C
      G0  = G
      AK0 = AK
      Z00 = Z0
      Z20 = Z2
      Z30 = Z3
      A10 = A1
      A20 = A2
      AM20 = AM2
C
C  INITIALIZE OTHER VARIBLES
C
```

```
      ISUB  = 1
      IDBUG = 0
C
C  INITIALIZE VARIABLES FOR SUB-TIME CUTTING
C
100  CONTINUE
C
      SIGEST  = SIGEFF
      SIGOLD  = SIGEST
      BOLD    = BETAEFF
      ZI      = ZI0
      ZIEST   = ZI0
      ZD      = ZD0
      ALPHA   = ALPHA0
      AVGEPS = AVGEPS0
C
C  RESTORE OLD MATERIAL CONSTANTS
C
      G  =  G0
      AK  =  AK0
      Z0  =  Z00
      Z2  =  Z20
      Z3  =  Z30
      A1  =  A10
      A2  =  A20
      AM2 = AM20
C
C  UPDATE NEW RATE MATERIAL CONSTANTS WITH SUBINCREMENT ISUB
C
      TFACTOR = 1./ ISUB
C
      DTSUB   = DTAU * TFACTOR
      DGSUB   = DG   * TFACTOR
      DKSUB   = DK   * TFACTOR
      DANSUB  = DAN  * TFACTOR
      DZ0SUB  = DZ0  * TFACTOR
      DZ2SUB  = DZ2  * TFACTOR
      DZ3SUB  = DZ3  * TFACTOR
      DA1SUB  = DA1  * TFACTOR
      DA2SUB  = DA2  * TFACTOR
      DAM2SUB = DAM2 * TFACTOR
C
C COMPUTE DEVITORIC STRESS AND SUBINCREMENTAL DEVITORIC AND
C       VOLUMETRIC STRAIN RATES
C
      SIGHYD = (STRES0(1) + STRES0(2) + STRES0(4)) / 3.0
      DO 20, I = 1,4
C
      EIN(I)     = EIN0(I)
      BETA(I)    = BETA0(I)
      BETAEST(I) = BETA0(I)
C
```

```
      IF (I.EQ.3) THEN
      FACT1 = 0.
      ELSE
      FACT1 = 1.
      ENDIF
C
      DEVSIG(I) = STRES0(I) - FACT1*SIGHYD
      DDEPS(I) = DEPS(I) * TFACTOR
      EPS(I)   = EPS0(I)
C
 20   CONTINUE
C
C
      DO 200 JSUB=1,ISUB
C
C     UPDATE ALL TEMPERATURE DEPENDENT MATERIAL CONSTANTS TO
C     END OF SUBTIME INCREMENT STEP
C
        G = G + DGSUB
       AK = AK + DKSUB
       AN = AN + DANSUB
       Z0 = Z0 + DZ0SUB
       Z2 = Z2 + DZ2SUB
       Z3 = Z3 + DZ3SUB
       A1 = A1 + DA1SUB
       A2 = A2 + DA2SUB
      AM2 = AM2 + DAM2SUB
C
      AVGEPS = (EPS(1)+DDEPS(1) +
     &        EPS(2)+DDEPS(2) +
     &        EPS(4)+DDEPS(4)) / 3.0
C
      DO 30 I = 1,4
      EPS(I) = EPS(I) + DDEPS(I)
C
      IF (I .EQ. 3) THEN
      DEVEPS(I) = EPS(I)
      ELSE
      DEVEPS(I) = EPS(I) - AVGEPS
      ENDIF
 30   CONTINUE
C
      ICOUNT = 0
C
 300  CONTINUE
C
      ZTOT = Z'EST + ZD
C
      IF(SIGEST .LE. 1.E-12) THEN
      SIGEST = 1.E-12
      DEIEFF = 0.0
      ELSE
```

```
      XTMP1 = (ZTOT/SIGEST)**2
      XTMP2 = -0.5*XTMP1**AN
      DEIEFF = D0*EXP(XTMP2)
      ENDIF
C
      XLAM = SQRT3 * DEIEFF/SIGEST
C
      DEIEFF = DEIEFF * DTSUB
C
      SIGNEW = 0.
      SSUM  = 0.
      PWDOT = 0.
C
        DO 40 I=1,4
          IF (I.EQ.3) THEN
            FACT1 = 2
            FACT2 = 0
            FACT3 = 0.5
          ELSE
            FACT1 = 1
            FACT2 = 1
            FACT3 = 1
          ENDIF
C
C ESTIMATE PLASTIC STRAINS AND STRESSES
C (ENGINEERING PLASTIC SHEAR STRAINS ARE COMPUTED)
C
          DEPSI(I) = XLAM * DEVSIG(I) * FACT1
C
C  COMPUTE THERMAL DIFFERENTIAL TERMS
C
       THETA3 = 0.
C
        EINEST(I) = EIN(I) + (DEPSI(I) * DTSUB) + THETA3
C
       DEVSIG(I)   = 2.*G*(DEVEPS(I)-EINEST(I)) * FACT3
C
       STRESS(I)  = DEVSIG(I) + FACT2 * AK * AVGEPS
C
       PWDOT = PWDOT + (STRESS(I)*DEPSI(I))
C
       SIGNEW  = SIGNEW + FACT1 * DEVSIG(I)**2
       SSUM    = SSUM   + FACT1 * STRESS(I)**2
C
 40   CONTINUE
C
      SIGNEW = SQRT(1.5*SIGNEW)
      SSUM   = SQRT(SSUM)
C
      IBCOUNT = 0
41    BNEW  = 0.0
C
```

```
      DO 42 I=1,4
        IF (I.EQ.3) THEN
          FACT1 = 2
        ELSE
          FACT1 = 1
        ENDIF
C
 42  BNEW   = BNEW   + FACT1 * BETAEST(I)**2
     BNEW   = SQRT(BNEW)
C
     DO 43 I=1,4
C
C  COMPUTE DRAG STRESS VECTORS
C
     V(I) = BETAEST(I)
     IF(BNEW .GT. 1.E-15) V(I) =  BETAEST(I)/ BNEW
     U(I) = STRESS(I)
     IF(SSUM .GT. 1.E-15) U(I) =  STRESS(I) / SSUM
C
 43  CONTINUE
C
     BSSUM1 = 0.
     BSSUM2 = 0.
     ZD     = 0.
C
     DO 44 I = 1,4
     IF (I.EQ.3) THEN
      FACT=2
     ELSE
      FACT=1
     ENDIF
C
     BTERM1 = (Z3 * U(I) - BETAEST(I)) * PWDOT
     BTERM2 =  Z1 * V(I) *((BNEW/Z1)**R2)
C
      THETA2 =  BETAEST(I) * DZ3SUB / Z3
C
      BETADOT(I) = AM2*BTERM1 - A2*BTERM2
C
      BETAEST(I) = BETA(I) + BETADOT(I)*DTSUB + THETA2
C
     ZD = ZD + BETAEST(I) * U(I) * FACT
CBIAXI     BSSUM1 = BSSUM1 + BETADOT(I) * BETADOT(I) * FACT
CBIAXI     BSSUM2 = BSSUM2 + BETAEST(I) * BETADOT(I) * FACT
 44  CONTINUE
C
C SPECIAL TRAP FOR BETA SNAP REVERSALS AT HIGH STRAIN RATES
C
      ZTOT = ZD + ZIEST
      IF(ZTOT .LT. 0) THEN
       BSCALE = ZIEST / ABS(ZD)
       ZD = ZD * BSCALE * 0.5
```

```
      DO 45 I = 1,4
 45     BETAEST(I) = BETAEST(I) * BSCALE * 0.5
     ENDIF
C
     BERROR = DABS(BNEW-BOLD2)
     BMAX   = DMAX1(BNEW,BOLD2)
      IF( BMAX .GT. TOLER)
     & BERROR = BERROR / BMAX
C
      BOLD2 = BNEW
C
      IF (BERROR .GT. TOLER)  THEN
        IBCOUNT = IBCOUNT + 1
       IF (IBCOUNT .GT. 5) THEN
        ISUB = ISUB * 2
       IF(ISUB .GT. 64) IDBUG = 1
        IF(ISUB .GT. 128) THEN
        WRITE (6,*) ' BETA REFUSES TO CONVERGE '
        STOP
        ELSE
        GOTO 100
        ENDIF
       ENDIF
      GO TO 41
      ENDIF
C
CBIAXI        IF(BSSUM1 .LT. 1.0E-12) BSSUM1 = 1.E-12
CBIAXI        VVDOT = BSSUM2 / BSSUM1
C
CBIAXI        IF(ABS(VVDOT) .LE. 1.)  THEN
CBIAXI          THETA = ACOS(VVDOT)
CBIAXI        ELSEIF( VVDOT .GT. 1.)  THEN
CBIAXI          THETA = 0.
CBIAXI        ELSE
CBIAXI          THETA = 3.141592654
CBIAXI        ENDIF
C
CBIAXI      ALDOT = AM2 * (ALPHA1*SIN(THETA)-ALPHA) * PWDOT
C
CBIAXI      ALEST = ALPHA + ALDOT * DTSUB
C
      ZTERM1 =  Z1*(ABS(ZIEST-Z2)/Z1)**R1
C
        THETA1 = DZ2SUB
C
CBIAXI      ZIDOT = AM1*(Z1+ALEST*Z3-ZIEST)*PWDOT
CBIAXI    &      - A1*ZTERM1
C
      ZIDOT = AM1*(Z1-ZIEST)*PWDOT
     &      - A1*ZTERM1
C
C   COMPUTE UPDATED ZTOT VALUE
```

```
C
     ZIEST = ZI +  ZIDOT*DTSUB + THETA1
C
C   NOW INVESTIGATE CONVERGENCE
C
     SERROR = DABS(SIGEST-SIGNEW)
     SIGMAX = DMAX1(SIGNEW,SIGEST)
      IF( SIGMAX .GT. TOLER)
    & SERROR = SERROR / SIGMAX
C
     DEPERR = DABS(DEIEFF-DOLD)
     DEPMAX = DMAX1(DEIEFF,DOLD)
      IF( DEPMAX .GT. TOLER)
    & DEPERR = DEPERR / DEPMAX
C
     ZERROR = DABS(ZIEST-ZOLD)
     ZMAX = DMAX1(ZIEST,ZOLD)
      IF( ZMAX .GT. TOLER)
    & ZERROR = ZERROR / ZMAX
C
      ERROR = SERROR + DEPERR + BERROR + ZERROR
C
     IF (IDBUG .EQ. 1) THEN
      WRITE(6,*) ' TIME KTR ICOUNT AND ISUB ',TIME, KTR,ICOUNT,ISUB
      WRITE(6,*)
      WRITE(6,*) 'SIG EST NEW SERR ', SIGEST,SIGNEW,SERROR
      WRITE(6,*) 'DEP OLD NEW EPERR ', DOLD,DEIEFF,DEPERR
      WRITE(6,*) 'B OLD NEW BERR ', BOLD,BNEW,BERROR
      WRITE(6,*) 'Z OLD NEW ZERR ', ZOLD,ZIEST,ZERROR
      WRITE(6,*)
     ENDIF
C
C  CHECK FOR CONVERGENCE
C
      IF( ERROR.GT. TOLER) THEN
C
C  NONCONVERGENT SOLUTION ARRIVES HERE
C
        SIGEST = SIGOLD*(1.-RELAX) + SIGNEW * (RELAX)
        SIGOLD = SIGEST
        DOLD  =  DEIEFF
        BOLD  =  BNEW
        ZOLD  =  ZIEST
C
C  UPDATE AND CHECK CONVERGENCE COUNT
C
        ICOUNT = ICOUNT+1
C
        IF (ICOUNT .GT. 30)  THEN
C
C  MAKE ADDITIONAL TIME-STEP CUTS
C
```

```
      ISUB = ISUB * 2
C
      IF (ISUB .EQ. 128) IDBUG = 1
C
C TERMINATE NONCONVERGE SOLUTIONS
C
      IF (ISUB .GT. 128) THEN
        WRITE(6,*) ' SOLUTION REFUSED TO CONVERGE '
        STOP
      ENDIF
C
C RESTART ITERATION SCHEME WITH NEW TIME-STEP CUT
C
      GOTO 100
C
      ENDIF
C
     GOTO 300
C
      ELSE
C
C CONVERGENCE SOLUTION ARRIVES HERE
C
     ALPHA = ALEST
     ZI    = ZIEST
C
      DO 50 I = 1, 4
      EIN(I)   = EINEST(I)
      BETA(I)  = BETAEST(I)
 50   CONTINUE
C
      ENDIF
C
C  END OF SUBTIME LOOPS
C
 200     CONTINUE
C
     SIGEFF  = SIGEST
     BETAEFF = BNEW
C
C
C DEBUG OUTPUT STATEMENTS
C
CJOE2      IF (IPT .EQ. 1 .AND. KTR .EQ. INTER) THEN
C
CJOE2  WRITE(6,*) ' TIME KTR ICOUNT AND ISUB ',TIME, KTR,ICOUNT,ISUB
CJOE2  WRITE(6,*)
CJOE2  WRITE(6,*) 'SIG EST NEW SERROR ', SIGEST,SIGNEW,SERROR
CJOE2  WRITE(6,*) 'DEP OLD NEW EPERROR ', DOLD,DEIEFF,DEPERR
CJOE2  WRITE(6,*) 'B OLD NEW BERROR ', BOLD,BNEW,BERROR
CJOE2  WRITE(6,*) 'Z OLD NEW ZERROR ', ZOLD,ZIEST,ZERROR
CJOE2  WRITE(6,*)
```

```
CJOE2 WRITE(6,*) ' FOR ELEMENT AND G POINT ', NEL, IPT
CJOE2 WRITE(6,*)
CJOE2 WRITE(6,*) ' EINEST    ', EINEST
CJOE2 WRITE(6,*) ' STRESS ', STRESS
CJOE2 WRITE(6,*) ' BETAEST ', BETAEST
CJOE2 WRITE(6,*) ' ZIEST, DEIEFF, ZD ', ZIEST, DEIEFF, ZD
C
CJOE2     ENDIF
C
   RETURN
   END
C
   SUBROUTINE STGET2(ARRAY)
C
   IMPLICIT DOUBLE PRECISION ( A-H,O-Z )
C
   REAL*8 ARRAY(60)
C
   COMMON /BPSTAT2/ EIN(4), ZI, SIGEFF, BETAEFF, EPEFF,  ALPHA,
  &           ZD, BETA(4), EPS0(4)
C
    EIN(1)  = ARRAY(1)
    EIN(2)  = ARRAY(2)
    EIN(3)  = ARRAY(3)
    EIN(4)  = ARRAY(4)
    BETA(1) = ARRAY(5)
    BETA(2) = ARRAY(6)
    BETA(3) = ARRAY(7)
    BETA(4) = ARRAY(8)
    ZI      = ARRAY(9)
    SIGEFF  = ARRAY(10)
    EPEFF   = ARRAY(11)
    BETAEFF = ARRAY(12)
    ALPHA   = ARRAY(13)
    ZD      = ARRAY(14)
    EPS0(1) = ARRAY(15)
    EPS0(2) = ARRAY(16)
    EPS0(3) = ARRAY(17)
    EPS0(4) = ARRAY(18)
C
    RETURN
    END
C
   SUBROUTINE STPUT2(ARRAY)
C
   IMPLICIT DOUBLE PRECISION ( A-H,O-Z )
C
   REAL*8 ARRAY(60)
C
   COMMON /BPSTAT2/ EIN(4), ZI, SIGEFF,  BETAEFF, EPEFF, ALPHA,
  &           ZD, BETA(4), EPS0(4)
C
```

```
      ARRAY(1) = EIN(1)
      ARRAY(2) = EIN(2)
      ARRAY(3) = EIN(3)
      ARRAY(4) = EIN(4)
      ARRAY(5) = BETA(1)
      ARRAY(6) = BETA(2)
      ARRAY(7) = BETA(3)
      ARRAY(8) = BETA(4)
      ARRAY(9) = ZI
      ARRAY(10)= SIGEFF
      ARRAY(11)= EPEFF
      ARRAY(12)= BETAEFF
      ARRAY(13)= ALPHA
      ARRAY(14)= ZD
      ARRAY(15)= EPS0(1)
      ARRAY(16)= EPS0(2)
      ARRAY(17)= EPS0(3)
      ARRAY(18)= EPS0(4)
C
      RETURN
      END
C
```

Appendix B

User-Defined Subroutines for Three-Dimensional Brick Elements
(file cuser3_dbeta.f)

```
      SUBROUTINE CUSER3 (NG,NEL,IPT,STRESS,EPS,STRAIN,DEPS,DEPST,
     A          THSTR1,THSTR2,KTR,INTER,SCP,ARRAY,IARRAY,D,
     B          ALFA,CTD,ALFAA,CTDD,CTI,TMP1,TMP2,TIME,DTAU,
     C          PHIST,PRST,PHIST1,DPSP,INTEG,ISUBM,INDNL,KEY)
C*I
C*I
C*I
C*I THIS SUBROUTINE IS TO BE SUPPLIED BY THE USER TO CALCULATE
C*I THE STRESSES AND CONSTITUTIVE MATRIX OF A SPECIAL MATERIAL.
C*I
C*I THIS SUBROUTINE IS CALLED IN USER2 FOR EACH INTEGRATION POINT
C*I FOR EACH 3-D SOLID ELEMENT TO PERFORM THE FOLLOWING OPERATIONS :
C*I
C*I   KEY.EQ.1  INITIALIZE THE WORKING ARRAYS DURING INPUT PHASE
C*I
C*I   KEY.EQ.2  CALCULATE ELEMENT STRESSES
C*I
C*I   KEY.EQ.3  CALCULATE THE STRESS/STRAIN MATRIX
C*!
C*I   KEY.EQ.4  PRINT CALCULATED STRESSES AND OTHER DESIRED
C*I             VARIABLES DURING STRESS PRINT-OUT
C*I
C*I
C*I THE FOLLOWING VARIABLES ARE USED TO PERFORM THE ABOVE OPERATIONS :
C*I
C*I   NG            ELEMENT GROUP NUMBER
C*I
C*I   NEL           ELEMENT NUMBER
C*I
C*I   IPT           INTEGRATION POINT NUMBER
C*I
C*I   STRESS(6)         STRESS COMPONENTS, RECEIVED AT TIME TAU
C*I                AND UPDATED BY USER-SUPPLIED CODING TO
C*I                CORRESPOND TO TIME TAU+DTAU
C*I
C*I   EPS(6)            TOTAL STRAIN COMPONENTS AT TIME T.
C*I                IN CASE OF LARGE STRAIN FORMULATION
C*I                (U.L.H.), EPS(4) ARE ELASTIC STRAINS AT
C*I                TIME T.
C*I
C*I   STRAIN(6)         TOTAL STRAIN COMPONENTS AT TIME T+DT.
C*I                IN CASE OF U.L.H. :
C*I                 A) FOR KEY.EQ.2 - ELASTIC STRAINS
C*I                   AT TIME TAU+DTAU (FOR KTR.EQ.0 -
```

```
C*I                    TRIAL ELASTIC STRAINS)
C*I                  B) FOR KEY.GT.2 - ELASTIC STRAINS AT
C*I                    TIME T+DT
C*I
C*I   DEPS(6)          SUBDIVIDED INCREMENTAL STRAIN COMPONENTS
C*I                 DEPS(I) = ( STRAIN(I) - EPS(I) )/INTER
C*I                        - DEPST(I)
C*I                 ( PASSED TO SUBROUTINE CUSER2 BY THE
C*I                  PROGRAM ADINA )
C*I
C*I   DPSP(6)          INCREMENT OF INELASTIC STRAIN ( PLASTIC
C*I                 AND/OR CREEP AND/OR VISCOPLASTIC, ETC.)
C*I                 IN THE SUBDIVISION. CALCULATED BY USER-
C*I                 SUPPLIED CODING AND PASSED TO THE PROGRAM
C*I                 ADINA. USED FOR U.L.H. ONLY.
C*I
C*I   DEPST(6)          COMPONENTS OF SUBINCREMENTAL THERMAL
C*I                 STRAIN
C*I                 ( PASSED TO SUBROUTINE CUSER2 BY THE
C*I                  PROGRAM ADINA )
C*I
C*I   THSTR1(6)         TOTAL THERMAL STRAIN AT TIME TAU
C*I                 FOR KEY.EQ.4 - THERMAL STRAIN AT TIME T
C*I
C*I   THSTR2(6)         TOTAL THERMAL STRAIN AT TIME TAU+DTAU
C*I                 FOR KEY EQ.4 - THERMAL STRAIN AT TIME T+DT
C*I
C*I   INTER            NUMBER OF SUBDIVISIONS FOR THE STRAIN
C*I                 INCREMENTS   (INTER = INT(PROP(123))
C*I
C*I   KTR             CURRENT SUBDIVISION NUMBER
C*I                 EQ.0     CALCULATION OF TRIAL ELASTIC
C*I                          STATE, IN CASE INTEG=1
C*I                 EQ.1     FOR FIRST SUBDIVISION
C*I                 EQ.INTER    FOR LAST SUBDIVISION
C*I
C*I   SCP(4)          SOLUTION CONTROL PARAMETERS
C*I
C*I   ARRAY(60)          WORKING ARRAY (REAL) , RECEIVED AT
C*I                 TIME TAU AND UPDATED BY USER-SUPPLIED
C*I                 CODING TO CORRESPOND TO TIME TAU+DTAU
C*I
C*I   IARRAY(2)          WORKING ARRAY (INTEGER) , RECEIVED AT
C*I                 TIME TAU AND UPDATED BY USER-SUPPLIED
C*I                 CODING TO CORRESPOND TO TIME TAU+DTAU
C*I
C*I   D(6,6)          STRESS/STRAIN MATRIX , TO BE CALCULATED
C*I                 BY USER-SUPPLIED CODING
C*I
C*I   ALFA             COEFFICIENT OF THERMAL EXPANSION AT
C*I                 TIME TAU
C*I
C*I   CTD(5)           TEMPERATURE-DEPENDENT MATERIAL CONSTANTS
C*I                 AT TIME TAU
```

```
C*I
C*I   ALFAA          COEFFICIENT OF THERMAL EXPANSION AT
C*I                  TIME TAU+DTAU
C*I
C*I   CTDD(5)        TEMPERATURE-DEPENDENT MATERIAL CONSTANTS
C*I                  AT TIME TAU+DTAU
C*I
C*I   CTI(8)         TEMPERATURE-INDEPENDENT MATERIAL CONSTANTS
C*I
C*I   TMP1           TEMPERATURE AT INTEGRATION POINT IPT AT
C*I                  TIME TAU. FOR KEY.EQ.4 - TEMPERATURE AT
C*I                  TIME T
C*I
C*I   TMP2           TEMPERATURE AT INTEGRATION POINT IPT AT
C*I                  TIME TAU+DTAU. FOR KEY.EQ.4 - TEMPERATURE
C*I                  AT TIME T+DT
C*I
C*I   TIME           TIME AT CURRENT STEP , T+DT
C*I
C*I   DT             TIME STEP INCREMENT , DT
C*I
C*I   PHIST(3,3)     MATRIX CONTAINING DIRECTION COSINES OF
C*I                  PRINCIPAL STRETCH DIRECTIONS, IN CASE OF
C*I                  U.L.H. FORMULATION
C*I
C*I   PRST(3)        PRINCIPAL STRETCHES, IN CASE OF U.L.H.
C*I
C*I   PHIST1(3)      DIRECTION COSINES OF THE FIRST PRINCIPAL
C*I                  STRETCH (STRETCH WHICH HAS MAXIMUM
C*I                  DEVIATION FROM 1.0)
C*I
C*I   INTEG          INTEGRATION PARAMETER FOR STRESS
C*I                  INTEGRATION
C*I                  EQ.0 - FORWARD INTEGRATION
C*I                  EQ.1 - BACKWARD INTEGRATION
C*I                     (RETURN MAPPING)
C*I
C*I   ISUBM          FLAG FOR CONTINUATION OF SUBDIVISION
C*I                  IN TIME STEP, APPLICABLE FOR INTEG.EQ.1
C*I                  EQ.0 -  CONTINUATION
C*I                  EQ.-1 - STOP OF SUBDIVISION
C*I                  IN THE USER-SUPPLIED CODING THE FLAG
C*I                  (INITIALLY EQ.0) MUST BE SET TO -1
C*I                  WHEN CRITERIA FOR STOPPING SUBDIVISIONS
C*I                  ARE REACHED
C*I
C*I   INDNL          FLAG FOR ELEMENT FORMULATION (NPAR(3))
C*I                  EQ.1 - MATERIALLY NONLINEAR ONLY (M.N.O.)
C*I                  EQ.2 - LARGE DISPLACEMENTS AND SMALL
C*I                     STRAINS (T.L.)
C*I                  EQ.3 - LARGE DISPLACEMENTS AND LARGE
C*I                     STRAINS (U.L.H.)
C*I
C*I
```

```
C*I   NOTE THAT THE VARIABLES PASSED TO THE SUBROUTINE WHEN KEY=3,4
C*I   ARE THESE CALCULATED LAST: I.E., CALCULATED IN THE LAST
C*I   SUBDIVISION WHEN KEY=2. HENCE THESE VARIABLES ARE NOT
C*I   CALCULATED WHEN KEY=3,4.
C*I
C*I
C*I
      IMPLICIT DOUBLE PRECISION ( A-H,O-Z )
C
      DIMENSION STRESS(6),STRAIN(6),DEPS(6),ARRAY(60),IARRAY(2),D(6,6),
     A         CTD(5),CTI(8),EPS(6),SCP(4)
      DIMENSION CTDD(5),DEPST(6),THSTR1(6),THSTR2(6)
      DIMENSION PHIST(3,3),PRST(3),PHIST1(3),DPSP(6)
C
C
      REAL*8     DTAU
C                TIME INCREMENT STEP, DT
      REAL*8     E
C                ELASTIC MODULUS
      REAL*8     G
C                SHEAR MODULOUS
      INTEGER    HINT
C            INTEGRATION POINT WRITTEN TO HISTORY FILE
      INTEGER    HELE
C            ELEMENT NUMBER ASSOCIATED WITH HIPT
      INTEGER    I
C            DO LOOP COUNTER
      INTEGER    J
C            DO LOOP COUNTER
C
C  DIRECTIONAL BODNER-PARTOM MATERIAL CONSTANTS:
C
C   TEMPERATURE INDEPENDENT CONSTANTS
C
       REAL*8   AM1
       REAL*8   Z1
       REAL*8   R1
       REAL*8   R2
       REAL*8   D0
       REAL*8   ALPHA1
C
C   TEMPERATURE INDEPENDENT CONSTANTS
C
       REAL*8   AN
       REAL*8   AM2
       REAL*8   Z0
       REAL*8   Z2
       REAL*8   Z3
       REAL*8   A1
       REAL*8   A2
C
C   TEMPERATURE DIFFERENTIALS
C
       REAL*8   DAM2
```

```
      REAL*8  DAN
      REAL*8  DZ0
      REAL*8  DZ2
      REAL*8  DZ3
      REAL*8  DA1
      REAL*8  DA2
C
      COMMON /MATCONST/ E, G, AK, DG, DK,
     &         AM1, AM2, ALPHA1, DAM2,
     &         Z1, Z3, R1, R2, D0,
     &         AN, Z0, Z2, A1, A2,
     &         DAN, DZ0, DZ2, DZ3, DA1, DA2
C
C   COMMON BLOCK FOR STATE VARIBLES
C
      COMMON /BPSTAT3/ EIN(6), ZI, SIGEFF, BETAEFF, EPEFF, ALPHA,
     &         ZD, BETA(6), EPS0(6)
C
C  RECALL STATE VARIBLES AT TIME TAU
C
      CALL STGET3(ARRAY)
C
      GO TO (1,2,3,4), KEY
C*I
C*I
C*I  K E Y = 1
C*I
C*I  INITIALIZE COMPONENTS OF REAL AND INTEGER WORKING ARRAYS
C*I  ( INITIALIZE ARRAY(60) AND IARRAY(2) )
C*I
    1 CONTINUE
C*I
C*I *** I N S E R T   U S E R - S U P P L I E D   C O D I N G
C*I
C*I
C    INITIALIZE STATE VARIBLES
C
      EIN(1)  = 0.0
      EIN(2)  = 0.0
      EIN(3)  = 0.0
      EIN(4)  = 0.0
      EIN(5)  = 0.0
      EIN(6)  = 0.0
      BETA(1) = 0.0
      BETA(2) = 0.0
      BETA(3) = 0.0
      BETA(4) = 0.0
      BETA(5) = 0.0
      BETA(6) = 0.0
      ZI      = CTD(3)
      SIGEFF  = 0.0
      EPEFF   = 0.0
      BETAEFF = 0.0
      ALPHA   = 0.0
```

```
      ZD     = 0.0
C
C       STORE STATE VARIBLES
C
      CALL STPUT3(ARRAY)
C
      RETURN
C*I
C*I
C*I  K E Y = 2
C*I
C*I  INTEGRATION OF ELEMENT STRESSES
C*I  ( CALCULATE STRESS(6) )
C*I
    2 CONTINUE
C*I
C*I  ***  I N S E R T   U S E R - S U P P L I E D   C O D I N G
C*I
C*I
C     B O D N E R - P A R T O M   M A T E R I A L
C
C     FOR 3-D SOLID ELEMENT
C
C     GET CURRENT VALUES OF MATERIAL CONSTANTS FOR GIVEN TEMPERATURE
C
C
      AM1    = CTI(1)
      Z1     = CTI(2)
      R1     = CTI(3)
      R2     = CTI(3)
      D0     = CTI(4)
      ANU    = CTI(5)
      ALPHA1 = CTI(8)
C
      A1  = CTI(6) * EXP(-CTI(7)/(TMP1+273.))
      A2  = A1
      DA1 = CTI(6) * EXP(-CTI(7)/(TMP2+273.))-A1
      DA2 = DA1
C
      G    =  CTD(1)/(2.*(1.+ ANU))
      DG   =  CTDD(1)/(2.*(1.+ ANU)) - G
C
      AK   =  CTD(1)/(1.-2.* ANU)
      DK   =  CTDD(1)/(1.-2.* ANU)  - AK
C
       AN  =  CTD(2)
      DAN  = CTDD(2)-CTD(2)
C
       Z2  = CTD(3)
      DZ0  = CTDD(3)-CTD(3)
      DZ2  = CTDD(3)-CTD(3)
C
       Z3  =  CTD(4)
      DZ3  = CTDD(4)-CTD(4)
```

```
C
  AM2 = CTD(5)
  DAM2 = CTDD(5)-CTD(5)
C
  RELAX = SCP(1)
  TOLER = SCP(2)
  HINT  = IFIX(SCP(3))
  HELE  = IFIX(SCP(4))
C
C  COMPUTE INCREMENTAL STRAINS WITH INCREMENTAL THERMAL STRAINS
C
  IF (KTR .EQ. 1) THEN
  DO 20 I = 1,6
20   EPS0(I) = EPS(I) - THSTR1(I)
  ENDIF
C
   DTAU1 = DTAU / FLOAT( INTER )
C
C  CALCULATE INCREMENTAL STRESSES AND UPDATE STRESS VECTOR
C
  CALL DBODNER3 ( DEPS, STRESS, DTAU1,
 +         NEL, IPT, KTR, TIME, INTER, RELAX, TOLER )
C
  DO 30  I = 1,6
30   EPS0(I) = EPS0(I) + DEPS(I)
C
C-----------------------------------------------------------
C  STORE STATE VARIBLES AT TIME TAU + DTAU
C
   CALL STPUT3(ARRAY)
C
  RETURN
C*I
C*I
C*I
C*I  K E Y = 3
C*I
C*I  FORM CONSTITUTIVE LAW
C*I  ( CALCULATE D(6,6) )
C*I
  3 CONTINUE
C*I
C*I  ***   I N S E R T   U S E R - S U P P L I E D   C O D I N G
C*I
C
C
C  THE TEMPERATURE DEPENDENT ELASTIC MATRIX IS DETERMINED
C  WITHOUT A PLASTIC STRAIN CORRECTION
C
    E  = CTDD(1)
    ANU =  CTI(5)
C
  DDT = DTAU/FLOAT(INTER)
C
```

```
      DO 315 I = 1,6
      DO 315 J = 1,6
  315 D(I,J) = 0.D0
      CM = E/(1. - 2.*ANU)
      AE = (1. + ANU)/E
      CP = AE
C
C   IN CASE CORRECTION FOR VISCOPLASTIC FLOW IN THE TIME STEP,
C   CORRECT THE CONSTANT CP
C
CJOE  IF (EST.GT.YLD) CP = CP + 1.5*DDT*GAMA*(EST/YLD - 1.)/EST
      CP = 1./CP
      C11 = (CM + 2.*CP)/3.
      C12 = (CM - CP)/3.
      D(1,1) = C11
      D(1,2) = C12
      D(1,3) = C12
      D(2,2) = C11
      D(2,3) = C12
      D(3,3) = C11
      D(4,4) = 0.5*CP
      D(5,5) = D(4,4)
      D(6,6) = D(5,5)
      DO 320 I = 1,3
      DO 320 J = I,3
  320 D(J,I) = D(I,J)
C
      RETURN
C*I
C*I
C*I  K E Y = 4
C*I
C*I  PRINTING OF ELEMENT RESPONSE
C*I  ( PRINT STRESS(6),STRAIN(6) )
C*I
    4 CONTINUE
C*I
C*I  ***  I N S E R T   U S E R - S U P P L I E D   C O D I N G
C*I
C*I
       IF((NEL.EQ.HELE).AND.(IPT.EQ.HINT)) THEN
 7006  FORMAT(2(1X,F8.1),12(1X,E11.4))
       ZMECHS = STRAIN(1) - THSTR2(1)
       ZTOT   = ZI+ZD
      WRITE(53,7006) TIME,TMP2,STRESS(1),STRESS(2),
     +          STRESS(3),ZTOT,ZI,ZD,ZMECHS,SIGEFF
       ENDIF
C
C   PRINT HEADING AND ELEMENT NUMBER
C
      IF  (NEL.EQ.1 .AND. IPT.EQ.1) THEN
          WRITE (6,9000)
      ENDIF
      IF (IPT.EQ.1) WRITE (6,9002) NEL
```

```
C
C   PRINT STRESSES
C
      WRITE (6,9003) IPT,STRESS,SIGEFF,STRAIN,TMP2
C
C   FORMAT STATEMENTS
C
9000 FORMAT (//,4X,3HNEL,2X,3HIPT,6X,9HSTRESS-XX,6X,
  1      9HSTRESS-YY,6X,9HSTRESS-ZZ,6X,9HSTRESS-XY,6X,
  2      9HSTRESS-XZ,6X,9HSTRESS-YZ,4X,
  2      11H EFF STRESS, /
  3     17X,9HSTRAIN-XX,6X,
  1      9HSTRAIN-YY,6X,9HSTRAIN-ZZ,6X,9HSTRAIN-XY,6X,
  2      9HSTRAIN-XZ,6X,9HSTRAIN-YZ,4X,
  2      11HTEMPERATURE, /)
9002 FORMAT (/ I7 /)
9003 FORMAT (7X,I5,7(2X,E13.6)/13X,7(2X,E13.6),/)
9004 FORMAT (7X,I5,8(2X,E13.6)/27X,3(2X,E13.6),30X,2(2X,E13.6) /
  1      27X, 3(2X,E13.6) /)
C
   CALL STPUT3(ARRAY)
C
   RETURN
C*FILE END
   END
C
   SUBROUTINE DBODNER3 ( DEPS, STRESS,  DTAU,
   +            NEL, IPT, KTR, TIME, INTER, RELAX, TOLER)
C
   IMPLICIT DOUBLE PRECISION ( A-H,O-Z )
C------------------------------------------------------------
C
   REAL*8    DEVEPS(6)
   REAL*8    EPS(6)
C
   REAL*8    AVGEPS
C           AVERAGE NORMAL STRAIN INCREMENT AT TIME T
   REAL*8    DDEPS(6)
C            SUBINCREMENTIAL DEVITORIC STRAINS
   REAL*8    DEIEFF
C            EFFECTIVE INELASTIC STRAIN INCREMENT
   REAL*8    DEPS(6)
C            INCREMENTAL TOTAL STRAIN VECTOR
   REAL*8    DEPSI(6)
C            INELASTIC PORTION OF THE DEVIATORIC STRAIN
C            VECTOR
   REAL*8    DEVSIG(6)
C            DEVIATORIC STRESS VECTOR OF THE STRESS STATE
C            AT TIME T
   INTEGER   ICOUNT
C            ITERATION LOOP COUNTER
   REAL*8    FACT1
C            INTEGER FACTOR
   REAL*8    FACT2
```

```
C              INTEGER FACTOR
   INTEGER   INTER
C              NUMBER OF SUBINCREMENTS
   INTEGER   IPT
C              GAUSS POINT
   INTEGER   ISUB
C              NUMBER OF SUBINCREMENTS WITHIN C
   INTEGER   KTR
C          CURRENT SUBDIVISION NUMBER
C              EQ. 1 FOR FIRST SUBDIVISION
C              EQ. 2 FOR LAST SUBDIVISION
   INTEGER   NEL
C          ELEMENT NUMBER
   REAL*8    RELAX
C              RELAXATION FACTOR FOR NEW STRESS ESTIMATE
   REAL*8    TOLER
C              TOLERANCE FOR CONVERGENCE
   REAL*8    SIGNEW
C              NEW EFFECTIVE STRESS
   REAL*8    SIGOLD
C              INITIAL EFFECTIVE STRESS
   REAL*8    SIGEST
C              EFFECTIVE STRESS FOR TIME STEP SIZE OF DT/JINT
   REAL*8    SIGHYD
C              HYDROSTATIC STRESS
   REAL*8    SIGMAX
C            MAXIMUM ABSOLUTE VALUE OF EITHER SIGEFF OR SIGEFZ
C            FOR INVESTIGATING CONVERGENCE
   REAL*8    STRES0(6)
C              STRESS VECTOR AT TIME TAU
   REAL*8    STRESS(6)
C              STRESS VECTOR AT TIME TAU+DTAUT
   REAL*8    TIME
C              CURRENT TIME VALUE
   REAL*8    EINEST(6)
C              ESTIMATED PLASTIC STRAINS
   REAL*8    EIN0(6)
C              PLASTIC STRAINS AT TIME TAU
   REAL*8    BETADOT(6)
C              BETA RATE VECTOR
   REAL*8    BETAEST(6)
C              ESTIMATED BETA VECTOR
   REAL*8    BETA0(6)
C              BETA VECTOR AT TIME TAU
   REAL*8    U(6), V(6)
C              DIRECTIONAL VECTORS
   REAL*8    ALEST
C              ESTIMATED ALPHA
   REAL*8    ZIEST
C              ESTIMATED ISOTROPIC DRAG STRESS
C
C-------------------------------------------------------------------
C
   COMMON /MATCONST/ E, G, AK, DG, DK,
```

```
    &          AM1, AM2,  ALPHA1, DAM2,
    &          Z1, Z3, R1, R2, D0,
    &          AN, Z0, Z2, A1, A2,
    &          DAN, DZ0, DZ2, DZ3, DA1, DA2
C
    COMMON /BPSTAT3/ EIN(6), ZI, SIGEFF, BETAEFF, EPEFF, ALPHA,
    &          ZD, BETA(6), EPS0(6)
C
    SQRT3 = SQRT(3.)
C
C   STORE OLD STATE VARIBLES
C
    DO 10 I=1,6
    STRES0(I) = STRESS(I)
    EIN0(I)  = EIN(I)
    BETA0(I) = BETA(I)
10  CONTINUE
C
    ZI0    = ZI
    ALPHA0  = ALPHA
    ZD0    = ZD
C
C  STORE OLD MATERIAL CONSTANTS
C
    G0  = G
    AK0 = AK
    Z00 = Z0
    Z20 = Z2
    Z30 = Z3
    A10 = A1
    A20 = A2
    AM20 = AM2
C
C   INITIALIZE OTHER VARIBLES
C
     ISUB  = 1
    IDBUG  = 0
C
C   INITIALIZE VARIABLES FOR SUB-TIME CUTTING
C
100  CONTINUE
C
    SIGEST  = SIGEFF
    SIGOLD  = SIGEST
    BOLD   = BETAEFF
    ZI     = ZI0
    ZIEST  = ZI0
    ZD     = ZD0
    ALPHA  = ALPHA0
    AVGEPS  = AVGEPS0
C
C  RESTORE OLD MATERIAL CONSTANTS
C
     G =  G0
```

```
      AK = AK0
      Z0 = Z00
      Z2 = Z20
      Z3 = Z30
      A1 = A10
      A2 = A20
      AM2 = AM20
C
C  UPDATE NEW RATE MATERIAL CONSTANTS WITH SUBINCREMENT ISUB
C
      TFACTOR = 1./ ISUB
C
      DTSUB  = DTAU * TFACTOR
      DGSUB  = DG   * TFACTOR
      DKSUB  = DK   * TFACTOR
      DANSUB = DAN  * TFACTOR
      DZ0SUB = DZ0  * TFACTOR
      DZ2SUB = DZ2  * TFACTOR
      DZ3SUB = DZ3  * TFACTOR
      DA1SUB = DA1  * TFACTOR
      DA2SUB = DA2  * TFACTOR
      DAM2SUB = DAM2 * TFACTOR
C
C COMPUTE DEVITORIC STRESS AND SUBINCREMENTAL DEVITORIC AND
C      VOLUMETRIC STRAIN RATES
C
      SIGHYD = (STRES0(1) + STRES0(2) + STRES0(3)) / 3.0
      DO 20, I = 1,6
C
      EIN(I)    = EIN0(I)
      BETA(I)   = BETA0(I)
      BETAEST(I) = BETA0(I)
C
      IF (I.GT.3) THEN
      FACT1 = 0.
      ELSE
      FACT1 = 1.
      ENDIF
C
      DEVSIG(I) = STRES0(I) - FACT1*SIGHYD
      DDEPS(I) = DEPS(I) * TFACTOR
      EPS(I)   = EPS0(I)
C
 20   CONTINUE
C
C
      DO 200 JSUB=1,ISUB
C
C   UPDATE ALL TEMPERATURE DEPENDENT MATERIAL CONSTANTS TO
C   END OF SUBTIME INCREMENT STEP
C
       G  = G  +  DGSUB
       AK = AK +  DKSUB
       AN = AN +  DANSUB
```

```
      Z0 = Z0 + DZ0SUB
      Z2 = Z2 + DZ2SUB
      A1 = A1 + DA1SUB
      A2 = A2 + DA2SUB
      AM2 = AM2 + DAM2SUB
C
      AVGEPS = (EPS(1)+DDEPS(1) +
     &        EPS(2)+DDEPS(2) +
     &        EPS(3)+DDEPS(3)) / 3.0
C
      DO 30 I = 1,6
      EPS(I) = EPS(I) + DDEPS(I)
C
      IF (I .GT. 3) THEN
      DEVEPS(I) = EPS(I)
      ELSE
      DEVEPS(I) = EPS(I) - AVGEPS
      ENDIF
 30   CONTINUE
C
      ICOUNT = 0
C
 300  CONTINUE
C
      ZTOT = ZIEST + ZD
C
      IF(SIGEST .LE. 1.E-12) THEN
      SIGEST = 1.E-12
      DEIEFF = 0.0
      ELSE
      XTMP1 = (ZTOT/SIGEST)**2
      XTMP2 = -0.5*XTMP1**AN
      DEIEFF = D0*EXP(XTMP2)
      ENDIF
C
      XLAM = SQRT3 * DEIEFF/SIGEST
C
      DEIEFF = DEIEFF * DTSUB
C
      SIGNEW = 0.
      SSUM  = 0.
      PWDOT = 0.
C
        DO 40 I=1,6
          IF (I.GT.3) THEN
            FACT1 = 2
            FACT2 = 0
            FACT3 = 1
          ELSE
            FACT1 = 1
            FACT2 = 1
            FACT3 = 2
          ENDIF
C
```

```
C ESTIMATE PLASTIC STRAINS AND STRESSES
C (ENGINEERING PLASTIC SHEAR STRAINS ARE COMPUTED)
C
      DEPSI(I) = XLAM *  DEVSIG(I) * FACT1
C
C  COMPUTE THERMAL DIFFERENTIAL TERMS
C
     THETA3 = 0.
C
    EINEST(I) = EIN(I) + (DEPSI(I) * DTSUB) + THETA3
C
    DEVSIG(I)  = FACT3*G*(DEVEPS(I)- EINEST(I))
C
    STRESS(I)   = DEVSIG(I) + FACT2 * AK * AVGEPS
C
    PWDOT = PWDOT + (STRESS(I)*DEPSI(I))
C
    SIGNEW  = SIGNEW + FACT1 * DEVSIG(I)**2
    SSUM    = SSUM   + FACT1 * STRESS(I)**2
C
 40   CONTINUE
C
   SIGNEW = SQRT(1.5*SIGNEW)
   SSUM   = SQRT(SSUM)
C
   IBCOUNT = 0
 41  BNEW  = 0.0
C
      DO 42 I=1,6
        IF (I.GT.3) THEN
          FACT1 = 2
        ELSE
          FACT1 = 1
        ENDIF
C
 42  BNEW    = BNEW   + FACT1 * BETAEST(I)**2
    BNEW    = SQRT(BNEW)
C
    DO 43 I=1,6
C
C  COMPUTE DRAG STRESS VECTORS
C
    V(I) = BETAEST(I)
    IF(BNEW .GT. 1.E-15) V(I) =  BETAEST(I)/ BNEW
    U(I) = STRESS(I)
    IF(SSUM .GT. 1.E-15) U(I) =  STRESS(I) / SSUM
C
 43  CONTINUE
C
    BSSUM1 = 0.
    BSSUM2 = 0.
    ZD     = 0.
C
    DO 44 I = 1,6
```

```
      IF (I.GT.3) THEN
       FACT=2
      ELSE
       FACT=1
      ENDIF
C
      BTERM1 = (Z3 * U(I) - BETAEST(I)) * PWDOT
      BTERM2 =  Z1 * V(I) *((BNEW/Z1)**R2)
C
      THETA2    =  BETAEST(I) * DZ3SUB / Z3
C
      BETADOT(I) = AM2*BTERM1 - A2*BTERM2
C
      BETAEST(I) = BETA(I) + BETADOT(I)*DTSUB + THETA2
C
      ZD = ZD + BETAEST(I) * U(I) * FACT
CBIAXI    BSSUM1 = BSSUM1 + BETADOT(I) * BETADOT(I) * FACT
CBIAXI    BSSUM2 = BSSUM2 + BETAEST(I) * BETADOT(I) * FACT
 44  CONTINUE
C
C SPECIAL TRAP FOR BETA SNAP REVERSALS AT HIGH STRAIN RATES
C
       ZTOT = ZD + ZIEST
       IF(ZTOT .LT. 0) THEN
        BSCALE = ZIEST / ABS(ZD)
        ZD = ZD * BSCALE * 0.5
       DO 45 I = 1,6
 45      BETAEST(I) = BETAEST(I) * BSCALE * 0.5
      ENDIF
C
      BERROR = DABS(BNEW-BOLD2)
      BMAX = DMAX1(BNEW,BOLD2)
       IF( BMAX .GT. TOLER)
     & BERROR = BERROR / BMAX
C
       BOLD2 = BNEW
C
       IF (BERROR .GT. TOLER)  THEN
         IBCOUNT = IBCOUNT + 1
        IF (IBCOUNT .GT. 5) THEN
         ISUB = ISUB * 2
          IF( ISUB .GT. 64) IDBUG = 1
          IF( ISUB .GT. 128) THEN
         WRITE (6,*) ' BETA REFUSES TO CONVERGE '
          STOP
          ELSE
          GOTO 100
          ENDIF
        ENDIF
       GO TO 41
       ENDIF
C
CBIAXI      IF(BSSUM1 .LT. 1.0E-12) BSSUM1 = 1.E-12
CBIAXI      VVDOT = BSSUM2 / BSSUM1
```

```
C
CBIAXI      IF(ABS(VVDOT) .LE. 1.) THEN
CBIAXI        THETA = ACOS(VVDOT)
CBIAXI      ELSEIF( VVDOT .GT. 1.) THEN
CBIAXI        THETA = 0.
CBIAXI      ELSE
CBIAXI        THETA = 3.141592654
CBIAXI      ENDIF
C
CBIAXI     ALDOT = AM2 * (ALPHA1*SIN(THETA)-ALPHA) * PWDOT
C
CBIAXI     ALEST = ALPHA + ALDOT * DTSUB
C
      ZTERM1 =  Z1*(ABS(ZIEST-Z2)/Z1)**R1
C
        THETA1 = DZ2SUB
C
CBIAXI      ZIDOT = AM1*(Z1+ALEST*Z3-ZIEST)*PWDOT
CBIAXI     &      - A1*ZTERM1
C
      ZIDOT = AM1*(Z1-ZIEST)*PWDOT
     &      - A1*ZTERM1
C
C     COMPUTE UPDATED ZTOT VALUE
C
      ZIEST = ZI +  ZIDOT*DTSUB + THETA1
C
C     NOW INVESTIGATE CONVERGENCE
C
      SERROR = DABS(SIGEST-SIGNEW)
      SIGMAX = DMAX1(SIGNEW,SIGEST)
       IF( SIGMAX .GT. TOLER)
     & SERROR = SERROR / SIGMAX
C
      DEPERR = DABS(DEIEFF-DOLD)
      DEPMAX = DMAX1(DEIEFF,DOLD)
       IF( DEPMAX .GT. TOLER)
     & DEPERR = DEPERR / DEPMAX
C
      ZERROR = DABS(ZIEST-ZOLD)
      ZMAX = DMAX1(ZIEST,ZOLD)
       IF( ZMAX .GT. TOLER)
     & ZERROR = ZERROR / ZMAX
C
       ERROR = SERROR + DEPERR + BERROR + ZERROR
C
      IF (IDBUG .EQ. 1) THEN
       WRITE(6,*) ' TIME KTR ICOUNT AND ISUB ',TIME, KTR,ICOUNT,ISUB
       WRITE(6,*) ' STRESS ' , STRESS
       WRITE(6,*) 'SIG EST NEW SERR ', SIGEST,SIGNEW,SERROR
       WRITE(6,*) 'DEP OLD NEW EPERR ', DOLD,DEIEFF,DEPERR
       WRITE(6,*) 'B OLD NEW BERR ', BOLD,BNEW,BERROR
       WRITE(6,*) 'Z OLD NEW ZERR ', ZOLD,ZIEST,ZERROR
       WRITE(6,*)
```

```
      ENDIF
C
C CHECK FOR CONVERGENCE
C
      IF( ERROR.GT. TOLER) THEN
C
C NONCONVERGENT SOLUTION ARRIVES HERE
C
        SIGEST = SIGOLD*(1.-RELAX) + SIGNEW * (RELAX)
        SIGOLD = SIGEST
        DOLD   = DEIEFF
        BOLD   = BNEW
        ZOLD   = ZIEST
C
C  UPDATE AND CHECK CONVERGENCE COUNT
C
        ICOUNT = ICOUNT+1
C
        IF (ICOUNT .GT.30)  THEN
C
C  MAKE ADDITIONAL TIME-STEP CUTS
C
          ISUB = ISUB * 2
C
          IF (ISUB .EQ. 128) IDBUG = 1
C
C TERMINATE NONCONVERGE SOLUTIONS
C
          IF (ISUB .GT. 128) THEN
              WRITE(6,*) ' SOLUTION REFUSED TO CONVERGE '
              STOP
          ENDIF
C
C RESTART ITERATION SCHEME WITH NEW TIME-STEP CUT
C
          GOTO 100
C
        ENDIF
C
      GOTO 300
C
        ELSE
C
C CONVERGENCE SOLUTION ARRIVES HERE
C
      ALPHA  = ALEST
      ZI     = ZIEST
C
        DO 50 I = 1, 6
        EIN(I)    = EINEST(I)
        BETA(I)   = BETAEST(I)
 50     CONTINUE
C
        ENDIF
```

```
C
C  END OF SUBTIME LOOPS
C
 200     CONTINUE
C
    SIGEFF  = SIGEST
    BETAEFF = BNEW
C
C
C  DEBUG OUTPUT STATEMENTS
C
CJOE2   IF (NEL .EQ. 1 .AND. IPT .EQ. 1 .AND. KTR .EQ. INTER) THEN
C
CJOE2  WRITE(6,*) ' TIME KTR ICOUNT ISUB IPT ',TIME, KTR,ICOUNT,ISUB,IPT
CJOE2  WRITE(6,*)
CJOE2  WRITE(6,*) 'SIG EST NEW SERROR ', SIGEST,SIGNEW,SERROR
CJOE2  WRITE(6,*) 'DEP OLD NEW EPERROR ', DOLD,DEIEFF,DEPERR
CJOE2  WRITE(6,*) 'B OLD NEW BERROR ', BOLD,BNEW,BERROR
CJOE2  WRITE(6,*) 'Z OLD NEW ZERROR ', ZOLD,ZIEST,ZERROR
CJOE2  WRITE(6,*)
CJOE2  WRITE(6,*) ' FOR ELEMENT AND G POINT ', NEL, IPT
CJOE2  WRITE(6,*)
CJOE2  WRITE(6,*) ' EINEST    ', EINEST
CJOE2  WRITE(6,*) ' STRESS ', STRESS
CJOE2  WRITE(6,*) ' DEVSIG ', DEVSIG
CJOE2  WRITE(6,*) ' STRAIN ', EPS
CJOE2  WRITE(6,*) ' BETAEST ', BETAEST
CJOE2  WRITE(6,*) ' ZIEST, DEIEFF, ZD ', ZIEST, DEIEFF, ZD
C
CJOE2   ENDIF
C
    RETURN
    END
C
    SUBROUTINE STGET3(ARRAY)
C
    IMPLICIT DOUBLE PRECISION ( A-H,O-Z )
C
    REAL*8 ARRAY(60)
C
    COMMON /BPSTAT3/ EIN(6), ZI, SIGEFF, BETAEFF, EPEFF,  ALPHA,
   &            ZD, BETA(6), EPS0(6)
C
     EIN(1)  = ARRAY(1)
     EIN(2)  = ARRAY(2)
     EIN(3)  = ARRAY(3)
     EIN(4)  = ARRAY(4)
     EIN(5)  = ARRAY(5)
     EIN(6)  = ARRAY(6)
     BETA(1) = ARRAY(7)
     BETA(2) = ARRAY(8)
     BETA(3) = ARRAY(9)
     BETA(4) = ARRAY(10)
     BETA(5) = ARRAY(11)
```

```
      BETA(6) = ARRAY(12)
      ZI     = ARRAY(13)
      SIGEFF  = ARRAY(14)
      EPEFF   = ARRAY(15)
      BETAEFF = ARRAY(16)
      ALPHA   = ARRAY(17)
      ZD      = ARRAY(18)
      EPS0(1) = ARRAY(19)
      EPS0(2) = ARRAY(20)
      EPS0(3) = ARRAY(21)
      EPS0(4) = ARRAY(22)
      FPS0(5) = ARRAY(23)
      EPS0(6) = ARRAY(24)
C
      RETURN
      END
C
      SUBROUTINE STPUT3(ARRAY)
C
      IMPLICIT DOUBLE PRECISION ( A-H,O-Z )
C
      REAL*8 ARRAY(60)
C
      COMMON /BPSTAT3/ EIN(6), ZI, SIGEFF,  BETAEFF, EPEFF, ALPHA,
     &           ZD, BETA(6), EPS0(6)
C
      ARRAY(1)  = EIN(1)
      ARRAY(2)  = EIN(2)
      ARRAY(3)  = EIN(3)
      ARRAY(4)  = EIN(4)
      ARRAY(5)  = EIN(5)
      ARRAY(6)  = EIN(6)
      ARRAY(7)  = BETA(1)
      ARRAY(8)  = BETA(2)
      ARRAY(9)  = BETA(3)
      ARRAY(10) = BETA(4)
      ARRAY(11) = BETA(5)
      ARRAY(12) = BETA(6)
      ARRAY(13) = ZI
      ARRAY(14) = SIGEFF
      ARRAY(15) = EPEFF
      ARRAY(16) = BETAEFF
      ARRAY(17) = ALPHA
      ARRAY(18) = ZD
      ARRAY(19) = EPS0(1)
      ARRAY(20) = EPS0(2)
      ARRAY(21) = EPS0(3)
      ARRAY(22) = EPS0(4)
      ARRAY(23) = EPS0(5)
      ARRAY(24) = EPS0(6)
C
      RETURN
      END
C
```

Appendix C

ADINAIN3.0 Input File for Verification Test Case 1
(file vcase1.in)

```
FEPROGRAM ADINA
DATABASE CREATE FILEUNIT=1
HEADING 'Test case 1 of Directional Bodner-Partom with Beta21s'
*
MASTER IDOF=100111 REACTIONS=YES MTOTK=500 NSTEP=48 DT=1.0,
   MODEX=EX IRINT=48
ANALYSIS TYPE=STATIC
ITERATION METHOD=FULL NEWTON LINE-SEARCH=YES
TOLERANCE TYPE=ED DNORM=1.e-5
* Displacement Loading with time (strain rate = 833.3e-6/s)
TIMEFUNCTION 1
 0.0  0.0
48.0  0.04
* Temperature with time
TIMEFUCTION 2
  0.0 25.0
 48.0 25.0
*
PRINTOUT VOLUME=MAX
*
PORTHOLE VOLUME=MAX FORMATTED=YES
*
SYSTEM 1 CARTESIAN
COORDINATES
ENTRIES NODE  ZL       YL
       1  0.100E+1  0.100E+1
       2  0.100E+1  0.000E-0
       3  0.000E-0  0.000E-0
       4  0.000E-0  0.100E+1
*
*
* Bodner Partom Test Material for b21S
*     with no Thermal Expansion
*
MATERIAL 1  USER CTI1=0.00  CTI2=1600. CTI3=3.    CTI4=1.e4 ,
          CTI5=0.34  CTI6=5.8e5 CTI7=1.37e4 CTI8=0.0 ,
          SCP1=0.75  SCP2=0.0001 SCP3=1     SCP4=1   ,
          XINTER=10    TREF=25.
*Temp(C) Expans(1/C) Elastic(MPa) n  z0(1/s) z3(MPa) m2(1/MPa)
23.000  0.0000e-6  112000.  4.8000 1550.0  100.0  0.350
260.00  0.0000e-6  108030.  3.5000 1300.0  300.0  0.350
315.00  0.0000e-6  106130.  3.0540 1250.4  390.0  1.502
365.00  0.0000e-6  104090.  2.6490 1205.4  500.0  2.549
415.00  0.0000e-6  101740.  2.2430 1160.4  660.0  3.597
465.00  0.0000e-6   99085.  1.8380 1115.3  960.0  4.644
```

```
482.00  0.0000e-6   98113.  1.7000  1100.0  1100.0   5.000
500.00  0.0000e-6   97045.  1.5000  1089.3  1300.0   5.763
525.00  0.0000e-6   95497.  1.2800  1074.4  1670.0   6.822
550.00  0.0000e-6   93873.  1.1000  1059.5  2100.0   7.881
575.00  0.0000e-6   92172.  0.9700  1044.6  2600.0   8.941
600.00  0.0000e-6   90395.  0.8200  1029.8  3700.0  10.000
650.00  0.0000e-6   86612.  0.7400  1000.0  3800.0  10.000
760.00  0.0000e-6   77216.  0.5800   600.0  4000.0  15.000
815.00  0.0000e-6   71964.  0.5500   300.0  4100.0  30.000
900.00  0.0000e-6   63122.  0.5500   300.0  4300.0  30.000
*
*
EGROUP 1 TWODSOLID SUBTYPE=AXISYMMETRIC MATERIAL=1
GSURFACE N1=1,N2=2,N3=3,N4=4,EL1=1,EL2=1,NODES=4
*
BOUNDARIES IDOF=110111 TYPE=NODES
2
*
BOUNDARIES IDOF=111111 TYPE=NODES
3
*
BOUNDARIES IDOF=101111 TYPE=NODES
4
*
INITIAL TEMPERATURES
1  25.  STEP 1 TO 4  25.
*
LOADS TEMPERATURE
1 1. 2 0 STEP 1 TO 4 1. 2 0
*
LOADS DISPLACEMENTS
1 3 1.0 1
2 3 1.0 1
*
* ADINA INPUT FILE CREATE
ADINA
END
```

Appendix D

Mathematica Results for Verification Test Case 1
(file vcase1.math)

Strain	Stress (MPa)
0.0000000	0.0000
0.0008333	93.330
0.0016666	186.66
0.0024999	279.99
0.0033332	373.32
0.0041665	466.65
0.0049998	559.98
0.0058331	653.31
0.0066664	746.64
0.0074997	839.97
0.0083330	933.30
0.0091663	1026.6
0.0099996	1083.9
0.010833	1098.6
0.011666	1110.4
0.012500	1119.6
0.013333	1126.7
0.014166	1132.0
0.014999	1136.0
0.015833	1138.9
0.016666	1141.1
0.017499	1142.7
0.018333	1143.8
0.019166	1144.6
0.019999	1145.2
0.020832	1145.6
0.021666	1145.9
0.022499	1146.2
0.023332	1146.3
0.024166	1146.4
0.024999	1146.5
0.025832	1146.6
0.026666	1146.6
0.027499	1146.7
0.028332	1146.7
0.029166	1146.7
0.029999	1146.7
0.030832	1146.7

Strain	Stress (MPa)
0.031665	1146.7
0.032499	1146.7
0.033332	1146.7
0.034165	1146.7
0.034999	1146.7
0.035832	1146.7
0.036665	1146.7
0.037499	1146.7
0.038332	1146.7
0.039165	1146.7
0.039998	1146.7

Appendix E

Mathematica Results for Verification Test Case 2
(file vcase2.math)

Strain	Stress (MPa)
0.0000000	0.0000
0.0008333	72.106
0.0016666	121.44
0.0024999	138.00
0.0033332	140.14
0.0041665	140.36
0.0049998	140.38
0.0058331	140.38
0.0066664	140.38
0.0074997	140.38
0.0083330	140.38
0.0091663	140.38
0.0099996	140.38
0.010833	140.38
0.011666	140.38
0.012500	140.38
0.013333	140.38
0.014166	140.38
0.014999	140.38
0.015833	140.38
0.016666	140.38
0.017499	140.38
0.018333	140.38
0.019166	140.38
0.019999	140.38
0.020832	140.38
0.021666	140.38
0.022499	140.38
0.023332	140.38
0.024166	140.38
0.024999	140.38
0.025832	140.38
0.026666	140.38
0.027499	140.38
0.028332	140.38
0.029166	140.38
0.029999	140.38
0.030832	140.38

Strain	Stress (MPa)
0.031665	140.38
0.032499	140.38
0.033332	140.38
0.034165	140.38
0.034999	140.38
0.035832	140.38
0.036665	140.38
0.037499	140.38
0.038332	140.38
0.039165	140.38
0.039998	140.38

Appendix F

Mathematica Results for Verification Test Case 3
(file vcase3.math)

Strain	Stress (MPa)
0.0000000	0.0000
0.0008333	72.106
0.0016666	121.44
0.0024999	138.00
0.0033332	140.14
0.0041665	140.36
0.0049998	140.38
0.0058331	140.38
0.0066664	140.38
0.0074997	140.38
0.0083330	140.38
0.0091663	140.38
0.0099996	140.38
0.010833	140.38
0.011666	140.38
0.012500	140.38
0.013333	140.38
0.014166	140.38
0.014999	140.38
0.015833	140.38
0.016666	140.38
0.017499	140.38
0.018333	140.38
0.019166	140.38
0.019999	140.38
0.020832	140.38
0.021666	140.38
0.022499	140.38
0.023332	140.38
0.024166	140.38
0.024999	140.38
0.025832	140.38
0.026666	140.38
0.027499	140.38
0.028332	140.38
0.029166	140.38
0.029999	140.38
0.030832	140.38

Strain	Stress (MPa)
0.031665	140.38
0.032499	140.38
0.033332	140.38
0.034165	140.38
0.034999	140.38
0.035832	140.38
0.036665	140.38
0.037499	140.38
0.038332	140.38
0.039165	140.38
0.039998	140.38

Appendix G

Mathematica Results for Verification Test Case 4 (file vcase4.math)

Strain	Stress (MPa)	Temp (°C)	Strain	Stress (MPa)	Temp (°C)
0.00000000	0.0000	25.000	0.032500	787.03	482.00
0.00083333	93.333	25.000	0.033333	787.03	482.00
0.0016667	186.67	25.000	0.034167	787.03	482.00
0.0025000	280.00	25.000	0.035000	787.03	482.00
0.0033333	373.33	25.000	0.035833	787.03	482.00
0.0041667	466.67	25.000	0.036667	787.03	482.00
0.0050000	560.00	25.000	0.037500	787.03	482.00
0.0058333	653.33	25.000	0.038333	787.03	482.00
0.0066667	746.67	25.000	0.039167	787.03	482.00
0.0075000	840.00	25.000	0.040000	787.03	482.00
0.0083333	933.33	25.000	0.040833	787.03	482.00
0.0091667	1026.7	25.000	0.041667	787.03	482.00
0.0100000	1083.9	25.000	0.042500	787.03	482.00
0.010833	1098.6	25.000	0.043333	787.03	482.00
0.011667	1110.4	25.000	0.044167	787.03	482.00
0.012500	1119.6	25.000	0.045000	787.03	482.00
0.013333	1126.7	25.000	0.045833	787.03	482.00
0.014167	1132.0	25.000	0.046667	787.03	482.00
0.015000	1136.0	25.000	0.047500	787.03	482.00
0.015833	1138.9	25.000	0.048333	787.03	482.00
0.016667	1141.1	25.000	0.049167	787.03	482.00
0.017500	1142.7	25.000	0.050000	787.03	482.00
0.018333	1143.8	25.000	0.050833	821.57	443.90
0.019167	1144.6	25.000	0.051667	843.77	405.80
0.020000	1145.2	25.000	0.052500	879.05	367.80
0.020833	1122.6	63.100	0.053333	913.54	329.70
0.021667	1097.2	101.20	0.054167	945.70	291.60
0.022500	1070.3	139.30	0.055000	975.69	253.50
0.023333	1041.9	177.30	0.055833	1008.1	215.40
0.024167	1011.7	215.40	0.056667	1038.7	177.30
0.025000	979.53	253.50	0.057500	1067.4	139.30
0.025833	949.64	291.60	0.058333	1094.6	101.20
0.026667	917.28	329.70	0.059167	1120.3	63.100
0.027500	882.53	367.80	0.060000	1144.7	25.000
0.028333	850.18	405.80	0.060833	1146.7	25.000
0.029167	825.88	443.90	0.061667	1146.7	25.000
0.030000	790.59	482.00	0.062500	1146.7	25.000
0.030833	787.05	482.00	0.063333	1146.7	25.000
0.031667	787.03	482.00	0.064167	1146.7	25.000

Strain	Stress (MPa)	Temp (°C)
0.065000	1146.7	25.000
0.065833	1146.7	25.000
0.066667	1146.7	25.000
0.067500	1146.7	25.000
0.068333	1146.7	25.000
0.069167	1146.7	25.000
0.070000	1146.7	25.000
0.070833	1146.7	25.000
0.071667	1146.7	25.000
0.072500	1146.7	25.000
0.073333	1146.7	25.000
0.074167	1146.7	25.000
0.075000	1146.7	25.000
0.075833	1146.7	25.000
0.076667	1146.7	25.000
0.077500	1146.7	25.000
0.078333	1146.7	25.000
0.079167	1146.7	25.000
0.080000	1146.7	25.000

Appendix H

Mathematica Results for Verification Test Case 5 (file vcase5.math)

Strain	Stress (MPa)	Temp (°C)	Strain	Stress (MPa)	Temp (°C)
0.0000	0.0000	650.00	0.016250	130.64	760.00
0.00041667	36.095	650.00	0.016667	129.59	760.00
0.00083333	72.190	650.00	0.017083	129.05	760.00
0.0012500	104.07	650.00	0.017500	128.76	760.00
0.0016667	133.08	650.00	0.017917	128.62	760.00
0.0020833	162.60	650.00	0.018333	128.54	760.00
0.0025000	192.14	650.00	0.018750	128.50	760.00
0.0029167	221.16	650.00	0.019167	128.48	760.00
0.0033333	249.09	650.00	0.019583	128.47	760.00
0.0037500	275.30	650.00	0.020000	128.46	760.00
0.0041667	299.15	650.00	0.020417	128.46	760.00
0.0045833	320.03	650.00	0.020833	128.46	760.00
0.0050000	337.52	650.00	0.021250	128.46	760.00
0.0054167	351.44	650.00	0.021667	128.46	760.00
0.0058333	361.94	650.00	0.022083	128.46	760.00
0.0062500	369.48	650.00	0.022500	128.46	760.00
0.0066667	374.65	650.00	0.022917	128.46	760.00
0.0070833	378.08	650.00	0.023333	128.46	760.00
0.0075000	380.30	650.00	0.023750	128.46	760.00
0.0079167	381.71	650.00	0.024167	128.46	760.00
0.0083333	382.60	650.00	0.024583	128.46	760.00
0.0087500	383.15	650.00	0.025000	128.46	760.00
0.0091667	383.49	650.00	0.025417	134.59	750.80
0.0095833	383.70	650.00	0.025833	144.80	741.70
0.0100000	383.83	650.00	0.026250	157.42	732.50
0.010417	374.19	659.20	0.026667	171.74	723.30
0.010833	357.33	668.30	0.027083	187.43	714.20
0.011250	336.61	677.50	0.027500	204.29	705.00
0.011667	314.21	686.70	0.027917	222.20	695.80
0.012083	291.34	695.80	0.028333	241.07	686.70
0.012500	268.60	705.00	0.028750	260.85	677.50
0.012917	246.28	714.20	0.029167	281.50	668.30
0.013333	224.60	723.30	0.029583	302.96	659.20
0.013750	203.67	732.50	0.030000	325.21	650.00
0.014167	183.61	741.70	0.030417	341.88	650.00
0.014583	164.50	750.80	0.030833	354.79	650.00
0.015000	146.43	760.00	0.031250	364.38	650.00
0.015417	136.98	760.00	0.031667	371.18	650.00
0.015833	132.73	760.00	0.032083	375.79	650.00

Strain	Stress (MPa)	Temp (°C)
0.032500	378.83	650.00
0.032917	380.78	650.00
0.033333	382.01	650.00
0.033750	382.78	650.00
0.034167	383.26	650.00
0.034583	383.56	650.00
0.035000	383.74	650.00
0.035417	383.86	650.00
0.035833	383.93	650.00
0.036250	383.97	650.00
0.036667	383.99	650.00
0.037083	384.01	650.00
0.037500	384.02	650.00
0.037917	384.03	650.00
0.038333	384.03	650.00
0.038750	384.03	650.00
0.039167	384.04	650.00
0.039583	384.04	650.00
0.040000	384.04	650.00

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